Entanglement Propagation

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Quantum Matter, Spacetime and Information
YITP Kyoto, June 2016

Based on:

M.R., Mukund Rangamani, Alexandre Vincart-Emard (arXiv:1512.03478, JHEP) + work in progress together with Anson Wong.

Outline:

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    - Hydrodynamical Evolution
3. Entanglement Propagation
    - Information Lightcone
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Introduction and Motivation

- Non-equilibrium many-body systems are interesting, gauge-gravity duality is one way to make general statements, in non-integrable systems.
- Entanglement entropy is an interesting quantity, quantifying the distribution of information in a quantum state.
- In holography, EE is easy to calculate using the RT, or more generally the HRT prescription.

  It is interesting to see the behaviour of the entanglement in non-equilibrium situation, where it goes one step further beyond local probes of the system. For example it is interesting to probe thermalization using the EE.
- Directly related (holographically and otherwise) to other measures of "scrambling" of quantum information.

Introduction and Motivation

- Much attention was given to entanglement growth following a global quench, resulting in a beautiful picture of "entanglement tsunami", quantified in terms of tsunami velocity v_T which expresses the rate of growth of EE following the quench.
- We are interested in entanglement *spread*, i.e the spatial propagation of EE following a localized excitation of the system.
- This is a spatially resolved version of the same experiment: start the system in an atypical, *locally excited* state, see how the entanglement on different scales returns to the equilibrium value.
- Some previous attacks on this problem involved taking certain limits (e.g. shock waves), or working in 1+1 dimensions. We will use an exact numerical solution of the bulk equations to access the exact results for a generic point in parameter space.

Quenching

We study a 2+1 dimensional strongly coupled CFT, dual to 3+1 dimensional asymptotically AdS4 spacetime. Starting with a thermal states of temperature T, dual to a bulk black brane, we source the metric by turning on a pulse of scalar field, dual to a relevant operator in the boundary CFT.

We choose the source function to be $\phi_0(t,x) = f(x)g(t)$ with

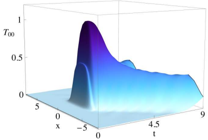
$$f(x) = \frac{\alpha}{2}\left[\tanh\left(\frac{x+\sigma}{4s}\right) - \tanh\left(\frac{x-\sigma}{4s}\right)\right], \qquad g(t) = \operatorname{sech}^2\left(\frac{t - t_q\Delta}{t_q}\right).$$

With it, we can ramp up the scalar field to reach its maximum value α at time $t = t_q\Delta$ before it drops and vanishes again. The parameters $\{s, t_q, \Delta\}$ are chosen to facilitate the numerics, whereas σ determines the spatial width of the perturbation and α is the amplitude.

Hydrodynamical Evolution

Soon after the injection of the localized pulse of energy the evolution of the energy-momentum tensor is described by hydrodynamics, because of fast (local) thermalization typical of holographic theories.

Since our perturbation excites the sound mode of the system, we have the initial energy-momentum perturbation dispersing at the speed of sound. Below we see a typical profile of the energy density as function of (x,t)



Curiously, the initial perturbation splits to two localized perturbations after some time; those follow the expected hydrodynamic evolution.

Evolution of the EE

In the situation described above, we look at the EE on different scales. We choose to look at boundary strips

$$\mathcal{A} = \{(x,y)\,|\,x \in (-L,L),\;\; y \in \mathbb{R}\}\,, \qquad \partial\mathcal{A} = \{(x,y)\,|\,x = \pm L,\;\; y \in \mathbb{R}\}$$

We track the entanglement (quantified as the difference from the entanglement in the unquenched thermal state), for strips centred around the location of the quench. This result in the function $\Delta S_A(L,t)$, depending on the size of the entangling surface and time.

We have 3 dimensionful parameters or equivalently 2 dimensionless ones.

- Temperature, or equivalently energy density.
- Quench amplitude α.
- Quench width σ.

The temperature T determine the range of sizes L which we can probe using our methods. Since we do not access the region behind the horizon, it follows that $L \gtrsim T$. This is sufficient to find some universal results.

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Information Lightcone

Our main result: For (nearly) all choices of parameters we find linear (ballistic) growth of $\Delta S_A(L,t)$. We quantify that by looking at

$$L_{peak}(t) = ArgMax_L \, \Delta S_A(L,t)$$

Note that unlike various limits taken previously, the maximum itself $Max_L \Delta S_A(L,t)$ is not constant.

Note that for $L > \sigma$ there is some time delay for which $\Delta S_A(L,t)$ exactly vanishes, by bulk causality. By linear dispersion we mean then that after that time t_0:

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Information Velocity

The information lightcone is analogous to the Lieb-Robinson velocity in spin systems. Like the LR velocity we find that v_E is state-dependent, i.e. depends on parameters of the quench.

Note that v_E is a priori different from other entanglement speeds (e.g. the tsunami velocity of Liu and Suh which has to do with entanglement growth rate).

We find lower and upper bounds for the information velocity, and some universal results in certain limits.

Information Velocity

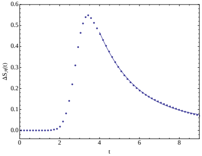
There is an interesting interplay between the size of the entangling surface L and the width of the pulse σ:

- When $L < \sigma$ the quench looks like a global one. When we further take sufficiently high T, we find that $v_E = 1$, regardless of the amplitude of quench (including values well within the non-linear regime). The speed of light is a bound for all cases, and the bound is saturated in this regime.
- When we increase L past σ, we enter a different linear regime with a different velocity. The change in slope when $L \sim \sigma$ is abrupt.
- When the quench is in the linear response regime, and for surfaces such that $L > \sigma$, we find that our information velocity is numerically very close to the tsunami velocity. However, increasing the amplitude of the quench increases the information velocity as well. The tsunami velocity is a lower bound for the parameter range we probe.

Entanglement Decay

Since we inject finite energy in infinite volume, our final state is the same as the initial state, and the EE returns to its original value.

We can study that decay process as function of time, a typical example is shown: $\Delta S_A(L,t)$ for fixed L (and quench parameters) and as function of time.



We see an initial time delay, rapid rise to the peak and decay. The best fit for the numerical results is exponential decay, with parameters depending on quench details.

The only other calculation we are aware of is in 1+1 dimensions, where the return to equilibrium is much slower. The difference is reminiscent of fast scrambling and thermalization which is typical of holographic theories.

Future Directions

In order of ambition:

- It would be nice to have a qualitative model reproducing some of the numerical results, making connection to other non-integrable models.
- Extension to other theories and states , e.g. massive theories and finite density black holes, especially in the extremal limit (in progress).
- Quench and annealing past critical points. In particular mutual information should be more sensitive to phase structure.
- Relation to other manifestations of information and scrambling: tsunami velocity, butterfly velocity and shock waves, Lieb-Robinson velocity.
- More generally: why does entanglement transport so simply. How is that related to microscopic chaos, and to other forms of transport?