Density dependence of Λ potential in nuclear matter from heavy-ion collisions and hypernuclear spectroscopy

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YITP Workshop on "Fundamentals in density functional theory" (DFT2022), Dec.7-20, 2022, YITP, Kyoto / Online (Hybrid).

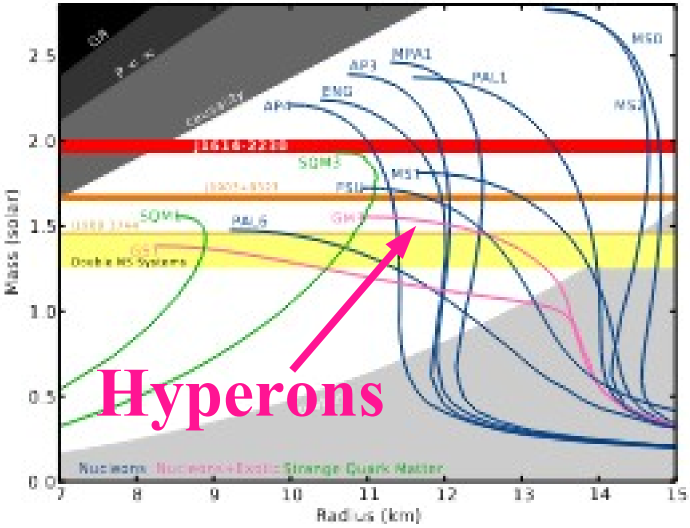
- **Introduction – Hyperon puzzle**
- **Directed flow of Λ**
- **Λ Hypernuclei**
- **Summary**


YITP Workshop (国内モレキュール型研究会)
"Fundamentals in density functional theory (DFT2022)"
07 December–20 December 2022
Yukawa Institute for Theoretical Physics, Kyoto University

Y.Nara, A. Jinno, K. Murase, AO, PRC106 ('22), 044902 [2208.01297];
A. Jinno, K. Murase, Y. Nara, AO, work in prog..

Hyperon Puzzle of Neutron Stars

- Hyperonic matter EOS cannot sustain $2M_\odot$ neutron stars.
- Proposed solutions
  - More repulsive hyperon potential ($U_\Lambda(\rho)$) at high density
  - Transition to quark matter before Λ appears
  - General relativity → Modified gravity
- Λ potential from chiral EFT
  - Three-body force may cause repulsive potential of Λ.



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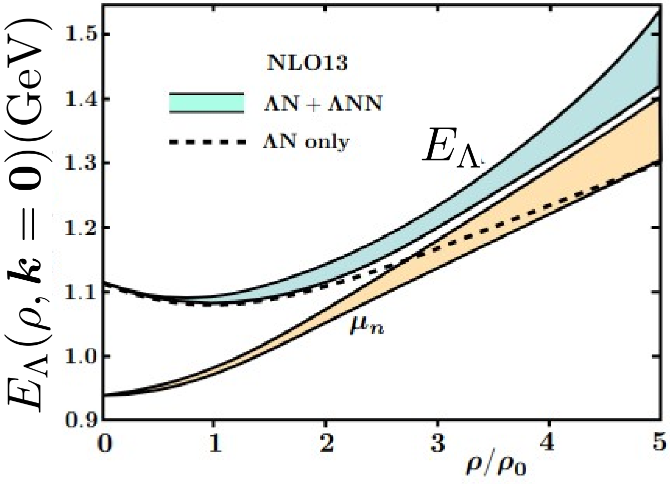
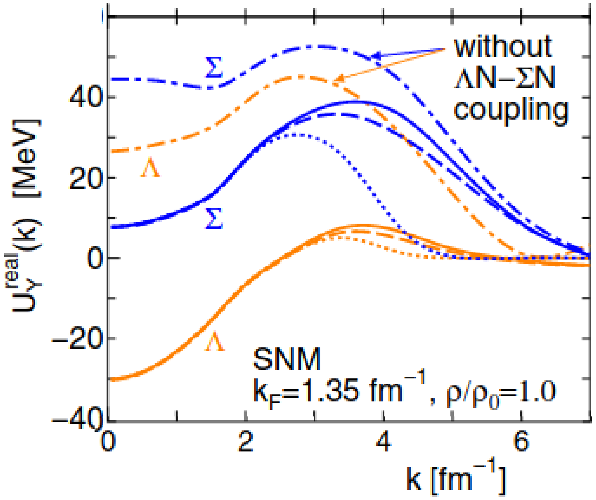


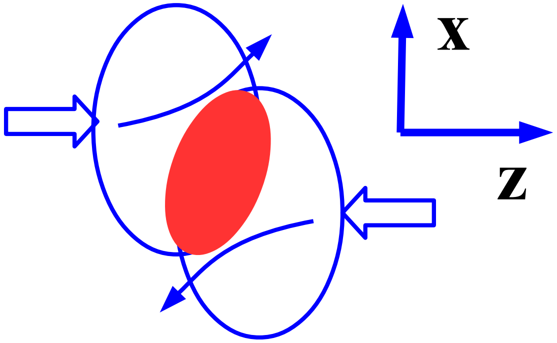
Gerstung, Kaiser, Weise ('20)



Kohno ('18)

Can we examine repulsive U_Λ at high densities ?

- **Candidate 1: Directed flow from heavy-ion collisions**
  - **Directed flow has been utilized to study EOS**



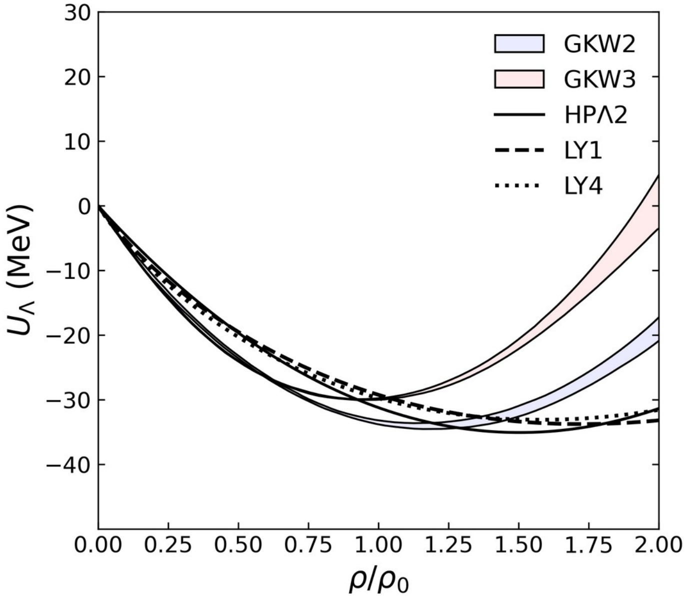
$$v_1 = \langle \cos\phi \rangle \ \text{(directed flow)}\,,\ \ \langle p_x \rangle\ \text{(side flow)}$$

E.g. Sahu, Cassing, Mosel, AO (nucl-th/9907002); Snellings+(nucl-ex/9908001); Danielewicz, Lacey, Lynch (nucl-th/0208016);

  - **How about v_1 of Λ ?**
- **Candidate 2: Hypernuclear Spectroscopy**
  - **Density dependence of U_Λ from chiral EFT is different from "Standard" potentials.**

E.g. Lanskoy, Yamamoto ('97)

  - **Does U_Λ from chiral EFT explain the separation energy of Λ ?**



Jinno+ (work in prog.)

Directed flow of Λ

U_Λ from Chiral EFT

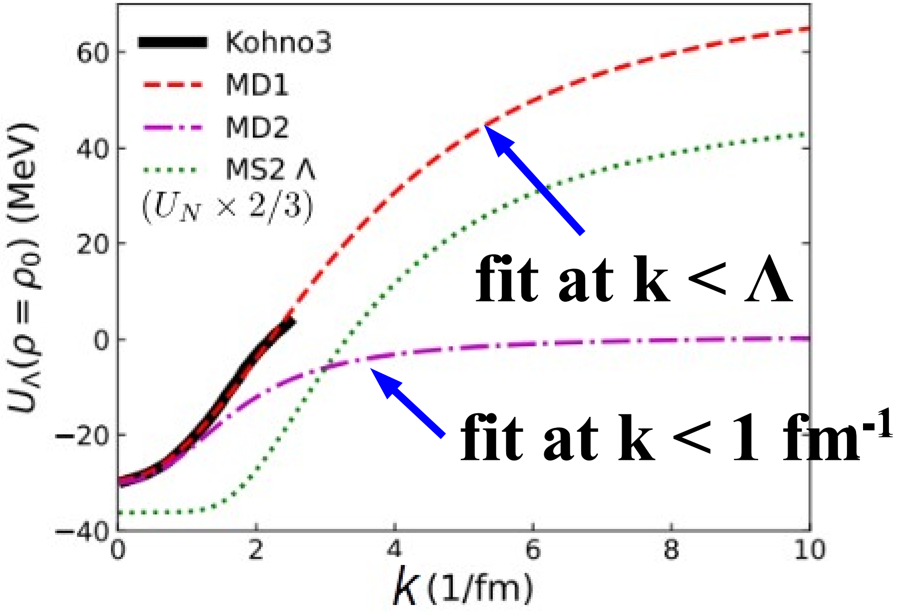
- **Chiral EFT with 3BF and hyperons**
  Gerstung+(2001.10563)(GKW, decuplet saturation model), Kohno (1802.05388)
  - **ρ-dep. potential using Fermi mom. expansion** *Tews+(1611.07133)*
    + momentum dep. fitted to Kohno ('18).

$$U_\Lambda(\rho,k) = a\frac{\rho}{\rho_0} + b\left(\frac{\rho}{\rho_0}\right)^{4/3} + c\left(\frac{\rho}{\rho_0}\right)^{5/3} + \sum_n \frac{C_n}{\rho_0}\int \frac{d\boldsymbol{k}'}{(2\pi)^3}\frac{f(\boldsymbol{r},\boldsymbol{k}')}{1+(\boldsymbol{k}-\boldsymbol{k}')^2/\mu_n^2}$$

  - **Range of fit**

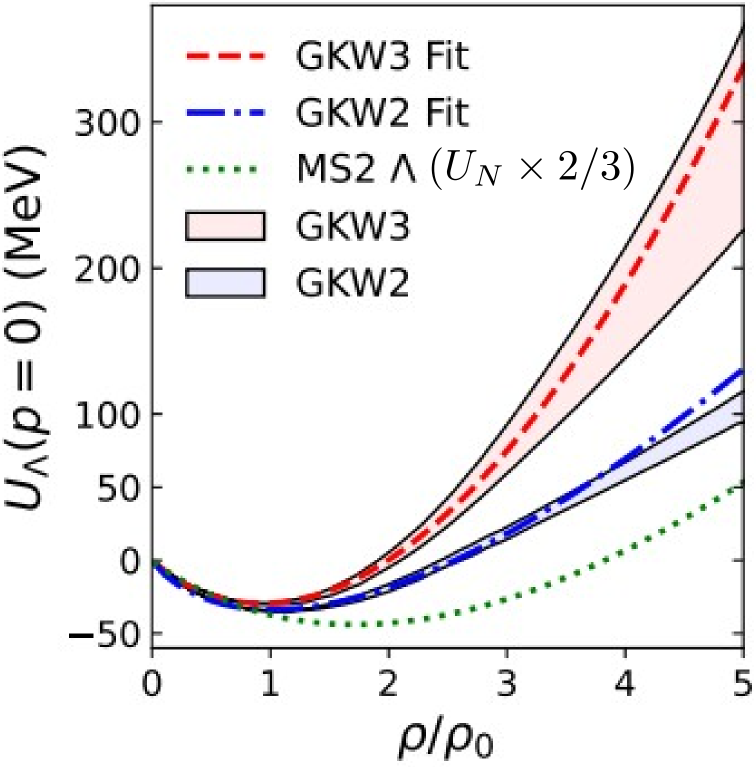
$$\rho \le 3.5\rho_0 \ \text{(unstable above } 3.5\rho_0\text{)}$$

$$k \le \Lambda \ \text{(MD1) or } k \le 1\ \text{fm}^{-1}\ \text{(MD2)}$$



fit at k < Λ

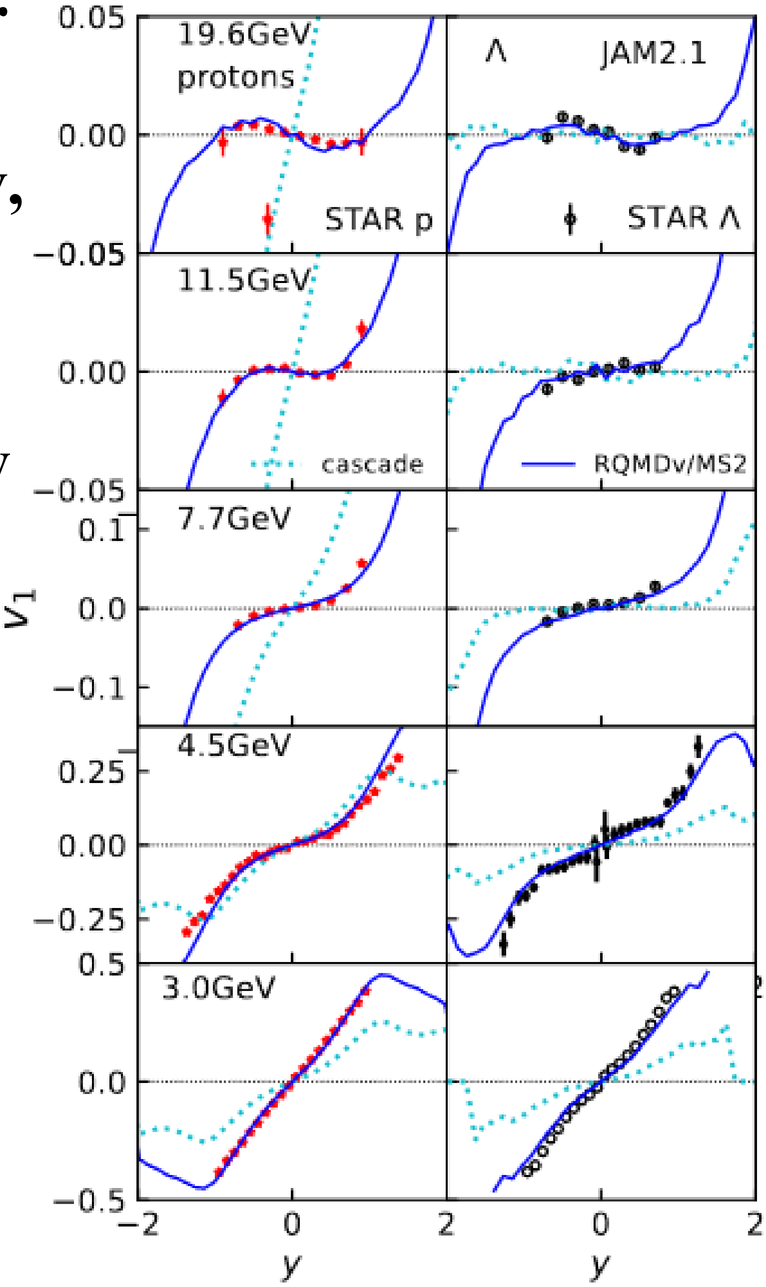
fit at k < 1 fm^{-1}

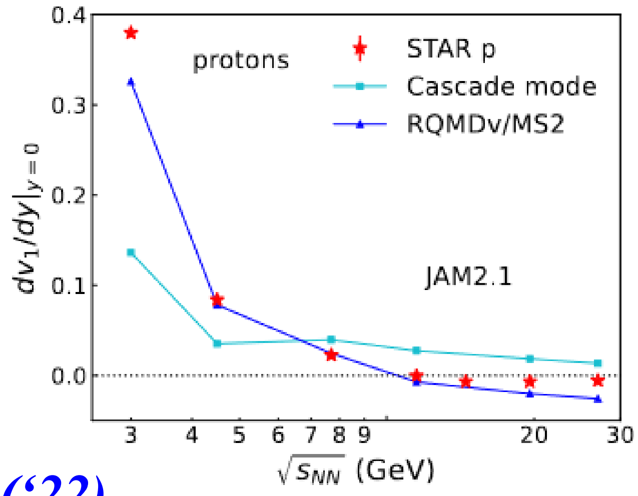


Nara, Jinno, Murase, AO ('22)

Directed flow of Λ

- **Calculation using JAM2 event generator**
  https://gitlab.com/transportmodel/jam2
  - **Potential effects are included in RQMDv, which solves the proton v1 puzzle. (Change of the v1 slope around 10 GeV)**
    Nara, AO, PRC105('22),014911[2109.07594]
  - **Directed flows of p and Λ are reasonably explained by using MS2 (momentum dep. soft potential) for non-strange baryons and MS2 x 2/3 for hyperons.**

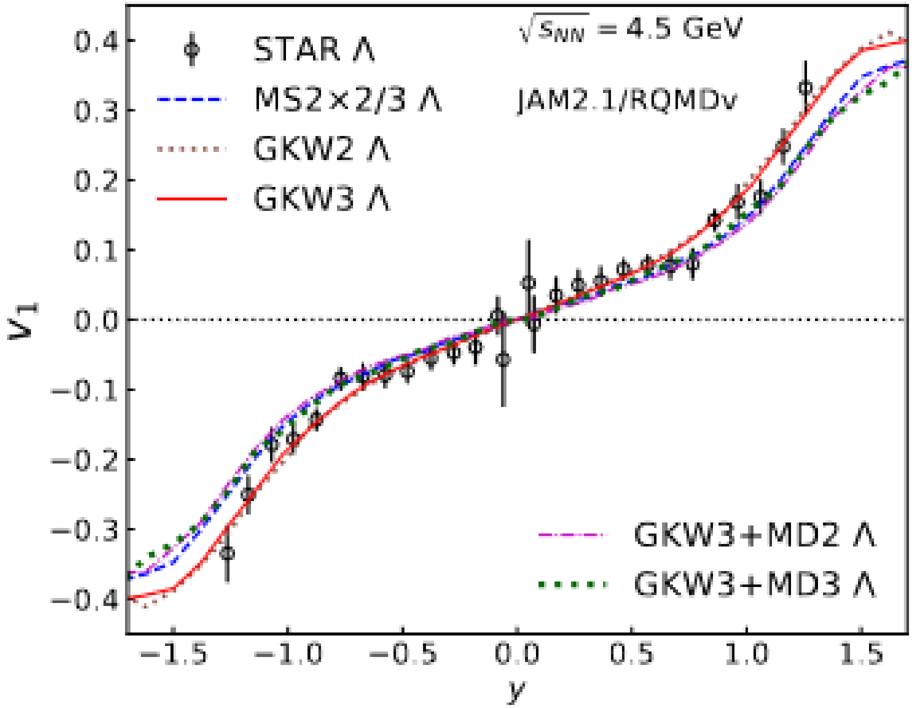




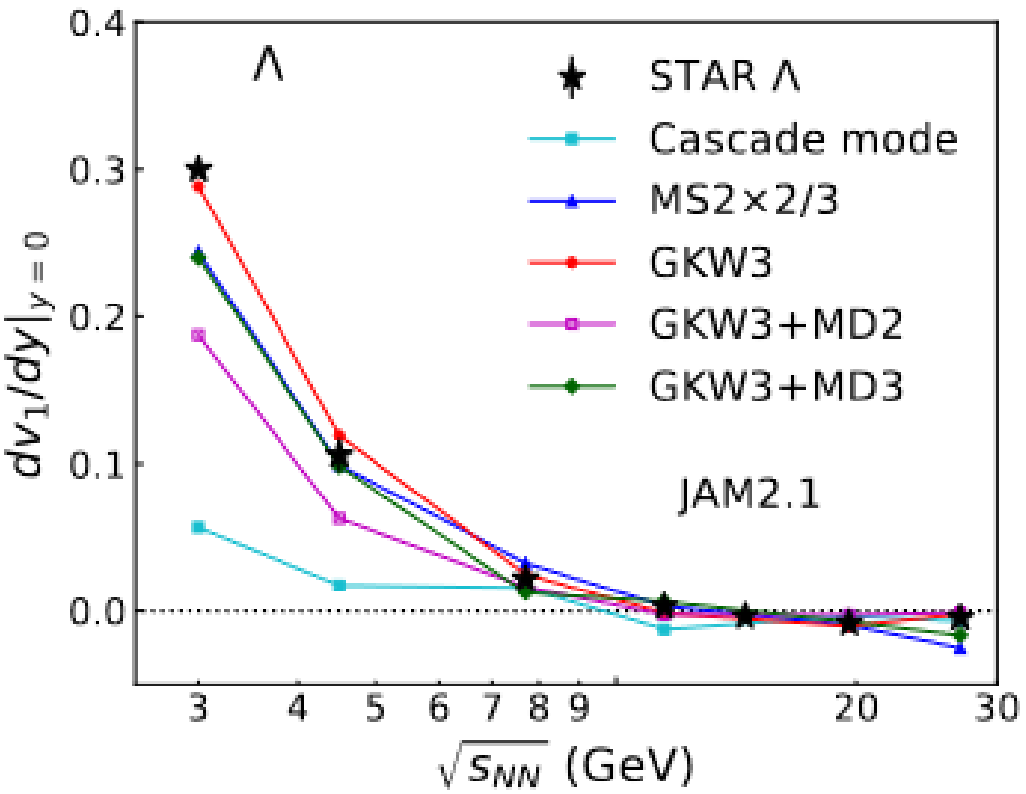
Nara, Jinno, Murase, AO ('22)

Directed flow of Λ with chiral EFT U_Λ

- **Λ potential from chiral EFT is adopted.**
  - **GKW2/3: Fit to GKW $U_\Lambda(\rho)$ without/with 3-body force.**
  - **MD2/3: Fit to Kohno('18) in the range k<500 and 200 MeV/c.**
- **GKW3 explains the data well.**
  - **Momentum dep. reduces v1 values.**
  - **GKW2 also explains the data.**



Nara, Jinno, Murase, AO ('22)



Repulsive Λ potential (GKW3) enhances dv1/dy and gives results closest to data, when momentum dependence is ignored.

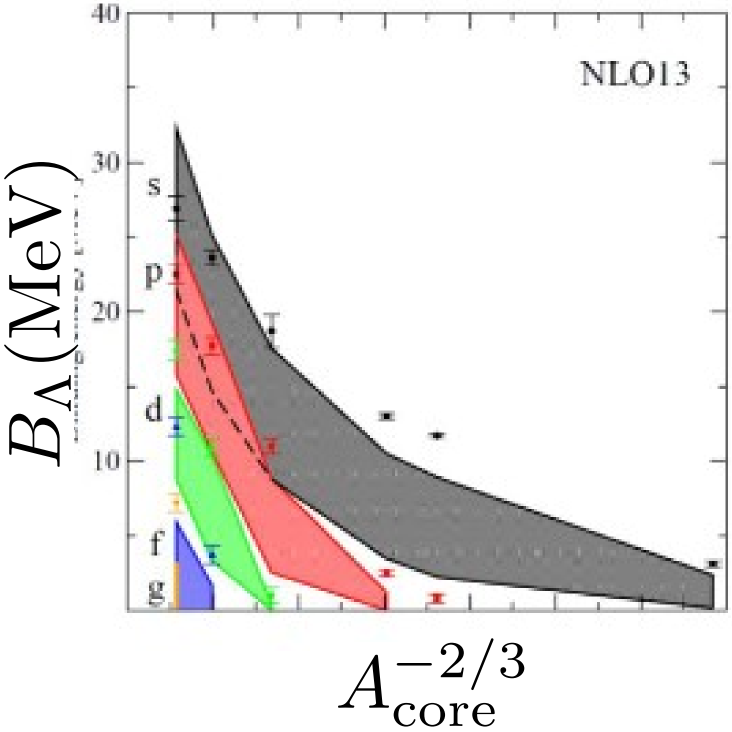
Momentum dep. of U_Λ reduces v1 (and dv1/dy), while the density dep. affects less.
(Why? We haven't understood the reason yet.)

Any other way to constrain the density and momentum dependence of U_Λ?
→ Hypernuclear spectroscopy

Λ Hypernuclei

Λ hypernuclei and Λ potential in nuclear matter

- **Λ potential from chiral EFT w/ 3BF has not been examined in hypernuclear spectroscopy**
  - **It needs to be verified (and tuned) including the density and momentum dependence !**
    (before applying it to heavy-ion collisions (?))
  - **NLO result has been tested but does not explain the data well.**
    Haidenbauer, Vidaña ('21)
  - **How about NNLO**
    with decouplet model ?



Haidenbauer, Vidaña ('21)

Skyrme Hartree-Fock for Λ hypernuclei

- **Previous breakthrough works (spherical SHF)**
  - **Rayet(‘76,‘81): Two-body SHF (w/o ρ dep.)**
  - **Lanskoy, Yamamoto (LY, ‘97): SHF w/ one ρ dep. term (as in standard HF for nucleons)**
  - **Choi, Hiyama et al. (‘22): SHF w/ two or more ρ dep. terms (significant improvement w/ two ρ dep. terms.)**
- **SHF for Λ hypernuclei**
  - **HF equation**

$$\left[-\boldsymbol{\nabla}\cdot\left(\frac{\hbar^2}{2m_B^*(\boldsymbol{r})}\right)\boldsymbol{\nabla}+U_B(\boldsymbol{r})-i\boldsymbol{W}_B(\boldsymbol{r})\cdot(\boldsymbol{\nabla}\times\boldsymbol{\sigma})\right]\psi_{iB}(\boldsymbol{r})=\varepsilon_i\psi_{iB}(\boldsymbol{r})$$

  - **HF potential**

$$\begin{aligned} U_\Lambda(\boldsymbol{r}) &= a_1^\Lambda\rho_N + a_2^\Lambda\tau_N + a_3^\Lambda\triangle\rho_N + a_4^\Lambda\rho_N^{4/3} + a_5^\Lambda\rho_N^{5/3} \\ \frac{\hbar^2}{2m_\Lambda^*} &= \frac{\hbar^2}{2m_\Lambda} + a_2^\Lambda\rho_N\ , \quad \tau_B = \sum_i \boldsymbol{\nabla}\psi_{iB}^*\cdot\boldsymbol{\nabla}\psi_{iB} \end{aligned}$$

Parameters from GKW

- **Density dependence at zero momentum**

$$U_\Lambda(\boldsymbol{r}) = a_1^\Lambda \rho + a_2^\Lambda \tau_N + a_3^\Lambda \triangle\rho + a_4^\Lambda \rho^{4/3} + a_5^\Lambda \rho^{5/3} \quad (\rho = \rho_N)$$

$$\to U_\Lambda(\rho, \boldsymbol{k}=0) = a_1^\Lambda \rho + a_4^\Lambda \rho^{4/3} + \widetilde{a}_5^\Lambda \rho^{5/3} \ \text{(uniform matter)}$$

$$\widetilde{a}_5^\Lambda = a_5^\Lambda + \alpha a_2^\Lambda \,, \ \alpha = \text{const.}$$

  - **Three parameters are tuned to reproduce GKW results.**

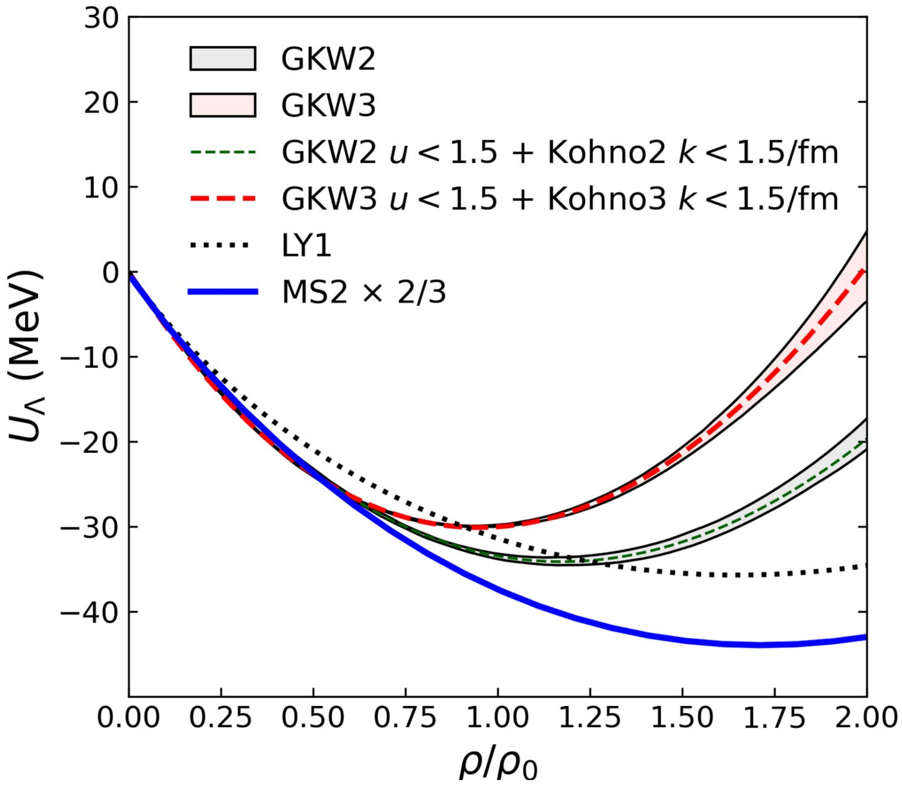
- **Momentum dependence**

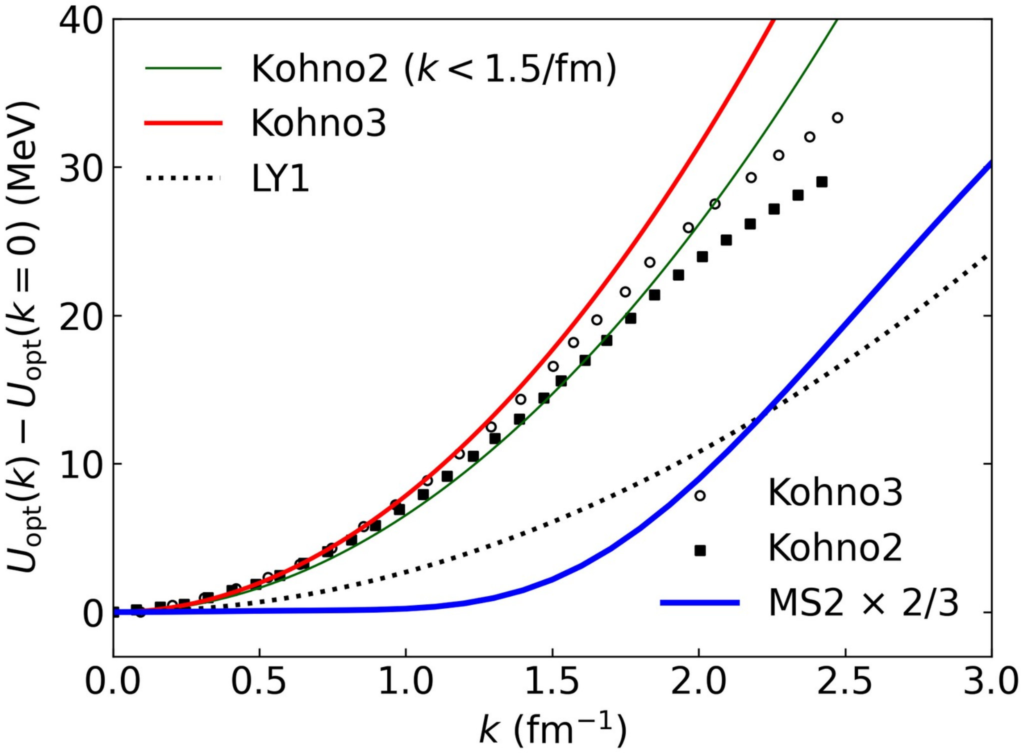
$$U_\Lambda(\rho, \boldsymbol{k}) = U_\Lambda(\rho) + a_2^\Lambda \boldsymbol{k}^2 \rho$$

  - **a_2^Λ is tuned to reproduce Kohno's results at low momentum.**

- **Finite range effects**
  - **a_3^Λ is tuned to reproduce the separation energy of Λ in $^{13}_{\Lambda}$C (even-even core nucleus)**

Parameters from GKW



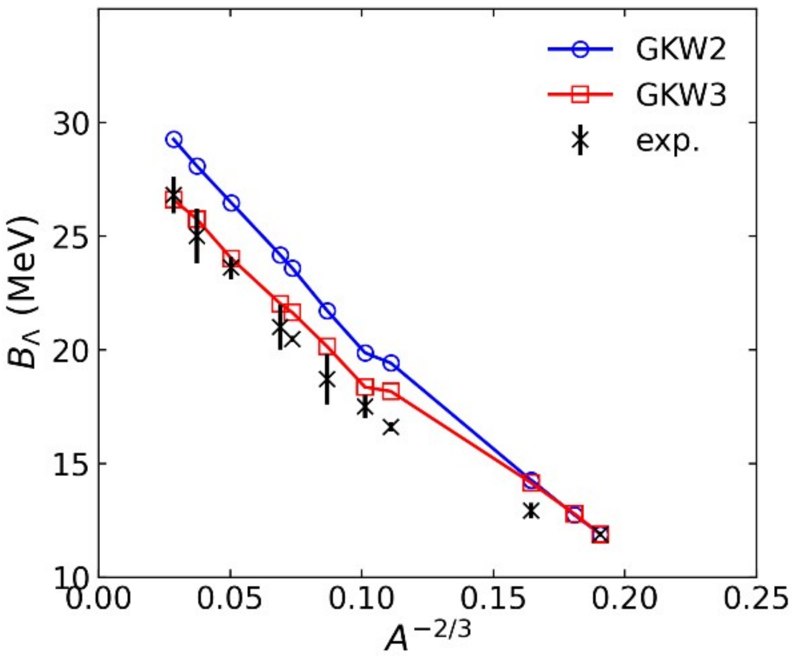


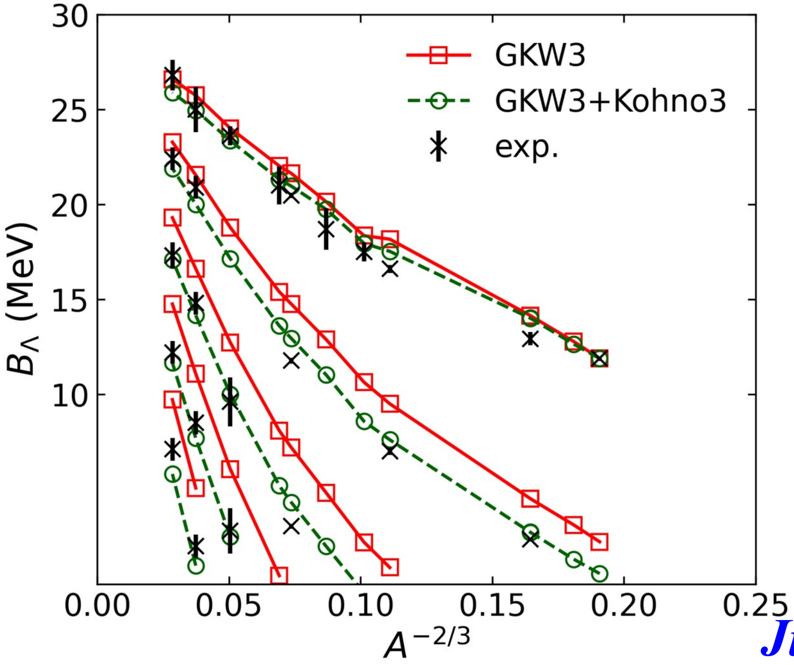
3 density-dep. terms + k^2 momentum dep. term are enough at $\rho/\rho_0 < 2$ and $k < 1.3$ fm^{-1}

Jinno+ (in prog.)

GKW2 and GKW3

- **NNLO chiral EFT with the decouplet saturation model without/with 3-body terms (GKW3 and GKW2)**
- **W/o mom. dep. (a_2^Λ=0), GKW2 overestimates B_Λ at large A.**
  - **Deeper potential at ρ_0. (~ -35 MeV)**
  - **Steeper A dep. is consistent with *Haidenbauer, Vidaña ('21)***
  - **Mom. dep. and finite range terms does not help.**
- **With mom. dep. term, GKW3+Kohno3 explains the data well.**





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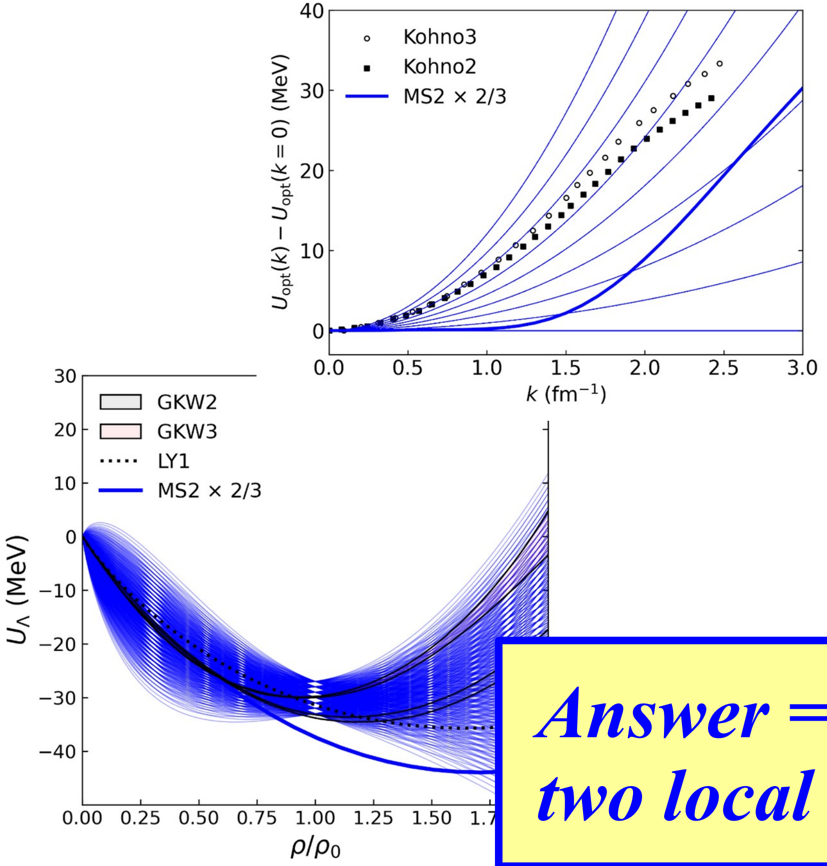
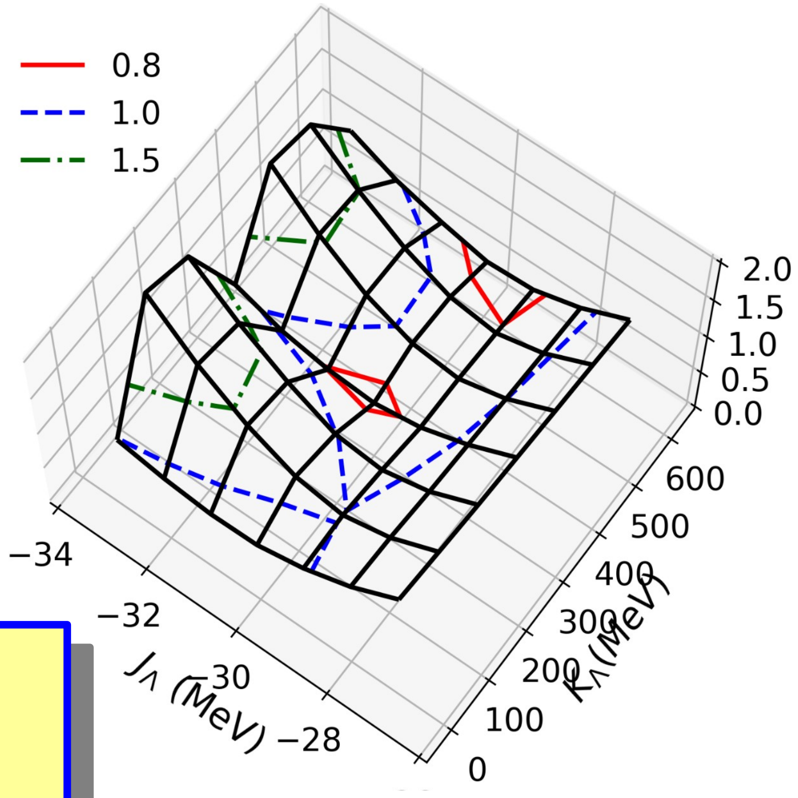
Question!

- **Chiral EFT and meson-exchange-based potentials have different density dependence. Why can both of them explain B_Λ?**
  → Global analysis using Taylor coeff. around ρ_0 !

$$U_\Lambda(\rho,k) = J_\Lambda + \frac{L_\Lambda}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_\Lambda}{9}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + a_2^\Lambda k^2 \rho + \mathcal{O}\left[\left(\frac{\rho-\rho_0}{\rho_0}\right)^3, k^4\right]$$



Answer = There are two local minima.



Jinno+ (in prog.)

Summary

- **Λ potential in nuclear matter from the chiral effective field theory with the 3-body force effect is examined via**
  the directed flow of Λ from heavy-ion collisions
  and the binding energies of Λ in hypernuclei.
  - **GKW2/3: $U_\Lambda(\rho,k=0)$ without/with 3-body force**

    Gerstung, Kaiser, Weise ('20)
  - **Kohno2/3: $U_\Lambda(\rho_0,k)$ without/with 3-body force** ***Kohno ('18)***
  - **With GKW3, repulsive U_Λ forbids Λ to appear in neutron stars and the hyperon puzzle is solved.**
- **Directed flow of Λ is well explained by GKW3 without mom. dep.**
  - **With strong mom. dep., v1(Λ) is underestimted.**
  - **GKW2 w/o mom. dep. also explains v1(Λ).**
- **GKW3+Kohno3 can explain the Λ binding energies (by tuning the finite range term).**
  - **GKW2 is too attractive and cannot explain B_Λ.**

Conclusion, Conjecture, and To do

- **Conclusion: Stiff U_Λ ($K_\Lambda \sim$ (500-600) MeV) having weak momentum dependence ($M_\Lambda^* \sim (0.8\text{-}0.9) M_\Lambda$)**
  explains both v1(Λ) in HIC and B_Λ in hypernuclei.
- **Conjecture: There may be two types of U_Λ which explains B_Λ.**
  - **Lanskoy-Yamamoto type: $K_\Lambda \sim$ 300 MeV**
  - **Chiral EFT type: $K_\Lambda \sim$ 600 MeV**
  - **Λ appears in neutron stars in the former,**
    but Λ does not appear in neutron stars in the latter.
- **To do**
  - **Comparison of the directed flows using U_Λ at two local minima.**
  - **Understand the reason for the insensitivity of v1(Λ) to the density dependence of U_Λ via time-dependent analysis.**
  - **More serious estimate of B_Λ using chiral EFT. (E.g. HypAMD)**
  - **Other SHF parameters are close to local minima ?**
    N.Guleria, S.K.Dhiman, R.Shyam(1108.0787)
  - **Why does chiral EFT show strong k-dep. in N(N)LO?**

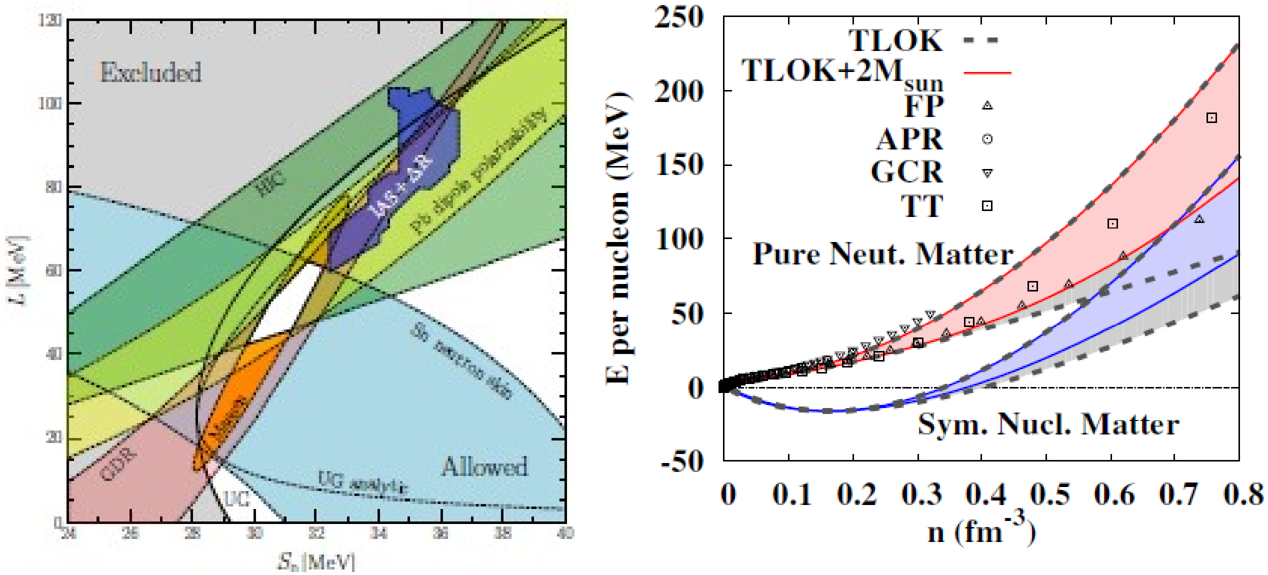
Thank you for your attention !

Fermi momentum expansion

- **Energy per nucleon would be expressed as the power series of k_F**

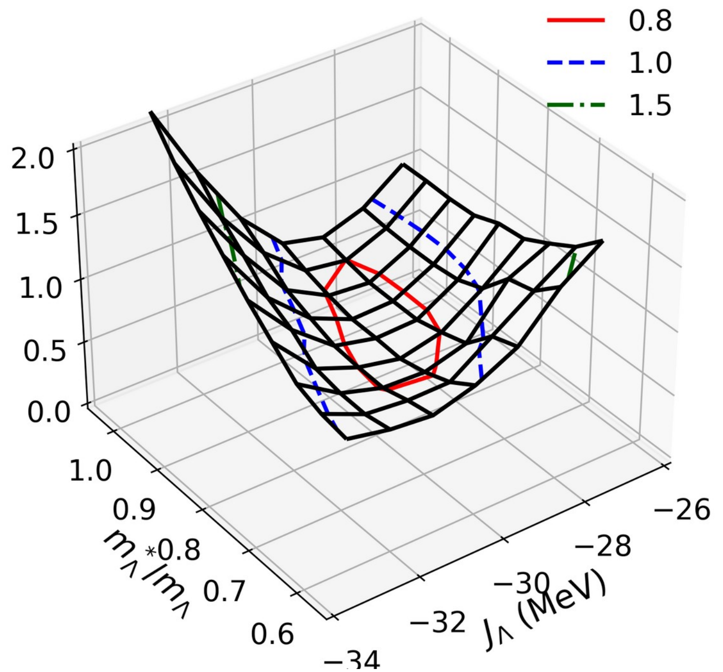
$$E = Tu^{2/3} + au + bu^{4/3} + cu^{5/3} + du^2$$
$$= J + \frac{L}{3}(u-1) + \frac{K}{18}(u-1)^2 + \frac{Q}{162}(u-1)^3 + \mathcal{O}((u-1)^4)$$
$$(u = \rho/\rho_0)$$

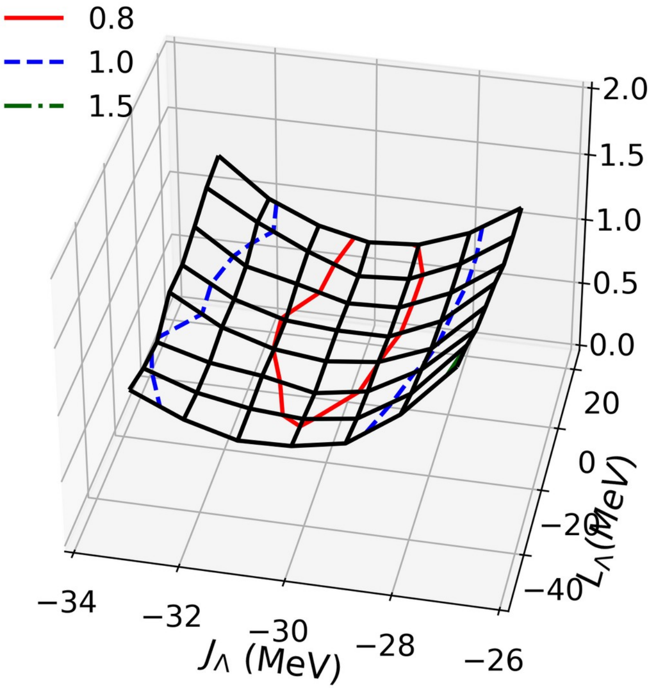
$$a = -4T + 20J - \frac{19}{3}L + K - \frac{1}{6}Q$$
$$b = 6T - 45J + 15L - \frac{5}{2}K + \frac{1}{2}Q$$
$$c = -4T + 36J - 12L + 2K - \frac{1}{2}Q$$
$$d = T - 10J + \frac{10}{3}L - \frac{1}{2}K + \frac{1}{6}Q$$



Tews, Lattimer, AO, Kolomeitsev ('17)

M*-J and J-L correlations





One local minimum

Parameter range

- $J_\Lambda = -33, -32, -31, \dots, -27\ \mathrm{MeV}$
- $L_\Lambda = -50, -40, -30, \dots, 20\ \mathrm{MeV}$
- $K_\Lambda = 0, 100, 200, \dots, 600\ \mathrm{MeV}$
- $m^*/m = 0.6, 0.65, 0.70, \dots, 1.0$
- 計3,528個のパラメータに対して、

  実験データとの平均誤差

$\left\langle \left(B_{\Lambda,\mathrm{exp}} - B_{\Lambda,\mathrm{HF}} \right)^2 \right\rangle^{1/2}$ を計算