Quantum entanglement and local excitations

Paweł Caputa

NORDITA

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Based on

“work in progress” with Marek M. Rams (Jagiellonian U.) …’16

1602.06542 [hep-th] with Tokiro Numasawa (YITP) and Alvaro Veliz-Osorio (WITS) ’16

previous works:

[PC, A. Veliz-Osorio’15]
[PC, J.Simon, A.Stikonas, T.Takayanagi, K.Watanabe ’15]
[PC, J.Simon, A.Stikonas, T.Takayanagi ’14]
[PC, M. Nozaki, T. Takayanagi ’14]

Outline

- Introduction (already in Nozaki's talk)
- Evolution of entanglement after local operator excitation in 2d CFTs
- RCFTs
- Numerics: Ising model
- Large c limit
- $SU(N)_k$ WZW
- Conclusions

Introduction

- Quantum Information gives a new perspective on quantum field theories
- New Tools in QFT: measures of entanglement, distance between states, complexity, OTO correlators etc.
- 2d CFT is a perfect playground for “defining”/exploring these tools
- Advantage: Symmetry (computation), numerics (critical points), RT (HRT)
- We can scan and classify 2d CFTs by the properties of these tools in various corners of parameter space. (Ideally holographic vs non-holographic CFTs).

Universality in CFT 1+1

[Holzhey,Larsen,Wilczek'94]

[Calabrese, Cardy'04]

[Calabrese, Cardy'16]

Ground state

$$S_A = \frac{c}{3}\log\frac{|A|}{a} + \tilde{c}_n$$

Quenches

$$S_A \sim \frac{c}{3}\log\frac{t}{\epsilon} \qquad S_A \sim c\,t \qquad S_A \sim c \times const$$

Useful to extract the central charge (numerics)!

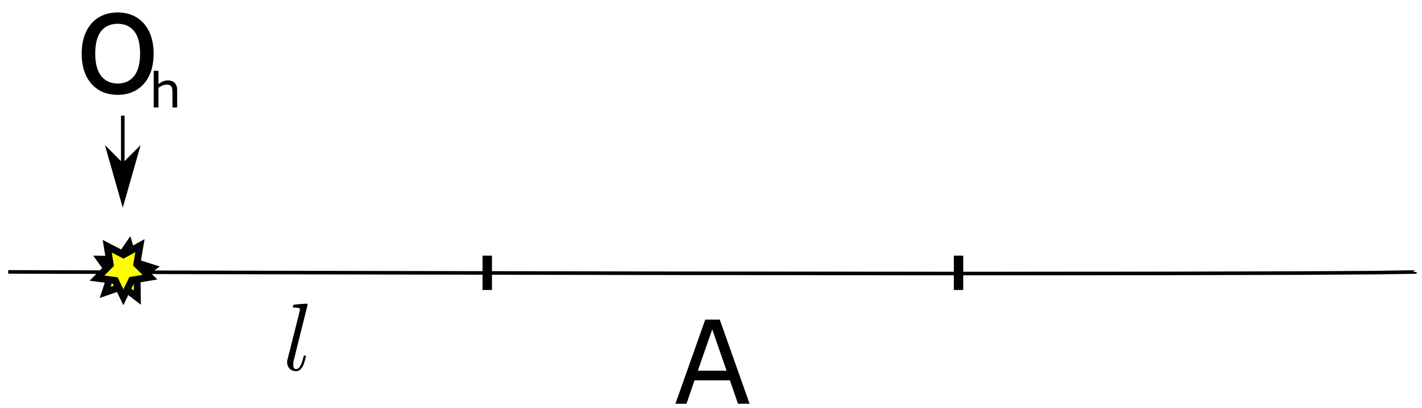
RT (HRT) results confirm the large c behaviour!

Can we be less universal but still under control?

Local Operator Excitations!

CFT in 1+1 d

[Nozaki,Numasawa,Takayanagi'13]



"Local quench" setup

$$\rho(t) = e^{-iHt} e^{-\epsilon H} O(-l)|0\rangle\langle 0|O^{\dagger}(-l) e^{-\epsilon H} e^{iHt}$$

How does this change (Renyi) entanglement entropies of A?

$$\rho_A(t) = Tr_{A^c}\left[\rho(t)\right] \qquad \Delta S_A^{(n)}(t) \quad ?$$

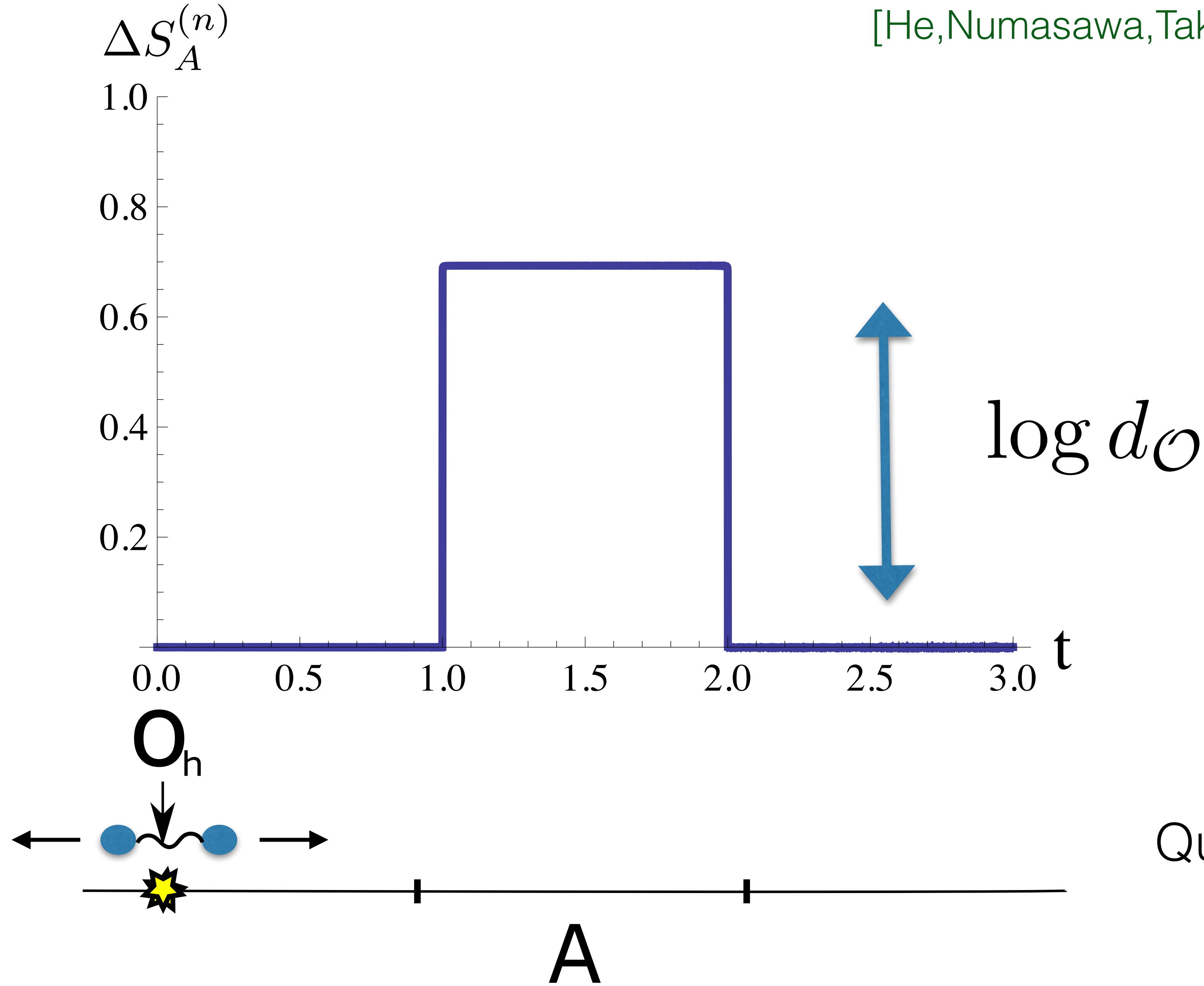
$$\Delta S_A^{(n)} = \frac{1}{1-n} \log \frac{\langle \mathcal{O}(w_1,\bar{w}_1)\mathcal{O}^\dagger(w_2,\bar{w}_2)...\mathcal{O}(w_{2n-1},\bar{w}_{2n-1})\mathcal{O}^\dagger(w_{2n},\bar{w}_{2n})\rangle_{\Sigma_n}}{\left(\langle \mathcal{O}(w_1,\bar{w}_1)\mathcal{O}^\dagger(w_2,\bar{w}_2)\rangle_{\Sigma_1}\right)^n}$$

- Q1: How much data about a CFT can we extract numerically ?
- Q2: Large c vs "holographic" behaviour?

This talk: Some modest progress in these directions

RCFTs

[He,Numasawa,Takayanagi,Watanabe'13]



Quasi-particle

for any member of a conformal family!

[P.C, Veliz-Osorio'15],[Chen,Guo,He,Wu'15]

2+1 d:

[Fradkin,Dong,Leigh,Nowling'08]

Can we see this numerically at the critical point?

Ising Model

$$H = \sum_i \left[-J\sigma_i^x \sigma_{i+1}^x + h\sigma_i^z \right]$$

critical for

$$g = h/J = \pm 1$$

CFT operators (families) $\mathbf{1}, \varepsilon, \sigma$

$$\Delta S_A^{(n)} = \log d_{\mathcal{O}} \qquad d_{\mathbf{1}} = d_\varepsilon = 1, \qquad d_\sigma = \sqrt{2}$$

lattice operators

$$\sigma(i) \sim \sigma_i^x \qquad \varepsilon(i) \sim \sigma_i^x \sigma_{i+1}^x - \sigma_i^z$$

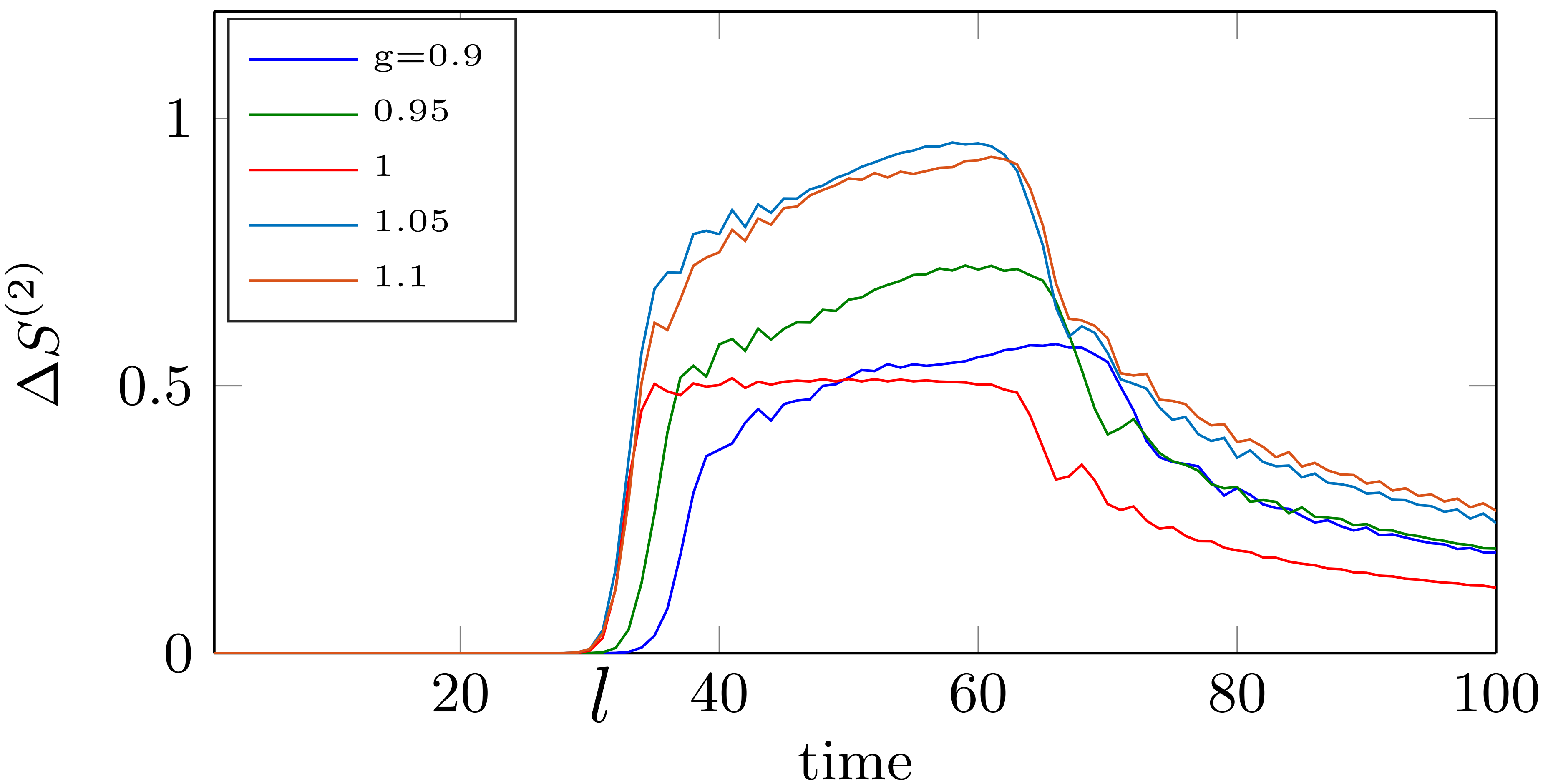
Jordan-Wigner map to free fermions and we can compute the reduced density matrix for our locally excited states (+ checks with MPS)

[Verstraete….Vidal…]

Second Renyi σ^x_{-l}

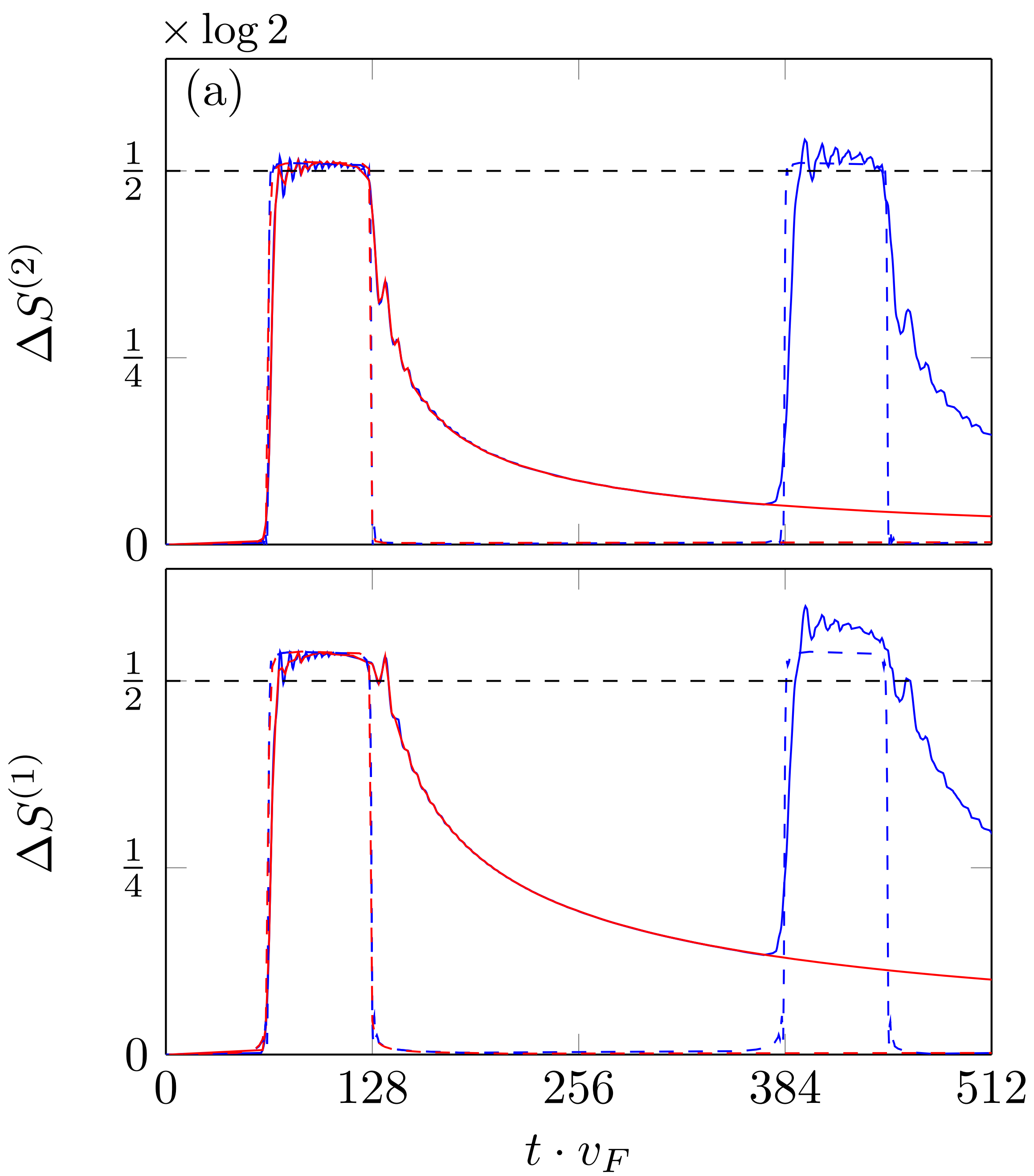
[PC,M.M. Rams'16.....]

matlab, $\log_2$



also for ε and $\partial\sigma$

CFT data:



$$H = -\sum_k \epsilon_k \left(\gamma_k^\dagger \gamma_k - \frac{1}{2} \right)$$

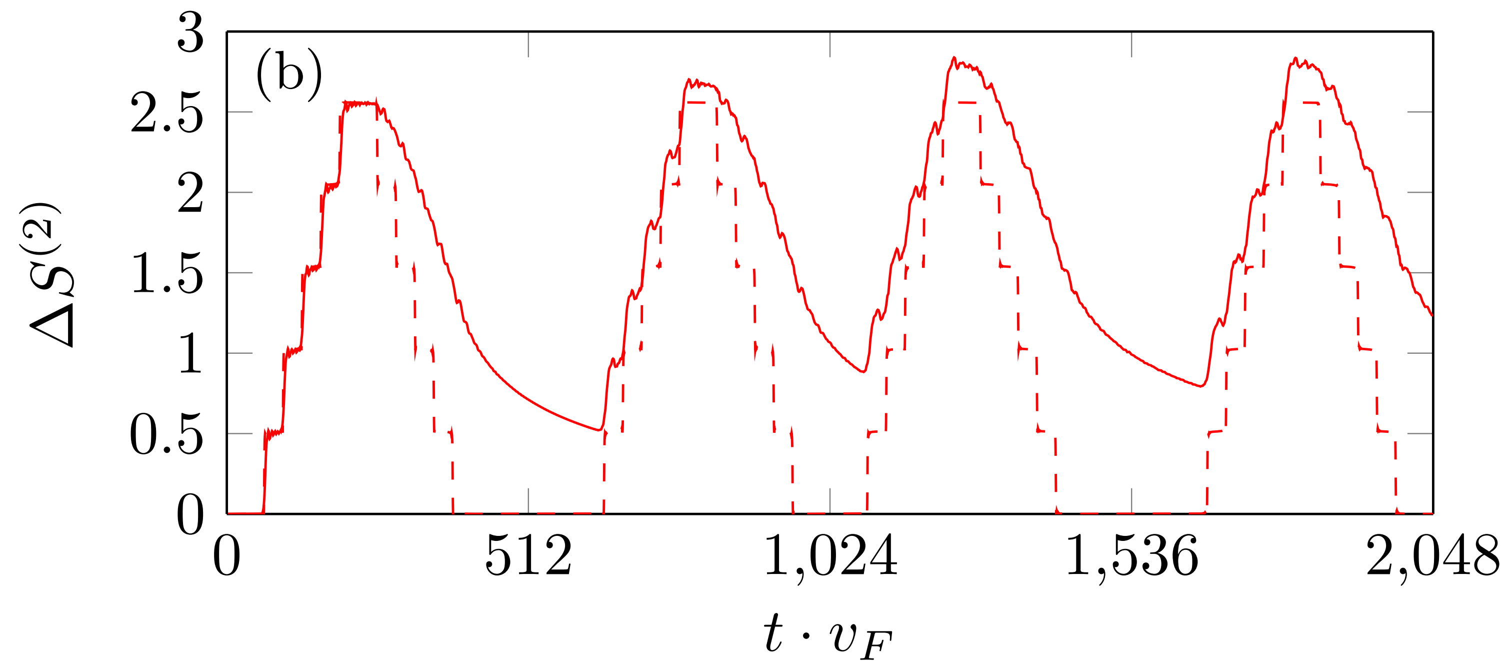
$$H^{lin} = -\sum_k \epsilon_k^{lin} \left(\gamma_k^\dagger \gamma_k - \frac{1}{2} \right)$$

$$\epsilon_k^{lin} = v_F k$$

General excitations (quasi-particle phenomenology)

$$|\psi\rangle = \mathcal{O}_i \ldots \mathcal{O}_k |0\rangle$$

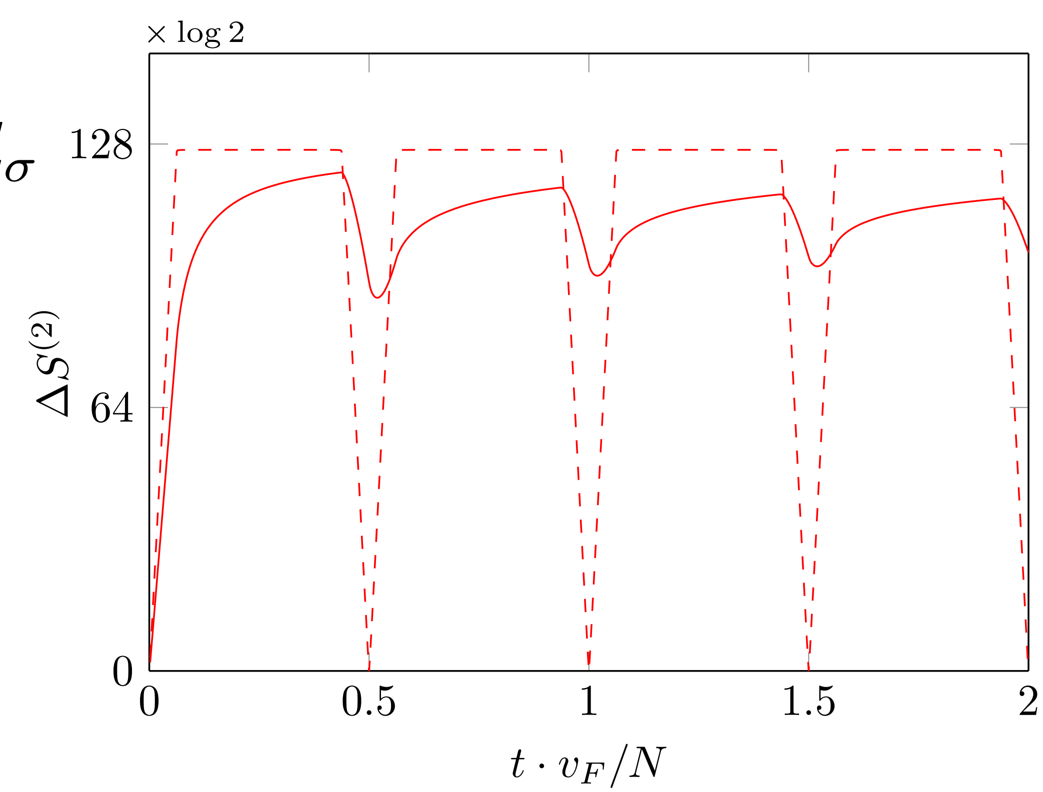
$$\Delta S_A^{(n)} = \sum_{i=1}^{k} \log d_{\mathcal{O}}$$



[CFT 2 operators: Numasawa 'to appear]

$$|\psi_G\rangle = \prod_{i=1}^{N} \sigma_i^x \, |0\rangle$$

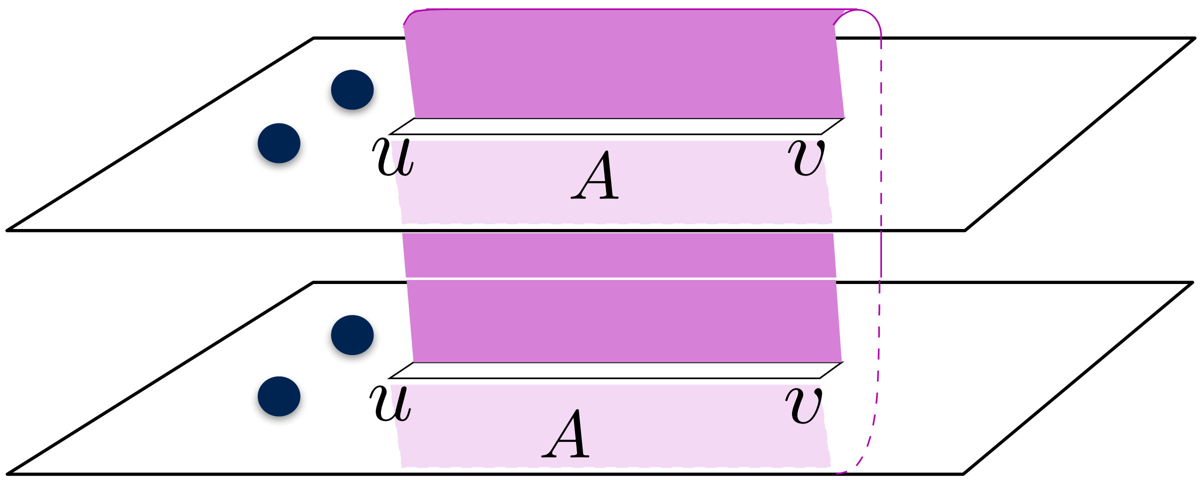
$2L \log d_\sigma$



large c: [Anous, Hartman, Rovai, Sonner '16]

Second Renyi entropy (Purity)

(n=2 and finite interval A,primary)



$$z(w) = \sqrt{\frac{w-u}{w-v}}$$

$$\boxed{\Delta S_A^{(2)}(z,\bar{z}) = -\log\left[|z(1-z)|^{4h}\mathcal{G}(z,\bar{z})\right]}$$

$$z = \frac{z_{12}z_{34}}{z_{13}z_{24}}$$

$$\mathcal{G}(z,\bar{z}) \equiv \langle \mathcal{O}(\infty)\mathcal{O}(1)\mathcal{O}(z,\bar{z})\mathcal{O}(0)\rangle$$

constant contribution:

$$\epsilon \to 0$$

$t \notin A \qquad (z,\bar{z}) \to (0,0)$

$t \in A \qquad (z,\bar{z}) \to (1,0)$

Rational CFT (diagonal)

$$\mathcal{H}_{tot} = \oplus_a \left(\mathcal{H}_a \otimes \bar{\mathcal{H}}_a\right)$$

In rational CFTs

$$\mathcal{G}(z,\bar{z}) = \sum_a \mathcal{F}(a|z)\bar{\mathcal{F}}(a|\bar{z})$$

finite

in order to extract the constant when $z \to 1 \quad \bar{z} \to 0$

$$\mathcal{F}(a|1-z) = \sum_b F_{ab}[\mathcal{O}]\mathcal{F}(b|z) \;\; \to$$

$$\boxed{\mathcal{G}(z,\bar{z}) \simeq F_{00}[\mathcal{O}](1-z)^{-2h}\bar{z}^{-2h}}$$

Finally

$$\boxed{\Delta S_A^{(2)} = -\log F_{00}[O] = \log d_O}$$

quantum dimension

$$d_a = \frac{S_{0a}}{S_{00}} = \frac{dim\left(\mathcal{H}_a\right)}{dim\left(\mathcal{H}_0\right)}$$

Information about the modular S-matrix of a CFT!

Large c

[PC,Nozaki,Takayanagi'14]

General arguments:

- conformal blocks exponentiate (factorisation) [Zamolodchikov]

$$\Delta S_A^{(2)} \simeq 4\Delta_O \cdot \log \frac{2t}{\epsilon}$$

- HRT in back-reacted geometry from a massive particle

$\Updownarrow$

- Heavy operator at large c CFT [Asplund,Bernamonti,Galli,Hartman'14]

$$\Delta S_A^{(1)} \sim \frac{c}{6} \log \frac{t}{\epsilon}$$

- Quasi-particle picture breaks-down [Asplund,Bernamonti,Galli,Hartman'15]

Origin of log(t) ?

SU(N)$_k$ WZW

[P.C,Numasawa,Veliz-Osorio'16]

State excited by the operator in the fundamental rep. $\square$

$$g^{\alpha}_{\beta}(-l)\,|0\rangle \qquad\qquad h=\bar{h}=\frac{N^2-1}{2N(k+N)}$$

4-point correlator from K-Z equations

$$\mathcal{G}(z,\bar{z})=\sum_{i,j}^{2} I_i \bar{I}_j \sum_n X_{nn}\mathcal{F}_i^{(n)}(z)\mathcal{F}_j^{(n)}(\bar{z})$$

affine blocks

Now take: $\epsilon\to 0 \qquad (z,\bar{z})\to(1,0)$

$$\boxed{\Delta S_A^{(2)}=\log[N]} \qquad [x]=\frac{q^{x/2}-q^{-x/2}}{q^{1/2}-q^{-1/2}} \qquad q=e^{-\frac{2\pi i}{N+k}}$$

Excitations respect level-rank duality N <-> k!

Large c in WZW

[P.C,Numasawa,Veliz-Osorio'16]

Parameters

$$c = \frac{k(N^2-1)}{k+N} \qquad \lambda = \frac{N}{k}$$

Correlator becomes (h~1/2)

$$\mathcal{G}(z,\bar{z}) \simeq \frac{1}{|z|^2} + \frac{1}{|1-z|^2} + \sqrt{\frac{\lambda}{c}}\left(\frac{1}{z(1-\bar{z})} + \frac{1}{(1-z)\bar{z}}\right)$$

$$\Delta S_A^{(2)}(t) \simeq 2h \log\left(\frac{2t}{\epsilon}\right) \qquad \Delta S_A^{(2)} \simeq \log\sqrt{\frac{c}{\lambda}}$$

Time-scale at which the log of quantum dimension is reached

$$t \simeq l + \frac{c^{1/4}}{2\lambda^{1/4}}\epsilon$$

Large c in WZW

[P.C,Numasawa,Veliz-Osorio'16]

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$$\Delta S_A^{(2)}(t) \simeq 2h \log\left(\frac{2t}{\epsilon}\right) \qquad \Delta S_A^{(2)} \simeq \log\sqrt{\frac{c}{\lambda}}$$

Time-scale at which the log of quantum dimension is reached

$$t \simeq l + \frac{c^{1/4}}{2\lambda^{1/4}}\epsilon$$

but no chaos!

Conclusions/Future:

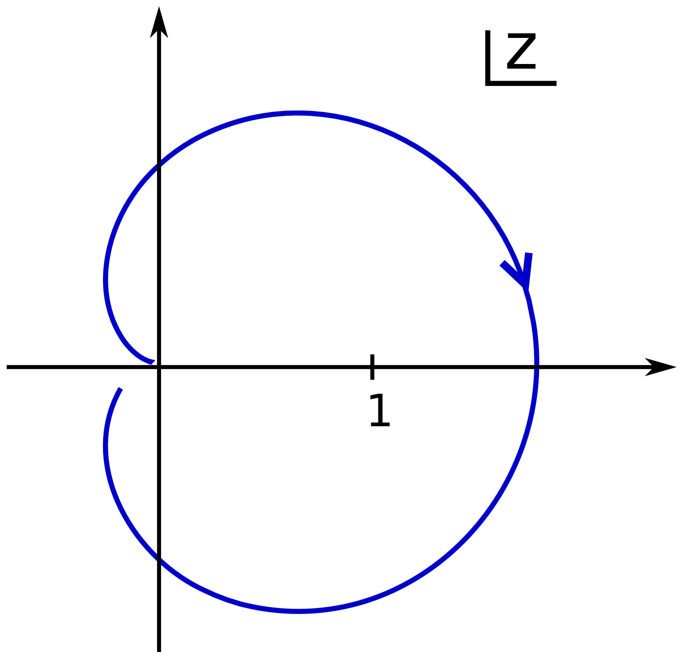
- Entanglement measures in field theory are very useful for characterising local operator excitations!
- Using QI tools we can extract more CFT data from the critical points (full S and modular T matrix?)
- How to see “holography” with QI tools (log(t)<->?)
- Numerics for other Hamiltonians (Potts?)
- General excitations at large c ?
- Measures of chaos and relative entropy ?

どうもありがとう!

OTO in RCFT

[P.C,Numasawa,Veliz-Osorio'16]
[Gu,Xi'16]

$$C_{ij}^{\beta}(t) \equiv \frac{\left\langle \mathcal{O}_i^{\dagger}(t)\mathcal{O}_j^{\dagger}\mathcal{O}_i(t)\mathcal{O}_j \right\rangle_{\beta}}{\left\langle \mathcal{O}_i^{\dagger}\mathcal{O}_i \right\rangle_{\beta} \left\langle \mathcal{O}_j^{\dagger}\mathcal{O}_j \right\rangle_{\beta}} = \mathcal{G}(z,\bar{z})$$



$$\mathcal{G}(z,\bar{z}) = \sum_p \mathcal{F}_{jj}^{ii}(p|z)\bar{\mathcal{F}}_{jj}^{ii}(p|\bar{z})$$

$$\mathcal{F}_{jj}^{ii}(p|z) \to \sum_q \mathcal{M}_{pq}\mathcal{F}_{jj}^{ii}(q|z)$$

$$\boxed{C_{ij}^{\beta}(t) \to \frac{1}{d_i d_j}\frac{S_{ij}^{*}}{S_{00}}}$$

$$\mathcal{M}_{00} = \frac{S_{ij}^{*}}{S_{00}}\frac{S_{00}}{S_{0i}}\frac{S_{00}}{S_{0j}}$$

"Monodromy scalar" anyon interferometry experiments

[Bonderson, Shtengel, Slingerland '06]