

Complexity Equals Action

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arXiv:1509.07876

Adam Brown, Daniel Roberts, Leonard Susskind, Brian Swingle, Ying Zhao

Complexity, Action, and Black Holes

arXiv:1512.04993

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holographic
duality



Complexity Equals Action

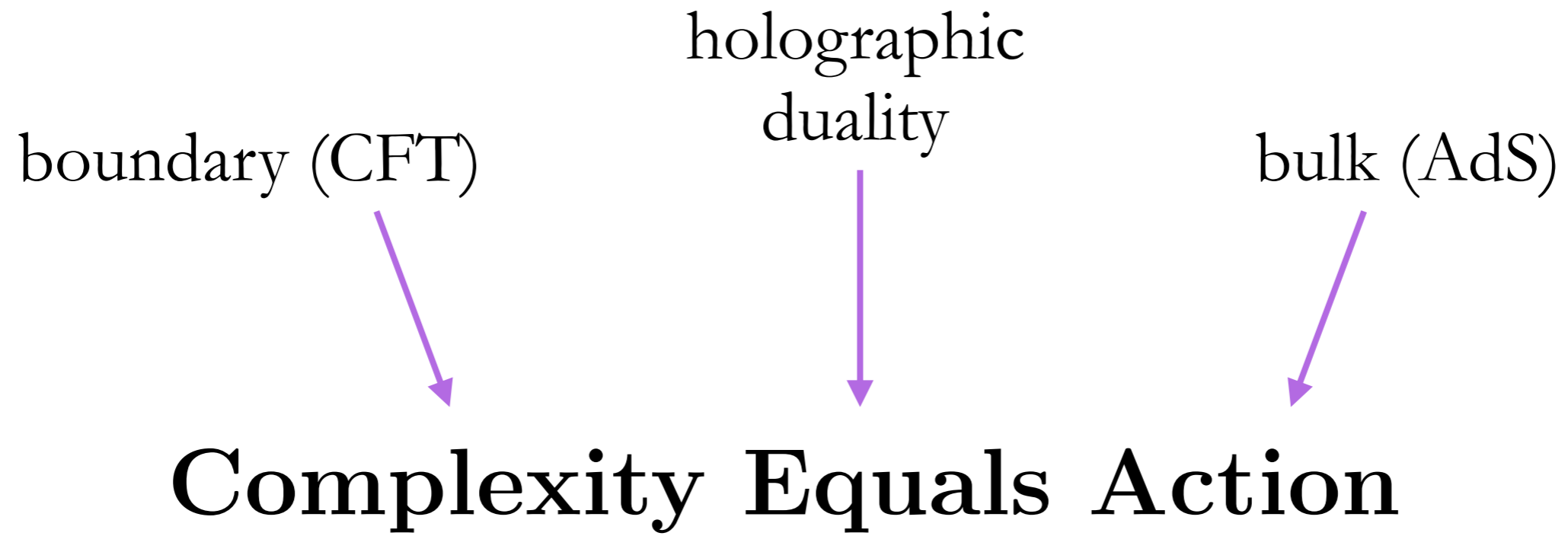
holographic
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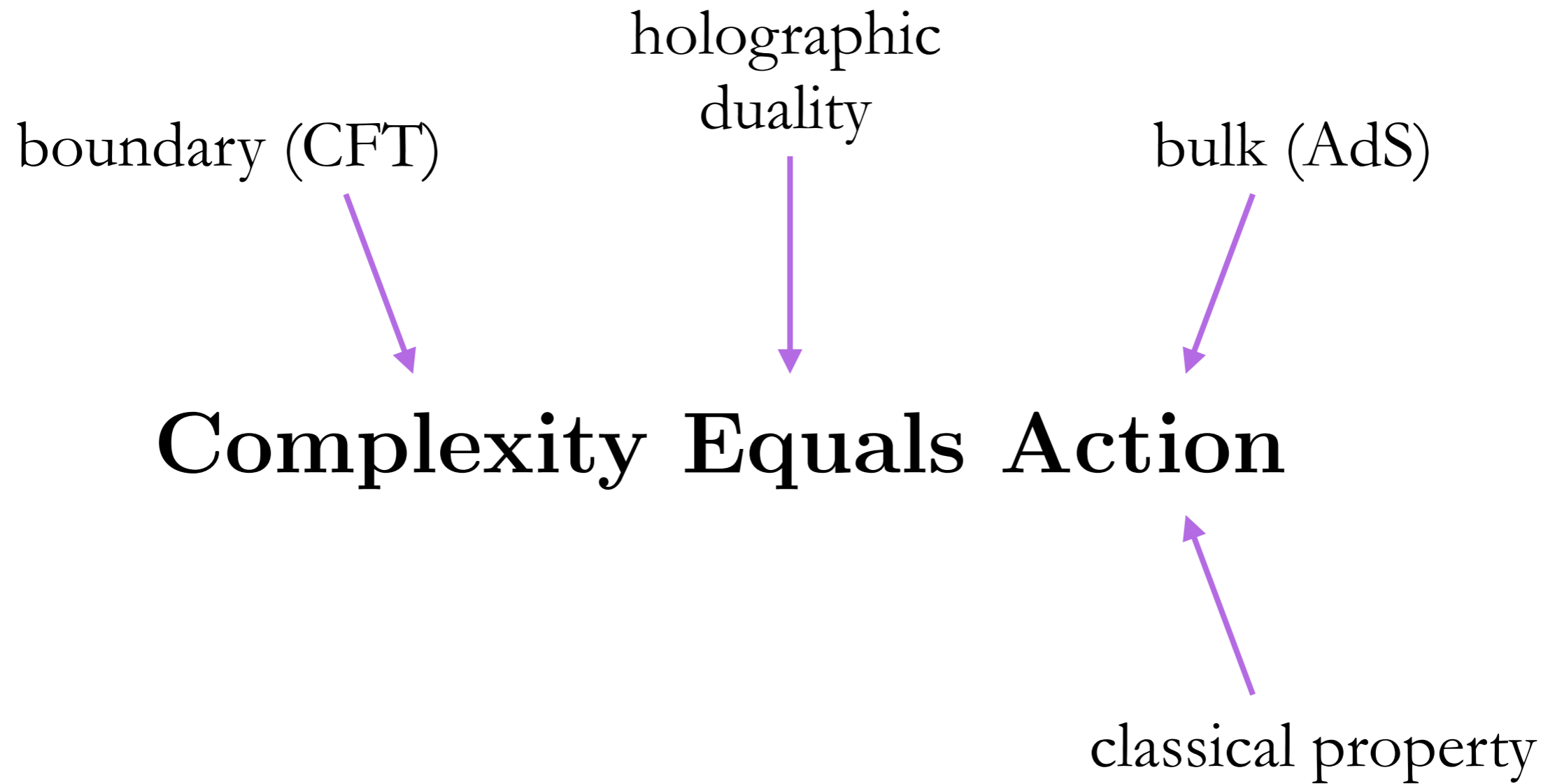


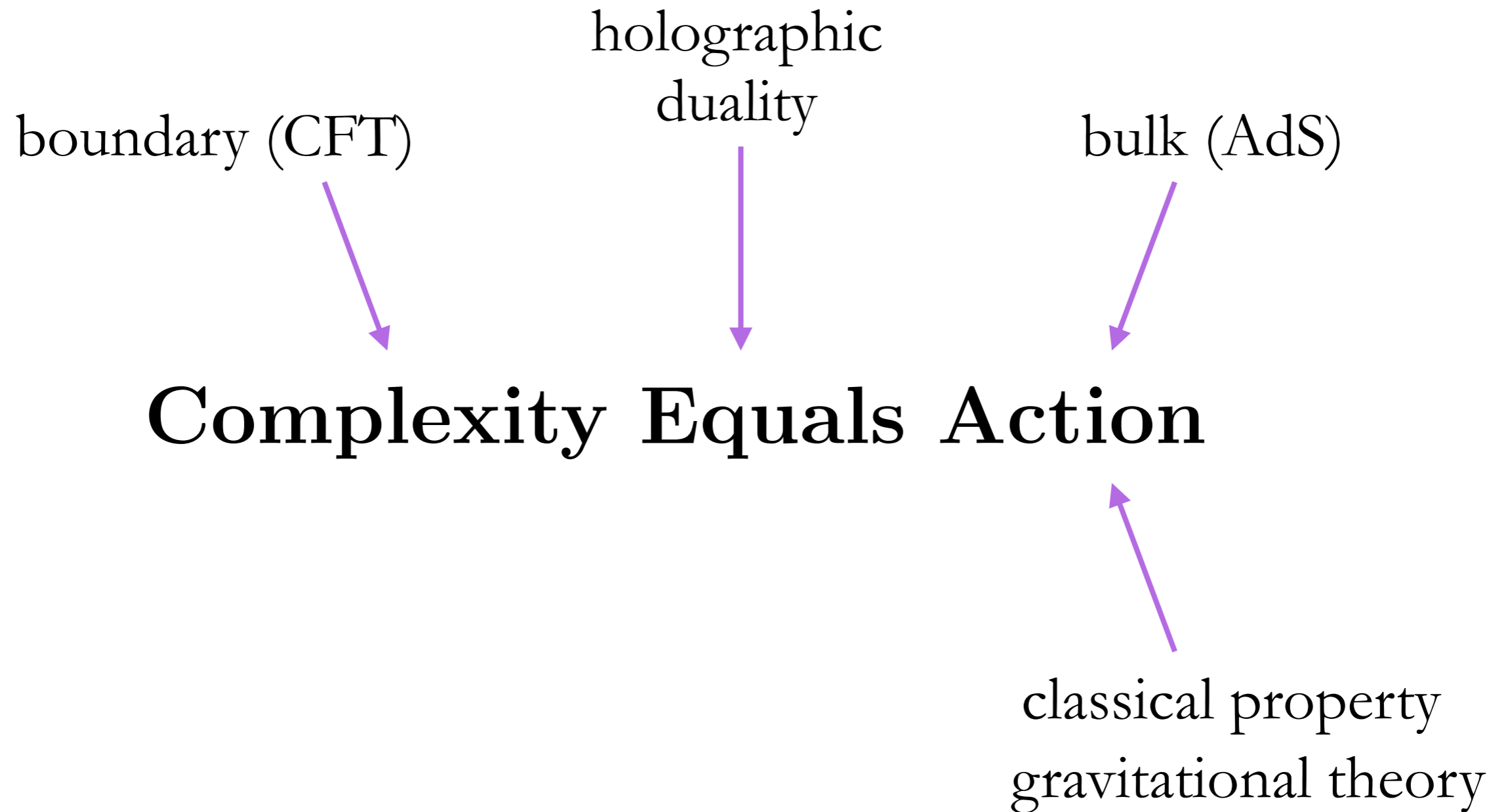
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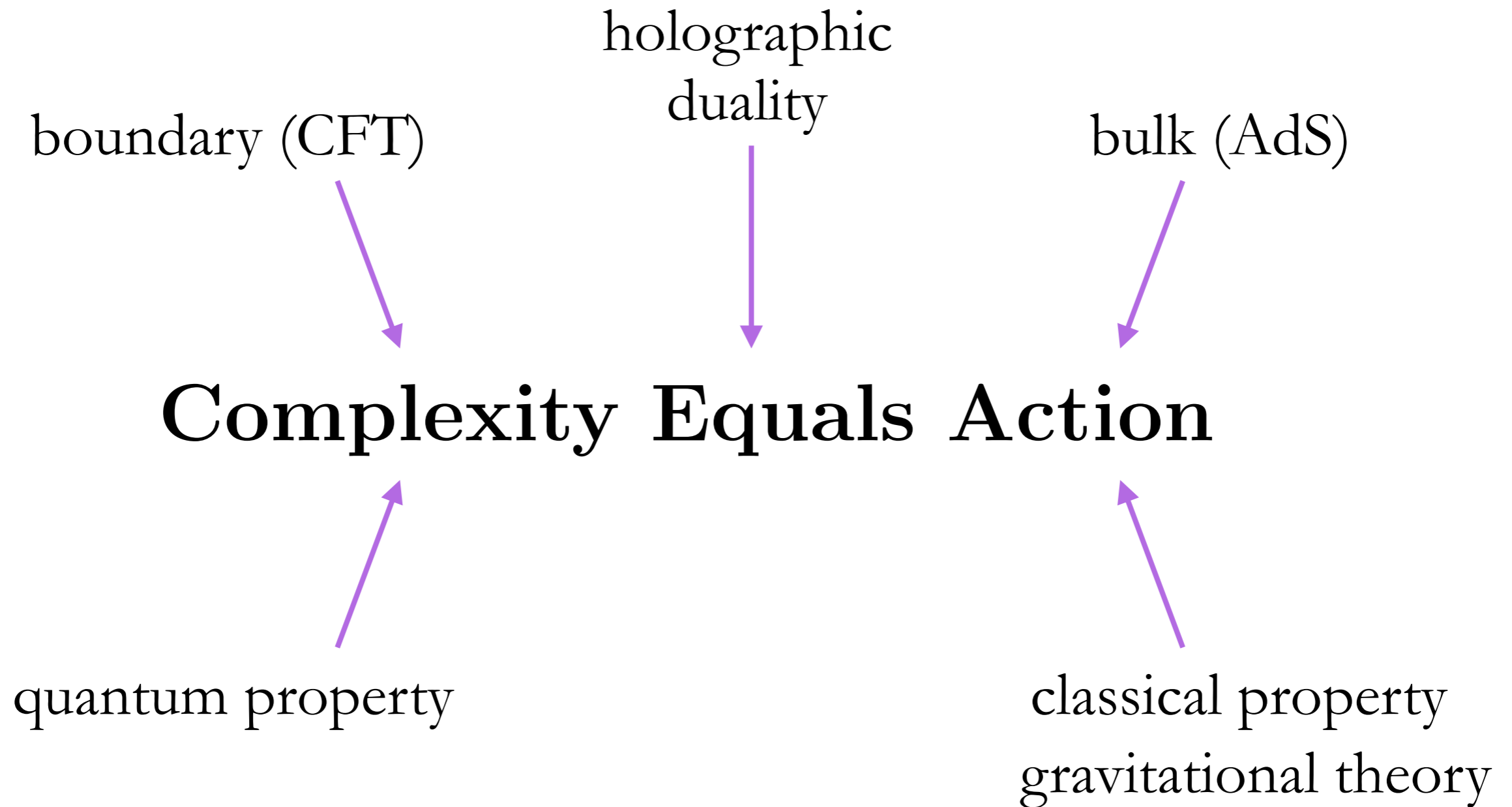


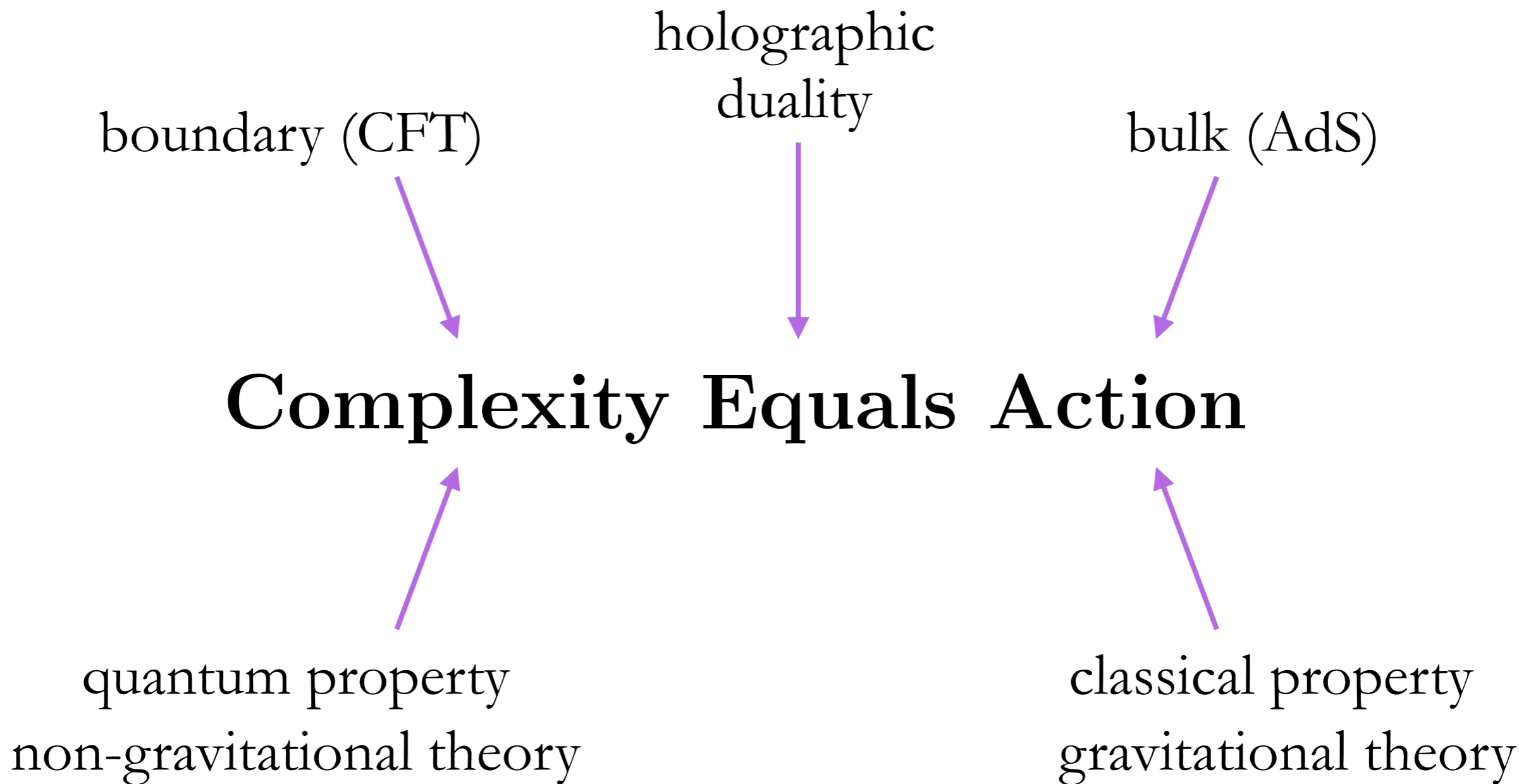
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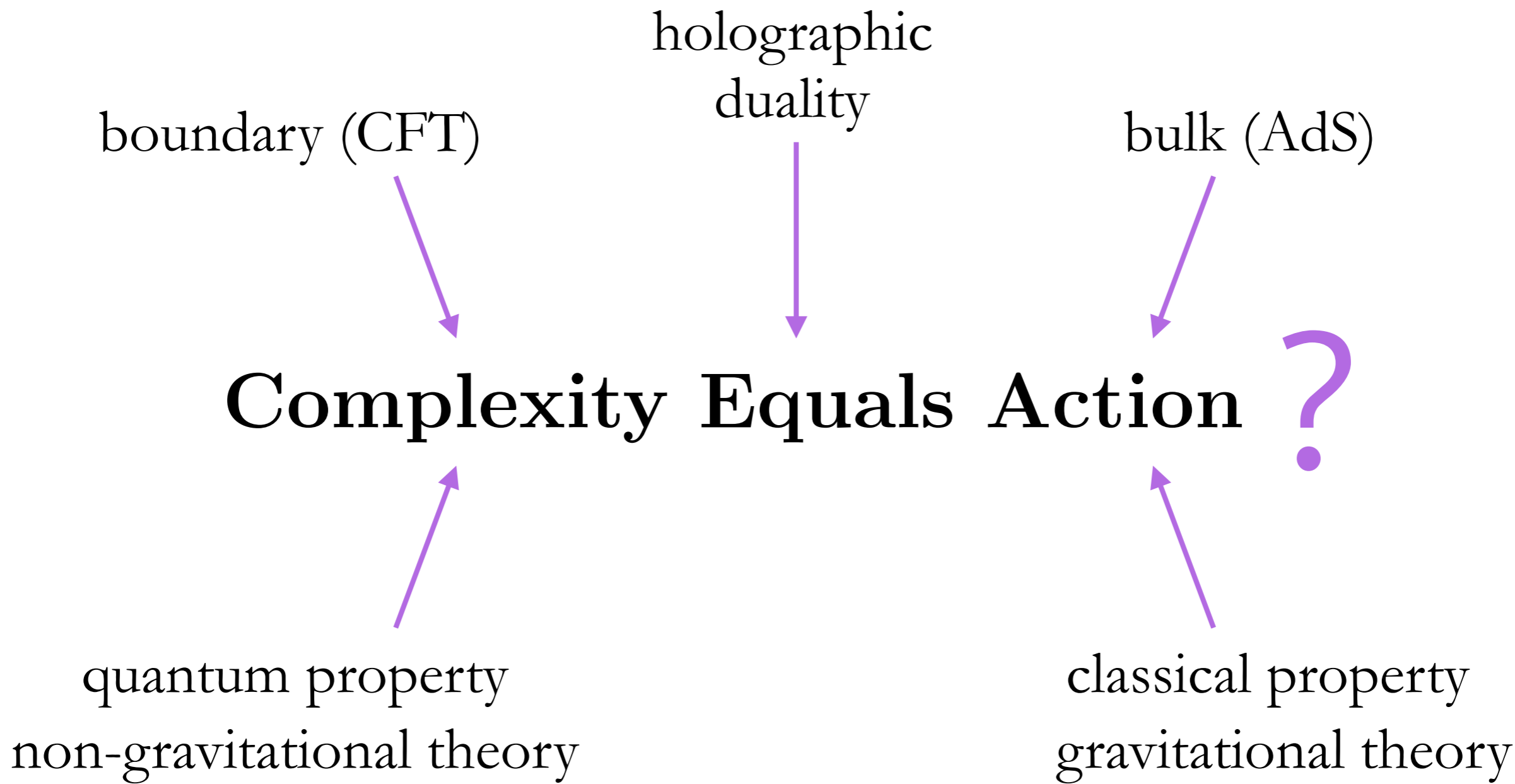












Complexity Equals Action

classical property
gravitational theory



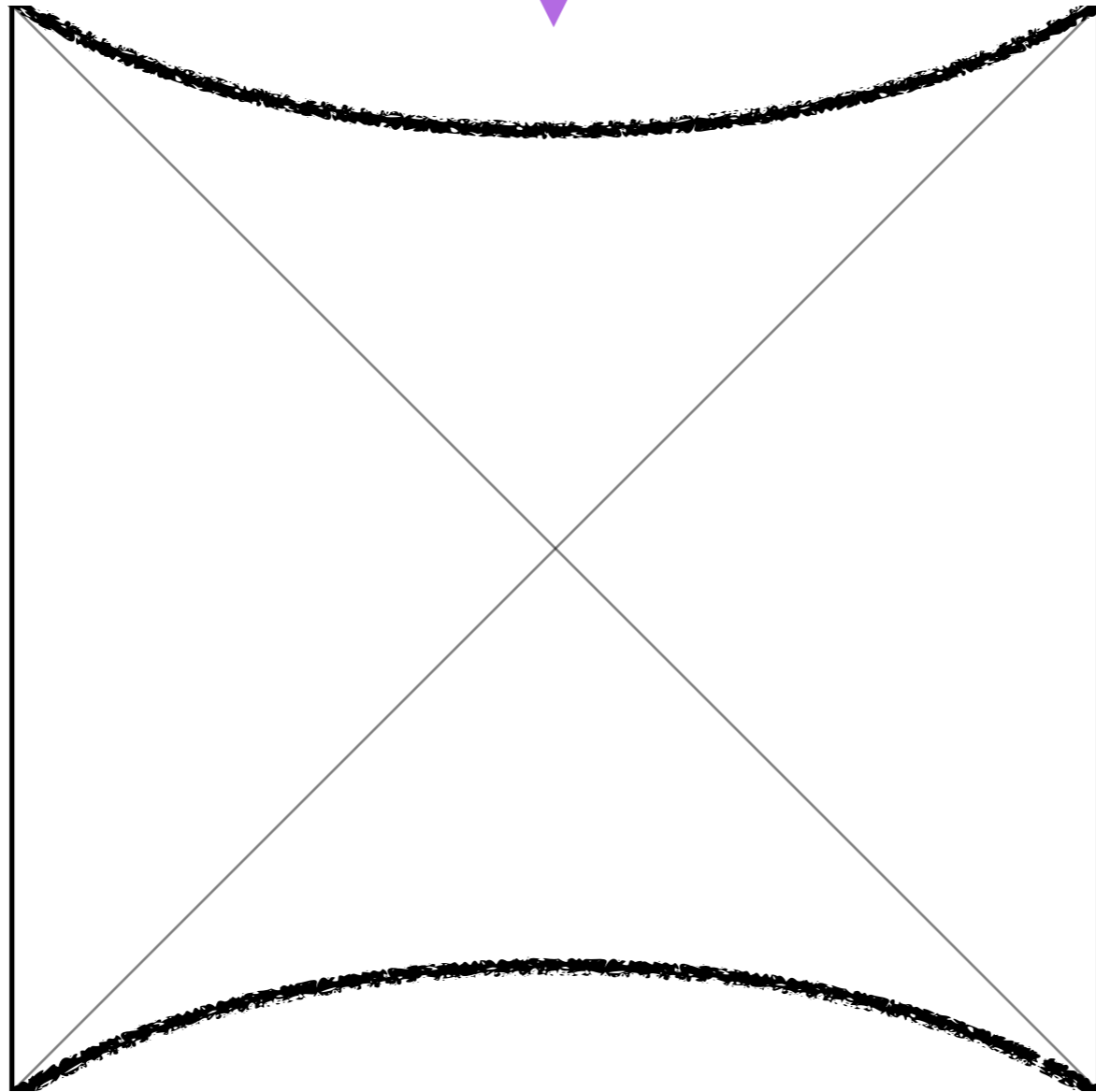
Complexity Equals Action

'size' of wormhole
for black hole in AdS

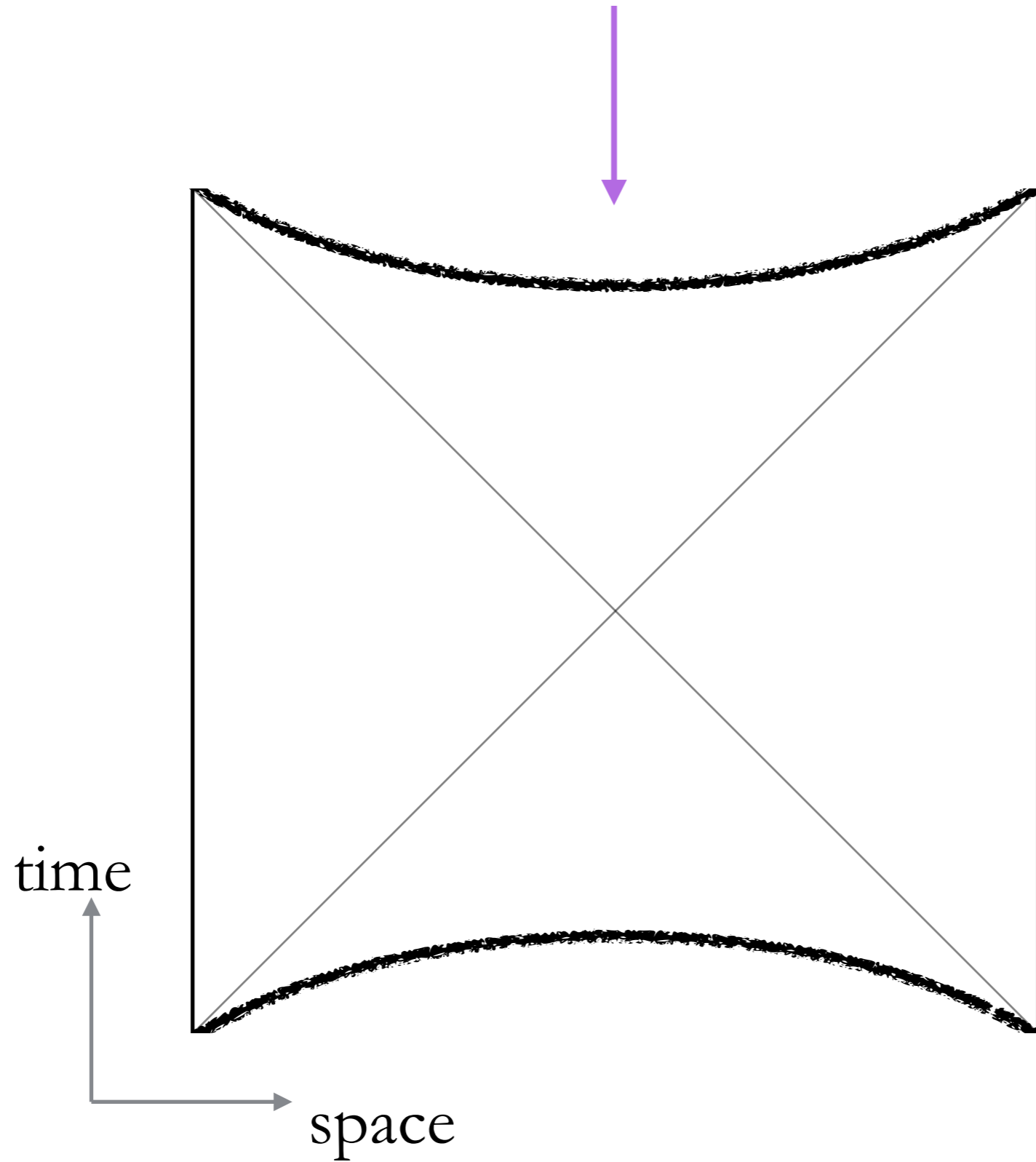
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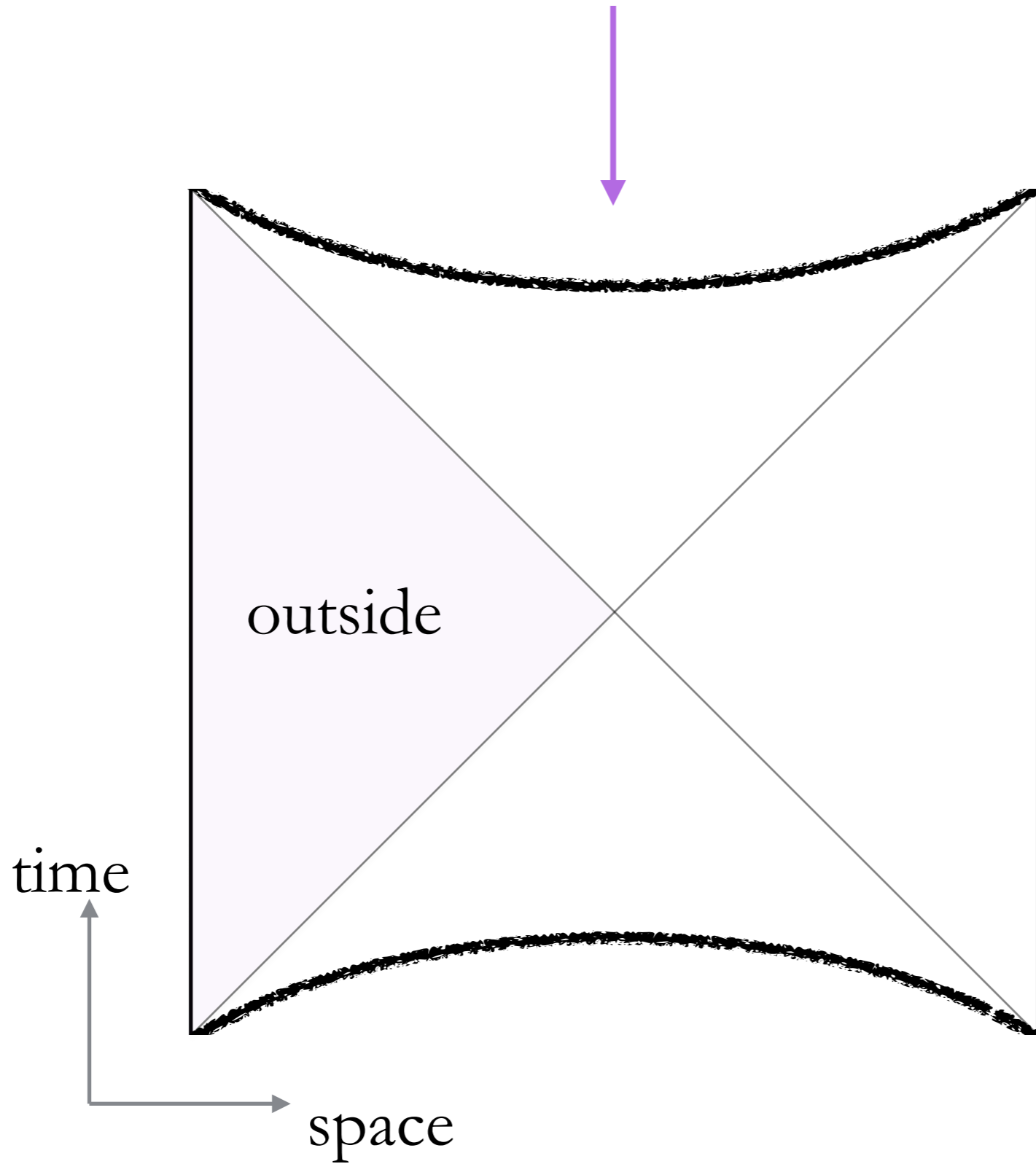
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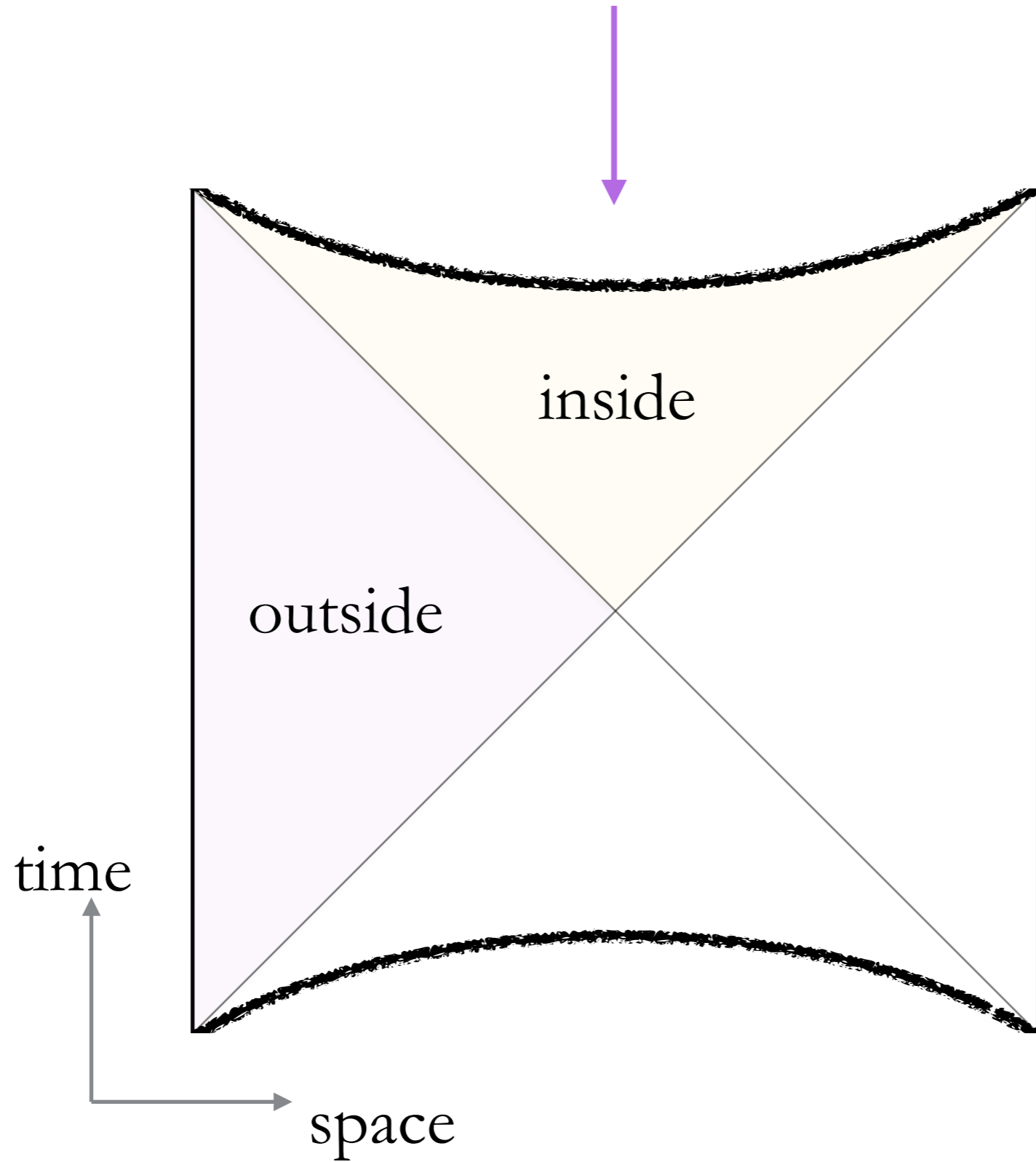
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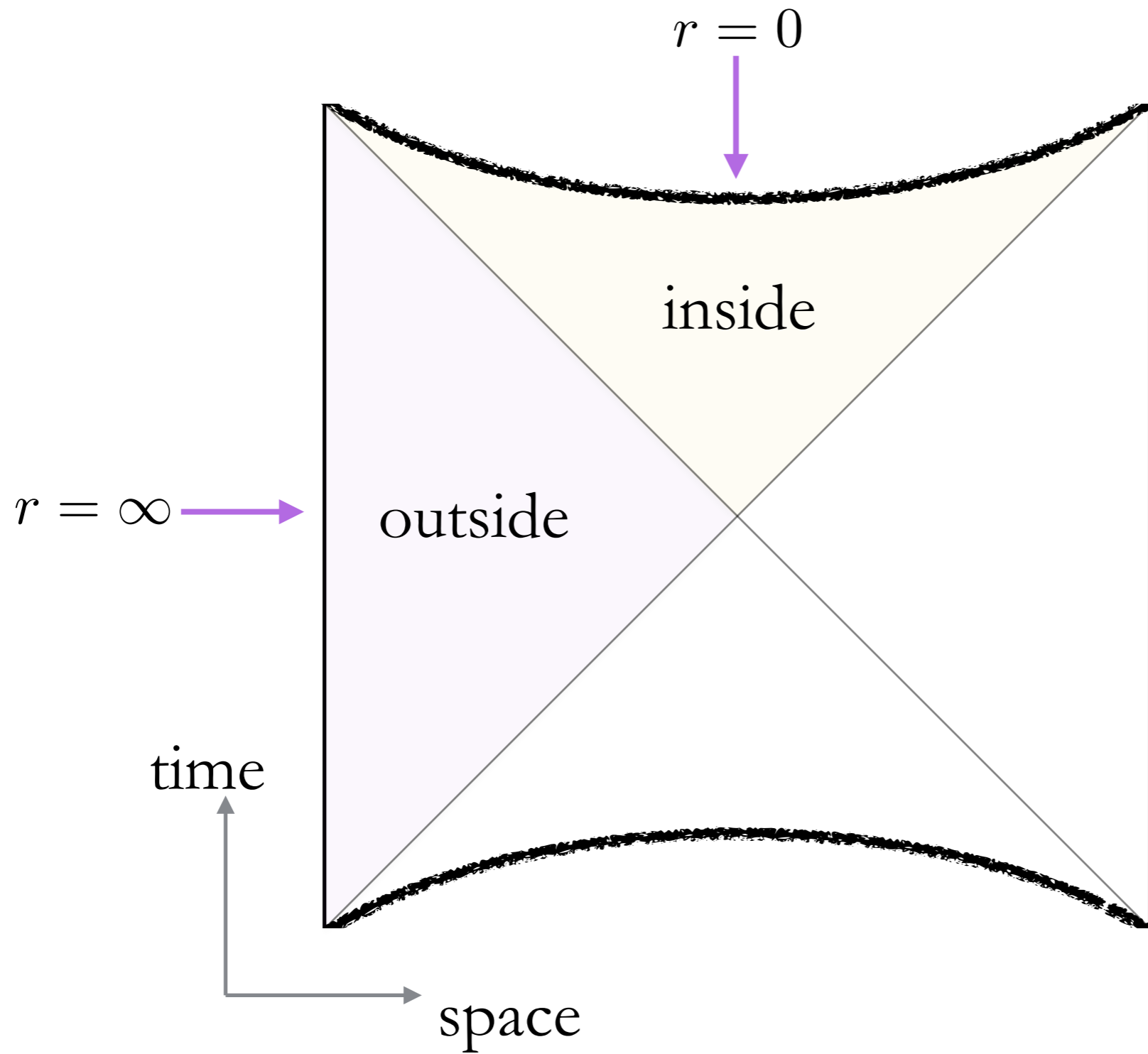
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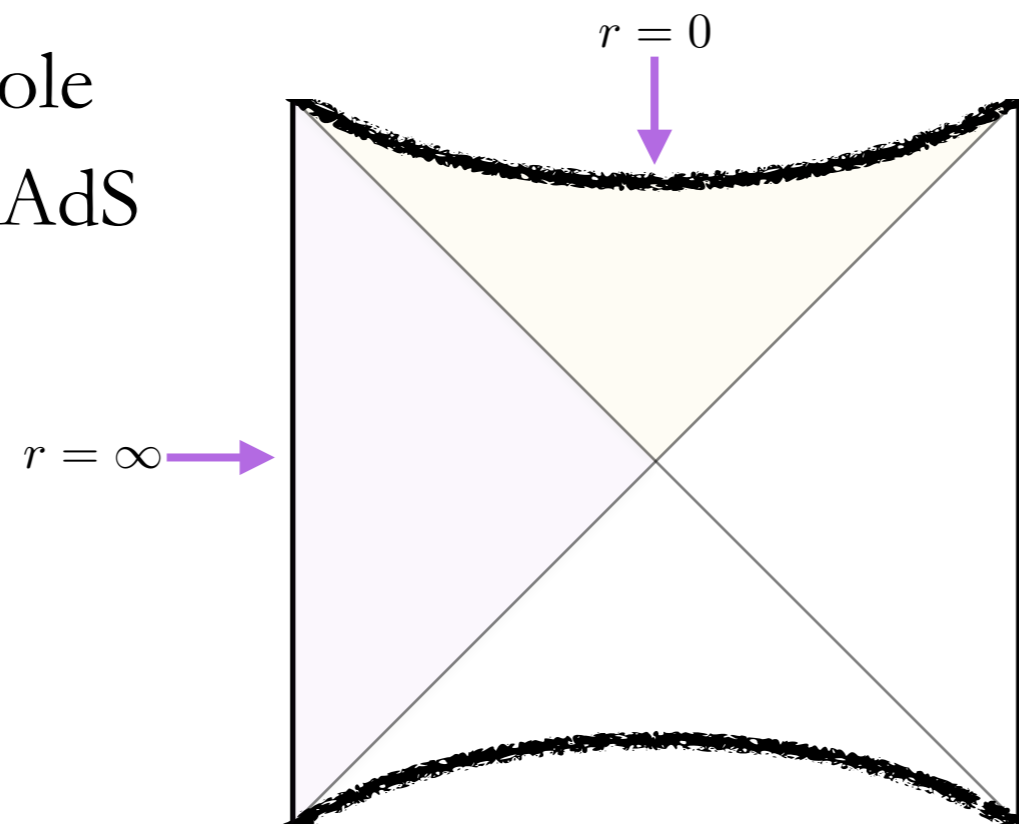
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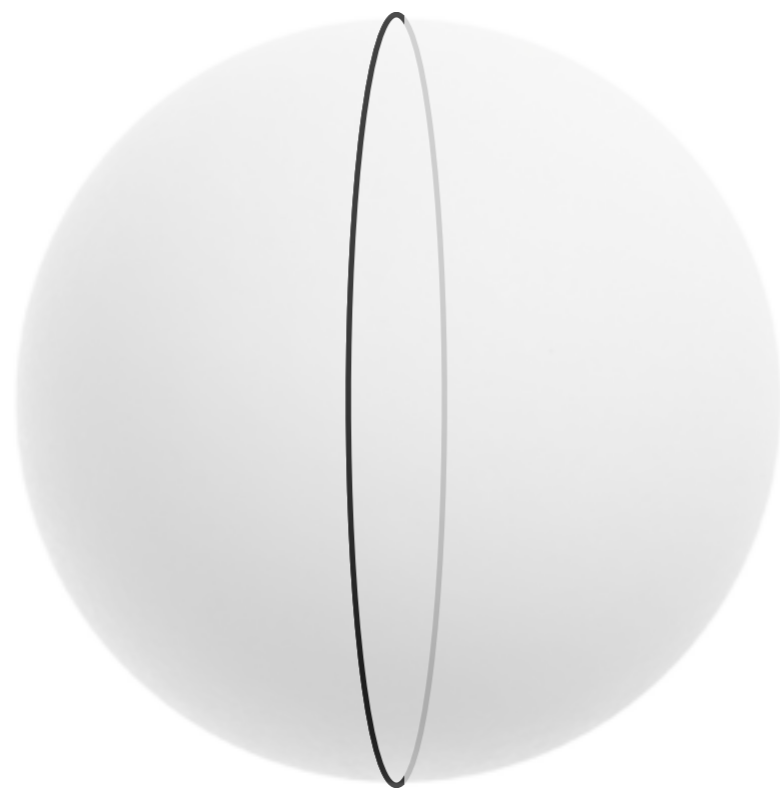
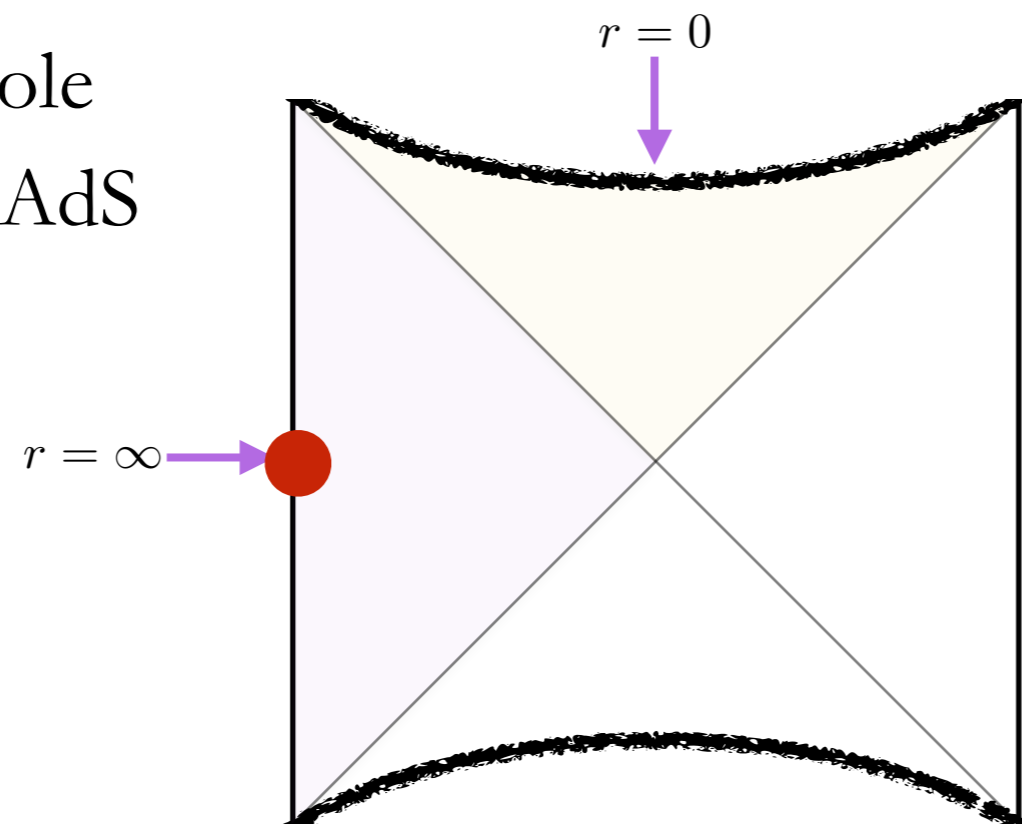
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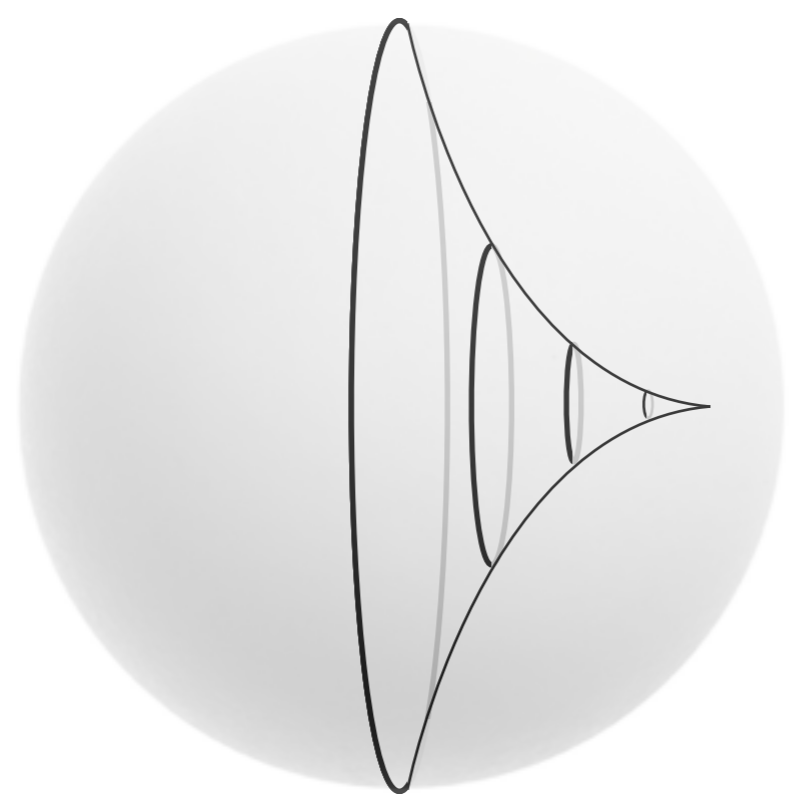
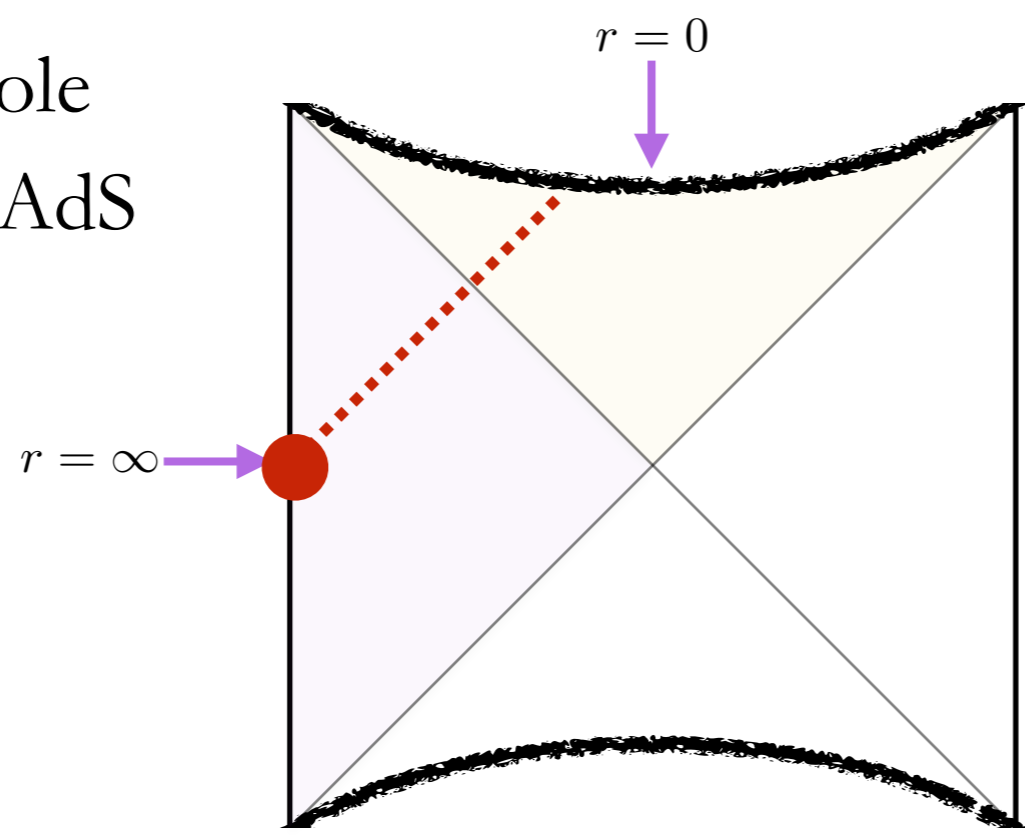
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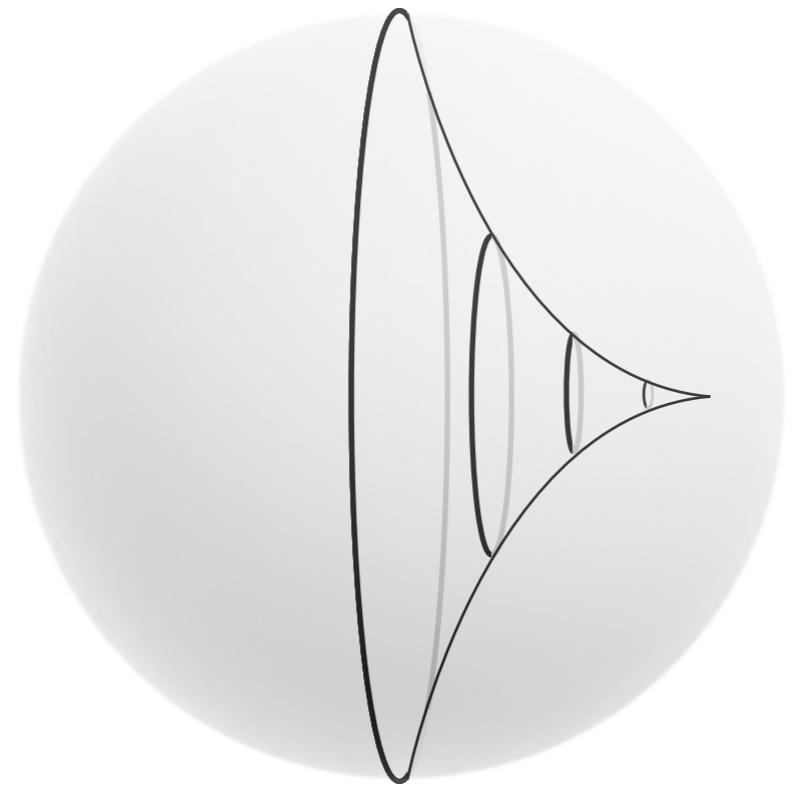
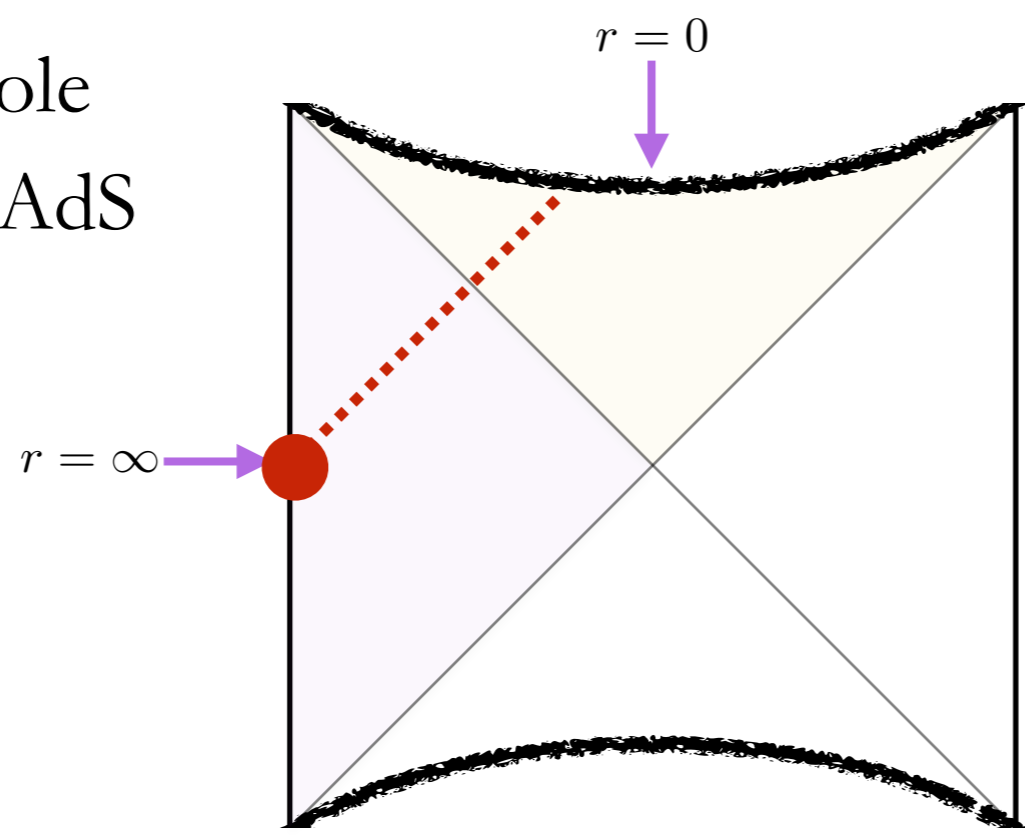
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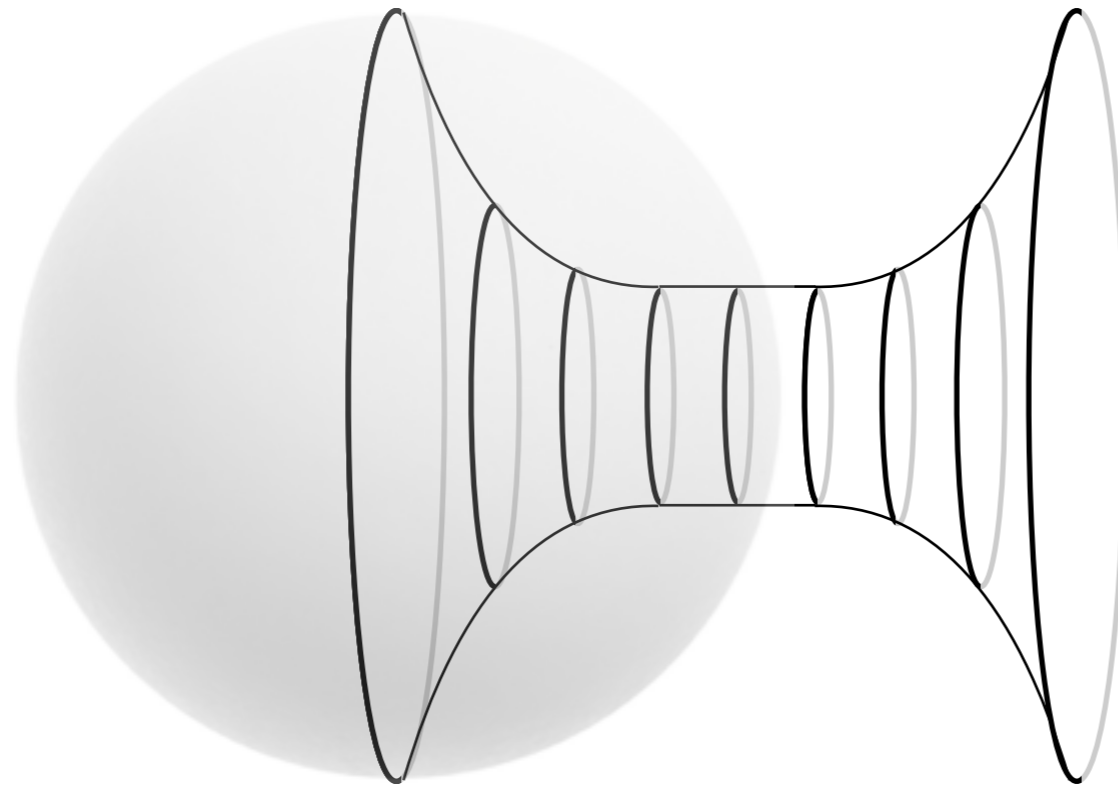
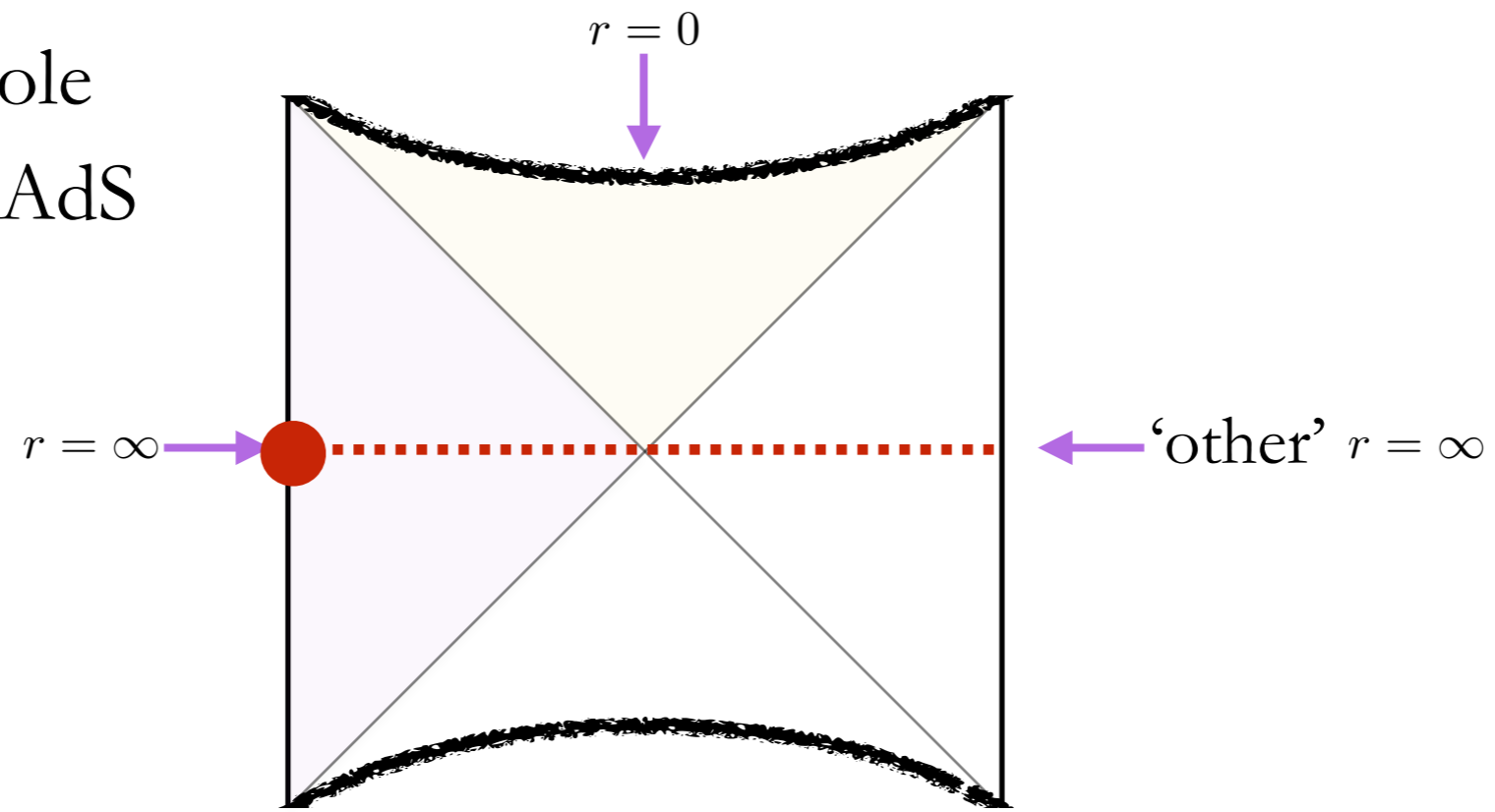
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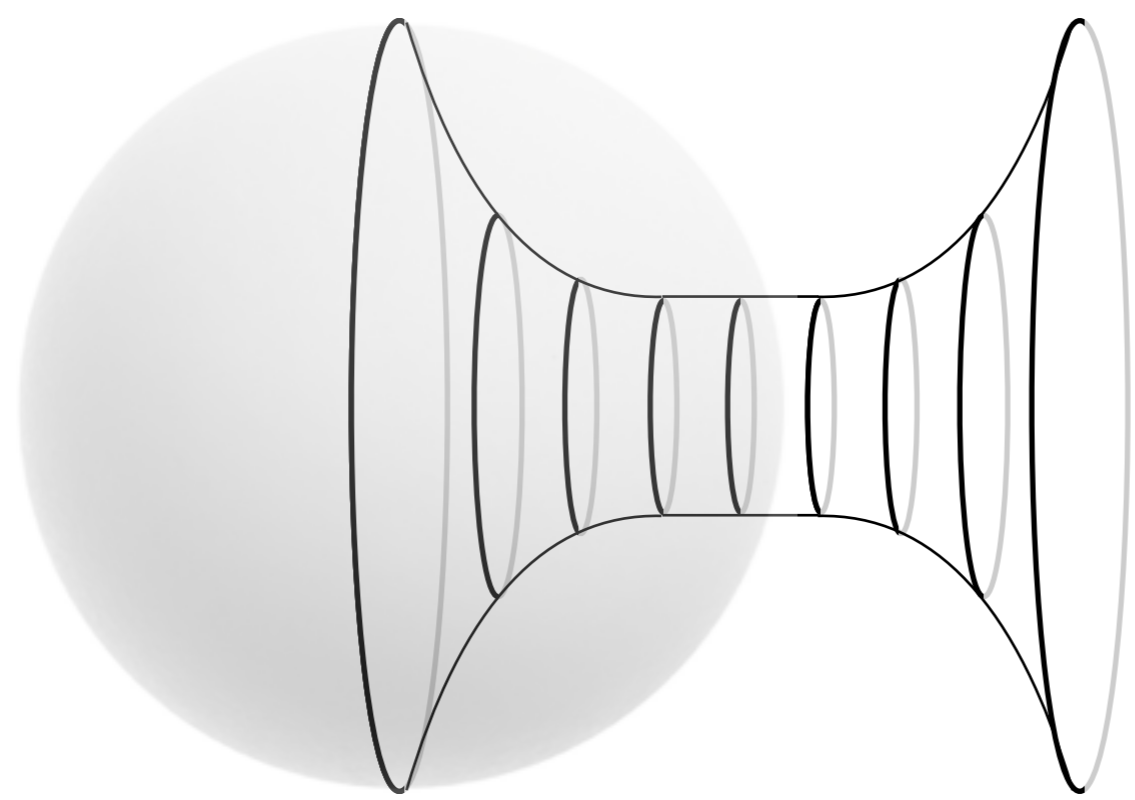
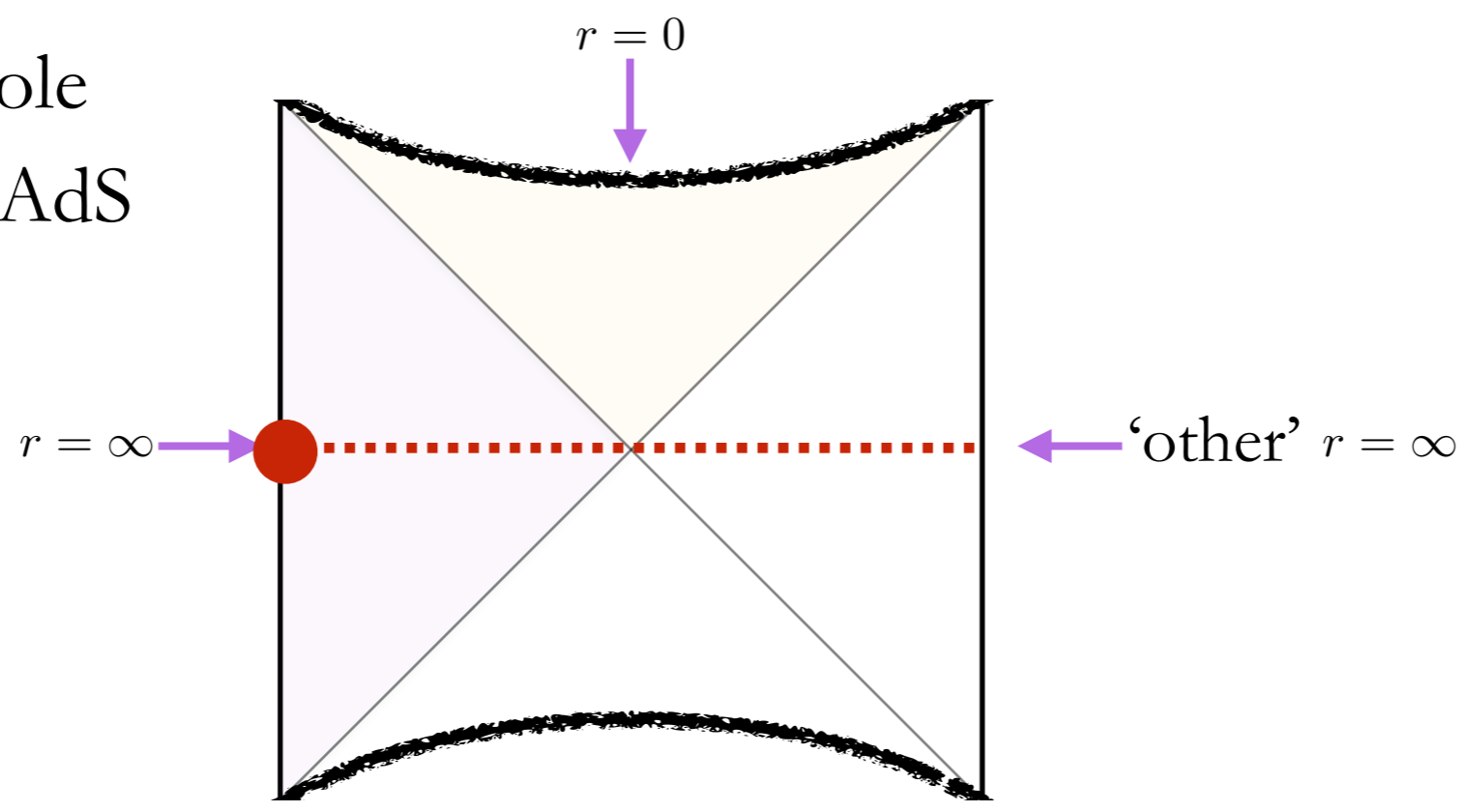
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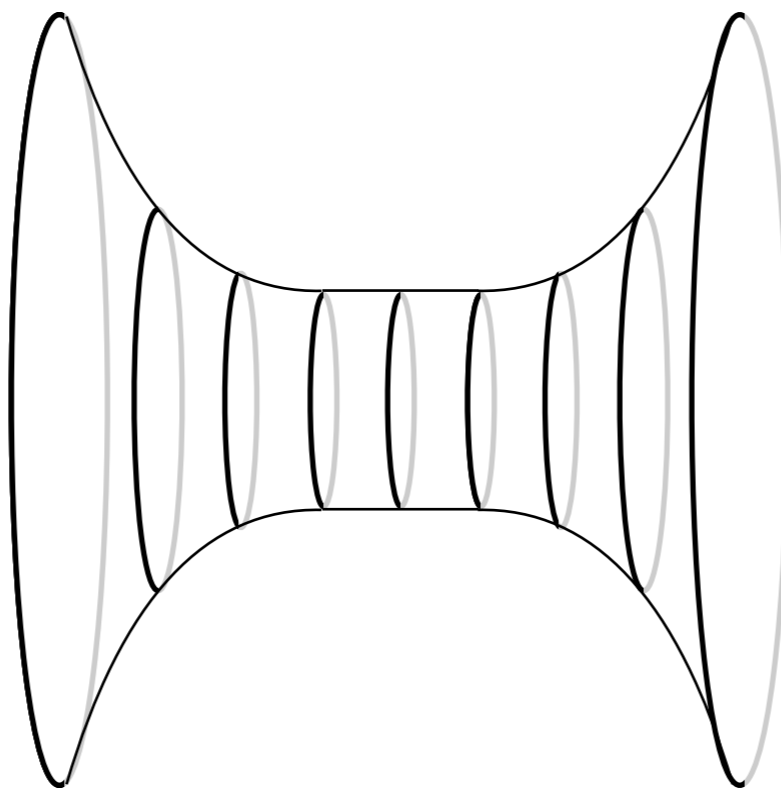
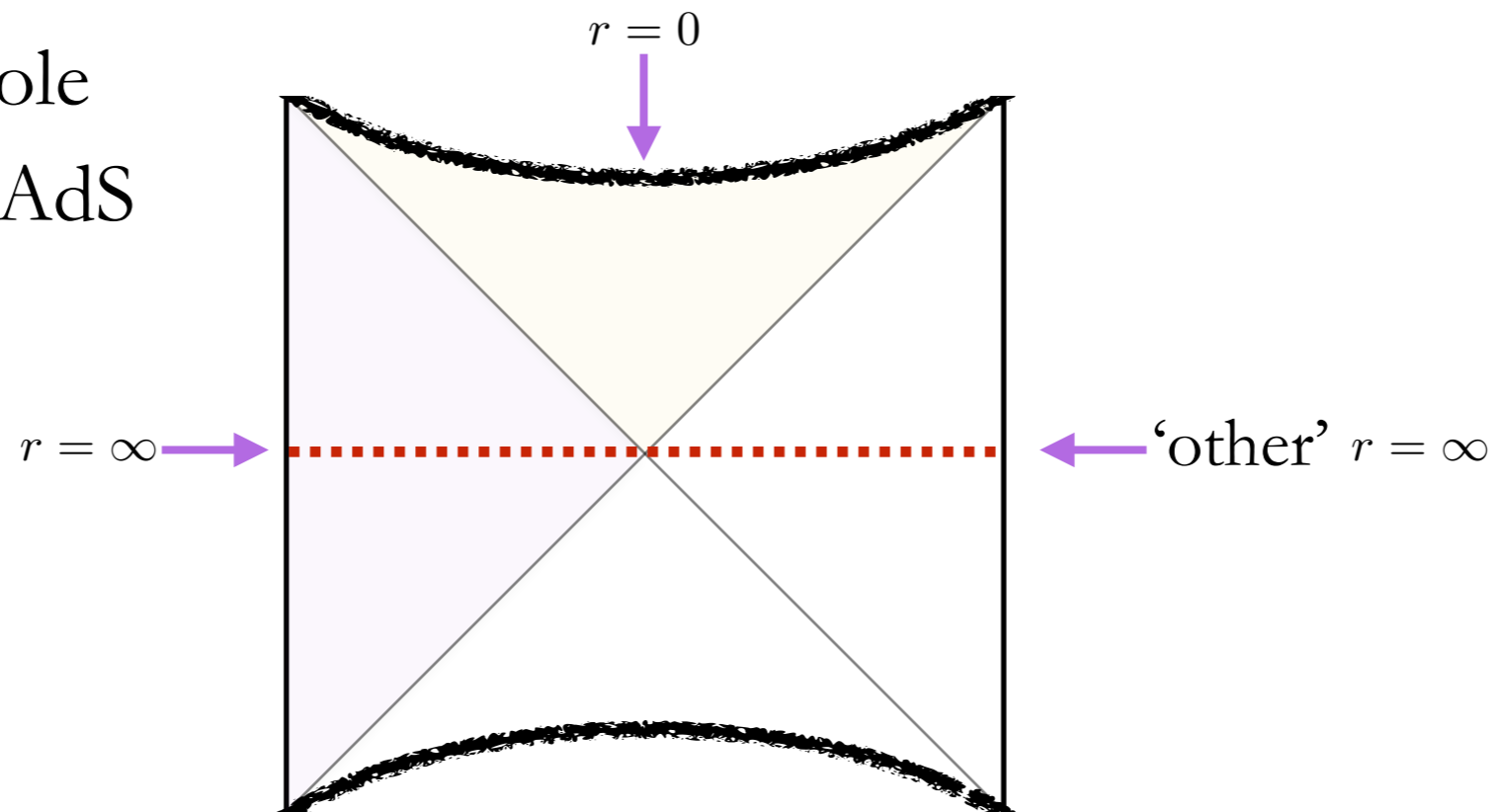
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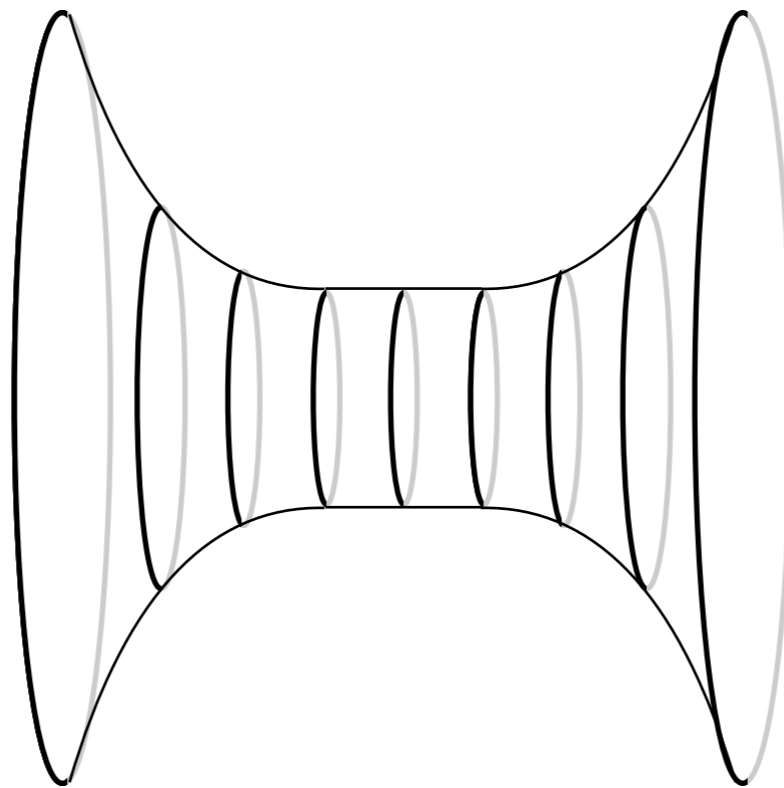
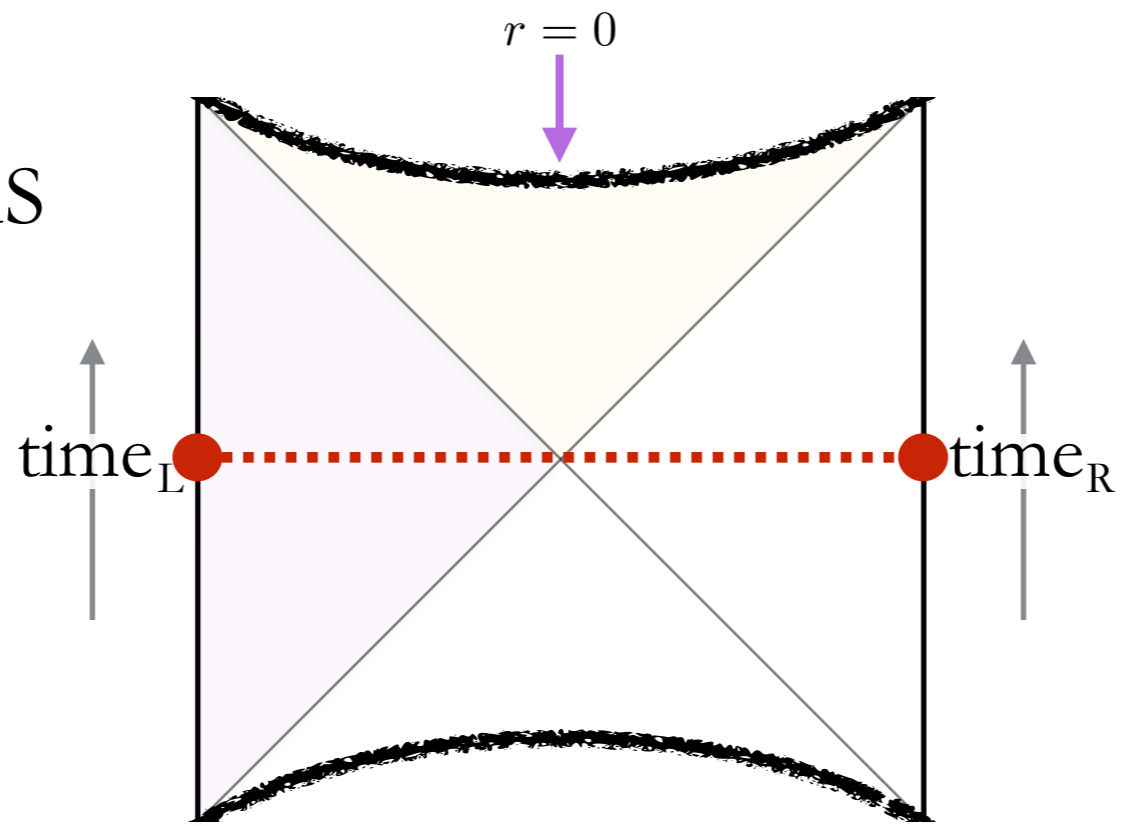


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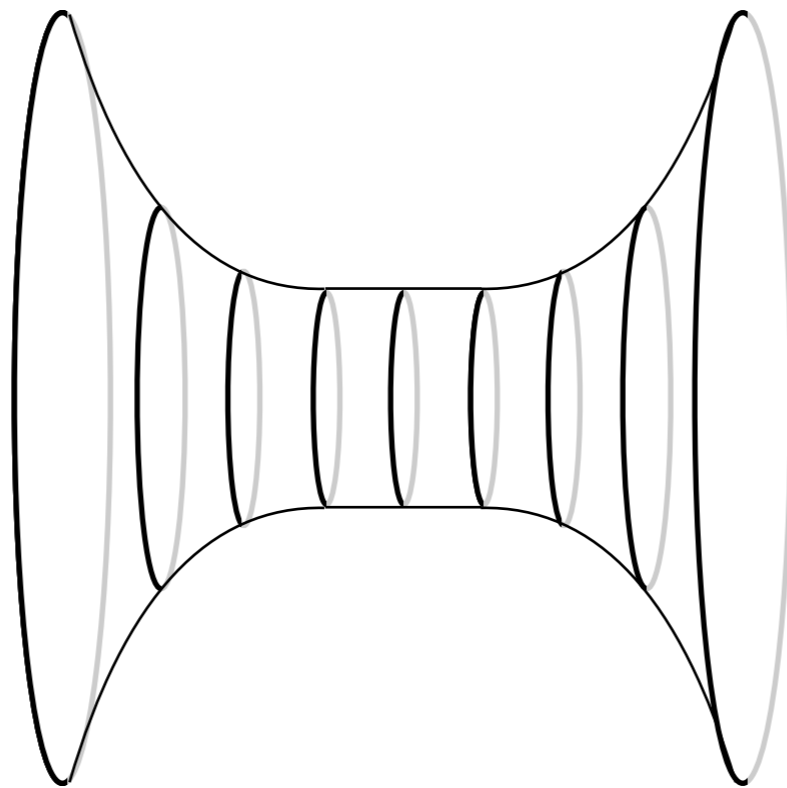
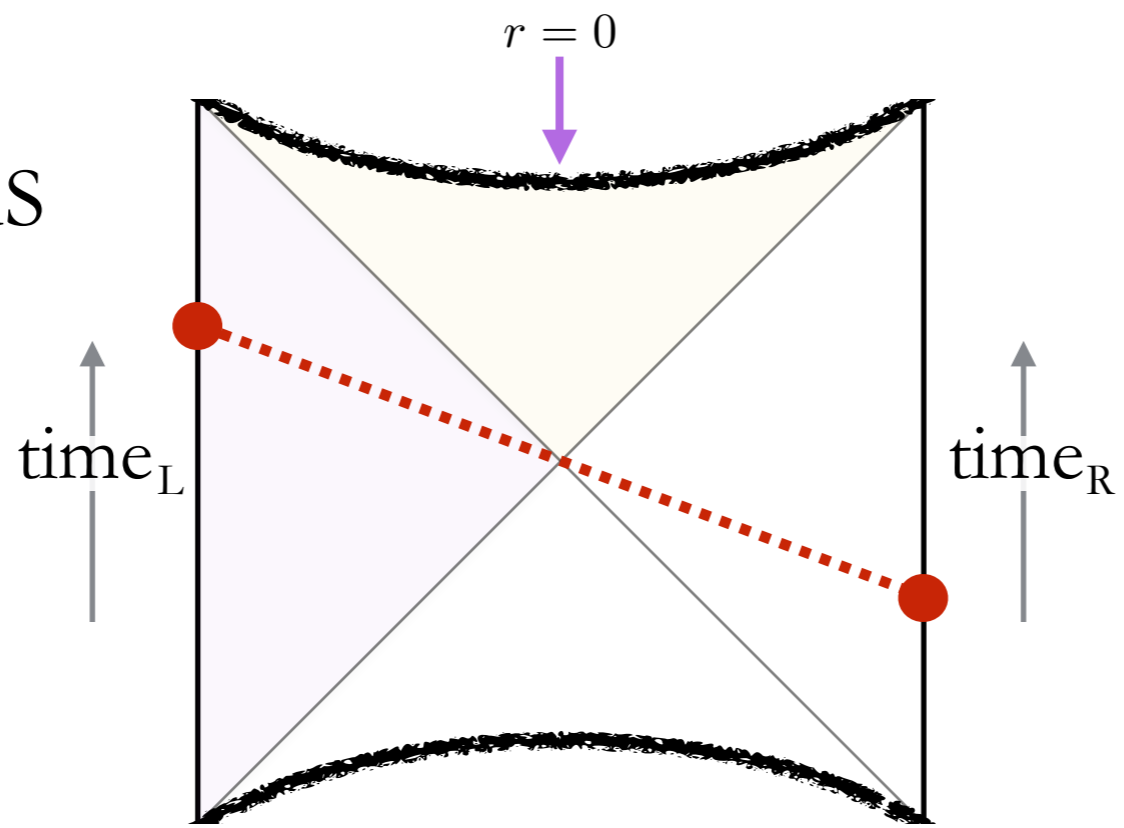
r_h

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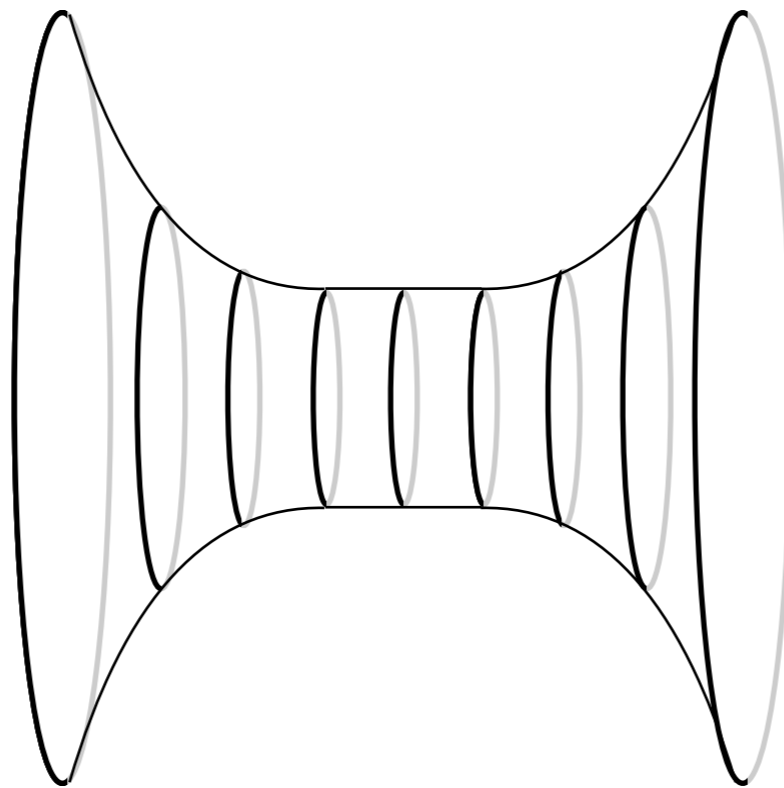
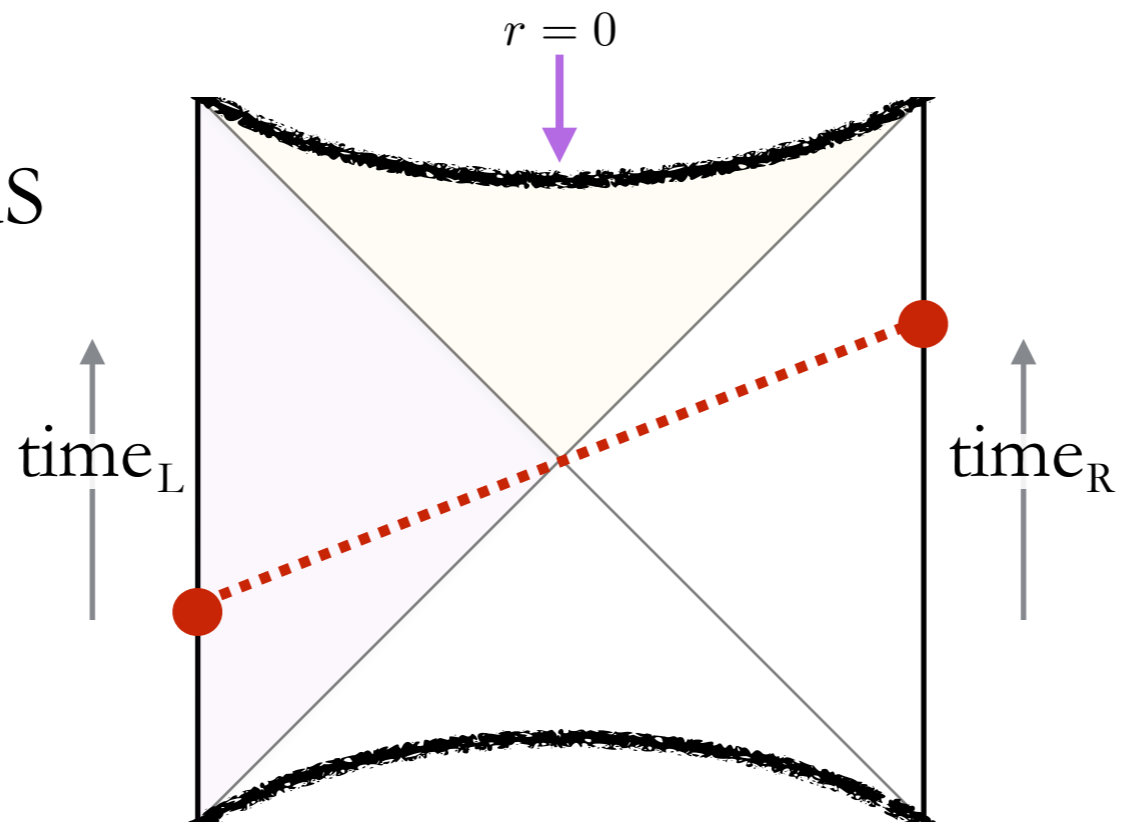
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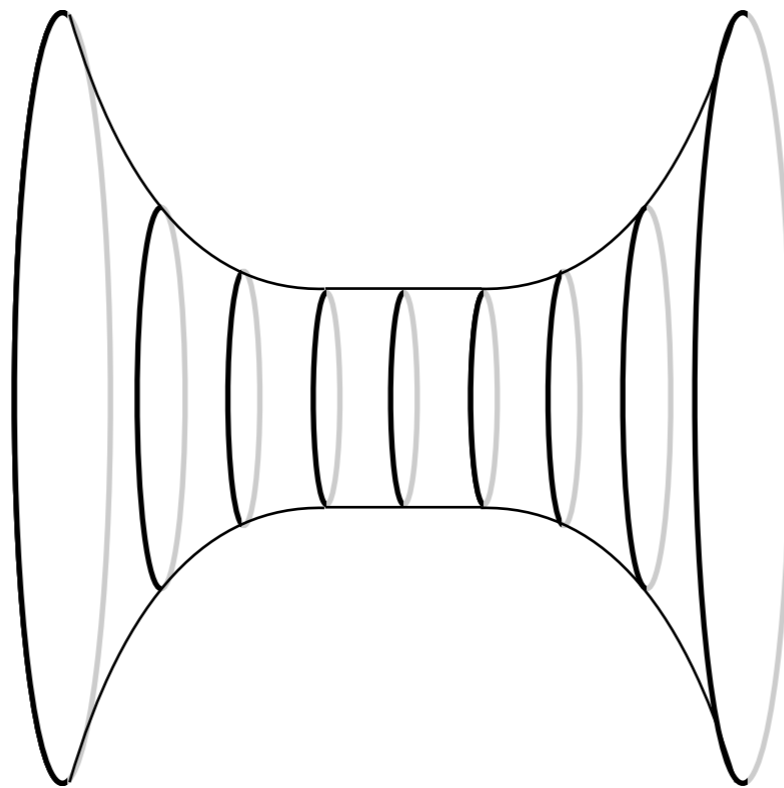
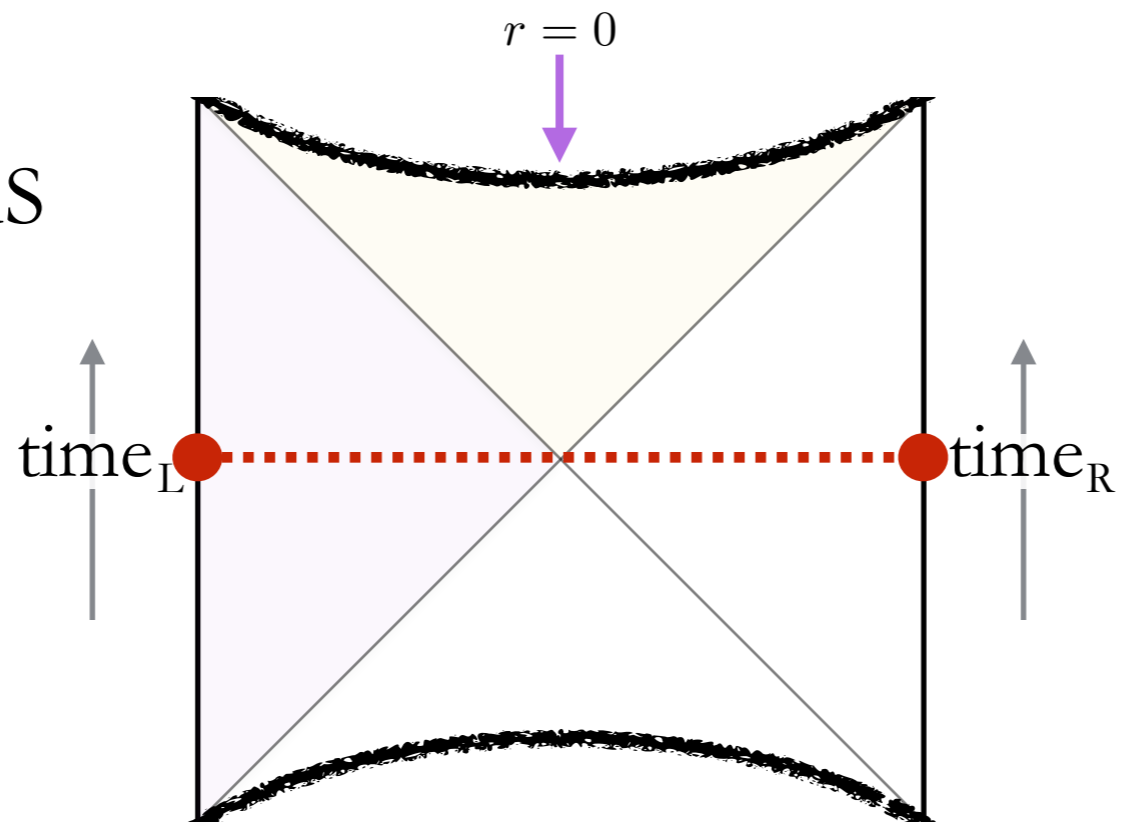
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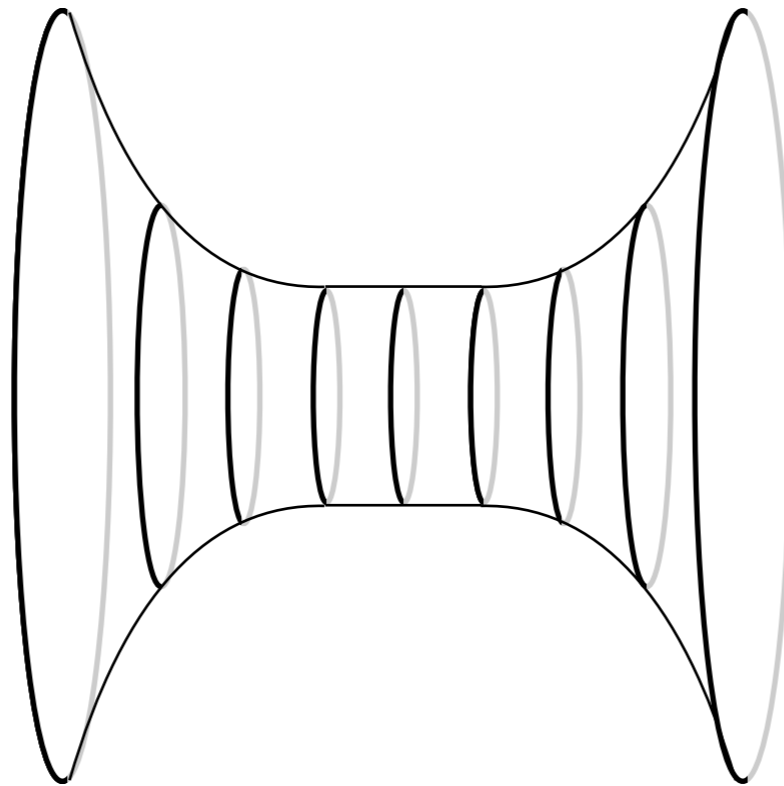
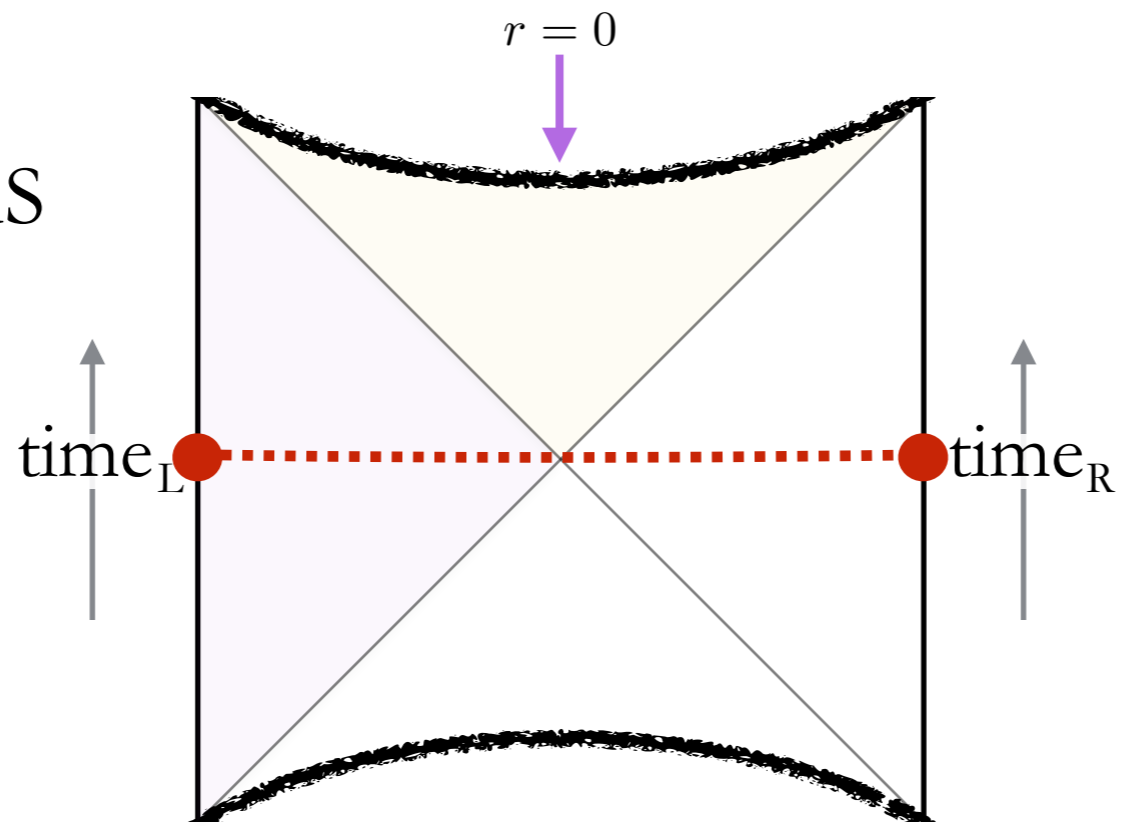
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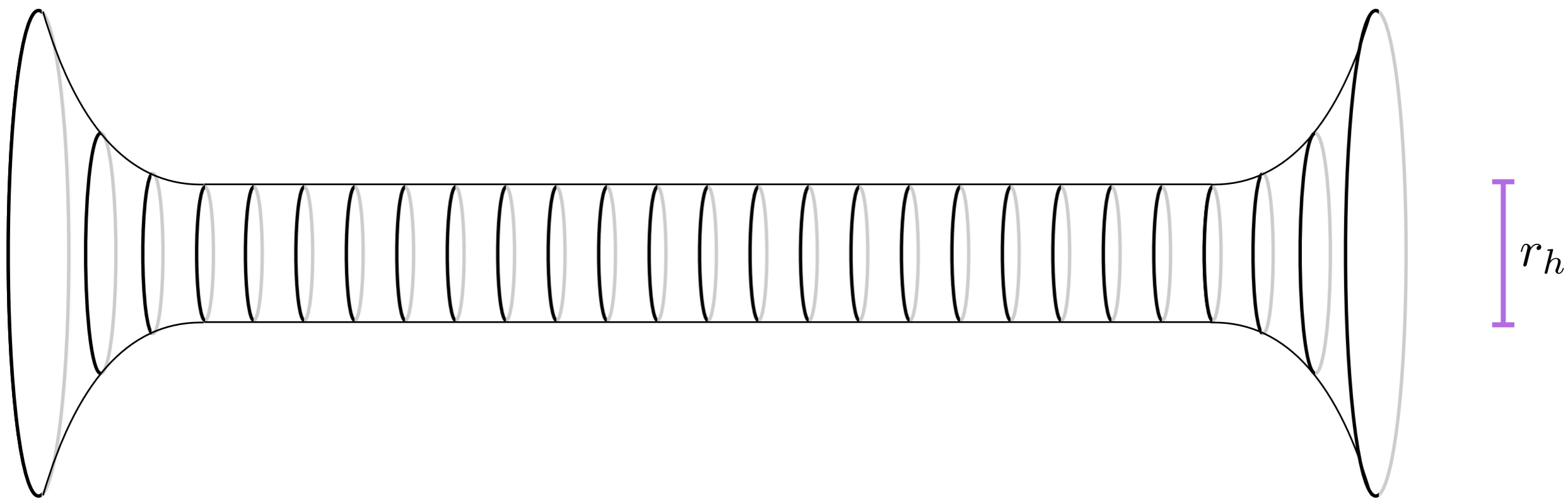
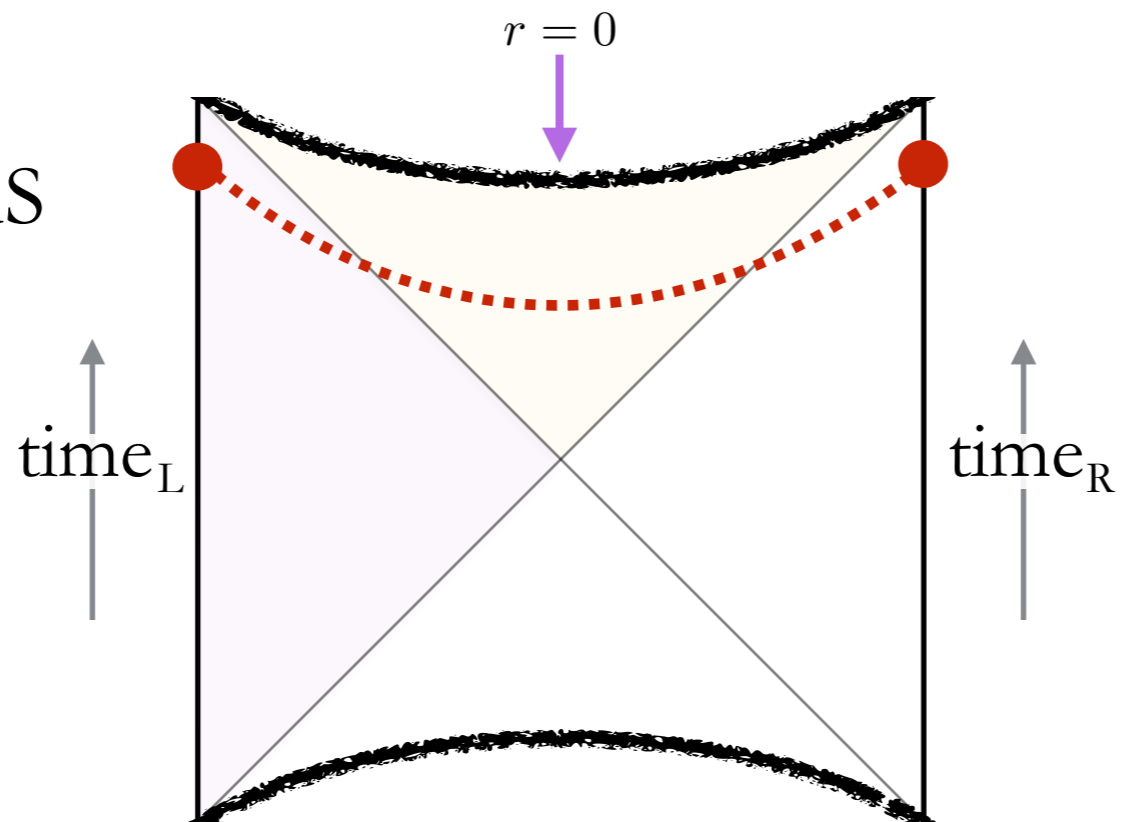
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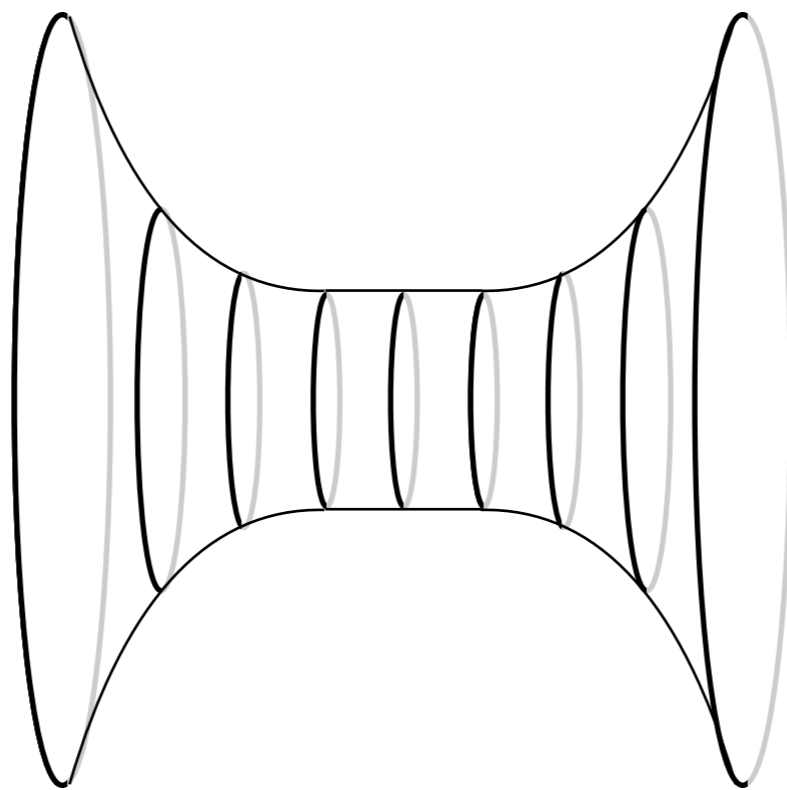
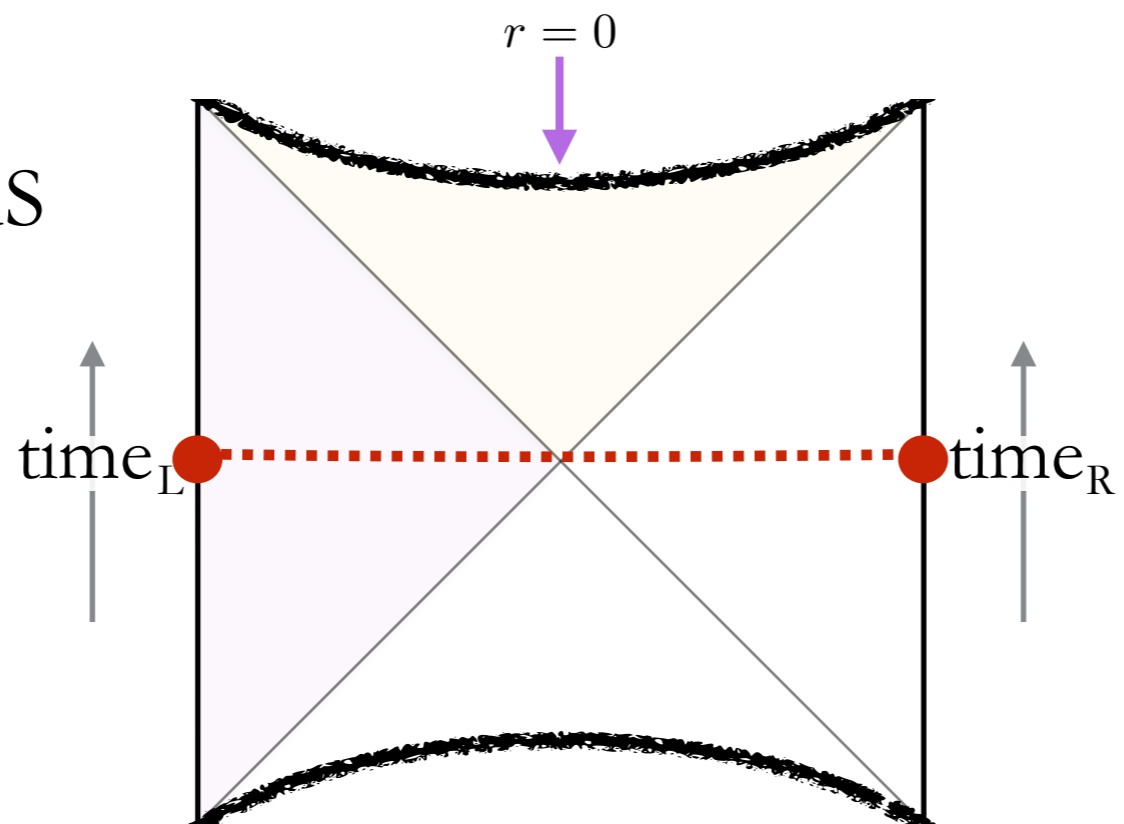
r_h

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wormhole length $\sim t_L + t_R$

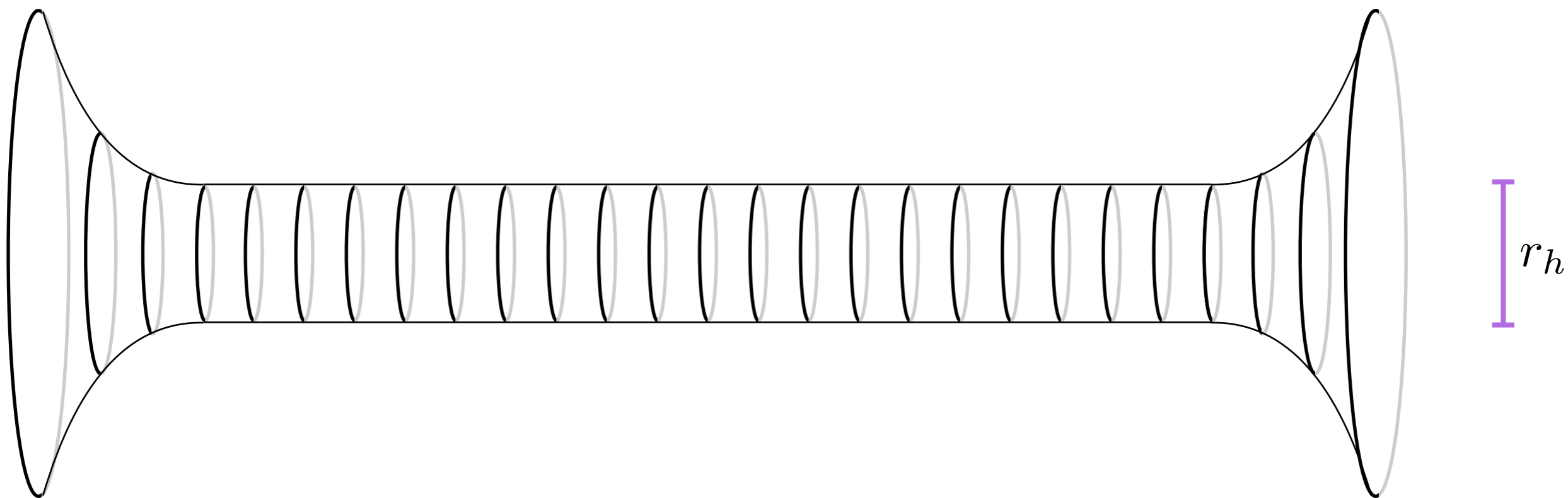
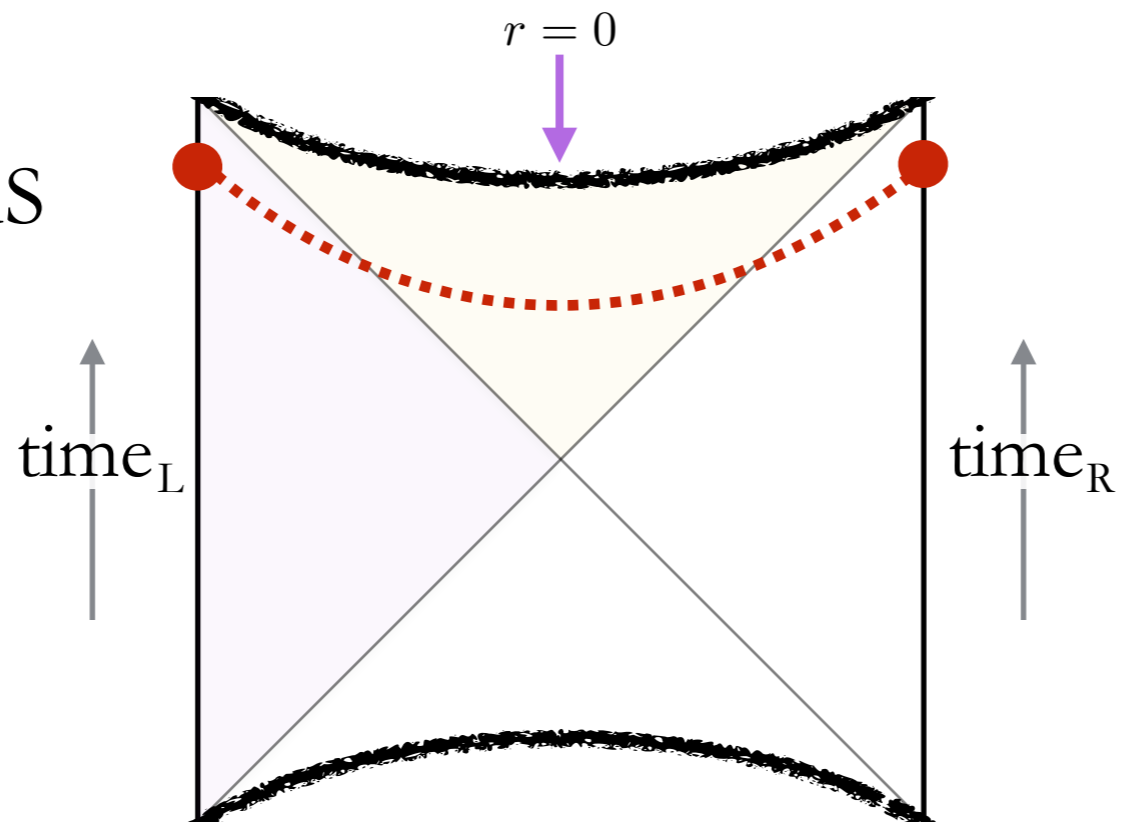
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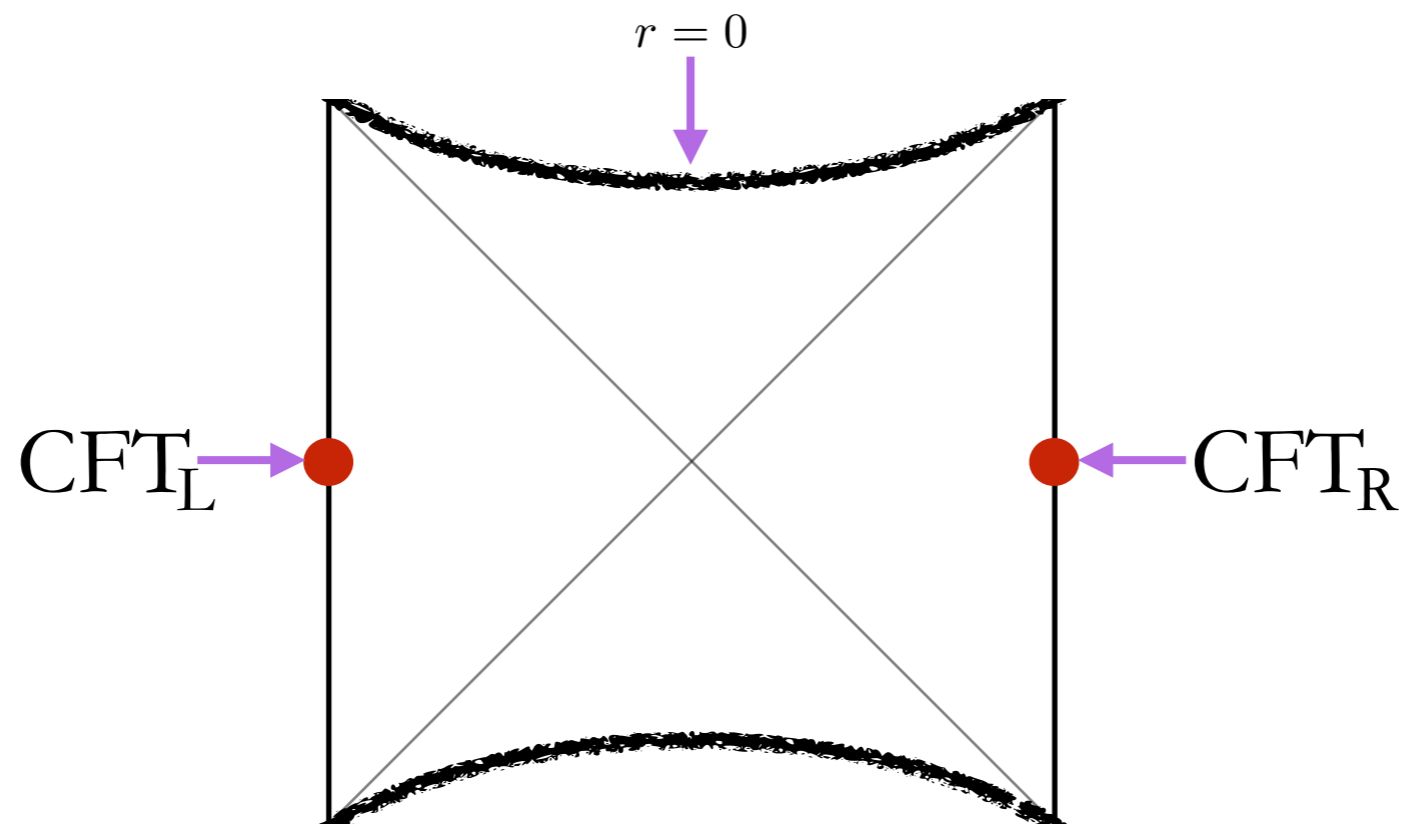
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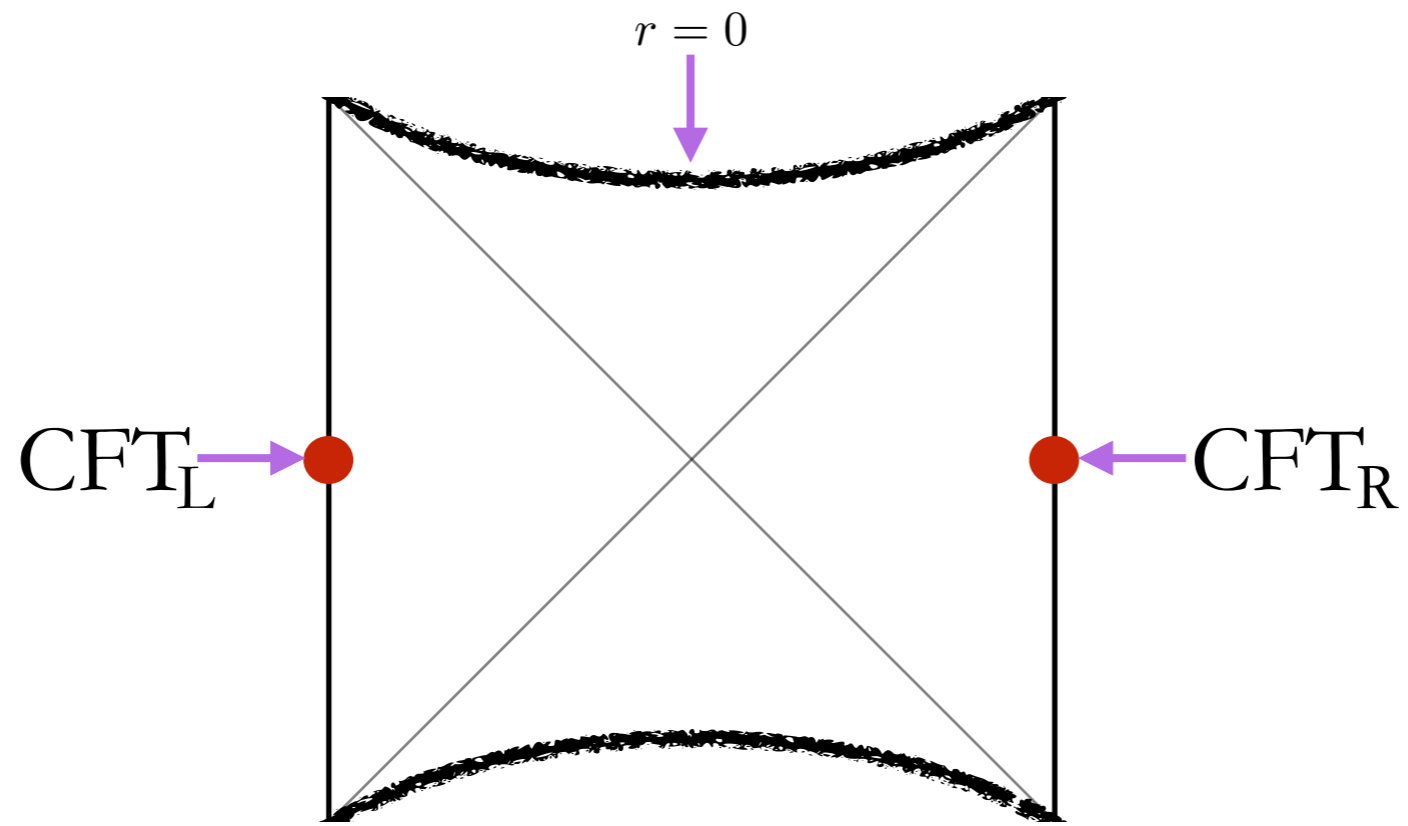
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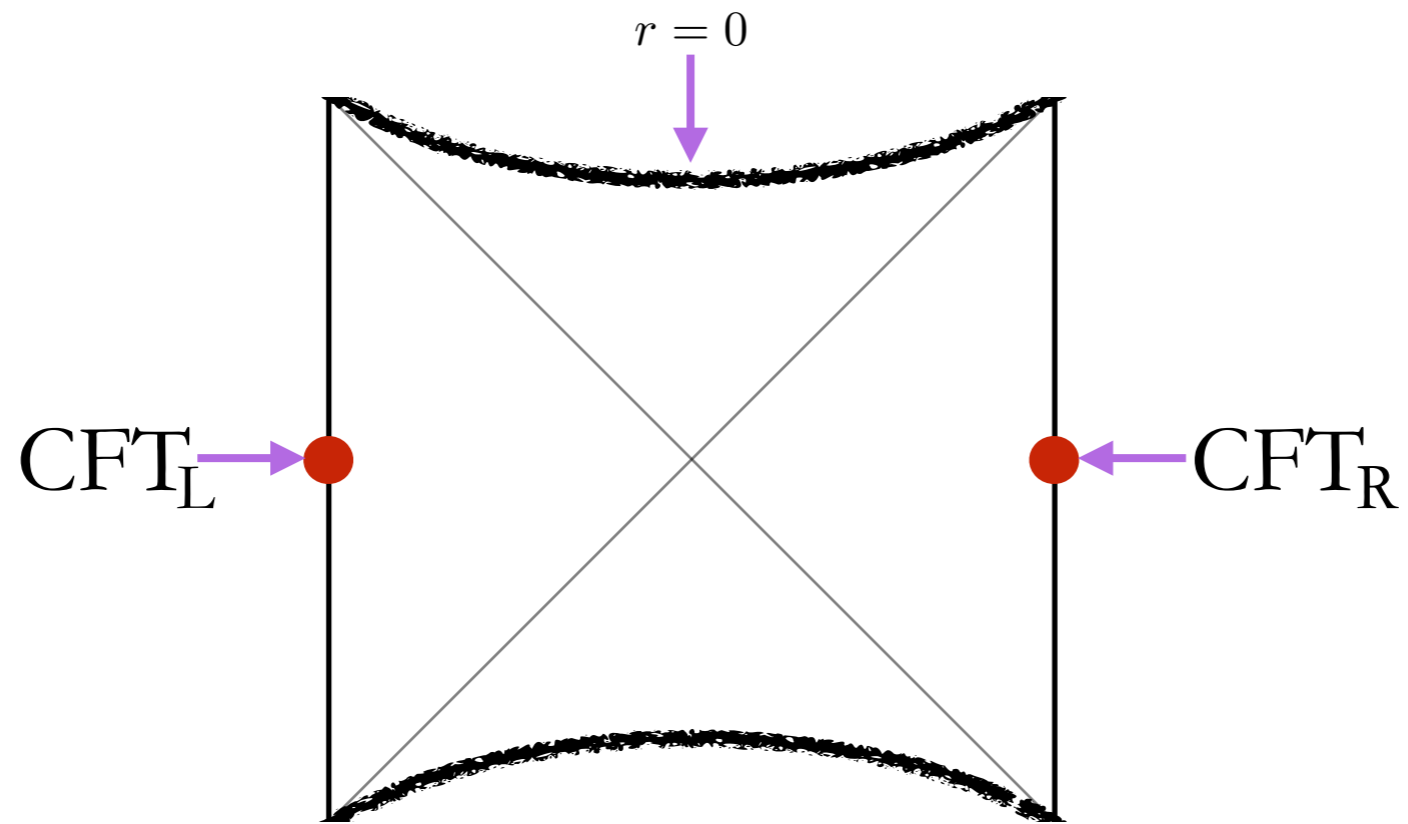


wormhole length $\sim t_L + t_R$



$$|\text{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$$

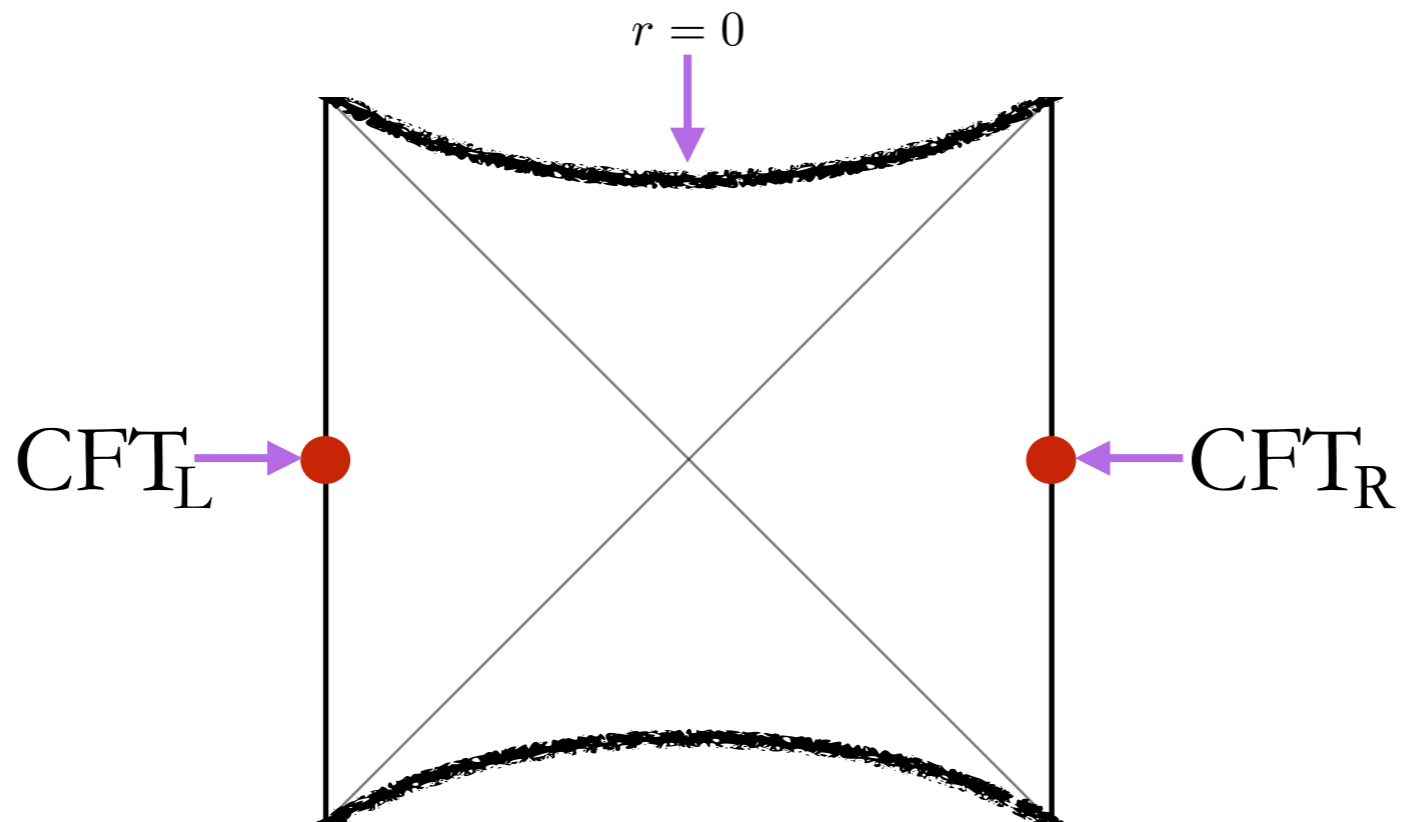
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$$|\psi(t_L, t_R)\rangle = \sum_i e^{-\beta E_i/2 + iE_i(t_L + t_R)} |E_i\rangle_L |E_i\rangle_R$$

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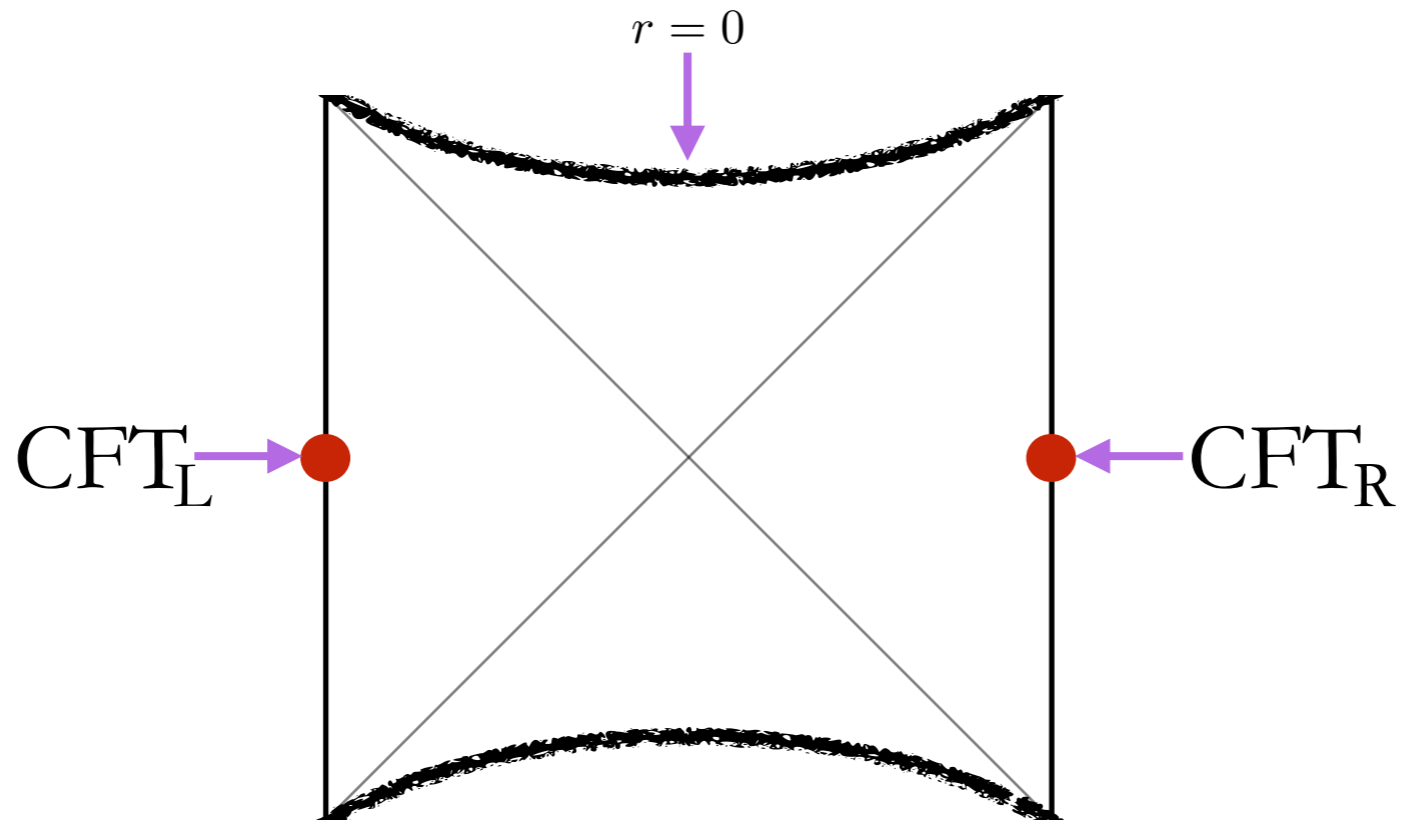


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What is CFT dual to linear growth of wormhole?

wormhole length $\sim t_L + t_R$



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COMPLEXITY?

wormhole length $\sim t_L + t_R$

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computational gate complexity of a quantum state

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DEFINITION?

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DEFINITION? starting in a reference state

$$\text{e.g. } |\text{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$$

how many fundamental gates

e.g. unitaries each of which
act only on two-qubits

are needed to make the state

e.g. to within an accuracy ϵ

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e.g. to within an accuracy ϵ

- can be exponentially large (Hilbert Space is huge)
- expected to grow linearly (at early times)

complexity \sim size of wormhole?

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EVIDENCE:

complexity \sim size of wormhole?

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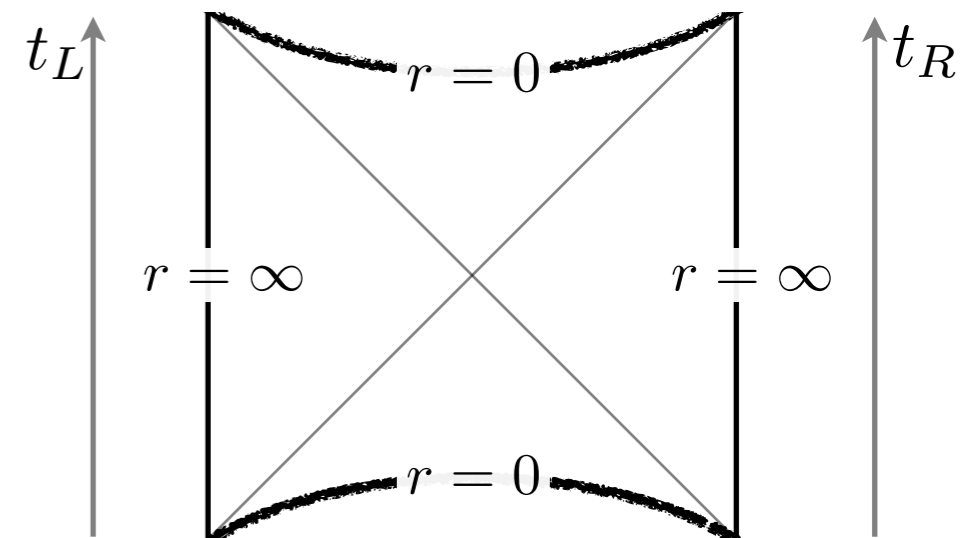
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 - entomb black hole in inert shell

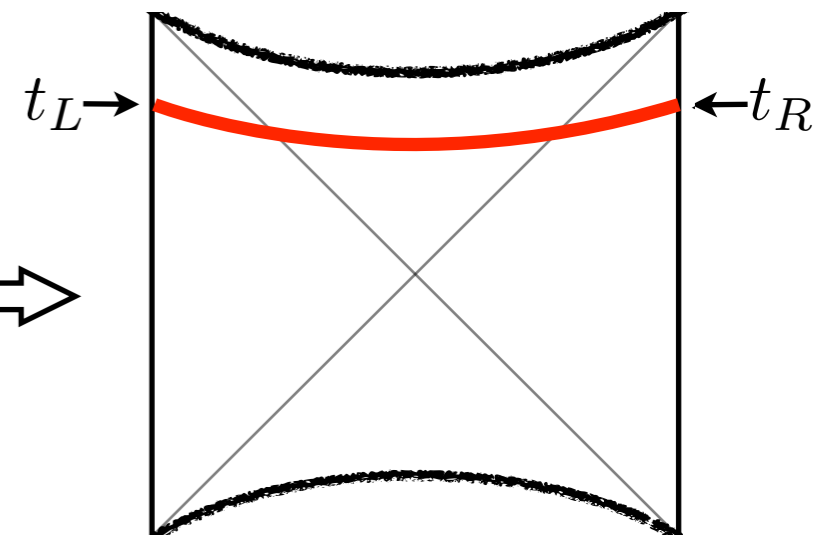
how to characterize size of wormhole?

$$\frac{1}{G\ell_{\text{AdS}}} \text{volume}$$

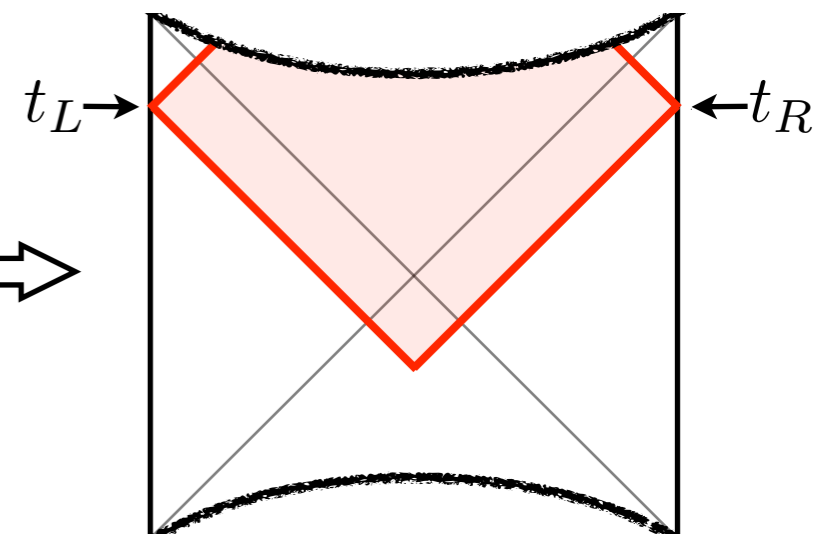
$$\frac{1}{\hbar} \text{action}$$



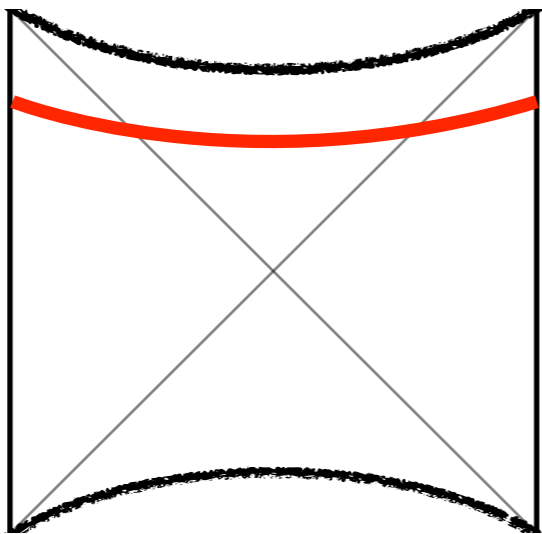
(old)
maximal slice \Rightarrow



(new/improved)
WdW patch \Rightarrow

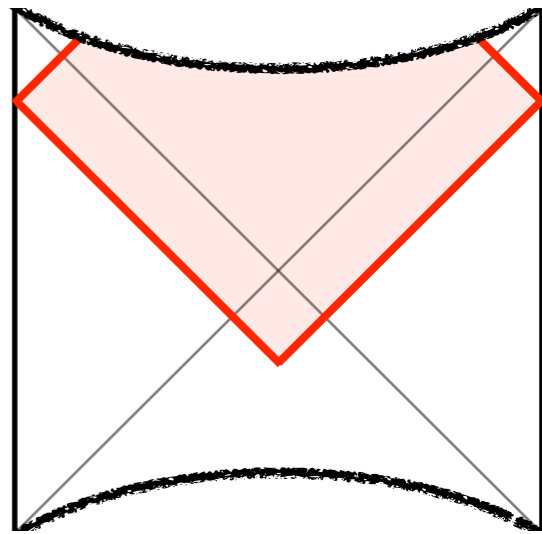


$\frac{1}{G l_{\text{AdS}}}$ **volume**

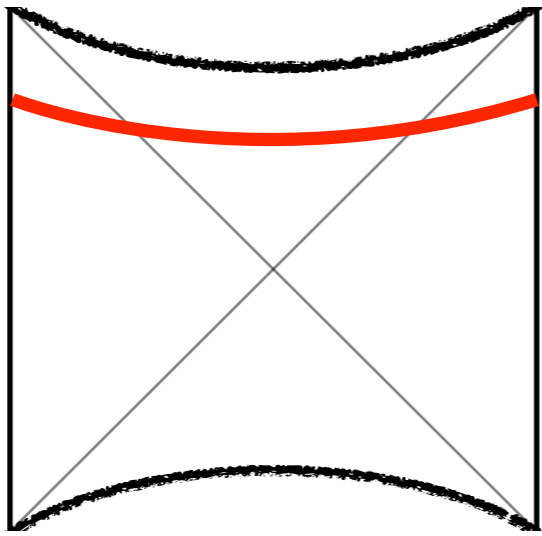


vs.

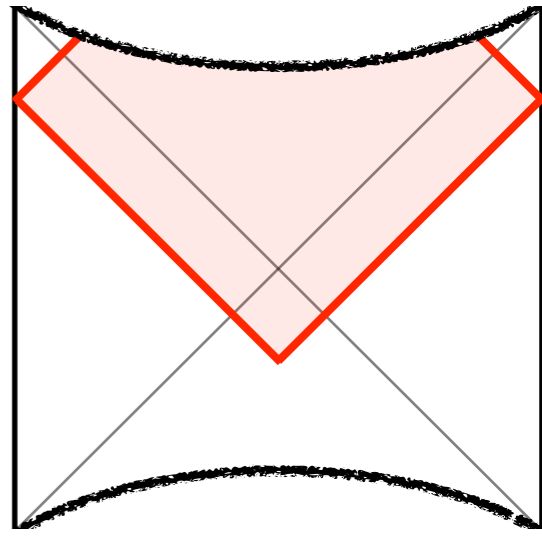
$\frac{1}{\hbar}$ **action**



$\frac{1}{G\ell_{\text{AdS}}}$ **volume**



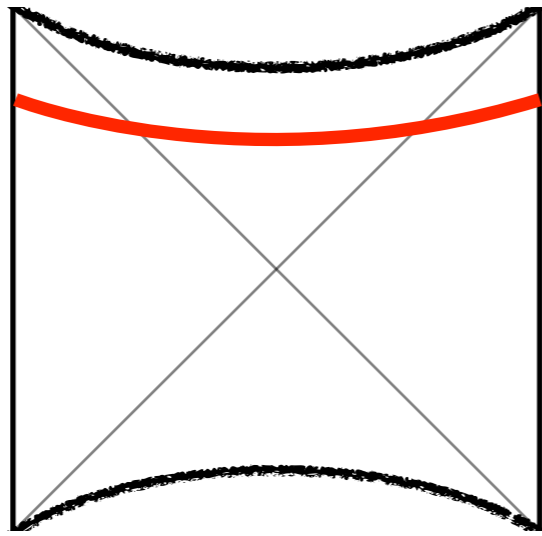
vs. $\frac{1}{\hbar}$ **action**



prefer action because:

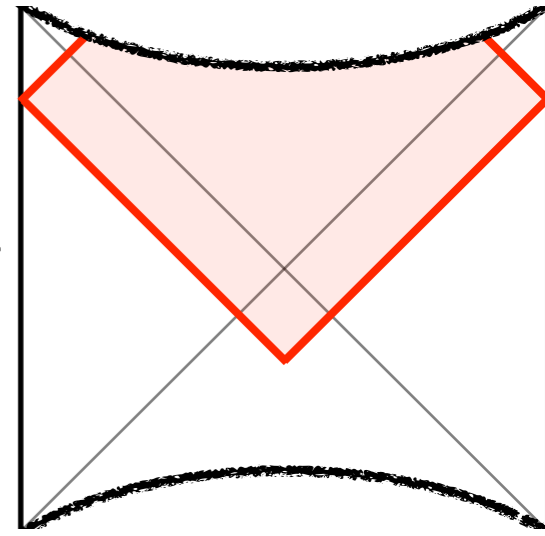
- naturally “dimensionless” (no arbitrary length scale)

$\frac{1}{G\ell_{\text{AdS}}}$ **volume**



vs.

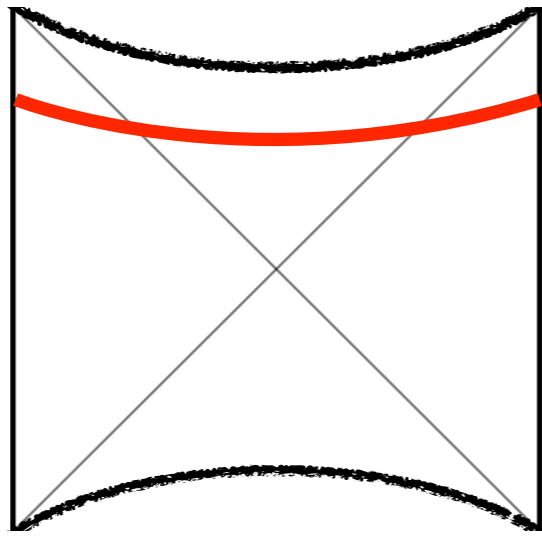
$\frac{1}{\hbar}$ **action**



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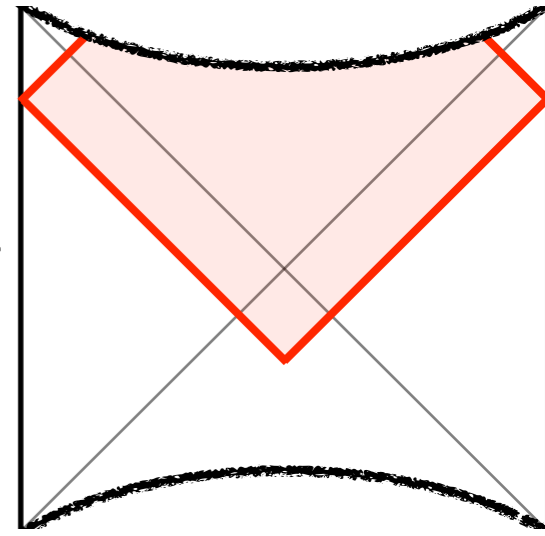
- naturally “dimensionless” (no arbitrary length scale)
- $d(\text{action})/dt = 2M$ (independent of size and dimension)

$\frac{1}{G\ell_{\text{AdS}}}$ **volume**



vs.

$\frac{1}{\hbar}$ **action**

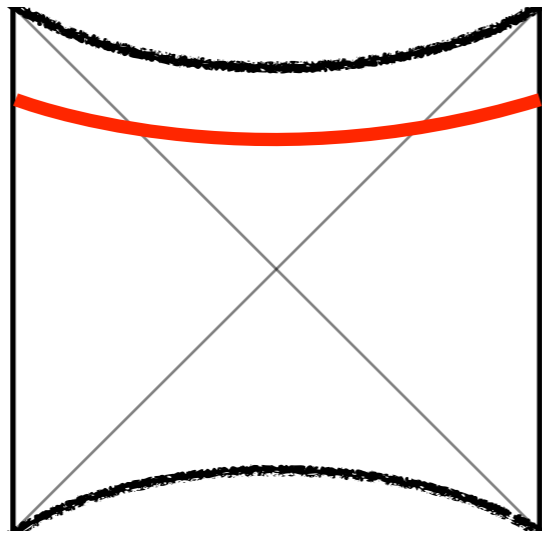


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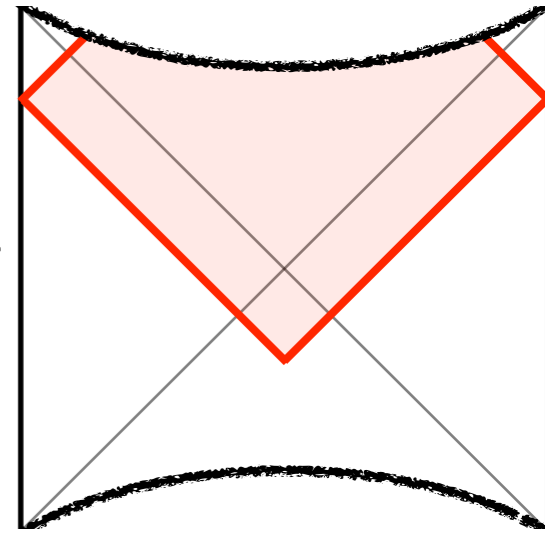
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connect to $\frac{d\text{Complexity}}{dt} \leq \frac{2M}{\pi\hbar}$

$\frac{1}{G\ell_{\text{AdS}}}$ **volume**



vs. $\frac{1}{\hbar}$ **action**



prefer action because:

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connect to $\frac{d\text{Complexity}}{dt} \leq \frac{2M}{\pi\hbar}$

black holes saturate limit implies

$$\text{Complexity} = \frac{\text{Action}}{\pi\hbar}$$

Complexity Equals Action ?

FURTHER WORK:

- precise definition of complexity?
- precise definition of action?
- relate imprecision in two definitions?
- reference state? (“complexity of formation”)
- classical proof that black holes maximize action?
- more general black holes?
- higher-derivative theories and singularities?
- principle of least computation?
- complexity and horizon transparency?
- lots of puzzles!