Holographic defect entropies and information flows

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Outline

- Kondo models from holography
  - Model — J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086
  - Entanglement entropy — J.E., Flory, Newrzella 1410.7811, JHEP 1501 (2015) 058
    J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666, Fortsch.Phys. 64 (2016)
  - Two-point functions — J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress
  - Quantum quenches — J.E., Flory, Newrzella, Wu in progress

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

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Motivation for study within gauge/gravity duality:

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

Motivation for study within gauge/gravity duality:

1. Simple model for a RG flow with dynamical scale generation (as in QCD)

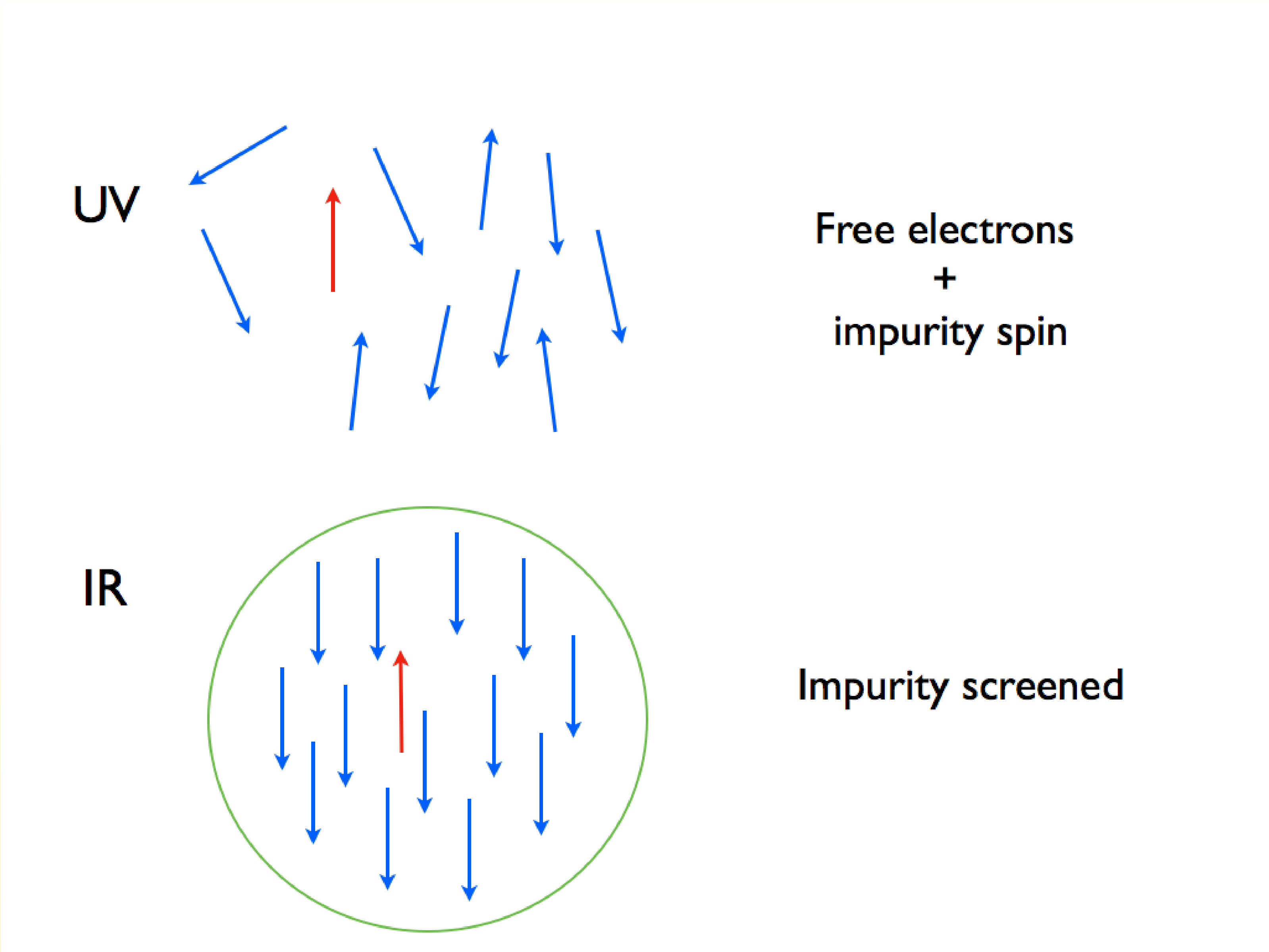
2. Example for holographic g-theorem

3. Relation to Sachdev-Ye-Kitaev model

4. New applications of gauge/gravity duality to condensed matter physics:

   - Magnetic impurity coupled to strongly correlated electron system
   - Entanglement entropy, quantum quench

Kondo effect



Kondo model

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Original Kondo model (Kondo 1964):
Magnetic impurity interacting with free electron gas

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Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Gauge/gravity requires large N: Spin group $SU(N)$

Kondo models from gauge/gravity duality

Gauge/gravity requires large N: Spin group $SU(N)$

In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = Q$

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Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192, PRB 58 (1998) 3794

Senthil, Sachdev, Vojta cond-mat/0209144, PRL 90 (2003) 216403

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

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Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation, screening
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of conductivity in IR with real exponent
- Holographic entanglement entropy from including backreaction
- Quantum quench: Late-time behaviour dominated by quasinormal modes

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

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Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

Near-horizon limit and field-operator map

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \to$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \to \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up gravity dual for Kondo model

Action:

$$S = S_{\text{Einstein-Hilbert}} + S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

$$S_{AdS_2} = -N \int d^3x\, \delta(x) \sqrt{-g} \left[\frac{1}{4} \mathrm{Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi\right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right]$$

$$V(\Phi) = M^2 \Phi^\dagger \Phi$$

Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z)\, dt^2 + dx^2 \right),$$

$$h(z) = 1 - z^2/z_H^2\,, \qquad T = 1/(2\pi z_H)$$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

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Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

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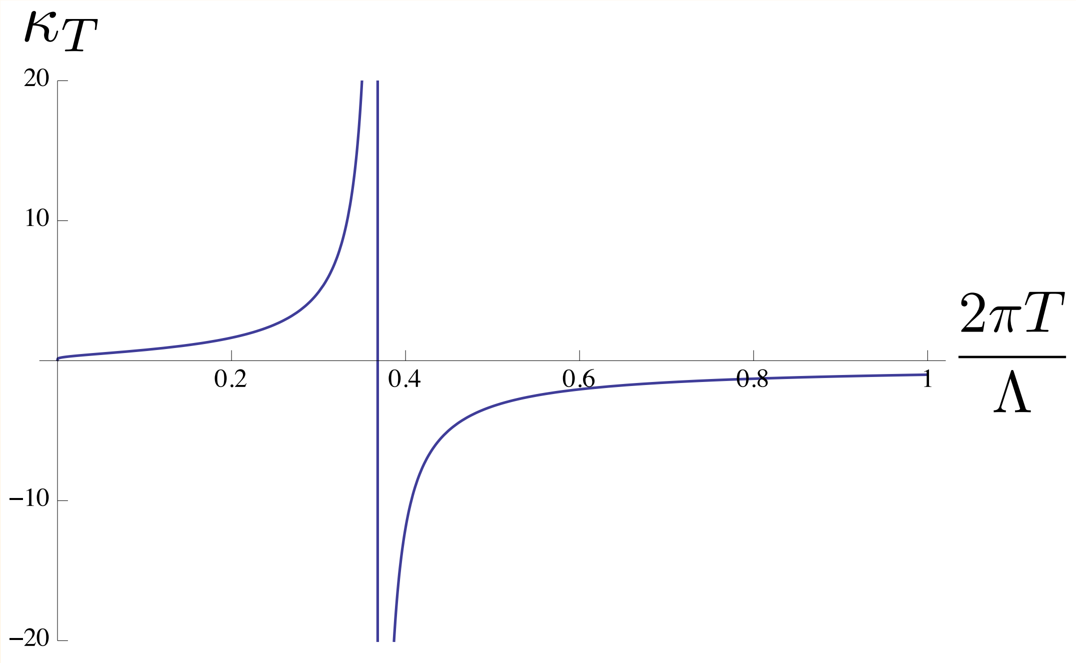
Dynamical scale generation

Scale generation

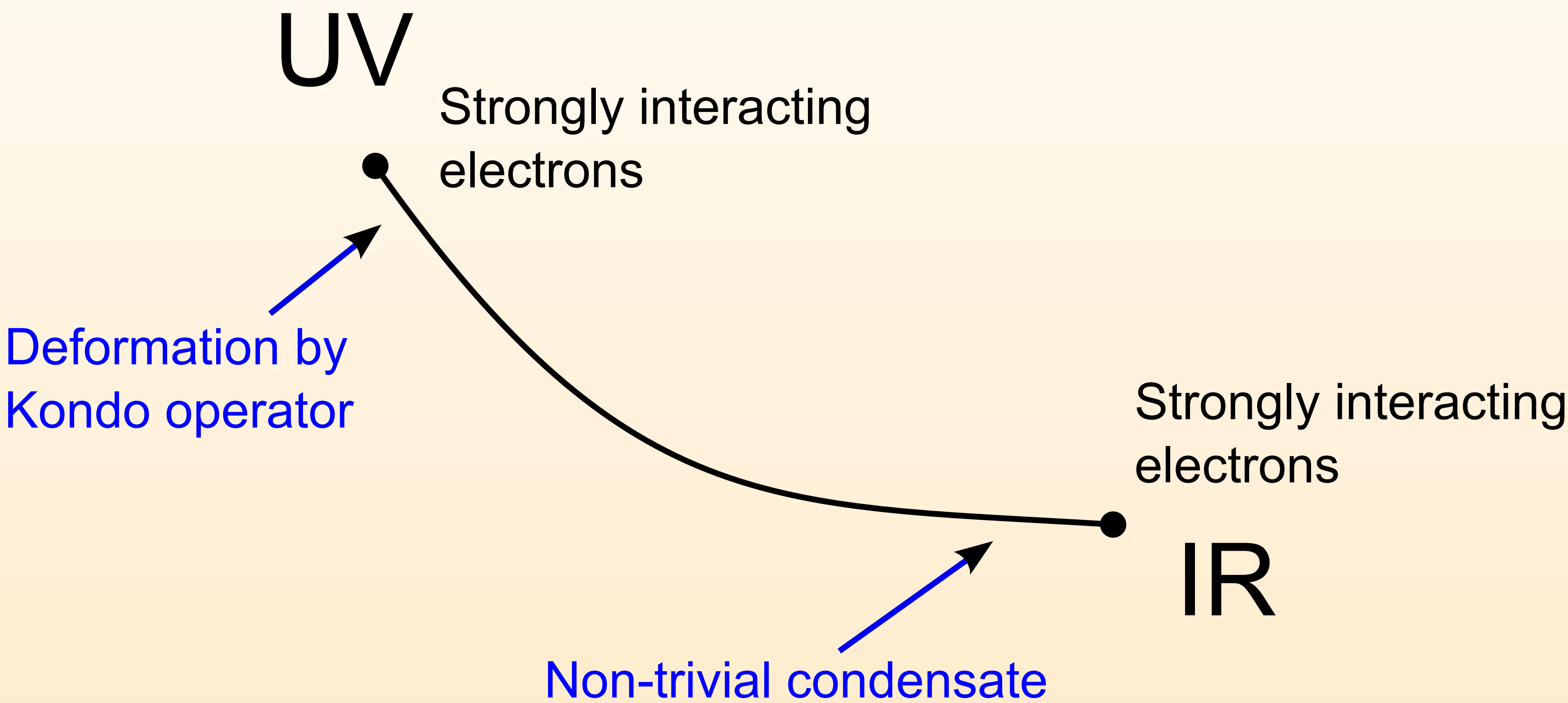


Divergence of Kondo coupling determines Kondo temperature T_K

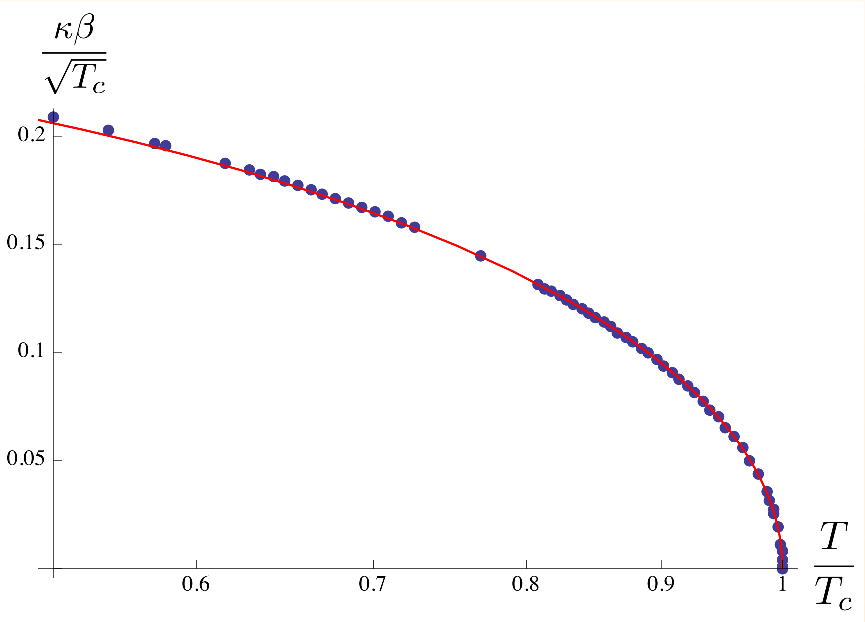
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

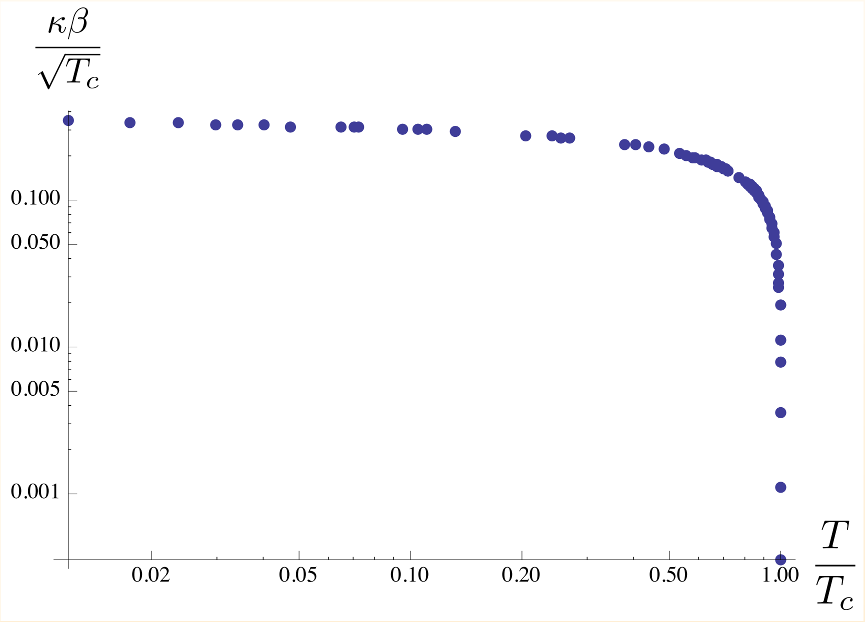
RG flow


UV
Strongly interacting
electrons
Deformation by
Kondo operator
Strongly interacting
electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)



(b)

Mean field transition

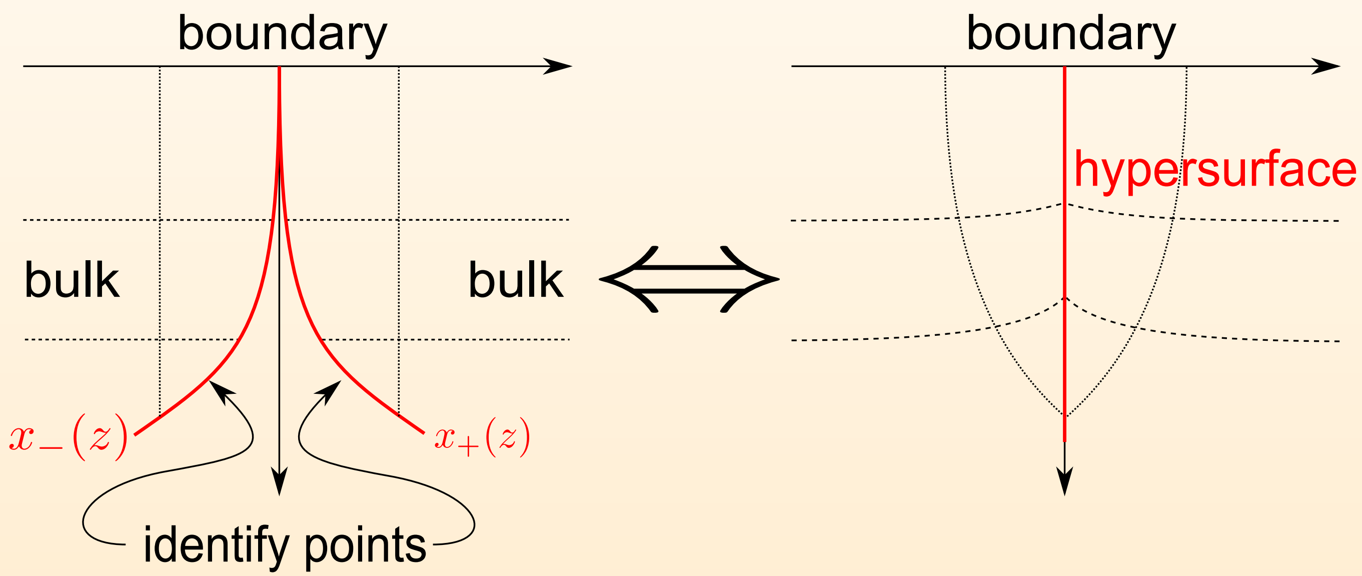
$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Entanglement entropy for magnetic impurity

(see poster by Mario Flory)

Including the backreaction using a thin brane and Israel junction conditions

Israel junction conditions $K_{\mu\nu} - \gamma_{\mu\nu} K = -\frac{\kappa}{2} T_{\mu\nu} \Leftrightarrow$ Energy conditions

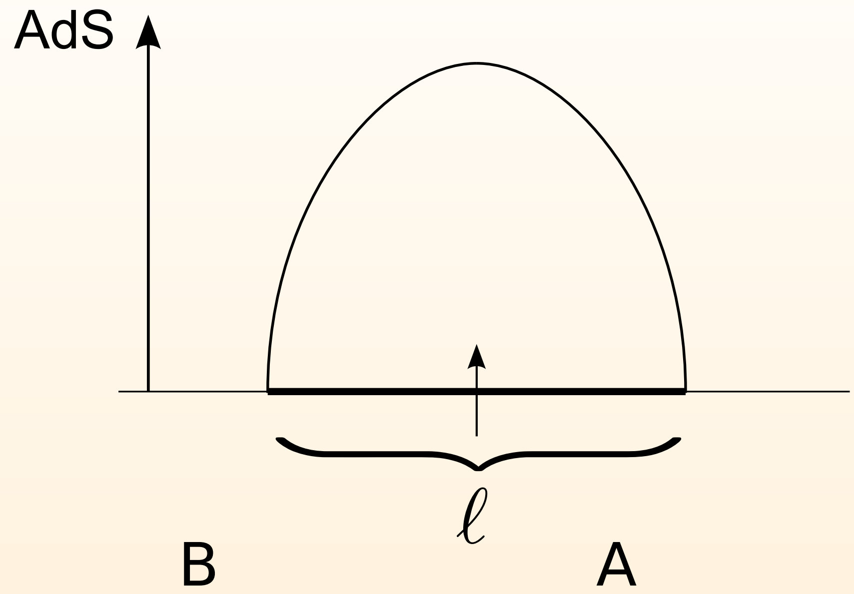


J.E., Flory, Newrzella 1410.7811

Cf. previous work on holographic BCFT

Takayanagi; Fujita, Takayanagi, Tonni 2011; Nozaki, Takayanagi, Ugajin 2012

Entanglement entropy for magnetic impurity



Impurity entropy:

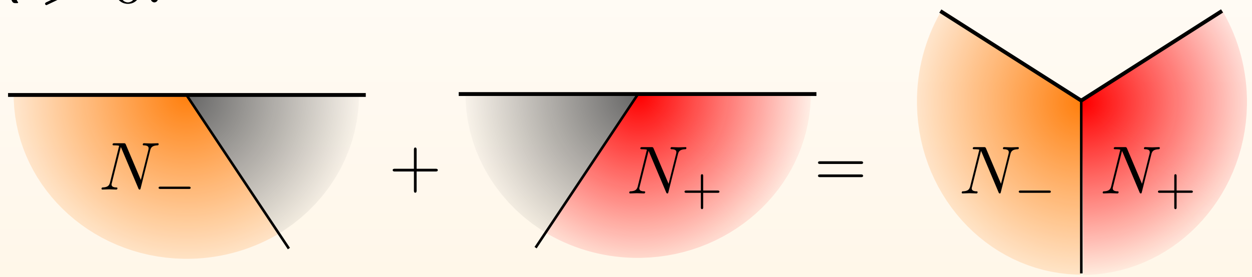
$$S_{\text{imp}} = S_{\text{condensed phase}} - S_{\text{normal phase}}$$

Subtraction also guarantees UV regularity

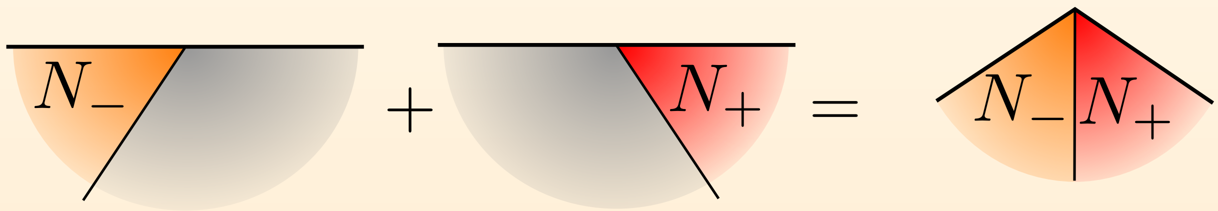
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811

$\lambda > 0$:



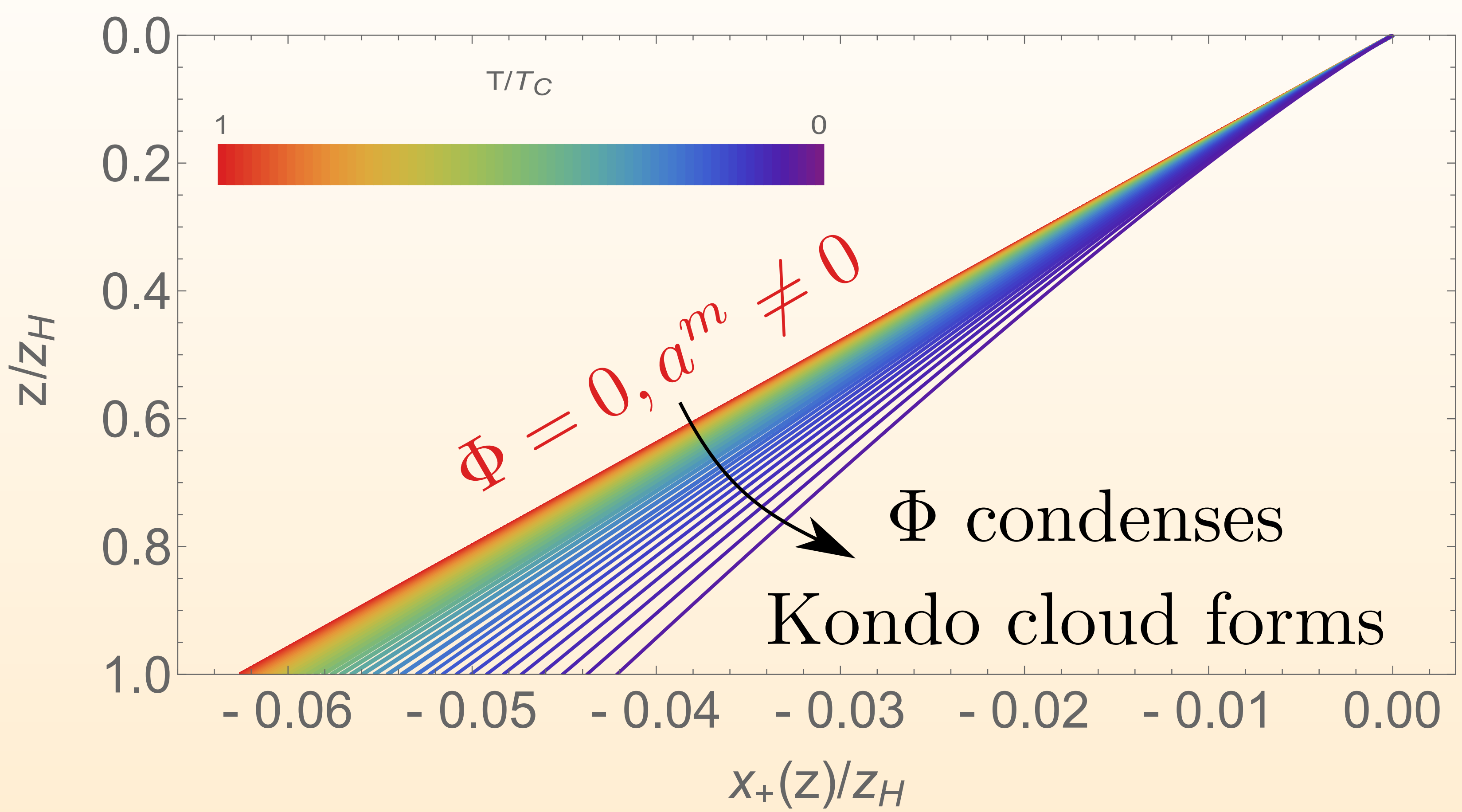
$\lambda < 0$:



Depending on the brane tension λ, the total space is enhanced or reduced

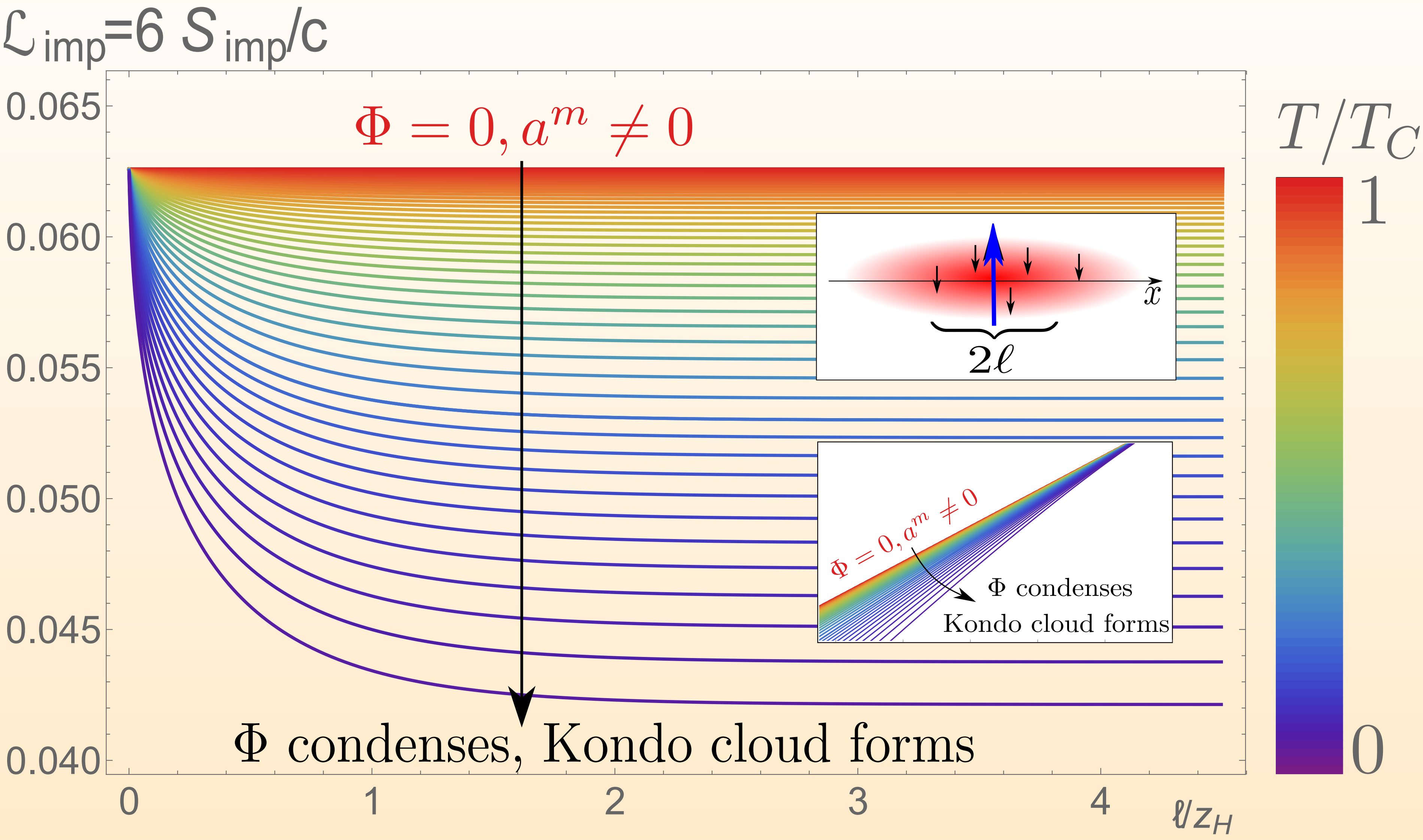
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811



The larger the condensate, the shorter the geodesic

Impurity entropy from gauge/gravity duality



Entanglement entropy for magnetic impurity: Comparison to field theory

Field-theory result

Sorensen, Chang, Laflorencie, Affleck 2007
(Eriksson, Johannesson 2011)

$$S_{\mathrm{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi\ell T)$$

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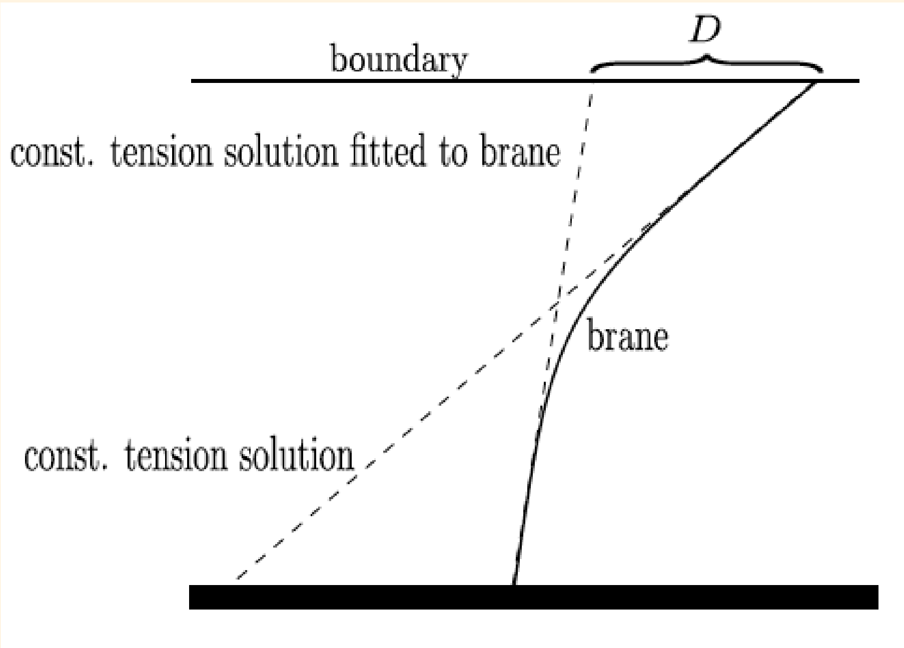
In our gravity approach:

J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu 1511.03666



Entanglement entropy for magnetic impurity: Comparison to field theory

On gravity side:

Impurity entropy from difference of entanglement entropies for constant tension branes

$$S_{\text{imp}}(\ell) = S_{BH}(\ell + D) - S_{BH}(\ell)$$

$$S_{BH}(\ell) = \frac{c}{3} \ln \left(\frac{1}{\pi \epsilon T} \sinh(2\pi \ell T) \right)$$

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For $D \ll \ell$:

$$S_{\text{imp}}(\ell) \sim D \cdot \partial_\ell S_{BH}(\ell) = \frac{2\pi D T}{3}\coth(2\pi\ell T)$$

Agrees with field theory result subject to identification $D \sim \xi_K$

Kondo model: Example for holographic g-theorem

Formation of Kondo cloud corresponds to
decrease in impurity degrees of freedom

$$S_{\mathrm{imp}}(\ell \to \infty) = -\frac{c}{3}\tilde{x}_+(z_H)$$

$\tilde{x}_+(z)$: Defect embedding scalar

g-theorem:

$$T \cdot \frac{\partial S_{\mathrm{imp}}(\ell \to \infty)}{\partial T} \geq 0$$

Due to null energy condition

Relation to Sachdev-Ye-Kitaev model

Sachdev-Ye-Kitaev model: Gaussian random couplings $J_{\alpha\beta,\gamma\delta}$ Sachdev+Ye 1993, Kitaev 2015

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} J_{\alpha\beta,\gamma\delta} \, \chi_\alpha^\dagger \chi_\beta \chi_\gamma^\dagger \chi_\delta - \mu \sum_\alpha \chi_\alpha^\dagger \chi_\alpha$$

May be obtained from two-dimensional model as follows:

(Bray, Moore J. Phys. C 1980; Georges, Parcollet, Sachdev PRB 63 92001)

Reduction to single site by averaging over disorder

$$H_S = -\sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$S_{\text{eff}} = -\frac{J^2}{2N} \int_0^\beta d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \, , \qquad Q(\tau - \tau') = \frac{1}{N^2} \langle \vec{S}(\tau) \vec{S}(\tau') \rangle$$

Use Abrikosov fermions χ as before, $S^a = \chi^\dagger T^a \chi$, and take large N limit

(see also Maldacena, Stanford arXiv:1604.07818)

Relation to Sachdev-Ye-Kitaev model

Similarly in Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192:

Reduction of large N-Kondo model to single-site model

by integrating out conduction electrons

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$\Rightarrow$ Spectral asymmetry of Green's functions

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by integrating out conduction electrons

$\Rightarrow$ Spectral asymmetry of Green's functions

Sachdev 1506.05111, Phys. Rev. X 5, 041025 (2015):

Spectral asymmetry also observed in SYK model

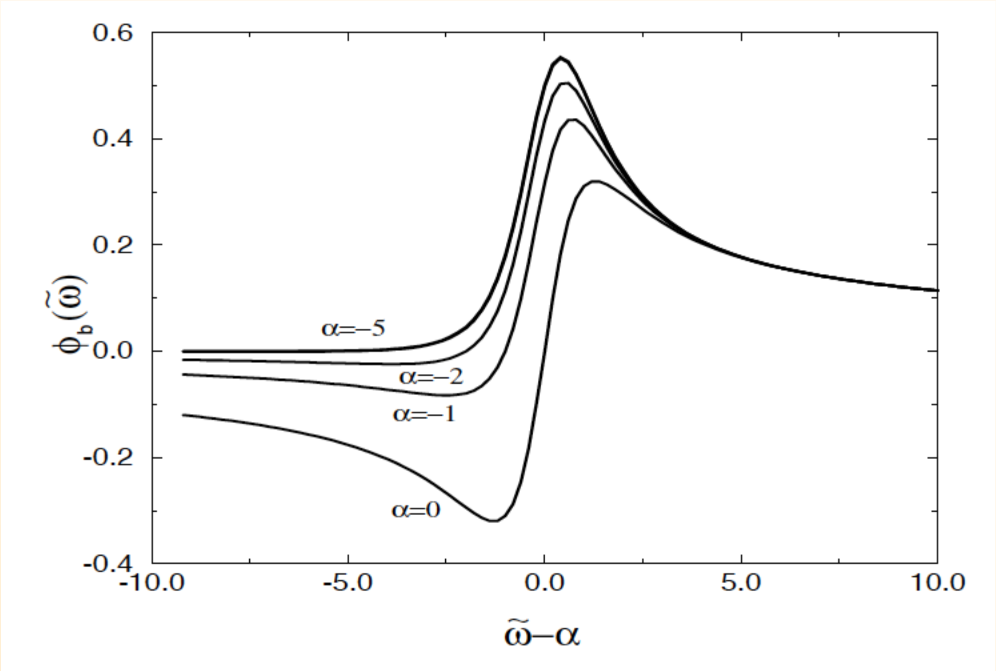
related to entropy of AdS_2 black hole

$$\omega_s = \frac{qT}{\hbar}\frac{\partial S}{\partial \mathcal{Q}}$$

Kondo model: Two-point functions

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192: Large N Kondo model

Spectral asymmetry ω_s: Particle-hole symmetry broken

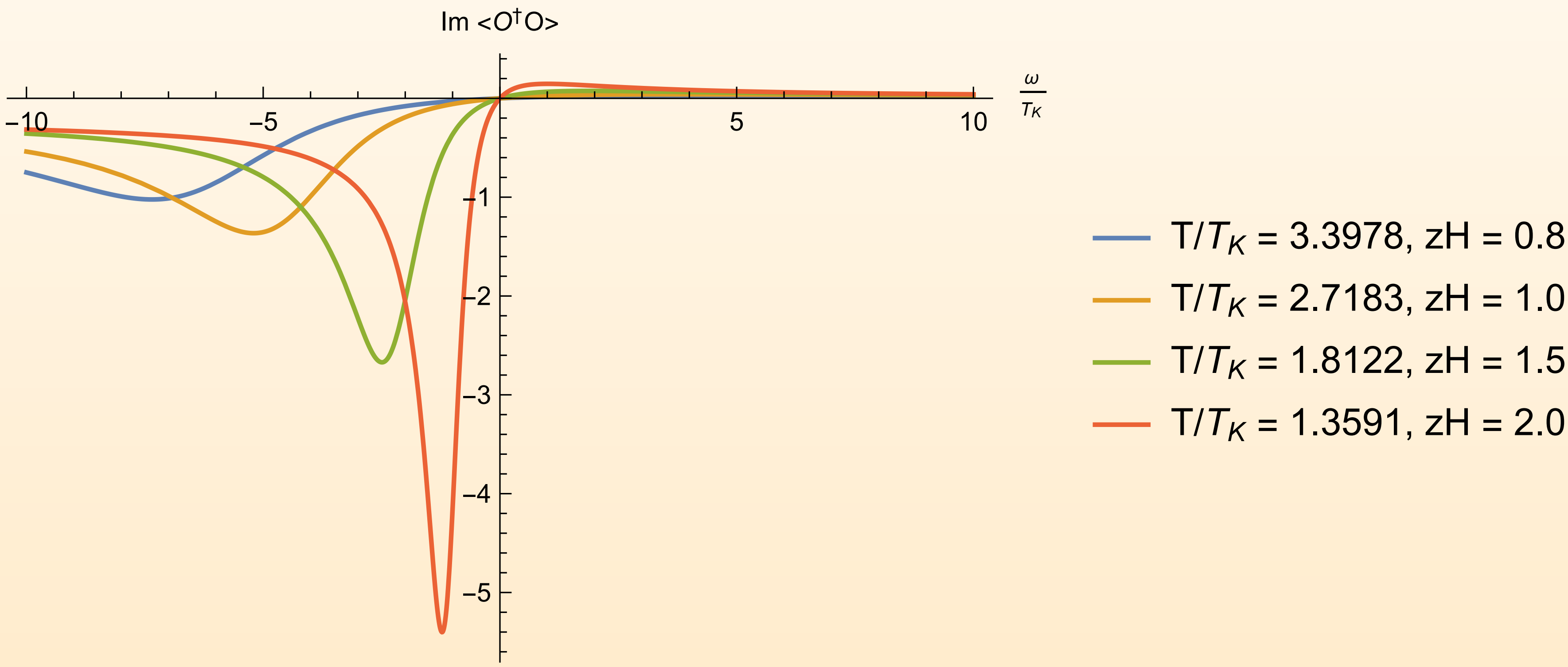


$-\mathrm{Im}G^R$ for bosonic $\langle \mathcal{O}\mathcal{O}^\dagger \rangle$

$\omega_s = \frac{qT}{\hbar}\frac{\partial S}{\partial Q}$ see also Sachdev 1506.05111, AdS_2 black hole (fermions)

J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress

Reduction to single-site automatic since equation of motion for 3d Chern-Simons field decouples from EOM's for 2d fields



Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

Allow for time dependence of the Kondo coupling and study response of the condensate

Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

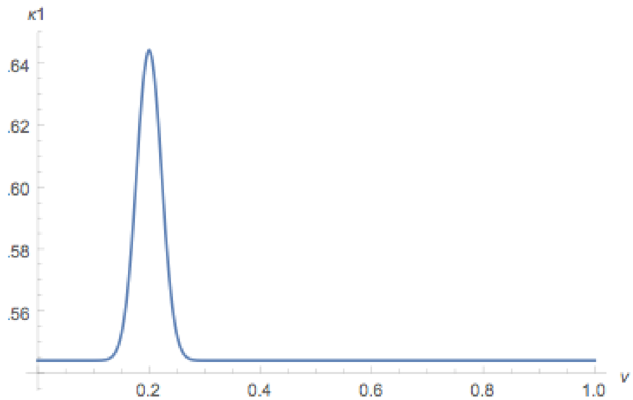
Observations:

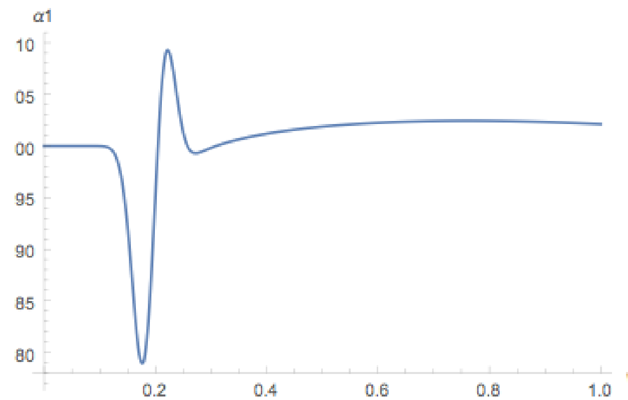
Different timescales depending on whether the condensate is asymptotically small or large

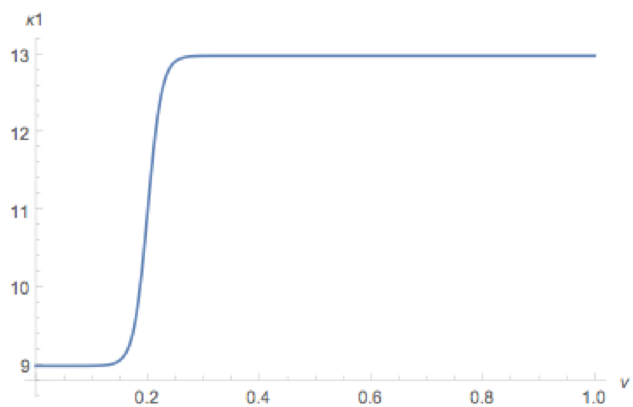
Timescales governed by quasinormal modes

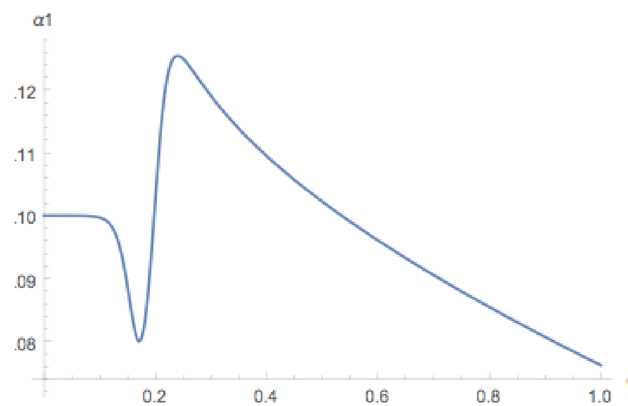
Time dependence

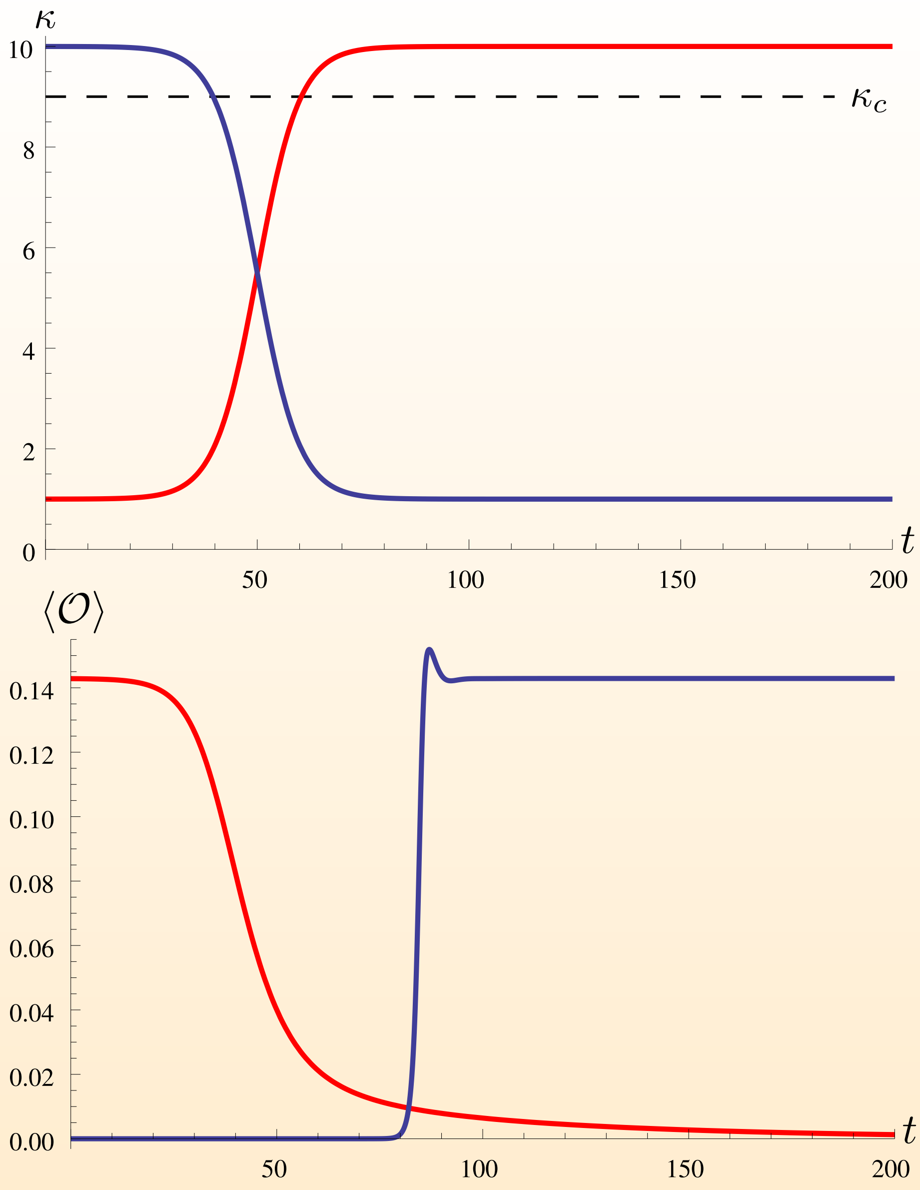
Kondo coupling → Condensate









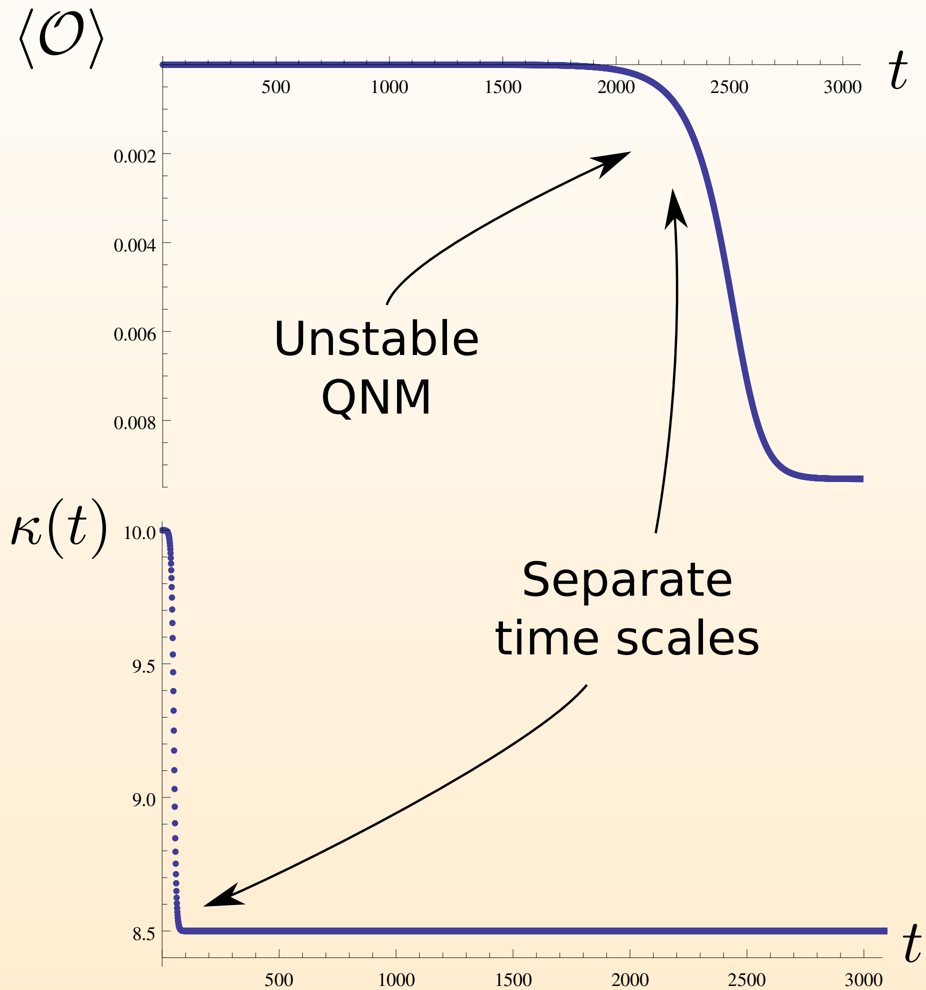


Quantum quenches in holographic Kondo model

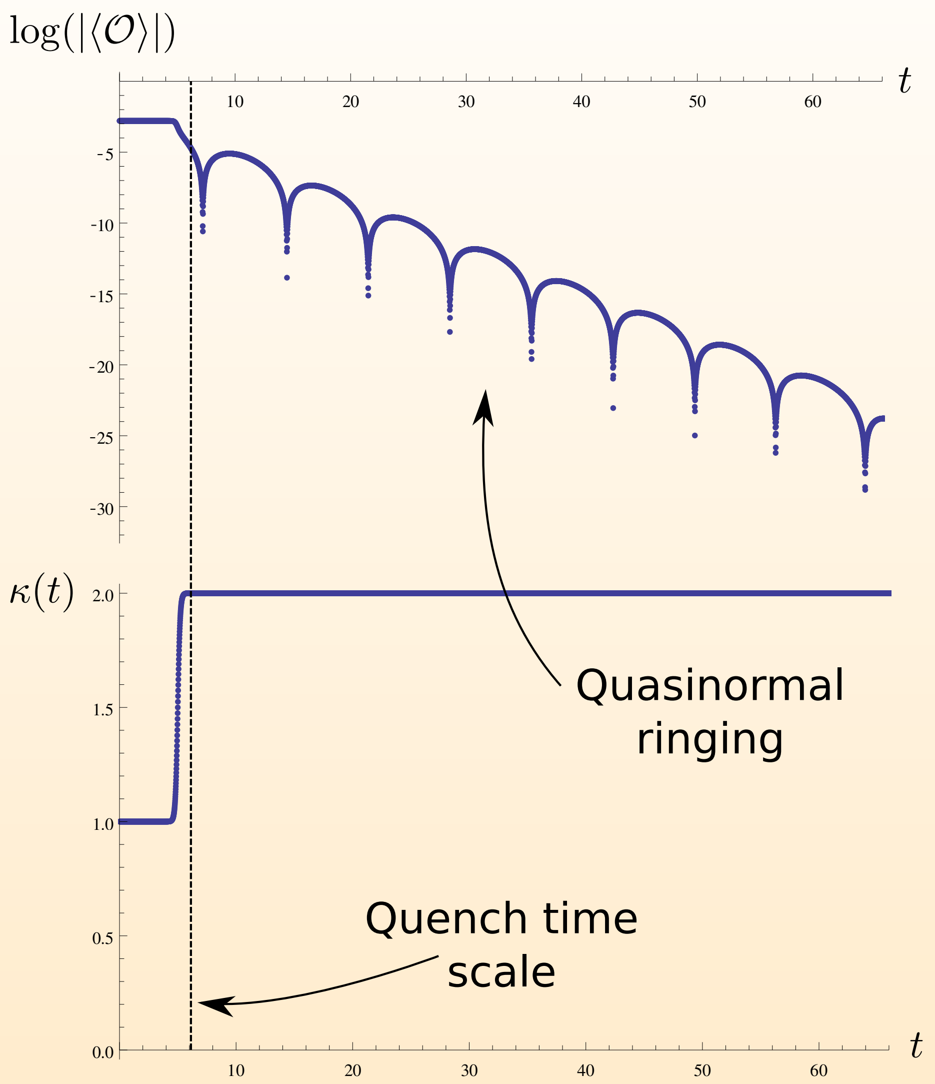
To and from condensed phase

Timescales determined by quasinormal modes

J.E., Flory, Newrzella, Strydom, Wu



$\langle \mathcal{O} \rangle$
t
500
1000
1500
2000
2500
3000
0.002
0.004
0.006
0.008
Unstable QNM
$\kappa(t)$
10.0
9.5
9.0
8.5
Separate time scales
t
500
1000
1500
2000
2500
3000


$\log(|\langle \mathcal{O} \rangle|)$
t
10
20
30
40
50
60
-5
-10
-15
-20
-25
-30
$\kappa(t)$
2.0
1.5
1.0
0.5
0.0
Quasinormal ringing
Quench time scale

Equilibration determined by quasinormal modes, which depend on T

Cf. Bayat, Bose, Johannesson, Sodano Phys. Rev. B 92, 155141 (2015) :

Quench in two-impurity Kondo model in spin-chain approach:

Late-time behaviour dominated by single-frequency oscillations,
independent of energy released

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Entanglement entropy
  - In agreement with g-theorem
  - Reproduces large N field theory result for large ℓ
  - Geometrical realization of Kondo correlation length
- Two-point functions
  - Spectral asymmetry
  - Relation to SYK model
- Quantum quenches
  - Dominated by quasinormal modes

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