Numerical study on the entropic c-function for SU(3) gauge theory

Etsuko Itou (KEK)

E.I., K. Nagata, Y. Nakagawa, A. Nakamura and V.I.Zakharov
PTEP 2016 (2016) no.6, 061B01
arXiv: 1512.01334 [hep-th]

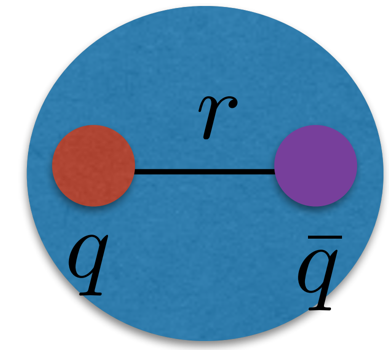
supercomputers

YKIS conference:"Quantum Matter, Spacetime and Information"
@ YITP, Kyoto University 2016/6/17

QCD

- The lagrangian is given by SU(3) gauge theory

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi$$

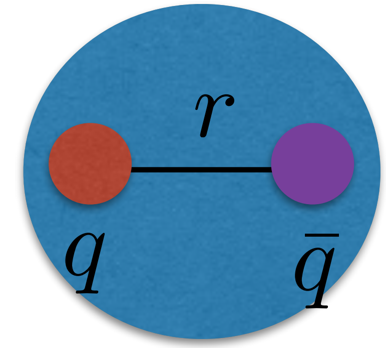


Hadron

"quenched" QCD

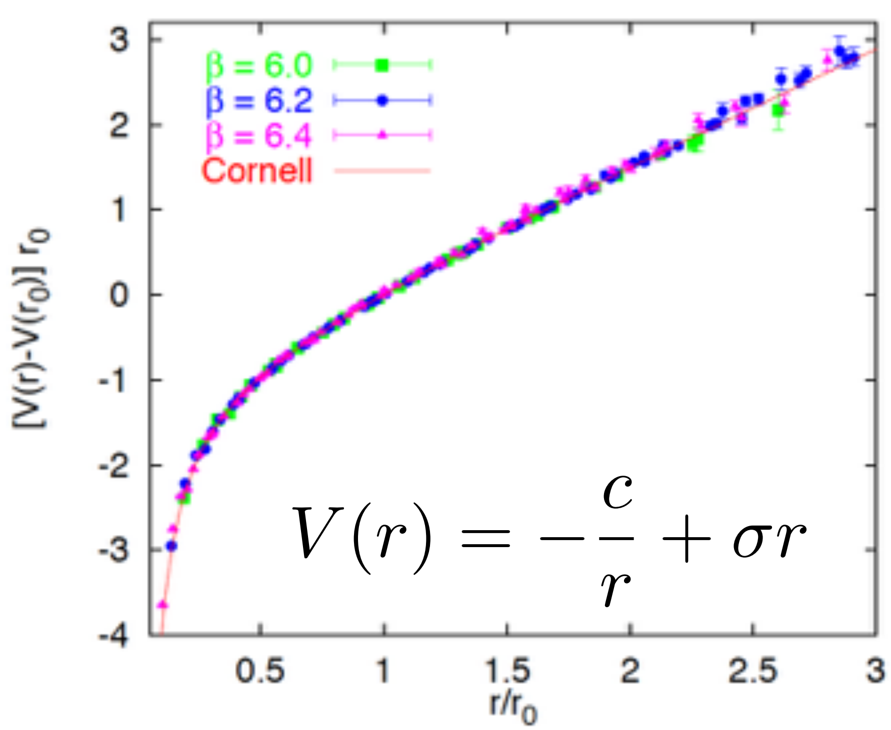
- The lagrangian is given by SU(3) gauge theory

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \cancel{\bar{\psi}(iD_\mu\gamma^\mu - m)\psi}$$



Hadron

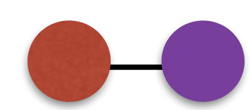
- It has the local SU(3) gauge invariance
- Lattice reg. is only known nonperturbative and gauge invariant reg. method from weak- to strong-coupling regimes

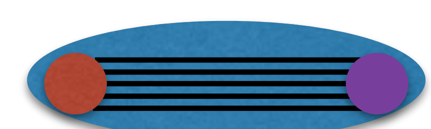


G. Bali Phys. Rept.343,(2001)1

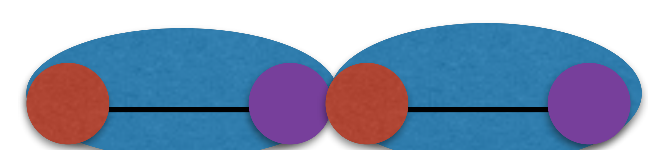
$q\bar{q}$ potential V(r)

interaction is very weak





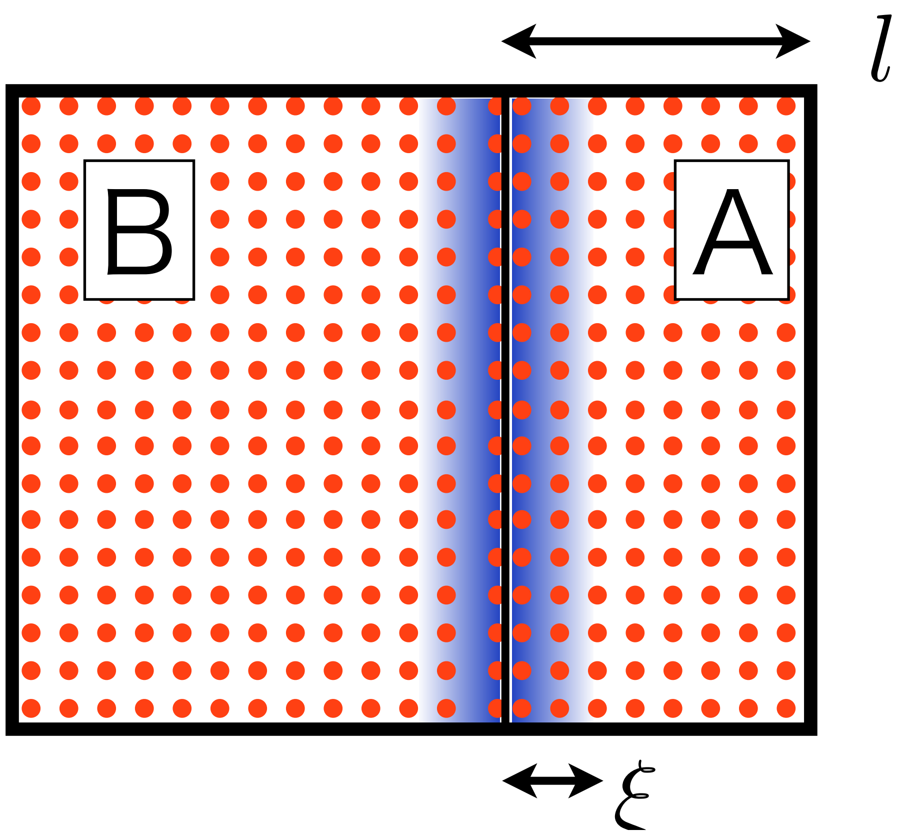
$r > \Lambda_{QCD}^{-1}$



Entanglement entropy for quantum system

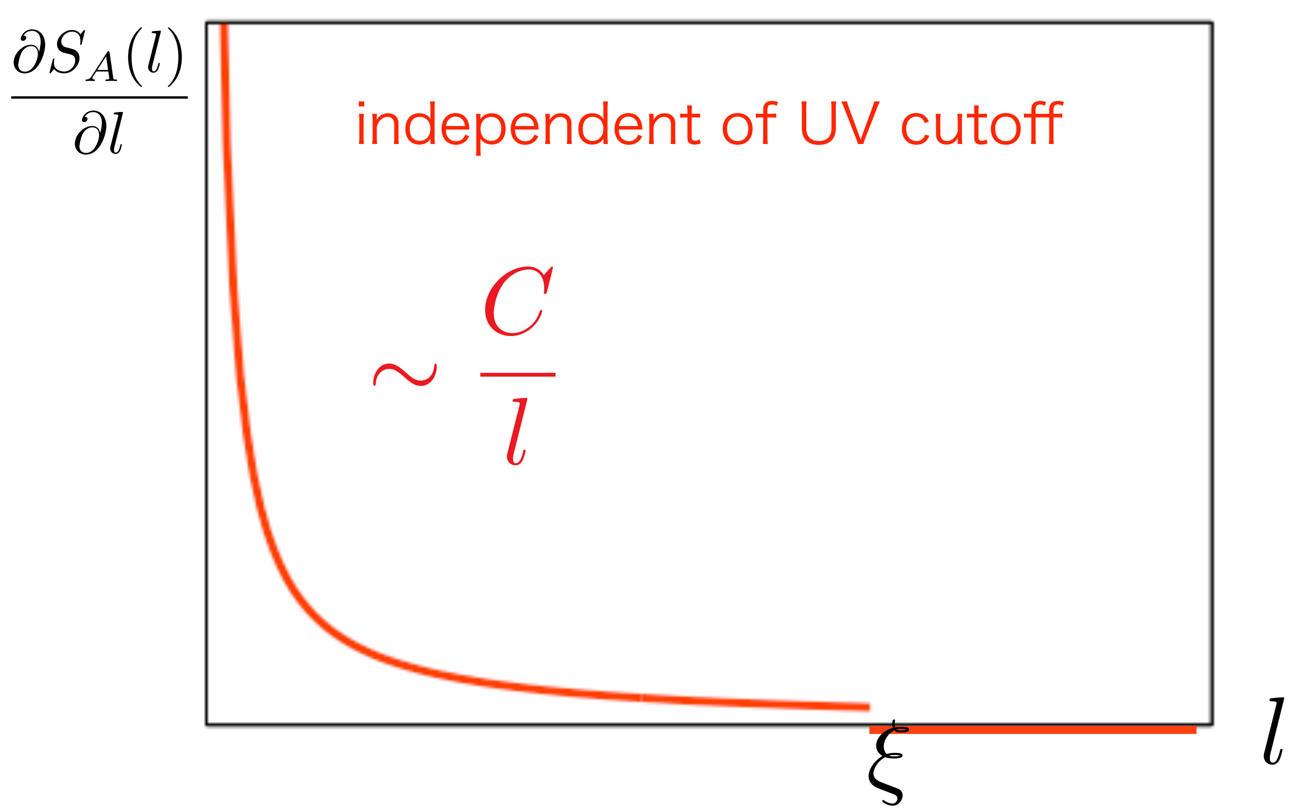
(1+1)-dim. model

Holzhey,Larsen and Wilczek: NPB424 (1994) 443
Calabrese and Cardy: J.S.M.0406(2004)P06002
Calabrese and Cardy, arXiv:0905.4013



ξ: correlation length of the system

l : subsystem(A) size



$$l \ll \xi \quad S_A(l) = \frac{c}{3} \log \frac{l}{a} + c_1$$

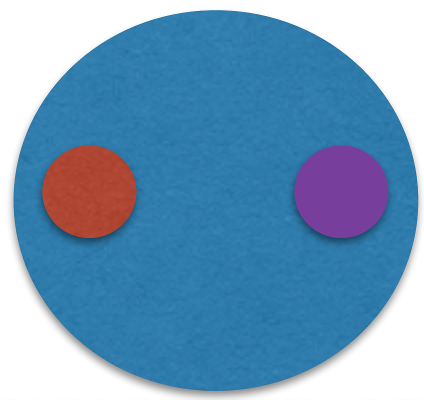
$$l \gg \xi \quad S_A(l) \to \frac{c}{3} \log \frac{\xi}{a}$$

QCD theory (T=0)

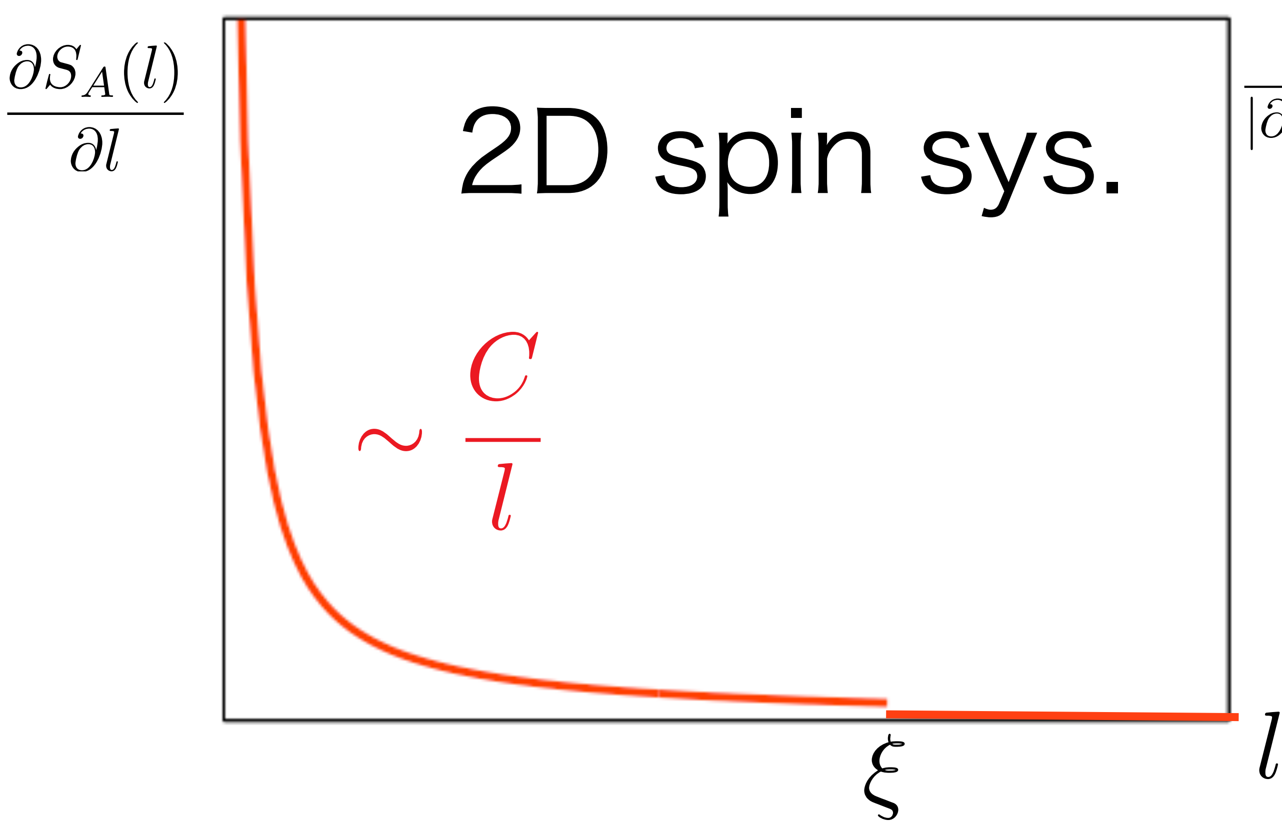
A color confinement changes the effective d.o.f of the system

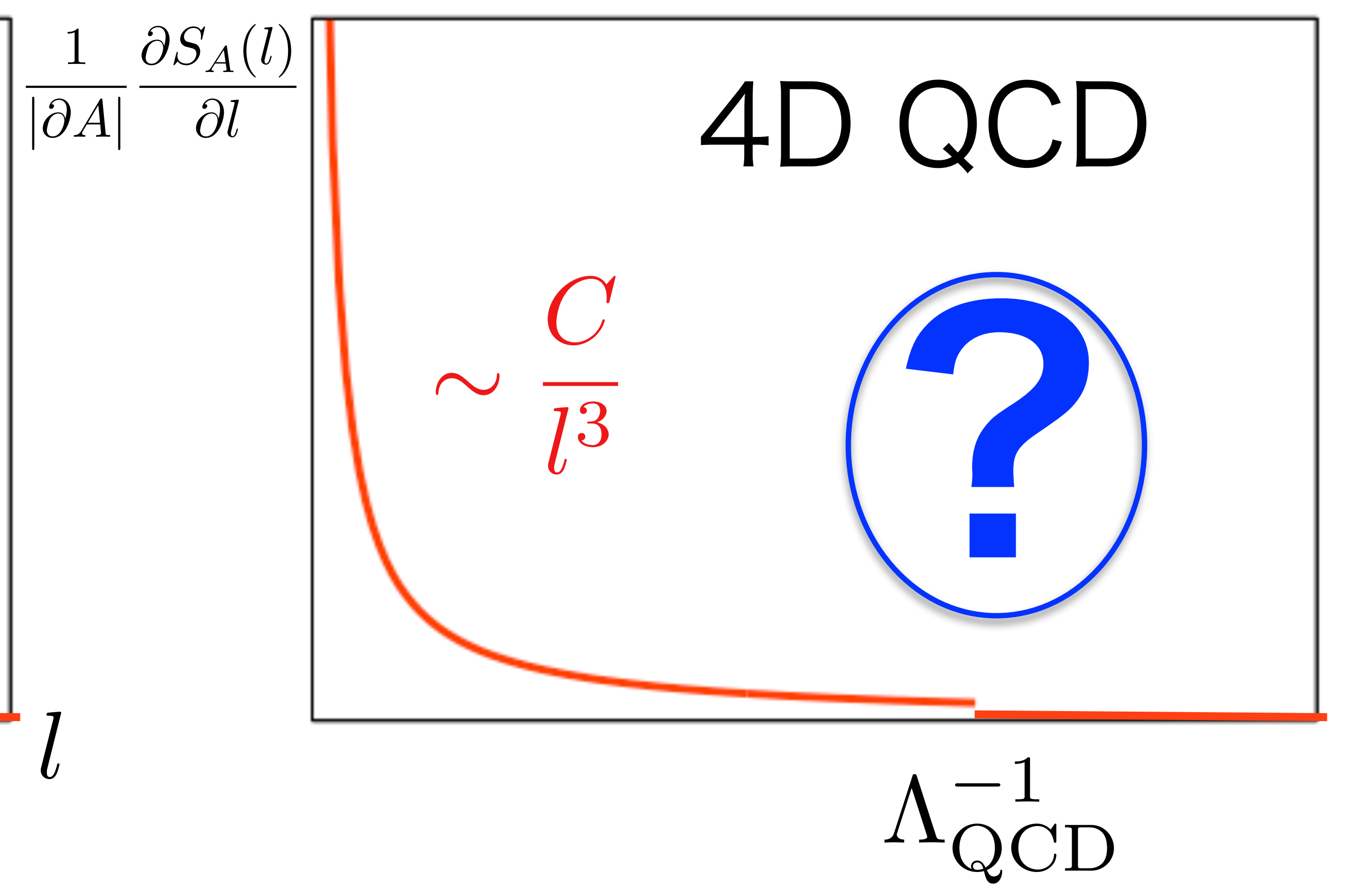
microscopically Λ_{QCD} macroscopically

colorful (gluons) $\sim O(N_c^2)$



colorless (singlet) $\sim O(1)$





Two issues to obtain E.E. in 4d gauge theory

- UV cutoff dependence of 4d E.E.

2d

$$S_A(l) = c_0 \log(l/a)$$

4d

$$S_A(l) = c_0 \frac{\text{Area}}{a^2} - c_0' \frac{\text{Area}}{l^2} + c_1 \log(l/a) + (\text{regular terms})$$

Entropic C-function $\quad C(l) = l^3 \frac{1}{|\partial A|} \frac{\partial S_A}{\partial l}$

Ryu and Takayanagi:PRL96(2006)181602
JHEP 0608(2006)045

cf) Holographic approach $\qquad$ C is obtained by AdS and QFT

- (local) gauge invariance

E.E. for gauge theory

- P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

  extended Hilbert space

- H.Casini, M.Muerta and J.A.Rosabal arXiv:1312.1183

  electric b.c.(electric center), magnetic center, trivial center

- D.Radicevic arXiv:1404.1391

  magnetic center

- W.Donnelly PRD85 (2012) 085004

  extended lattice construction

- S.Ghosh, R.M.Soni,S.P.Trivedi arXiv:1501.02593

- S.Aoki, T.Iritani, M.Nozaki, T.Numasawa, N.Shiba, H.Tasaki arXiv:1502.04267

  maximally gauge invariant reduced density matrix

E.E. for gauge theory

- P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

  extended Hilbert space

- H.Casini, M.Muerta and J.A.Rosabal arXiv:1312.1183

  electric b.c.(electric center), magnetic center, trivial center

- D.Radicevic arXiv:1404.1391

  magnetic center

- W.Donnelly PRD85 (2012) 085004

  extended lattice construction

- S.Ghosh, R.M.Soni,S.P.Trivedi arXiv:1501.02593

- S.Aoki, T.Iritani, M.Nozaki, T.Numasawa, N.Shiba, H.Tasaki arXiv:1502.04267

  maximally gauge invariant reduced density matrix

red definitions are inadequate for E.E. or rhoA

E.E. for gauge theory

- P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

  extended Hilbert space

Our work

- H.Casini, M.Muerta and J.A.Rosabal arXiv:1312.1183

  electric b.c.(electric center), magnetic center, trivial center

- D.Radicevic arXiv:1404.1391

  magnetic center

- W.Donnelly PRD85 (2012) 085004

  extended lattice construction

- S.Ghosh, R.M.Soni,S.P.Trivedi arXiv:1501.02593

- S.Aoki, T.Iritani, M.Nozaki, T.Numasawa, N.Shiba, H.Tasaki arXiv:1502.04267

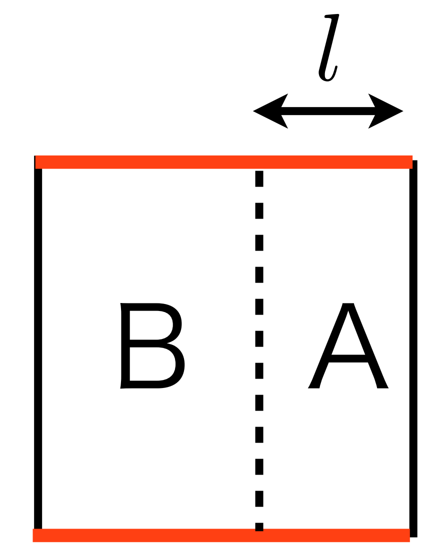
  maximally gauge invariant reduced density matrix

red definitions are inadequate for E.E. or rhoA

Strategy of our simulation

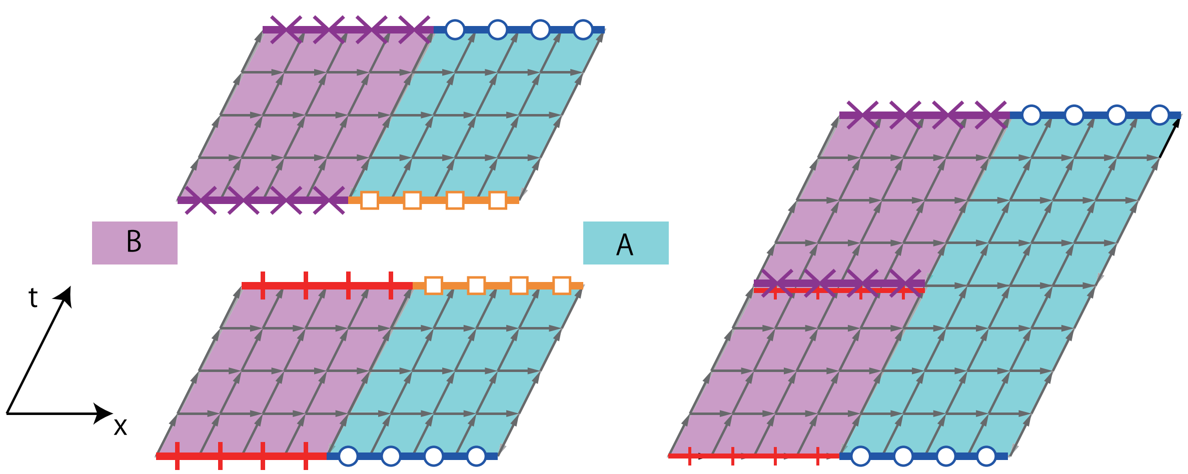
(Gauge fixing is unnecessary)

- gauge configuration generation (on the n=2 replica lattice)
- measurement of observable (the action density)



$$\frac{\partial S_A(l)}{\partial l} = \lim_{n \to 1} \frac{\partial}{\partial l} \frac{\partial}{\partial n} F[l,n] \;= \int_0^1 d\alpha \langle S_{l+a}[U] - S_l[U] \rangle_\alpha$$

Fodor (2009)



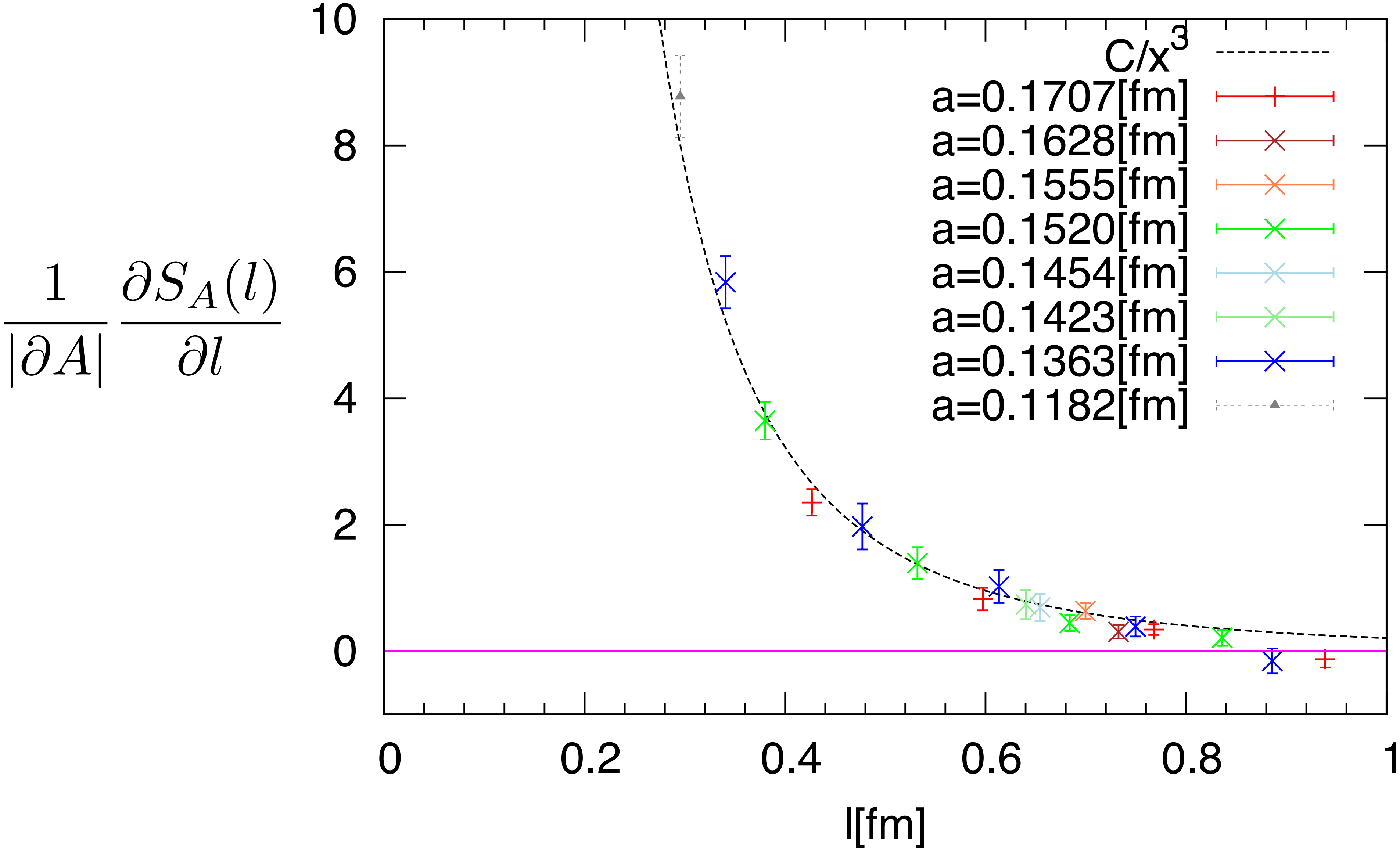
n=2 replica lattice

- Period in temporal dir. on the replica lattice depends on the x-coordinate.

Simulation results

Lattice results for SU(3) gauge theory

T=0, quenched QCD



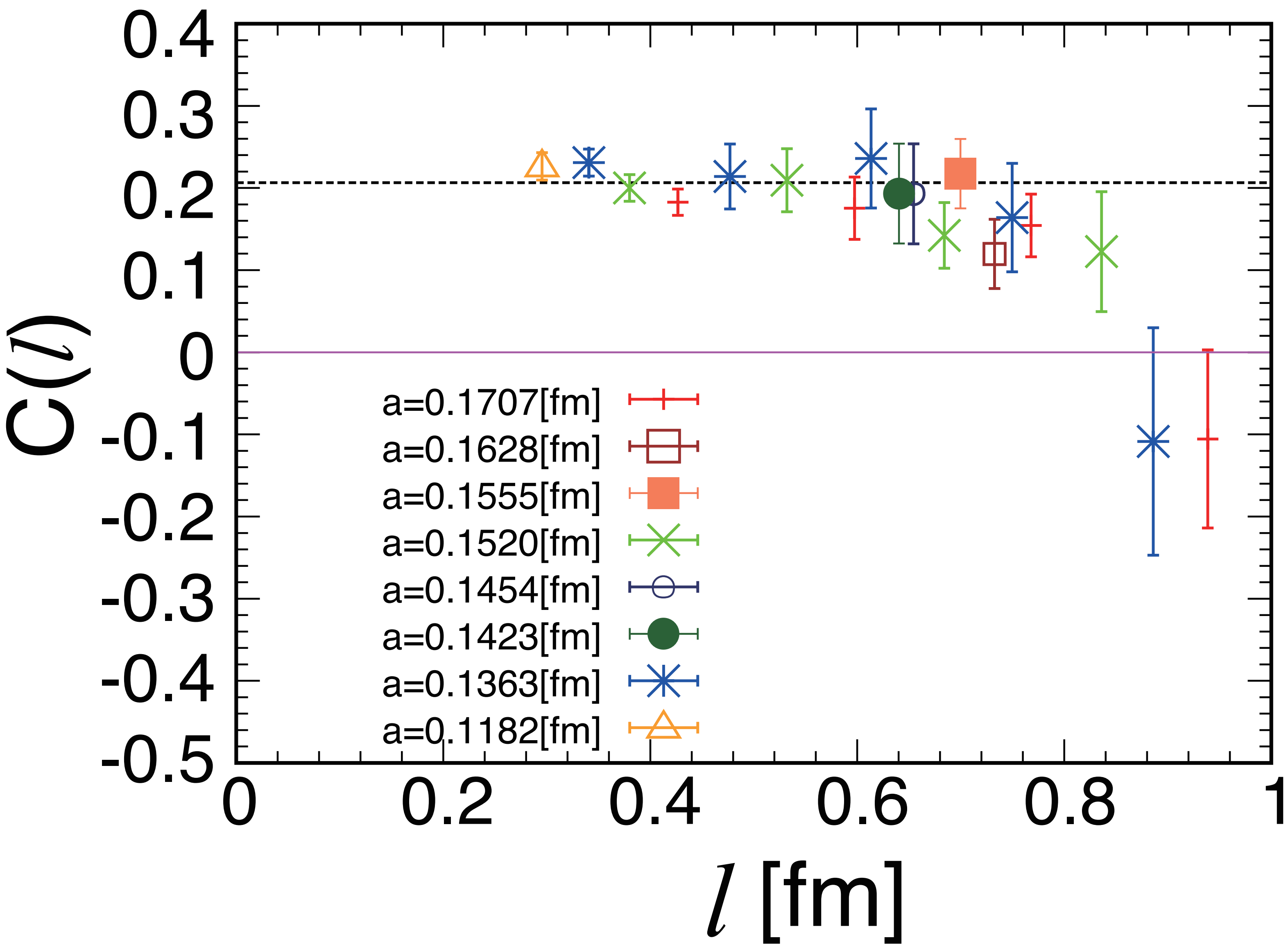
4d

$$S_A(l) = c_0 \frac{\text{Area}}{a^2} - c_0' \frac{\text{Area}}{l^2} + c_1 \log(l/a) + (\text{regular terms})$$

- in short range, 1/l^3 scaling
- confirm the area law
- leading term of l is dominated

Entropic C-function

$$C(l) = l^3 \frac{1}{|\partial A|} \frac{\partial S_A}{\partial l}$$



- In short l region, C is constant.
- Best fit value: C=0.2064(73)

- The discontinuity is not clear.

cf.)

$$\frac{1}{\Lambda_{\rm QCD}} \left(\frac{1}{T_c} \right) = 0.7 - 0.8[\rm fm]$$

Comparison with free field approximation

Free field calculation:

(3+1)-dim. CFT $$\frac{1}{|\partial A|}S_A(l) = c\frac{N_c^2}{a^2} - c'\frac{N_c^2}{l^2}$$

Estimation for non-abelian gauge theory

Ryu and Takayanagi:JHEP 0608(2006)045

$$C_{\mathrm{gauge}} \sim 2c' \cdot 2 \cdot 8 \sim 0.1568$$

$c' \sim 0.0049$ for free real scalar theory $$A_\mu^a \ \left\{ \begin{array}{l} a = 1, \cdots, 8 \\ \mu = 1, \cdots, 4 \end{array} \right.$$

Casini and Huerta:
Phys. Rev. D 93, 105031 (2016), J. Phys. A42, 504007 (2009)

$$C_{\mathrm{gauge}} \sim 0.177$$

$$(c' \sim 0.00553...)$$

Our numerical result

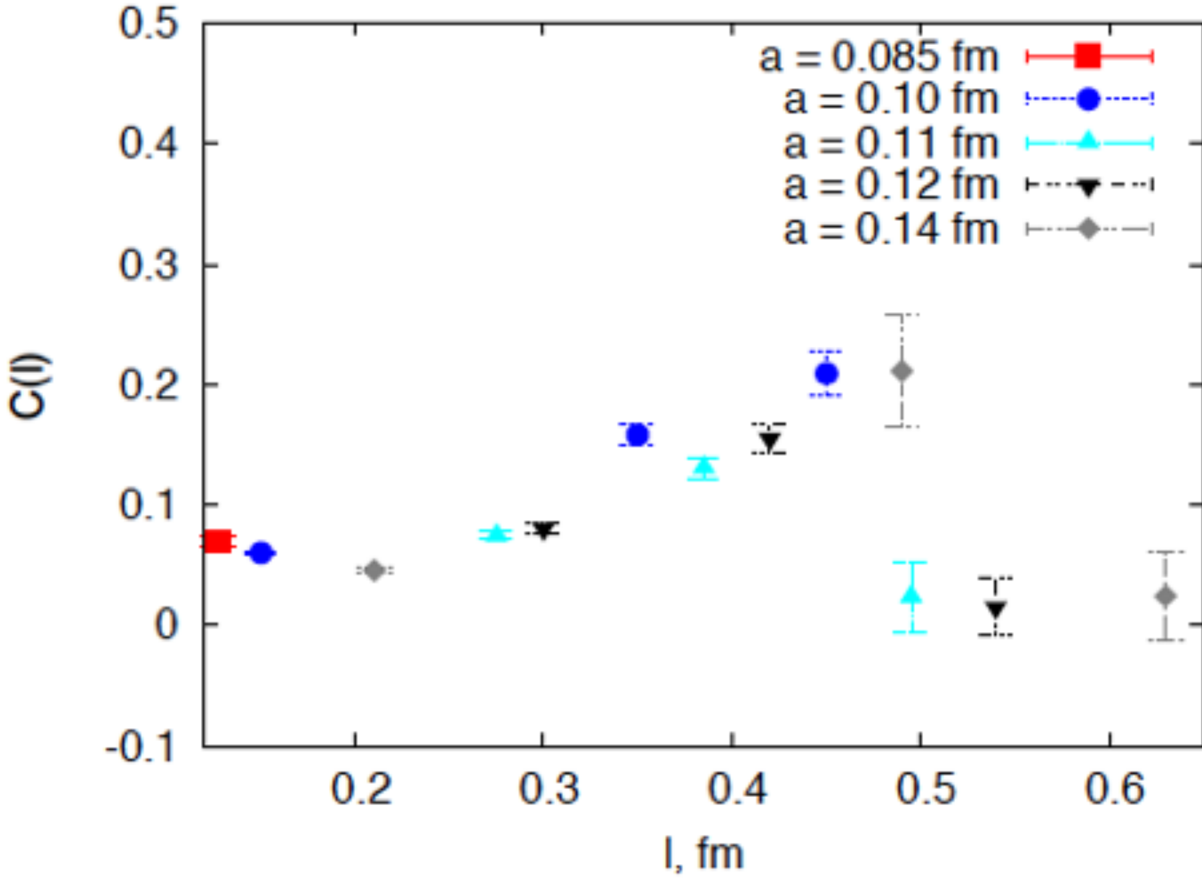
$$C_{\mathrm{gauge}} \sim 0.2064$$

Almost consistent

just below QCD scale, the free field app.
is well described.

Comparison with other results

SU(2) gauge theory



P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

- l<0.3[fm] C(l) = 0.07 — 0.08

Our SU(3) result: C=0.2064(73)

Scaling down to SU(2): $0.2064 \times \frac{3}{8} \sim 0.0774$

- enhancement around l ~0.5[fm]
- discontinuity exists
- # of configs. ~ 100 cf) SU(3):50,000

Hagedorn-type model

I. R. Klebanov, D. Kutasov and A. Murugan; Nucl.Phys. B796 (2008) 274

At large-Nc, confining theory can be described by free glueballs

sum of all glueballs spectrum $$C_{\rm total} \sim \int m^{\beta} e^{(\beta_H - 2l)m}$$

- critical length $l_c \sim T_H^{-1}/2$
- discontinuity exists

☆finite Nc correction?
☆instanton effect?
☆model is too naive?

Summary

- This is the first precise determination of E.E. for quenched QCD
- c-fn. in short regime correctly counts # of gluons
- Discontinuity around lc is unclear as contrast with SU(2)/ large-N results

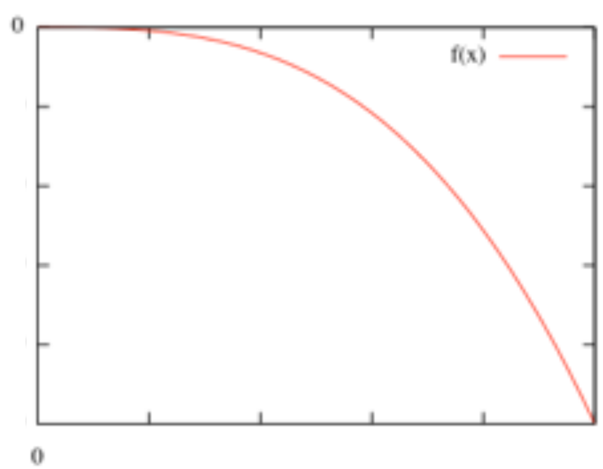
Future directions

If "it looks very promising (or trivial)"

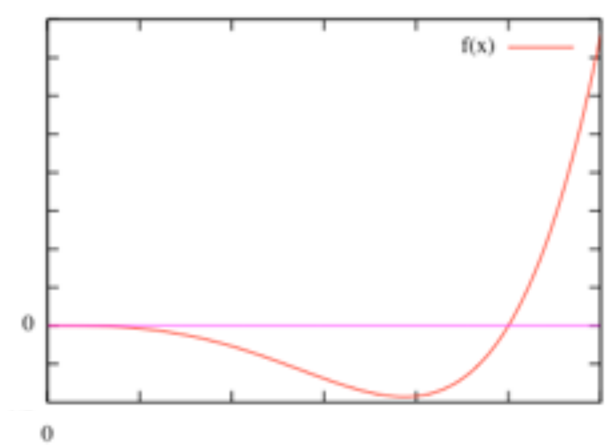
- Real QCD including quarks
- Conformal window in 4d SU(3) many Nf theory (a- /or c- fn.)

beta fn.

Nf<9



9<Nf<17



nontrivial IR fixed point is found by lattice simulation cf) E.I. PTEP(2013)083B01

If not…

theoretical side

1/Nc correction
nonperturbative vacuum
improved Hagedorn model

Lattice side

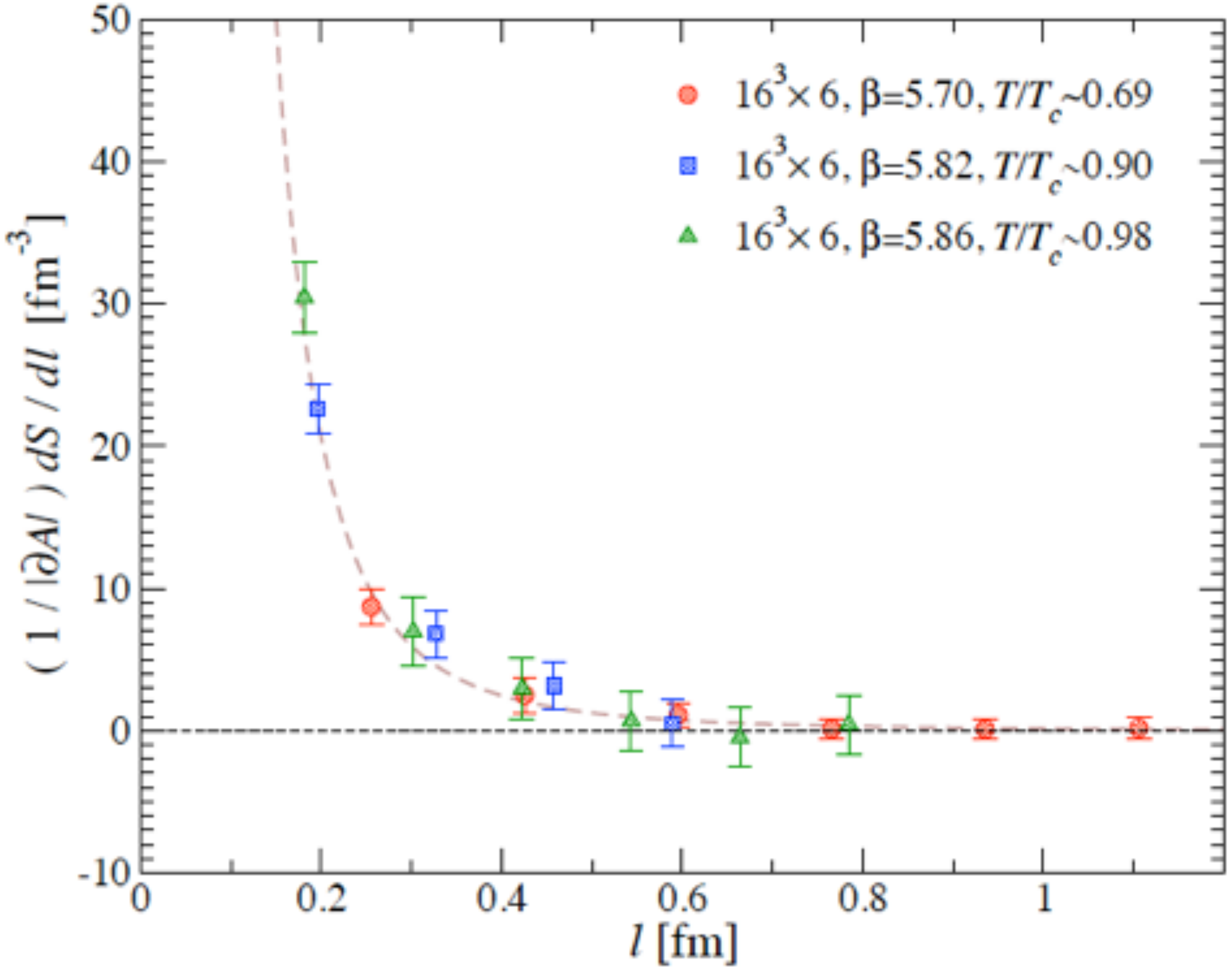
- UV cutoff independence?
- replica number (n ->1) dependence

backup

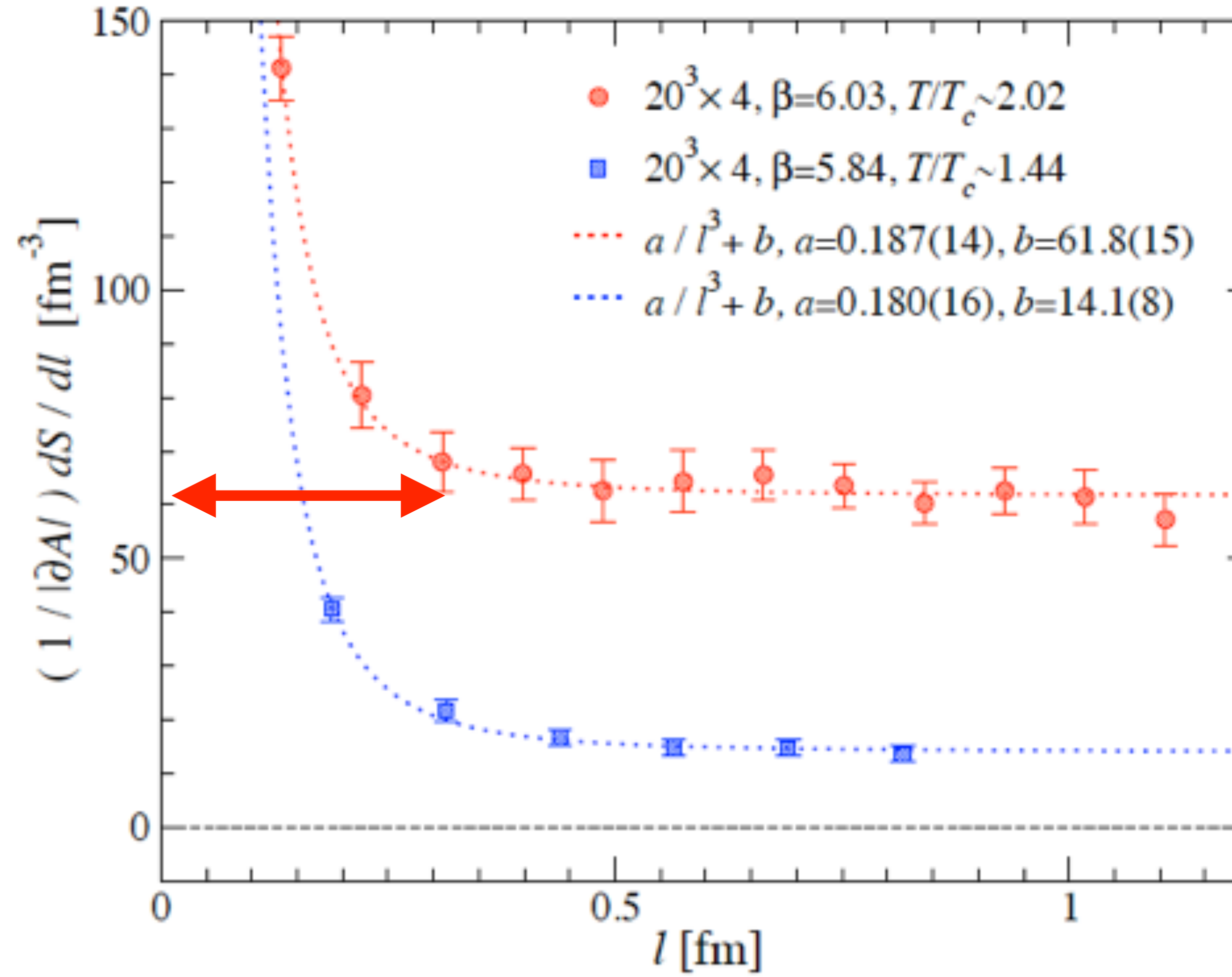
finite T for quenched SU(3)

arXiv:1104.1011: Y.Nakagawa et al.

T< Tc



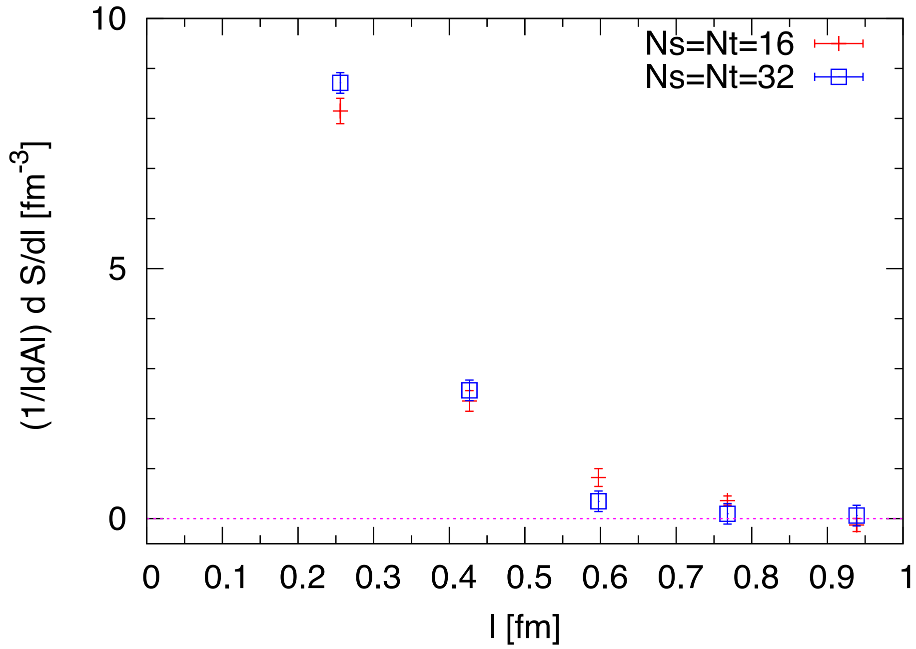
T >Tc



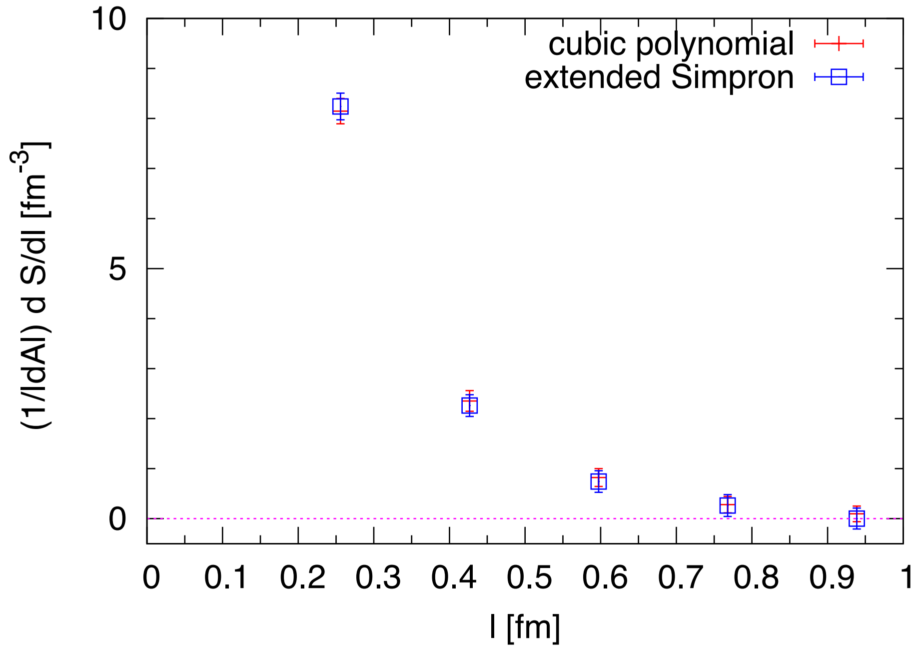
thermal entropy	replica method	integration method (2% error)
		Boyd et al.:NPB469,419(1996)
$T\sim 1.44T_c$	$s\sim 14.1(8)$	17
$T\sim 2.02T_c$	$s\sim 61.8(15)$	56

Detail analyses

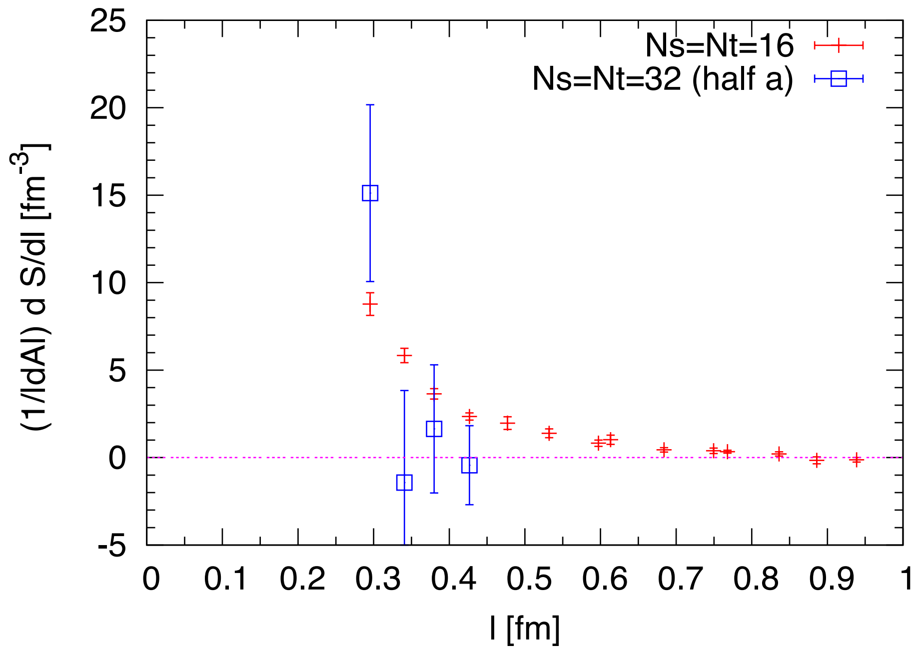
finite vol. effect



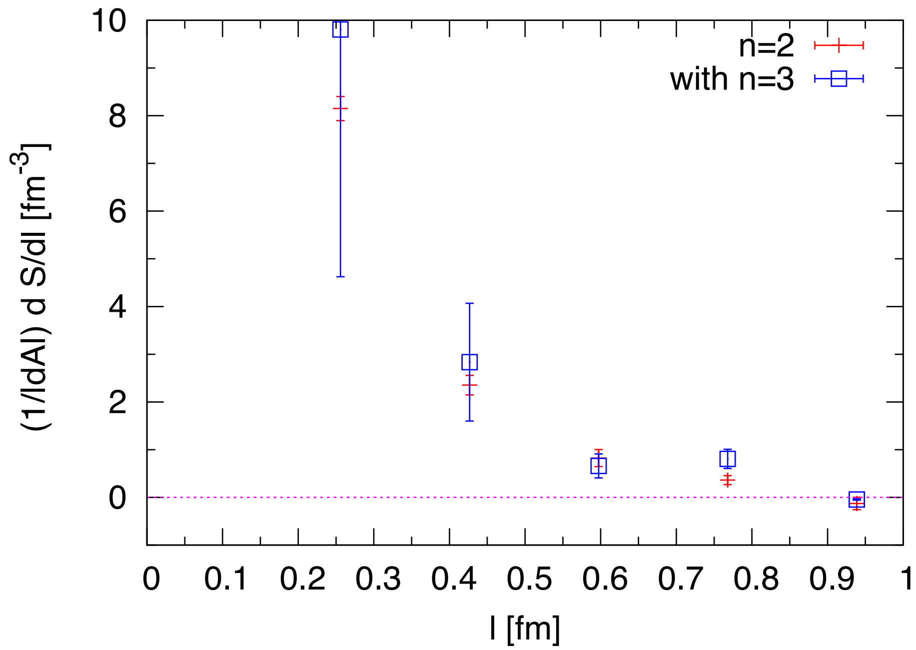
algorithm dependence of the numerical integration



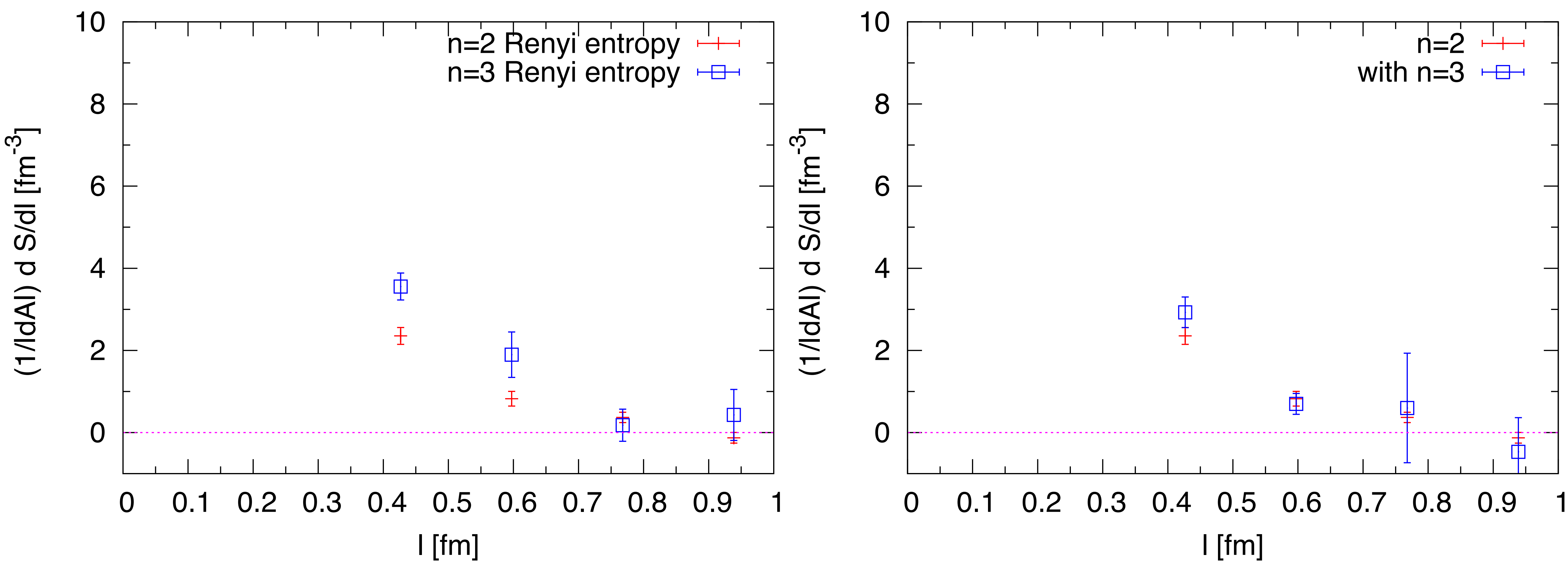
UV cutoff dependence



replica number dependence



replica number dependence



$$\lim_{n\to 1}\frac{\partial}{\partial n}F[l,n] = (F[l,n=2]-F[l,n=1])/\Delta n|_{\Delta n=1}$$

$$-\frac{\Delta n}{2}(F[l,n=3]-2F[l,n=2]+F[l,n=1])|_{\Delta n=1}$$