Matrix product approximations to multipoint functions in two-dimensional conformal field theory

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Goal

- Understand the entanglement structure of quantum field theories using tensor network methods
  - Tensor networks model the entanglement properties of many body systems and are successfully applied in condensed matter physics
  - What about quantum field theories?
- States of the quantum field theory and tensor network states live in different Hilbert spaces: how to measure closeness?

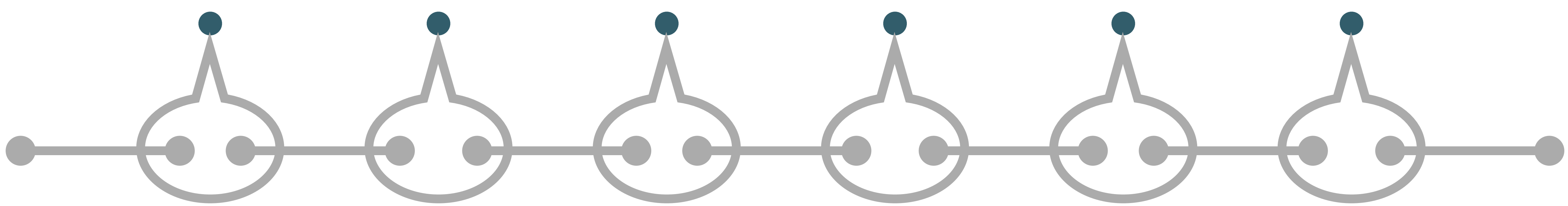
How to approximate a quantum field theory?

- Focus on physical quantities: correlation functions
- If tensor networks can approximately reproduce correlation functions of quantum field theories, then we can use them to understand the entanglement structure of quantum field theories.
- This talk: Conformal Field Theories and Matrix product states

Recap: Matrix product states

- Tensor network states for spin chains:

maximally entangled pairs of (bond) dimension D are placed between the physical particles (physical dimension d) and are contracted by a DxDxd dimensional tensor



- Correlation functions can be computed efficiently (in D) and reduce to the computation of a sequence of completely positive maps on matrices of dimension D

Main result

Correlation functions of Conformal Field Theories can be arbitrarily well approximated by correlations functions of Matrix Product states.

Scaling of Parameters:
number of fields n, UV cutoff d (measured in terms of energy), approximation error ε, C constant depending on CFT (not necessarily central charge)

scaling of bond dimension	fixed n, UV cutoff d	fixed ε
$\log(D)$	$\sim \log(1/\varepsilon)\, C\, n/d$	$\sim \sqrt{C\, n}$

Achievements & shortcomings

- We obtain a sequence of explicit Tensors describing Matrix product states, which better and better satisfy the conformal symmetry
- Proof is mathematically rigorous and constructive; holds for most unitary Conformal Field Theories
- However, the parameter scaling is worse than would could be expected from entropic arguments [Cardy&Calabrese, Holzhey et. al.,..]
- Uses the language of Vertex operator algebras: first introduced by Borcherds in his proof of the Moonshine conjecture

More Symmetries: Wess-Zumino-Witten models

- In addition to conformal symmetries, WZW models possess an additional local symmetry given by an affine Lie algebra based on a simple compact Lie group

- These additional symmetries carry over to the MPS Tensors; leads to a group invariant MPS

- Moreover, the interactions (fusion rules) are completely described already in the lowest level; the higher order Tensors are only needed to model the conformal and affine symmetries

Proof sketch: regularization

- identify states with Hilbert-Schmidt operators on the chiral theory; field operators become linear maps: need to approximate by finite-dimensional ones

- a finite UV cutoff regularises the unbounded field operators and turns them into bounded operators

$$\langle \phi_1(x_1) \longleftrightarrow \phi_2(x_2) \quad \dots\dots \quad \phi_n(x_n) \rangle$$

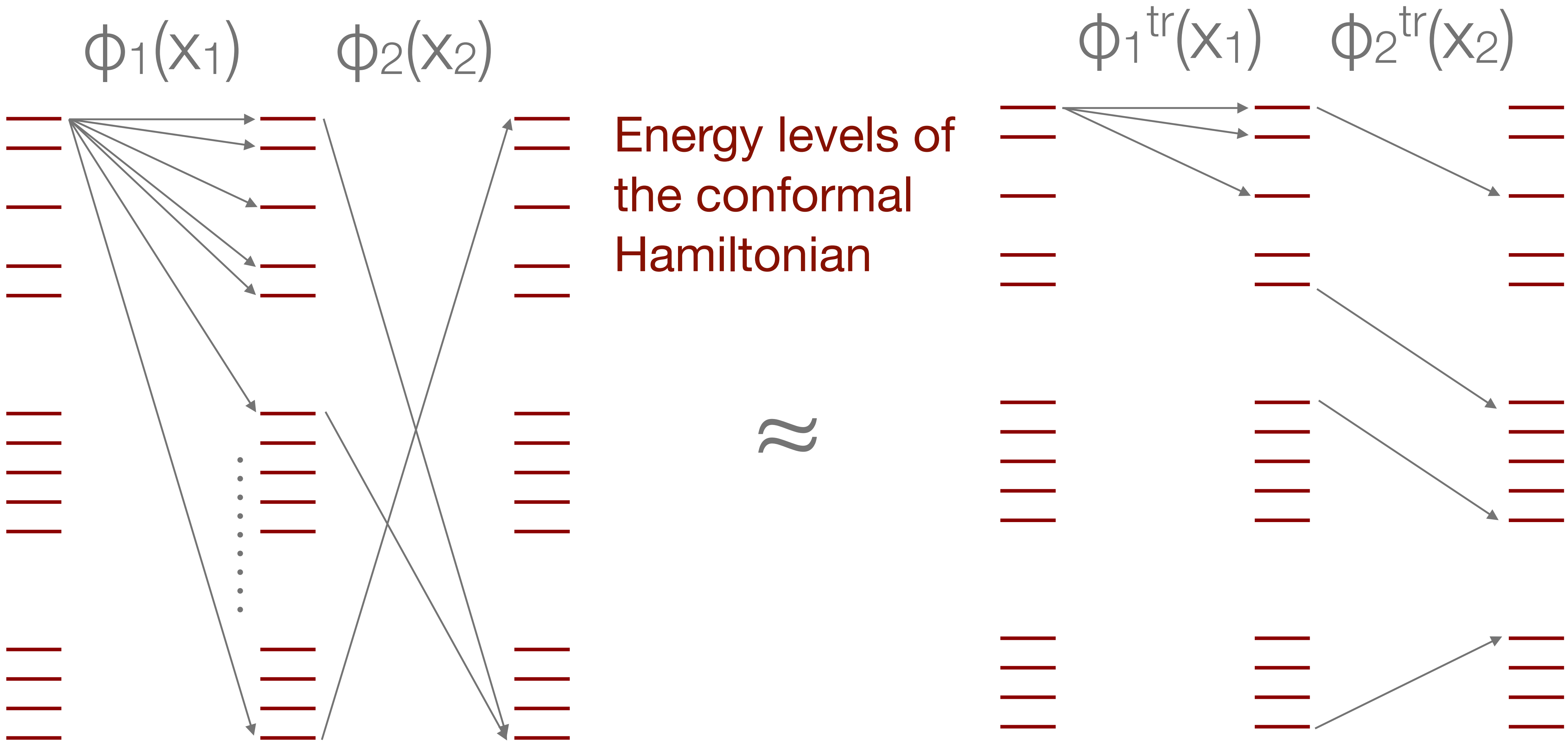
$$|x_1 - x_2| > d$$

- techniques: use results of Wassermann for WZW models (explicit bounds), and the existence of genus-1 correlation functions for general CFTs [Zhu, Huang]

Proof sketch: renormalization

Bounded field operator $\phi(x)$: can change the energy by an arbitrary amount

Precision Truncated bounded field operator $\phi^{tr}(x)$ can only change the energy by a fixed amount



Summary & Outlook

- Correlation functions of CFTs can be approximated by those from MPS
- Our Approximations are constructive, provide rigorous error bounds and respect additional symmetries (WZW)
- Can be analysed further to understand low energy states of CFTs in quantum information theoretic terms (connection to quantum error correction?)
- Generalisation to MERA (multiscale entanglement renormalization Ansatz) seems possible and may provide better parameter scaling