A precision test of AdS/CFT with flavor

Talk by Andreas Karch, (UW Seattle) at "Quantum Matter, Spacetime and Information" conference

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work in collaboration with Brandon Robinson and Christoph Uhlemann

How do we know AdS/CFT is correct?

It is difficult to prove AdS/CFT.
Equality between what?

How do we know AdS/CFT is correct?

N=4 SYM with gauge group SU(N)

=

Type IIB string theory on $AdS_5 \times S^5$

How do we know AdS/CFT is correct?

N=4 SYM with gauge group SU(N)

This is a gauge theory. In principle defined on lattice. Well posed problem.

=

Type IIB string theory on $AdS_5 \times S^5$

How do we know AdS/CFT is correct?

N=4 SYM with gauge group SU(N)

=

We basically only know this as a perturbative expansion.
These days we say its non-perturbatively defined via AdS/CFT.
Therefore true by assumption?

Type IIB string theory on AdS_5 x S^5

How do we know AdS/CFT is correct?

N=4 SYM with gauge group SU(N)

=

Practical question: does IIB SUGRA + classical strings describe the strong coupling, large N limit of N=4?

Type IIB string theory on $AdS_5 \times S^5$

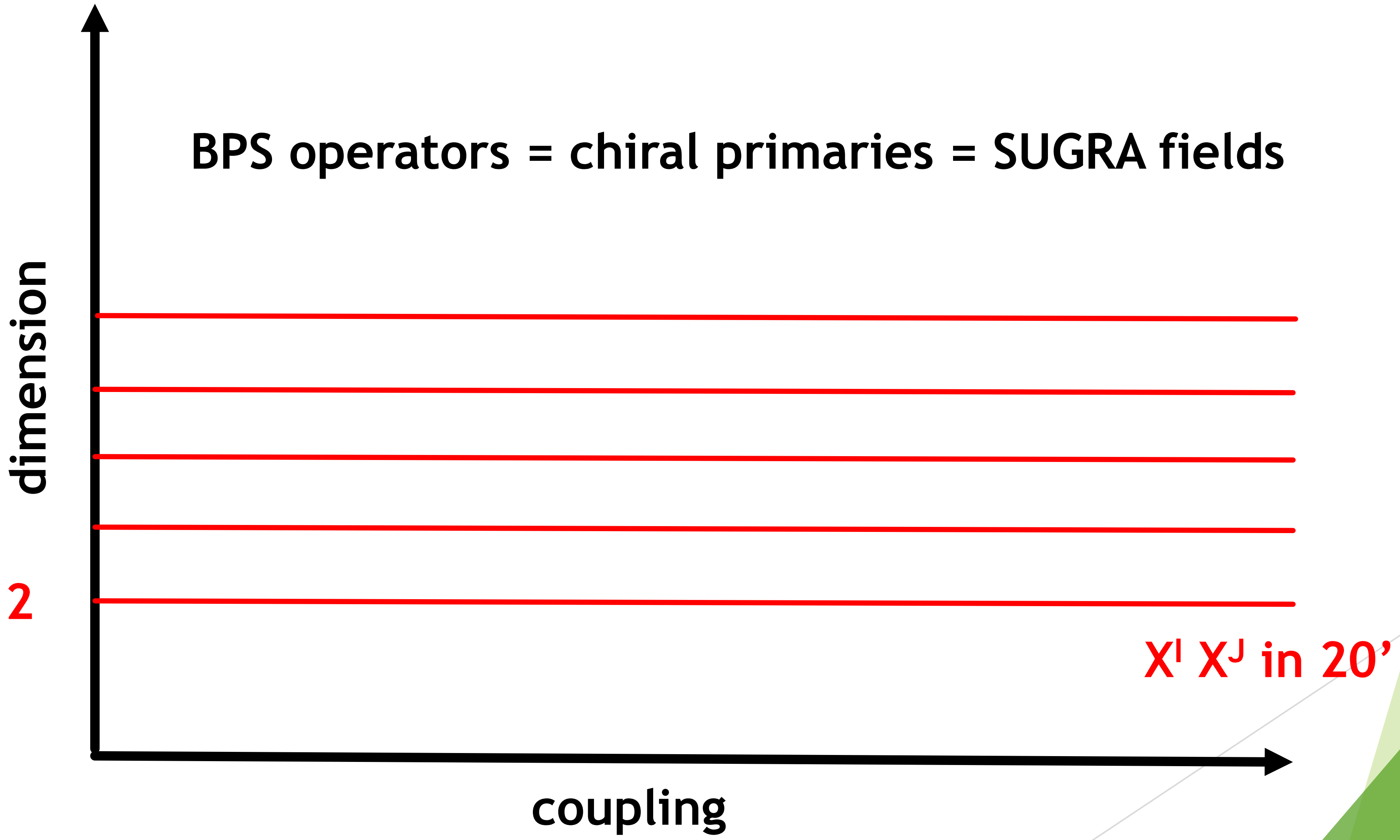
Does IIB SUGRA describe strongly coupled N=4?

Established beyond reasonable doubt.

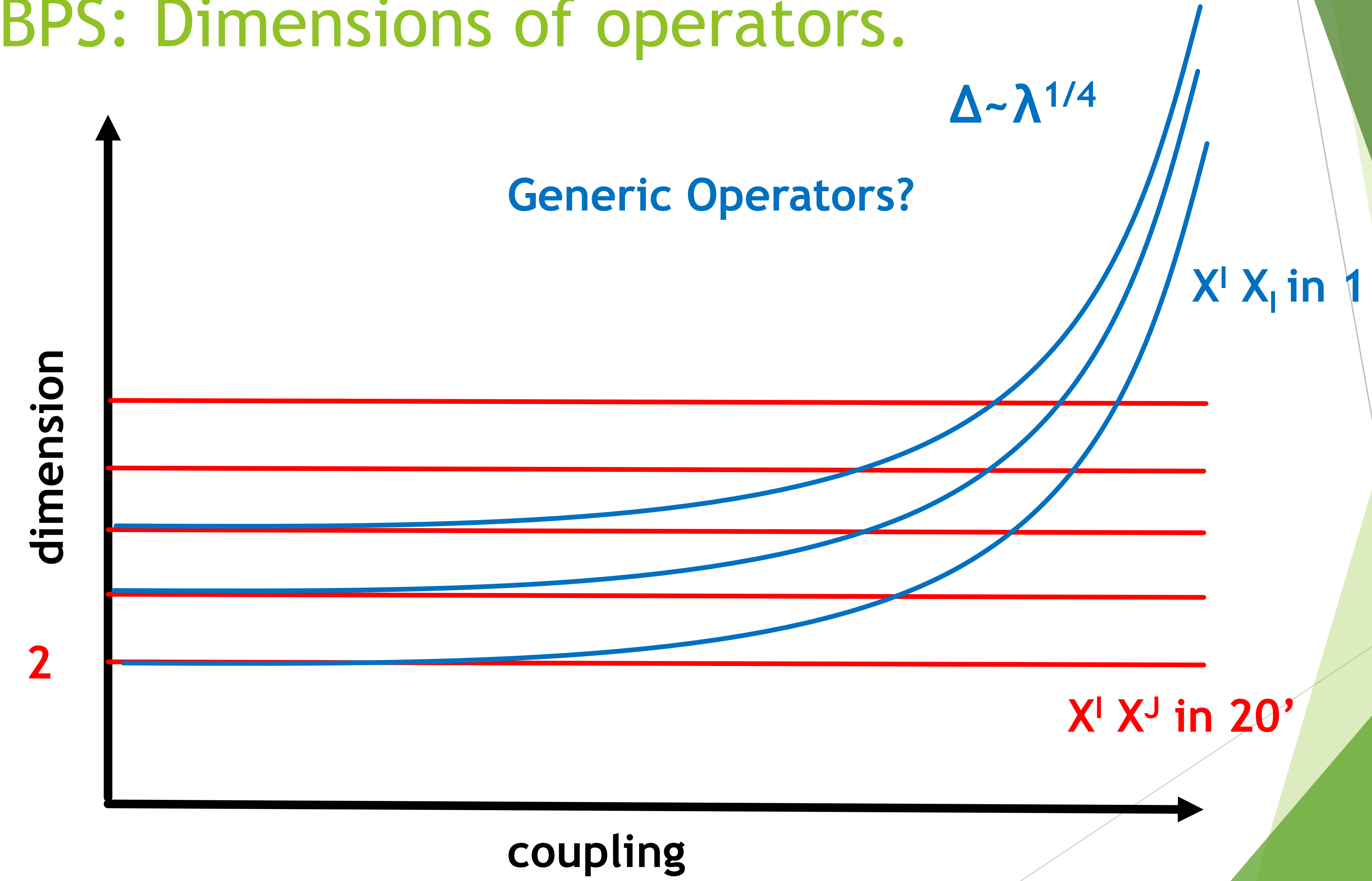
Early evidence:

BPS quantities. Take the same value at all couplings.

BPS: Dimensions of operators.



BPS: Dimensions of operators.



Non-BPS evidence also exists!

About 10 years after the AdS/CFT proposal

Integrability

Beisert, Eden, Staudacher → Ansatz for dimension of certain operatos as function of coupling.

Inspired guess work

BES conjecture matches both weak and strong expansion.

Additional evidence:

- Qualitative Predictions sensible.

  - Thermodynamics
  - Entanglement Structure
  - Correlation functions
  - Real time dynamics

Additional evidence:

- Numerical checks (in low dimensions).

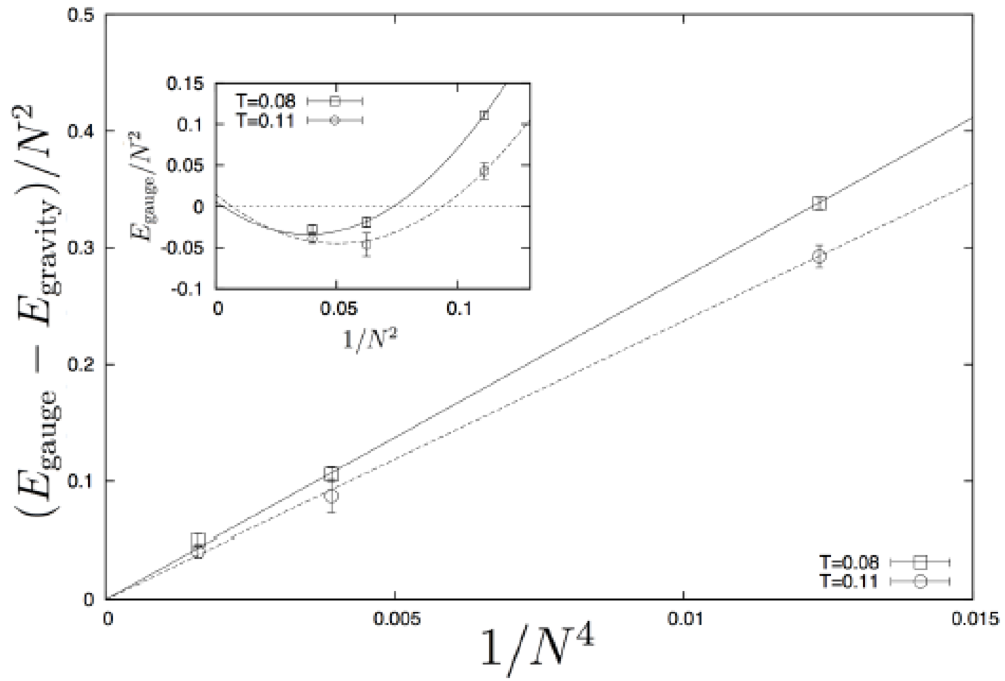


Figure 4: **The difference $(E_{\mathbf{gauge}} - E_{\mathbf{gravity}})/N^2$ as a function of $1/N^4$.** We show the results for $T = 0.08$ (squares) and $T = 0.11$ (circles). The data points can be nicely fitted by straight lines passing through the origin for each T. In the small box, we plot $E_{\rm gauge}/N^2$ against $1/N^2$ for $T = 0.08$ and $T = 0.11$. The curves represent the fits to the behavior $E_{\rm gauge}/N^2 = 7.41\, T^{2.8} - 5.77\, T^{0.4}/N^2 + \text{const.}/N^4$ expected from the gravity side.

(Hanada, Hyakutake, Ishiki, Nishimura, published in SCIENCE)

Monte-Carlo simulation of D0 brane quantum mechanics.

Rigorous checks from localization.

The probably most compelling checks performed to date probably come using the technique of (supersymmetric) localization.

Rigorous checks from localization.

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NOT

Localization:

Starting point: Nil-potent symmetry generator:

$$Q^2 = 0$$

Easily found in theories with extended supersymmetry

Plays role of exterior derivative operator on field space:

$$d^2 = 0$$

Localization:

In particular, "Stokes theorem" for Path integrals now reads:

$$\int \mathcal{D}\phi \, Q\,(\ldots) \;=0$$

- Integral of total derivative vanishes
- Need to be able to drop boundary terms.
- … includes $e^{(\text{-action})}$ term that suppresses "boundaries of field space"

Localization

Want: $$\mathcal{Z} = \int D\phi e^{-S[\phi]}$$ ∞-dim configuration space

$$Q^2 V = QS = 0$$ (meaning, Q is a symmetry)

Define: $$\mathcal{Z}(t) = \int D\phi e^{-S[\phi]-tQV}$$

with: $$\partial_t \mathcal{Z}(t) = \int D\phi\, Q\left(V e^{-S[\phi]-tQV}\right) = 0$$

Localization.

$$\mathcal{Z}(t) = \int D\phi e^{-S[\phi]-tQV}$$

independent of t.

(also true if we insert any operators A with QA=0)

At t $\rightarrow$ infinity the path integral is dominated by saddle

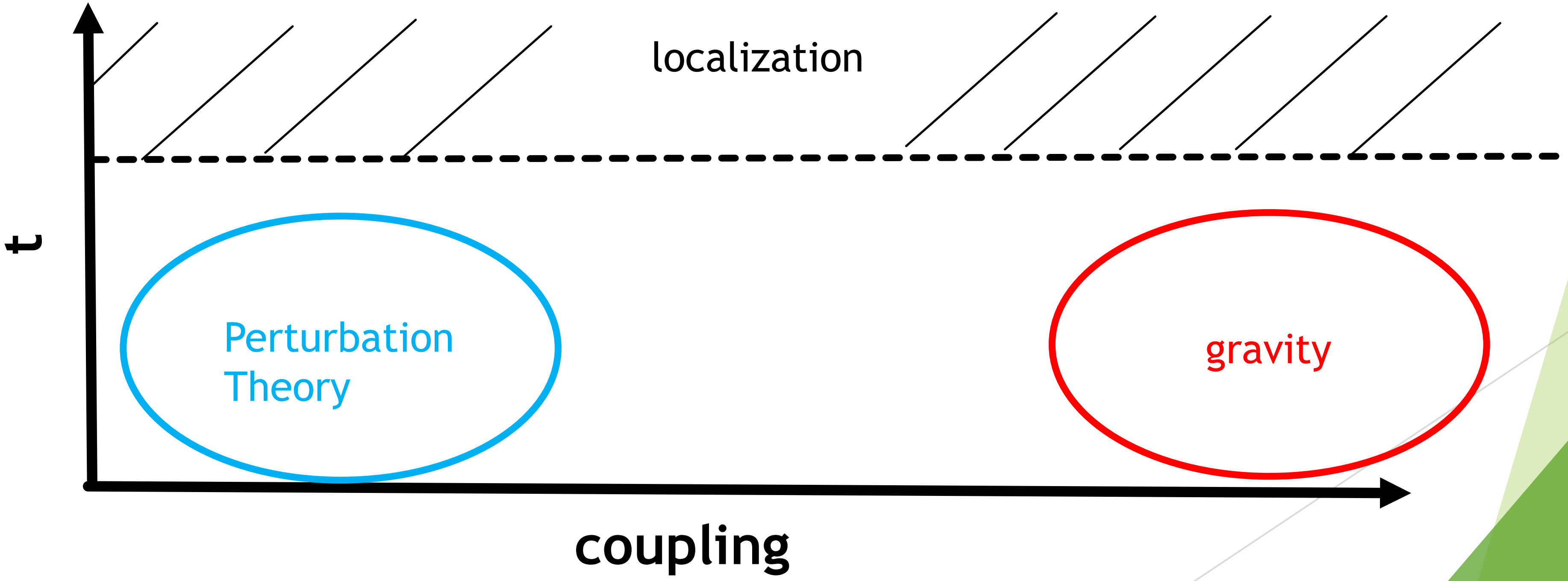
Path integral localizes to zeroes of QV.

$$\mathcal{Z} = \lim_{t\to\infty} \mathcal{Z}(t) = \int D\phi_{QV}\, Z_{1-\text{loop}}\, e^{-S[\phi]-tQV|_{t\to\infty}}$$

Often path integral reduces to sum/ordinary integral

Localization and AdS/CFT

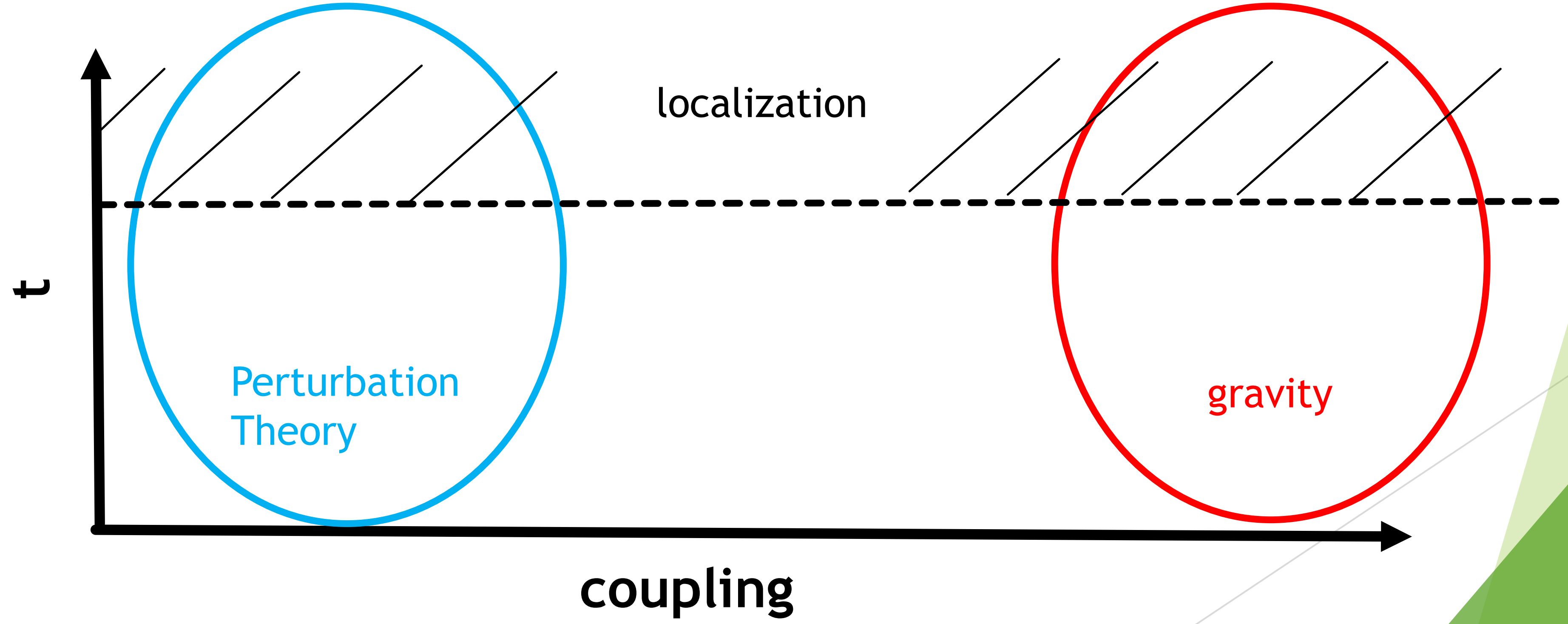
For generic quantities, localization is just another limit.



Localization and AdS/CFT

But free energy of N=4 SYM independent of t !!

Also works for expectation value of SUSY Wilson loops.



Free energy of N=4 on S^4

Can calculate free energy of N=4 SYM on S^4 at any coupling and compare to supergravity.

$$F = -\log Z$$

But: scheme dependent!

$$S = \ldots + \int R^2$$

Finite counterterms.

- no dynamical field
- local in "sources" (here metric)
- only affect contact terms
- coefficient ambiguous

Free energy of N=2* (massive adjoint hypermultiplet) on S^4

[Pestun '07]

Need second mass scale (in addition to radius).

$$F = F(m * L)$$

Finite number of scheme dependent terms.

[Bobev,Elvang,Freedman,Pufu '13]

Perfect agreement.

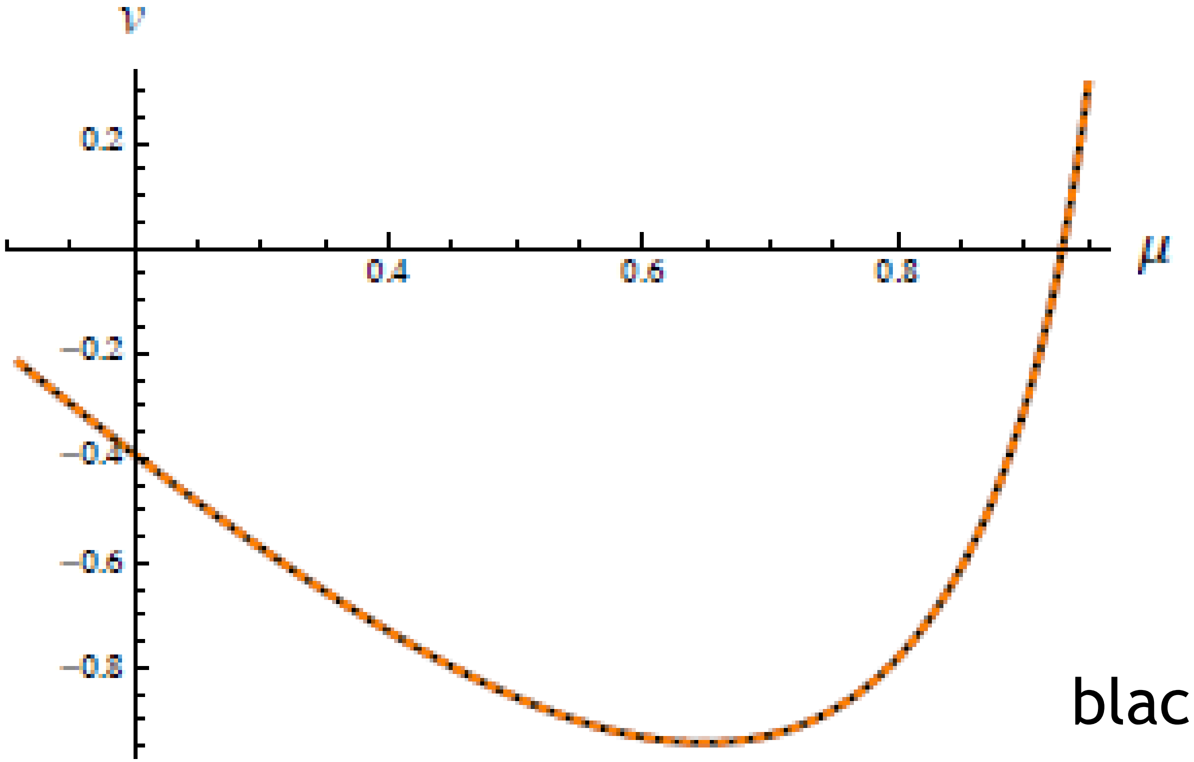
Alternatively: Wilson loops in N=4

[Ericksson,Semenoff,Zarembo '00]

[Pestun '07]

Free energy of N=2*

[Bobev,Elvang,Freedman,Pufu '13]



orange:
numerical sugra solution

black:

$$v(\mu) = -2\mu - \mu\log(1-\mu^2)$$

analytic answer from localization.

? We should be able to do better ?

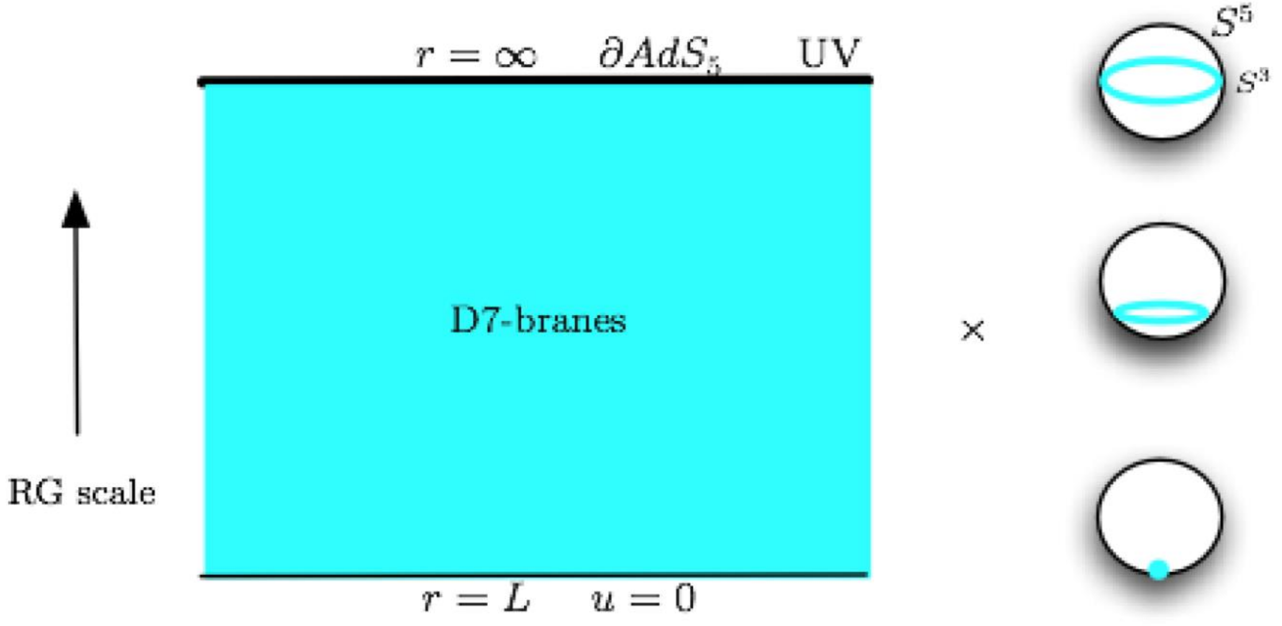
Does IIB SUGRA describe strongly coupled N=4?

Established beyond reasonable doubt.

Testing flavored holography

Flavored Holography

(Katz, AK)



(picture from CLMRW-review, 2011)

Add fundamental matter quarks via probe branes!

N, not N^2 ; quenched

tension negligible;
no backreaction

Numerous applications

- No QCD without quarks
- Wilson lines
- Simplest model of dissipation
- Holographic lattices
- charged matter for CM applications
- non-equilibrium steady states
- interacting topological states
- single EPR pairs
- ….

Extra subtleties.

Many questions beyond the probe limit:

- Asymptotic freedom lost
- are there still branes in backreacted geometry?
- can the probe be completely geometrized?
- …..

Could this all be wrong???

Extra subtleties.

Many questions beyond the probe limit:

- Asymptotic freedom lost
- are there still branes in backreacted geometry?
- can the probe be completely geometrized?
- …..

In any case: not nearly as well tested as N=4/AdS

Goal:

Calculate the free energy of a massive fundamental representation
$N=2$ supersymmetric hypermultiplet coupled to $N=4$ SYM on S^4 using localization
and compare, in the large N strong coupling limit, to the probe brane answer.

Supersymmetry on curved space.

Challenge 1:

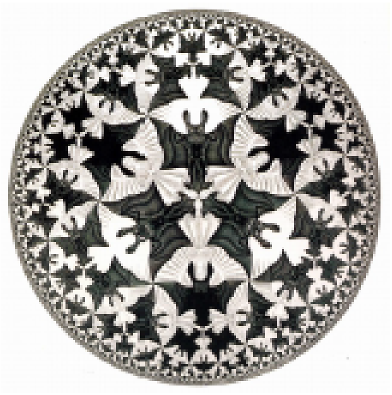
Generically SUSY completely broken by connection terms in action.

For superconformal theories SUSY obviously preserved for spaces that are conformally flat:

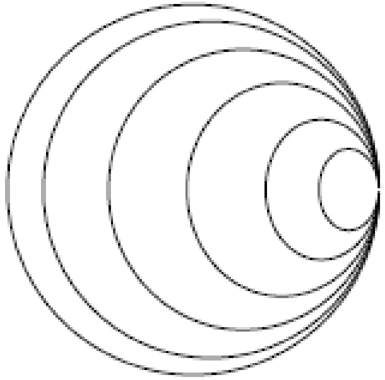
AdS_4 S^4

Conformal field theory on conformally flat spaces.

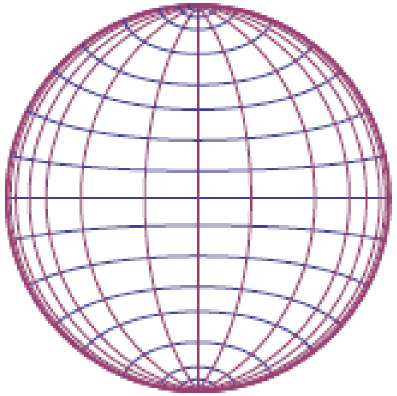
In simple cases: just choose different coordinates on AdS_5, different representative of boundary conformal structure



$$\frac{dz^2 + (1 - \frac{z^2}{4})g_{S^4}}{z^2}$$



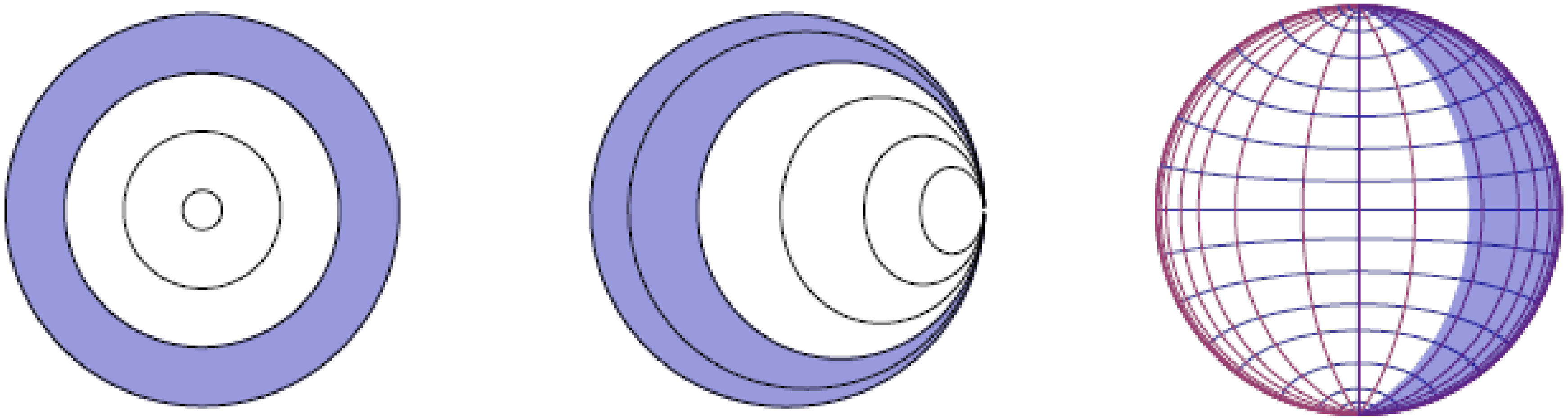
$$\frac{dz^2 + \eta}{z^2}$$



$$\frac{dz^2 + (1 + \frac{z^2}{4})g_{AdS_4}}{z^2}$$

Boundary geometry: S^4, $\mathbb{R}^{1,3}$ and two copies of AdS_4

Generically mass terms break SUSY.



$$\frac{dz^2 + (1 - \frac{z^2}{4}) g_{S^4}}{z^2} \qquad \frac{dz^2 + \eta}{z^2} \qquad \frac{dz^2 + (1 + \frac{z^2}{4}) g_{\mathrm{AdS}_4}}{z^2}$$

Boundary geometry: S^4, $\mathbb{R}^{1,3}$ and two copies of AdS_4

A distinction without much of a difference for conformal theories.
Not with massive flavors: geometrically different D7 embeddings.

Non-supersymmetric embeddings

These non-supersymmetric embeddings have been constructed before.

(AK, O'Bannon, Yaffe, ...)

Exhibit interesting topology changing phase transition with universal, calculable exponents:

Not suited for our purpose.

a	$\beta_\pm$
7	$-3 \pm \sqrt{2}$
6	$-\frac{5}{2} \pm \frac{1}{2}$
5	$-2 \pm i$
4	$-\frac{3}{2} \pm i\frac{\sqrt{7}}{2}$
3	$-1 \pm i\sqrt{2}$
2	$-\frac{1}{2} \pm i\frac{\sqrt{7}}{2}$

Restoring SUSY.

To restore (at least some) SUSY we need to add new terms to the action. Compensating terms.

Ex: “topological twisting” (Witten)

Compensating term = background R-charge gauge field equal to spin connection.

Keeps some SUSY alive on any curved space (creates a scalar supercharge).

Restoring SUSY.

For special spaces, simpler compensating terms suffice:

(Pestun)

superpotential mass accompanied with purely scalar mass.

AdS_4 real mass

S^4 imaginary mass $\rightarrow$ unitarity lost

Can be understood as due to auxiliary terms in non-dynamical supergravity background.

(Festucchia, Seiberg)

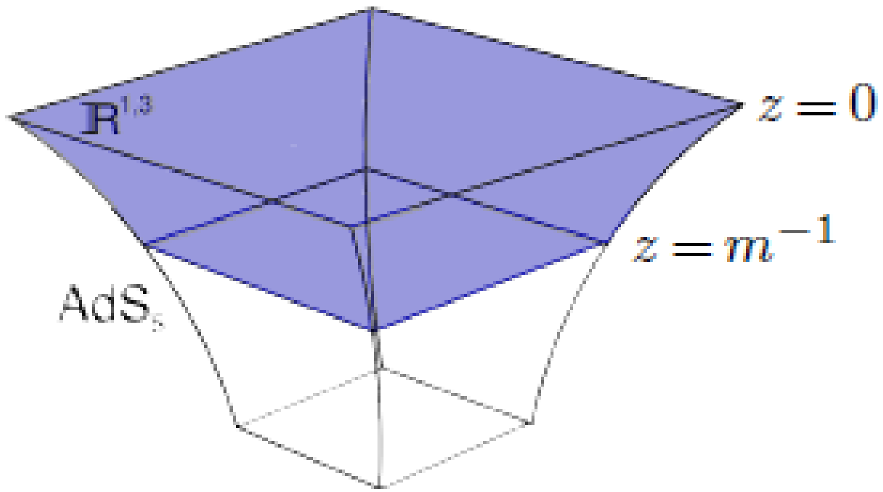
Holographic compensating terms.

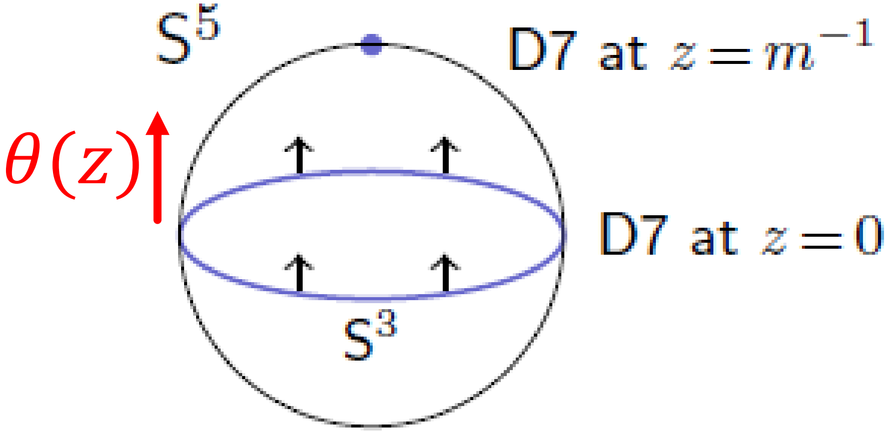
To find holographic duals to SUSY theories, compensating terms are crucial.

N=2* on Minkowski: 2 scalars in 5d gauged sugra turned on

N=2* on sphere: **3** scalars in 5d gauged sugra turned on

Mass terms for flavor branes.

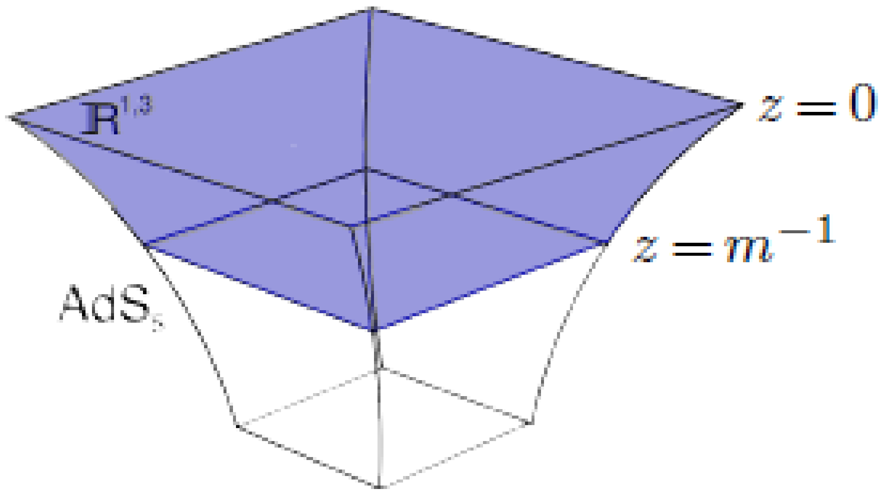


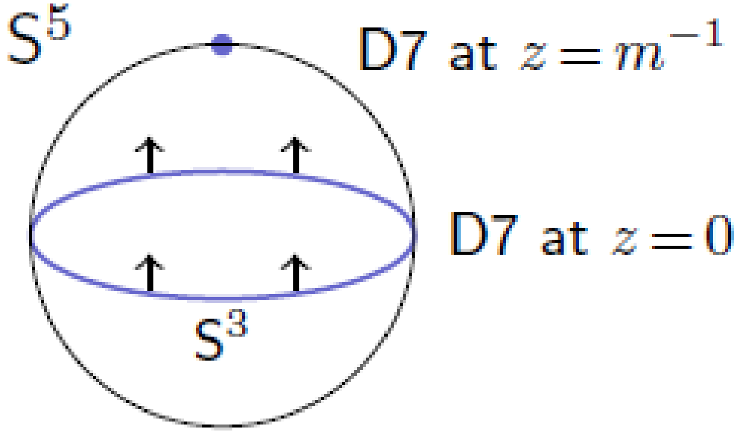


Superpotential mass = slipping mode

$\theta(z)$

Mass terms for flavor branes.





compensating scalar mass = internal gauge field

$$A = f(z)\omega$$

particular spherical harmonic on S^3

(Kruczenski, Mateos, Myers, Winters)

Mass terms for flavor branes.

SUSY embedding completely characterized by two scalar functions:

$$\theta(z), \mathrm{f}(z)$$

(f purely imaginary for flavors on S^4)

DBI action for D7-branes with gauge field in that background:

$$S_{\mathrm{D7}} = -T_7 \int d^8\xi \sqrt{-\det(g+F)} + 2T_7 \int C_4 \wedge F \wedge F$$

$$\begin{aligned}
0 = &\frac{1}{4}\theta' \sin\theta \tanh\rho \left(32 f^2 - 4\cos(2\theta) + \cos(4\theta) + 3\right) \left(2f'^2 - \left(\theta'^2 + 1\right)\cos(2\theta) + \theta'^2 + 1\right) \\
&- \cos\theta \left(2\sin^2\theta \left(2f^2 \left(\theta'^2 + 1\right) + f'^4\right) + f'^2 \left(\theta'^2 + 5\right)\sin^4\theta + 3\left(\theta'^2 + 1\right)\sin^6\theta\right) \\
&+ 4f^2 f'^2 \cos\theta \left(\theta'^2 - 1\right) + 4ff'\sin\theta \left(ff'\theta'' - ff''\theta' + f'^2\theta'\right) \\
&+ 4f\sin^3\theta \left(f\theta'' + f'\left(\theta'^3 + \theta'\right)\right) + f'\sin^5\theta(f'\theta'' - f''\theta') + \theta''\sin^7\theta
\end{aligned}$$

$$\begin{aligned}
0 = &8f\cosh\rho \left(f'^2 + \left(\theta'^2 + 1\right)\sin^2\theta\right) \sqrt{\left(4f^2 + \sin^4\theta\right)\left(f'^2 + \left(\theta'^2 + 1\right)\sin^2\theta\right)} \\
&+ \sin^3\theta\cosh\rho \left(f'\cos\theta \left(2f'^2\theta' + \left(\theta'^3 + \theta'\right)\sin^2\theta\right) - \sin^6\theta \left(f'\theta'\theta'' - f''\theta'^2 - f''\right)\right) \\
&+ 2f^2 \left(2\sin\theta\cosh\rho \left(\sin\theta \left(-f'\theta'\theta'' + f''\theta'^2 + f''\right) - f'\left(\theta'^3 + \theta'\right)\cos\theta\right)\right) \\
&- 2f\cosh\rho \left(f'^2 + \left(\theta'^2 + 1\right)\sin^2\theta\right)\left(2\theta'^2\sin^2\theta - \cos(2\theta) + 1\right) \\
&+ 4\left(4f^2 + \sin^4\theta\right) f'\sinh\rho \left(f'^2 + \left(\theta'^2 + 1\right)\sin^2\theta\right)
\end{aligned}$$

Finding analytic solution hopeless. But good consistency check.

Supersymmetry to the rescue

IIB sugra background: δfermions=0 $\rightarrow$ BPS/Killing spinor eq.

Adding probe D-branes:

- no effect on background or Killing spinor eq. @LO
- superspace embedding $\rightarrow$ too many fermions
- fermionic κ gauge symmetry for #bosons = #fermions
  [Aganagic,Popescu,Schwarz; Cederwall et al.; Bergshoeff,Townsend '96]

Background with D-brane preserves supersymmetries that are generated by Killing spinors and compatible with κ-symmetry.

κ- symmetry for D-branes

Supersymmetries compatible with κ-symmetry: [Bergshoeff, Townsend]

$$\Gamma_\kappa \epsilon = \epsilon$$

$\nearrow$ projector, encodes D7 embedding

$\nwarrow$ background Killing spinor

This equation yields:

- Projection condition on SUSY preserved
- 1^{st} order equation on background fields

The devil is in the details:

$$\Gamma_\kappa = \frac{1}{\sqrt{\det(1+g^{-1}F)}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \dots j_n k_n} F_{j_1 k_1} \dots F_{j_n k_n} J^{(n)}_{(p)}$$

$$J^{(n)}_{(p)} = (-1)^n \, (\sigma_3)^{n+(p-3)/2} \, i\sigma_2 \otimes \Gamma_{(0)}$$

$$\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{-\det g}} \varepsilon^{i_1 \dots i_{p+1}} \gamma_{i_1 \dots i_{p+1}} \,, \qquad \gamma_m = e^a_\mu \Gamma_a \partial_m X^\mu$$

Projector

Background Killing spinor

$$\epsilon = e^{\frac{\theta}{2} i\Gamma^{\underline{\psi}}\Gamma_{\vec{\chi}}} \, e^{\frac{\psi}{2} i\Gamma_{\vec{\chi}}\Gamma^{\underline{\theta}}} \, e^{\frac{1}{2}\chi_1 \Gamma^{\underline{\theta\chi_1}}} \, e^{\frac{1}{2}\chi_2 \Gamma^{\underline{\chi_1\chi_2}}} \, e^{\frac{1}{2}\chi_3 \Gamma^{\underline{\chi_2\chi_3}}}$$

$$\times \, e^{\frac{\rho}{2} i\Gamma_{\underline{\rho}}\Gamma_{\mathrm{AdS}}} \left[e^{\frac{r}{2} i\Gamma_{\underline{r}}\Gamma_{\mathrm{AdS}}} + i e^{r/2} x^\mu \Gamma_{\underline{x_\mu}} \Gamma_{\mathrm{AdS}} P_{r-} \right] P_{\mathrm{L}} \epsilon_0$$

With just a little bit of algebra.....

$$\cos\theta(z) = 2\cos\left(\frac{2\pi k + \cos^{-1}\tau(z)}{3}\right) ,$$

k=2

$$\tau(z) = \frac{96z^3(c - m\log\frac{z}{2}) + 6mz(z^4 - 16)}{(z^2-4)^3} ,$$

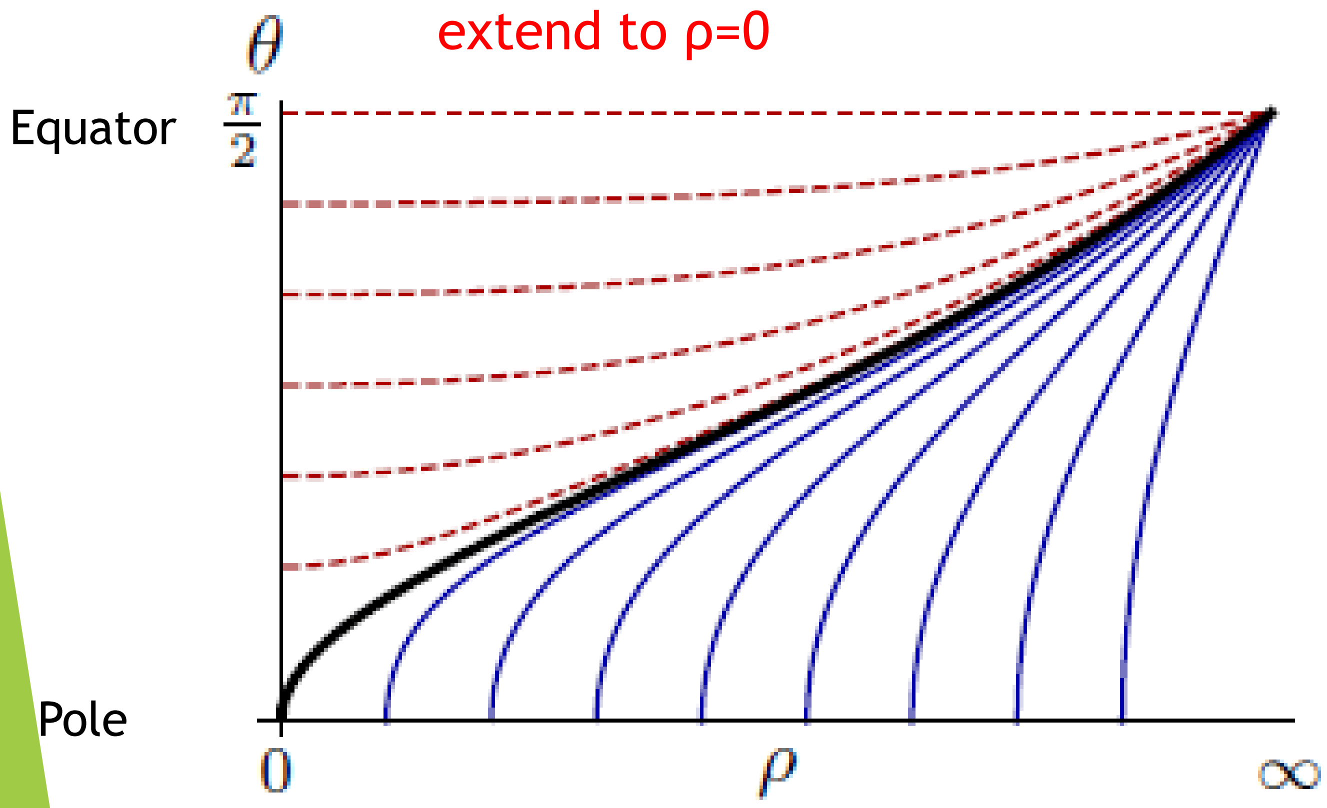
$$f(z) = -i\sin^3\theta\,\frac{z(z^2-4)\theta' - (z^2+4)\cot\theta}{8z} .$$

c fixed in terms of m by regularity condition

This simple embedding indeed solves the complicated DBI EOM ✓

$m \sim$ flavor mass $M = m\sqrt{\lambda}/2\pi$, $c \sim$ chiral condensate $\langle\overline{\psi}\psi\rangle$

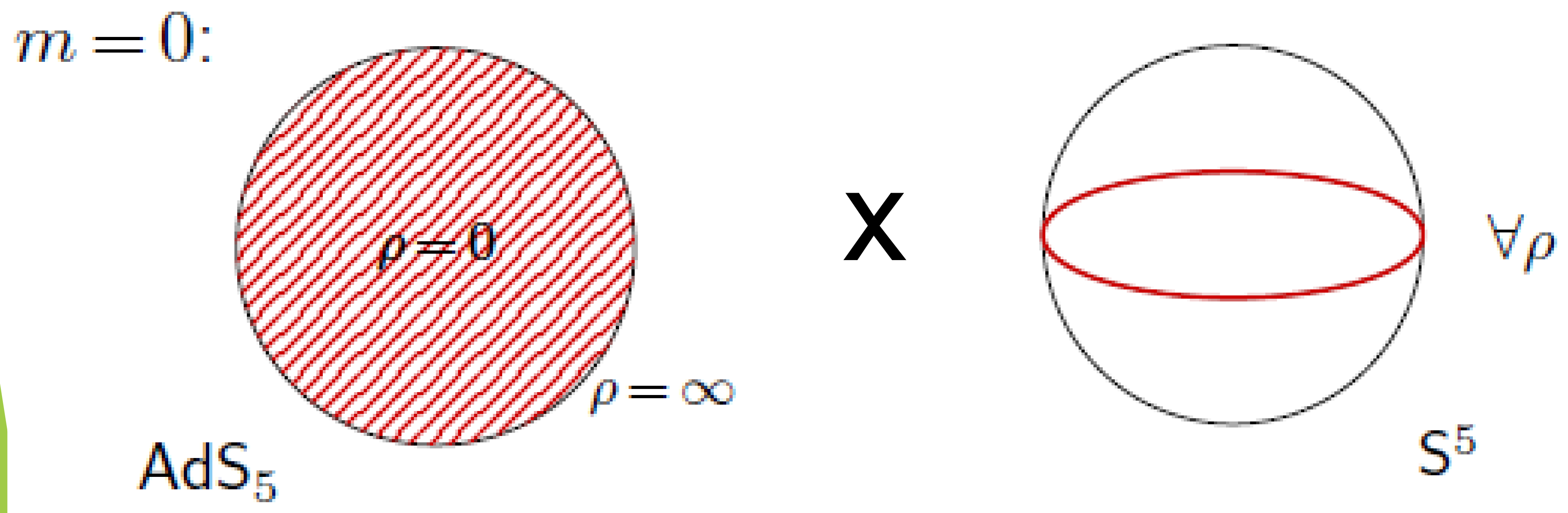
Phase Diagram of D7 embeddings on S^4



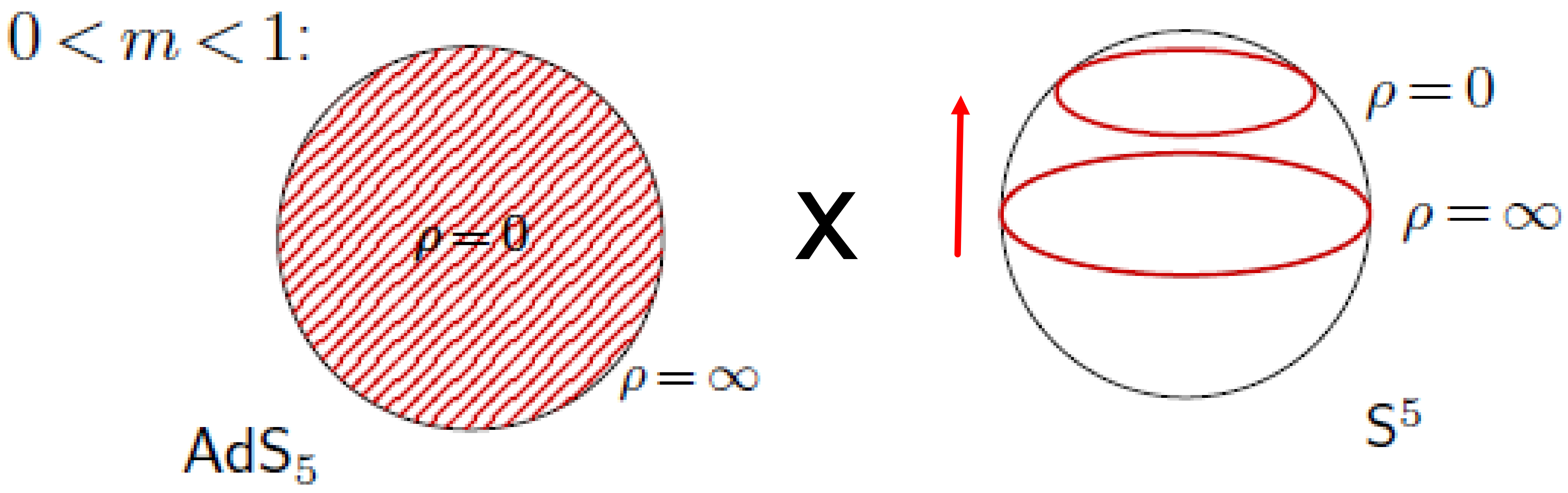
Phase transition at m=1

terminate at finite ρ

Small mass embeddings (m<1):

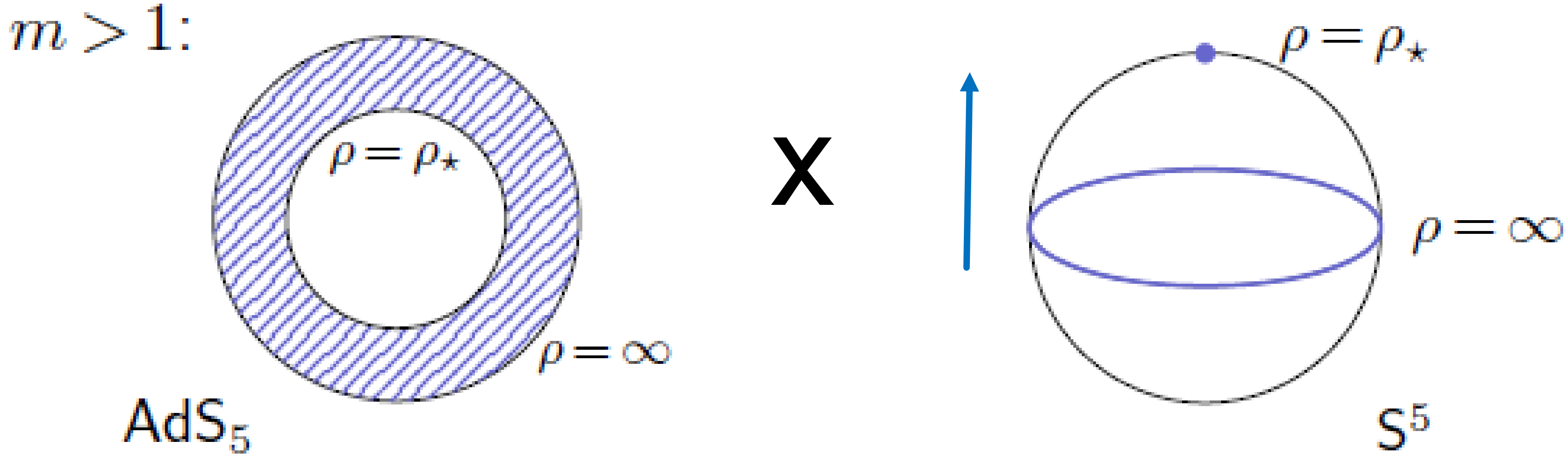


Small mass embeddings (m<1)



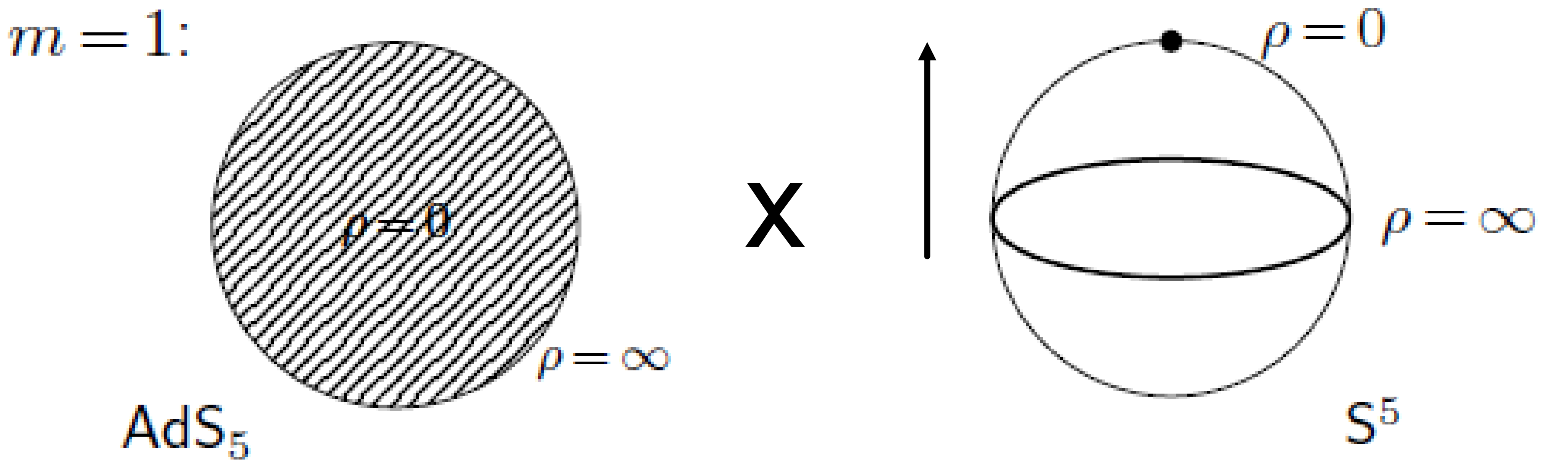
Brane slides off.
Reaches finite angle at center

Large mass embeddings



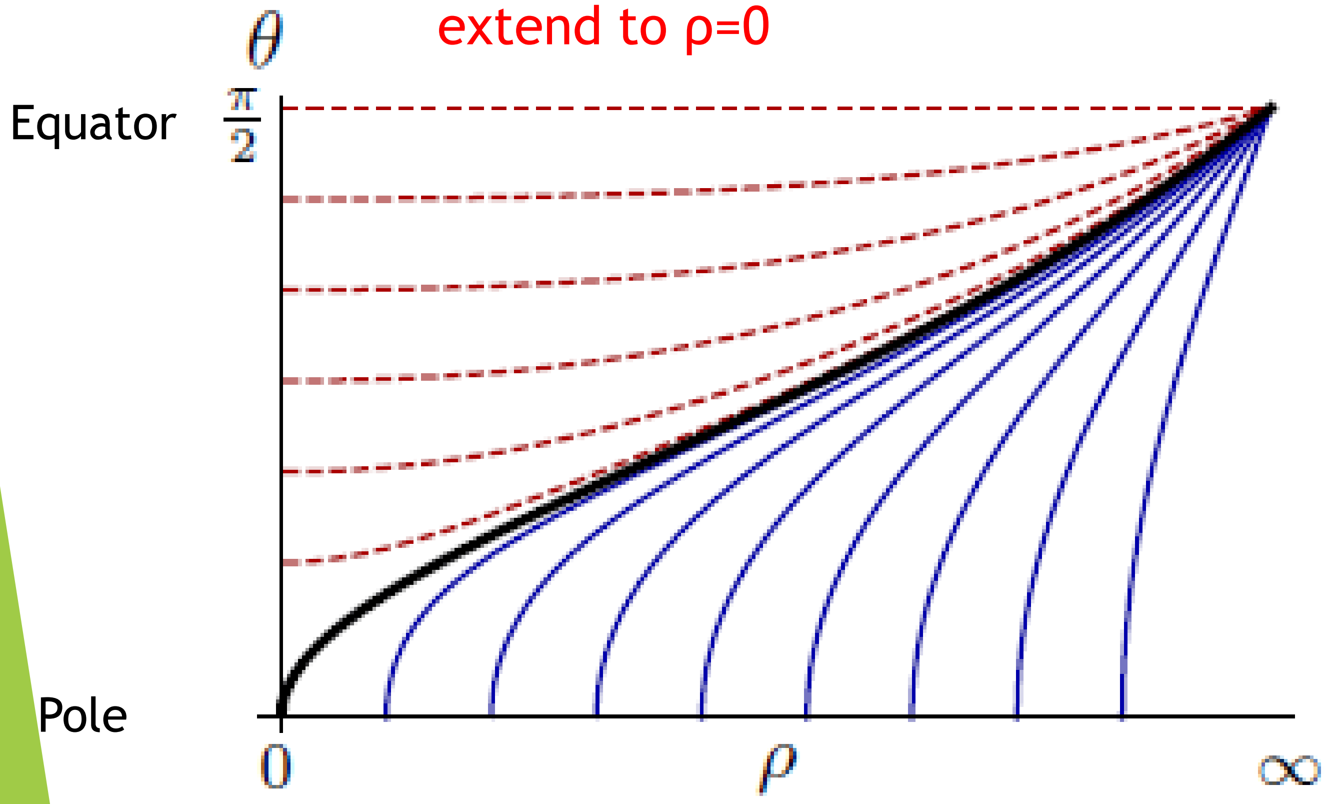
Brane smoothly caps off
at finite value of ρ

Critical embedding at m=1



For critical embedding brane caps of exactly at center, ρ=0

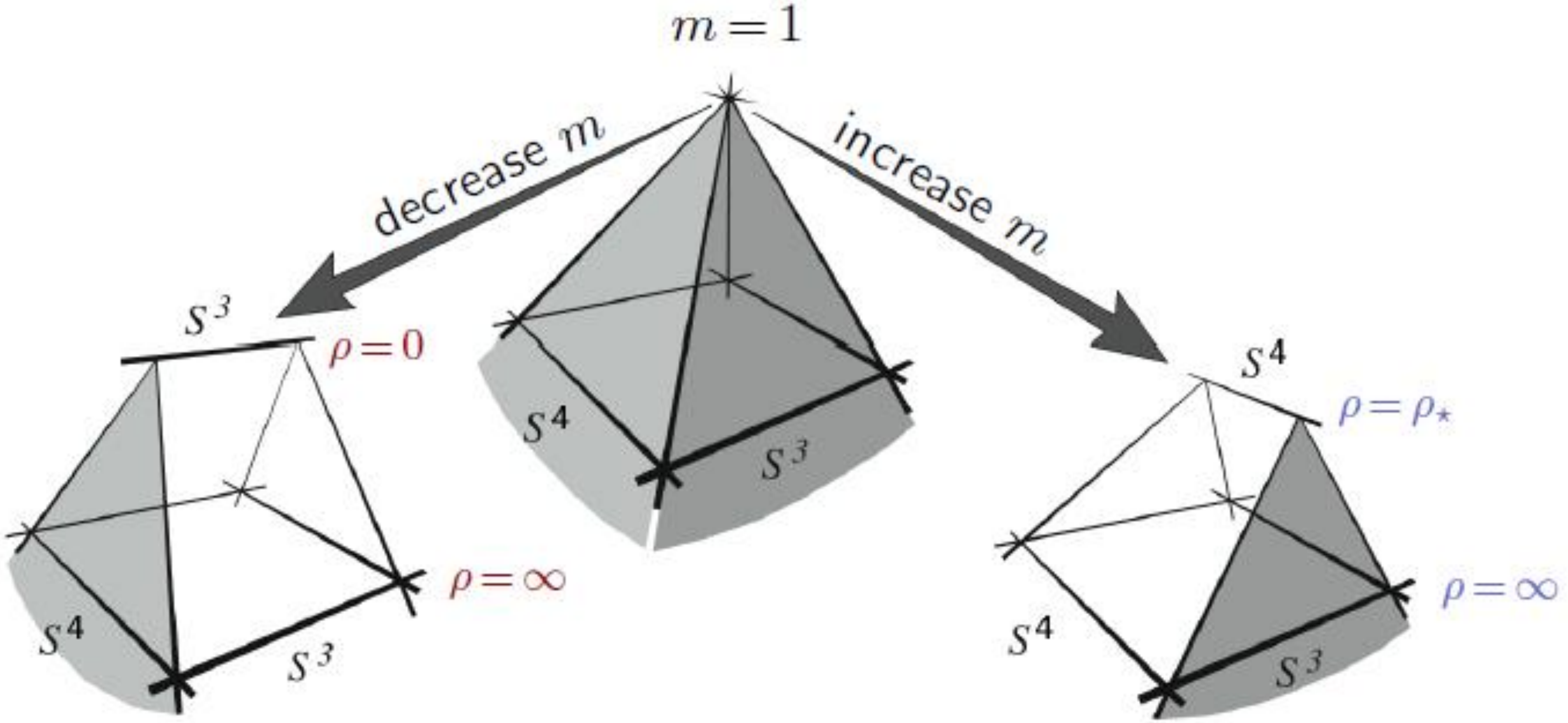
Phase Diagram of D7 embeddings on S^4



Phase transition at m=1

terminate at finite ρ

Geometry of the Phase Transition



S^4 in AdS_5 degenerates at $\rho = 0$

S^3 in S^5 degenerates at $\rho = \rho_\star$

Free energy and critical exponents

Two one-point functions from holographically renormalized on-shell action ($\mu \equiv \sqrt{\lambda}/2\pi$):

$$\mu\langle\mathcal{O}_\theta\rangle = -\frac{1}{\sqrt{g_{S^4}}}\frac{\delta S_{\mathrm{D7,ren}}}{\delta\theta^{(0)}} \qquad \mu\langle\mathcal{O}_f\rangle = -\frac{1}{\sqrt{g_{S^4}}}\frac{\delta S_{\mathrm{D7,ren}}}{\delta f^{(0)}}$$

Varying *within* susy configurations: $\delta\theta^{(0)} = i\delta f^{(0)}$.

$\rightarrow$ flavor contribution to free energy $F^{(1)}$:

$$\langle\mathcal{O}_s\rangle := \langle\mathcal{O}_\theta\rangle + i\langle\mathcal{O}_f\rangle = \frac{1}{V_{\mathrm{S}^4}}\frac{dF^{(1)}}{dM}$$

Free energy and critical exponents

Finite counterterms $\sim M^4, M^2R^{-2}$ introduce scheme dependence

$$V_{S^4}\langle \mathcal{O}_s\rangle = \frac{2}{3}\mu N_f N \left[3c + \frac{2+12\alpha_1}{3}m^3 - \frac{7+4\beta}{2}m\right]$$

$\alpha_1 = -\frac{5}{12}$ to preserve susy, term linear in m scheme dependent

The interesting part is c, determined from IR regularity:

$$c_{m>1} = \frac{m^2+2}{3}\sqrt{m^2-1} + m\log\left(m - \sqrt{m^2-1}\right)$$

$$c_{m\leq 1} = 0$$

Free energy and critical exponents

Condensate $\langle \mathcal{O}_s \rangle$ non-analytic at $m = 1$. For $m = 1 + \epsilon$:

$$\langle \mathcal{O}_s \rangle = \frac{\mu N_f N_c}{V_{S^4}} \left[\frac{1}{3} - (1+\epsilon) \log \frac{\mu^2}{4} - \epsilon - 2\epsilon^2 + \frac{16\sqrt{2}}{15} \epsilon^{5/2} + \ldots \right]$$

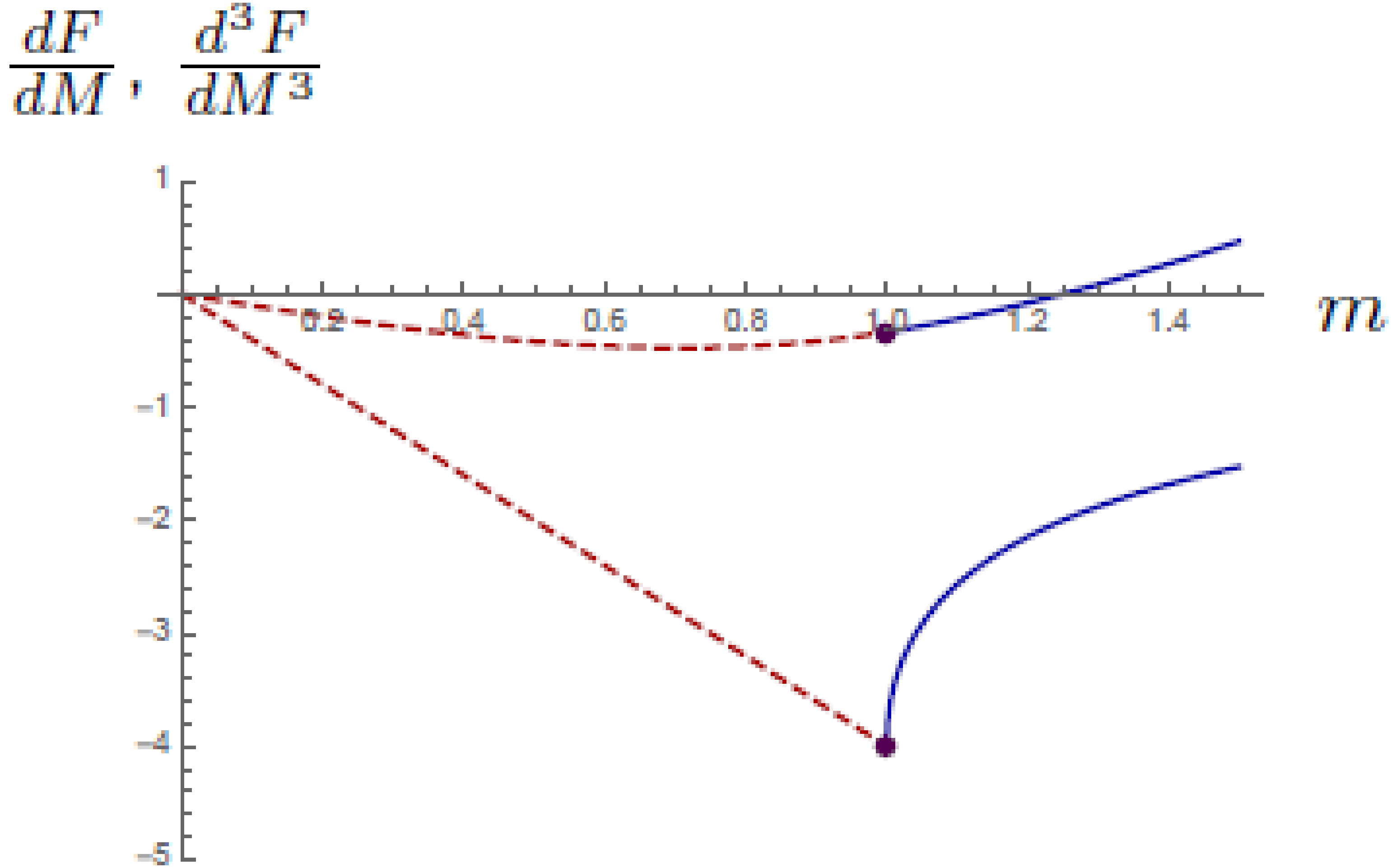
$\rightarrow$ first non-analytic term $\propto \epsilon^{5/2}$

Compare to non-susy embeddings: [Karch,O'Bannon,Yaffe '09]

$$\langle \mathcal{O}_\theta \rangle = \text{analytic} + \#\epsilon^{\alpha} + \ldots \qquad \alpha = \frac{4+\sqrt{2}}{4-\sqrt{2}}$$

Imaginary gauge field changes scaling analysis – susy's different.

Free energy and critical exponents:



Localization for flavored SYM

Start with N=4. Vanishing locus of QV:

single (position independent) scalar.

Path integral → Matrix model

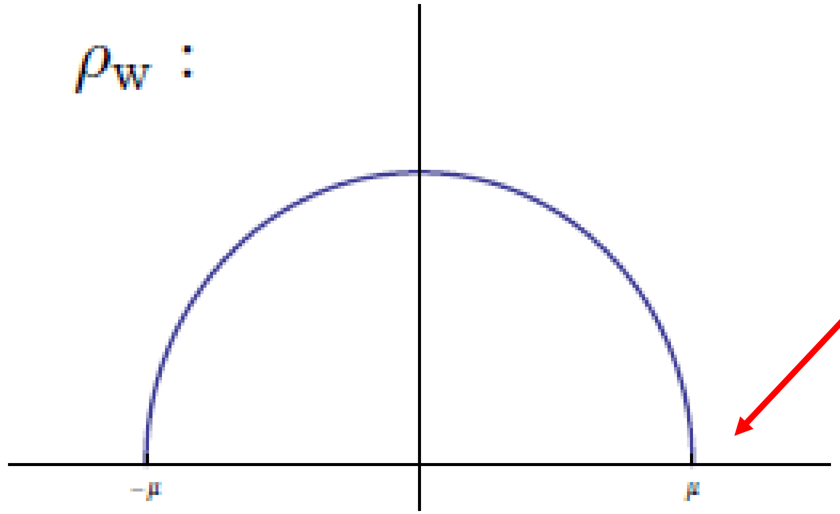
Action from 1-loop determinant around vanishing locus:

Gaussian matrix model

$$\mathcal{Z} = \int da^{N-1} \prod_{i<j} a_{[ij]}^2 \, e^{S_0} \,, \qquad S_0 = -\frac{8\pi^2}{\lambda} N \sum_i a_i^2$$

Localization at large N

At large N Matrix Model solved by saddle point approximation.
Can find eigenvalue distribution. N=4: Wigner semi-circle



endpoint ~ $\lambda^{1/2}$

Localization with flavors

Massive flavors enter via 1-loop factor.
Modify action of Matrix model

$$\mathcal{Z} = \int d^{N-1}a \, \frac{\prod_{i<j} a_{[ij]}^2}{\prod_i \sqrt{H_+^{N_f}(a_i) H_-^{N_f}(a_i)}} \, e^{S_0} =: \int d^{N-1}a \, e^{\hat{S}}$$

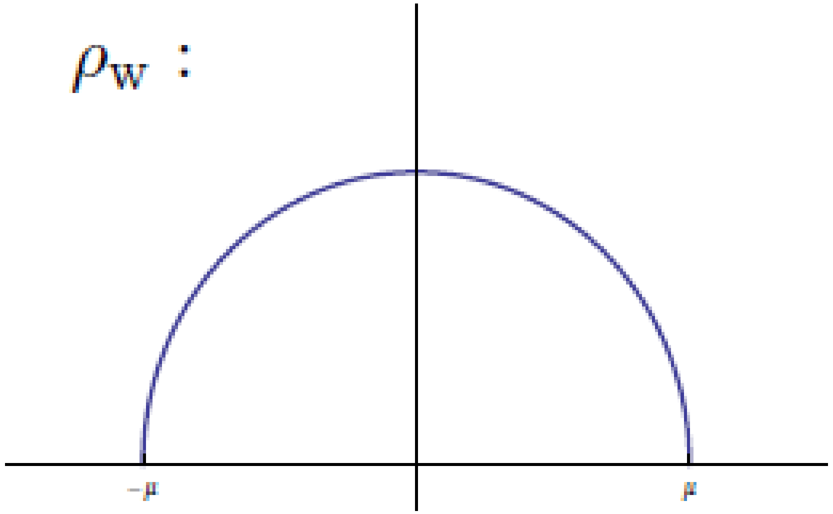
$$H_\pm(x) = H(x \pm M), \; H(x) = G(1 - ix)G(1 + ix)$$

(Pestun)

Nightmare even at large N! Potential involves Barnes G.

(Russo, Zarembo)

The Matrix Model in the Probe Limit



For a finite number
of fundamental rep
hypers, semi-circle unchanged.

Calculate flavor contribution
to F with this eigenvalue density.

Free energy = “integrals of Barnes G”

… and large λ

argument of G = eigenvalue ± m ~ $\lambda^{1/2}$

can use asymptotic form of G: logs

Leading-order correction to F' with $M, \lambda \gg 1$:

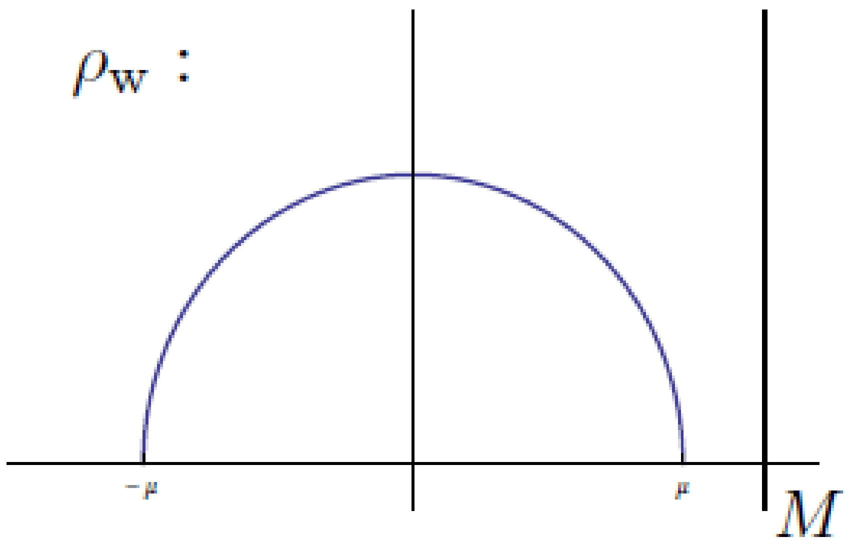
$$\frac{dF^{(1)}}{dM} = \frac{N_f N}{2} \int_{-\mu}^{\mu} \underbrace{dx \rho_{\mathrm{w}}(x) \left[4M - x_+ \log x_+^2 - x_- \log x_-^2\right]}_{I(x)}$$

Wigner semicircle: $\rho_{\mathrm{w}} = \frac{2}{\pi \mu^2} \sqrt{\mu^2 - x^2}$

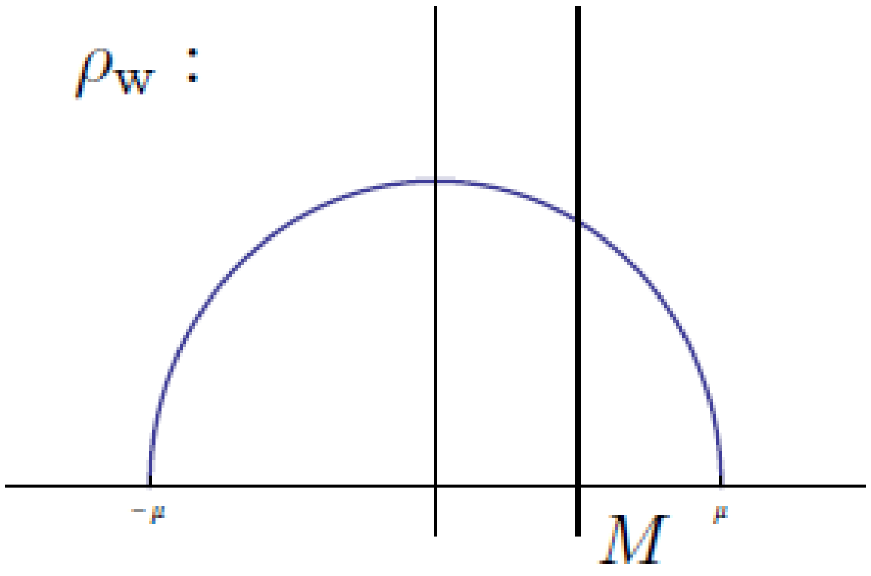
Phase transition: $M > \mu = \sqrt{\lambda}/2\pi$ vs. $M < \mu$

Phase transition at large λ

m>1



m<1



Showdown….

Showdown….

Evaluating the matrix-model integral:

$$M > \mu: \quad F'^{(1)} = \frac{N_f N}{3\mu^2}\Big[2\sqrt{M^2-\mu^2}(M^2+2\mu^2) - 2M^3 + 3M\mu^2\Big(1 - 2\log\frac{M+\sqrt{M^2-\mu^2}}{2}\Big)\Big]$$

$$M < \mu: \quad F'^{(1)} = N_f N\left[M - \frac{2}{3}\mu^2 M^3 - 2M\log\frac{\mu}{2}\right]$$

Showdown....

Evaluating the matrix-model integral:

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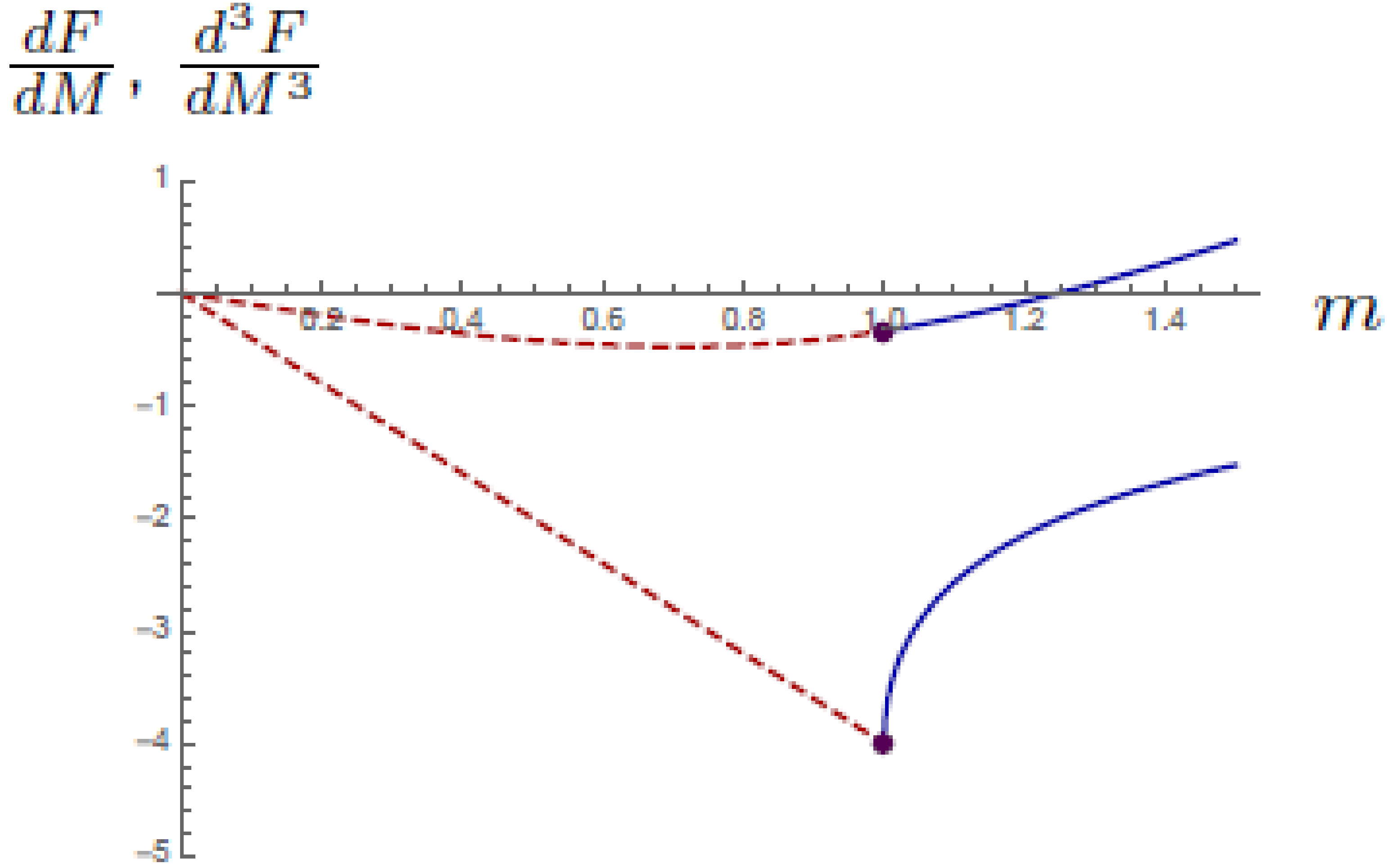
$$M < \mu : \quad F'^{(1)} = N_f N\left[M - \frac{2}{3}\mu^2 M^3 - 2M\log\frac{\mu}{2}\right]$$

IDENTICAL TO PROBE BRANE ANSWER !!!

Free energy and critical exponents:



Conclusions:

FLAVORED HOLOGRAPHY LIVES.

Interesting things to do for the future:

- Understand phase structure of AdS4 flavors
- Find analytic solution for N=2*. Maybe possible for AdS4
- Finite N, finite λ
- Holographic backgrounds for topologically twisted theories
- Holography at finite t. Non-BPS quantities?
- other probe branes: D3/D5