Effective Field Theory of Dissipative Fluids

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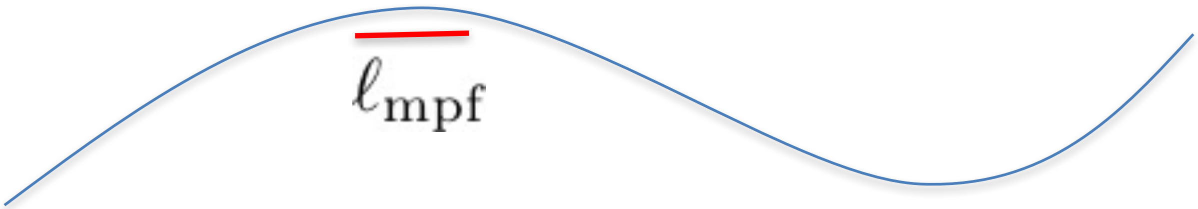
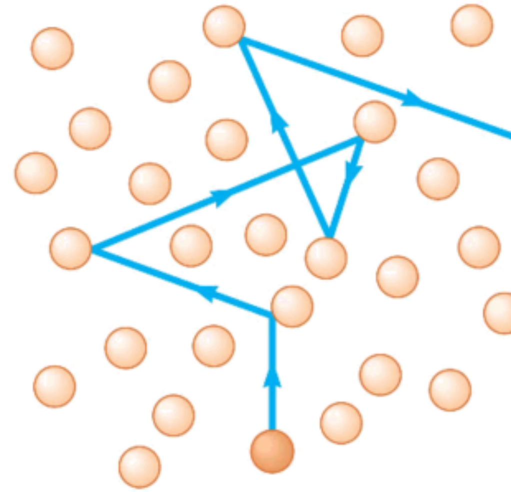
Conserved quantities

Consider a long wavelength disturbance
of a system in thermal equilibrium





non-conserved quantities: relax locally, $\tau_{\mathrm{relax}} \sim \tau_{\mathrm{mfp}}$

conserved quantities: cannot relax locally, only via transports

$$\lambda \to \infty, \quad \Rightarrow \quad \tau_{\mathrm{relax}} \to \infty$$

Conserved quantities

Gapless and
long-lived
modes

(only ones in
a medium)

There should exist a universal low energy effective theory.

Hydrodynamics

Thermal equilibrium: $$\rho = \frac{1}{Z} \exp\left[\int d\vec{x}\, \beta(u_\mu T^{0\mu} + \mu J^0)\right]$$

Promote these quantities to dynamical variables: (local equilibrium)

$$\beta(t,\vec{x}),\ u^\mu(t,\vec{x}),\ \mu(t,\vec{x})$$

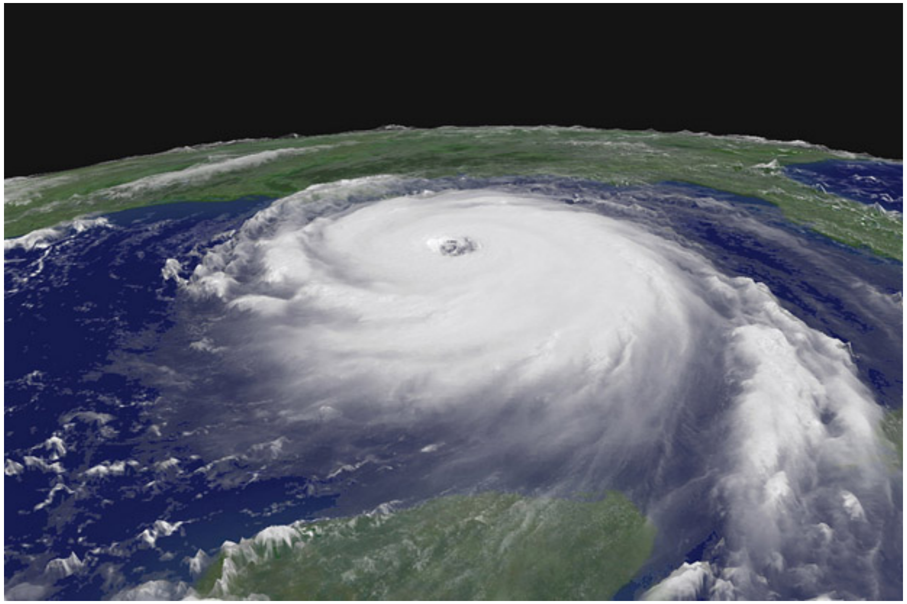
slowly varying functions of spacetime

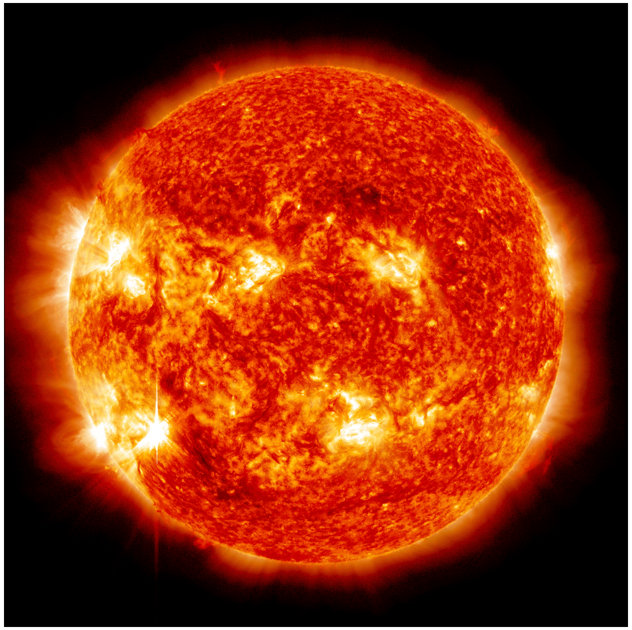
Express expectation values of the stress tensor and conserved current in terms of derivative expansion of these variables: constitutive relations.

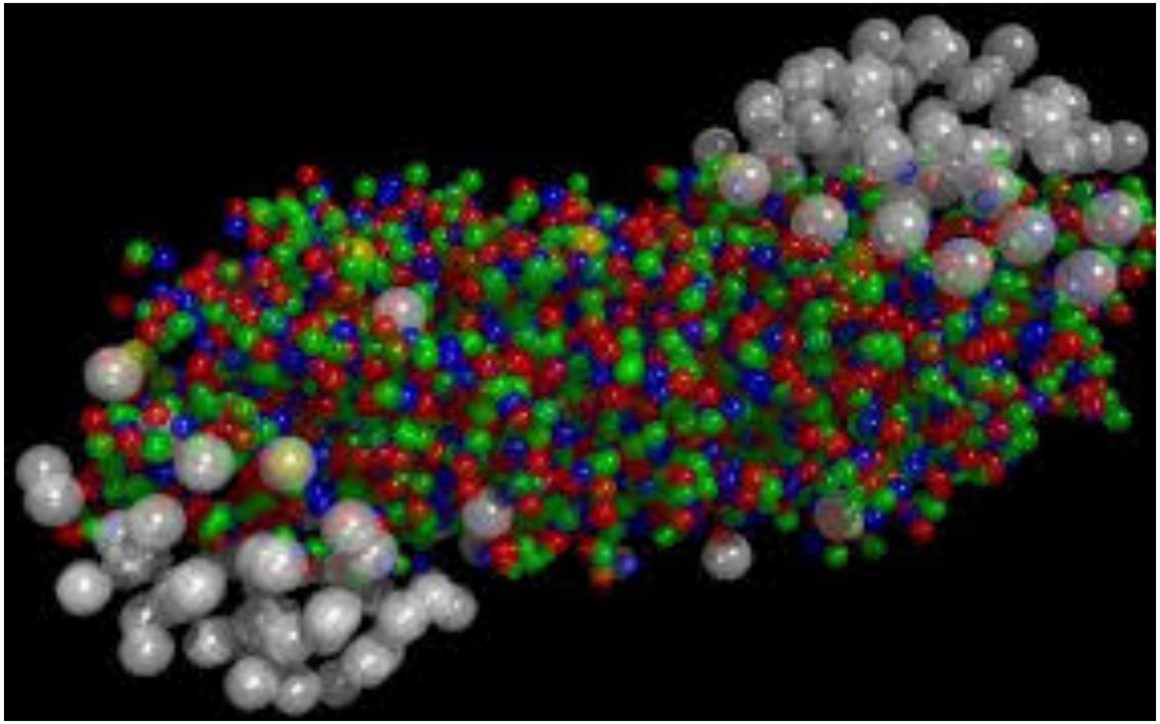
Equations of motion:

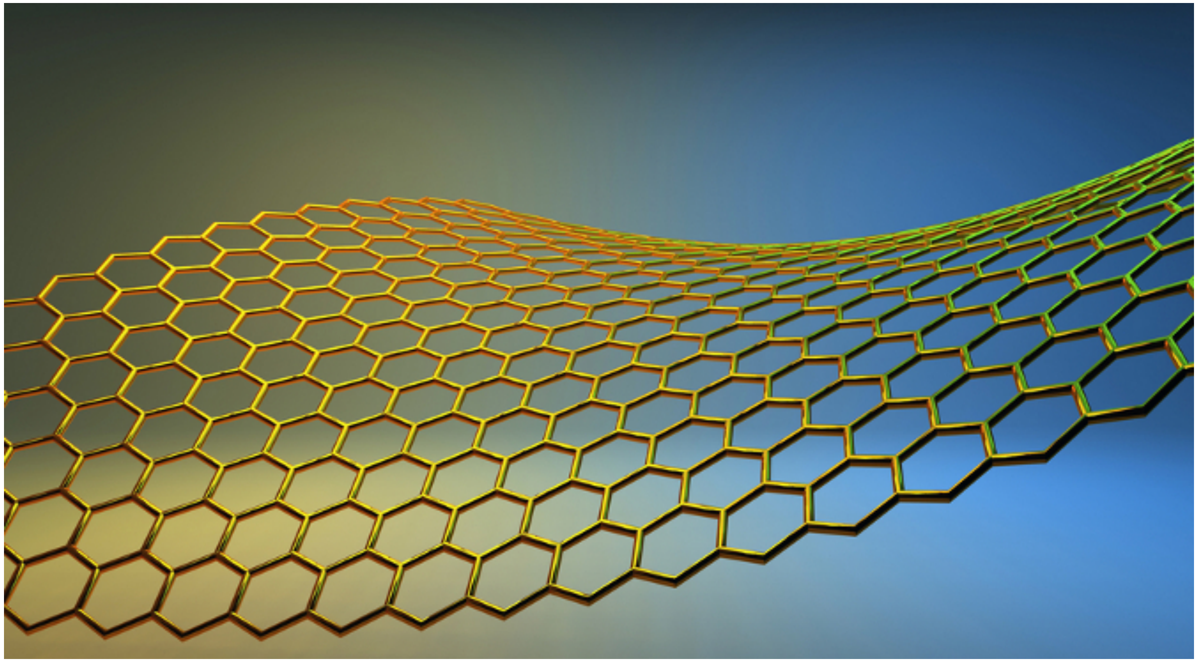
$$\partial_\mu \langle T^{\mu\nu}\rangle = 0, \qquad \partial_\mu \langle J^\mu\rangle = 0$$

d+1 variables, d+1 equations









Despite the long and glorious history of hydrodynamics

It does not capture fluctuations.

Fluctuations

There are always statistical fluctuations .....

Important in many contexts:

Long time tail

transports,

dynamical aspects of phase transitions,

non-equilibrium states,

turbulence,

finite size systems ....

At low temperatures, quantum fluctuations can also be important.

Phenomenological level: stochastic hydro (Landau, Lifshitz)

$$\partial_\mu \langle T^{\mu\nu} \rangle = \xi^\nu, \quad \partial_\mu \langle J^\mu \rangle = \zeta$$

ξ^μ, ζ : noises with local Gaussian distribution

Expect:

1. interactions among noises
2. interactions between dynamical variables and noises
3. fluctuations of dynamical variables themselves

particularly important for non-equilibrium situations.

Until now not known how to treat such nonlinear effects systematically. Not even clear it is a good question.

Constraints

Current formulation of hydrodynamics is awkward.

Constitutive relations : not enough to just write down the most general derivative expansion consistent with symmetries.

Phenomenological constraints: solutions should satisfy:

1. Entropy condition $\partial_\mu S^\mu \geq 0$

2. Onsager relations: linear response matrix must be symmetric

awkward: use solutions to constrain equations of motion

Microscopic derivation? Are these complete?

develop hydrodynamics as a *bona fide* low energy effective field theory of a general many-body system at finite temperature

Action principle which incorporates both dissipations and noises

1. gives a full interacting theory of noises.

2. Microscopic origin and completeness of phenomenological constraints

3. New constraints (nonlinear Onsager relations)

Should be distinguished from an action which just reproduces standard eoms (which may not capture fluctuations correctly)

Effective theory approach may also make it easier to generalize hydrodynamics EOM to less familiar situations, say with momentum dissipations, anomalies.....

Searching for an action principle for hydrodynamics has been a long standing open problem, dating back at least to G. Herglotz in 1911 ....

All results at non-dissipative level ....

Many activities since 70's to understand hydrodynamic fluctuations ....

Results

Approach: put a relativistic quantum many-body ssystem in a curved spacetime

1. Hydrodynamics with classical statistical fluctuations

is described by a supersymmetric quantum field theory

$$\hbar_{\text{eff}} \propto \frac{1}{s} \qquad s : \text{entropy density}$$

See also Haehl, Loganayagam, Rangamani

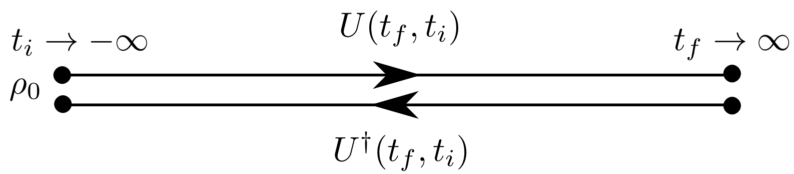
2. Hydrodynamics with quantum fluctuations also incorporated

is described by a "quantum-deformed" (supersymmetric) quantum field theory.

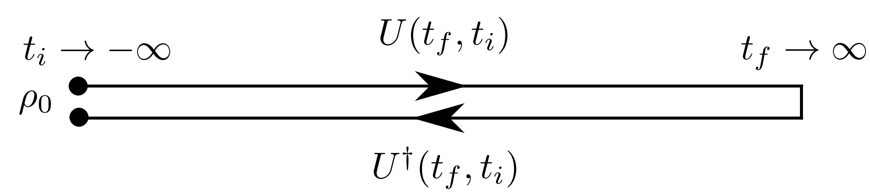
Part II: formulation

Transition amplitudes v.s. expectation values

We are interested in an effective theory describing nonlinear dynamics around a state.



(b)



$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i) \qquad \mathrm{Tr}\left(\rho_0 \cdots\right)$$

Closed time path (CTP) or Schwinger-Keldysh contour

Should double all degrees of freedom

Should be contrasted with EFT describing transition amplitudes,

$$\langle f| \cdots |i\rangle$$

Hydro effective field theory

At long distances and large times:

All correlation functions of the stress tensor and conserved currents in thermal equilibrium

$$= \int D\chi_1 D\chi_2 \, e^{iS_{\text{hydro}}[\chi_1,\chi_2]} \ldots$$

hydrodynamic modes

EFT approach:

1. What are χ ? $\beta(t,\vec{x}), u^\mu(t,\vec{x}), \mu(t,\vec{x})$ do not work

2. What are the symmetries of S_{hydro} ?

3. Integration measure?

Dynamical variables: integrating in

Toy example: a single conserved current J^μ

$$e^{W[A_{1\mu},A_{2\mu}]} = \mathrm{Tr}\left(\rho_0 \mathcal{P} e^{i\int d^dx\, A_{1\mu}J_1^\mu - i\int d^dx\, A_{2\mu}J_2^\mu}\right),$$

1. Current conservation:

$$W[A_{1\mu},A_{2\mu}] = W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2]$$

2. W must be nonlocal : Non-locality solely due to integrating out hydro modes

Integrate in hydro modes: $e^{W[A_1,A_2]} = \int D\phi_1 D\phi_2\, e^{iI[A_1,A_2;\phi_1,\phi_2]}$

(a): $I[A_1,A_2;\phi_1,\phi_2]$ local (b): Ensure 1 is satisfied

(c): ϕ EOMs must be equivalent to current conservations

Proposal: (use the usual Stueckelberger trick)

$$e^{W[A_{1\mu},A_{2\mu}]} = \int D\phi_1 D\phi_2 \, e^{iI[B_{1\mu},B_{2\mu}]}$$

$$B_{1\mu} \equiv A_{1\mu} + \partial_\mu \phi_1, \quad B_{2\mu} \equiv A_{2\mu} + \partial_\mu \phi_2$$

$I[B_{1\mu}, B_{2\mu}]$ is a local action. $\phi_{1,2}$: hydro modes

Satisfy the following consistency requirements:

1. $W[A_{1\mu}, A_{2\mu}] = W[A_{1\mu} + \partial_\mu \lambda_1, A_{2\mu} + \partial_\mu \lambda_2]$

2. Eoms of $\phi_{1,2}$ are equivalent to current conservations.

Dynamical variables (II)

For stress tensor, we put the system in a curved spacetime

$$e^{W[g_{1\mu\nu}, g_{2\mu\nu}]} = \mathrm{Tr}\left[U_1(+\infty, -\infty; g_{1\mu\nu})\rho_0 U_2^\dagger(+\infty, -\infty; g_{2\mu\nu})\right]$$

Conservation of stress tensor:

$$W[g_1, g_2] = W[\tilde{g}_1, \tilde{g}_2] \qquad \tilde{g}_{1\mu\nu}(x) = \frac{\partial y_1^\sigma}{\partial x^\mu} g_{1\sigma\rho}(y_1(x)) \frac{\partial y_1^\rho}{\partial x^\nu}$$

Integrate in hydro modes:

Promote spacetime coordinates to dynamical fields

$$e^{W[g_1, g_2]} = \int DX_1 DX_2 D\tau_1 D\tau_2 \, e^{iI[h_1, \tau_1; h_2, \tau_2]}$$

$$h_{1ab}(\sigma) = e^{-2\tau_1(\sigma)} \frac{\partial X_1^\mu}{\partial \sigma^a} g_{1\mu\nu}(X_1(\sigma)) \frac{\partial X_1^\nu}{\partial \sigma^b}$$

1. $W[g_1, g_2] = W[\tilde{g}_1, \tilde{g}_2]$

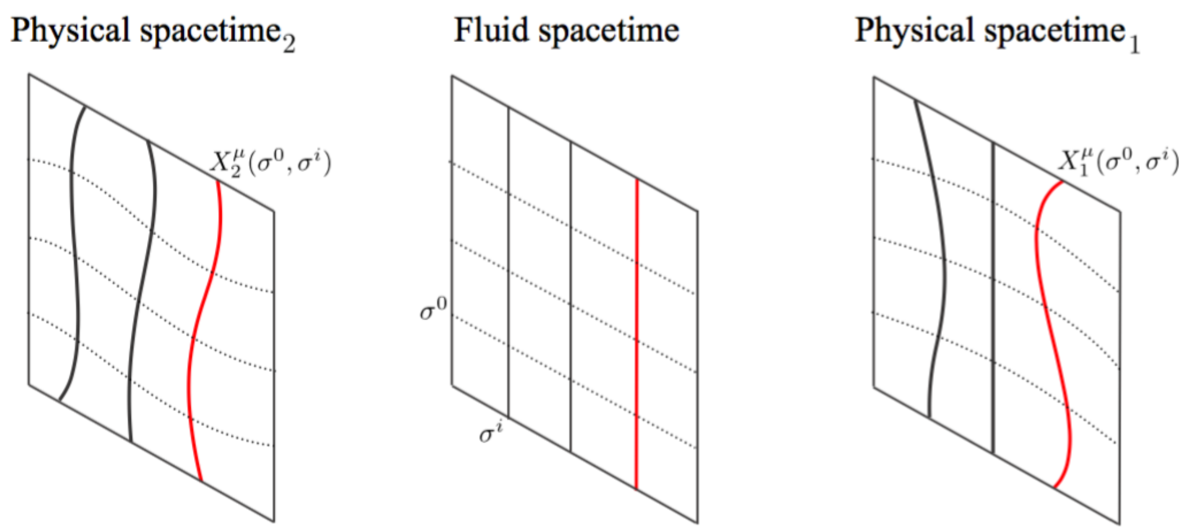
2. X eoms are equivalent to conservation of stress tensor

an emergent spacetime with coordinates σ^a

$$h_{1ab}(\sigma) = e^{-2\tau_1(\sigma)} \frac{\partial X_1^\mu}{\partial \sigma^a} g_{1\mu\nu}(X_1(\sigma)) \frac{\partial X_1^\nu}{\partial \sigma^b}$$

Interpretation of σ^a : σ^i label individual fluid elements, σ^0 internal time

$X^\mu(\sigma^a)$: motion of a fluid element in physical spacetime



So we just recovered the Lagrange description of a fluid!

As a starting point, we could simply double the degrees of freedom in the Lagrange description.

A bit history:

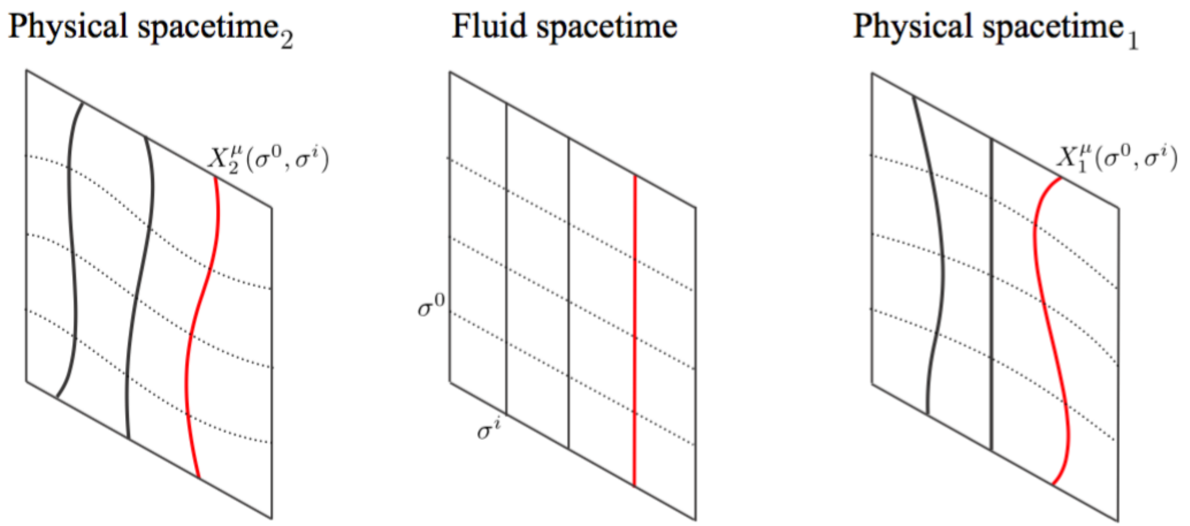
Using a single copy of $\sigma^i(x^\mu)$ as dynamical variable for an ideal fluid action dated back to G. Herglotz in 1911.

Covariant $\sigma^a(x^\mu)$ was used by Taub in 1954.

Rediscovered in 2005 by Dubovsky, Gregoire, Nicolis and Rattazzi in hep-th/0512260 and further developed by Dubovsky, Hui, Nicolis and Son in arXiv:1107.0731 , ...

Nickel and Son showed the covariant version arises naturally from holography (arXiv:1103.2137).

Doubled copies appeared in Haehl, Loganayagam, Rangamani arXiv:1502.00636, and Crossley, Glorioso, HL, Wang arXiv:1504.07611.



Standard hydro variables (which are now derived quantities)

$$u^\mu = \frac{1}{b}\frac{\partial X^\mu}{\partial \sigma^0}, \quad e^{-\tau} = \frac{T}{T_0}, \quad \mu = u^\mu B_\mu$$

A significant challenge: ensure the eoms from the action of X and ϕ can be solely expressed in terms of these velocity type of variables. (e.g. solids v.s. fluids)

Symmetries (I)

Now need to specify the symmetries of $I[h_1, \tau_1, B_1; h_2, \tau_2, B_2]$

$$e^{W[g_1, A_1; g_2, A_2]} = \int DX_1 DX_2 D\tau_1 D\tau_2 D\phi_1 D\phi_2 \, e^{iI[h_1, \tau_1, B_1; h_2, \tau_2, B_2]}$$

Note that it is defined in fluid spacetime σ^a

Interpretation of σ^a : σ^i label individual fluid elements, σ^0 internal time

Require the action to be invariant under:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

$$B_{1i} \to B'_{1i} = B_{1i} - \partial_i \lambda(\sigma^i), \quad B_{2i} \to B'_{2i} = B_{2i} - \partial_i \lambda(\sigma^i)$$

define what is a fluid!

It turns out these symmetries indeed do magic for you:

at the level of equations of motion, they ensure all dependence on dynamical variables can be expressed in $\beta(t,\vec{x}), u^\mu(t,\vec{x}), \mu(t,\vec{x})$

→ Recover standard formulation of hydrodynamics

(modulo phenomenological constraints)

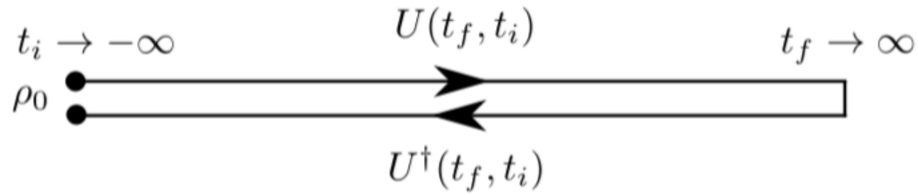
Full non-linear fluid fluctuating dynamics encoded in non-trivial differential geometry:

$$D_0,\ D_i,\ R^{(1)}_{ij},\ R^{(2)}_{ij},\ \mathfrak{t}_{ij}$$

This would be the full the story in a usual situation.

Symmetries (II)

We are considering EFT for a system defined with CTP:



The generating functional has the following properties:

- Reflectivity condition:

$$W^*[g_1, A_1; g_2, A_2] = W[g_2, A_2; g_1, A_1]$$

- KMS condition plus PT imply a Z_2 symmetry on W:

$$W[g_1(x), A_1(x); g_2(x), A_2(x)] = W[g_1(-x), A_1(-x); g_2(-t - i\beta_0, -\vec{x}), A_2(-t - i\beta_0, -\vec{x})]$$

- Unitarity condition:

$$W[g, A; g, A] = 0$$

Full bosonic theory

Reflectivity condition can be easily imposed, leading to a complex action.

→ Imaginary part of the action non-negative

Imposing KMS condition is very tricky.

proposal: local KMS condition, a Z_2 symmetry on the action

→ All the constraints from entropy current condition and linear Onsager relations

New constraints on equations of motion from nonlinear Onsager relations.

Fermions and Supersymmetry

See also Haehl et al
arXiv: 1510.02494
1511.07809

Unitarity condition: $$W[g, A; g, A] = 0$$

is a "topological" condition on the measure of path integrals

Introduce fermionic partners ("ghost" fieds) for dynamical variables and require the action to have a BRST type symmetry.

At a quadratic level in dynamical fields, one finds that local KMS condition leads to an emergent fermionic symmetry.

$$\delta^2 = 0, \quad \bar{\delta}^2 = 0, \quad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon 2 \tanh \frac{i\beta_0 \hbar \partial_t}{2}$$

But not clear how to write down a nonlinear action with such an algebra.

Requires a "quantum-deformed" SUSY

Classical limit: $\hbar \to 0$

$$\delta^2 = 0, \qquad \bar{\delta}^2 = 0, \qquad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon i\beta_0\partial_t$$

become standard supersymmetry in time direction.

In this limit one can write down a supersymmetric completion of the full bosonic hydrodynamic action.

Note that in the classical limit, path integral remains, capturing statistical fluctuations.

Example: nonlinear stochastic diffusion

Consider the theory for a single conserved current, where the relevant physics is diffusion.

Dynamical variables: $\varphi_{1,2}$ (or φ_a, φ_r)

Roughly, φ_r : standard diffusion mode, φ_a: the noise.

$$\mathcal{L} = iT(\partial_i\varphi_a)^2(\sigma + \sigma_1\partial_0\varphi_r) + \partial_0\varphi_a\partial_0\varphi_r(\chi + \chi_1\partial_0\varphi_r) - \partial_i\varphi_a\partial_0\partial_i\varphi_r(\sigma + \sigma_1\partial_0\varphi_r)$$
$$+c_a(\chi\partial_0 - \sigma\partial_i^2)\partial_0 c_r - \chi_1\partial_0 c_a\partial_0\varphi_r\partial_0 c_r - \sigma_1\partial_i^2 c_a\partial_0\varphi_r\partial_0 c_r$$
$$-iT\sigma_1(\partial_i c_a\partial_i\varphi_a\partial_0 c_r + (\partial_0 c_a\partial_i\varphi_a - \partial_i c_a\partial_0\varphi_a)\partial_i c_r),$$

If ignoring interactions of noise

$$(\partial_0 - D\partial_i^2)n - \left(\lambda_1\partial_0 - \frac{\lambda}{2}\partial_i^2\right)n^2 = \xi$$

A variation of Kardar-Parisi-Zhang equation

Summary

An EFT for general dissipative fluids.

Recovers the standard hydrodynamics as equations of motion, constitutive relations, constraints.

Encodes quantum and thermal fluctuations systematically in a path integral expansion.

Full non-linear fluid fluctuating dynamics encoded in non-trivial differential geometry.

Fermionic excitations and Emergent supersymmetry.

Future directions

Formalism:

Non-relativistic limit ,
superfluids,
Anisotropic, inhomogeneous,
“quantum-deformed” Supersymmetry

....

Applications:

Longtime tails, running of viscosities,

Non-equilibrium steady states, dynamical flows of QGP

Dynamical aspects of classical and
quantum phase transitions

Scaling behavior in hydro behavior via fixed points
of QFTs, such as KPZ scaling, turbulence ….

…………

Thank You