ExU collaboration

Linking quantum information to physics

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The University of Tokyo & JST PRESTO

March 7th @ Annual meeting of ExU collaboration

Outline of this talk

A journey of a thousand miles begins with a single step

Information, Chaos, and Black Holes in the Quantum regime.

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1. Information and Chaos in the classical regime.

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3. Beyond the Hayden-Preskill.

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1. Information and Chaos in the classical regime.
2. The Hayden-Preskill thought experiment.
3. Beyond the Hayden-Preskill.
4. Conclusion and Outlooks.

1. Information and Chaos in the classical regime.

1. Quantum Error Correction (QEC)
2. Chaotic dynamics and Error Correction in classical
3. Quantum chaos and QEC?

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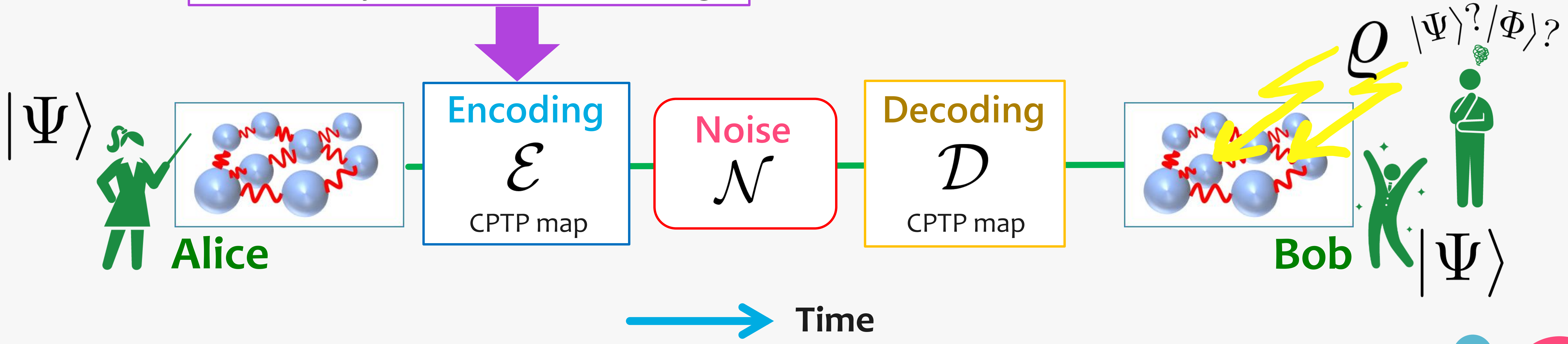
Quantum Error Correction

An introduction in a nutshell

What is Quantum Error Correction (QEC)?

- **QEC** is a method to effectively cancel quantum noise by **ENCODING** and **DECODING**.

Today, we consider to use chaotic dynamics for encoding!



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \mathrm{id}$$

1. Information and Chaos in the classical regime.

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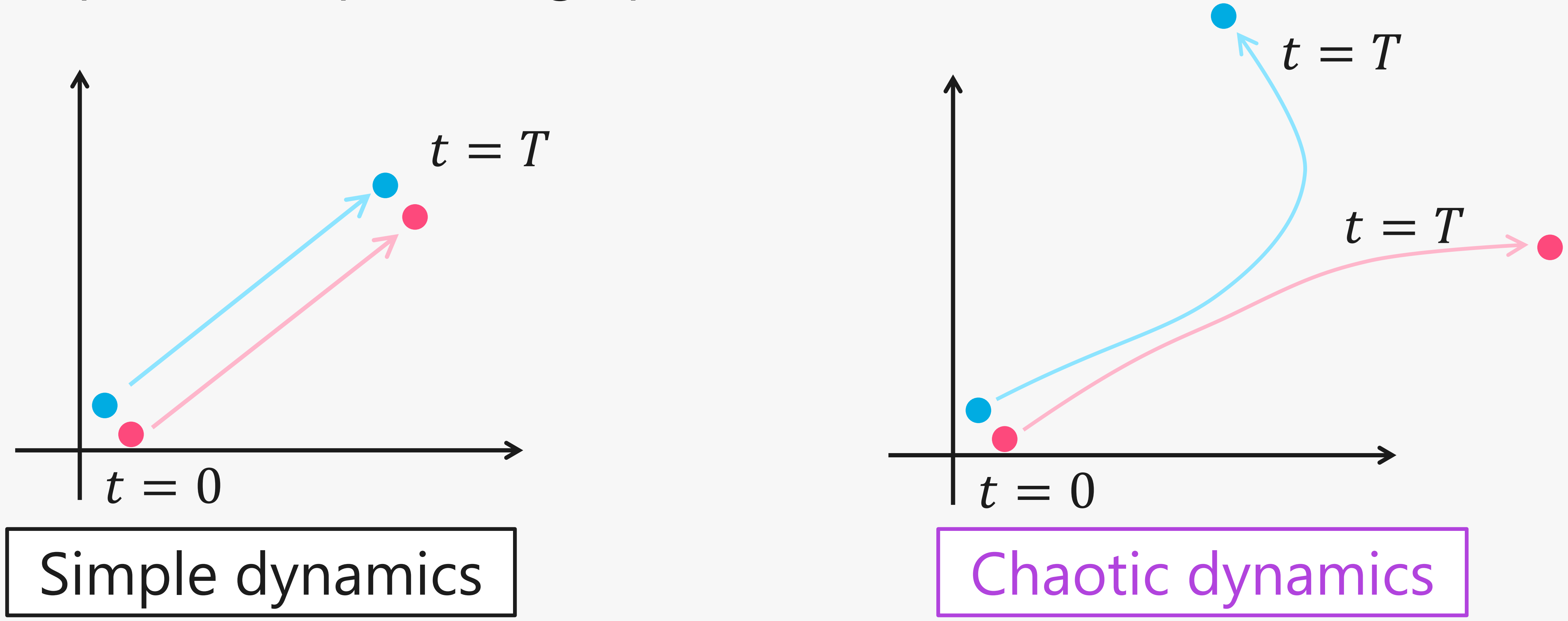
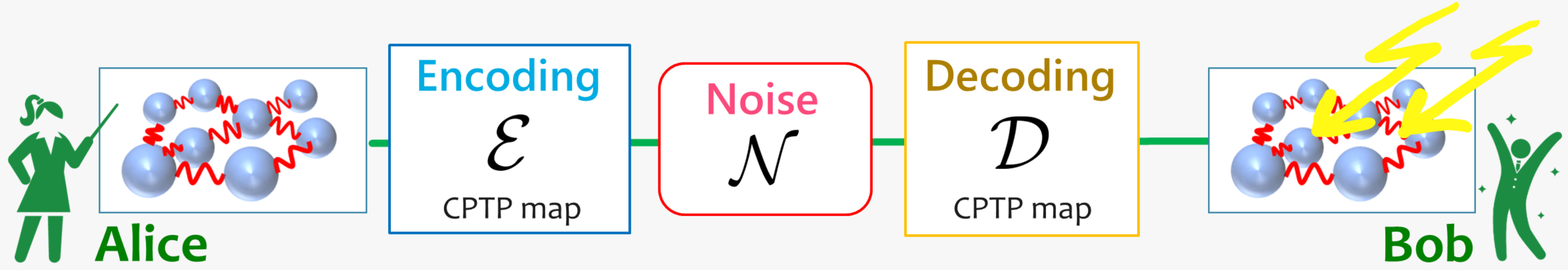
Chaos and Error Correction in classical

How can we use chaotic dynamics for correcting errors?

What is chaos in the classical regime?

- **Chaos** is a dynamical system highly sensitive to initial conditions.





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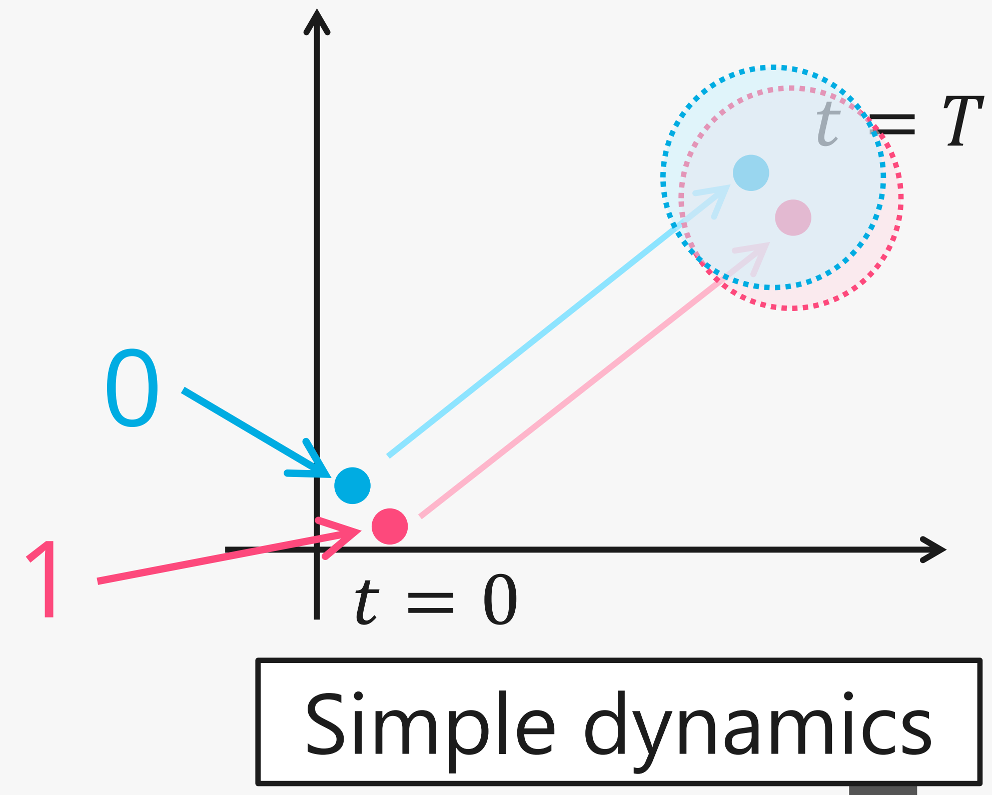
QEC in a nutshell

Chaos and Error Correction in classical

How can we use chaotic dynamics for correcting errors?

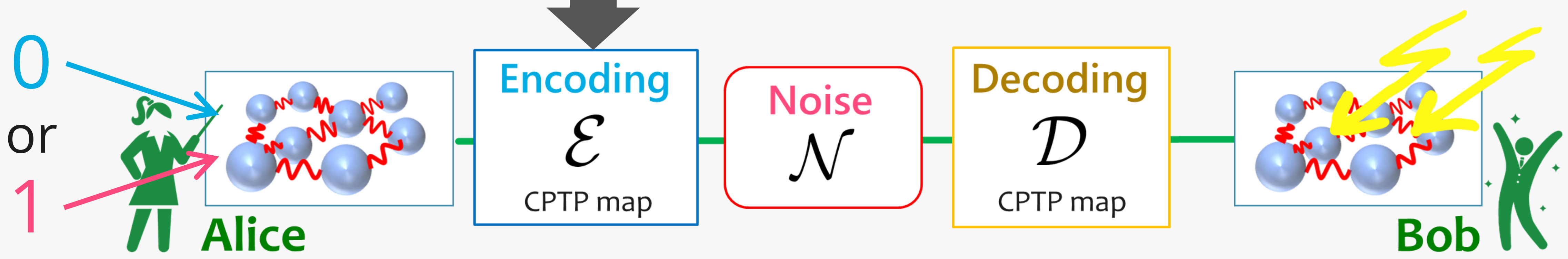
Chaos and Error Correction: intuition 1

- **Chaos** is a dynamical system highly sensitive to initial conditions.



1. Write 0 or 1 into the system.
2. "Time evolution" of the information by the **simple** dynamics.
3. Due to the uncertainty induced by **the noise**, the info. gets **unclear**.
4. The **huge overlap** makes it difficult to **decode** the info. (0 or 1).

→ **Failure of decoding!**



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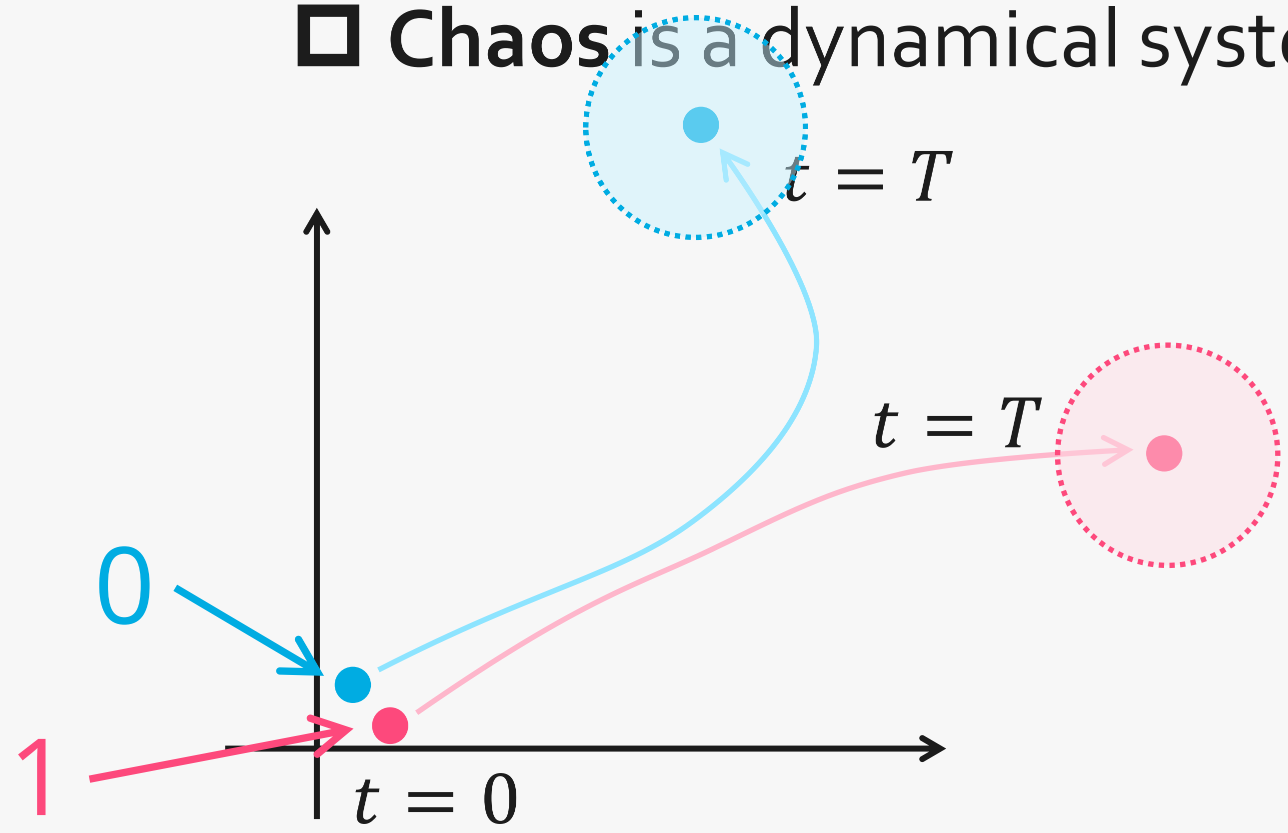
QEC in a nutshell

Chaos and Error Correction in classical

How can we use chaotic dynamics for correcting errors?

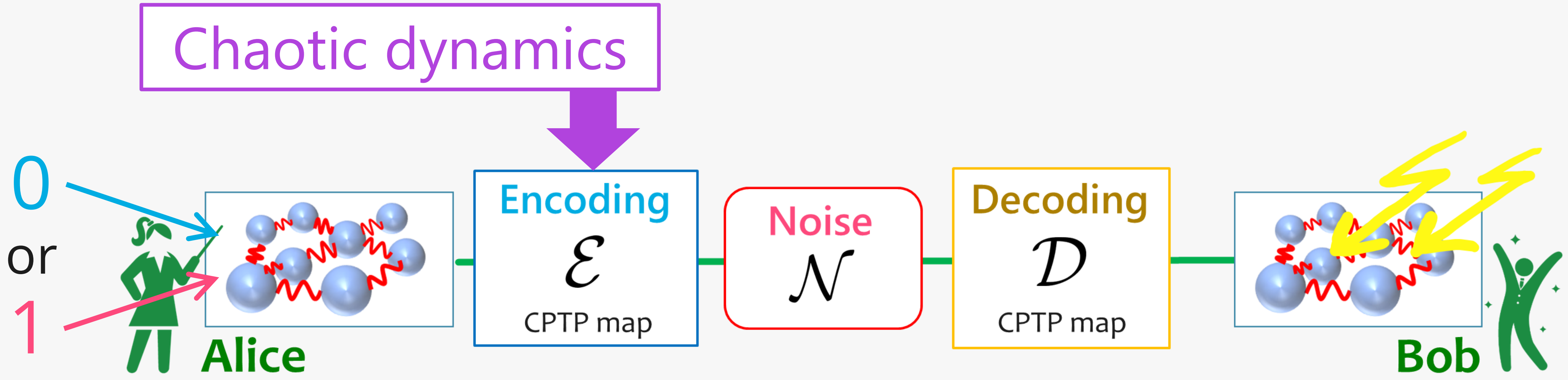
Chaos and Error Correction: intuition 2

☐ **Chaos** is a dynamical system <u>highly sensitive to initial conditions</u>.



1. Write 0 or 1 into the system.
2. “Time evolution” of the information by the **chaotic** dynamics.
3. Due to the uncertainty induced by **the noise**, the info. gets **unclear**.
4. **No overlap** makes it easy to **decode** the info. (0 or 1).

→ **Success of decoding!**



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QEC in a nutshell

Chaos and Error Correction in classical

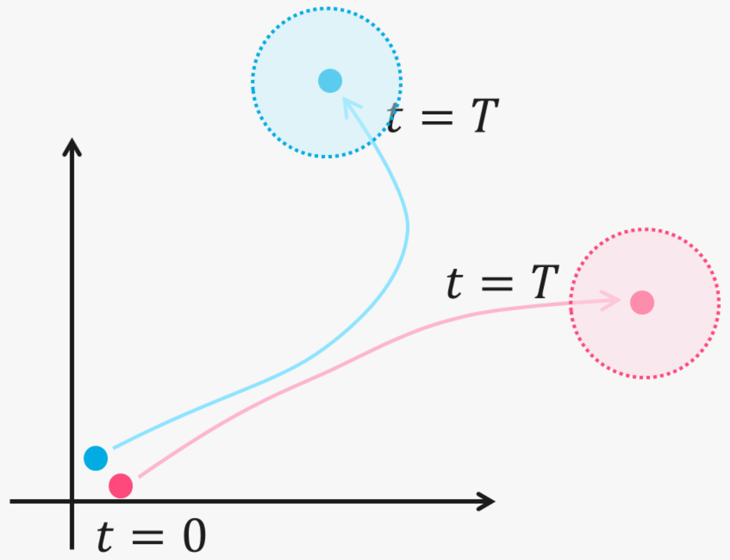
How can we use chaotic dynamics for correcting errors?

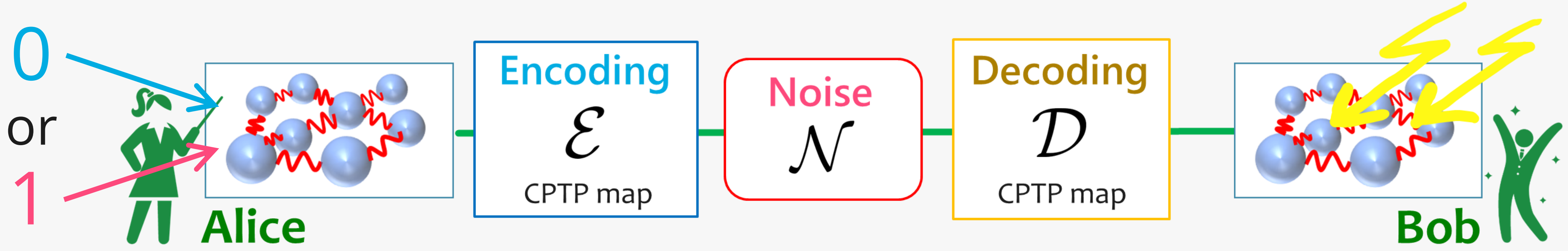
Important remark on Chaos and Error Correction

- The **sensitivity of chaos to the initial condition** makes it easy to **read out information**!

One needs to know all the details of the dynamics!

- **In physics**, it is common to assume that the details of the system are **our of control**. Hence, **chaos** is **unpredictable.**
- **In quantum information**, everything is **under control**.
  - ➢ **Chaotic dynamics is a powerful tool to encode information!**





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QEC in a nutshell

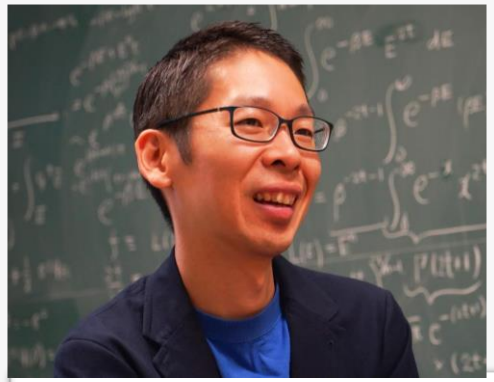
1. Information and Chaos in the classical regime.

Chaos and Error Correction in classical

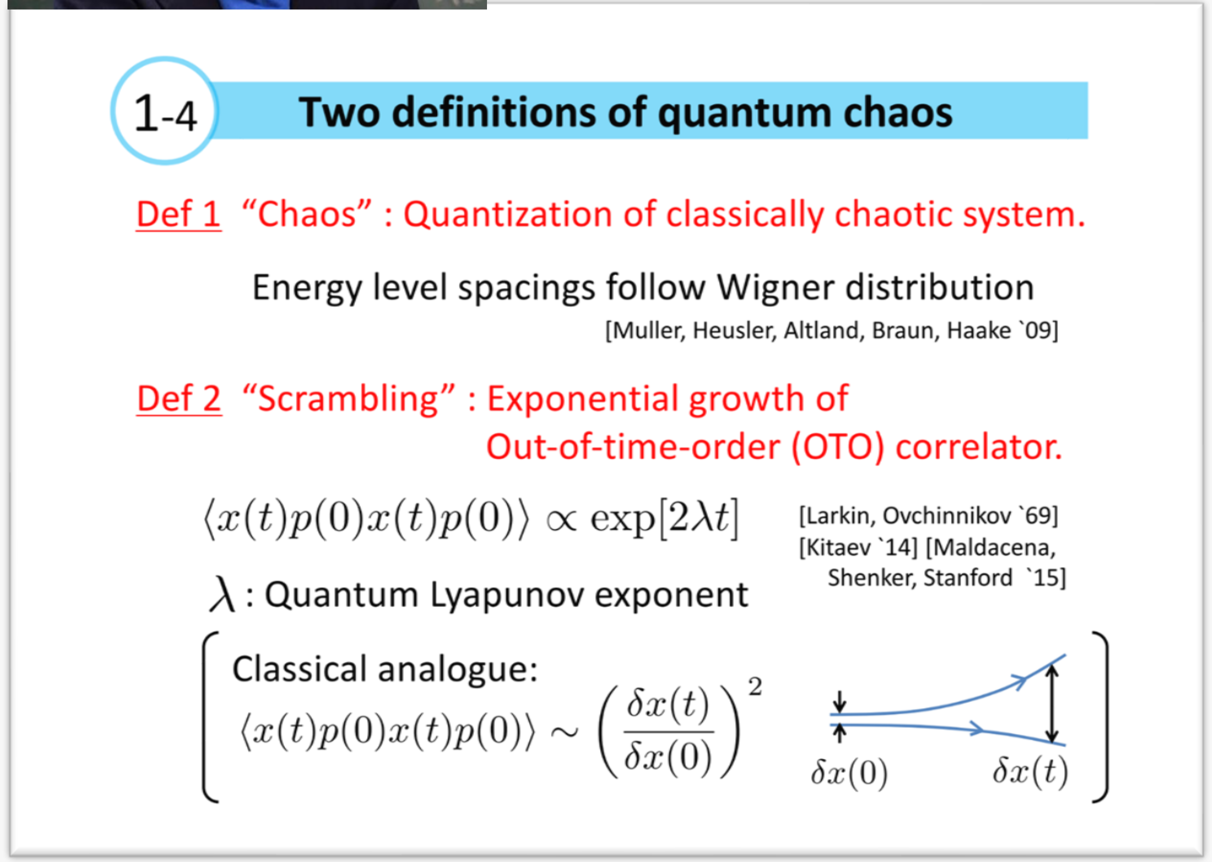
How can we use chaotic dynamics for correcting errors?

Chaos in the **Quantum** regime 1

- "Quantum chaos" is **non-trivial**.
  - (At least) two inequivalent defs of "**quantum chaos**".



By Prof. Hashimoto
3rd Colloquium of ExU
(available on the ExU website)



1-4 Two definitions of quantum chaos

Def 1 "Chaos" : Quantization of classically chaotic system.

Energy level spacings follow Wigner distribution [Muller, Heusler, Altland, Braun, Haake \`09]

Def 2 "Scrambling" : Exponential growth of Out-of-time-order (OTO) correlator.

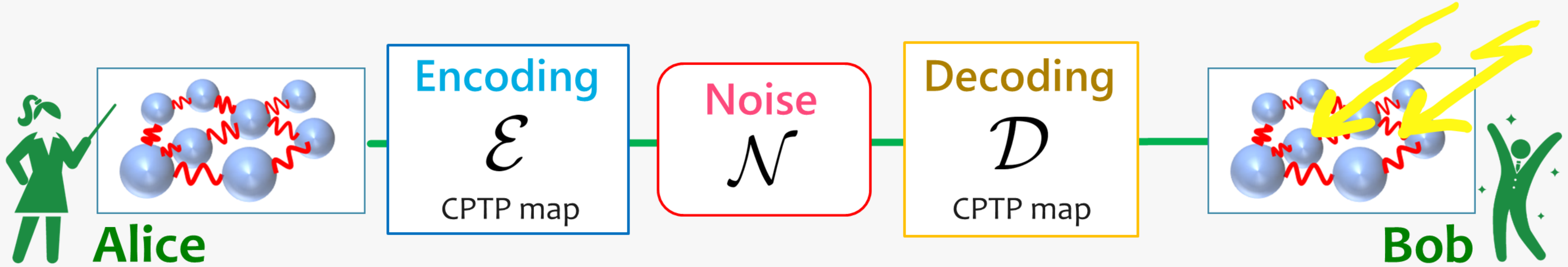
$$\langle x(t)p(0)x(t)p(0)\rangle \propto \exp[2\lambda t]$$

[Larkin, Ovchinnikov \`69] [Kitaev \`14] [Maldacena, Shenker, Stanford \`15]

λ : Quantum Lyapunov exponent

Classical analogue:

$$\langle x(t)p(0)x(t)p(0)\rangle \sim \left(\frac{\delta x(t)}{\delta x(0)}\right)^2$$



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \mathrm{id}$$

QEC in a nutshell

Chaos and Error Correction in classical

How can we use chaotic dynamics for correcting errors?

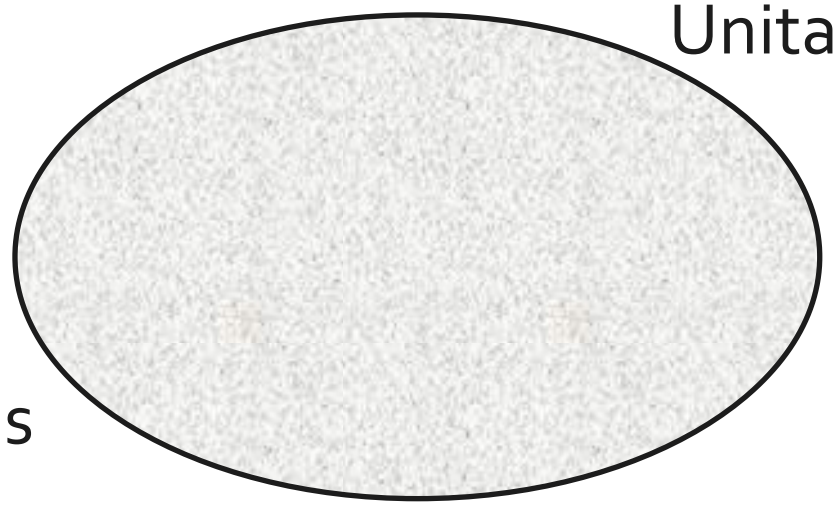
Chaos in the **Quantum** regime 2

- "Quantum chaos" is **non-trivial**.
  - We model the "chaotic dynamics" by a Haar random unitary (a.k.a. circular unitary ensemble).

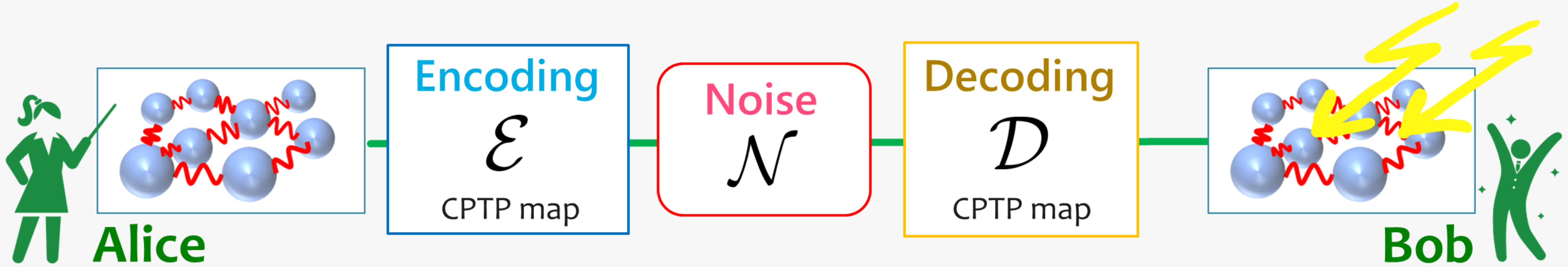
A Haar random unitary = Circular Unitary Ensemble (CUE)

- A random unitary uniformly distributed over a unitary group
  - A typical dynamics of quantum chaos.

Unitary group



A Haar random unitary
= uniform distribution of unitaries



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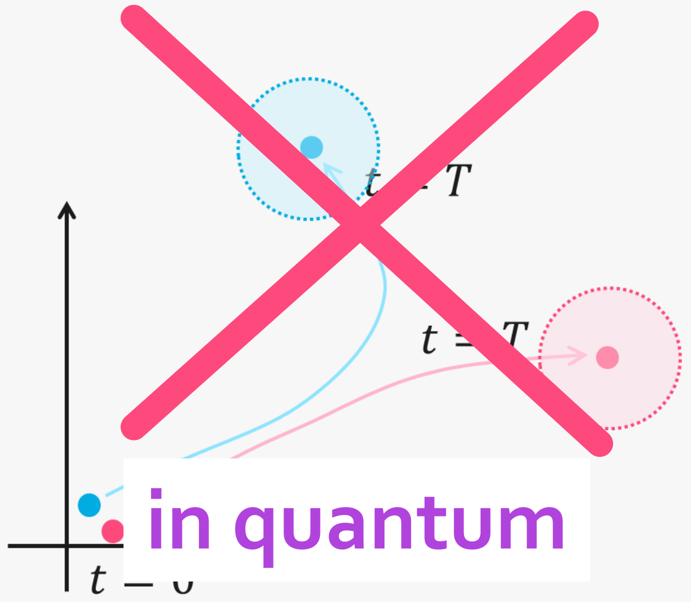


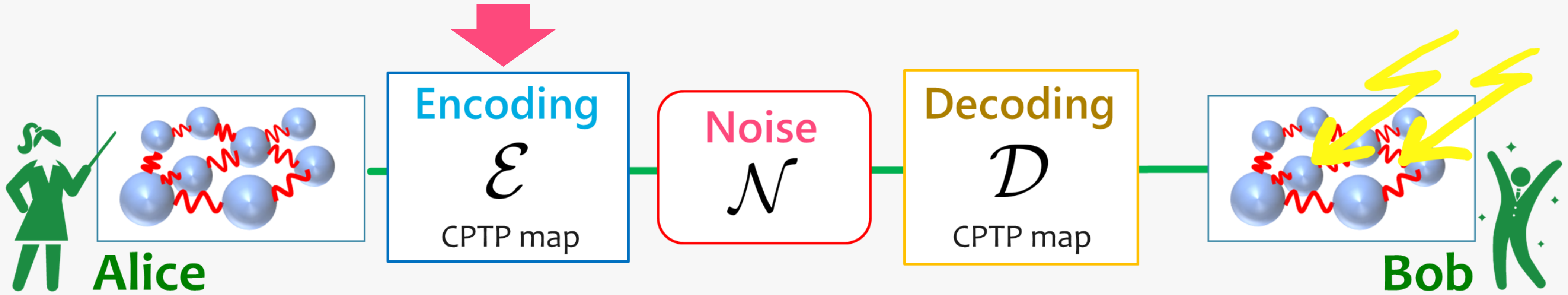
in quantum

Chaos and Quantum Error Correction

- ☐ "Quantum chaos" is **non-trivial**.
  - ➢ We model the "chaotic dynamics" by a Haar random unitary (a.k.a. circular unitary ensemble).
- ☐ **"QEC by chaotic dynamics"** is **non-trivial**.
  1. Unitarity preserves the distance between two states.
  2. We need to recover an unknown quantum state (e.g. $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$), not 0 or 1.
- ☐ It turns out that **quantum chaotic dynamics** is useful for **QEC**. [Dupuis et al, CMP '14]
  - ➢ Finally, established by the "**one-shot decoupling theorem**".

A Haar random unitary

As an example, we see the **Hayden-Preskill thought experiment.**



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QEC in a nutshell

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A journey of a thousand miles begins with a single step

Information, Chaos, and Black Holes in the Quantum regime.

1. Information and Chaos in the classical regime.
2. **The Hayden-Preskill thought experiment.**
3. Beyond the Hayden-Preskill.
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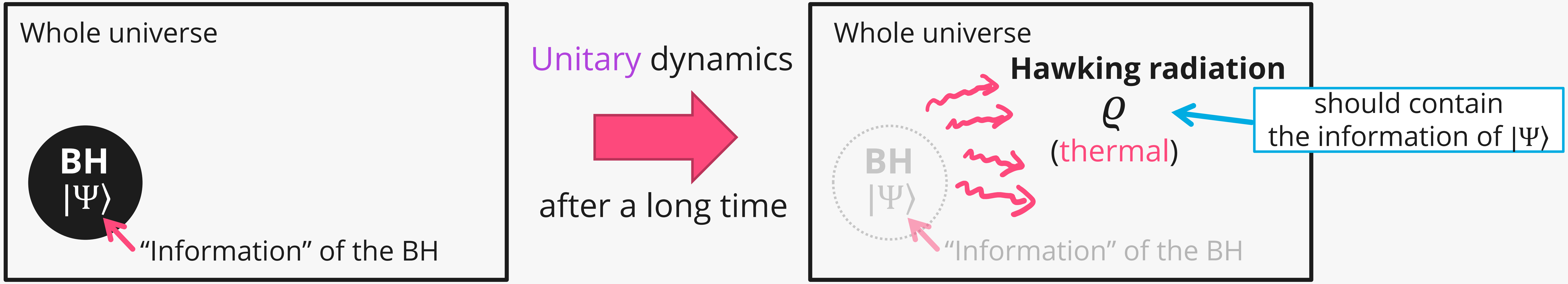
The … ment

Informati…

Informati…

How?

Informati…



If you look at the beginning and the end, you will notice that

Apparent contradiction

a **pure** state $|\Psi\rangle$ becomes a **mixed** state ϱ by **unitary** dynamics.

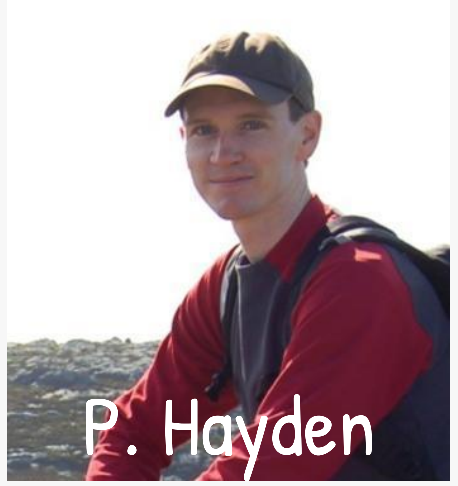
The state $|\Psi\rangle$ should be recoverable from the radiation (if we are in the quantum side)

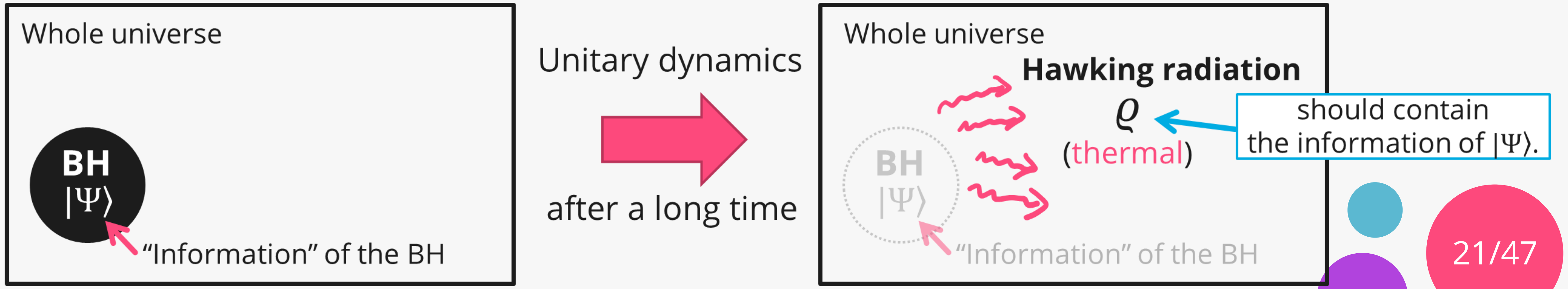
The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

How can we recover quantum information from the radiation?

- In 2007, Hayden and Preskill addressed this question based on **a qubit-toy model of a "BH"**.



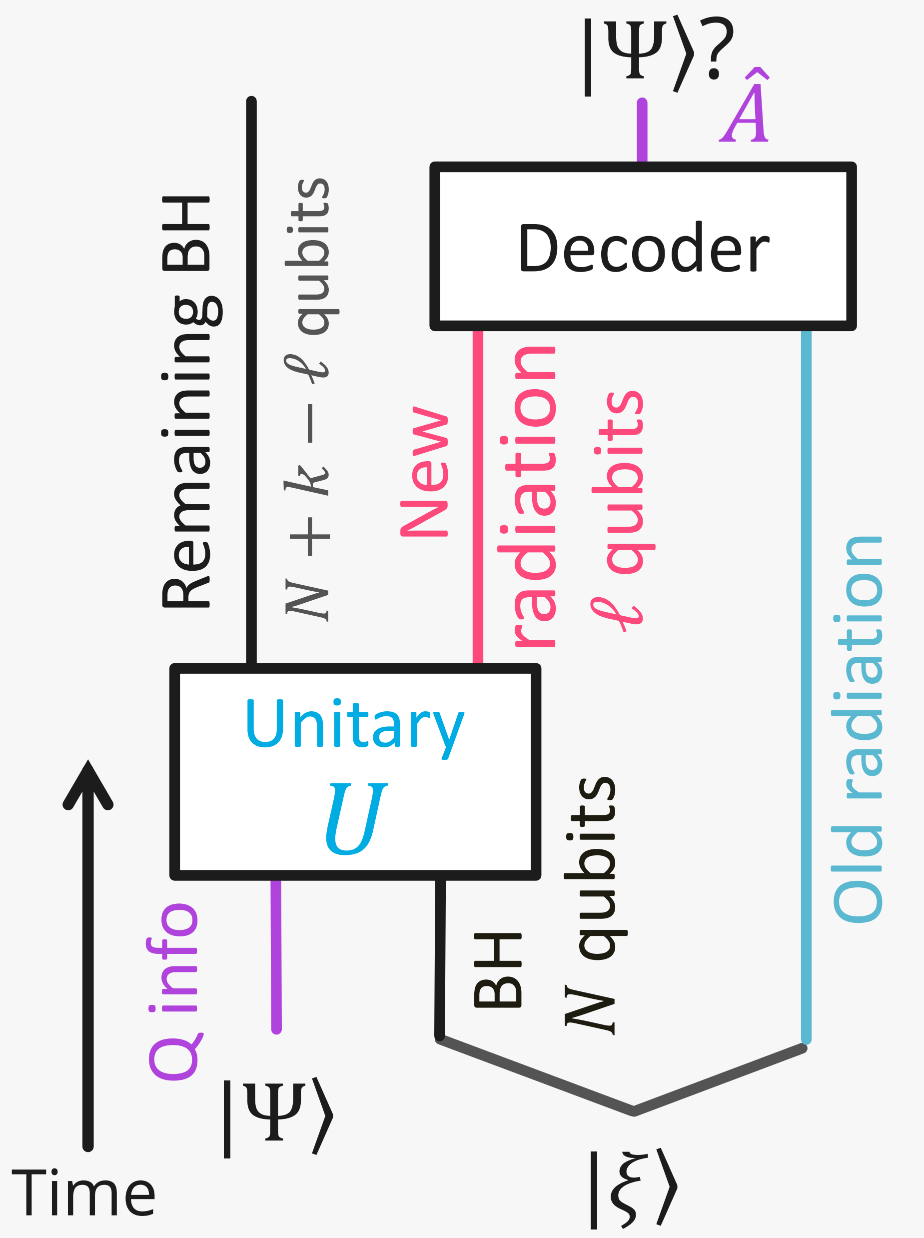


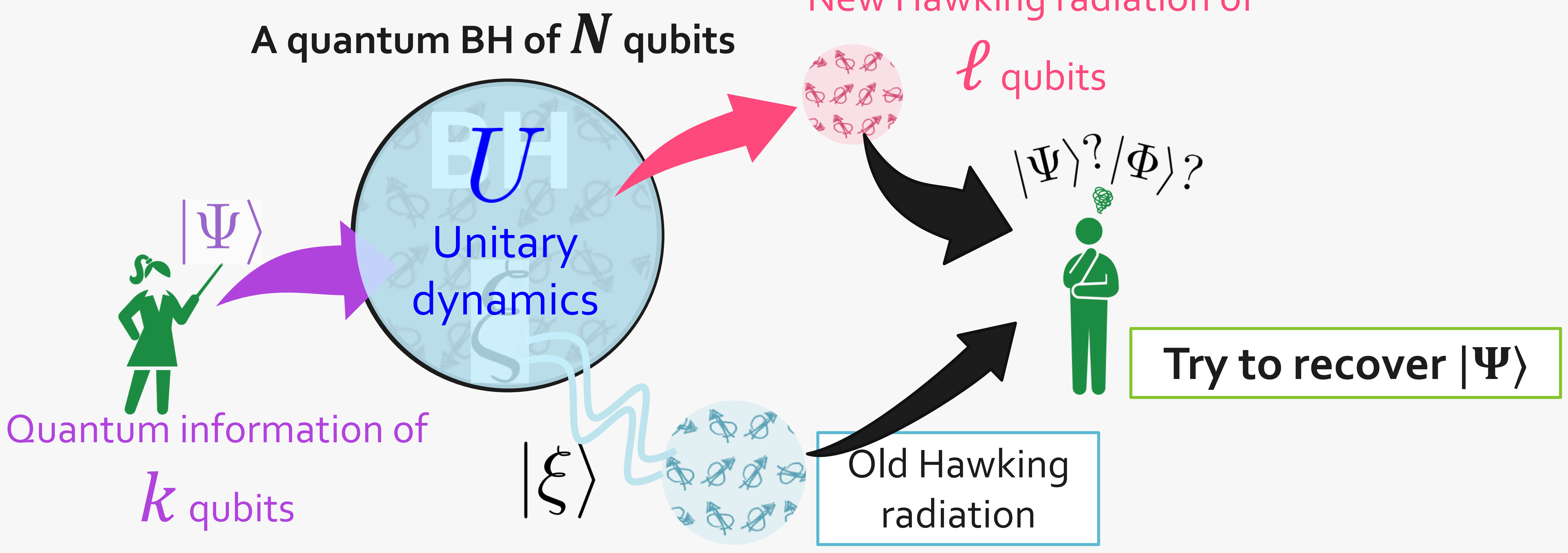
The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

Information paradox and the Hyden-Preskill thought experiment

- A qubit-toy model of a quantum "BH".





To decode the k-qubit quantum info., how large should ℓ be?

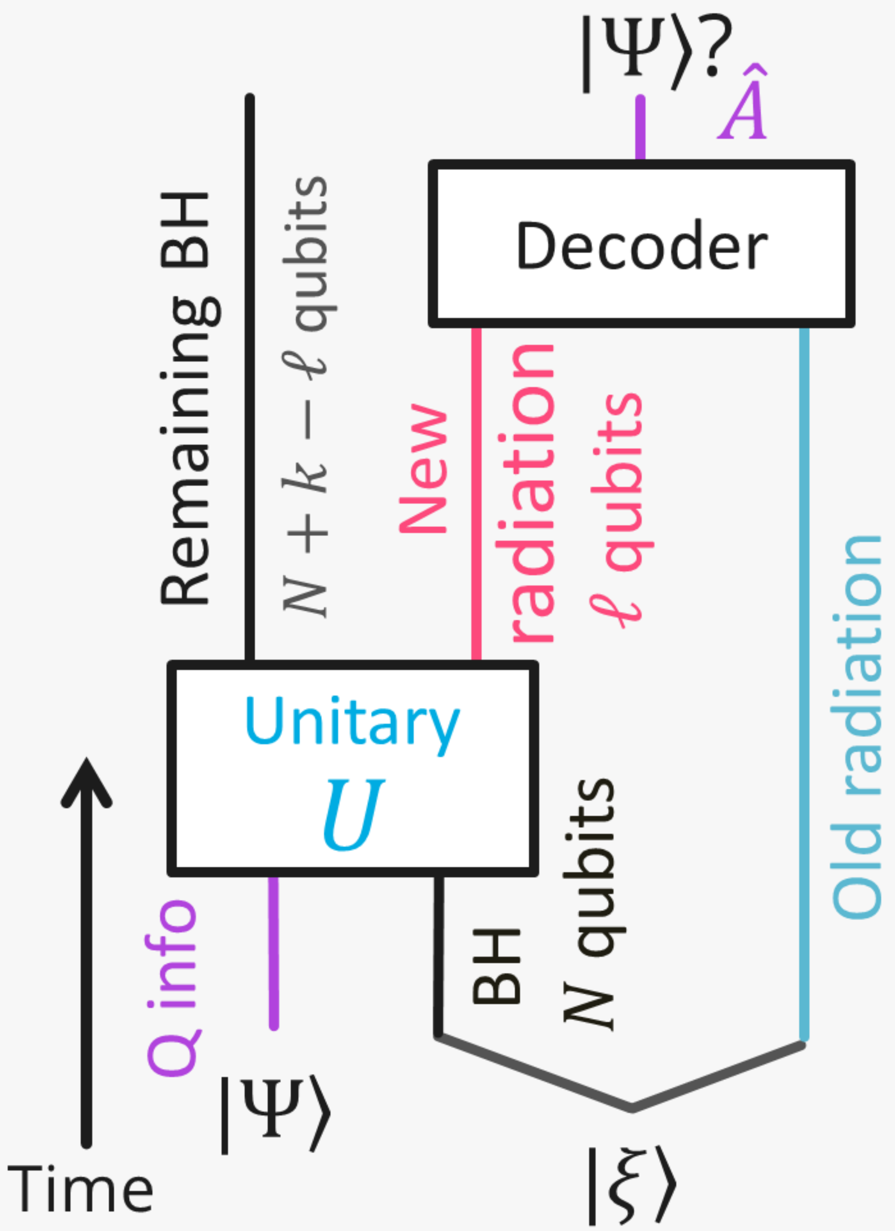
The Hayden-Preskill thought experiment

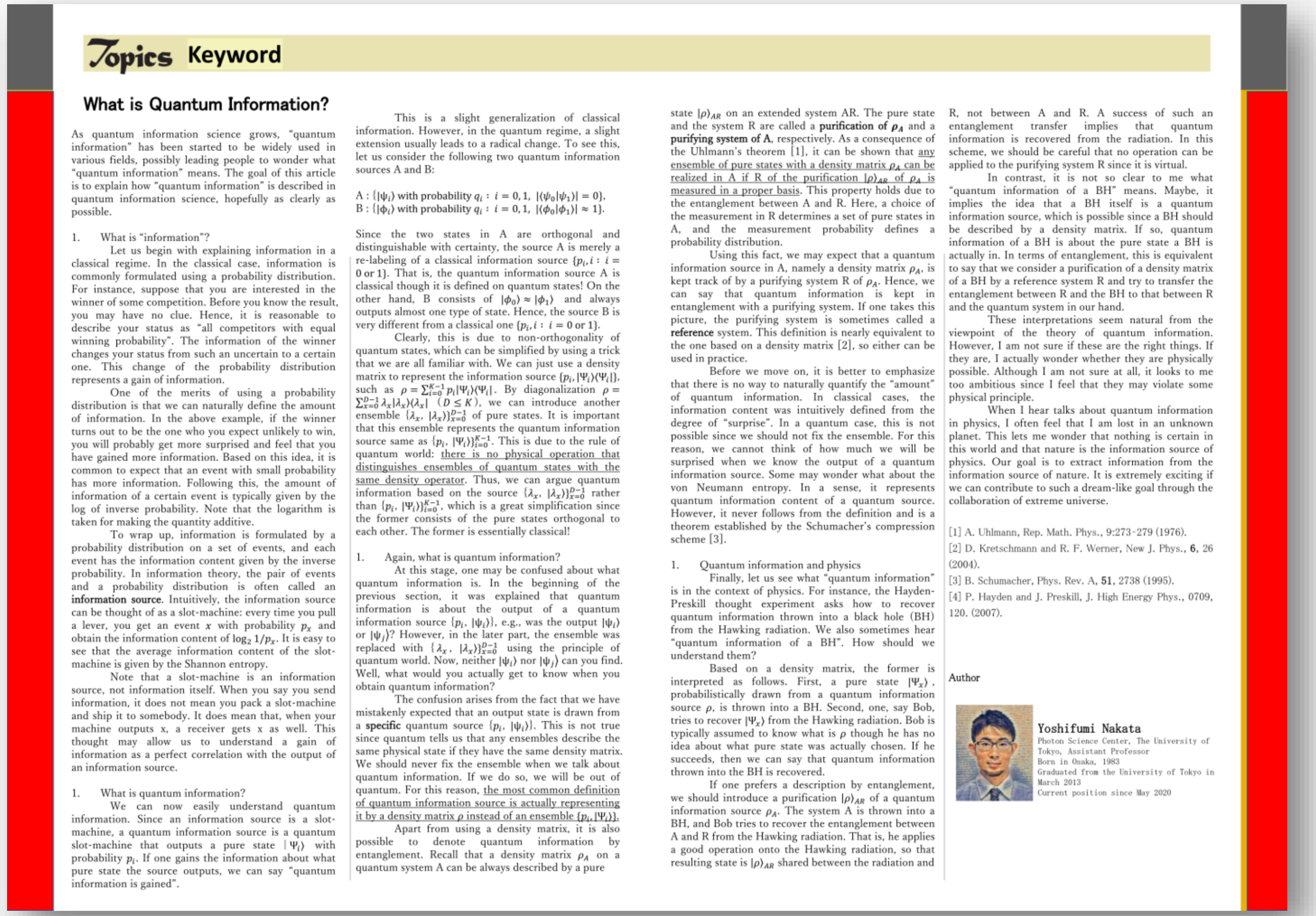
Information-theoretic toy model of quantum black holes

Hayden-Preskill thought experiment 1

To decode the k-qubit quantum info., how large should ℓ be?

1. What is quantum information?
   - See the newsletter of ExU (available at the end of March).





Topics Keyword

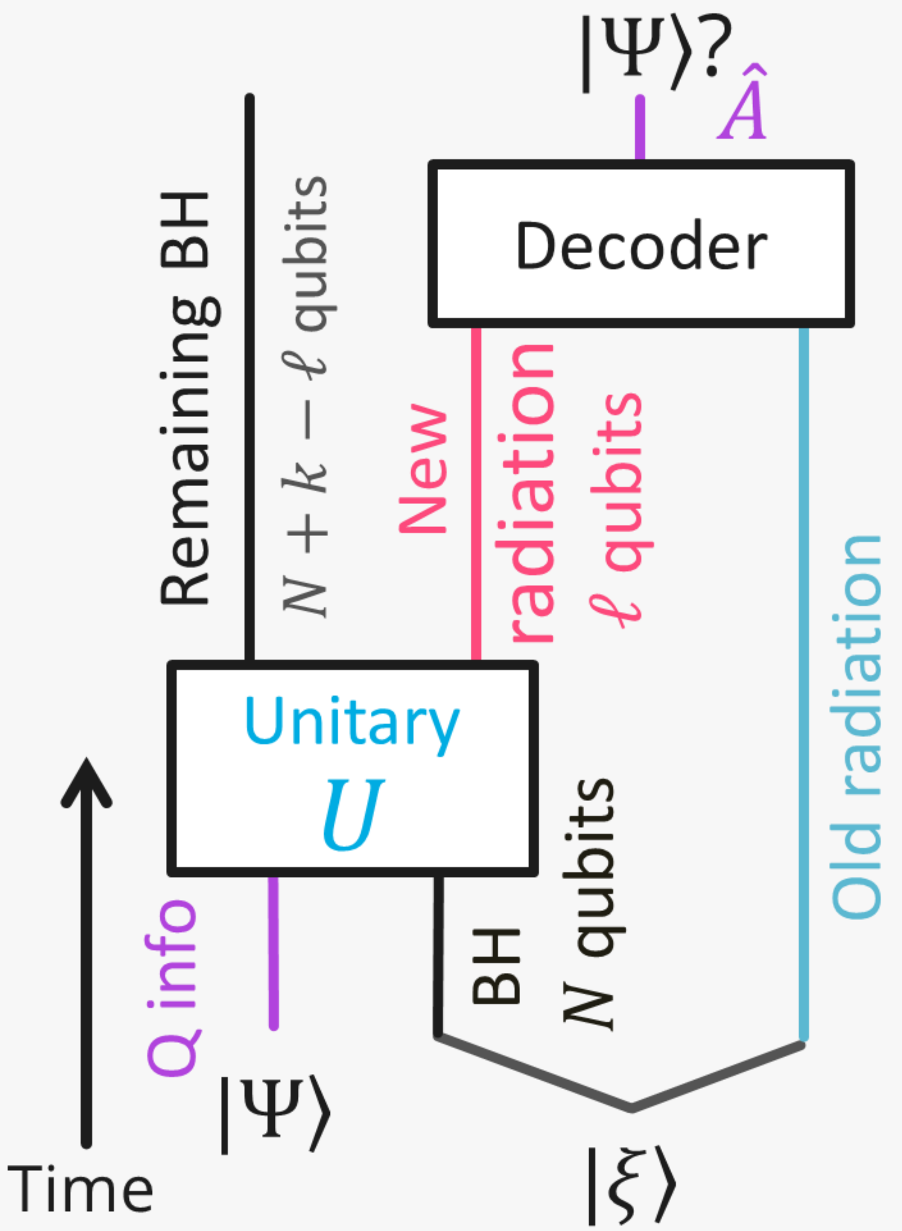
What is Quantum Information?

Author

Yoshifumi Nakata

Information-theoretic toy model of quantum black holes

Hayden-Preskill thought experiment 1



To decode the k-qubit quantum info., how large should ℓ be?

1. What is quantum information?
   - See the newsletter of ExU (available at the end of March).

   Recovery error: $\Delta(\ell|\xi, U)$ $\quad (0 \leq \Delta(\ell|\xi, U) \leq 1)$

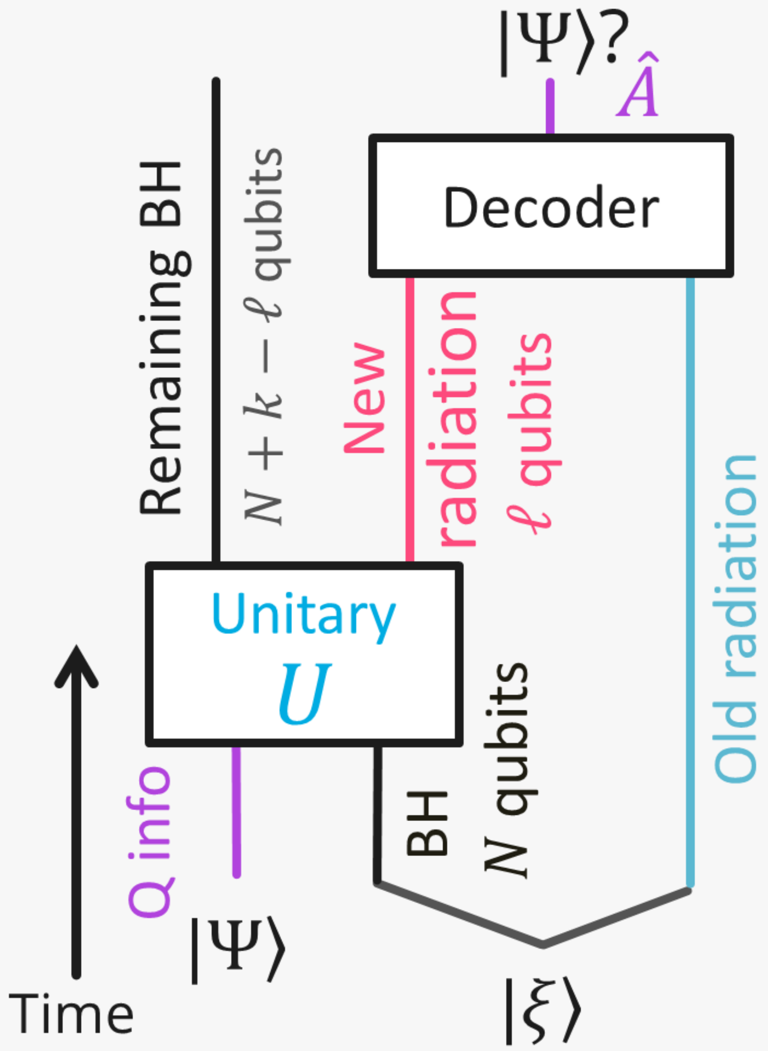
   How does this scale with ℓ?

2. How do we model the unitary dynamics U of the BH?
   - The unitary U is assumed to be fully **chaotic** (a **Haar random** unitary).

The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

Hayden-Preskill thought experiment 2



If the dynamic of the BH is chaotic (Haar random), [Hayden & Preskill, JHEP, '07]

$$\mathbb{E}_{U_{\mathrm{Haar}}}[\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\mathrm{th}}-\ell)}$$

Here, $\ell_{\mathrm{th}} = k + \frac{N - H_2(\xi)}{2}$, and $H_2(\xi) = -\log \mathrm{Tr}[\xi^2]$ is the collision entropy of the initial black hole B_{in}.

- If $\ell \gg \ell_{\mathrm{th}} = k + \frac{N - H_2(\xi)}{2}$, k-qubit quantum information is recoverable.
  - Entropy of the initial BH determines the threshold ℓ_{th}.

The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

Hayden-Preskill thought experiment 3

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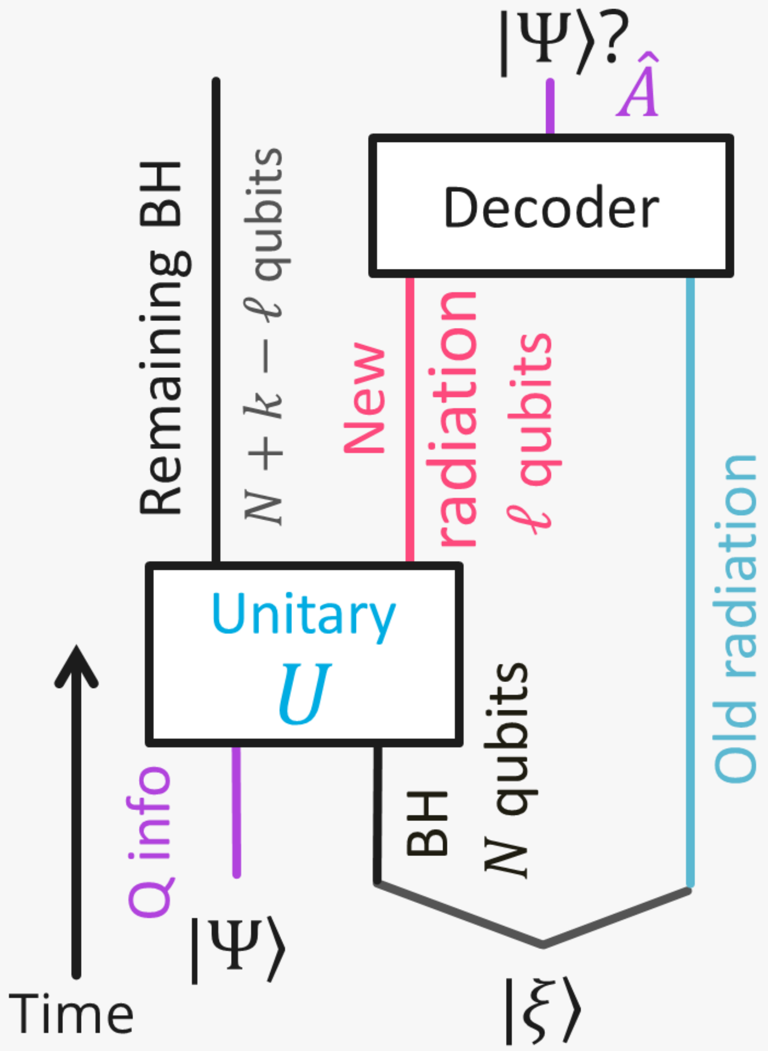


- If $\ell \gg \ell_{\mathrm{th}} = k + \frac{N - H_2(\xi)}{2}$, k-qubit quantum information is recoverable.
  - Entropy of the initial BH determines the threshold ℓ_{th}.
  - If the BH is initially pure ($T = 0$), $H_2(\xi) = 0$ and $\ell_{\mathrm{th}} = k + N/2$.
  - If the BH is initially completely mixed ($T = \infty$), $H_2(\xi) = N$ and $\ell_{\mathrm{th}} = k$.

Independent of N

➡ The k-qubit info is recoverable when $O(k)$ qubits are radiated.

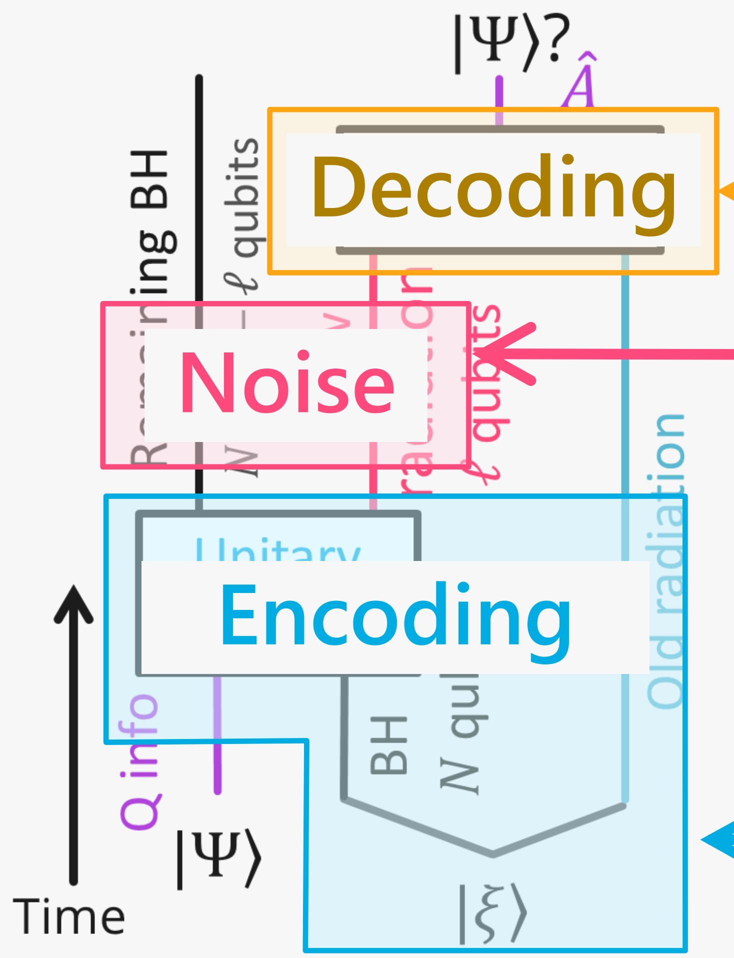
The Hayden-Preskill recovery

The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

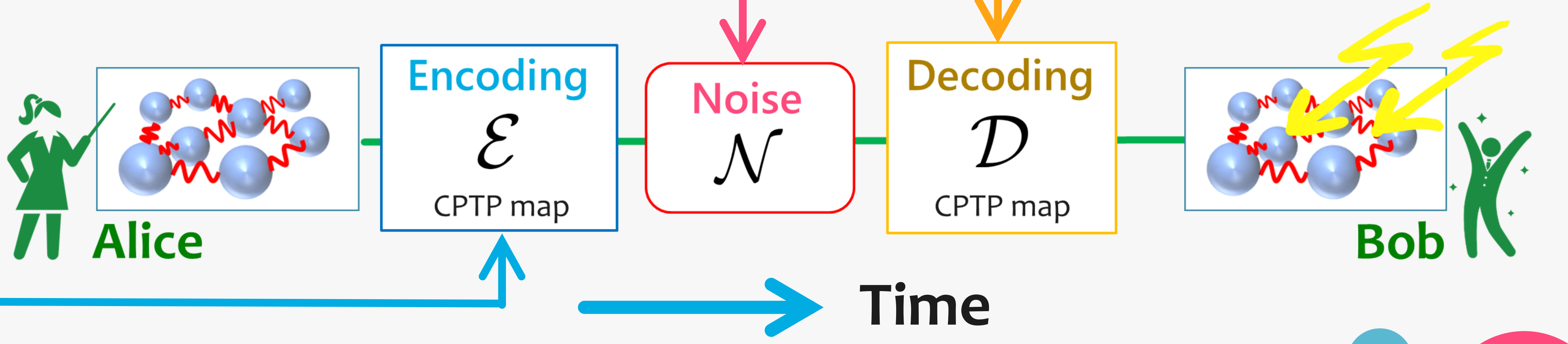
Hayden-Preskill & Quantum Error Correction

- The Hayden-Preskill recovery is a special case of **QEC** by chaotic dynamics.
  - **Noise** = Partial trace over the remaining BH.
  - **Encoding** = Adding a state $|\xi\rangle$ and applying a chaotic (Haar random) unitary U.



Hayden-Preskill

We "lost" the remaining BH.
Noise = Partial trace



QEC in a nutshell

The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

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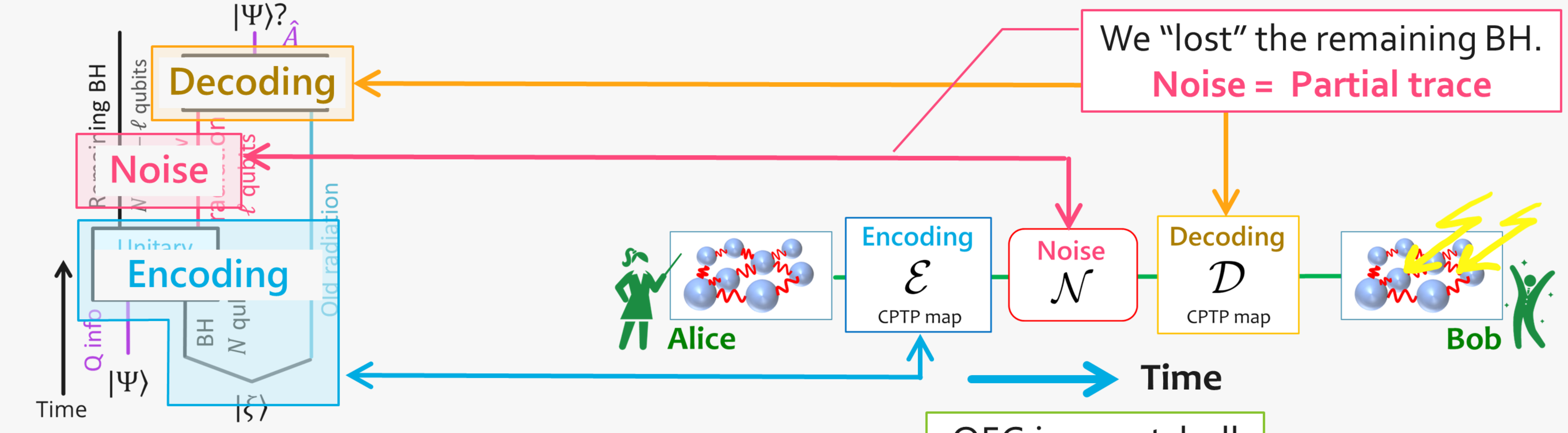
The Hayden-Preskill thought experiment is a good playground over
quantum error correction, quantum chaos, and a quantum black hole!!



Hayden-Preskill

QEC in a nutshell

Outline of this talk

A journey of a thousand miles begins with a single step

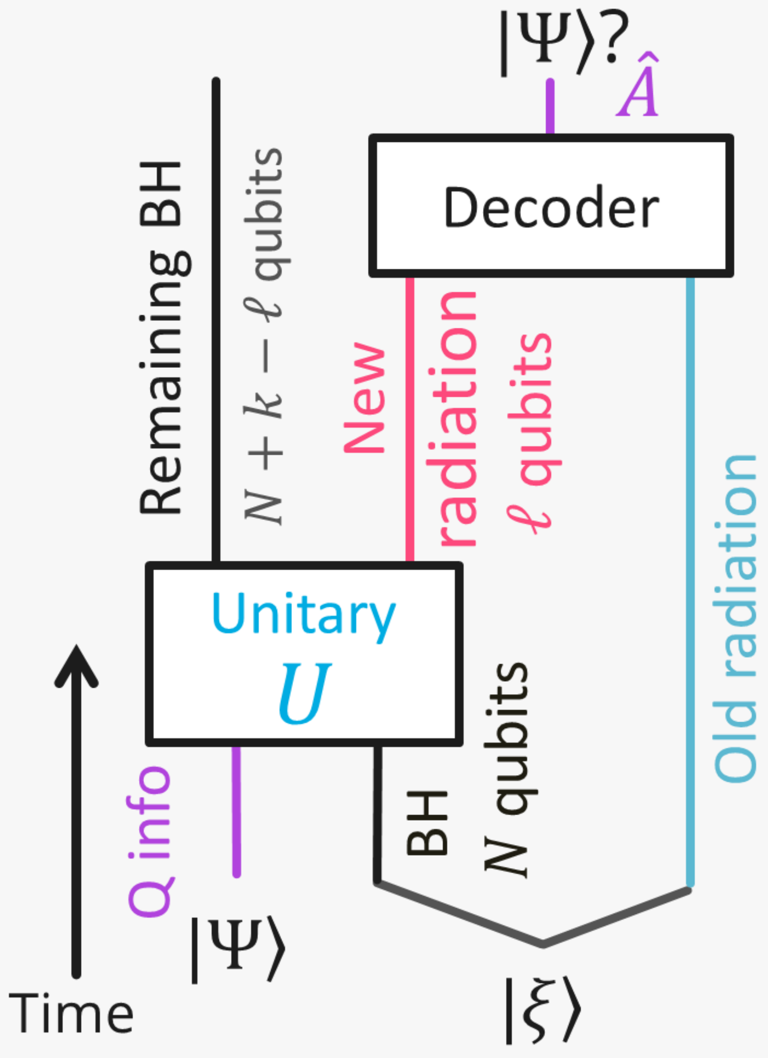
Information, Chaos, and Black Holes in the Quantum regime.

3. Beyond the Hayden-Preskill.

Beyond the Hayden-Preskill

Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, **and a quantum black hole!!**



Beyond the Hayden-Preskill

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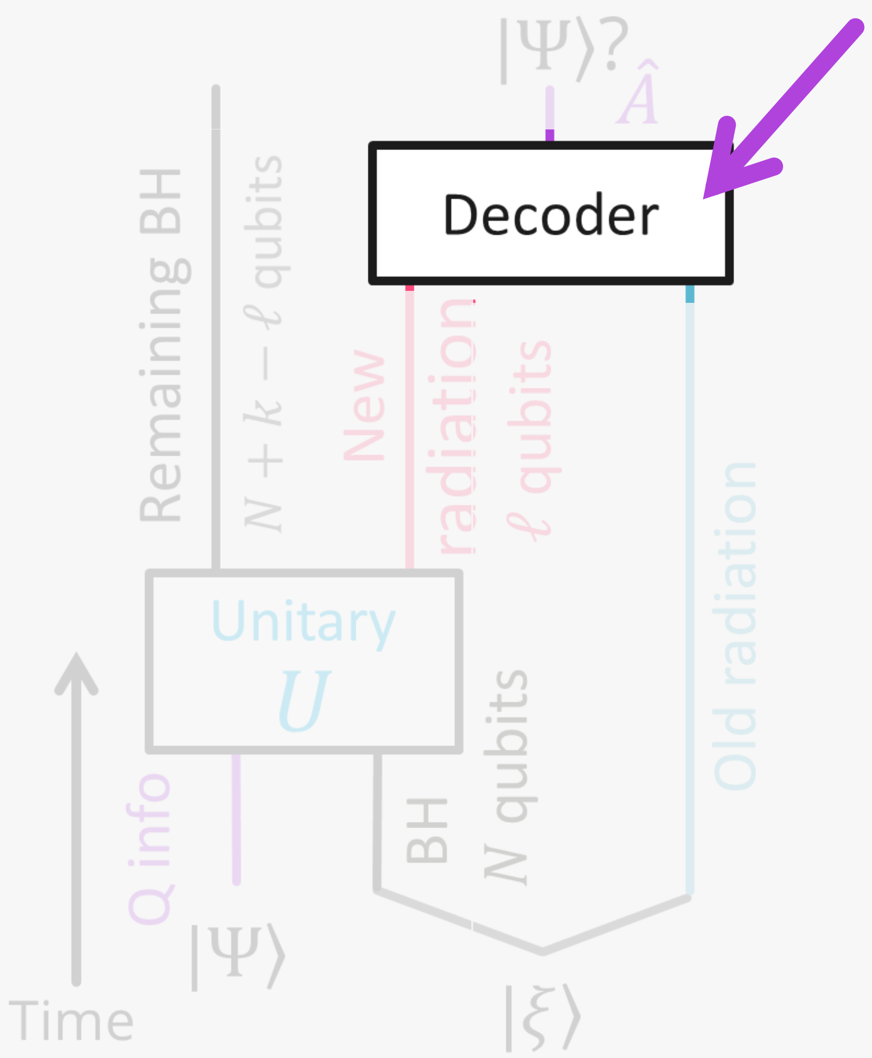


1. **No decoder is given.**
   - ➢ **How can we actually decode information?**

[YN & Koashi, in prep]

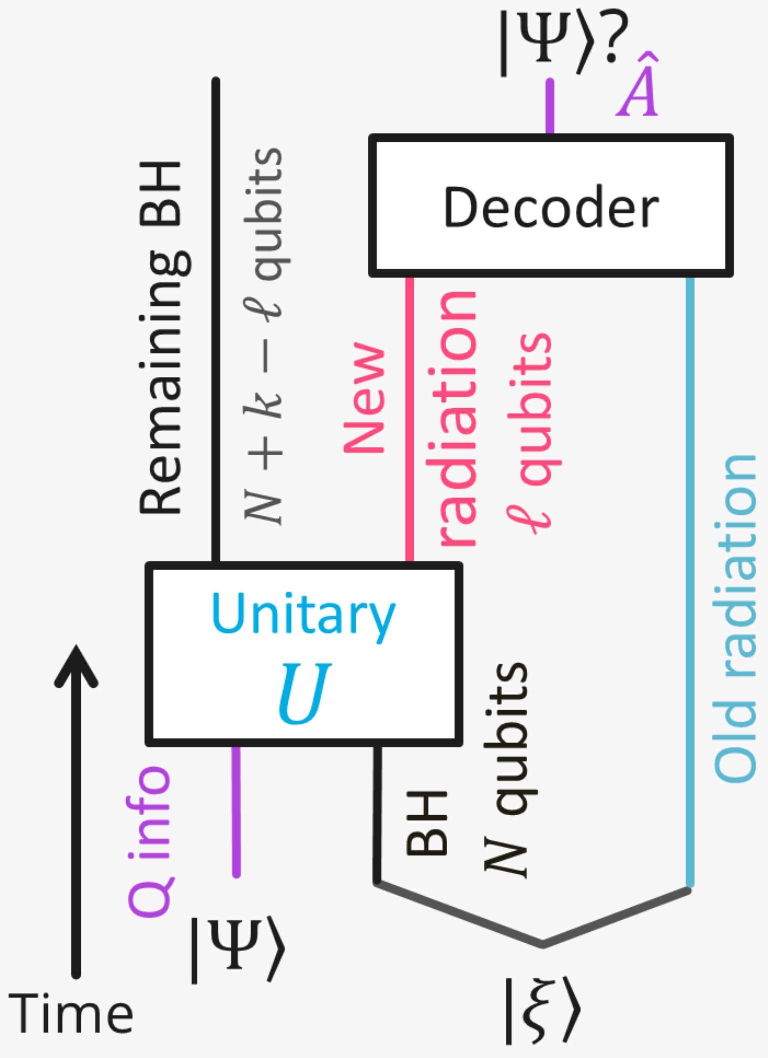
Decoding the Hayden-Preskill

Beyond the Hayden-Preskill 1

How do we decode information?

- In the original work by Hayden&Preskill, no decoder was provided.
  - How can we explicitly construct a decoder?

- Two decoders in the literature, and their relations to physics.
  - Teleportation-type decoder for special cases, related to OTOC [Yoshida & Kitaev '17].
  - Petz decoders, related to spacetime geometry [Penington, et al'19].



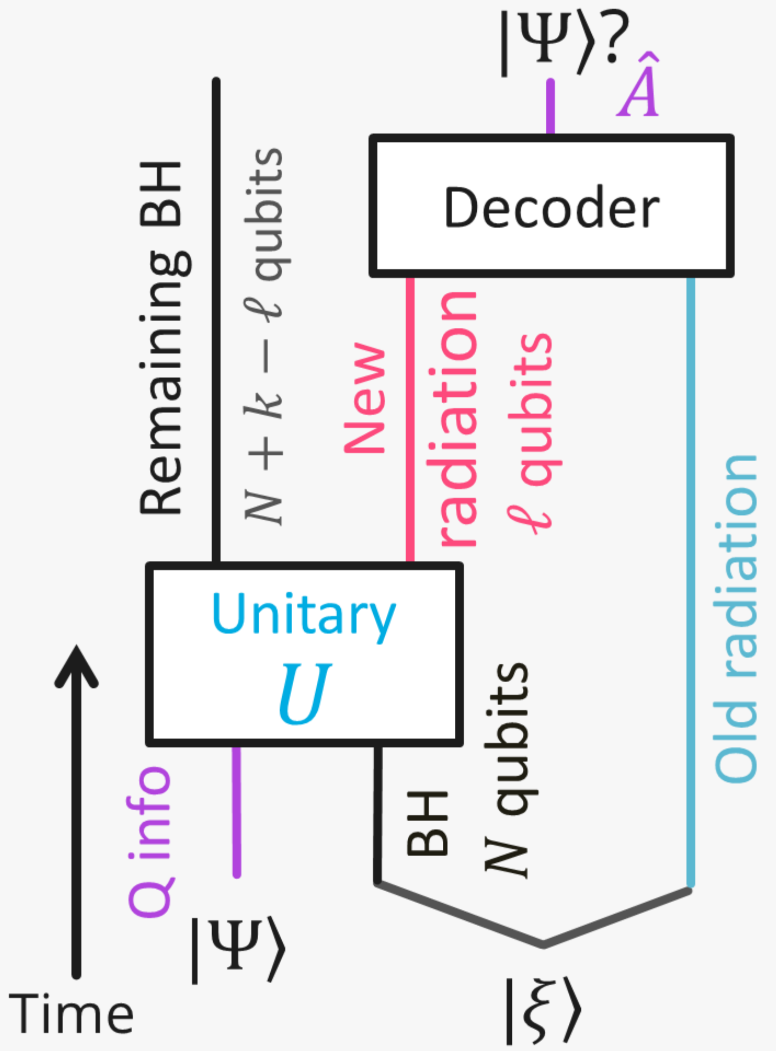
A decoder is a key link to bridge quantum information to physics.

Decoding the Hayden-Preskill

Beyond the Hayden-Preskill 1

New results by the Classical-to-Quantum decoder

- We propose an intuitive decoder, called a **Classical-to-Quantum decoder**.
  - It is universal since it works for general noise.
  - It is related to the Renyi-2 entropy.
- A chaotic BH **simultaneously** emits classical and quantum information.
  - There is **NO difference b/t C and Q info**" in the **QEC by chaotic dynamics**.



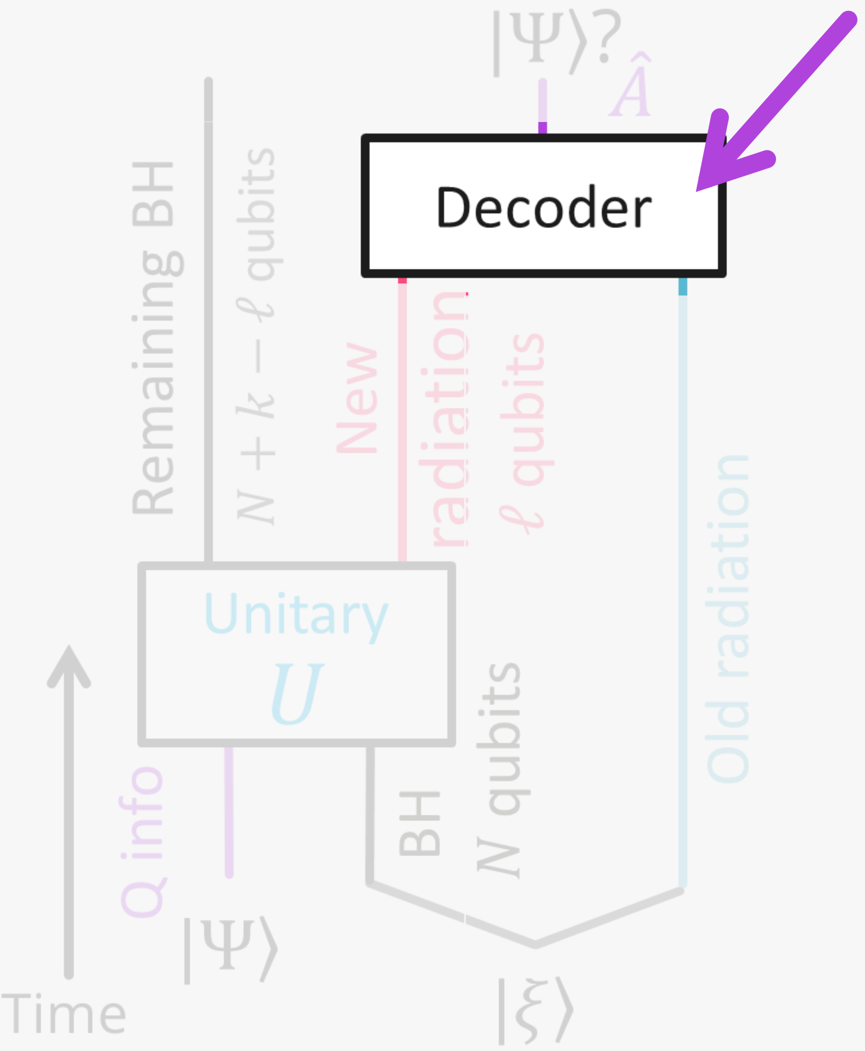
	Recovery error for quantum information	Recovery error for classical information
Hayden & Preskill '07	$\leq 2^{\frac{1}{2}(\ell_{\mathrm{th}}-\ell)}$	N.A.
Our results	$\leq 2^{\ell_{\mathrm{th}}-\ell}$	$\leq 4^{\ell_{\mathrm{th}}-\ell}$

Here, $\ell_{\mathrm{th}} = k + \dfrac{N - H_2(\xi)}{2}$

Beyond the Hayden-Preskill

Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, **and a quantum black hole!!**



1. **No decoder is given.**
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A decoder gives a "link" b/t quantum information and physics.

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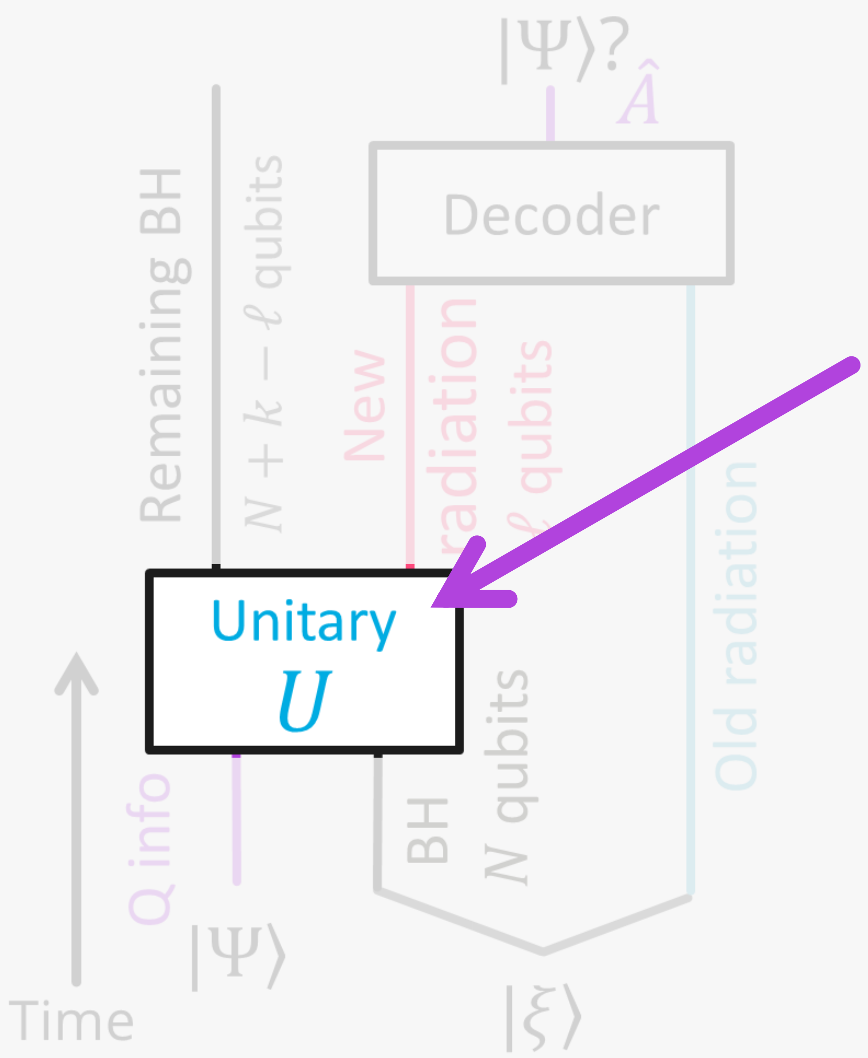


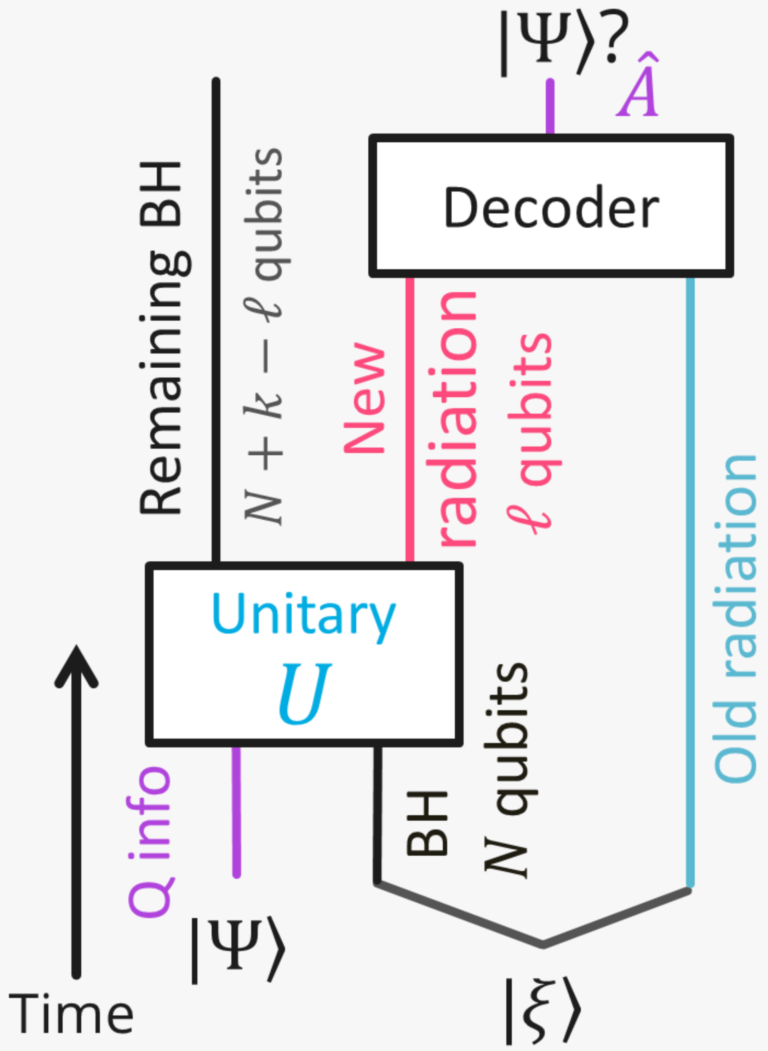
1. **No decoder is given.**
   - **How can we actually decode information?** [YN & Koashi, in prep]

   A decoder gives a "link" b/t quantum information and physics.

2. **No energy conservation** due to the Haar assumption.
   - **What if something is conserved?** [YN, Wakakuwa & Koashi, '19&'20]

The HP with energy conservation

Haar random dynamics "scrambles" everything



- Hayden & Preskill assume a fully chaotic dynamics = a Haar random unitary.

$$\mathbb{E}_{U_{\mathrm{Haar}}}[\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\mathrm{th}}-\ell)}$$

  - Such a unitary U_{Haar} scrambles everything = **nothing (e.g. energy) is conserved**.

- What if a conservation law is taken into account?
  - Energy conservation (in some sense) was heuristically discussed in [Yoshida, PRD100:086001 '19] and [Liu, PRR 2:043164, '20].
  - Comprehensive analysis was done in [YN, Wakakuwa, and Koashi,'19 &'20].

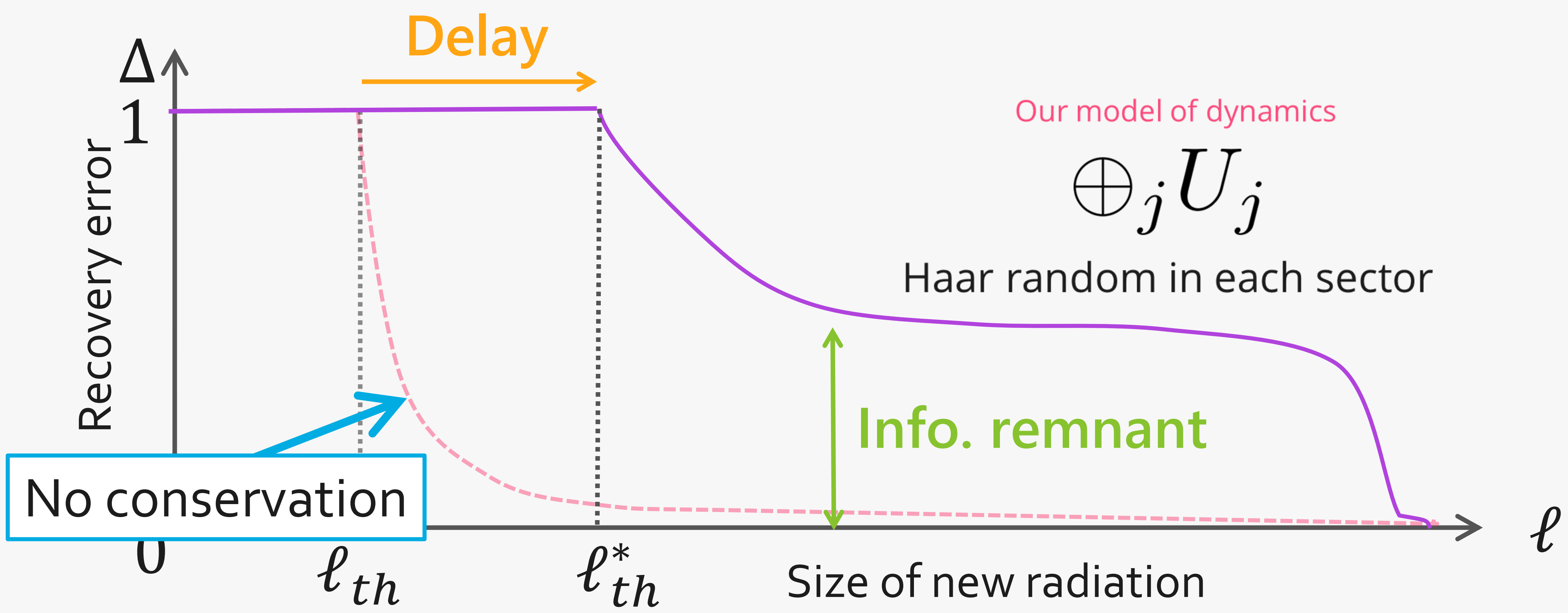
Hayden & Preskill		Our model of dynamics	
U	→	$\bigoplus_j U_j$	j: conserved quantity
Haar random		Haar random in each sector	

The HP with energy conservation

Beyond the Hayden-Preskill 2

What changes would occur in the HP recovery? 1

- Two big changes by symmetry.
  1. **Delay** of the threshold: the threshold ℓ_{th} changes to ℓ^*_{th}.
  2. **Information remnant**: some information cannot be recovered from the radiation.



[Hayden & Preskill, '07]

$$\mathbb{E}_{U_{\rm Haar}}[\Delta(\ell|\xi,U)] \leq 2^{\frac{1}{2}(\ell_{\rm th}-\ell)}$$

$$\mathbb{E}_U[\Delta] \lesssim 2^{\frac{1}{2}(\ell^*_{\rm th}-\ell)} + \Delta_{rem}$$

A formal expression is given by entropic quantities

The HP with energy conservation

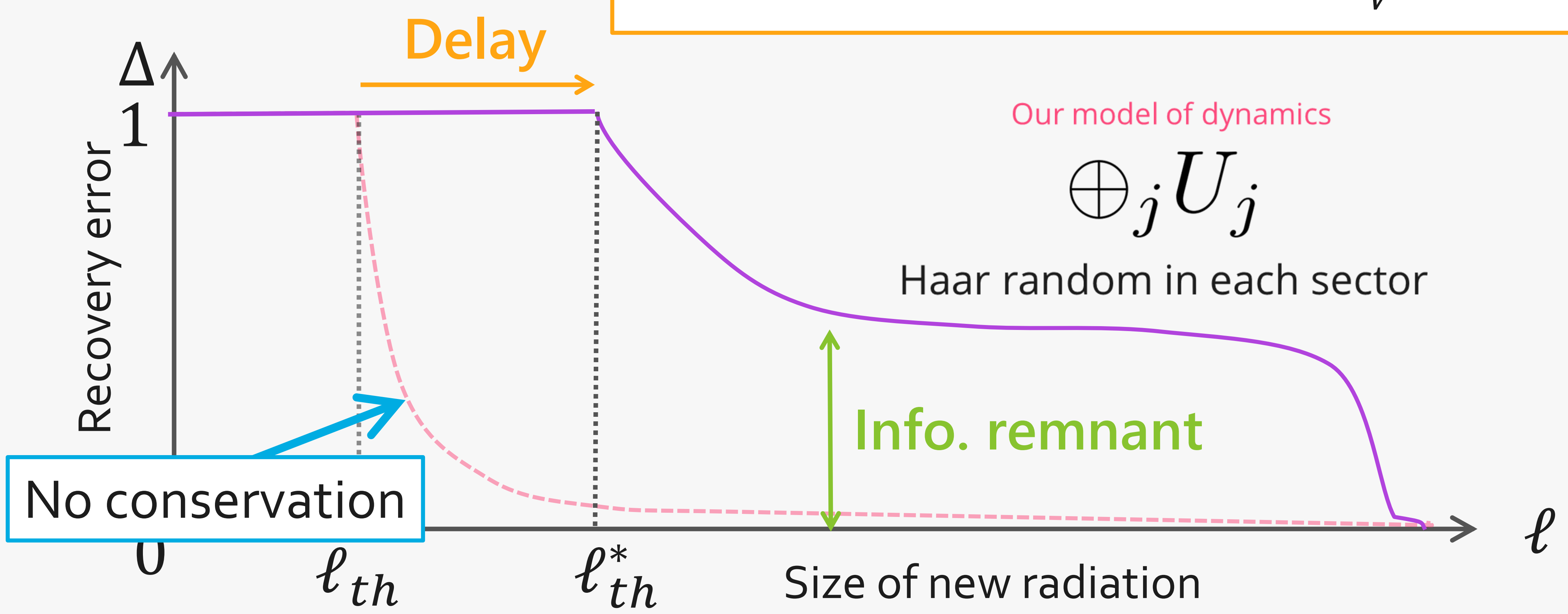
Beyond the Hayden-Preskill 2

What changes would occur in the HP recovery? 2

- Two big changes by symmetry.
  1. **Delay** of the threshold: the threshold ℓ_{th} changes to ℓ^*_{th}.
     - In the case of energy conservation, it is characterized by the head capacity of the BH.

$$\text{delay} \propto \frac{E}{S(E)}\sqrt{|C_V|}$$

- E: Energy of the BH
- $S(E)$: Entropy of the initial BH
- C_V: Heat capacity of the BH



Our model of dynamics

$$\bigoplus_j U_j$$

Haar random in each sector

$$\mathbb{E}_{U_{\text{Haar}}}[\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\text{th}} - \ell)}$$

$$\mathbb{E}_U[\Delta] \lesssim 2^{\frac{1}{2}(\ell^*_{\text{th}} - \ell)} + \Delta_{rem}$$

A formal expression is given by entropic quantities

The HP with energy conservation

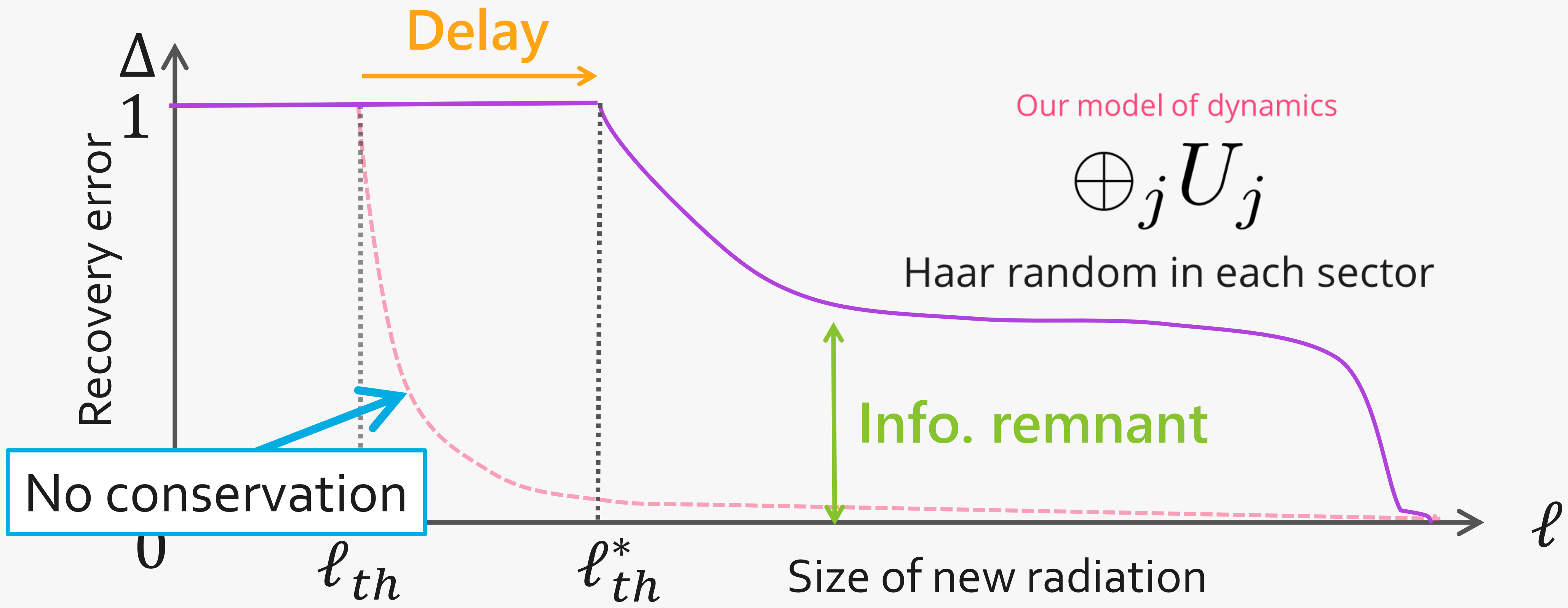
Beyond the Hayden-Preskill 2

What changes would occur in the HP recovery? 3

- Two big changes by symmetry.
  1. **Delay** of the threshold: the threshold ℓ_{th} changes to ℓ_{th}^*.
  2. **Information remnant**: a certain amount of information cannot be recovered from the radiation.
     - Amount of the remnant ∝ (symmetry breaking of the initial BH)$^{-1}$ ≈ Variance of energy

$$\text{delay} \propto \frac{E}{S(E)}\sqrt{|C_V|}$$

Interesting relations between information and physics



$$\mathbb{E}_{U_{\mathrm{Haar}}}[\Delta(\ell|\xi,U)] \leq 2^{\frac{1}{2}(\ell_{\mathrm{th}}-\ell)}$$

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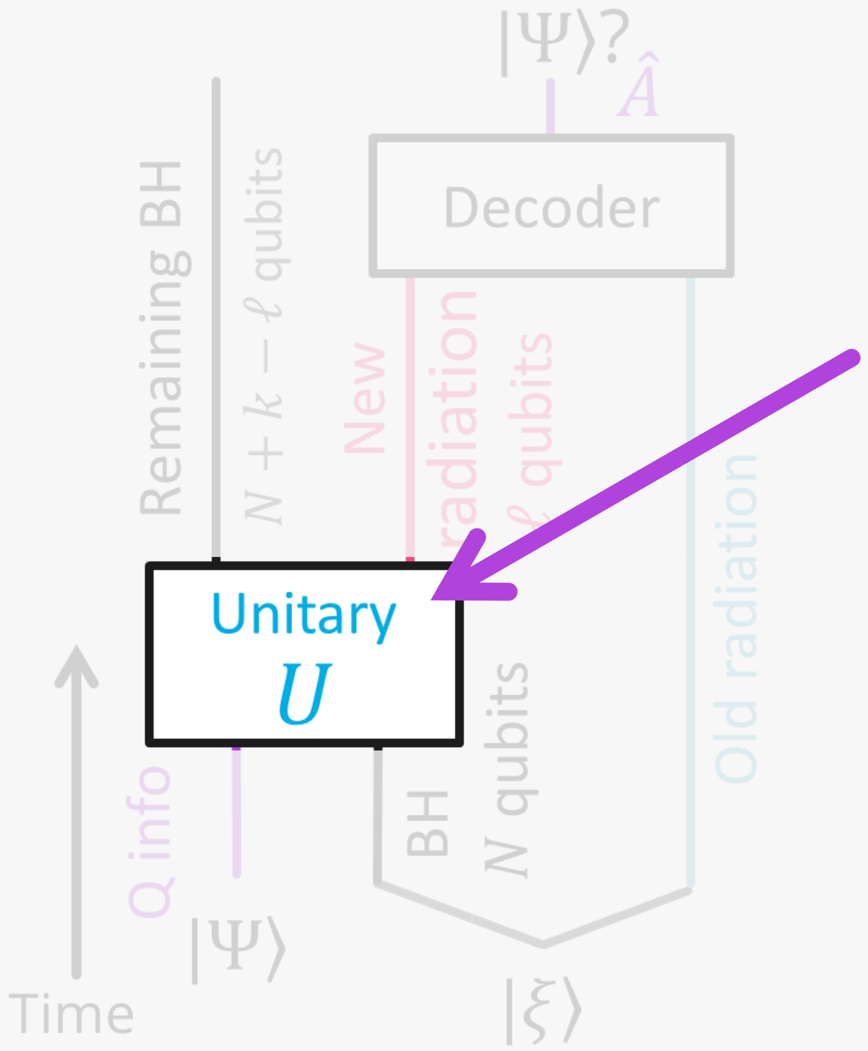
A formal expression is given by entropic quantities

Beyond the Hayden-Preskill

Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over
quantum error correction, **quantum chaos**, **and a quantum black hole!!**



1. **No decoder is given.**
   - **How can we actually decode information?** [YN & Koashi, in prep]

     A decoder gives a "link" b/t quantum information and physics.
2. **No energy conservation** due to the Haar assumption.
   - **What if something is conserved?** [YN, Wakakuwa & Koashi, '19&'20]

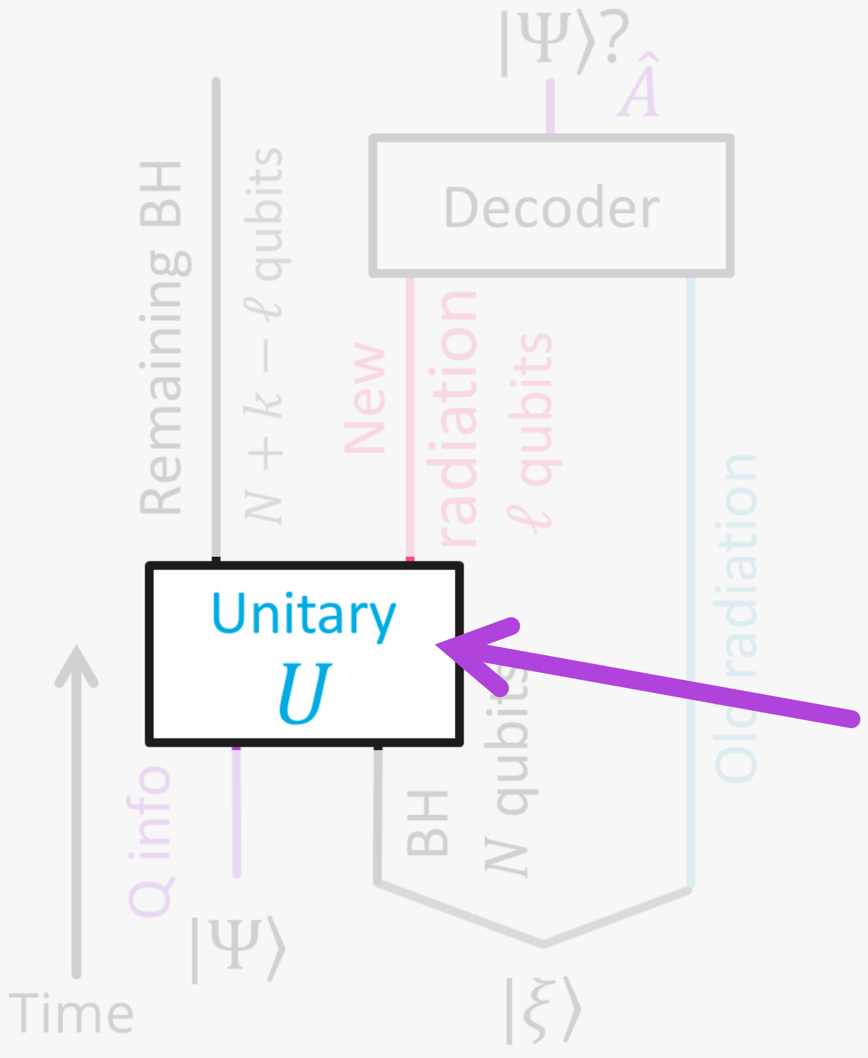
     Non-trivial "links" b/t quantum information and physics are revealed.

Beyond the Hayden-Preskill

Beyond the Hayden-Preskill 1

Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, **and a quantum black hole!!**



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     Non-trivial "links" b/t quantum information and physics are revealed.
3. **No Hamiltonian (or Lagrangian)**.
   - **What if the dynamics is given by a Hamiltonian dynamics?**

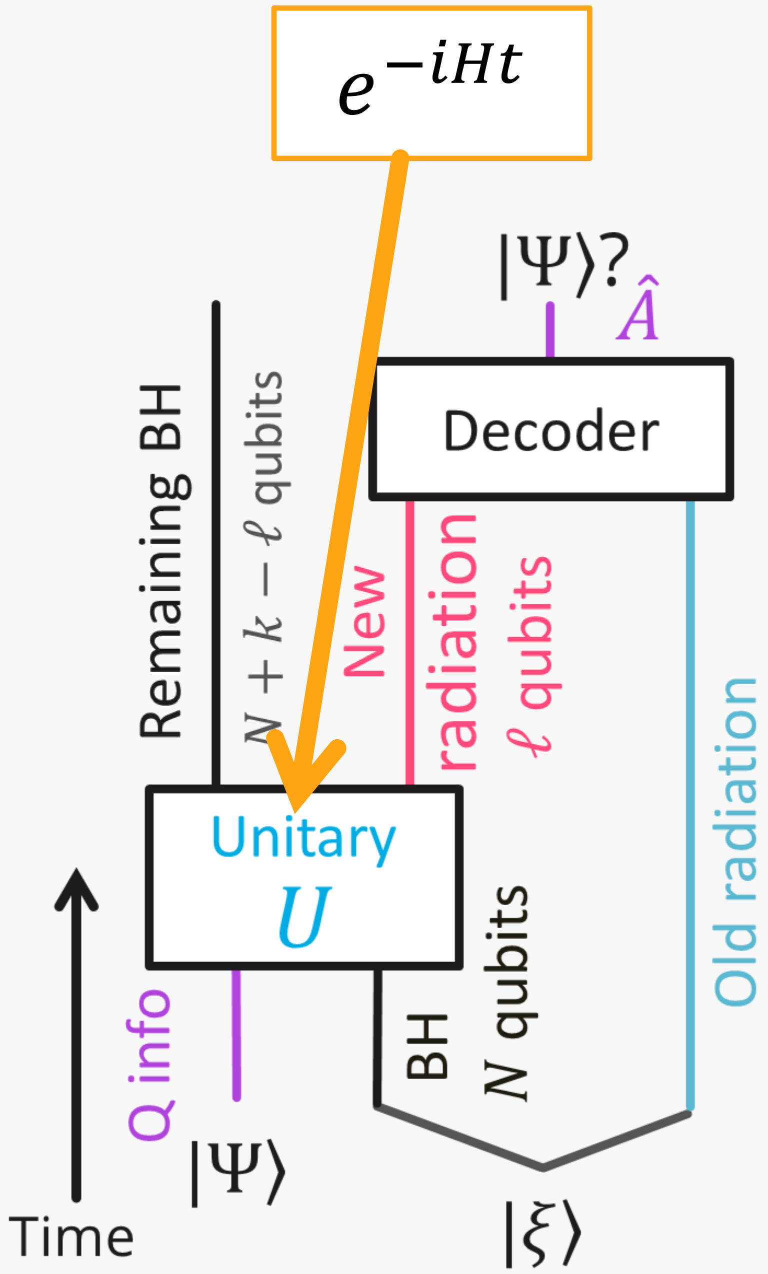
[YN & Tezuka, in prep]

The Hayden-Preskill recovery in SYK model

Beyond the Hayden-Preskill 3

Hayden-Preskill recovery by a Hamiltonian dynam

STAY TUNED



- **No Hamiltonian (or Lagrangian) s**
  - The analysis is basically base
  - What if the dynamics is given $= e^{-iHt}$?

This is a non-trivial question.

- A Haar random unitary cannot be generated by a time-indep. Hamiltonian.
  - No formal proof (to my knowledge), but highly believed.

In collaboration with M. Tezuka

......however, who knows the Hamiltonian of a black hole?

Sachedev-Ye-Kitaev (SYK) model as a quantum dual!

Outline of this talk

A journey of a thousand miles begins with a single step

Information, Chaos, and Black Holes in the Quantum regime.

1. Information and Chaos in the classical regime.
2. The Hayden-Preskill thought experiment.
3. Beyond the Hayden-Preskill.
4. **Conclusion and Outlooks.**

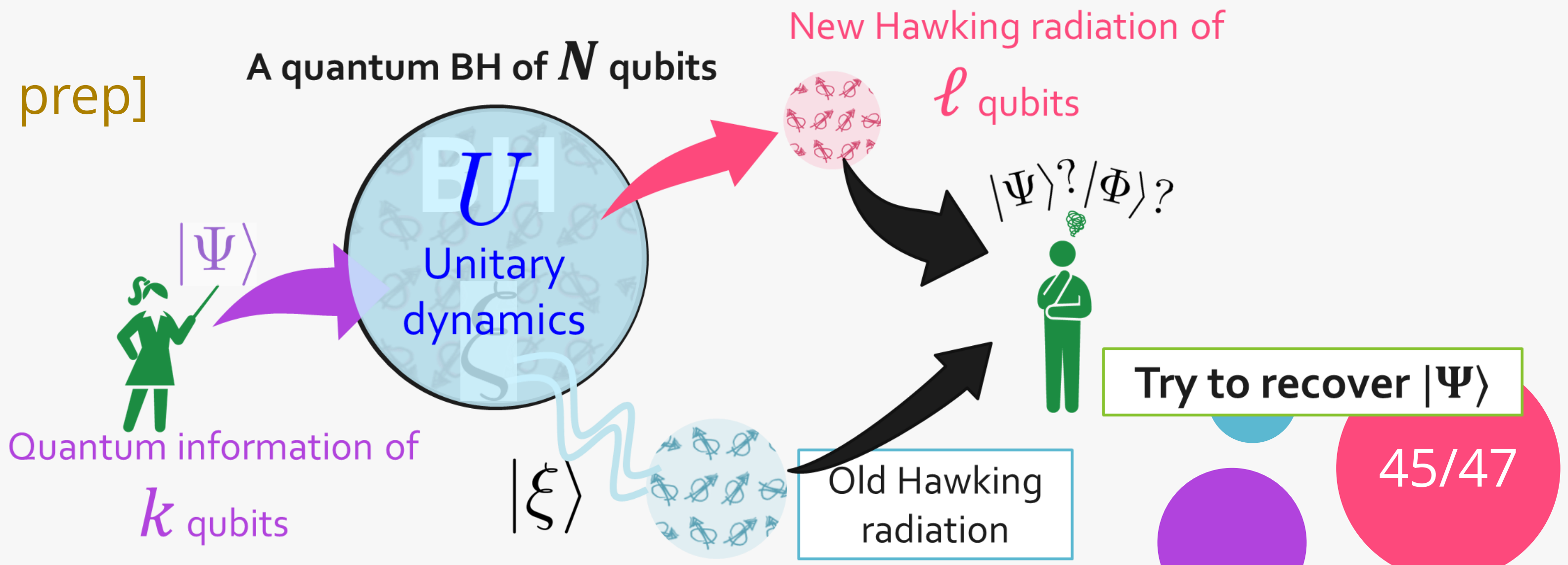
Conclusion and outlooks

Towards the collaboration over QI, condensed-matter, and high energy physics

Conclusion

Error Correction, Chaos, and Black Holes in the Quantum regime.

- Chaotic dynamics (a Haar random unitary) is useful for correcting quantum noise.
- The Hayden-Preskill thought experiment is a canonical toy model about quantum error correction, quantum chaos, and a quantum black hole.
  1. Decoding the Hayden-Preskill [YN & Koashi, in prep]
  2. Energy conservation and the Hayden-Preskill recovery [YN, Wakakuwa & Koashi, '19&'20]
  3. The Hayden-Preskill recovery and SYK models [YN & Tezuka, in prep]



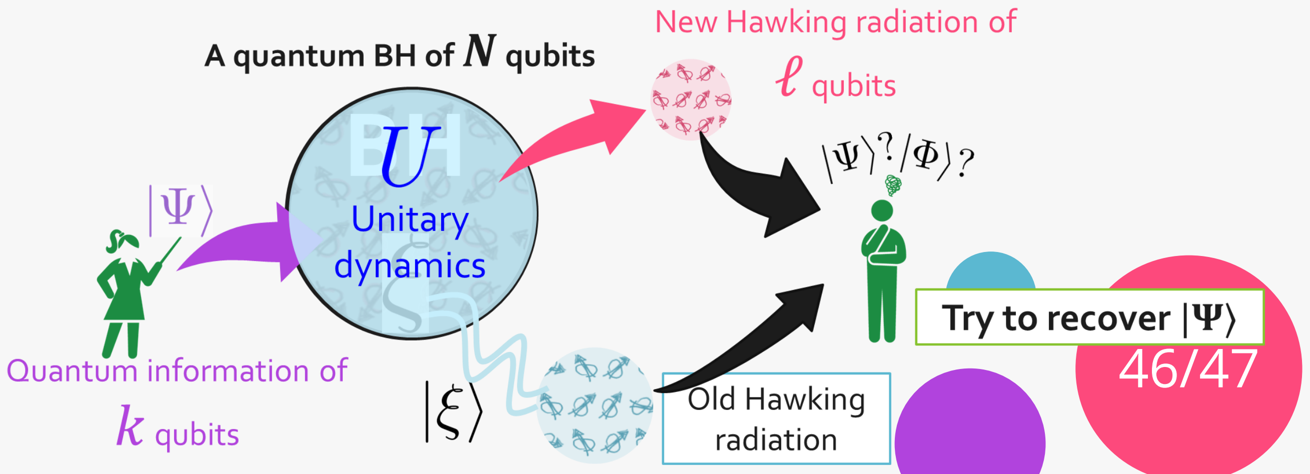
Conclusion and outlooks

Towards the collaboration over QI, condensed-matter, and high energy physics

Open questions

Error Correction, Chaos, and Black Holes in the Quantum regime.

1. The Hayden-Preskill thought experiment has a little flavor of black holes….
   - It's about quantum error correction, quantum chaos, and a quantum black hole.
   - More flavor of quantum black holes?
2. Quantum phase vs the Hayden-Preskill recovery?
   - The HP recovery may succeed/fail in chaotic, MBL, and integrable phases.
   - Characterize quantum phases by the recovery?



Thank you

…

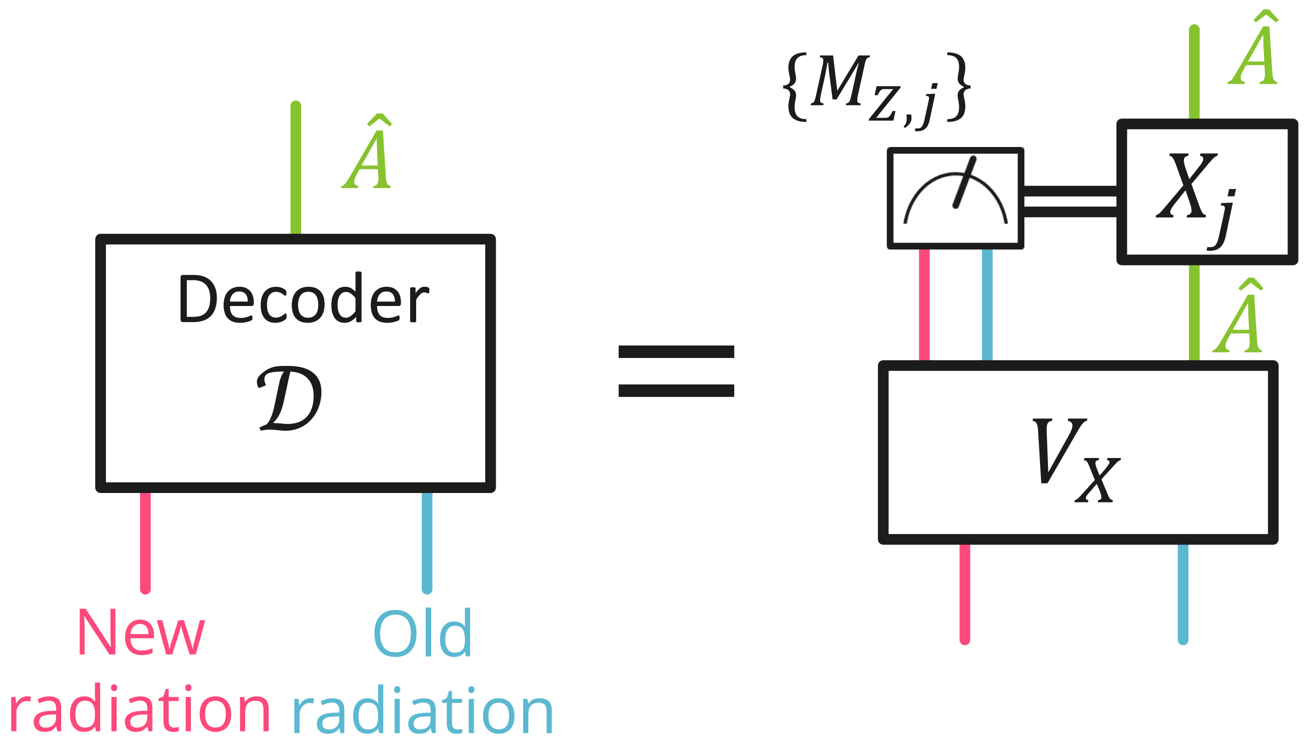
for your attention

Classical-to-Quantum decoder

Decoding Hayden-Preskill

Classical-to-Quantum decoder 1

Classical-to-Quantum decoder



Correcting errors by M_Z

A coherent meas. constructed from M_X

$$V_X := \sum_j \sqrt{M_{X,j}} \otimes |j_X\rangle$$