Many-body quantum physics through the lens of quantum entanglement

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Many-body quantum systems

$$H = \sum_{i=1}^{N>10^{23}} \left[\frac{\mathbf{p}_i^2}{2m} + v(\mathbf{r}_i) \right] + \sum_{i,j} V(|\mathbf{r}_i - \mathbf{r}_j|)$$

$$H = J \sum_{i,j}^{n.n.} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} \left(\gamma_\mu D^\mu - m \right) \psi - \frac{1}{2} \operatorname{tr} G_{\mu\nu} G^{\mu\nu}$$

$$\vdots$$

- May exhibit a rich varieties of phenomena: spontaneous symmetry breaking, topological phases of matter, non-equilibrium physics, eigenstate thermalization, quantum information scrambling, etc.
- Known to be very hard: $\dim \mathcal{H} \sim e^N$
- May exhibit (ground) states with non-trivial quantum entanglement

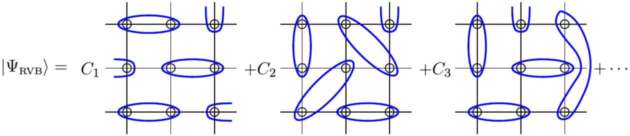
Quantum entanglement

- Product states v.s. entangled states:

$$|\Psi\rangle = |\uparrow\rangle|\uparrow\rangle \quad \text{vs.} \quad |\Psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

- Conventional v.s. topological order:

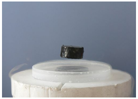
$$|\Psi\rangle = |\uparrow\uparrow\uparrow\uparrow\cdots\rangle$$

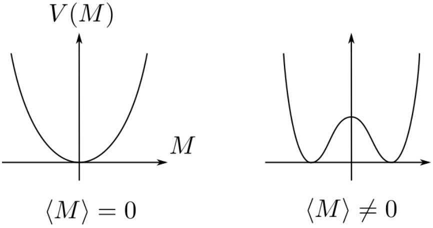


Resonating valence bond (RVB) state [Anderson (73)]

“Conventional” phases of matter

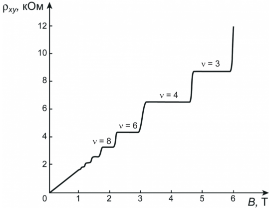
- Many phases of condensed matter can be characterized by spontaneous symmetry breaking and local order parameters.





Topological phases of matter

- Topological phases escape from the symmetry breaking paradigm (i.e., no order parameter).
- Instead, characterized by their quantized responses and/or existence of exotic excitations
- E.g., the Hall conductance in the quantum Hall effect

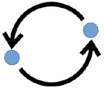


- Other examples: topological insulators, topological superconductors, Haldane spin chain, etc.

Topologically-ordered phases in (2+1)D

- Phases that support anyons
- Anyons are neither bosons nor fermions; have non-trivial exchange (braiding) statistics

$e^{i\theta}$

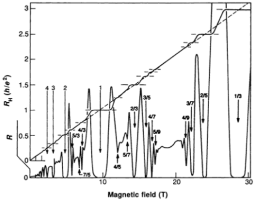


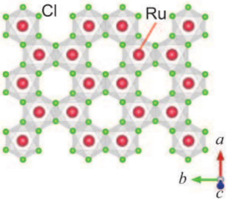
- Abelian/Non-Abelian topological order
- Not characterized by the symmetry-breaking paradigm, but by the properties of anyons (fusion, braiding, etc.)

	SSB phases	Topologically-ordered phases
Ground states	Degeneracy w/ SSB	Topological degeneracy
Excitations	Nambu-Goldstone bosons	Anyons
Effective theory	Landau-Ginzburg	Topological field theory

- Bulk-boundary correspondence

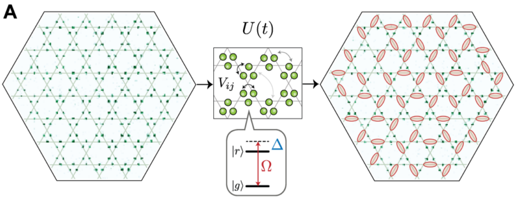
Fractional quantum Hall effect (↓)



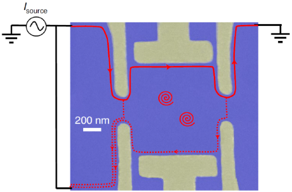


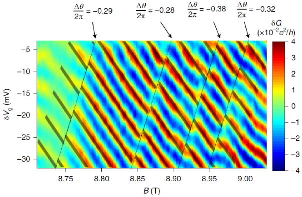
Spin-orbital-assisted Mott insulator α-RuCl$_3$ (“Kitaev spin liquid”) (↑)

Rydberg atoms on a Kagome lattice (↓) [Semeghini1 et al (21)]



- Conventional phases can be detected by conventional probes: (local) correlation functions $\langle O_1 O_2 \cdots \rangle$
- How can we extract/measure topological data?
  Direct observation of abelian braiding statistics [Nakamura et al (20), Bartolomei et al (20)]





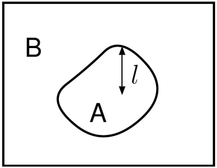
- Other measurable quantities? Topological invariants?

Entanglement entropy

- The von-Neumann entanglement entropy:

$$S_A := -\mathrm{Tr}_A(\rho_A \log \rho_A)$$

for the reduced density matrix $\rho_A = \mathrm{Tr}_B\, |\Psi\rangle\langle\Psi|$



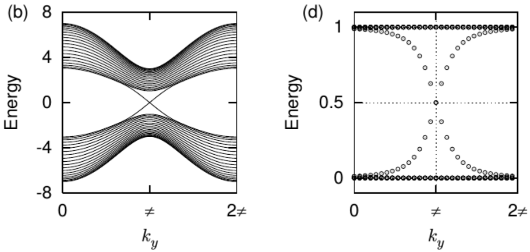
- For topologically-ordered states in (2+1)D [Levin-Wen, Kitaev-Preskill (05)]

$$S_A = const. \times \ell - \log \mathcal{D}$$

"Topological entanglement entropy" $\gamma = \log \mathcal{D}$ carries universal data

Entanglement spectrum

- The spectrum of the reduced density matrix ρ_A
- $\rho_A \sim e^{-\epsilon H_{edge}}$ for topological phases. "Bulk-boundary correspondence"
  [Li-Haldane (08)]
- E.g. Chern insulators (single-particle entanglement spectrum)
  [SR-Hatsugai (06)]

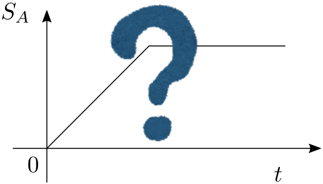


Entanglement as a probe for dynamics

- Time evolution of von Neumann or Rényi entropy

$$S_A = -\mathrm{Tr}\,\rho_A \log \rho_A, \quad S_A^{(2)} = -\log \mathrm{Tr}\,\rho_A^2$$

in non-equilibrium process. E.g., quantum quench: $|\Psi(t)\rangle = e^{-iHt}|\Psi_{init}\rangle$

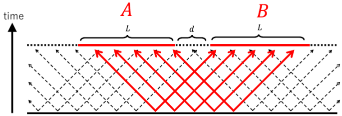


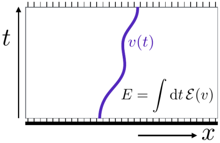
- Thermalization, quantum information scrambling, many-body localization, etc.

"Hydrodynamics" of entanglement

Two canonical models for entanglement propagation:

Quasi-particle picture v.s Membrane picture

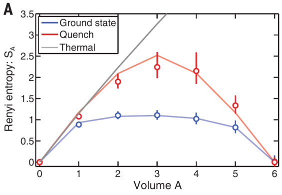


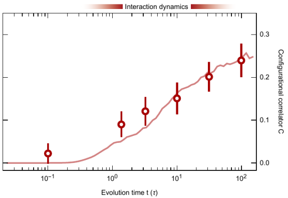


[Figures: Jonay-Huse-Nahum (18)]

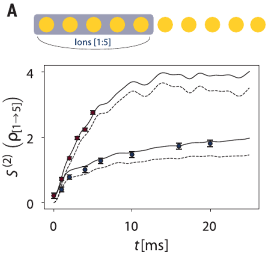
Can we measure entanglement entropy?

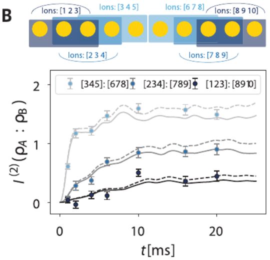
- [Islam et al (15)] [Kaufman et al (16)] [Lukin et al (18)]





- Randomized measurement [Brydges (19)]

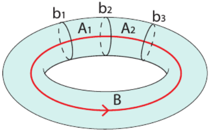


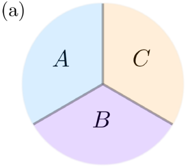


- Introduction; topology and dynamics in quantum many-body systems detected by entanglement
- Negativity and reflected entropy
- Tripartitioning topological liquid
- Summary and outlook

Going beyond entanglement entropy for bipartition

- Go beyond bipartition, and study entanglement quantities





- Multipartite entanglement?

$$\frac{1}{\sqrt{2}}\left[|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right], \quad \frac{1}{\sqrt{3}}\left[|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right],$$

- Entanglement negativity and reflected entropy

Entanglement negativity

- How to quantify quantum entanglement between A and B when $\rho_{A\cup B}$ is *mixed* ? E.g., finite temperature; A, B is a part of bigger system.
- The entanglement entropy is an entanglement measure only for pure states (not monotone under LOCC).
- *Entanglement negativity* and *logarithmic negativity*, using *partial transpose*
  [Peres (96), Horodecki-Horodecki-Horodecki (96), Vidal-Werner (02), Plenio (05) ...]

$$\mathcal{N}(\rho) := \sum_{\lambda_i<0} |\lambda_i| = \frac{1}{2}\left(||\rho^{T_B}||_1 - 1\right),$$

$$\mathcal{E}(\rho) := \log(2\mathcal{N}(\rho)+1) = \log||\rho^{T_B}||_1.$$

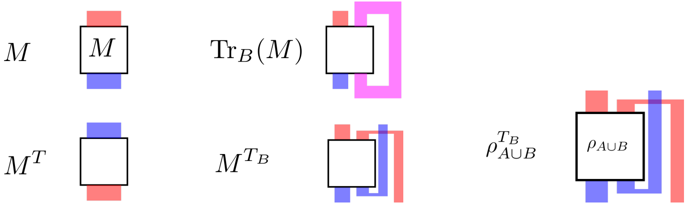
- Good entanglement measure since LOCC monotone.
- For mixed states, negativity can extract quantum correlations only.

- Definition: for an operator M, its partial transpose M^{T_B} is

$$\langle e_i^{(A)} e_j^{(B)} | M^{T_B} | e_k^{(A)} e_l^{(B)} \rangle := \langle e_i^{(A)} e_l^{(B)} | M | e_k^{(A)} e_j^{(B)} \rangle$$

where $|e_i^{(A,B)}\rangle$ is the basis of $\mathcal{H}_{A,B}$.

- For fermionic (anyonic) systems, we need to take into account particle statistics properly. [Shapourian-Shiozaki-SR (16); Shapourian-SR (18); Shapourian-Mong-SR (20)]



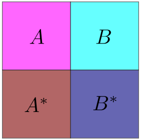
Reflected Entropy

[Dutta-Faulkner (19)]

- The von-Neumann entropy of a canonical purification:

$$\rho_{AB} = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|_{AB}$$

$$\to \left|\sqrt{\rho}_{AB}\right\rangle_{AA^*BB^*} \equiv \sum_i \sqrt{p_i} \left|\psi_i\right\rangle_{AB} \left|\psi_i^*\right\rangle_{A^*B^*} .$$



$$R_{A:B} := S_{vN}(A \cup A^*)$$

- Satisfies $I_{A:B} \leq R_{A:B} \leq 2\min\left[S(A), S(B)\right]$.
- Admits holographic dual $R_{A:B} = 2E_W$ [Dutta-Faulkner (19)]

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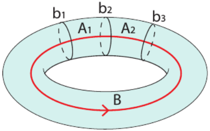
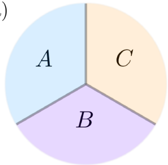
Going beyond entanglement entropy for bipartition

- What can we learn by studying entanglement negativity and reflected entropy?
- Will show results for the following two setups:



[Wen-Matsuura-SR (16)]
[Liu-Sohal-Kudler-Flam-SR (21)]

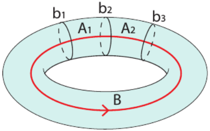
(a)



Negativity for topological liquid

[Lee-Vidal (13), Castelnovo (13), Wen-Matsuura-SR (16), Wen-Chang-SR (16) Lim-Asasi-Teo-Mulligan (21)]

- Generic state on a torus: $|\psi\rangle = \sum_a \psi_a |\mathfrak{h}_a\rangle\rangle$



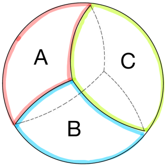
- Mutual information and negativity:

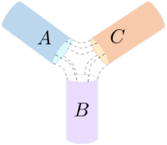
$$I_{A_1:A_2} = \frac{\pi c}{12}\frac{l_2}{\epsilon} - 2\ln\mathcal{D} + 2\sum_a |\psi_a|^2 \ln d_a - \sum_a |\psi_a|^2 \ln|\psi_a|^2$$

$$\mathcal{E}_{A_1:A_2} = \frac{\pi c}{16}\frac{l_2}{\epsilon} - \ln\mathcal{D} + \ln\Big(\sum_a |\psi_a|^2 \ln d_a\Big)$$

$\mathcal{E}$ is dependent on ψ_a only for non-Abelian topological order (for Abelian topological order, $d_a = 1$ for all a).

[Liu-Sohal-Kudler-Flam-SR (21)]

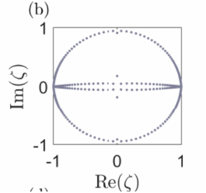




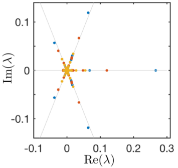
- Studied chiral p-wave superconductor (the Ising TQFT) with $c = 1/2$ and the integer quantum Hal state with $c = 1$.

Negativity spectrum

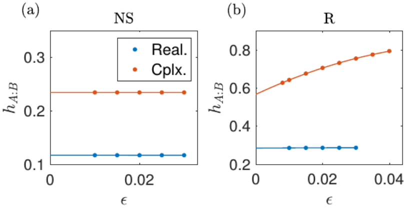
- For fermionic systems, the eigenvalues of $\rho_{A\cup B}^{T_B}$ are complex.
- Nontrivial distribution of the eigenvalues of $\rho_{A\cup B}^{T_B}$



- C.f. (1+1)D fermionic CFTs (6-fold structure) [Shapourian-Ruggiero-SR-Calabrese (19)] (1+1)D topological superconductor (8-fold structure) [Inamura-Kobayashi-SR (19)]



Reflected entropy



- The "Markov gap" $h_{A:B} = R_{A:B} - I_{A:B}$ seems universal and given by

$$h_{A:B} = \frac{c}{3} \ln 2$$

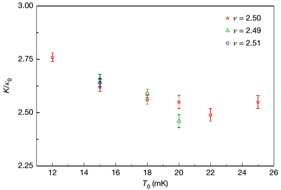
where c is the (total) central charge. Agrees with the recent claim [Zou, Siva, Soejima, Mong, Zaletel (2110.11965)] $(h_{A:B} = (c/3) \ln 2 = 0.116, 0.231$ for $c = 1$ and $c = 1/2)$.

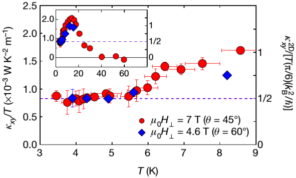
Central charge c

- Chiral central charge can be measured by the thermal conductance in the edge [Kane-Fisher (96)] :

$$\kappa = \frac{\pi k_B^2 T}{6} \times c$$

E.g. half-filled Landau level [Banerjee (18)] , Kitaev spin liquid [Kasahara (18)]

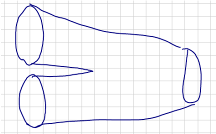




- Modular commutator $i\langle[K_{AB}, K_{BC}]\rangle = \pi c/3$: Kim-Shi-Kato-Albert (21)

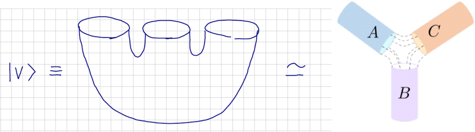
Multipartition topological phases using string field theory

- String field theory = many-body (second quantized) string theory
- Interaction vertices in string field theory



[Witten (86), Gross-Jevicki (87), LeClair-Peskin-Preitschopf (89), ...]

- Vertex state $|V\rangle \simeq$ Topological ground state $|\Psi\rangle$ near the entangling boundary by utilized bulk-boundary correspondence

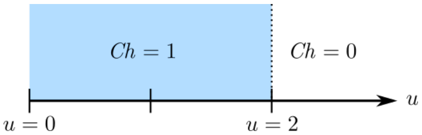


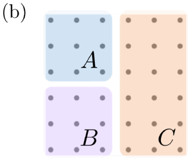
- Generalized [Qi-Katsura-Ludwig (12)] for tripartition geometry

Lattice Chern insulator calculations

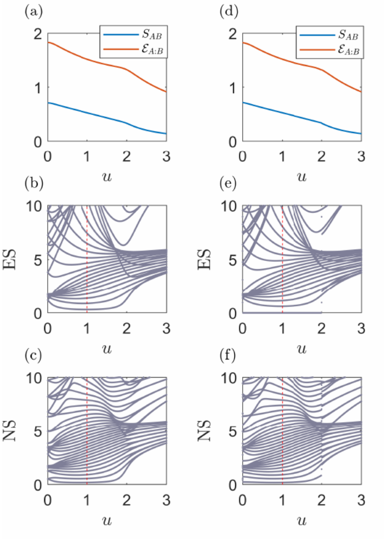
- Lattice fermion model:

$$H = \frac{-i}{2} \sum_{\mathbf{r}} \sum_{\mu=x,y} \left[f_{\mathbf{r}}^{\dagger} \tau_{\mu} f_{\mathbf{r}+\mathbf{a}_{\mu}} - f_{\mathbf{r}+\mathbf{a}_{\mu}}^{\dagger} \tau_{\mu} f_{\mathbf{r}} \right]$$
$$+ \frac{1}{2} \sum_{\mathbf{r}} \sum_{\mu=x,y} \left[f_{\mathbf{r}}^{\dagger} \tau_{z} f_{\mathbf{r}+\mathbf{a}_{\mu}} + f_{\mathbf{r}+\mathbf{a}_{\mu}}^{\dagger} \tau_{z} f_{\mathbf{r}} \right] + u \sum_{\mathbf{r}} f_{\mathbf{r}}^{\dagger} \tau_{z} f_{\mathbf{r}},$$

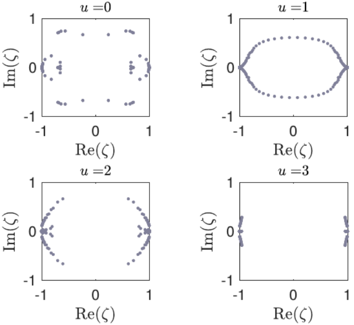




Entropy and Negativity



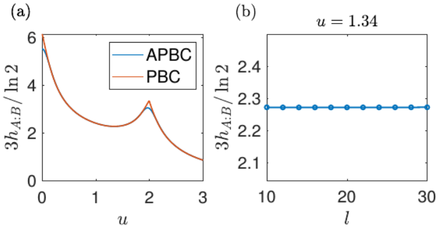
Negativity spectrum



- "Trivial" for $|u| \to \infty$
- "Circular distribution" deep in the Ch=1 phase

Reflected entropy

- $h_{A:B}$ is minimal in the topological phase around $u = 1.34$
- $h_{A:B} \sim (c/3) \ln 2 \times 2.3$
- Note that there are four trijunctions, as opposed to two in the edge theory calculations. May result in a factor of 2.



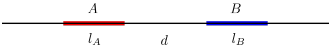
- See also: [Zou, Siva, Soejima, Mong, Zaletel (2110.11965)]

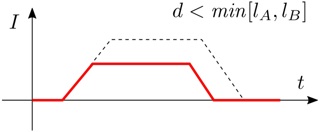
Summary/Outlook

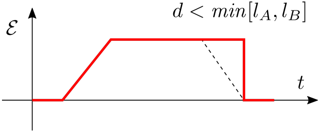
- New tripartition setup and new calculations of entanglement quantities
- They may capture topological data beyond topological entanglement entropy, e.g., Abelian v.s. Non-abelian, total central charge.
- Finite-T topological transition can be detected by negativity [Hart-Castelnovo(18);Lu-Hsieh-Grover(19)]
- May have an implication on numerics (tensor-networks)
- May have an implication in string field theory?
- Other entanglement quantities, such as odd entropy, entanglement of purification, etc?
- Experiments: Many-body interference or randomized measurements [Islam et al (15)] [Kaufman et al (16)] [Lukin et al (18)] [Brydges (19)]

Negativity/reflected entropy in quantum dynamics

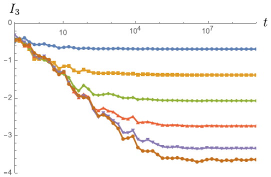
- Quantum quench in integrable/chaotic (1+1)D CFT [Kudler-Flam-Kusuki-SR (20)]

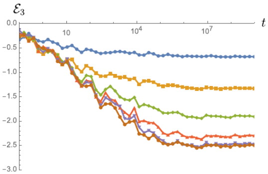






- Many-body localizing dynamics [MacCormack-Tan-Kudler-Flam-SR (19)]





Thank you for your attention!

- Paper: arXiv:2110.11980
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- Thanks also:
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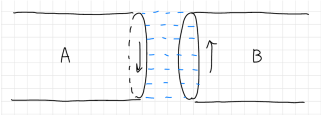
Edge theory approach to entanglement



- ρ_A obtained from a ground state $|GS\rangle$ by tracing out half-space can be obtained from conformal boundary state $|B\rangle$: [Qi-Katsura-Ludwig (12)]

$$[T(\sigma) - \bar{T}(\sigma)]\,|B\rangle = 0$$

- Near the entangling boundary, $|GS\rangle \sim e^{-\epsilon H_{edge}}|B\rangle$ so that the reduced density matrix is

$$\rho_A \propto \mathrm{Tr}_B\left[e^{-\epsilon H_{edge}}|B\rangle\langle B|e^{-\epsilon H_{edge}}\right]$$

- "Physical picture": healing the cut; gapped edge by potential