Matrix Quantum Mechanics from Qbits

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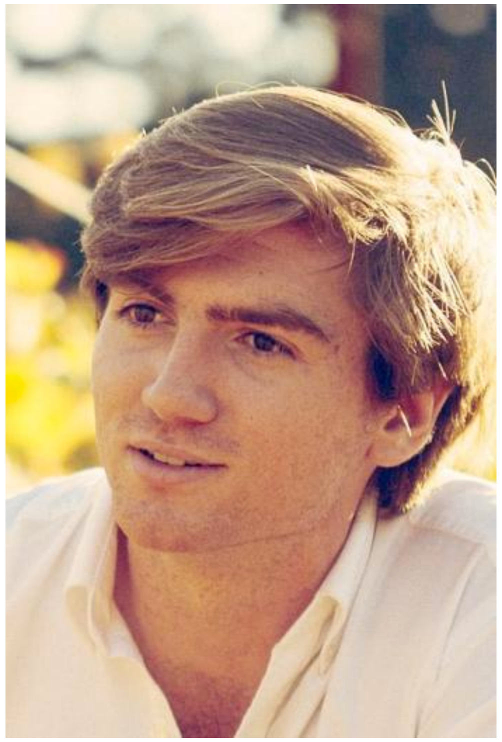

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Preliminary comments

- Deep aspects of quantum gravity (Bekenstein-Hawking, Ryu-Takayanagi, …) have been gleaned from the semi-classical path integral.

- Nonetheless, ultimately, understanding the emergence of spacetime, ‘It from Qbit’, will involve a characterization of an underlying microscopic entanglement structure that supports emergent local dynamics.

- Emergent locality is somewhat different from MPS, MERA, PEPS, etc, whose role is to put manifest locality to a powerful use.

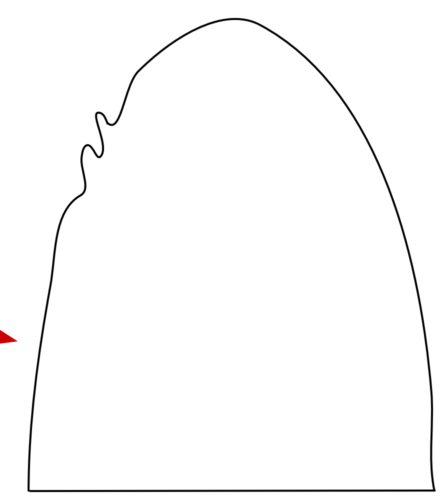
“It from Qbit”, explicit version

- This talk is about an explicit realization of emergent local dynamics from a quantum spin system.
- Logic: map the spin system onto a matrix quantum mechanics that is known to lead to emergent local dynamics at a quantum critical point.

Ising spins

$$\int \mathcal{D}\Phi e^{i\int dt\, \mathrm{tr}\left(\dot{\Phi}^2 - V(\Phi)\right)}$$

Matrix quantum mechanics



Spacetime

“It from Qbit”, explicit version

- The emergent local dynamics will be rather simple for the model we solve: a scalar field in 1+1 dimensions.

- However, start to get a feel for various nontrivial steps that arise in obtaining a matrix quantum mechanics structure from ‘atoms of entanglement’ (Qbits/spins).

- Previously shown how the accumulation of entanglement necessary for emergent locality arises in this case: it is contained in the Fermi Sea structure of the eigenvalue dynamics [1504.07985, SAH + Mazenc; Das ’95].

Transverse field Ising models

- The usual transverse field Ising model is

$$H = -h \sum_i S_i^x + v_2 \sum_{<ij>} S_i^z S_j^z$$

$$| \rightarrow \rightarrow \cdots \rightarrow \rangle \qquad\qquad | \uparrow \downarrow \uparrow \cdots \uparrow \rangle$$

$\mathbb{Z}_2$ symmetry broken

Continuous quantum phase transition

$\Rightarrow$ Gapless excitations $\Rightarrow$ Continuum limit described by CFT.

Transverse field Ising models

- A symmetric matrix of N^2 spins, $\vec{S}_{AB}$, interacting nonlocally:

$$H = -h \sum_{AB} S^x_{AB} + \frac{v_4}{N} \sum_{ABCD} S^z_{AB} S^z_{BC} S^z_{CD} S^z_{DA}$$

- Symmetries:
  (1) flipping the spin of rows and columns: $\mathbb{Z}_2^N$
  (2) permuting rows and columns: S_N

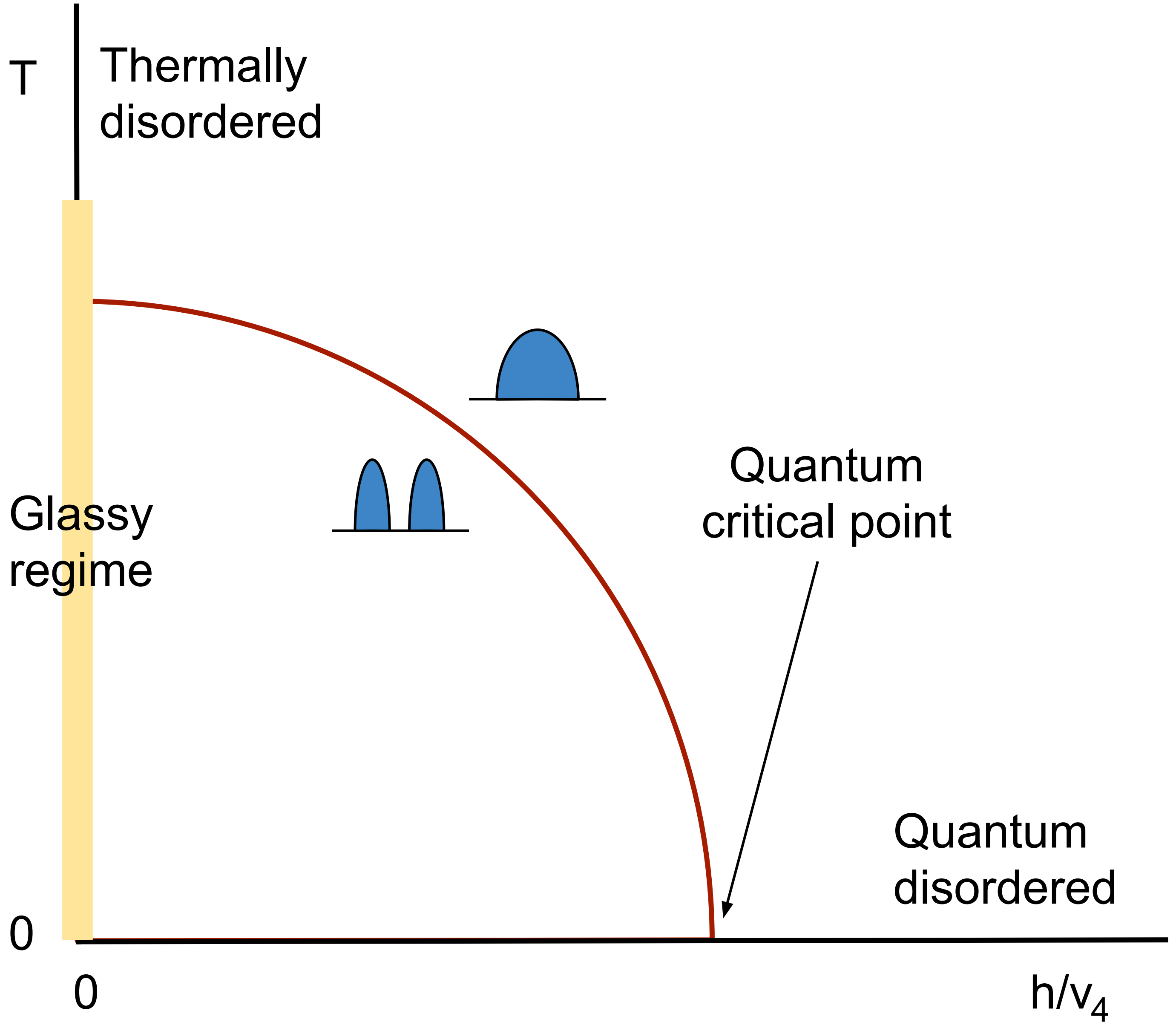
- Classical model (h=0) studied by [Cugliandolo-Kurchan-Parisi-Ritort, cond-mat/9407086] has 'crystalline' order at low T, preempted by glassiness.

Transverse field Ising models

Our results for this model at large N:

- First order symmetry breaking transition at critical v_4/h.
- Disordered large N state is described by a matrix quantum mechanics for all couplings.
- Disordered large N state undergoes a continuous quantum phase transition as a function of v_4/h.
- At the quantum critical point, the low energy excitations are described by massless scalar field in 1+1 dimensions.

Transverse field Ising models



Step 1: Suzuki-Trotter decomposition

Path integral for spins:

$$
\begin{aligned}
Z &= \mathrm{Tr}\, e^{-\beta H} \\
&= \sum_{\sigma_1 \in \{\pm 1\}^{N^2}} \cdots \sum_{\sigma_M \in \{\pm 1\}^{N^2}} \prod_{m=1}^{M} \langle \sigma_m | e^{-\epsilon H} | \sigma_{m+1} \rangle \,, \qquad \epsilon \equiv \frac{\beta}{M} \ll 1 \\
&\approx \sum_{\sigma_1 \in \{\pm 1\}^{N^2}} \cdots \sum_{\sigma_M \in \{\pm 1\}^{N^2}} \prod_{m=1}^{M} e^{-\epsilon\, \mathrm{tr} V(\sigma_m)} \prod_{A,B} \langle \sigma_m | 1 + \epsilon\, h\, S^x_{AB} | \sigma_{m+1} \rangle \\
&= \sum_{\sigma_1 \in \{\pm 1\}^{N^2}} \cdots \sum_{\sigma_M \in \{\pm 1\}^{N^2}} \exp\left\{ -\epsilon \sum_{m=1}^{M} \left(\mathrm{tr}\, V(\sigma_m) + \frac{\widetilde{K}}{2} \sum_{A,B} \frac{\left(\sigma_m^{AB} - \sigma_{m+1}^{AB}\right)^2}{\epsilon^2} \right) \right\} .
\end{aligned}
$$

Step 2: Constrained bosons

Introduce continuous boson field:

$$\sum_{\sigma_1 \in \{\pm 1\}^{N^2}} \cdots \sum_{\sigma_M \in \{\pm 1\}^{N^2}} F(\sigma) = \int \prod_{m,A,B} d\Phi_m^{AB} \delta\left(\left(\Phi_m^{AB}\right)^2 - 1\right) F(\Phi)$$

$$= \int \prod_{m,A,B} d\Phi_m^{AB} d\mu_m^{AB} e^{i\mu_m^{AB}\left(\left(\Phi_m^{AB}\right)^2 - 1\right)} F(\Phi)\,,$$

Step 3: Continuum limit (in time)

Send time step to zero, but remember time step was introduced by hand, not a microscopic parameter:

$$\frac{\widetilde{K}}{2}\sum_{A,B}\frac{\left(\Phi_m^{AB}-\Phi_{m+1}^{AB}\right)^2}{\epsilon^2}\to\frac{K}{2}\left(\frac{d\Phi}{d\tau}\right)^2+\frac{K'}{2}\frac{1}{h^2}\left(\frac{d^2\Phi}{d\tau^2}\right)^2+\cdots.$$

Microscopic energy scale to flip a spin

Derivative expansion can be truncated when excitations with energy ΔE « h are present: quantum critical point.

Crucially, can also truncate at weak coupling (free spins) and strong coupling (classical spins).

Step 4: Matrix Quantum Mechanics Saddle!

So far:

$$Z = \int \mathcal{D}\Phi \mathcal{D}\mu \exp\left\{ -\int_0^\beta d\tau \left[\mathrm{tr}\left(\frac{K}{2} \left(\frac{d\Phi}{d\tau} \right)^2 + V(\Phi) \right) + i \sum_{AB} \mu_{AB} \left(\left(\Phi^{AB} \right)^2 - 1 \right) \right] \right\} .$$

Not SO(N) invariant

Import a trick from [Cugliandolo-Kurchan-Parisi-Ritort, '94]:

- At large N, a consistent saddle point has $\mu_{AB} = \mu$.
- In classical spin model, this saddle was shown to describe the high T disordered phase.
- Perhaps captures the quantum disordered wavefunction?

Step 4: Matrix Quantum Mechanics Saddle!

Thus:

$$Z = \int \mathcal{D}\Phi \mathcal{D}\mu \exp\left\{ - \int_0^\beta d\tau \, \mathrm{tr}\left(\frac{K}{2}\left(\frac{d\Phi}{d\tau}\right)^2 + \mu\Phi^2 + V(\Phi) - \mu N \right) \right\}$$

Expectation: If there is continuous QPT in the disordered state, its critical dynamics is described by the above MQM.

Remainder:

- Find the ground state of the above.
- Show that it correctly describes the disordered state.
- Show that there is a continuous QPT.
- Show that low energy excitations: 1+1 scalar field.

Step 5: Hamiltonian and ground state

- Hamiltonian: Exercise in constrained quantization.
  [Dirac brackets, secondary constraints etc.]

- Go to eigenvalue basis (non-singlet modes decouple at the quantum critical point [Gross-Klebanov '91]):

$$\Phi = O^T \Lambda O \,, \quad \Lambda_{ij} = \lambda_i \delta_{ij}$$

- Change variables to collective field [Jevicki-Sakita '80]:

$$\rho(\lambda, t) = \sum_{i=1}^{N} \delta\left(\lambda - \lambda_i(t)\right)$$

Step 5: Hamiltonian and ground state

- The collective field Hamiltonian and constraints are:

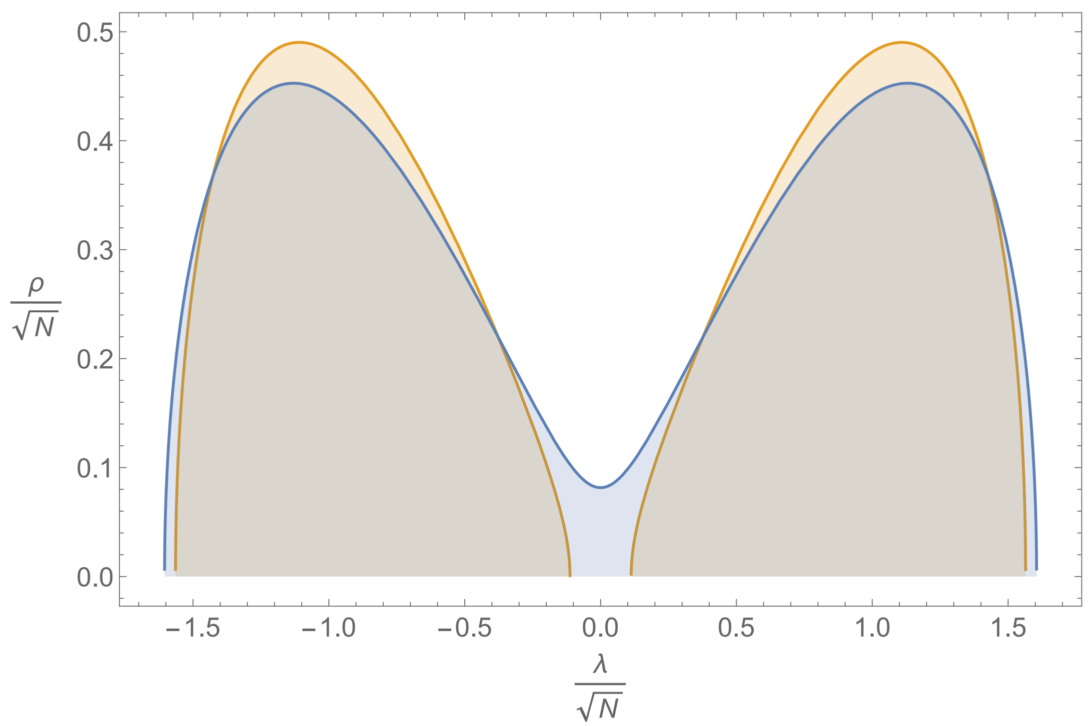
$$H = \int d\lambda \rho(\lambda) \left(\frac{1}{2K} \left[\partial_\lambda \pi(\lambda) \partial_\lambda \pi(\lambda) + \frac{\pi^2}{12} \rho(\lambda)^2 \right] + V(\lambda) \right) - \frac{1}{2KN^2} \left(\int d\lambda \rho(\lambda) \lambda \partial_\lambda \pi(\lambda) \right)^2$$

$$Q_1 = \int d\lambda \rho(\lambda) = N$$

$$Q_2 = \int d\lambda \lambda^2 \rho(\lambda) = N^2$$

Ensures $dQ_2/dt = 0$

- Ground state eigenvalue distribution has $\pi=0$.



Step 6: Existence of continuous QPT

- As $v_4 \to 0$ (quantum disordered spins), E_{MQM} matches the spin system ground state energy to 3rd order in perturbation theory [Expect disagreement at 4th order]:

$$\frac{1}{16h}\frac{E_0}{N^2} = -\frac{1}{16} + 2\frac{v_4}{16h} - 8\left(\frac{v_4}{16h}\right)^2 + 256\left(\frac{v_4}{16h}\right)^3 - 14848\left(\frac{v_4}{16h}\right)^4 + \cdots$$

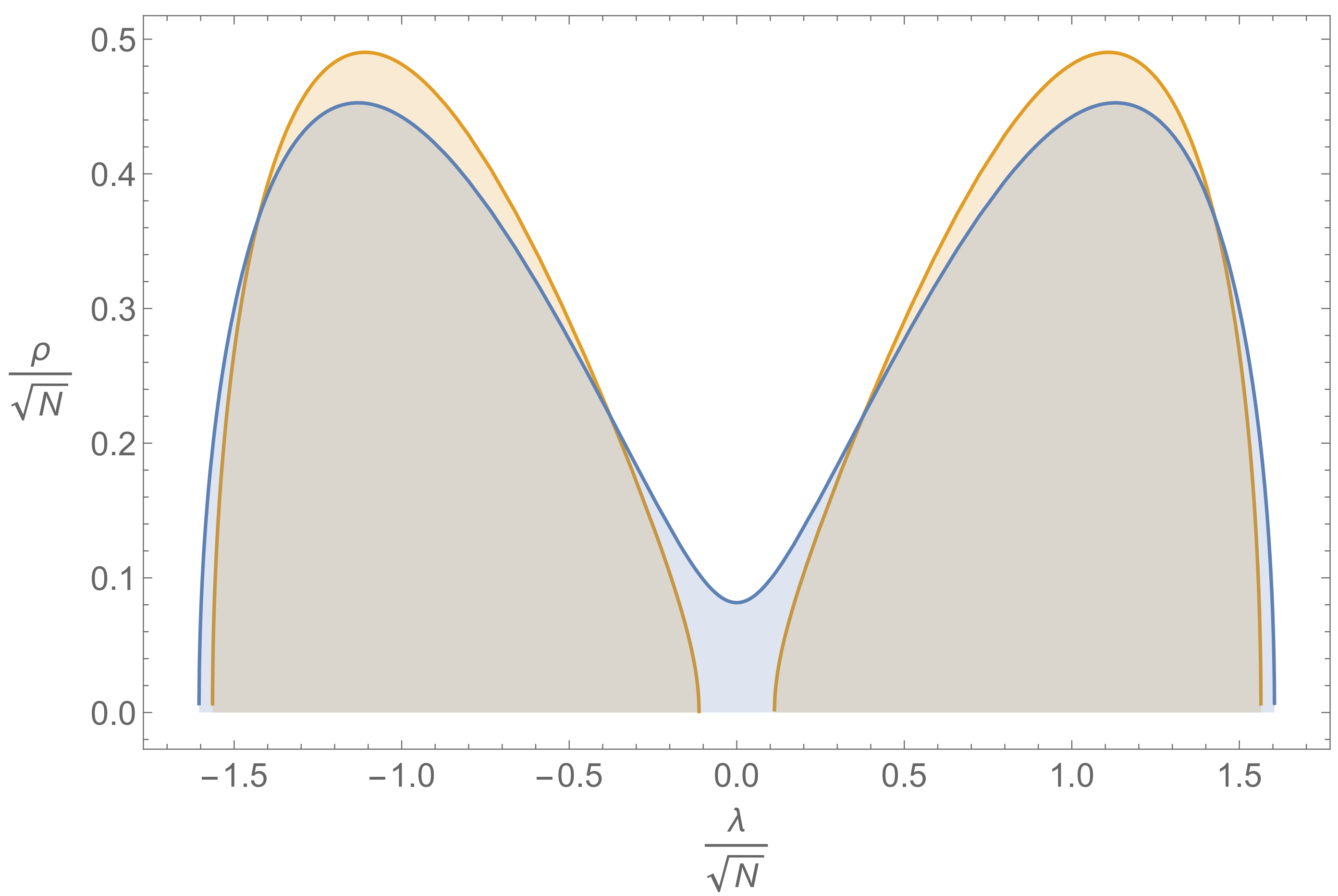
- Confirms MQM describes the disordered state.

- As $v_4 \to \infty$, E_{MQM} matches the energy of (subdominant) classical disordered state of Cugliandolo et al.:

$$\frac{E_0}{N^2} = v_4 + \cdots$$

Step 6: Existence of continuous QPT

- MQM eigenvalue distribution is connected at small v_4 and disconnected at large v_4.
  $\Rightarrow$ 'Topological' QPT at intermediate coupling.



Step 7: Emergent local dynamics

- Critical dynamics of topological phase transitions in MQM are well understood [eg. Das-Jevicki '90], ripples of the eigenvalue distribution:

$$\rho = \rho(\lambda) + \delta\rho(\lambda, t)$$

- Key step: canonical transformation on the fluctuations

$$\begin{aligned} \delta\rho &\sim \partial_q \phi \,, \\ \partial_\lambda \delta\pi &\sim \pi_\phi \,. \end{aligned}$$

- Quadratic Hamiltonian becomes:

$$H^{(2)} = \frac{\pi}{4K} \int dq \left((\bar{P}\pi_\phi)^2 + (\partial_q \phi)^2 \right)$$

- Excitations described by free 1+1 scalar field!

Step 7: Emergent local dynamics

- At weak coupling, fluctuations of the collective field can be related to particular spin excitations.

- Given by S_N singlet states with $n << N$ spins flipped along diagonal:

$$\left| \begin{matrix} \leftarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \leftarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \end{matrix} \right\rangle + \left| \begin{matrix} \leftarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \leftarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \end{matrix} \right\rangle + \left| \begin{matrix} \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \leftarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \rightarrow \\ \rightarrow & \rightarrow & \rightarrow & \rightarrow & \leftarrow \end{matrix} \right\rangle + \cdots$$

Conclusions

- Shown that Matrix Quantum Mechanics can arise at quantum critical points in transverse field Ising models.
- Analogous to how Ising CFTs arise, but dynamics is not in space but in the large N matrix interactions.
- Critical dynamics of these systems: free scalar field in emergent 1+1 dimensional spacetime.
- In the disordered phase, N^2 constraints can be relaxed to a single constraint:

$$(S^z_{AB})^2 = 1 \qquad \to \qquad \sum_{AB} (S^z_{AB})^2 = N^2 \,.$$