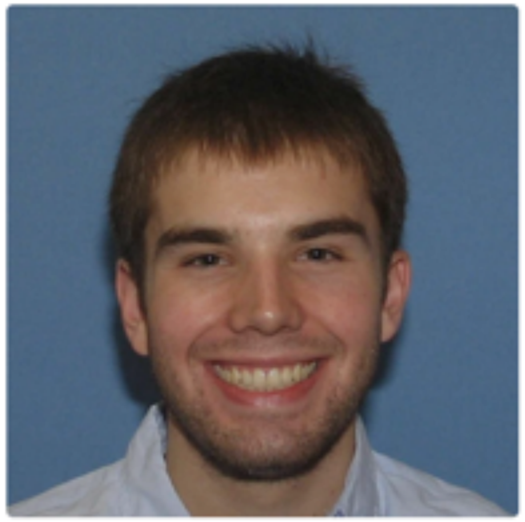
Detecting unparticles and anomalous dimensions in the Strange Metal

Thanks to: NSF, EFRC (DOE)

Brandon Langley



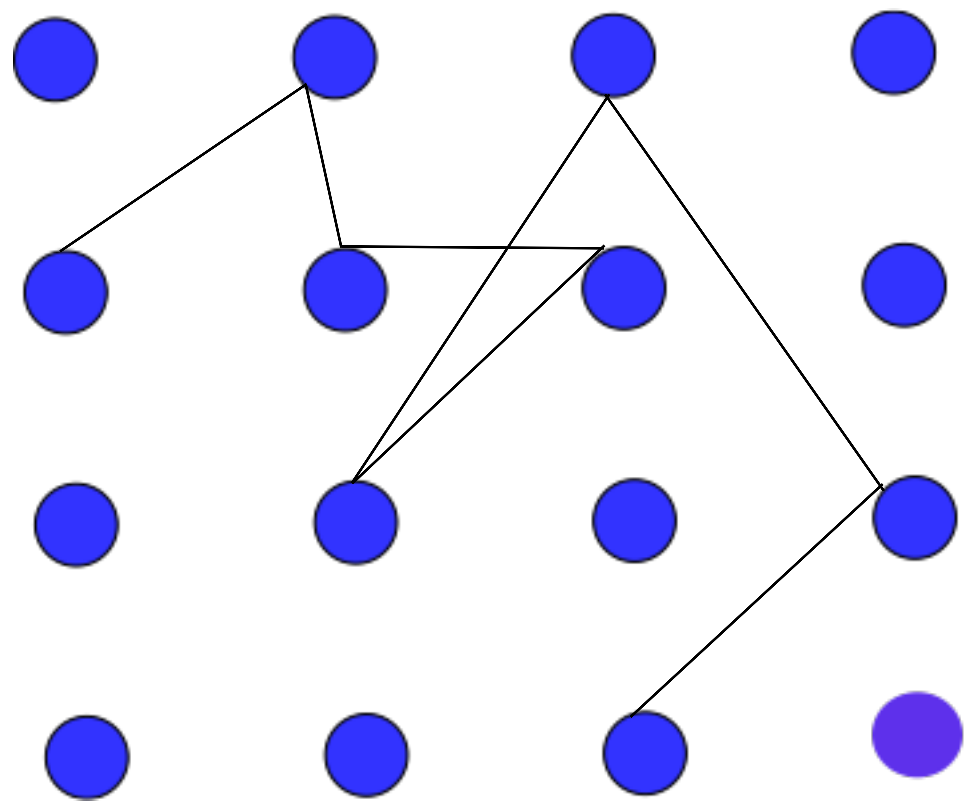
Garrett Vanacore

Kridsangaphong Limtragool

Andreas Karch

Gabriele La Nave

Drude metal



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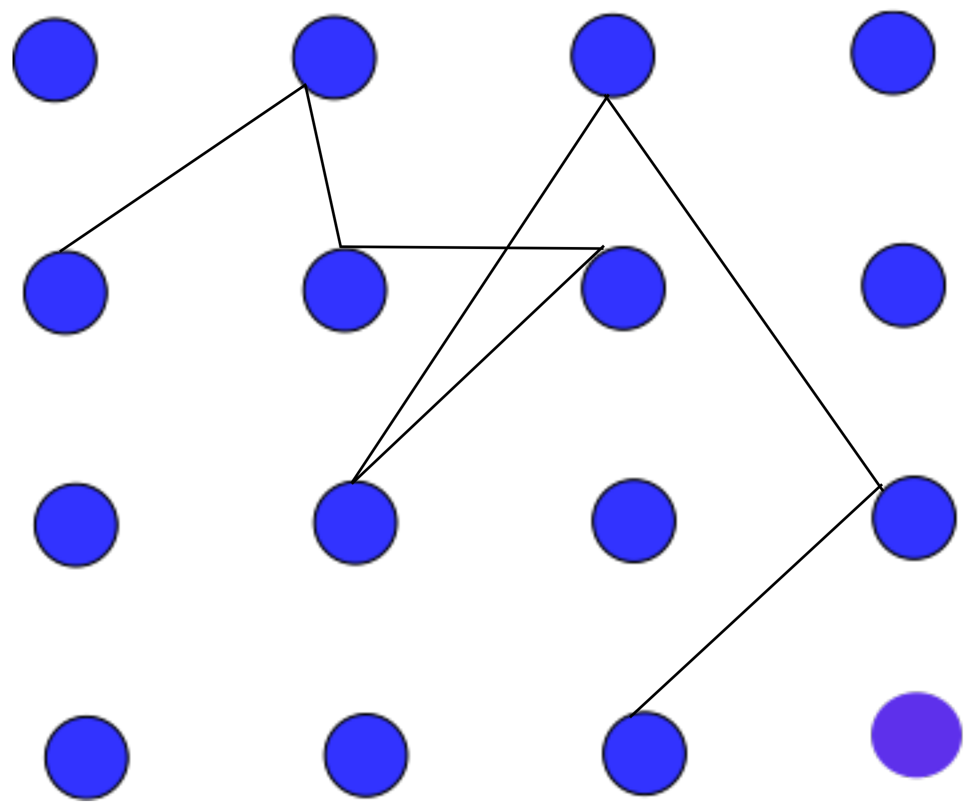
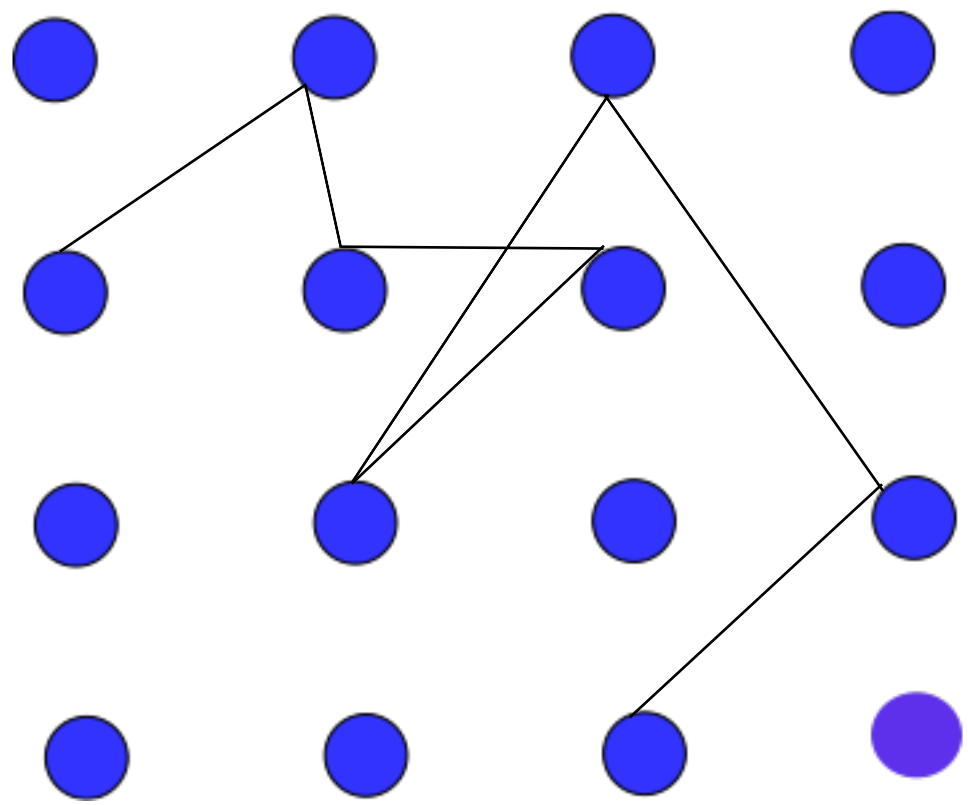


$$\dot{p} = e(E + \frac{p \times B}{m}) - \frac{p}{\tau}$$

momentum relaxation

Drude metal



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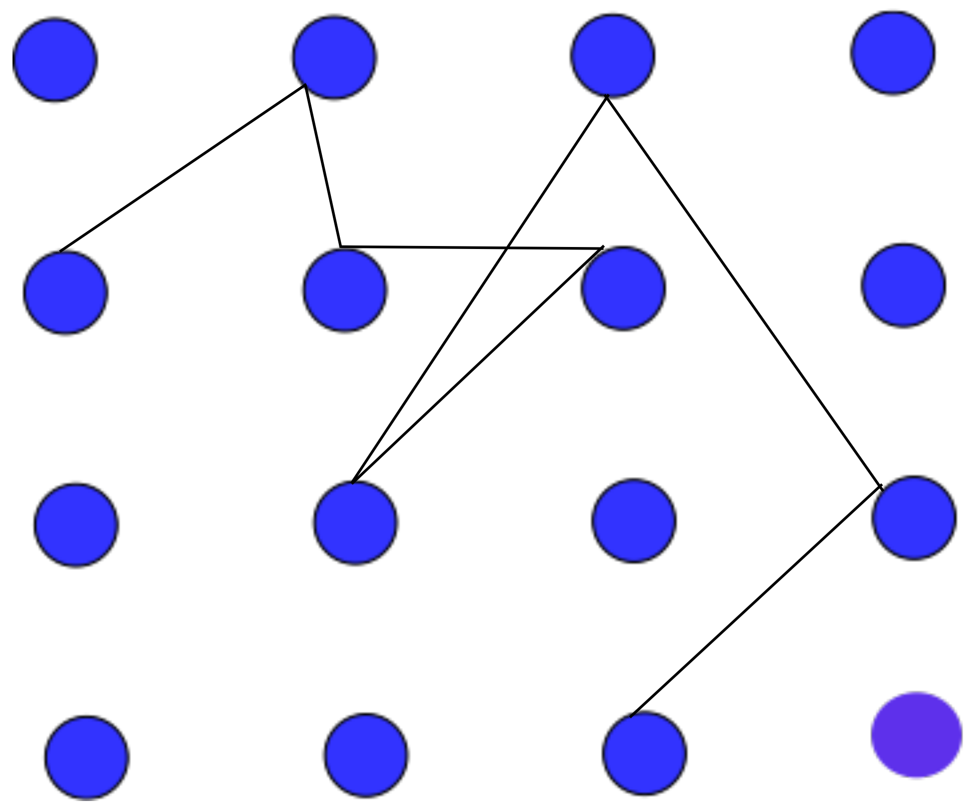

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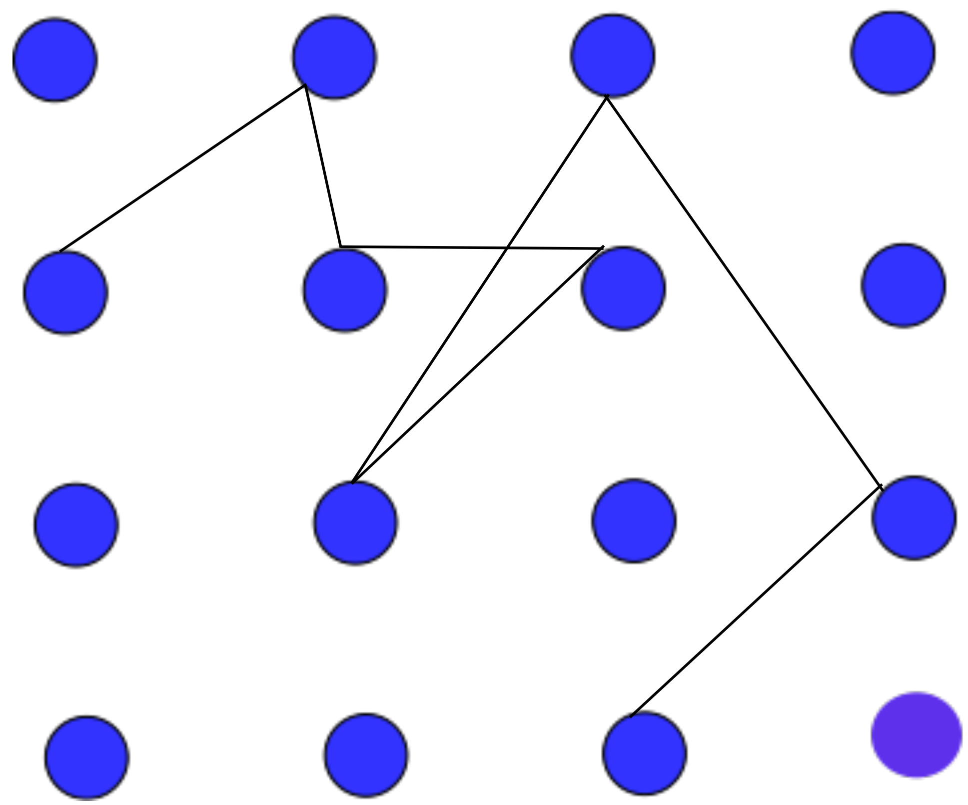

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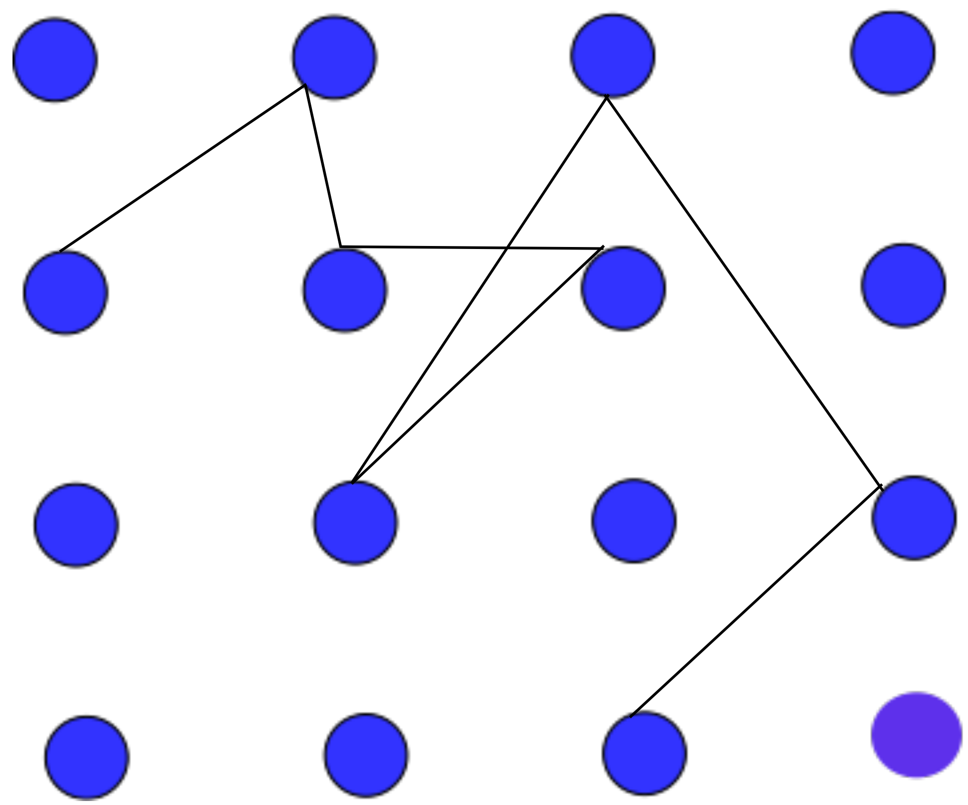

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massive graviton

standard metals

resistivity

$$\rho \propto T^2$$

Weidemann-Franz law

$$\frac{\kappa_{xx}}{T\sigma_{\rm xx}} = \frac{\pi^2}{3}$$

optical conductivity

$$\Re\sigma \propto 1/\omega^2$$

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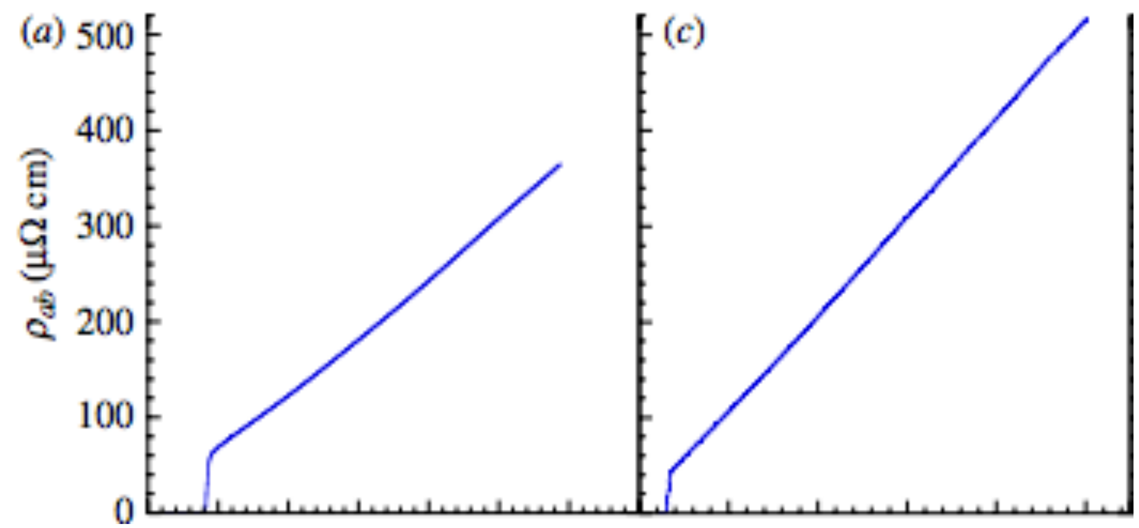
strange metal: experimental facts

T-linear resistivity

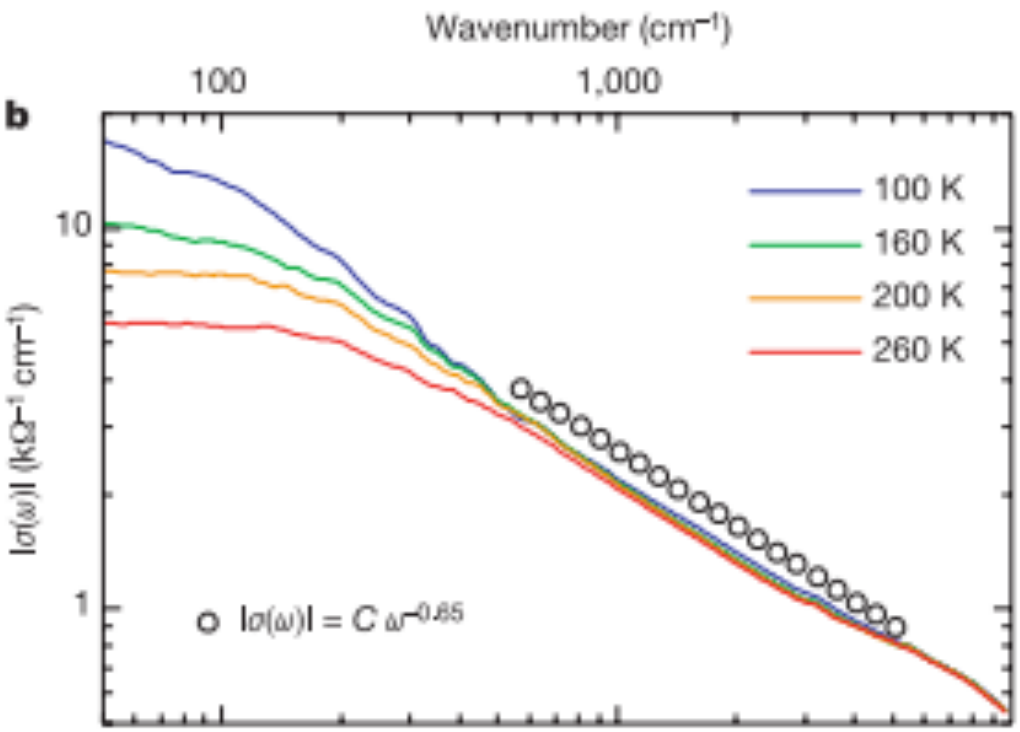


strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

[1]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2]*Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3]*Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*



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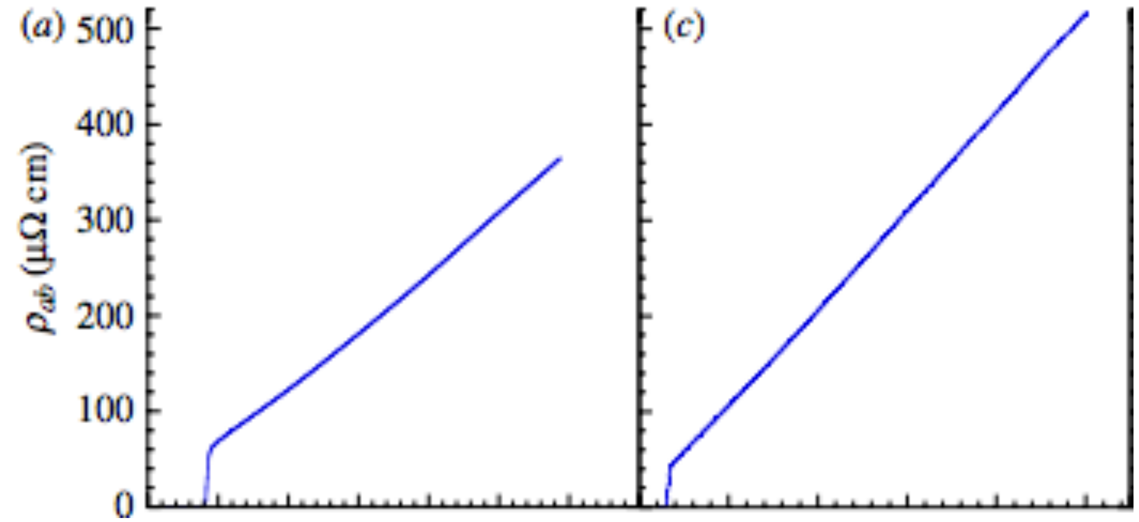
$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

T-linear resistivity



strange metal: experimental facts

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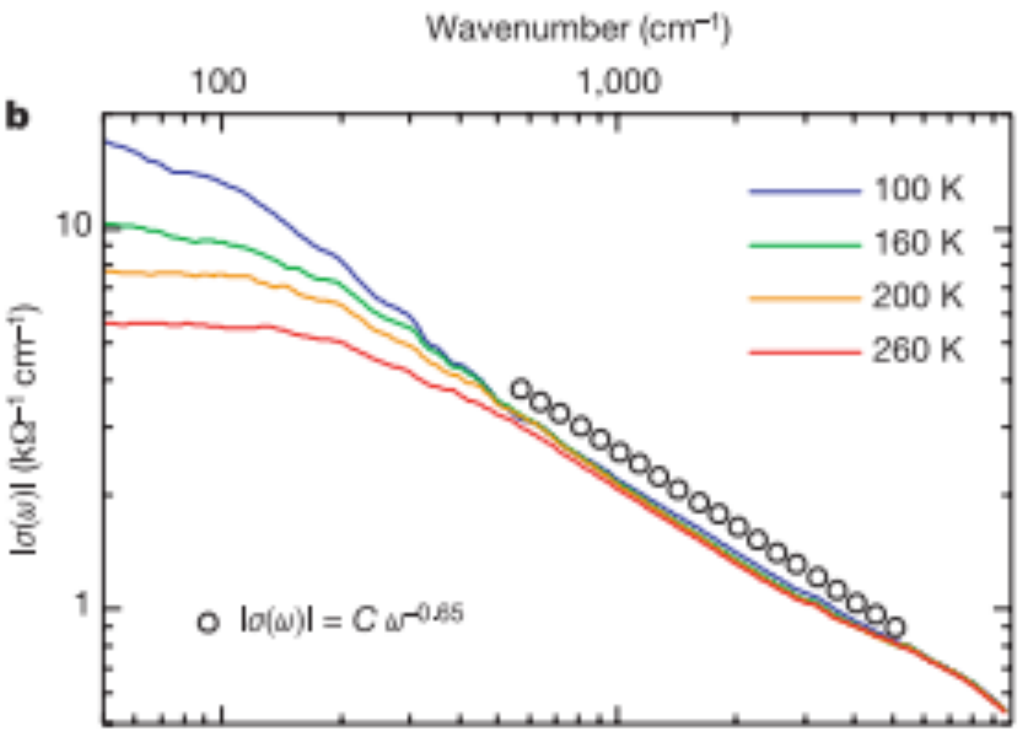

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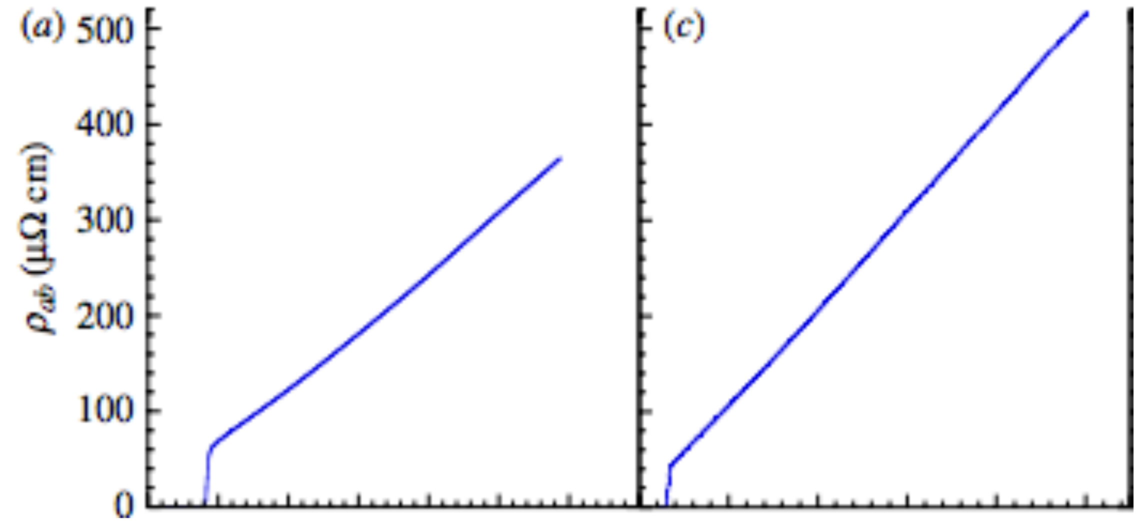
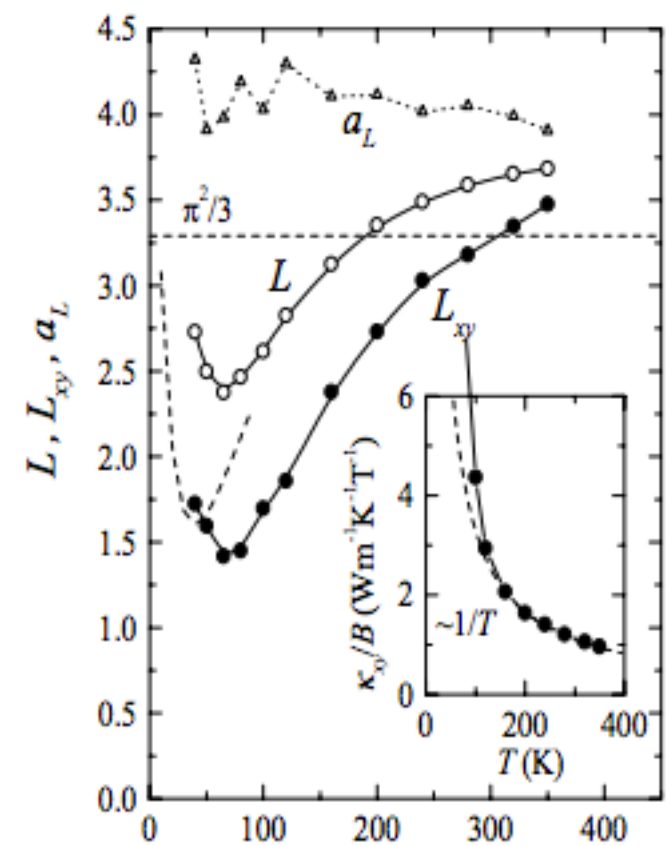
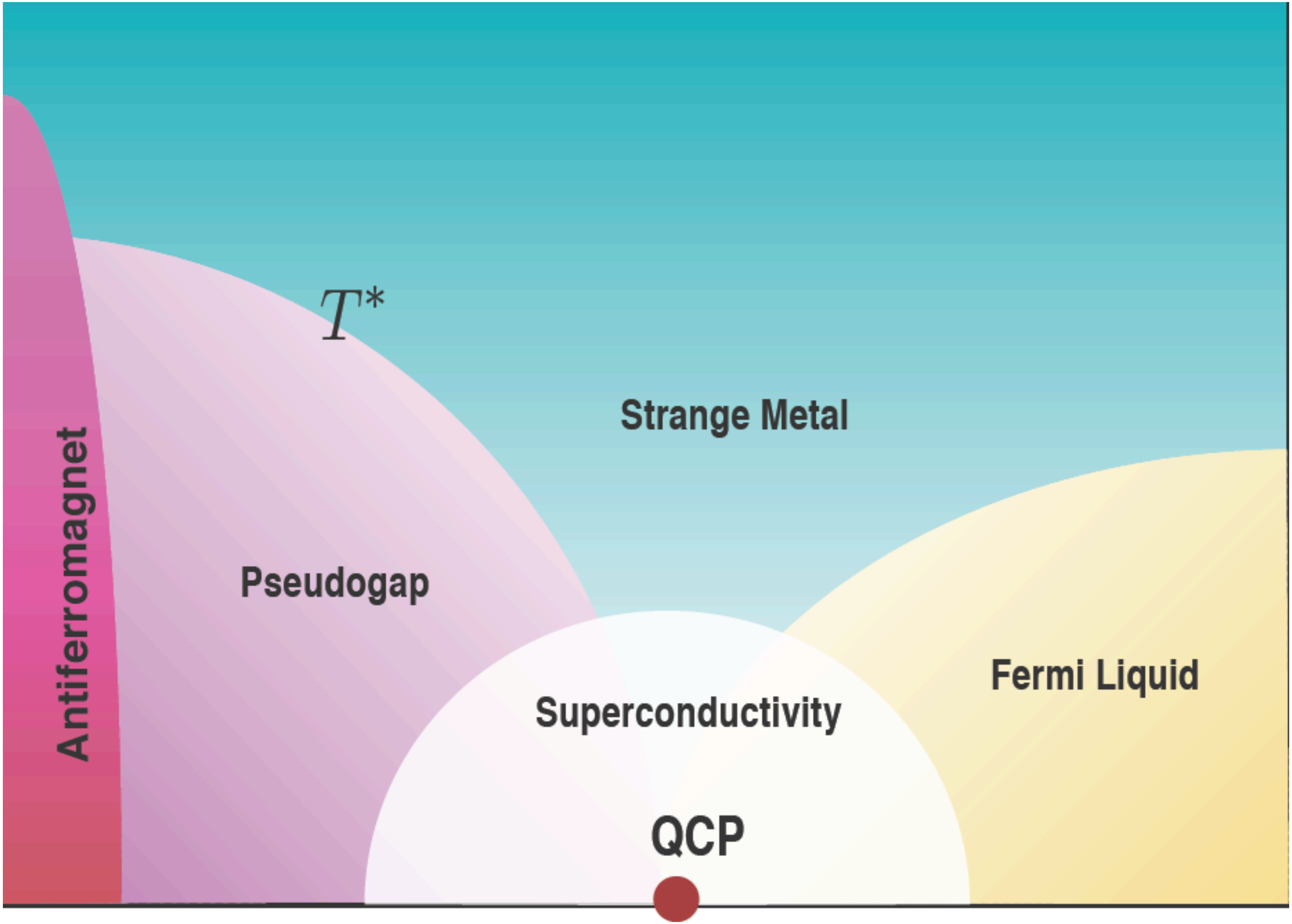


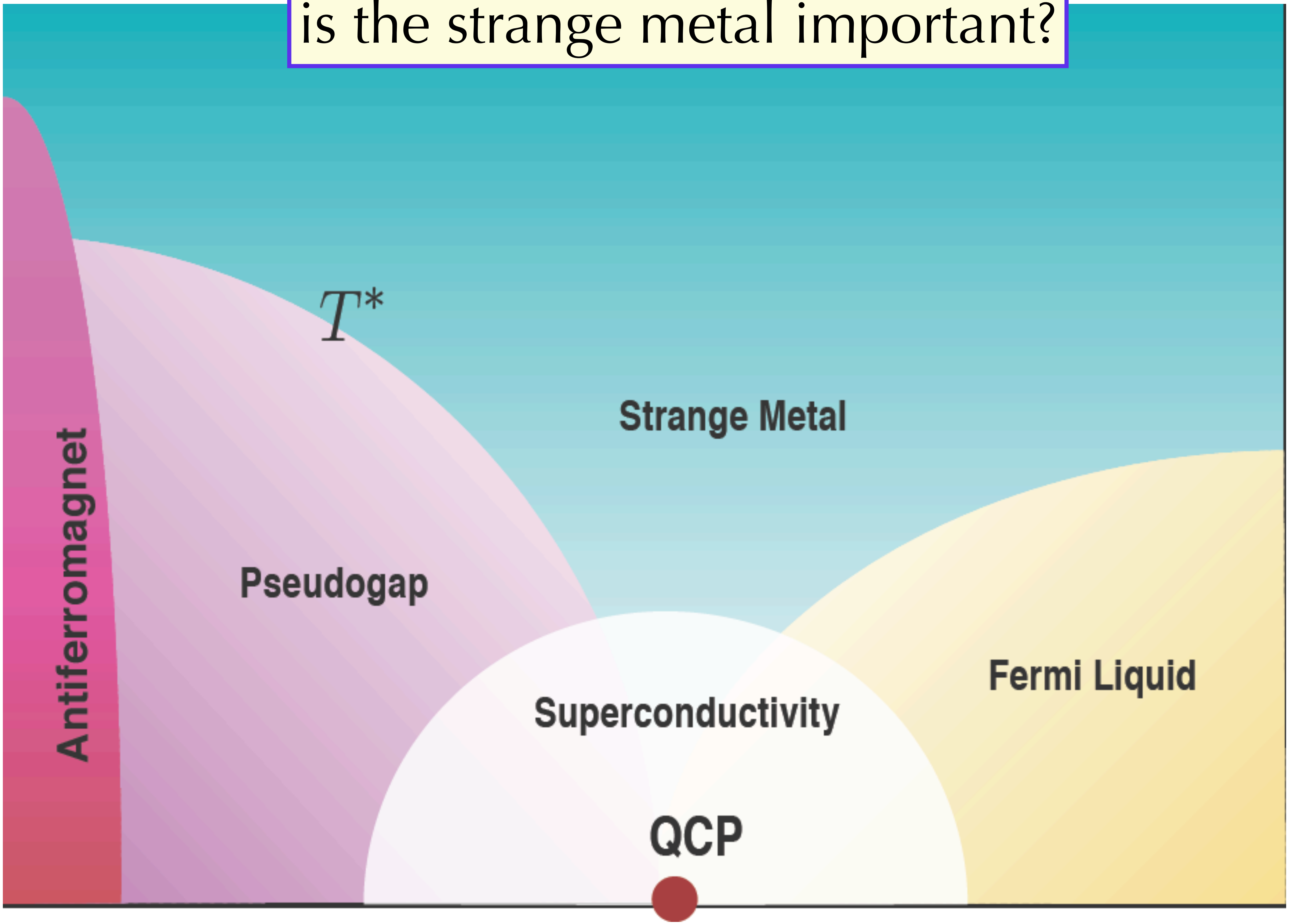
$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$





T^*
Strange Metal
Antiferromagnet
Pseudogap
Fermi Liquid
Superconductivity
QCP

is the strange metal important?



$$\frac{\text{Theories of cuprates}}{\text{Theories of strange metal}} = \infty$$

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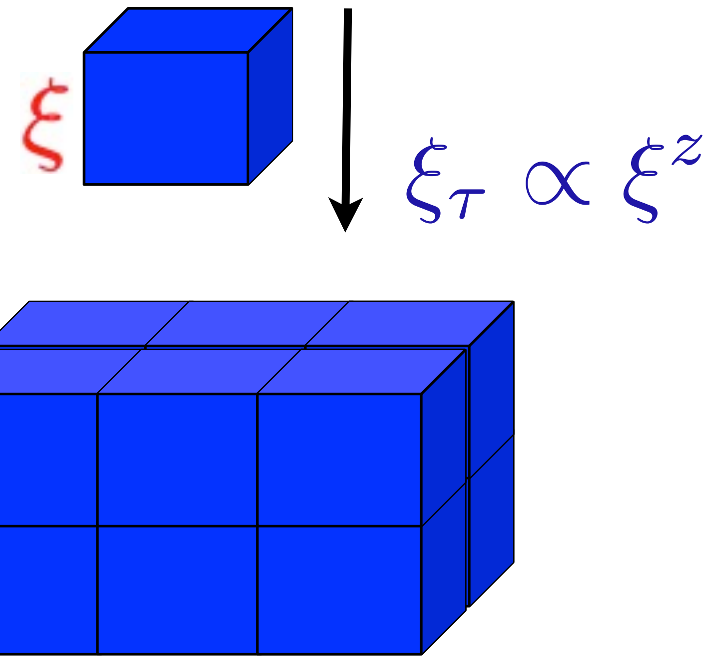
why is the problem hard?

single-parameter scaling

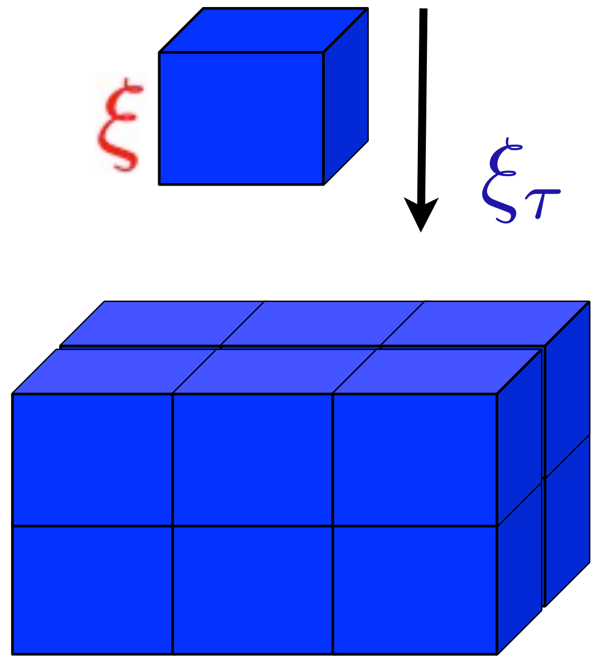


$$\rho \propto T^{(2-d)/z}$$

$$\sigma(\omega, T) \propto \omega^{(d-2)/z}$$

$$C_v \propto T^{d/z}$$

single-parameter scaling



$$\xi_\tau \propto \xi^z$$

1

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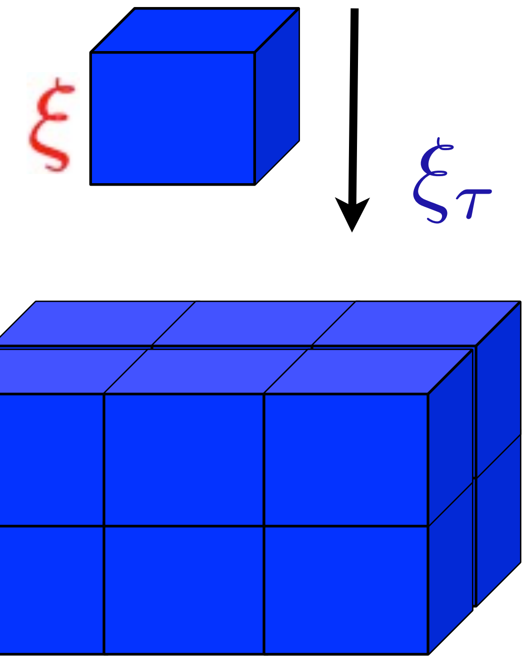

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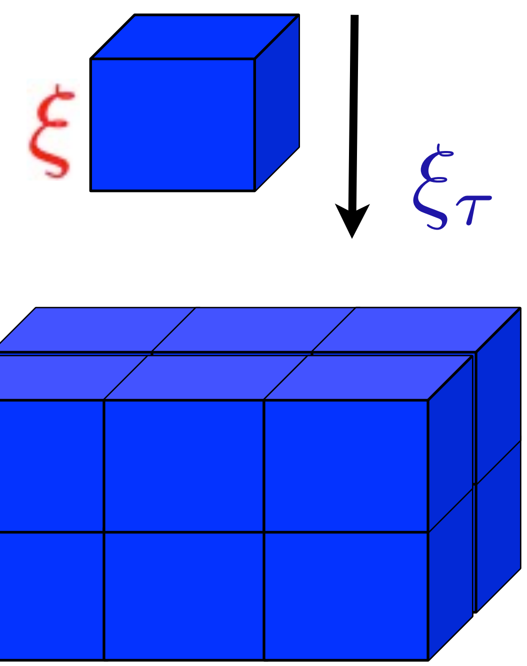

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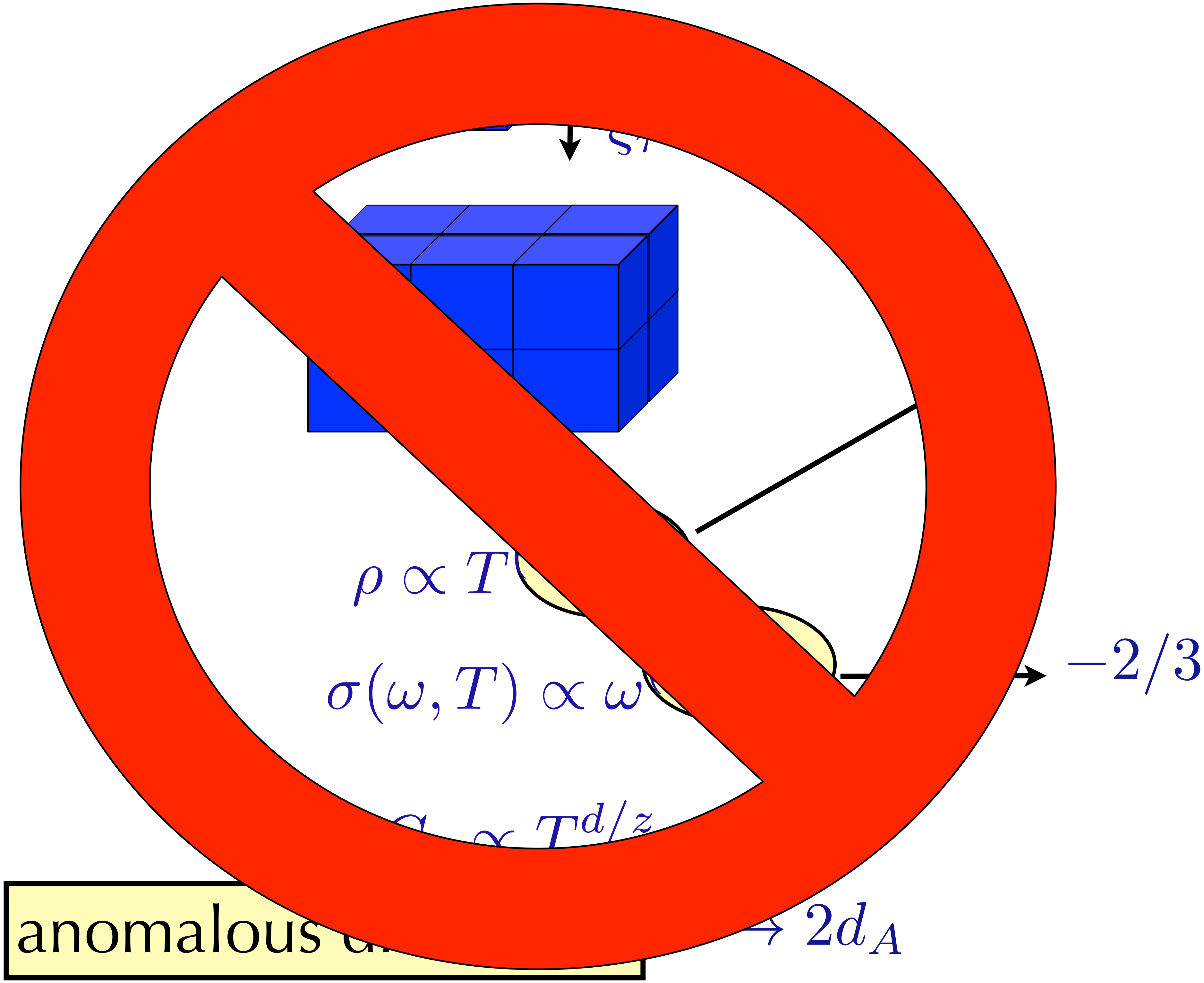
anomalous dimension $2 \to 2d_A$

single-parameter scaling



new length scale?

strange metal

strange metal

multi-scale sector

strange metal

multi-scale sector

$$[J] = \Phi$$

strange metal

multi-scale sector

$[J] = \Phi$ fractional not d-1

strange metal

multi-scale sector

$$[J] = \Phi$$

fractional
not d-1

$$[A_\mu] = d_A \neq 1$$

strange metal

multi-scale sector

$$[J] = \Phi$$

fractional
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$$[A_\mu] = d_A \neq 1$$

probe by fractional Aharonov-Bohm effect

optical conductivity

$$N_{\mathrm{eff}}(\Omega) = \frac{2mV_{\mathrm{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

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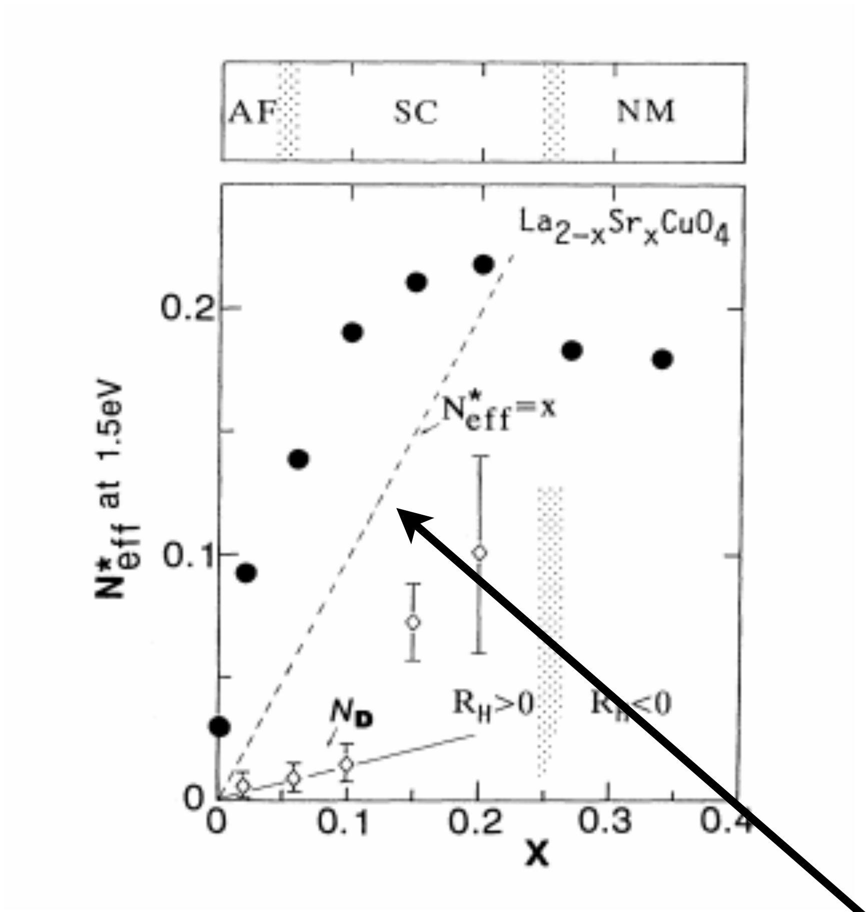
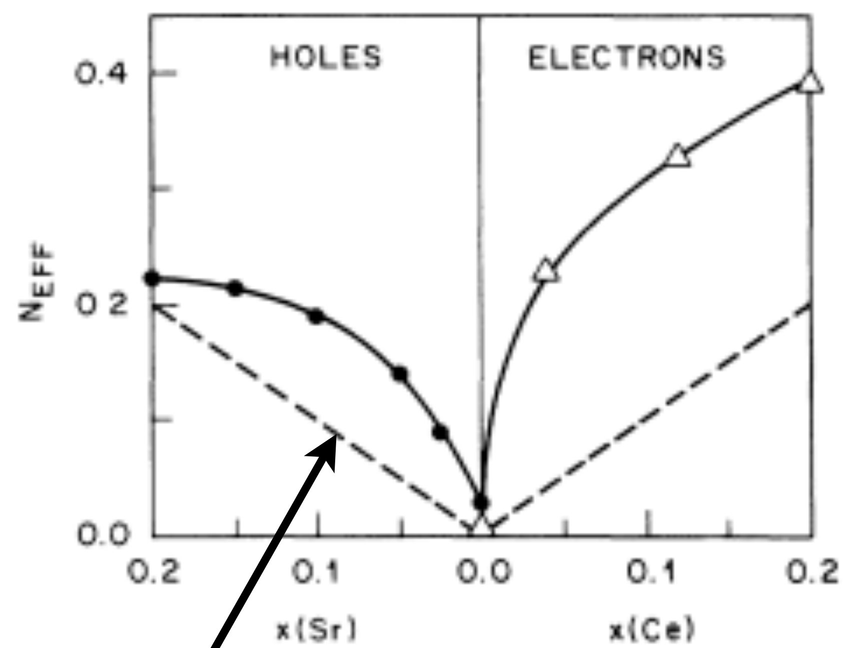
Uchida, et al.

Cooper, et al.





x

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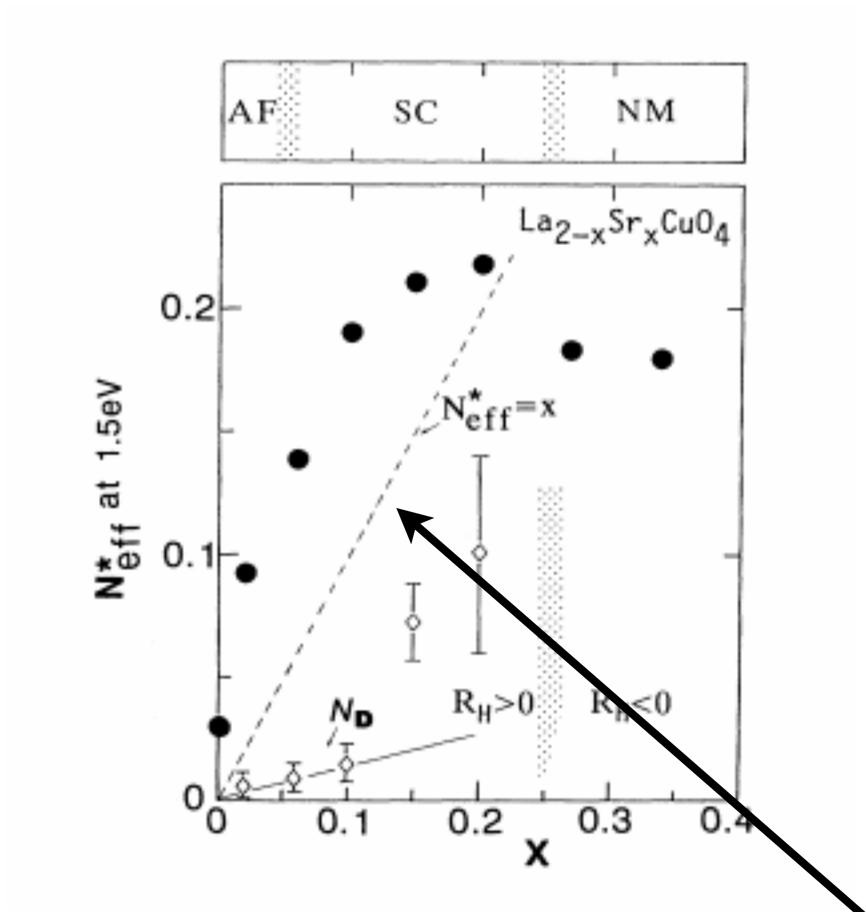

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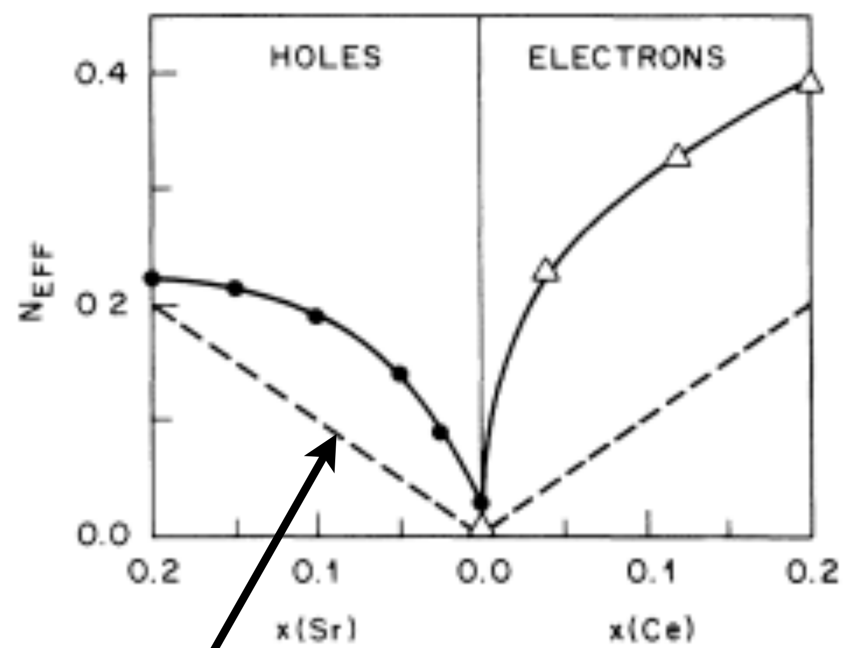

x

low-energy model for $N_{\text{eff}} > x$??

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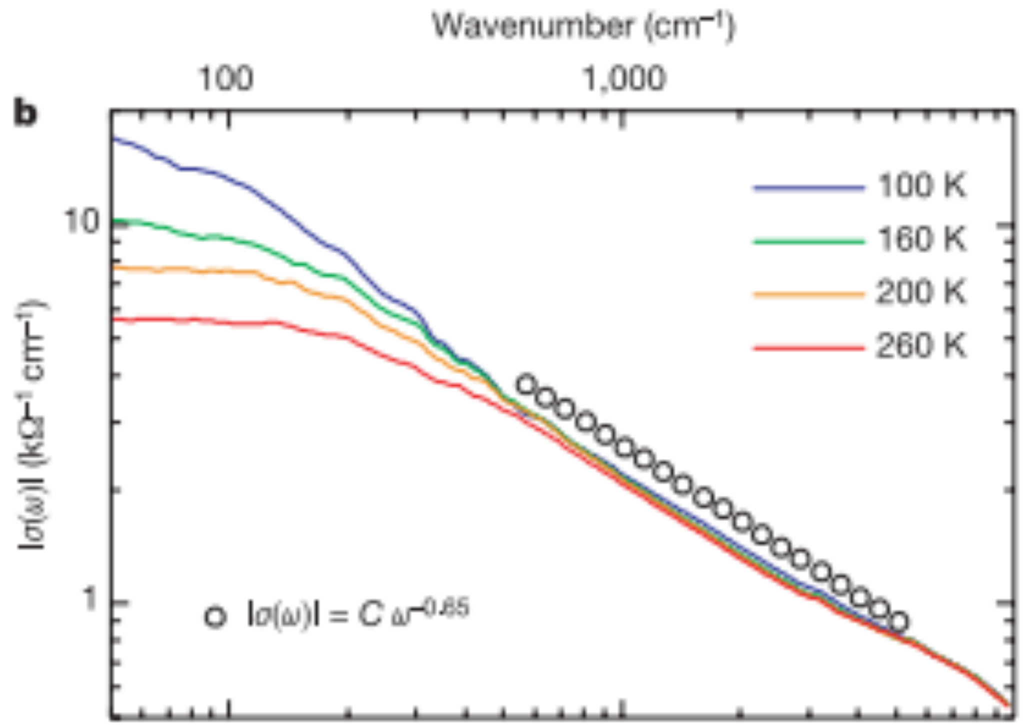

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$
$$\gamma = 1.35$$

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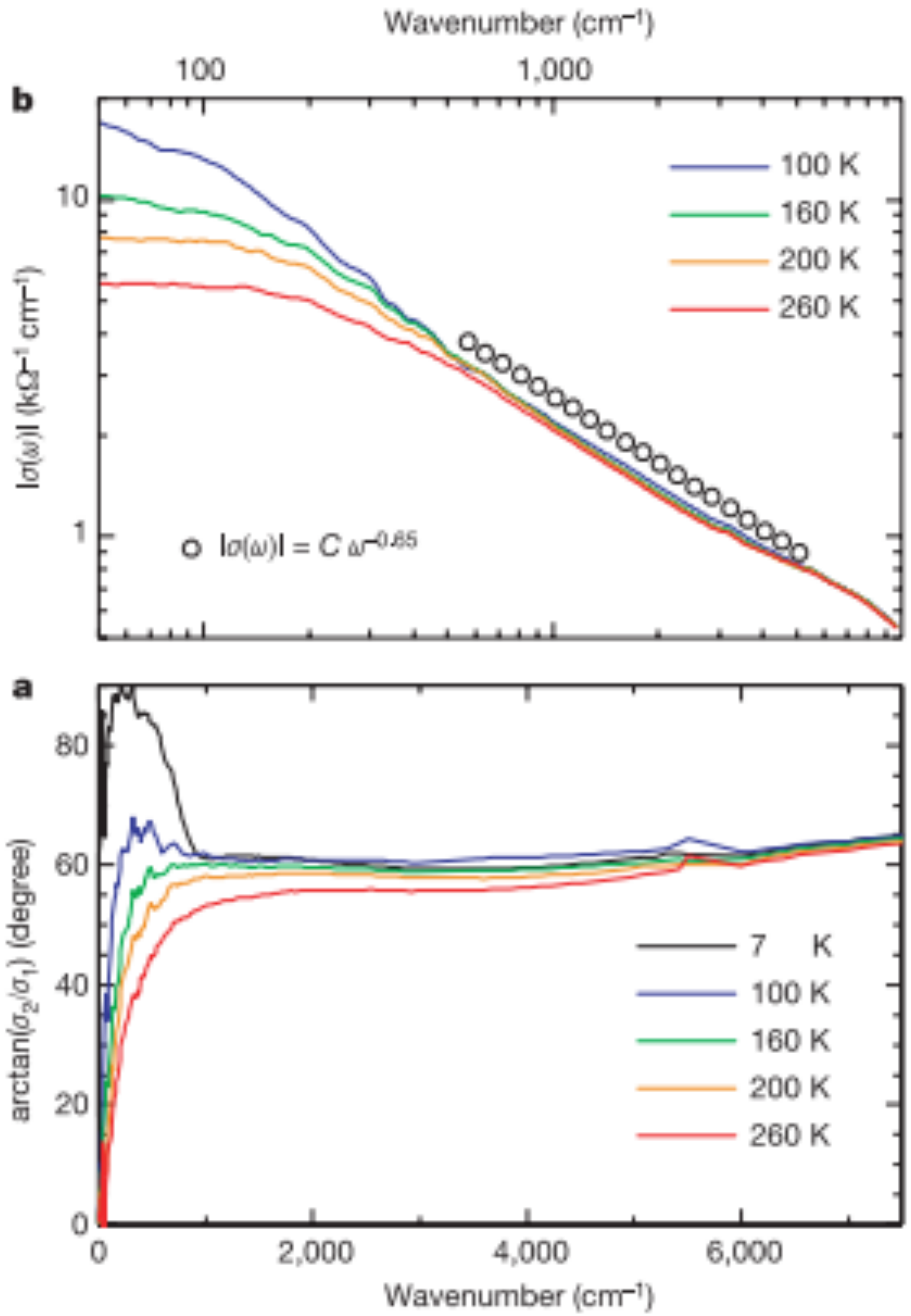

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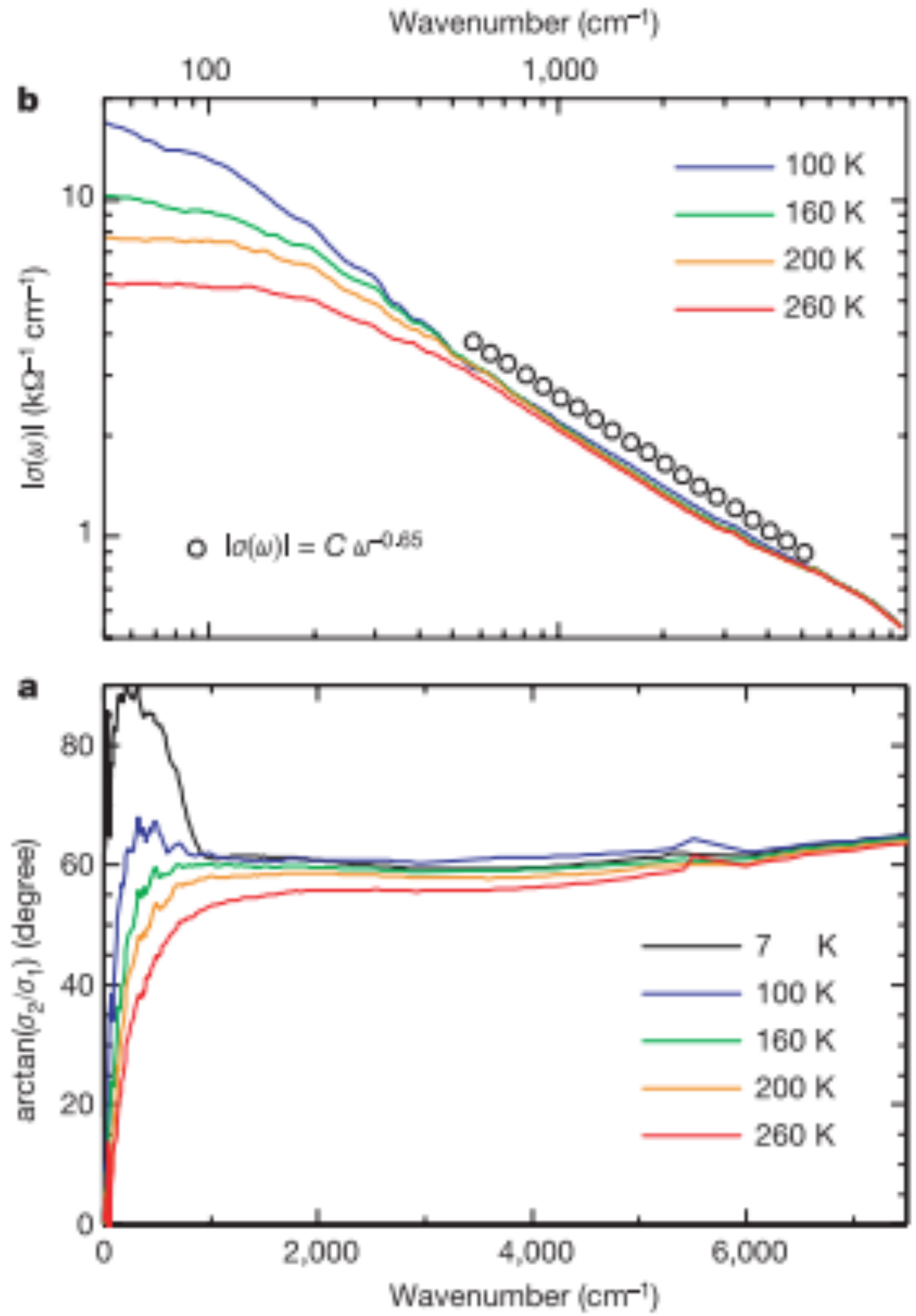

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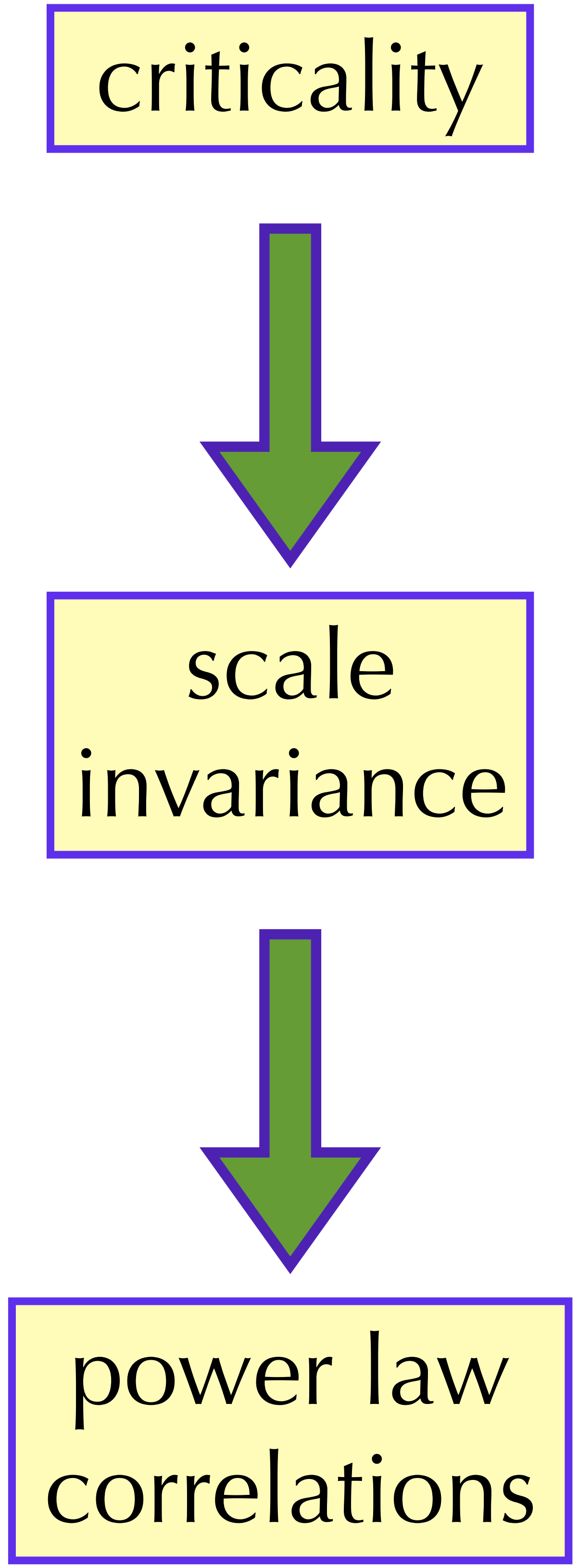
$$\tan\sigma_2/\sigma_1 = \sqrt{3}$$
$$\theta = 60^\circ$$


criticality
scale invariance
power law correlations

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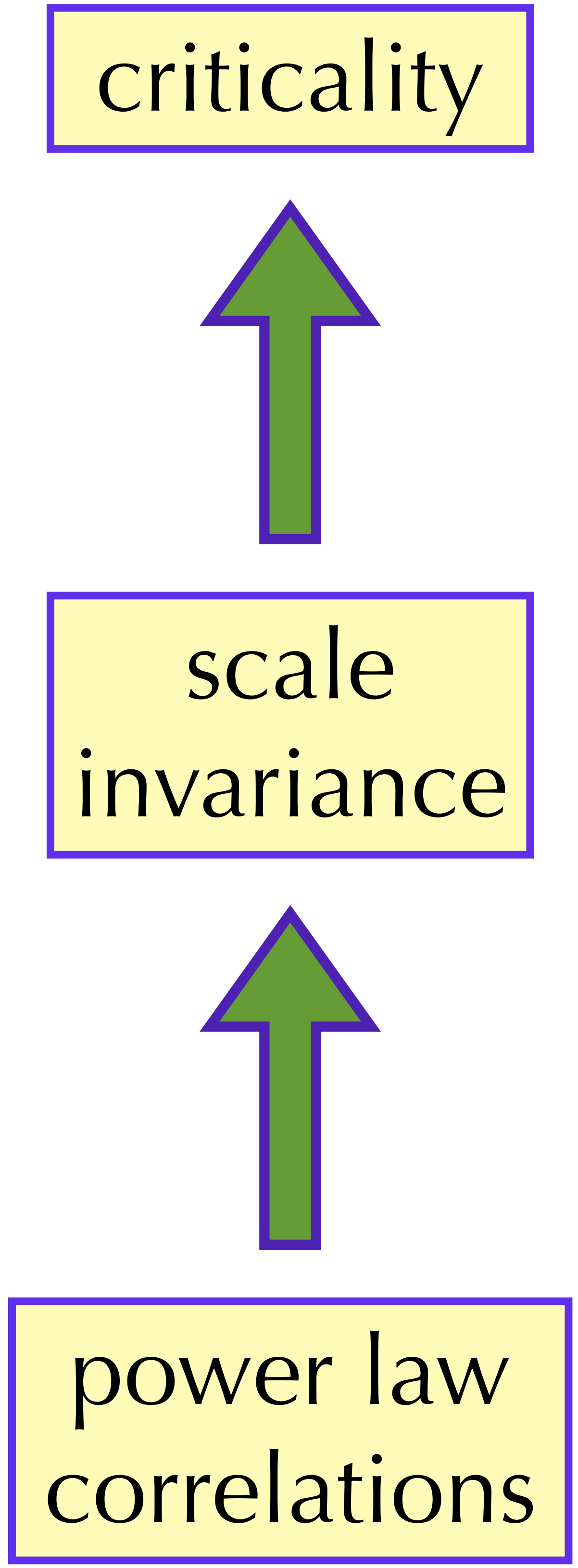
criticality
scale invariance
power law correlations

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^\eta}$$

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

1.) cuprates are not 1-dimensional

2.) vertex corrections matter

problems

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2.) vertex corrections matter

$$\left.\begin{aligned} &\sigma \propto G\left(G(\Gamma^\mu)^2, \Gamma^{\mu\nu}\right) \\ &[G] = L^{d+1-2d_U} \\ &[\Gamma^\mu] = L^{2d_U-d} \\ &[\Gamma^{\mu\nu}] = L^{2d_U-d+1} \end{aligned}\right\}$$

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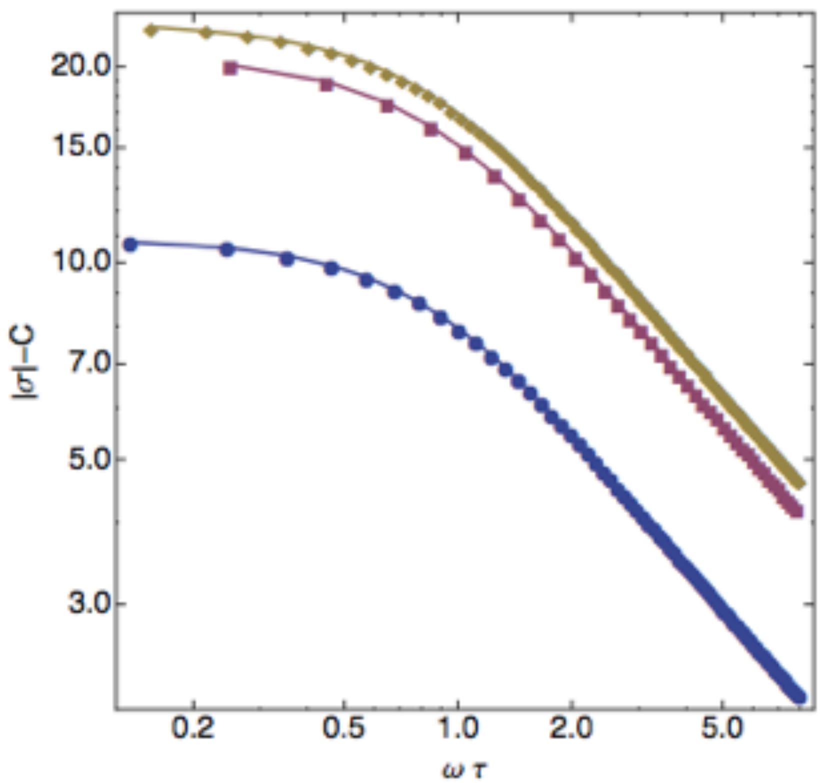
\end{aligned}\right\} \quad [\sigma] = L^{2-d}$$

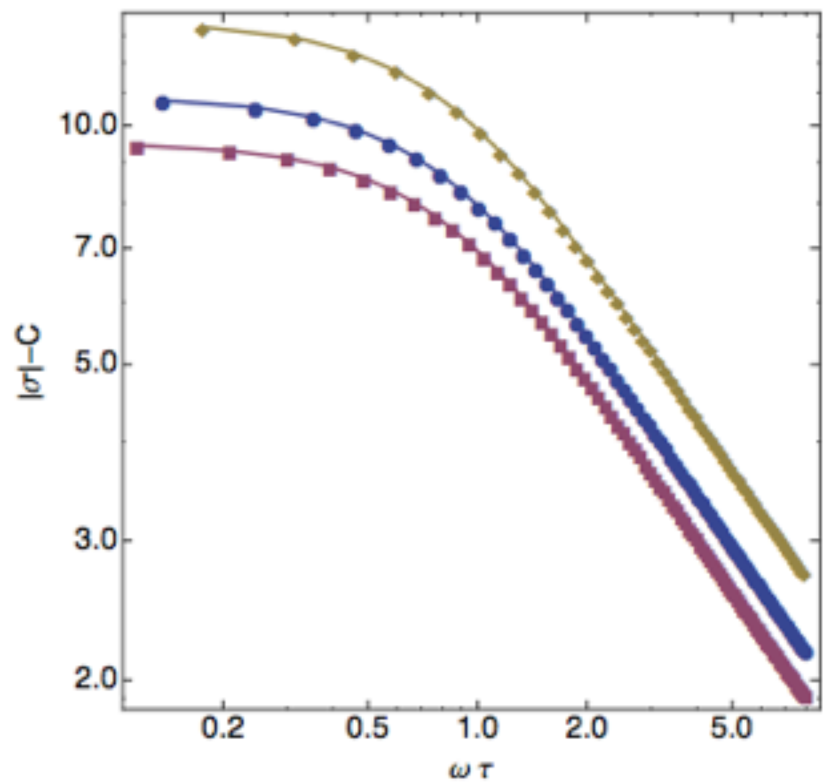
independent of d_U


1997
Anderson


1997
Anderson
2012
string theory
AdS/CFT

optical conductivity from a gravitational lattice





log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al.*, Journal of High Energy Physics, 2012

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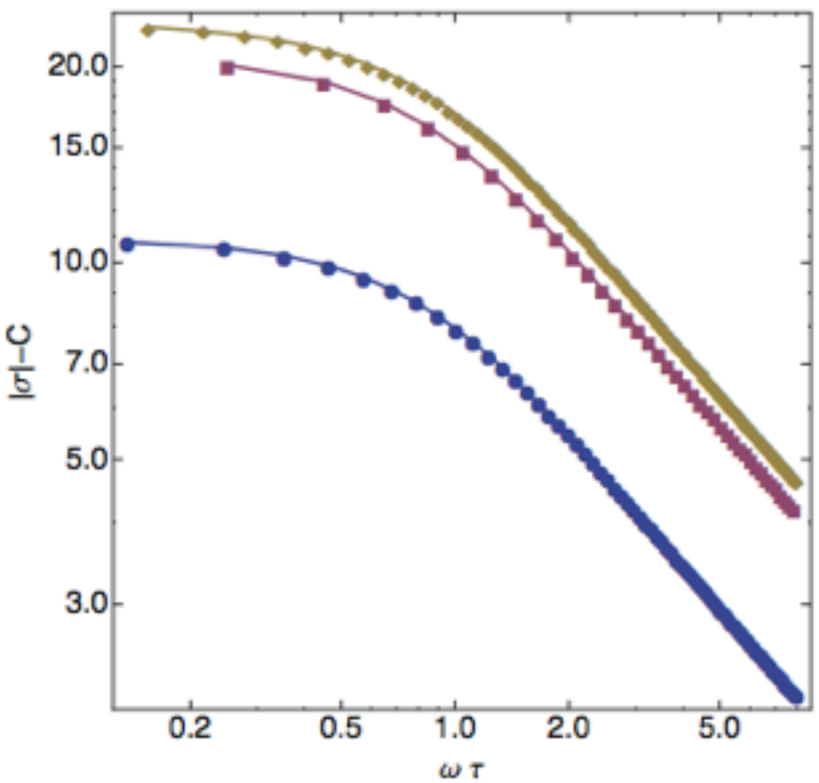

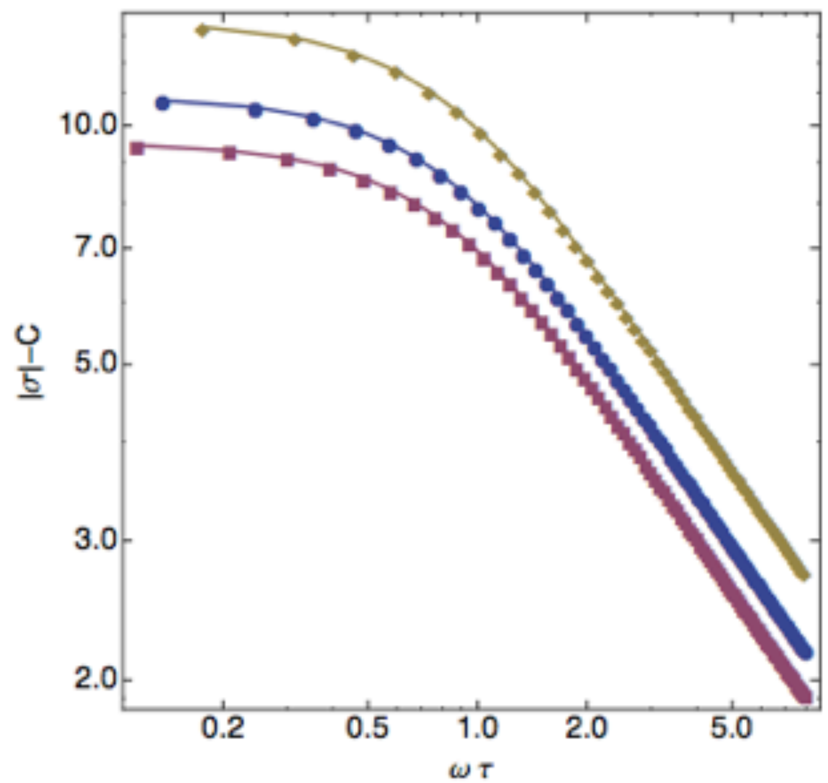

log-log plots for various parameters

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for $0.2 \lesssim \omega\tau \lesssim 0.8$

a remarkable claim!
replicates features of the strange metal? how?

G. Horowitz *et al*., Journal of High Energy Physics, 2012

$\boxed{\text{new equation!}}$

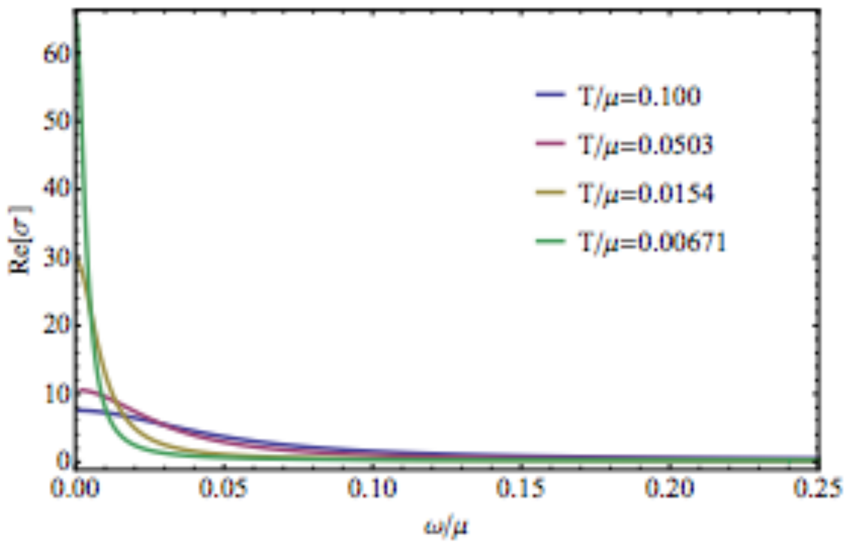
$\boxed{\text{Einstein-Maxwell equations}} + \boxed{\text{non-uniform charge density}} = B\omega^{-2/3}$

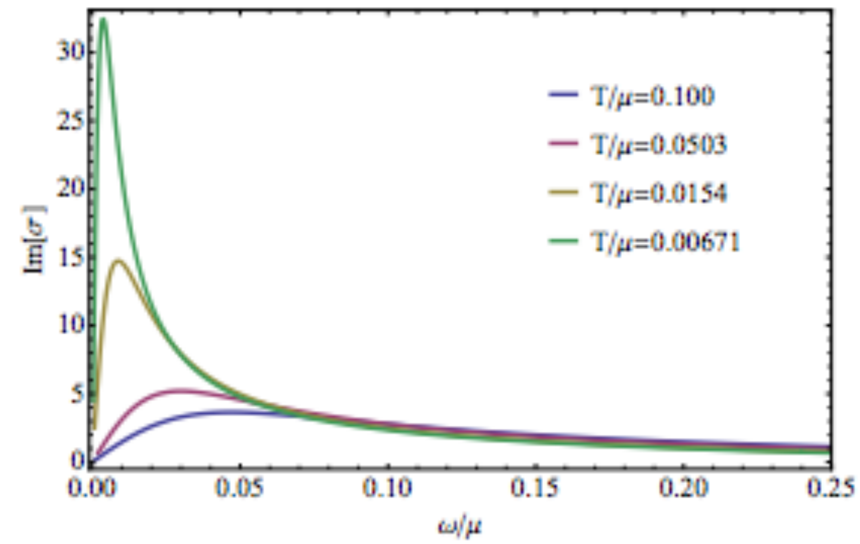
not so fast!

Donos and Gauntlett
(Q-lattice)

Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$





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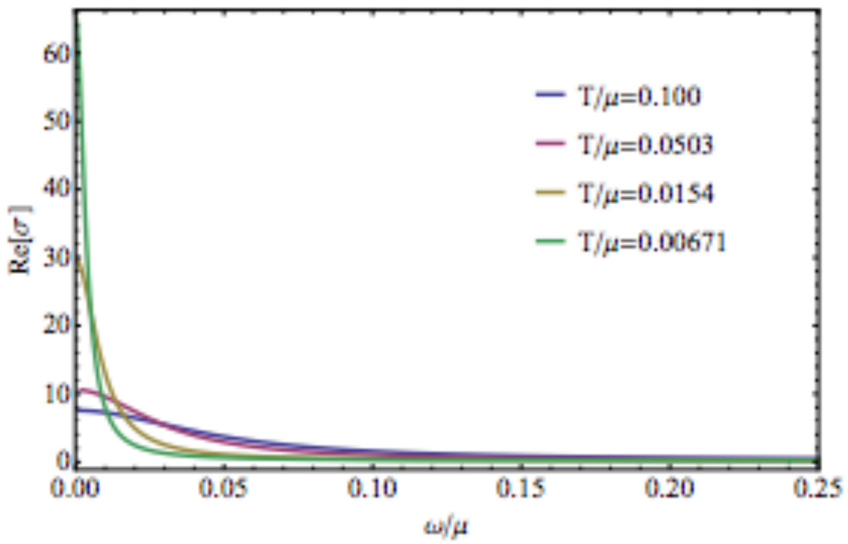

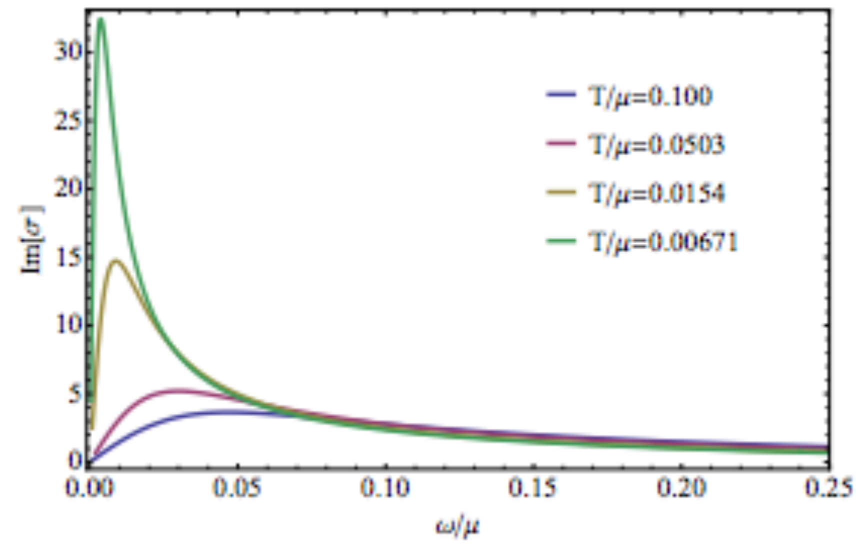

$$-\frac{2}{3}=1+\omega\frac{|\sigma|''}{|\sigma|'}$$

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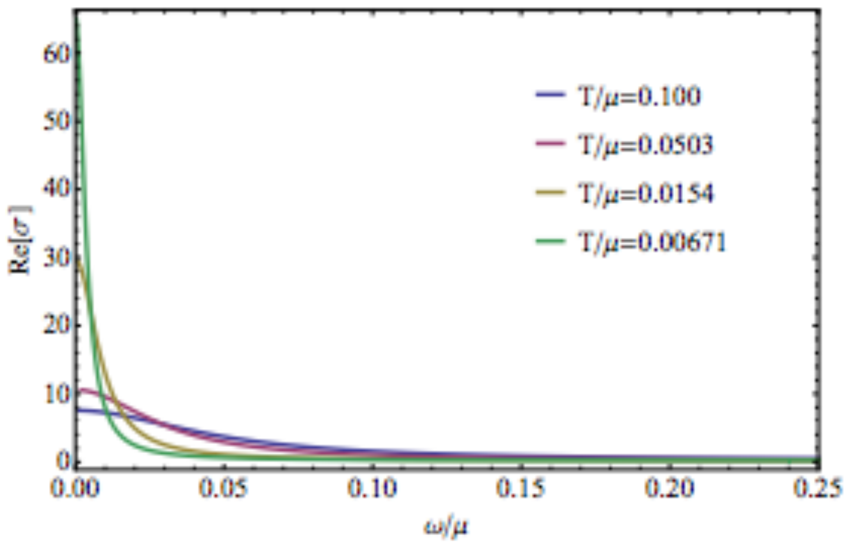

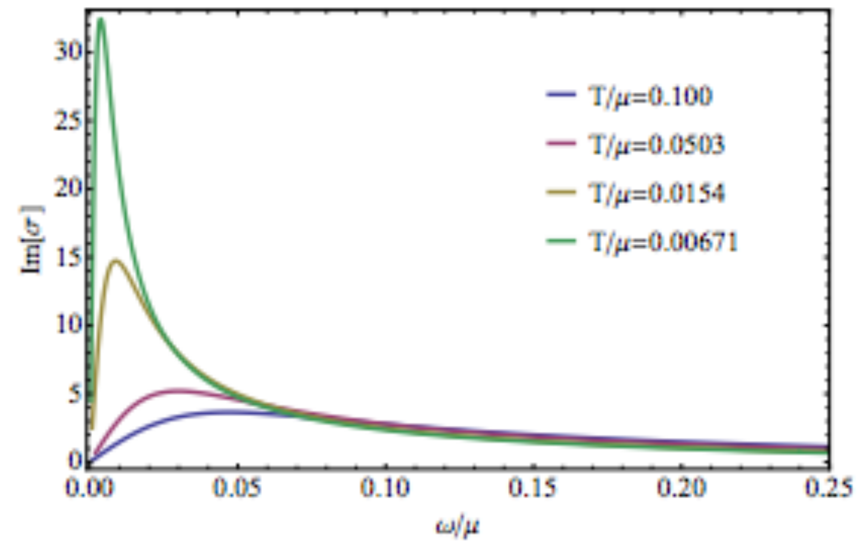

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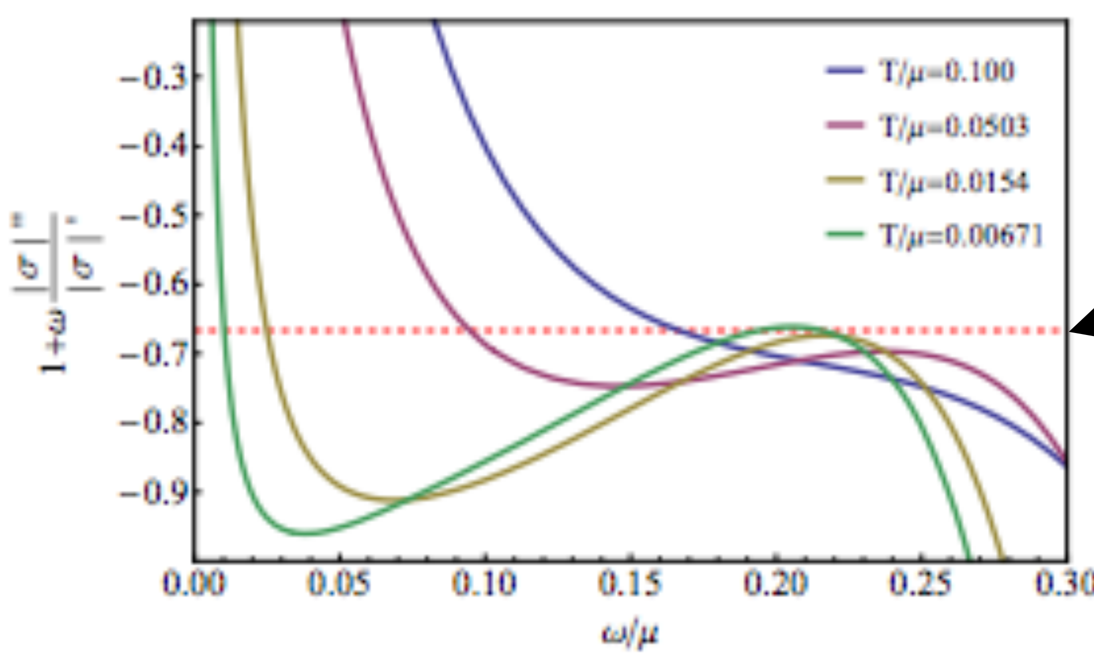

$B\omega^{-2/3}$

Donos and Gauntlett (Q-lattice)

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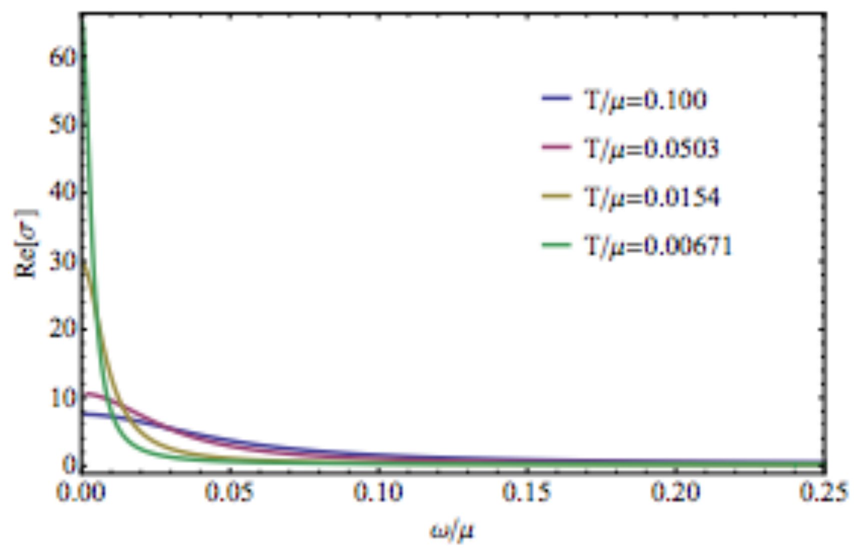

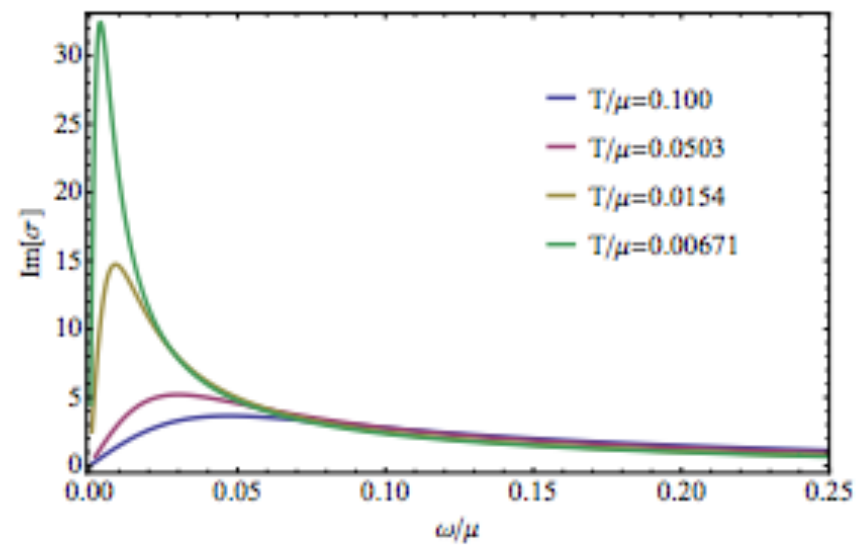

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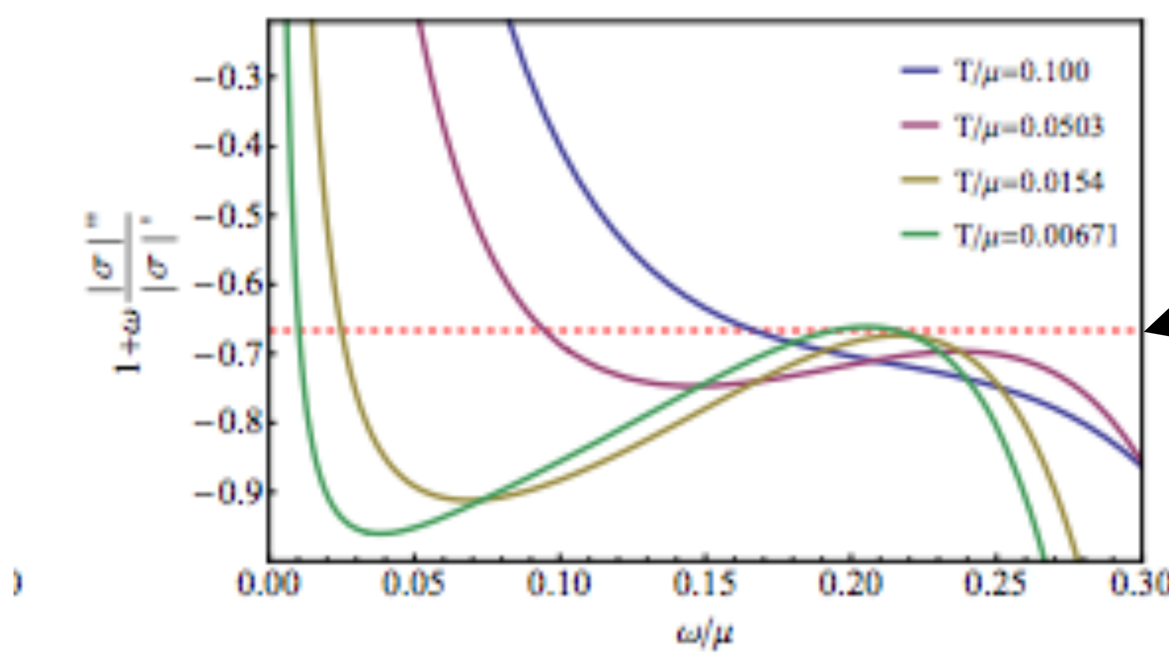


$B\omega^{-2/3}$

no power law!!

who is correct?

who is correct?

let's redo the calculation

model

action = gravity + EM + lattice

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(\boxed{R-2\Lambda}-\frac{1}{2}F^2\right),$$

$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(\phi)]$$

model

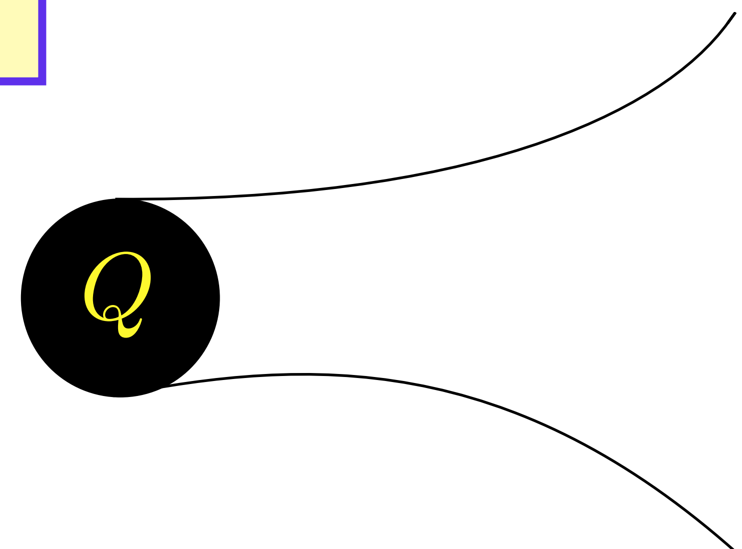
action = gravity + EM + lattice

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(\boxed{R-2\Lambda}-\frac{1}{2}F^2\right),$$

$$\mathcal{L}(\phi) = \sqrt{-g}[-|\partial\phi|^2 - V(\phi)]$$

conductivity
within AdS

$$(g_{\mathrm{ab}}\,,V(\Phi),A_t)$$
$$(\mathrm{metric}\,,\mathrm{potential}\,,\mathrm{gaugefield})$$



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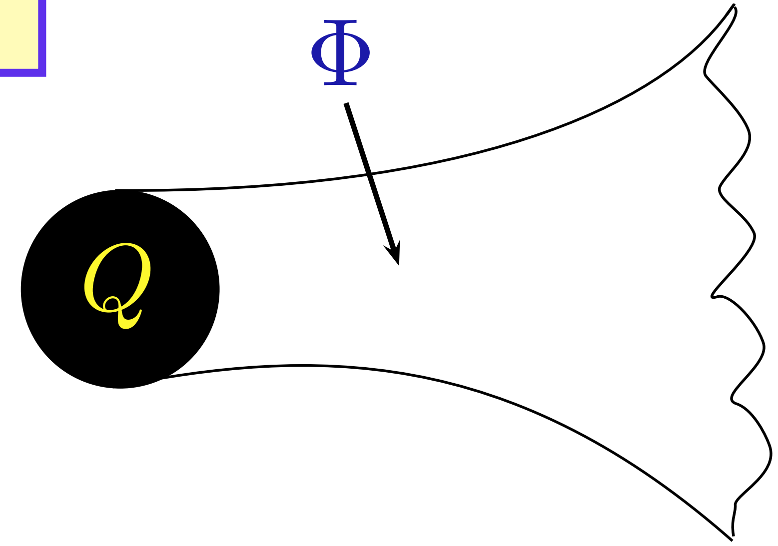

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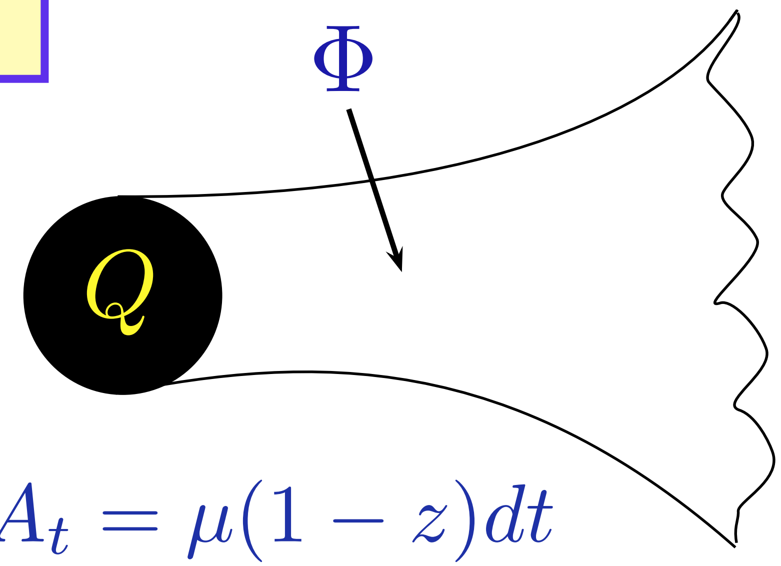

$$A_t = \mu(1-z)dt$$

$$\rho = \lim_{z\to 0}\sqrt{-g}F^{tz}$$

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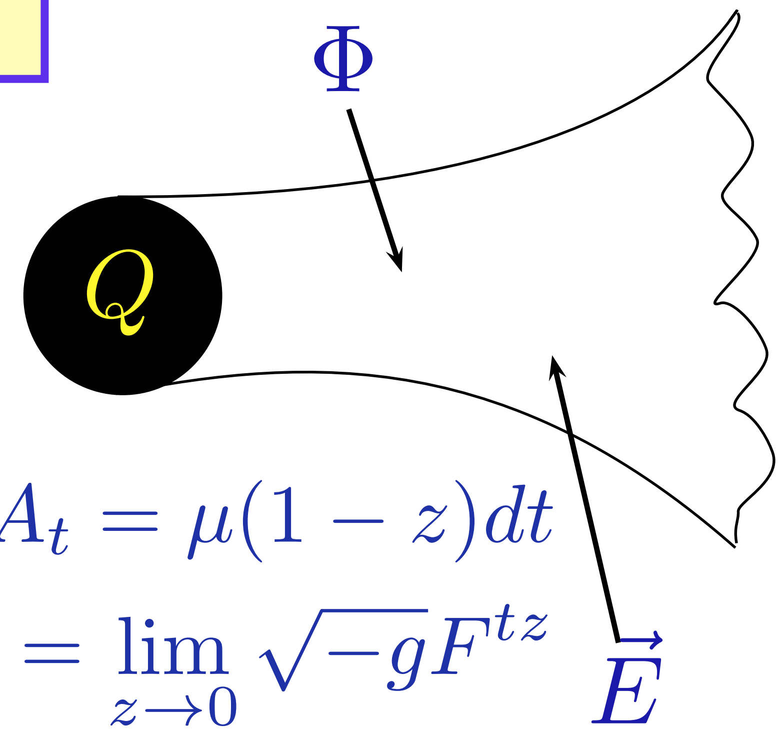
perturb with
electric field



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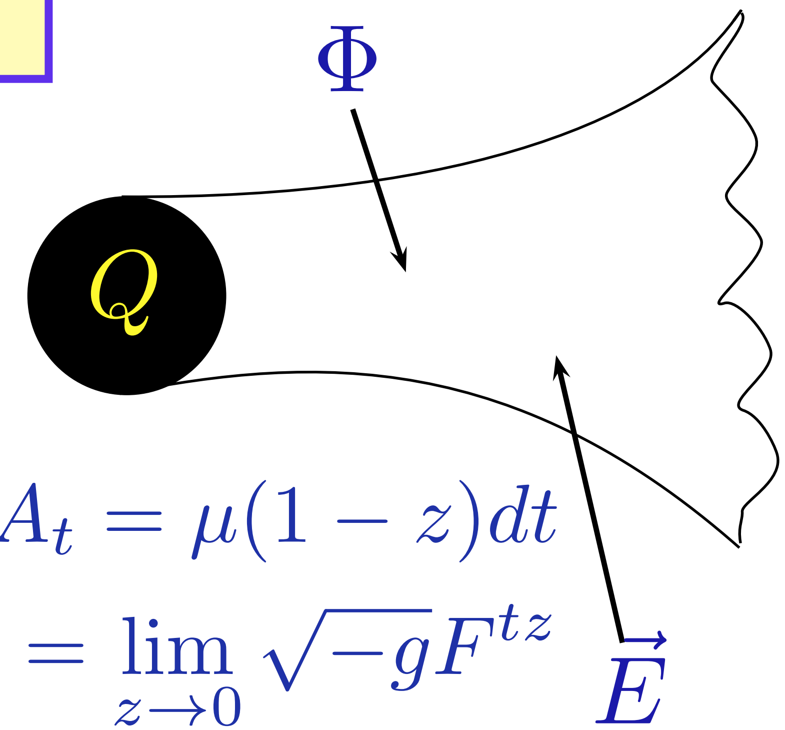

$$A_t = \mu(1-z)dt$$

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perturb with electric field

$$g_{\mathrm{ab}} = \bar{g}_{\mathrm{ab}} + h_{\mathrm{ab}}$$
$$A_a = \bar{A}_a + b_a$$
$$\Phi_i = \bar{\Phi}_i + \eta_i$$

conductivity within AdS

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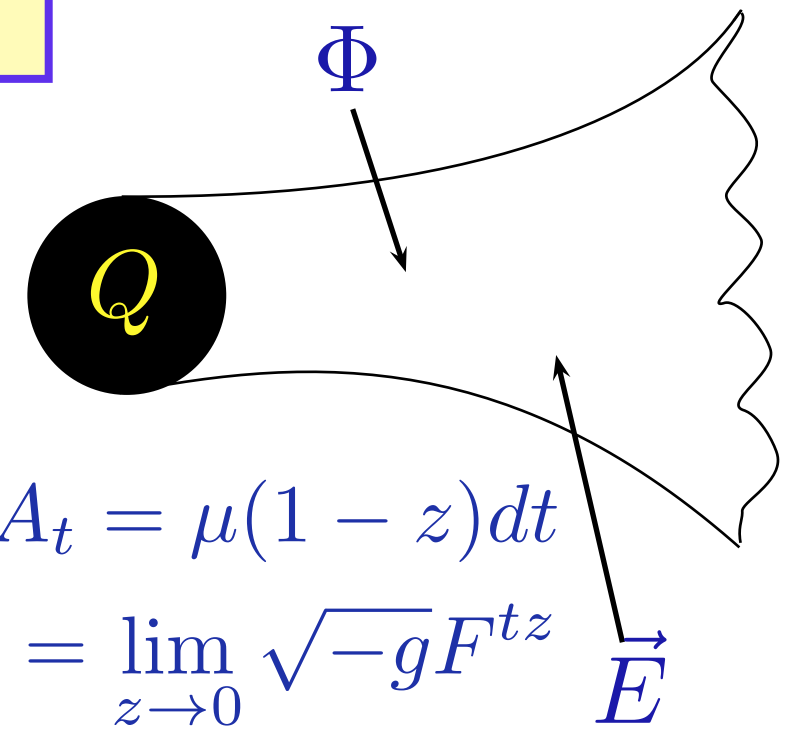

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$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega)z + O(z^2)$$

$$\sigma = J_x(x,\omega)/E$$

conductivity within AdS

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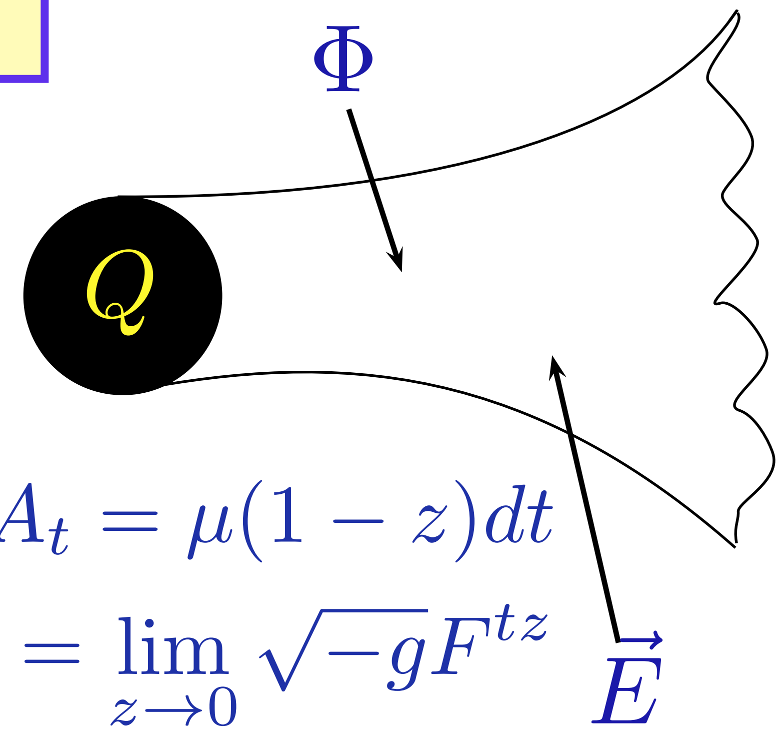

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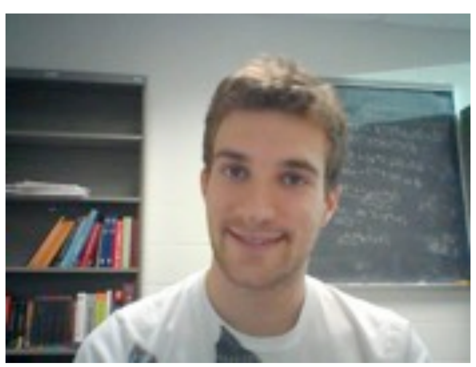

Brandon Langley

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solve equations of motion with gauge invariance (without mistakes)

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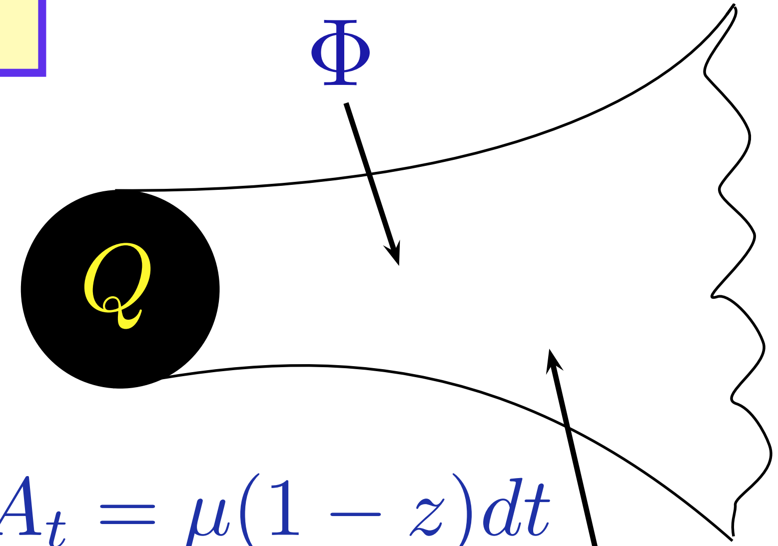

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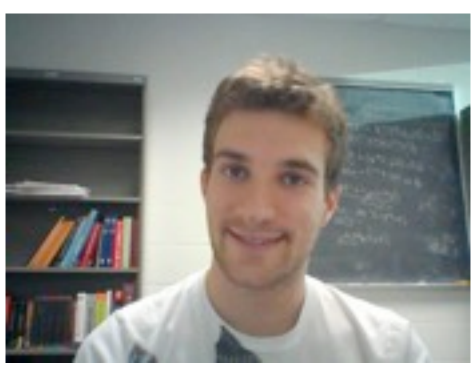
$\vec{E}$

perturb with
electric field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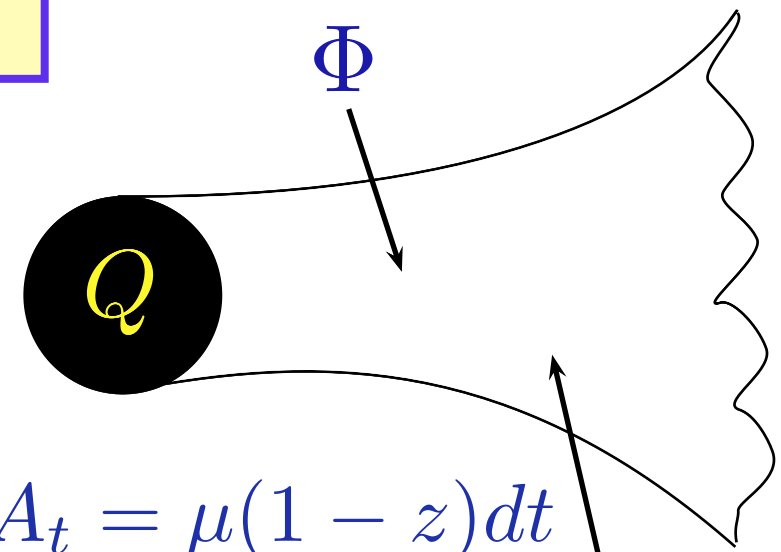

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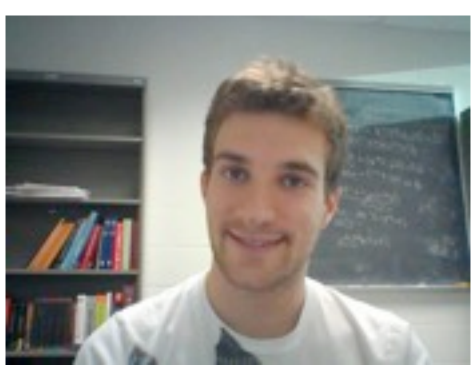

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solve equations of motion with gauge invariance (without mistakes)

$$\sigma = J_x(x,\omega)/E$$

HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots \, ,$$
$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

HST vs. DG

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de Donder gauge

DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

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$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

radial gauge

Our Model

$$\mathcal{L}_{\Phi} = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right),$$

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right),$$

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Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right),$$

$$\Phi_2 = A_0 \cos\left(kx + \frac{\theta}{2}\right)$$

$$\theta = 0$$

HST

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

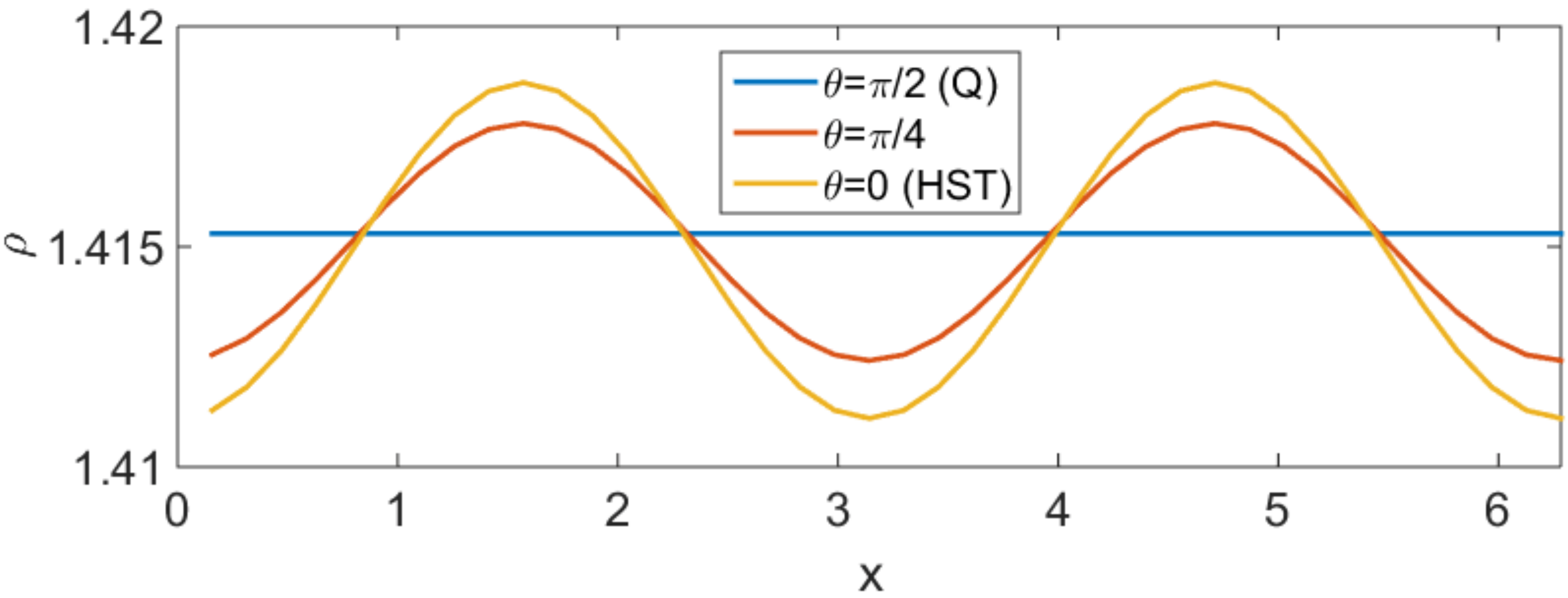
$$\Phi_1(x) = A_0 \cos\left(kx - \frac{\theta}{2}\right), \qquad \Phi_2 = A_0 \cos\left(kx + \frac{\theta}{2}\right)$$

$\theta = 0$ → **HST**

$\theta = \frac{\pi}{2}$ → **DG**

charge density

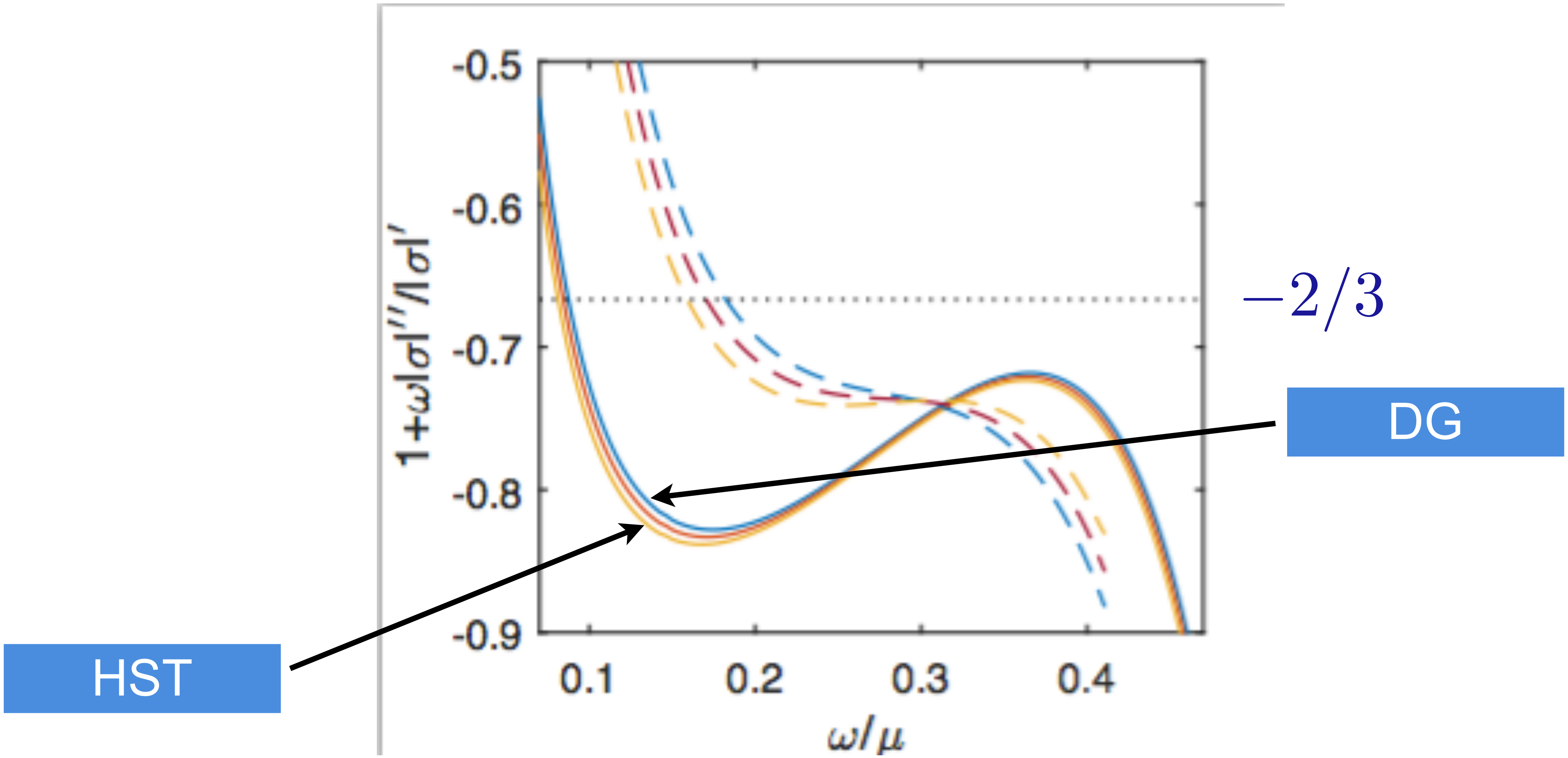
$$\rho = \lim_{z\to 0} \sqrt{-g} F^{tz}$$



is there a power law?

is there a power law?

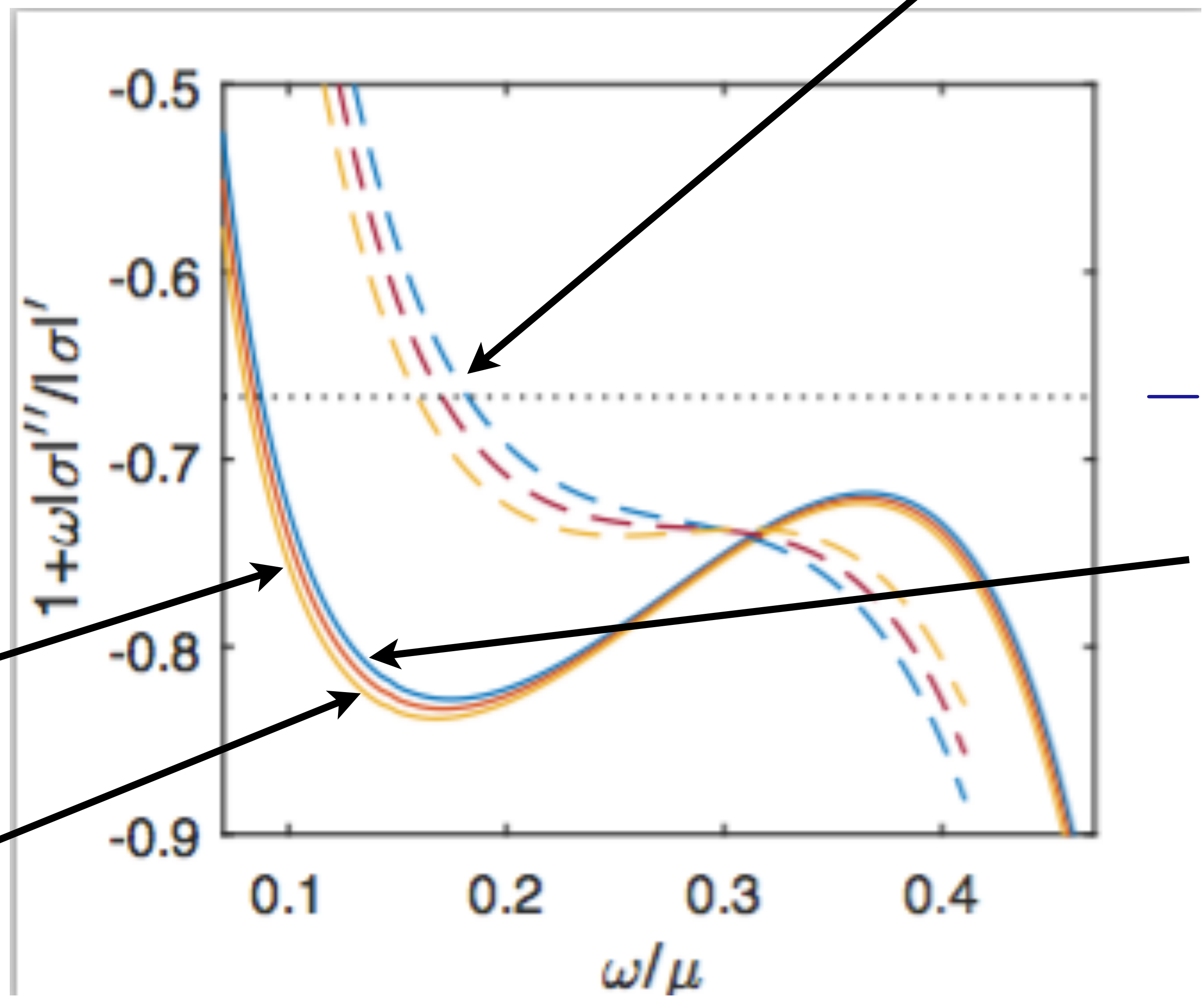
Results



is there a power law?

Results

k=2



$-2/3$

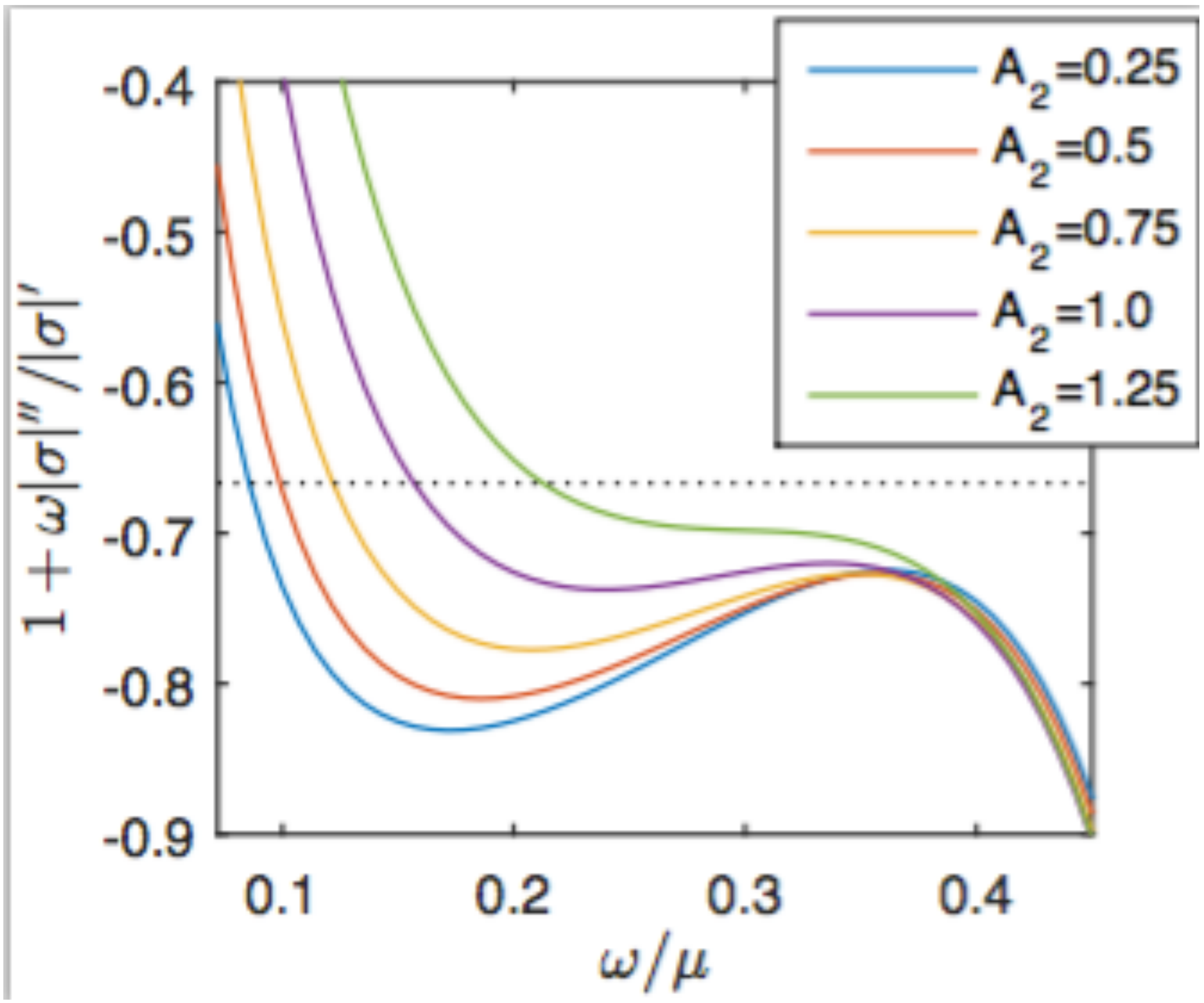
DG

k=1

HST

Results

$$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$$



similar results: Rangamani, Rozali, Smyth arxiv: 1505.05171

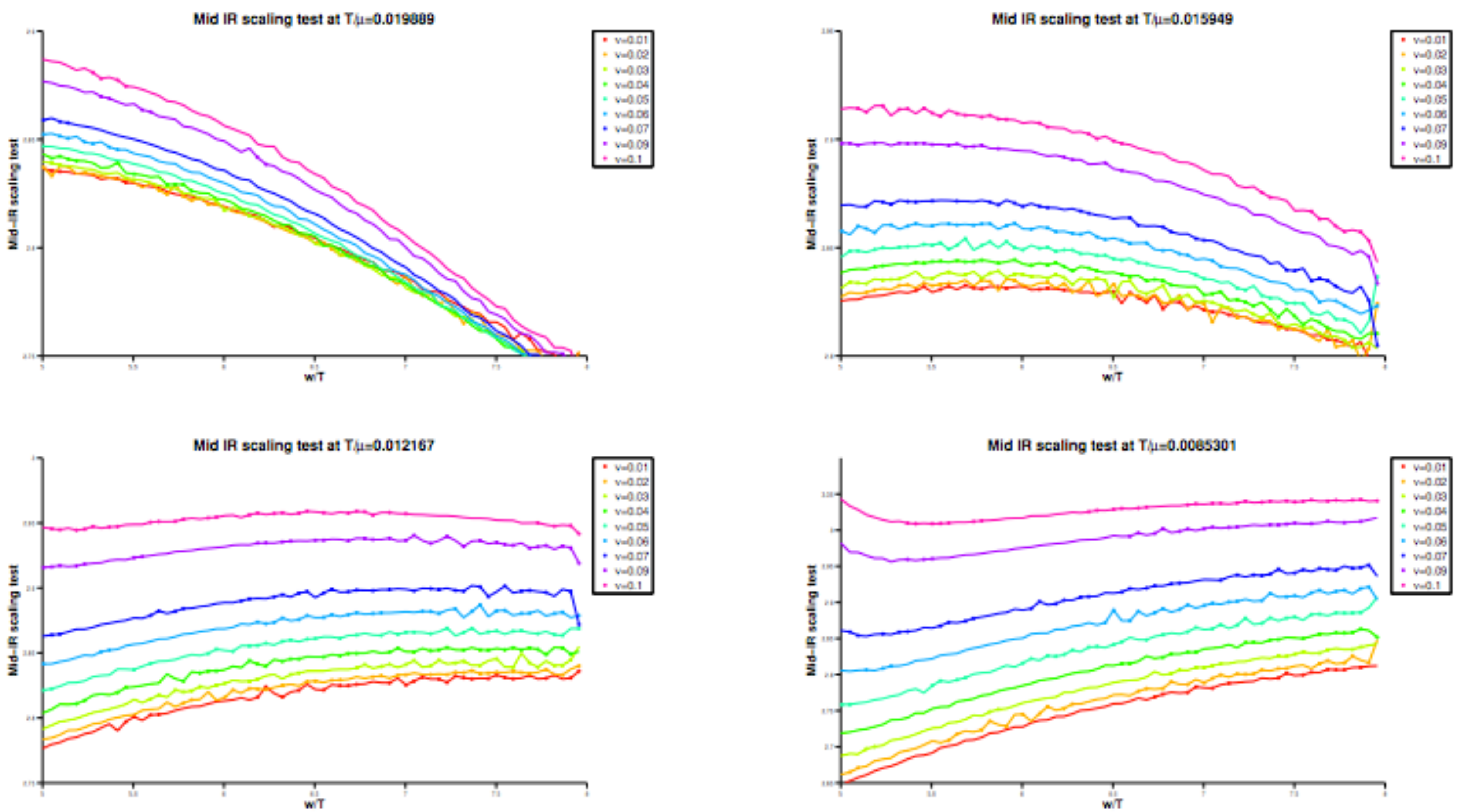
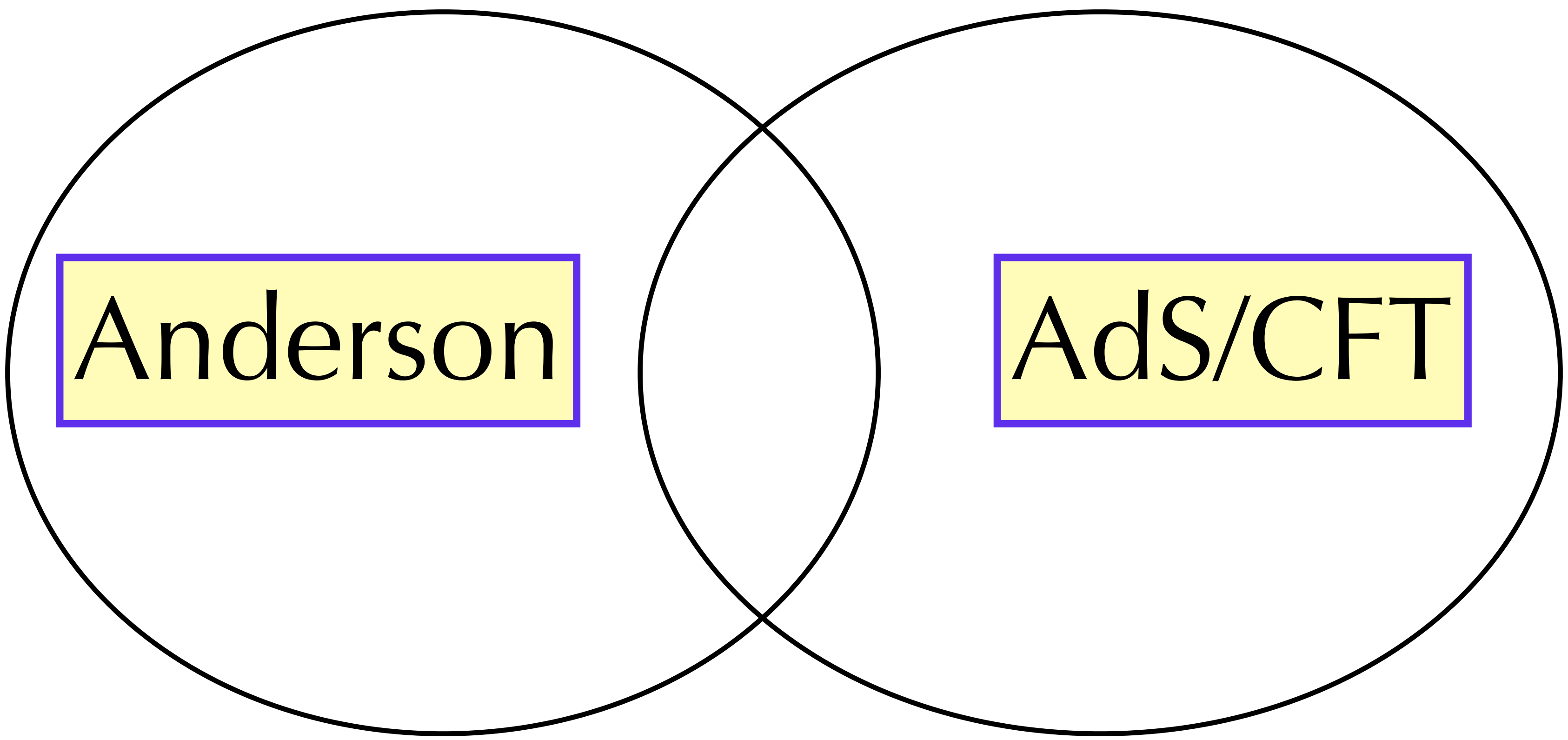
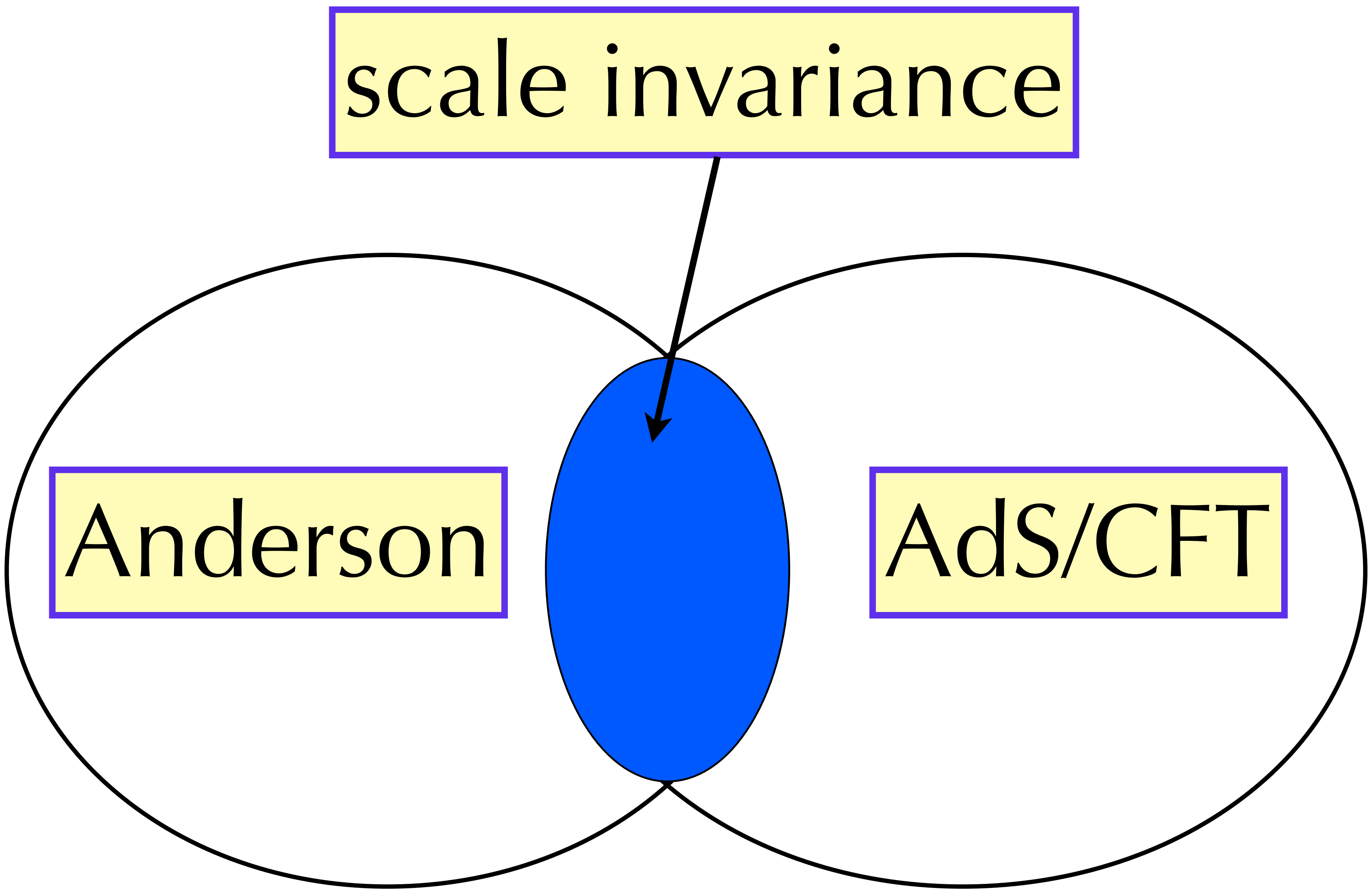


Fig. 9: The behaviour of the diagnostic function $F(\varpi)$ as we scan for mid-range scaling behaviour as a function of υ. At low temperatures, and for appropriately chosen values of υ, the existence of a scaling regime is possible. For

No


Anderson
AdS/CFT


scale invariance
Anderson
AdS/CFT

beyond single-parameter scaling

multiple scale sector

incoherent stuff (all energies)

beyond single-parameter scaling

multiple scale sector

unparticles (GFF)

no well-defined mass

incoherent stuff (all energies)

$$\mathcal{L} = \quad \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)$$

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)\rho(m^2)dm^2$$

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theory with all possible mass!

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)\rho(m^2)dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

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$$\mathcal{L} \to \Lambda^{4+d_\rho}\mathcal{L}$$

scale invariance is restored!!

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not particles

unparticles

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propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

propagator

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$$d_U - 2$$

$\boxed{\text{propagator}}$

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$$d_U - 2$$

$\boxed{\text{continuous mass}}$

$\phi(x, m^2)$

$\boxed{\text{flavors}}$

propagator

$$\left(\int_0^\infty dm^2 m^{2\gamma} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2|\gamma|}$$

$$d_U - 2$$

continuous mass

$\phi(x, m^2)$ $\Longleftrightarrow$ $e^2(m)$

Karch, 2015

flavors

multi-bands

take experiments seriously

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

take experiments seriously

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continuous mass

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

continuous mass

$$\sigma(\omega) = \int_0^M \frac{\rho(m) e^2(m) \tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

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$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega\tau_0 \frac{m^c}{M^c}}$$

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perform integral

$$\frac{a + 2b - 1}{c} = -\frac{1}{3}$$

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$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega\tau_0} dx \; \frac{x^{-\frac{1}{3}}}{1-ix}$$

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$$\Big\downarrow \; \omega\tau_0 \to \infty$$

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$$\Bigg\downarrow \ \omega\tau_0 \to \infty$$

$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega\tau_0} dx \; \frac{x^{-\frac{1}{3}}}{1-ix}$$

$$\Big\downarrow \; \omega\tau_0 \to \infty$$

$$\boxed{\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}}$$

$$\boxed{\begin{gathered}\tan\sigma = \sqrt{3} \\ 60^\circ\end{gathered}}$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

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$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

hyperscaling violation

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

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anomalous dimension

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\rho(m) = \rho_0 \frac{m^{a+1}}{M^a}$$

hyperscaling violation

$$e(m) = e_0 \frac{m^b}{M^b}$$

anomalous dimension

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

momentum loss

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

hyperscaling violation

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

anomalous dimension

$$e(m) = e_0 \frac{m^{b}}{M^b}$$

momentum loss

$$\tau(m) = \tau_0 \frac{m^{c}}{M^c}$$

$$c = 1$$

$$a + 2b = 2/3$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

hyperscaling violation

$$e(m) = e_0 \frac{m^b}{M^b}$$

anomalous dimension

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

momentum loss

$$c = 1$$

$$a + 2b = 2/3$$

$$\Longrightarrow$$

$$b = 0$$

$$a = 2/3$$

No

No

but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

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Hartnoll/Karch

$$\Phi = bz = -2/3$$

No

but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

Hartnoll/Karch

$$\Phi = bz = -2/3$$

$$\rho \propto T^{-2\Phi/z}$$

gauge invariance

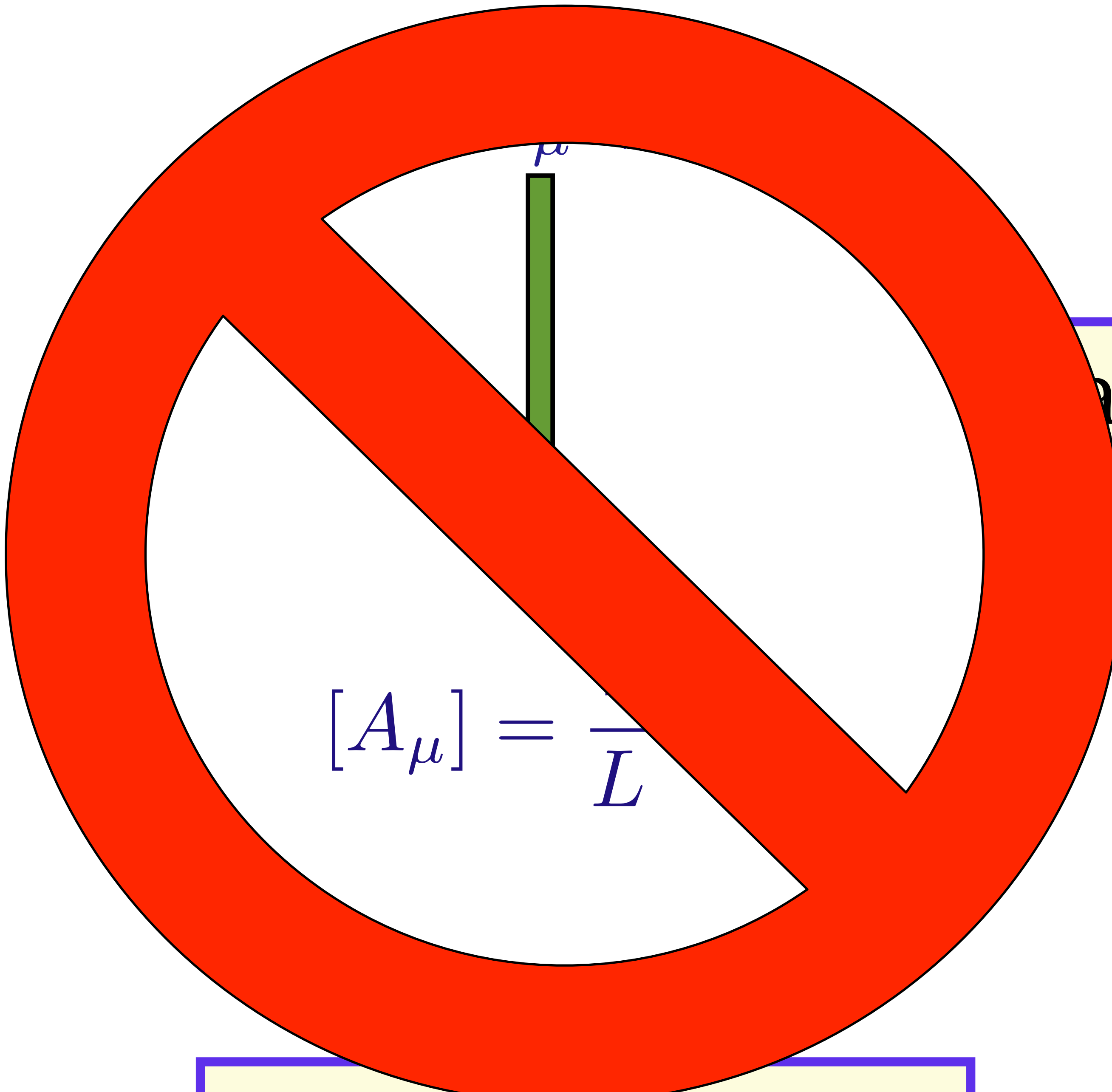
$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu \mathcal{G}$$

has no units

$$[A_\mu] = \frac{1}{L} = 1$$

if $[A_\mu] \neq 1$

gauge invariance



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if $[A_\mu] \neq 1$

How can $\mathcal{A}_\mu$ have an anomalous dimension?

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$$\vec{\nabla}^\alpha \times \vec{A} = \vec{B}$$

no Stokes' theorem

$$\oint \vec{A} \cdot d\ell \neq \int_S B \cdot d\vec{S}$$

Aharonov-Bohm Effect must change

flux through ring

$$\Delta\Phi = \frac{e}{\hbar}\oint \vec{A}\cdot d\ell$$

$$\Delta\Phi = \frac{eB\pi r^2}{\hbar}$$

Stokes' theorem

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$$\Delta \ldots \frac{e}{\hbar} \oint \vec{A} \cdot d\ell$$

$$\Delta\Phi = - \ldots$$

Stokes' theorem

$$\oint \vec{A} \ldots \int_S \ldots d\vec{S}$$

$\boxed{\text{physical gauge connection}}$

$$a_i \equiv [\partial_i, I_i^\alpha A_i] = \partial_i I_i^\alpha A_i$$

$\boxed{\text{compute AB phase}}$

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$$-\frac{\hbar^2}{2m}(\partial_i - i\frac{e}{\hbar}a_i)^2\psi = i\hbar\partial_t\psi.$$

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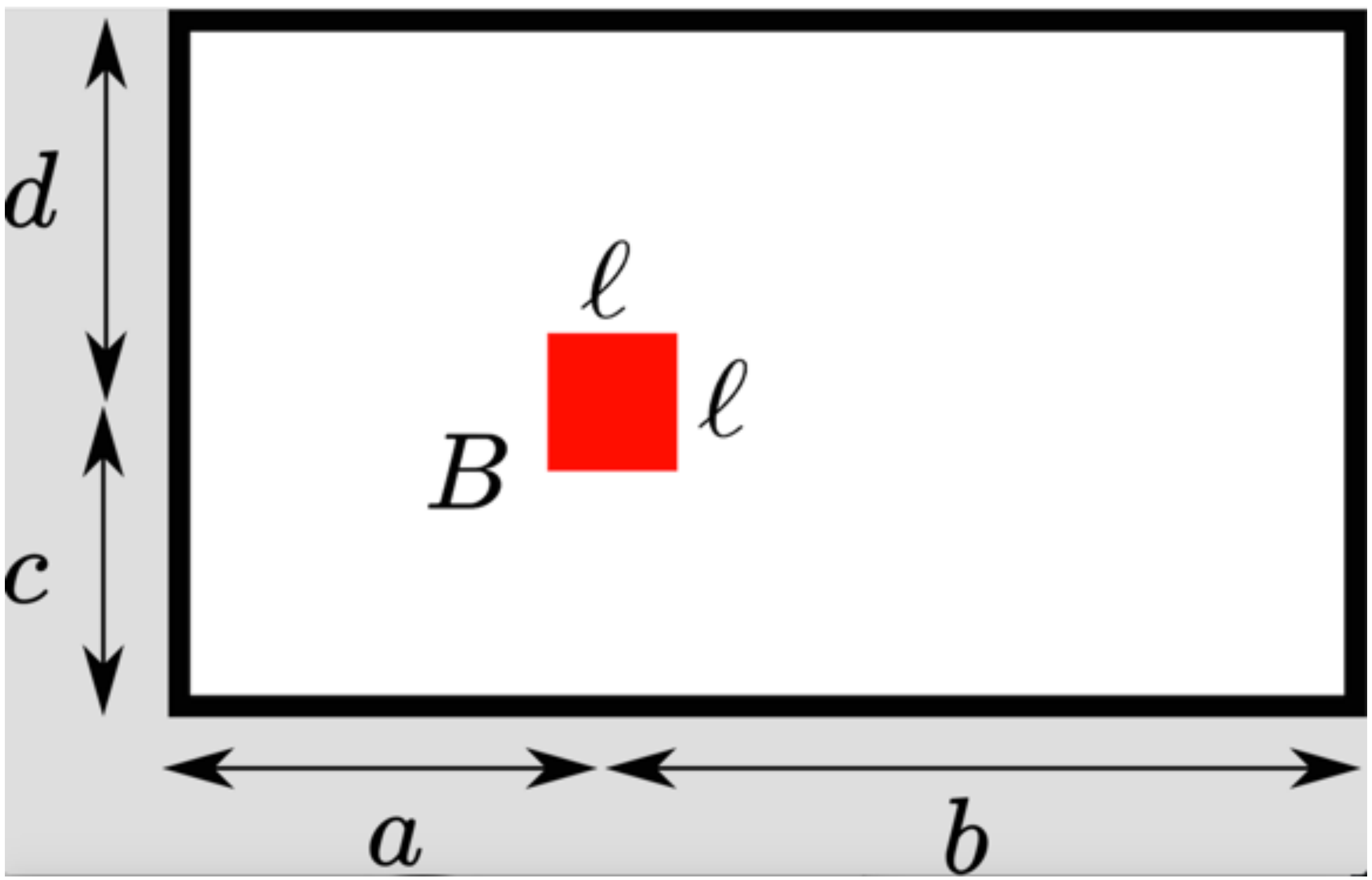

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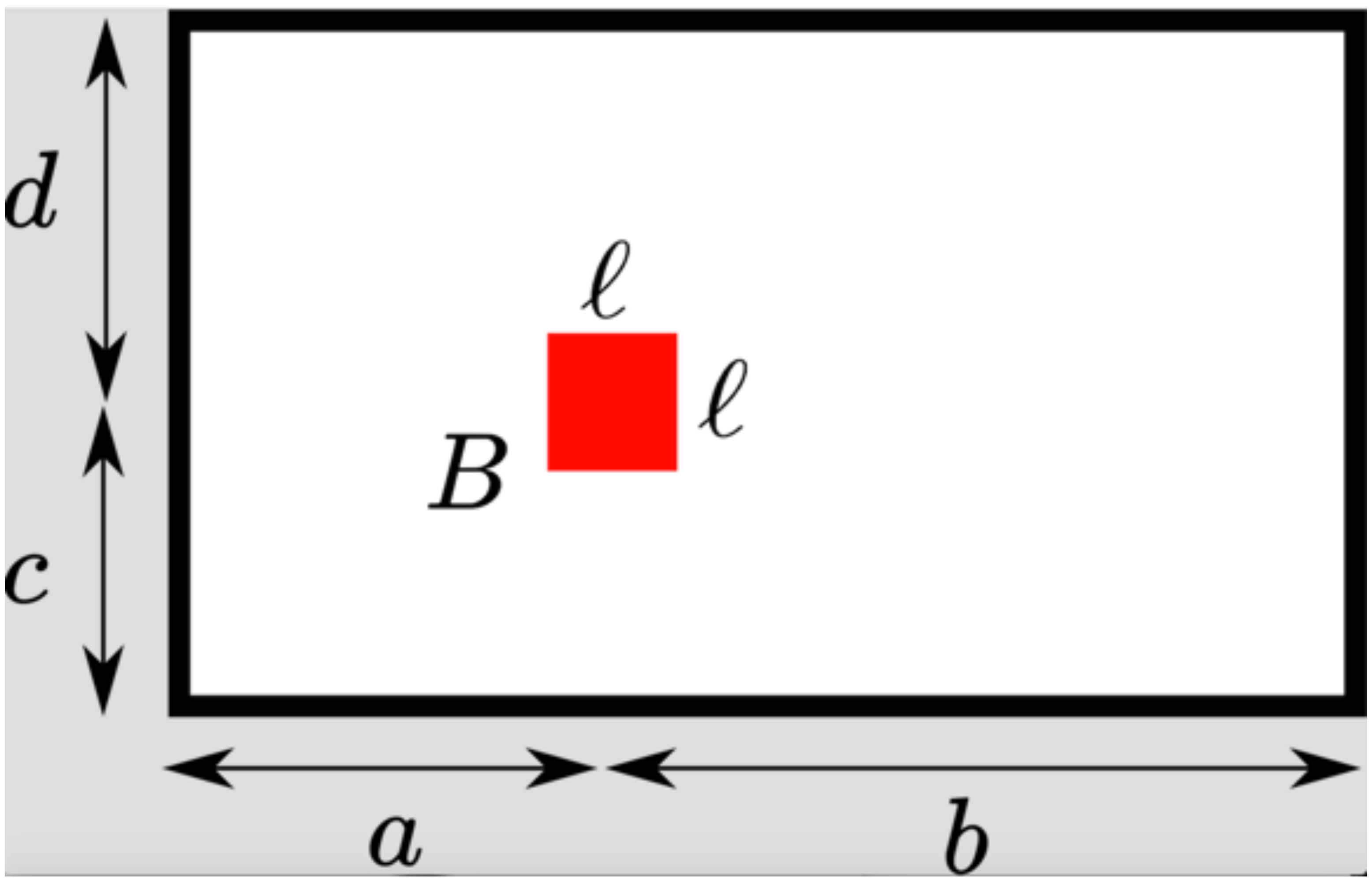

use fractional calculus

$$\Delta\phi_{\mathrm{R}} = \frac{eB\ell^2}{\hbar}\left(\frac{b^{\alpha-1}d^{\alpha-1}}{\Gamma^2(\alpha)}\right) c \gg l, d \gg l$$

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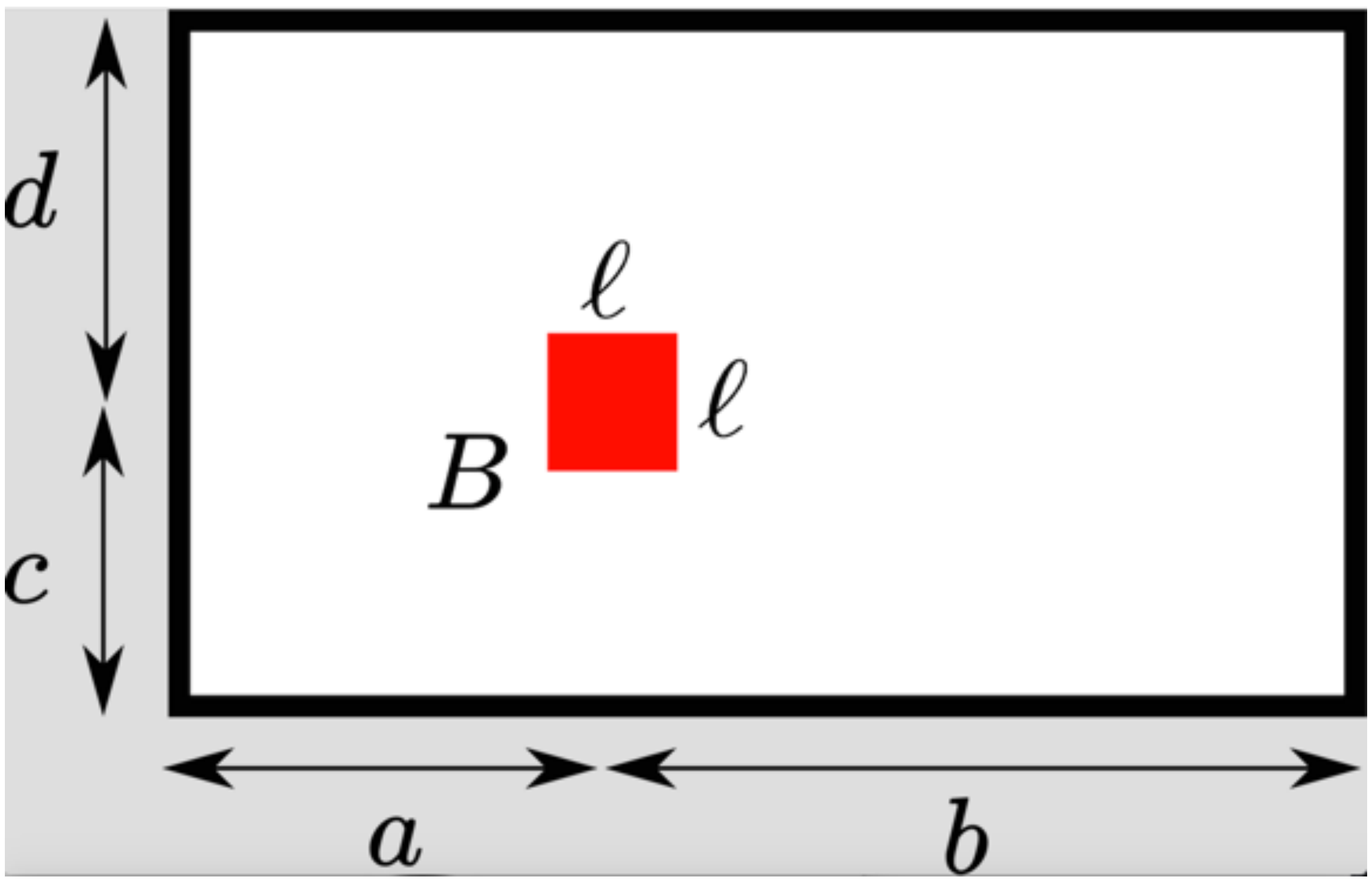

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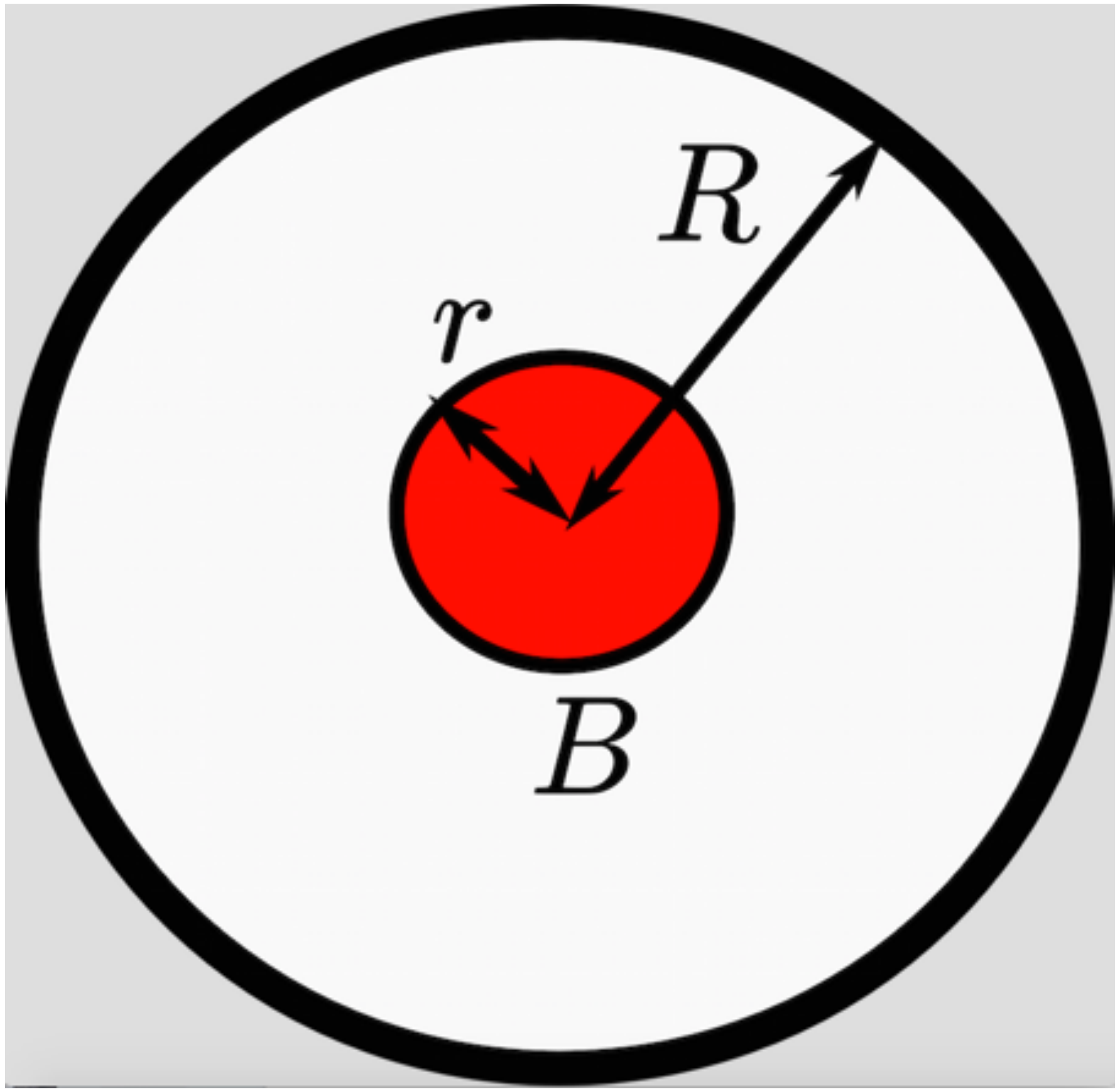
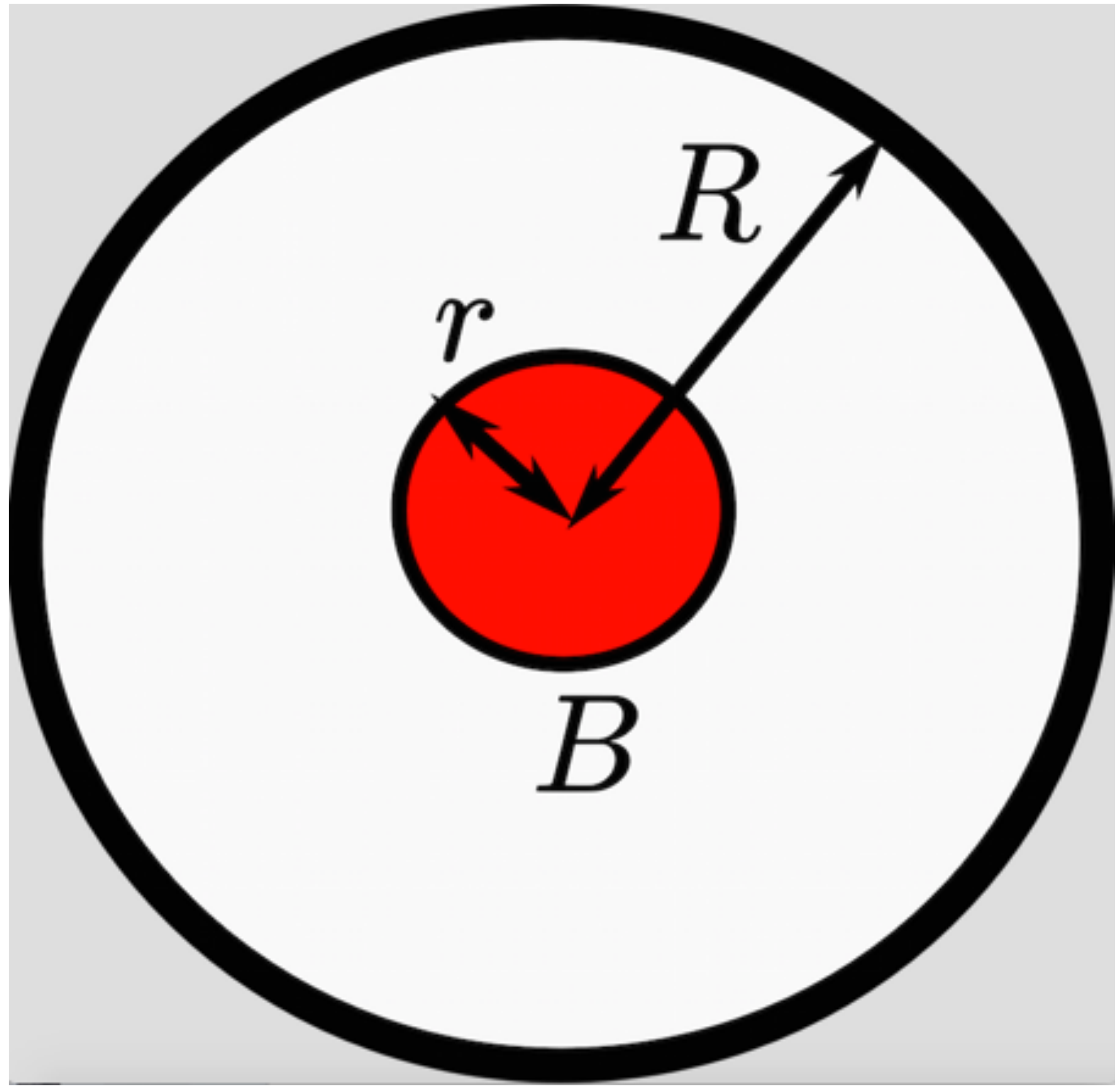


$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha, 2-\alpha; 2; \frac{r^2}{R^2})\right)$$



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$\boxed{\text{is the correction large?}}$

$\Downarrow$ $\alpha = 1 + 2/3 = 5/3$

is the correction large?

$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar}\boxed{L^{-5/3}/(0.43)^2}$$

yes!

what is the mechanism for the anomalous dimension?

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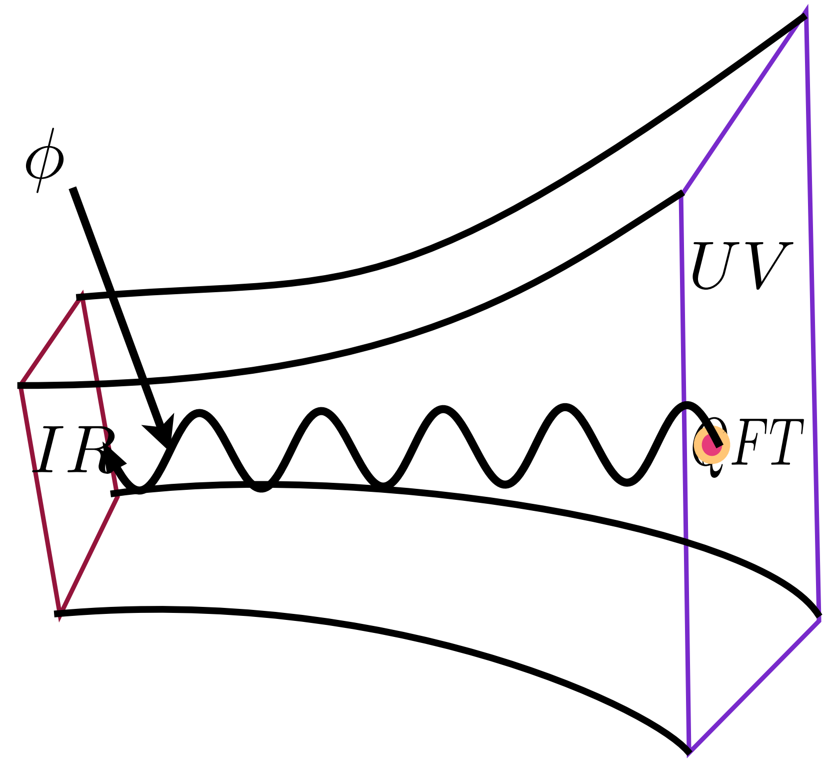
is the non-locality real?

bulk-boundary operator correspondence

operators
$\mathcal{O}$

$(\partial_\mu \phi)^2 + m^2 \phi^2$



$e^{\int d^d x \phi_{\mathcal{O}} \mathcal{O}}$

bulk-boundary operator correspondence

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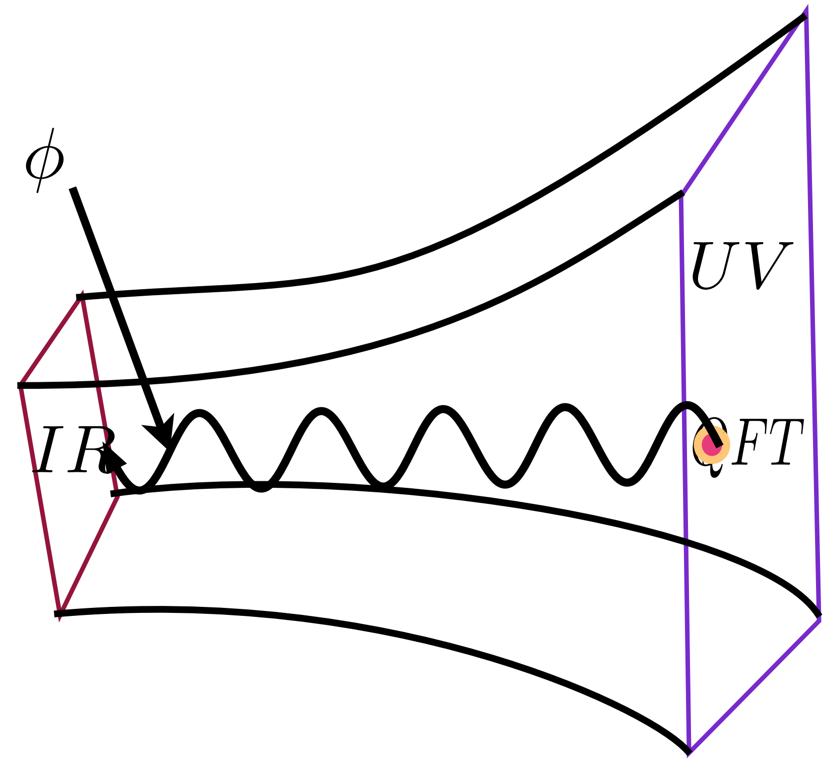

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Local bulk operators in AdS/CFT:
a boundary view of horizons and locality

Alex Hamilton[1], Daniel Kabat[1], Gilad Lifschytz[2] and David A. Lowe[3]

$$\phi_0(x) \leftrightarrow \mathcal{O}(x) \; .$$

This implies a correspondence between local fields in the bulk and *non-local* operators in the CFT.

$$\phi(z,x) \leftrightarrow \int dx' \, K(x'|z,x)\mathcal{O}(x') \; .$$

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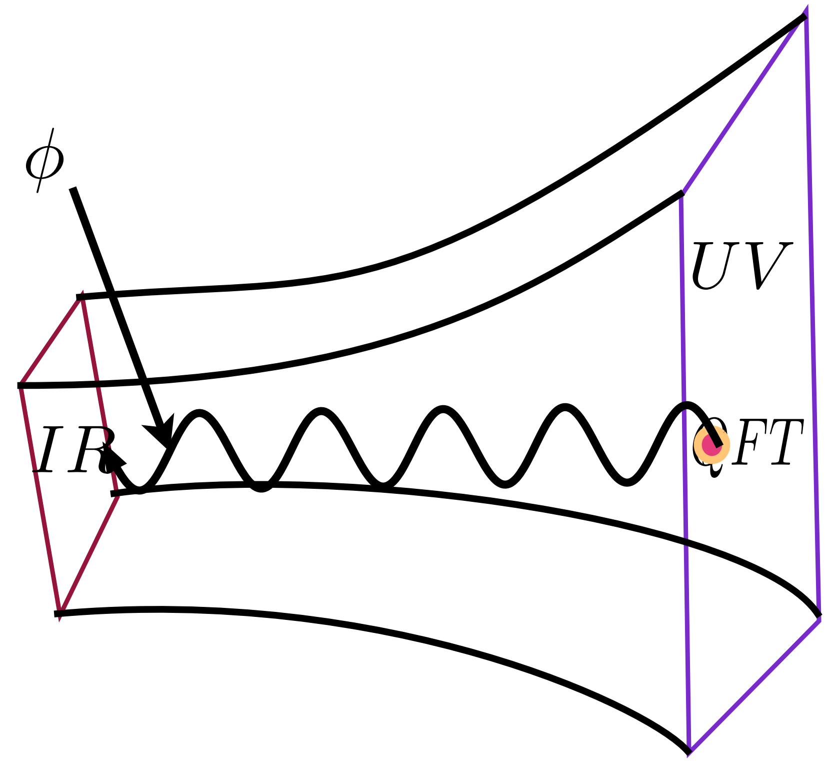

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smearing function

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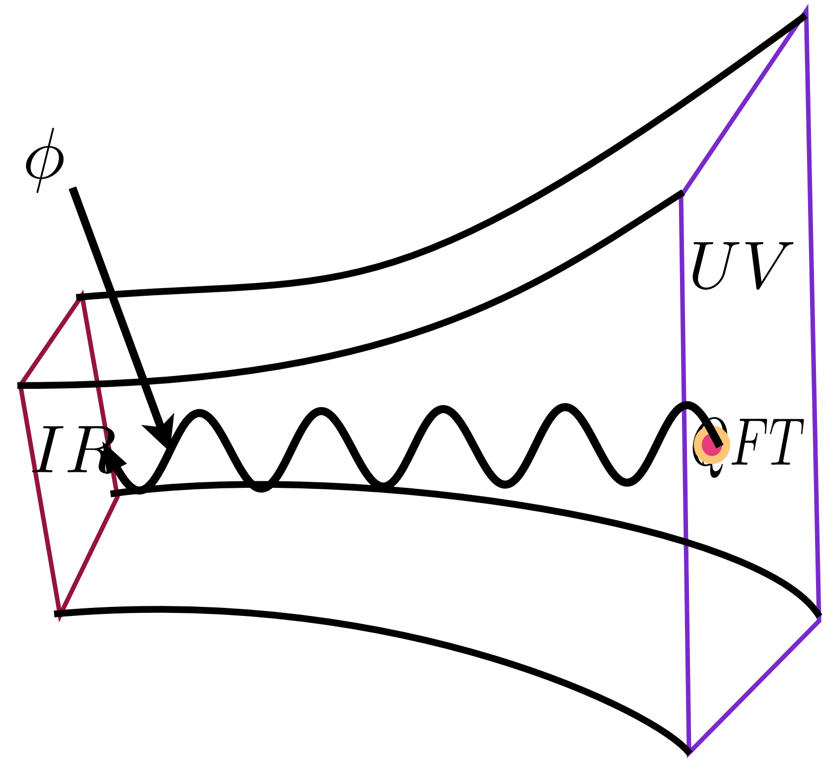
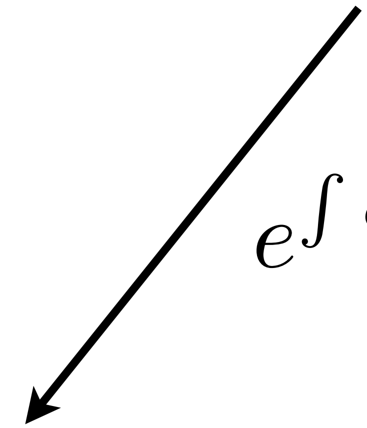


operators $\mathcal{O}$

$e^{\int d^d x \phi_{\mathcal{O}} \mathcal{O}}$



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see also K. Rehren (2000)

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use Caffarelli/
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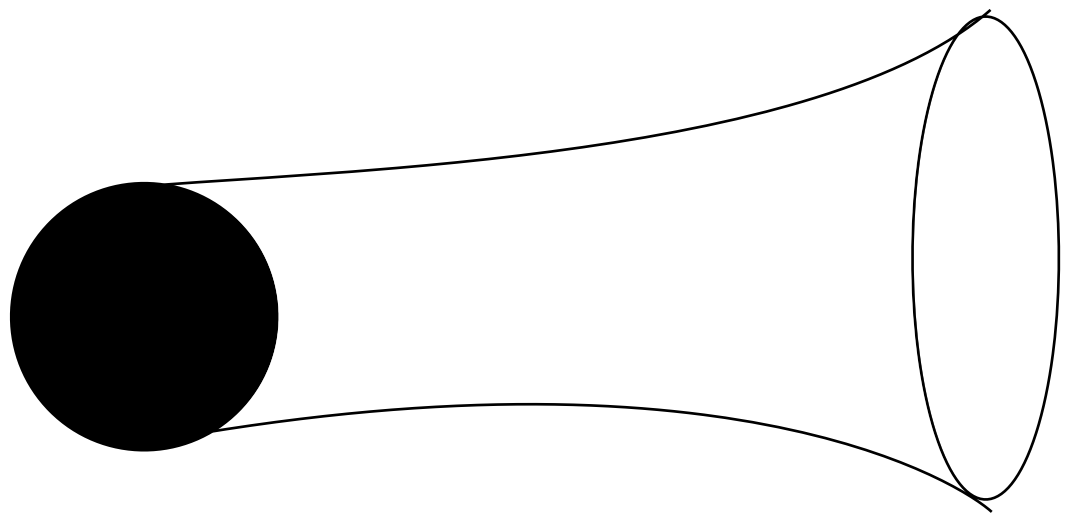
$$\gamma = \frac{1-a}{2}$$

$$\mathcal{O} = (-\Delta)^\gamma \phi_0$$

$$\gamma = \sqrt{4m^2+d^2}/2$$

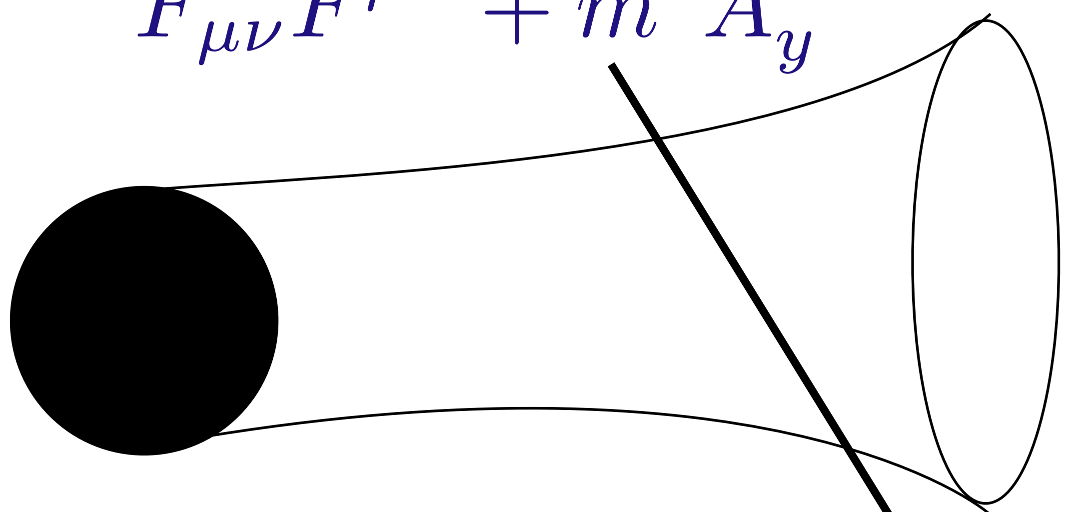
the $\mathcal{O}$ for massive scalar field

mechanism: fractional gauge fields



mechanism: fractional gauge fields

$$F_{\mu\nu}F^{\mu\nu} + m^2 A_y^2$$



dynamical `Higgs' mode

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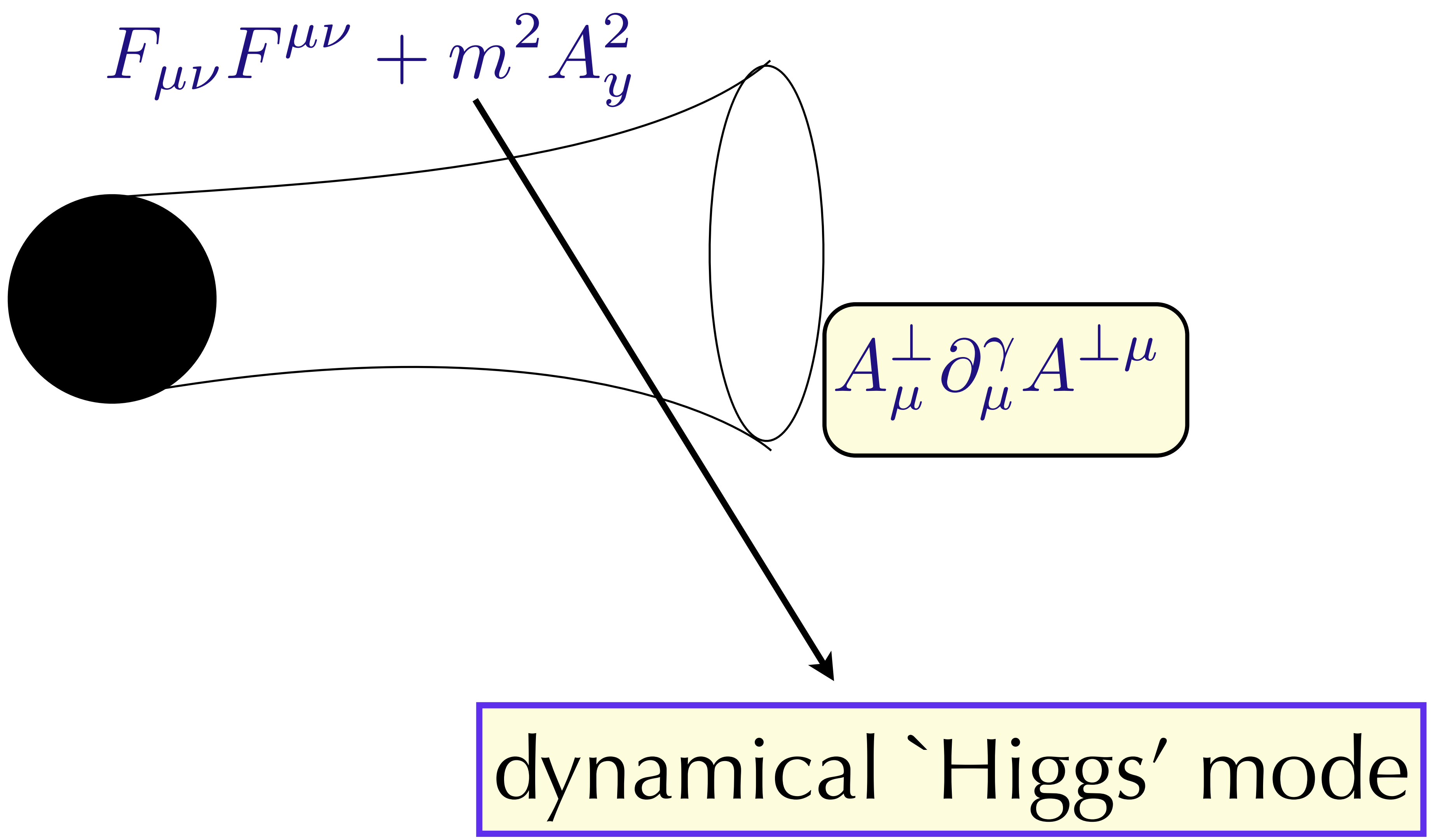

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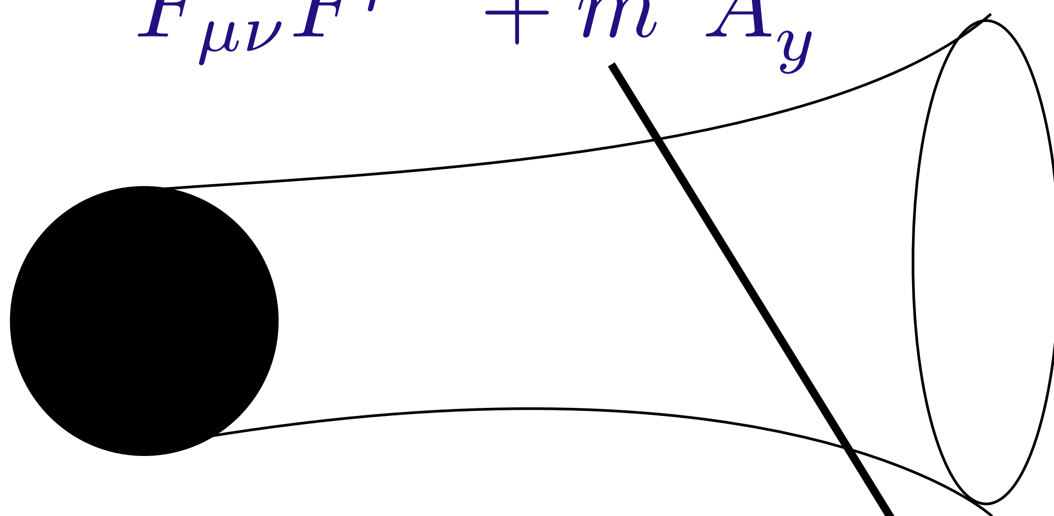


$$A_\mu^\perp \partial_\mu^\gamma A^{\perp\mu}$$

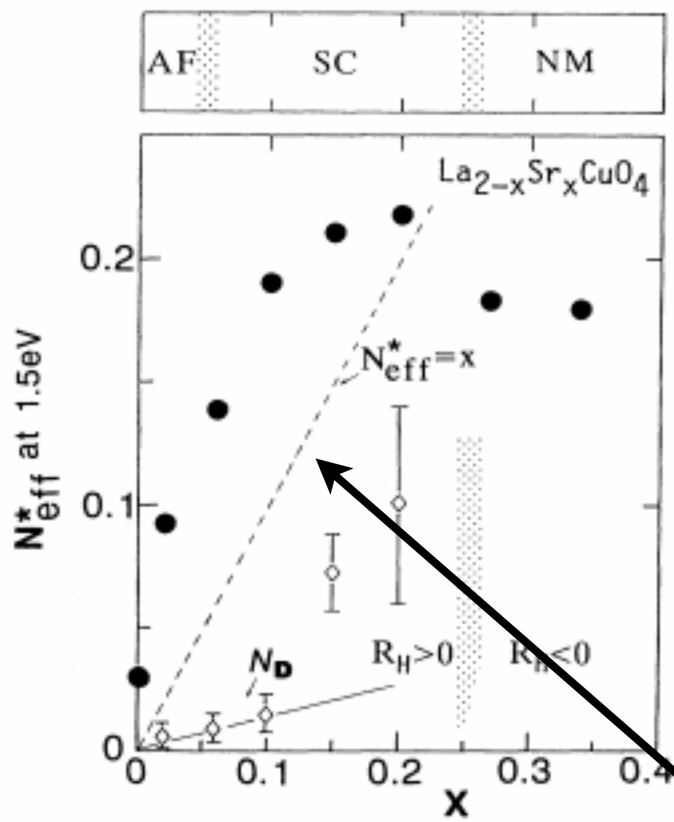
dynamical `Higgs' mode

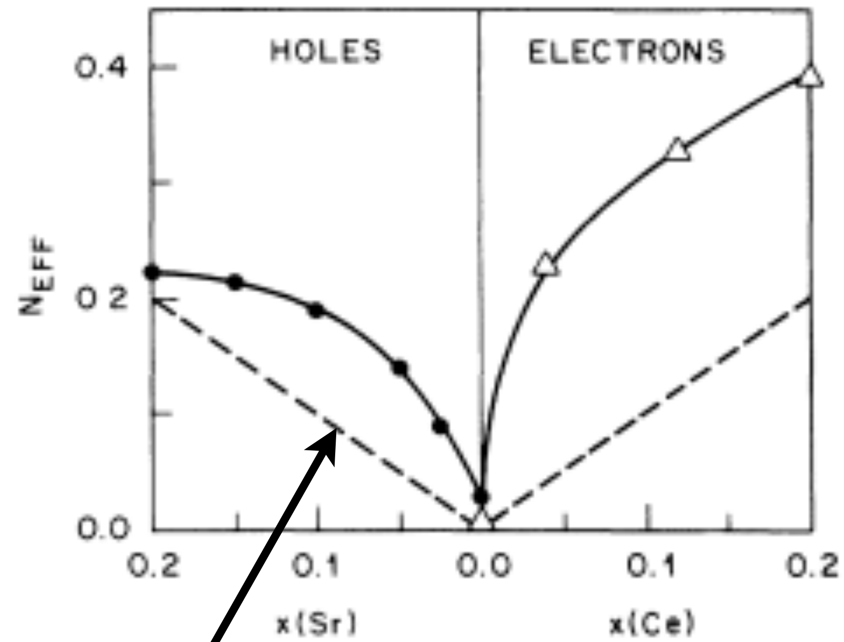
new length scale

$$N_{\rm eff}(\Omega) = \frac{2mV_{\rm cell}}{\pi e^2}\int_0^{\Omega} \sigma(\omega)d\omega$$

Uchida, et al.

Cooper, et al.





x

low-energy model for $N_{\rm eff} > x$??

what if?

$$\text{K.E.} \propto (\partial_\mu^2)^\alpha$$

f-sum rule

$$\frac{W(n,T)}{\pi c e^2} = A n^{1+\frac{2(\alpha-1)}{d}} + \cdots$$

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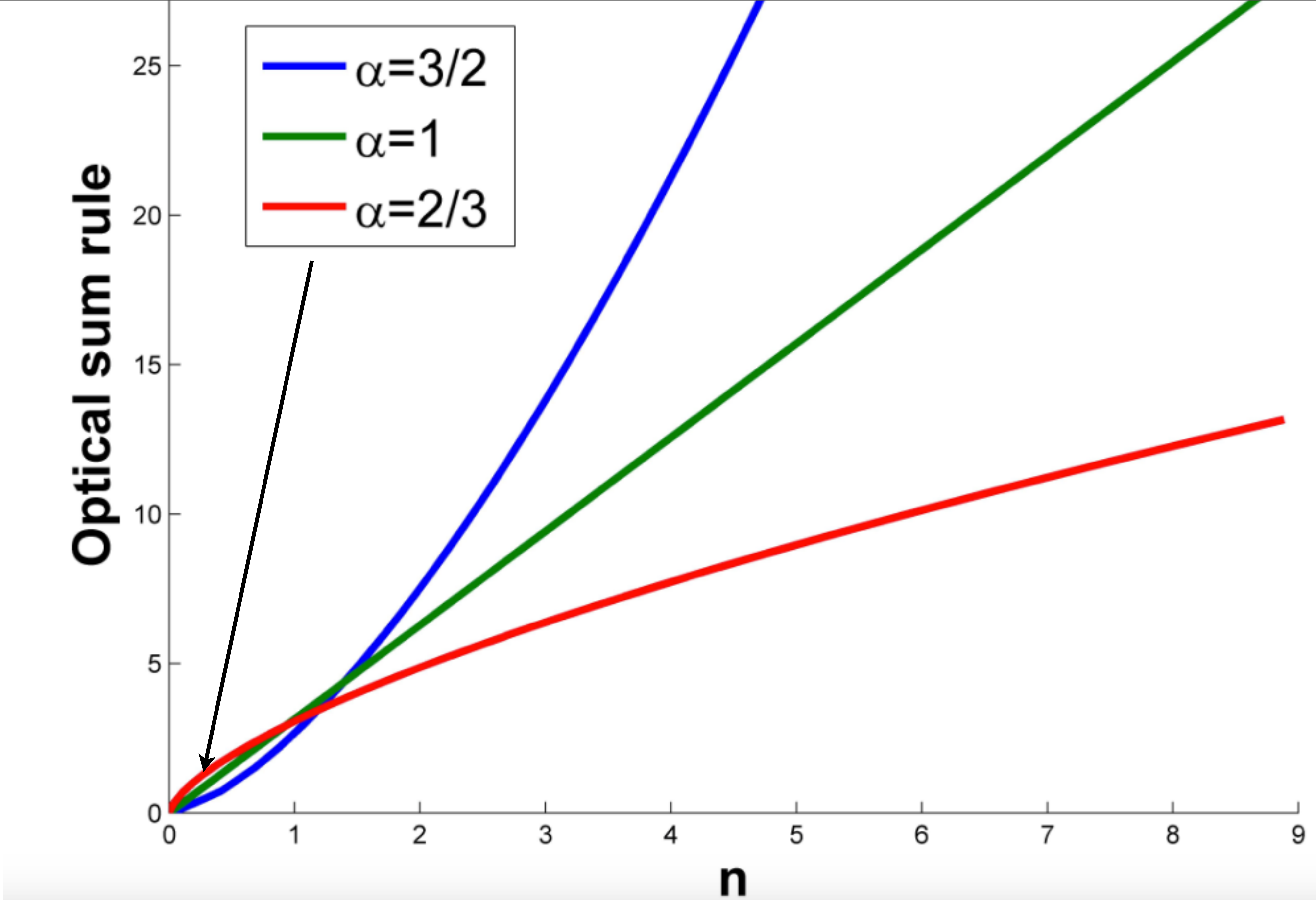
W>n if $\alpha < 1$



α=3/2
α=1
α=2/3
Optical sum rule
n
0
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9

combine AC
+DC transport

fixes all exponents
a,b,c

$$[J] = d_U$$

combine AC
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probe with fractional
Aharonov-Bohm effect

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boundary
non-local action

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