

ExU collaboration



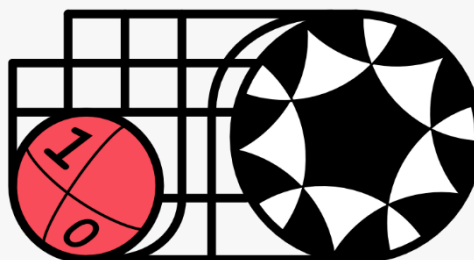
# Linking quantum information to physics

**Yoshifumi Nakata**

The University of Tokyo & JST PRESTO



March 7<sup>th</sup> @ Annual meeting of ExU collaboration



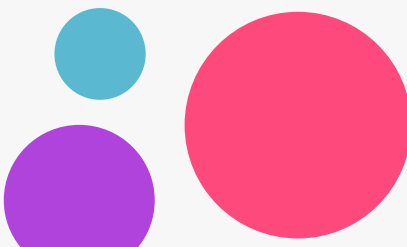


# Outline of this talk

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*A journey of a thousand miles begins with a single step*

Information, Chaos, and Black Holes in the **Quantum** regime.





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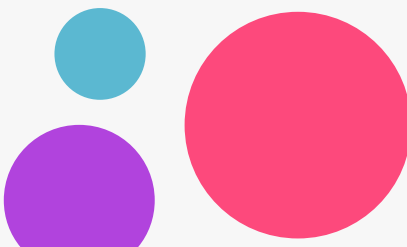
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*A journey of a thousand miles begins with a single step*

Information, Chaos, and Black Holes in the Quantum regime.



1. Information and Chaos in the classical regime.





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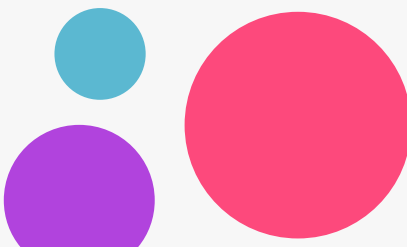
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*A journey of a thousand miles begins with a single step*

Information, Chaos, and Black Holes in the Quantum regime.

1. Information and Chaos in the classical regime.

2. The Hayden-Preskill thought experiment.





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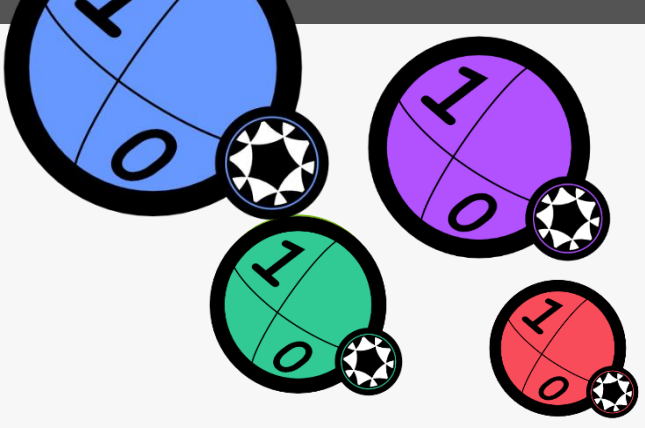
1. Information and Chaos in the classical regime.

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3. Beyond the Hayden-Preskill.

4. Conclusion and Outlooks.

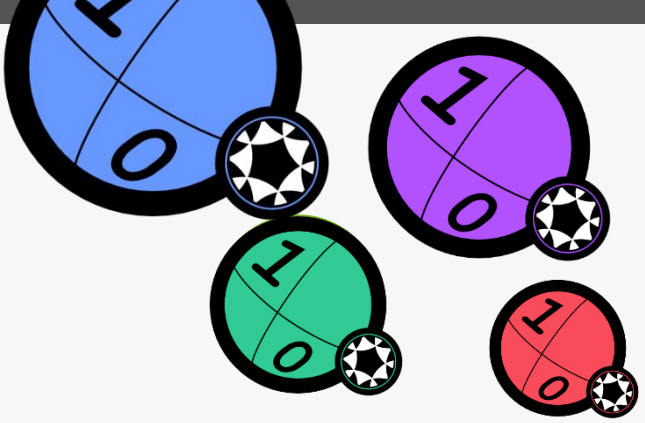




# 1. Information and Chaos in the classical regime.

1. Quantum Error Correction (QEC)
2. Chaotic dynamics and Error Correction in classical
3. Quantum chaos and QEC?





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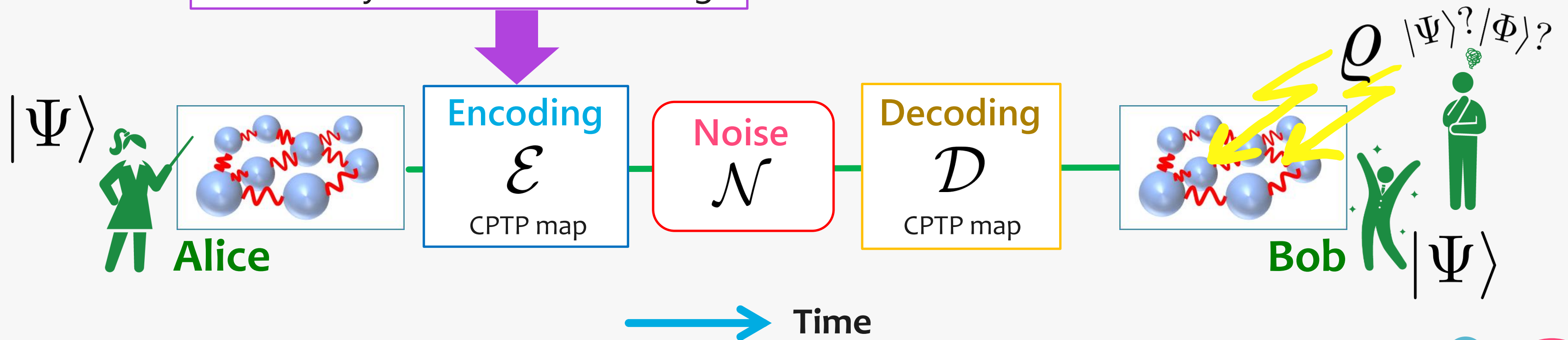
# Quantum Error Correction

*An introduction in a nutshell*

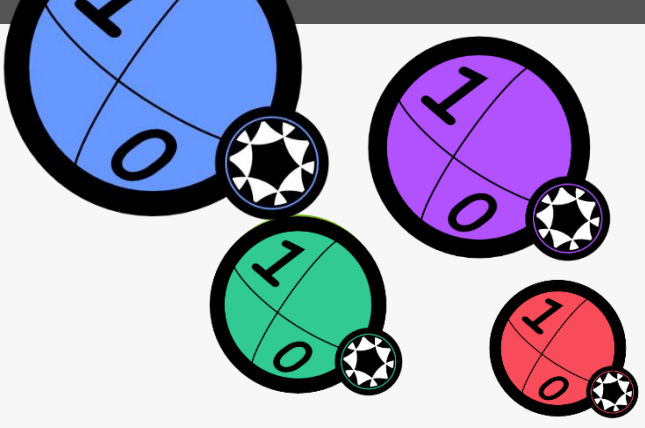
## What is Quantum Error Correction (QEC)?

□ **QEC** is a method to effectively cancel quantum noise by **ENCODING** and **DECODING**.

Today, we consider to use  
chaotic dynamics for encoding!



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$



# 1. Information and Chaos in the classical regime.

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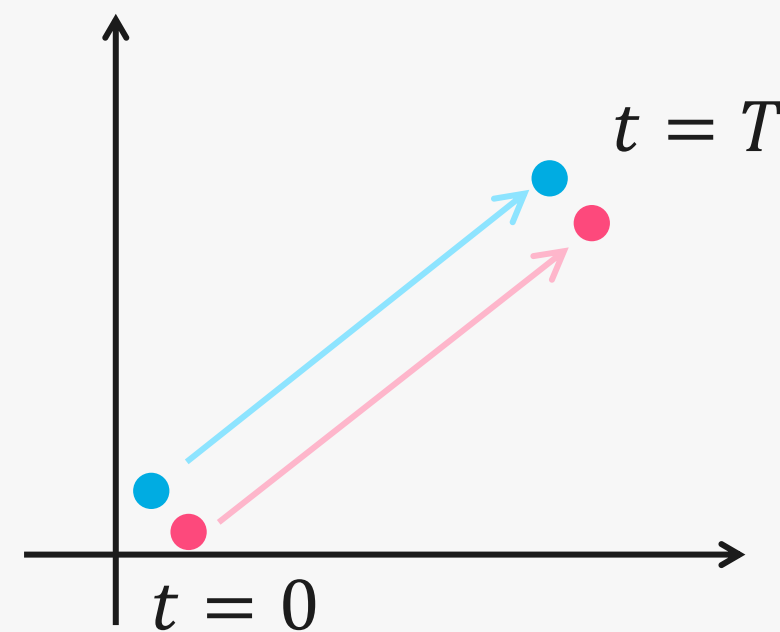


# Chaos and Error Correction in classical

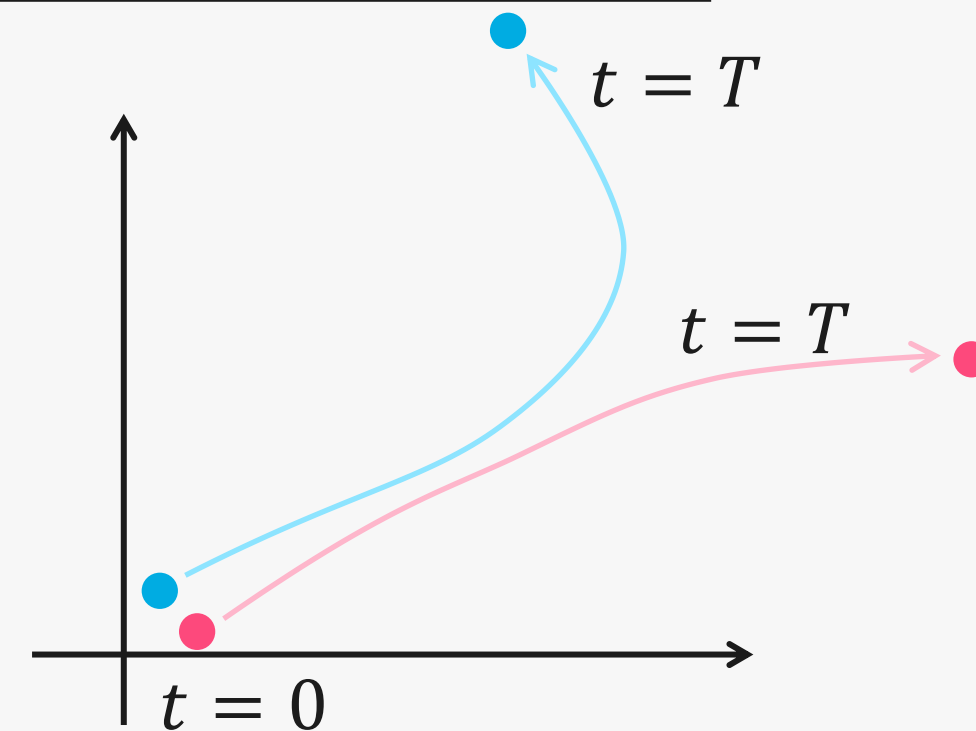
*How can we use chaotic dynamics for correcting errors?*

## What is chaos in the classical regime?

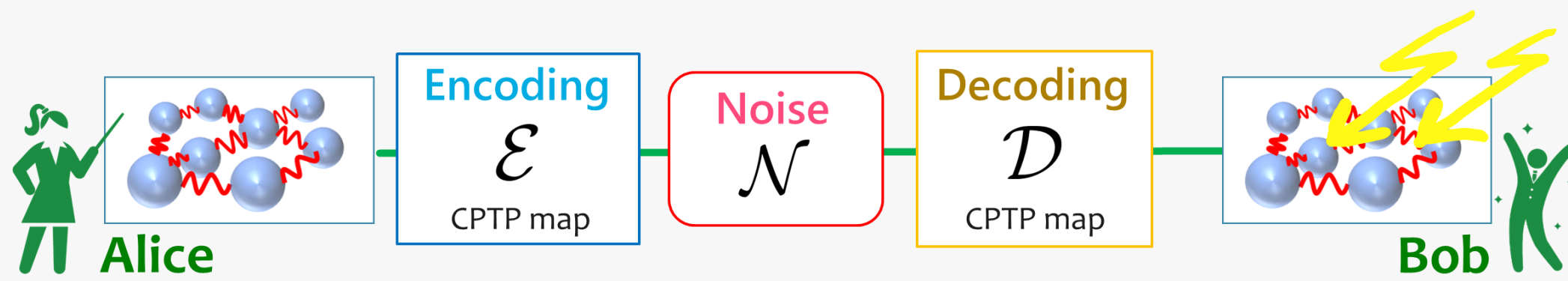
□ Chaos is a dynamical system highly sensitive to initial conditions.



Simple dynamics



Chaotic dynamics



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

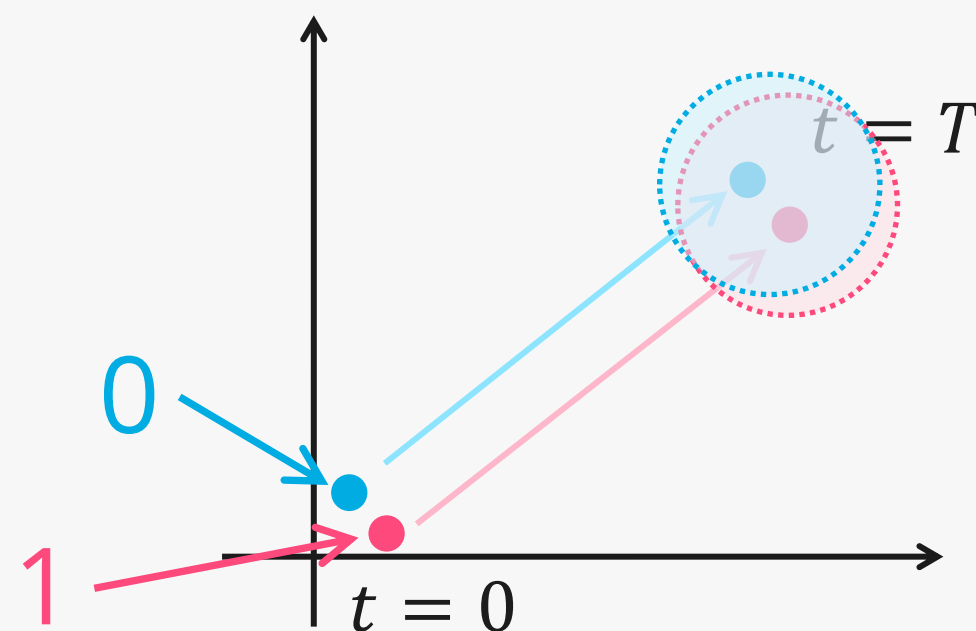
QEC in a nutshell

# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

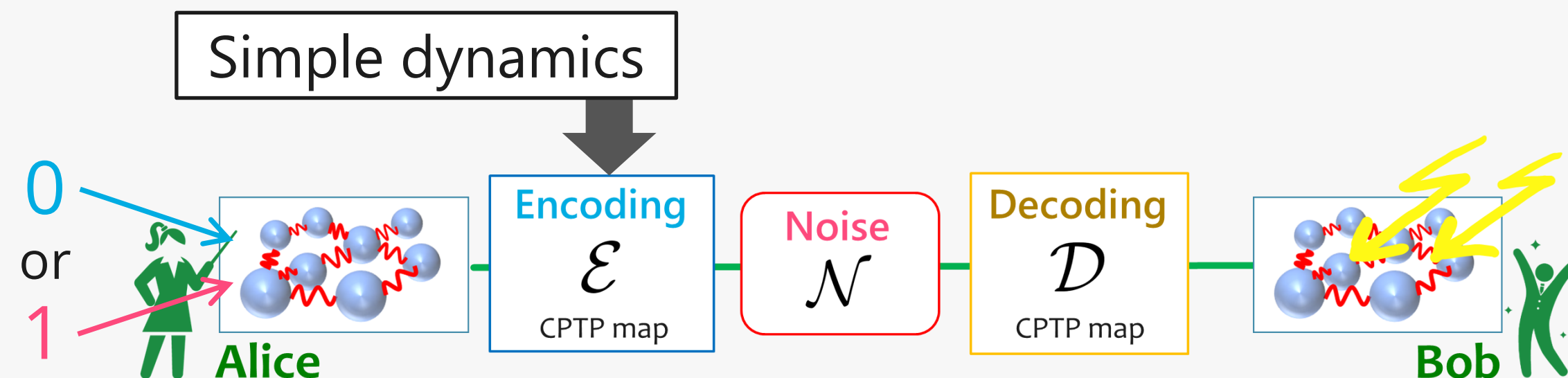
## Chaos and Error Correction: intuition 1

□ **Chaos** is a dynamical system highly sensitive to initial conditions.



1. Write 0 or 1 into the system.
2. "Time evolution" of the information by the **simple** dynamics.
3. Due to the uncertainty induced by **the noise**, the info. gets **unclear**.
4. The **huge overlap** makes it difficult to **decode** the info. (0 or 1).

→ **Failure of decoding!**



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

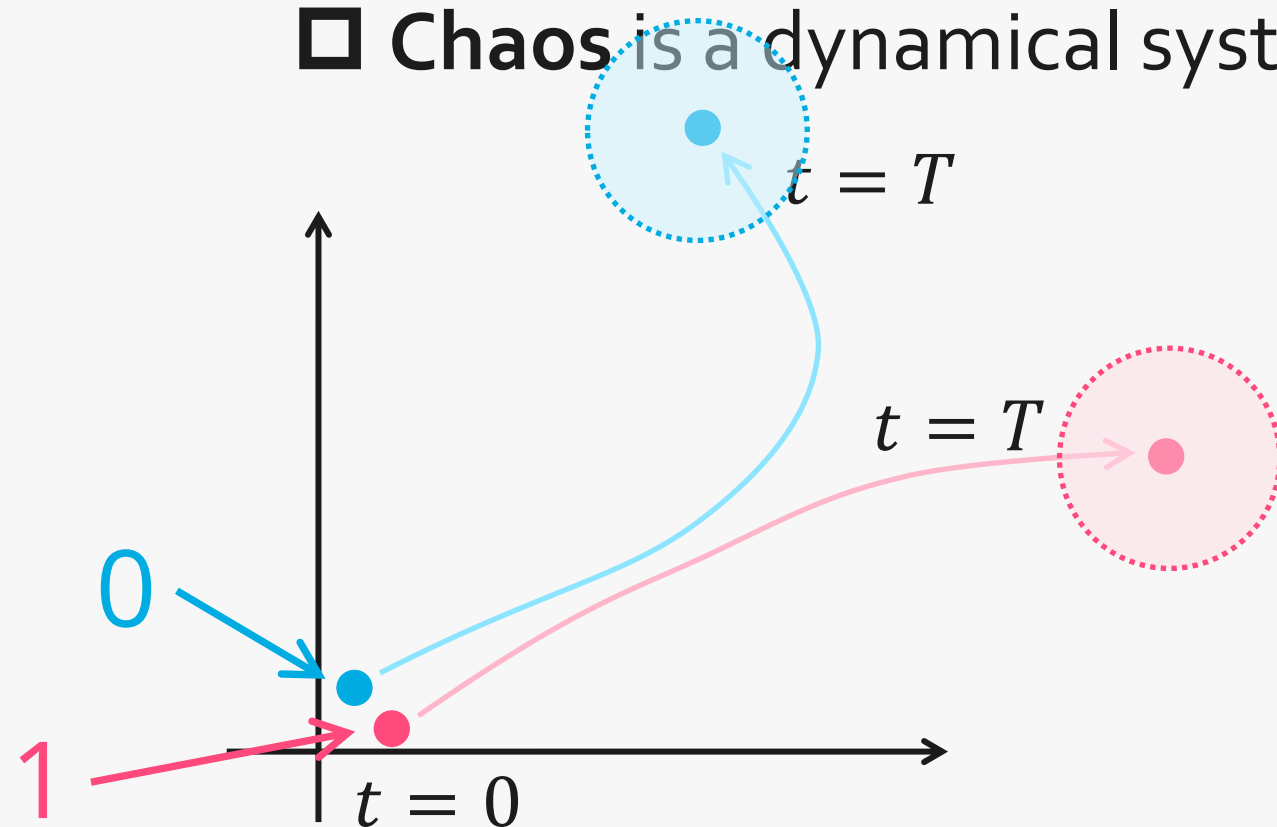
QEC in a nutshell

# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

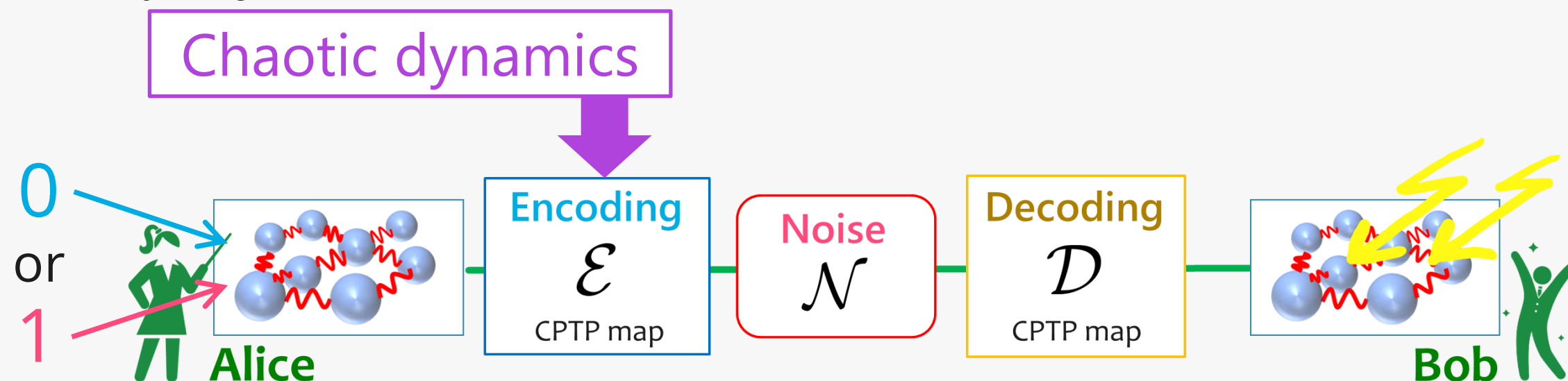
## Chaos and Error Correction: intuition 2

□ **Chaos** is a dynamical system highly sensitive to initial conditions.



1. Write **0** or **1** into the system.
2. "Time evolution" of the information by the **chaotic** dynamics.
3. Due to the uncertainty induced by **the noise**, the info. gets **unclear**.
4. **No overlap** makes it easy to **decode** the info. (**0** or **1**).

→ **Success of decoding!**



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

QEC in a nutshell

# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

## Important remark on Chaos and Error Correction

❑ The sensitivity of chaos to the initial condition makes it easy to read out information!

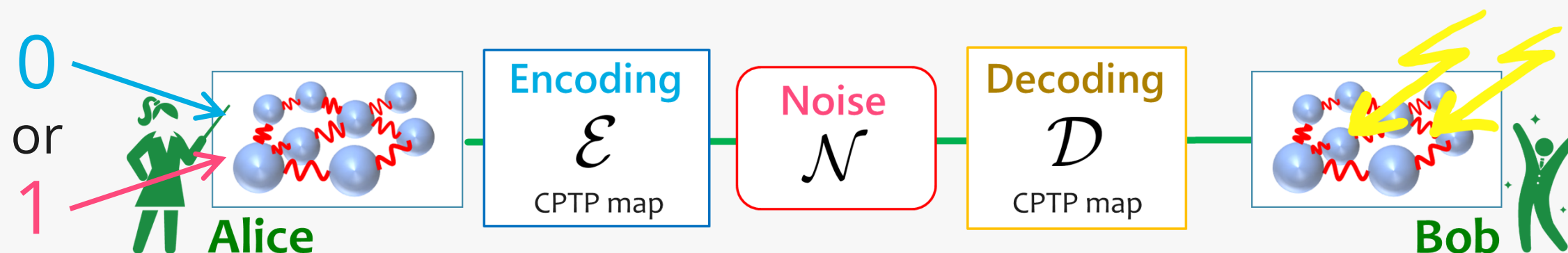
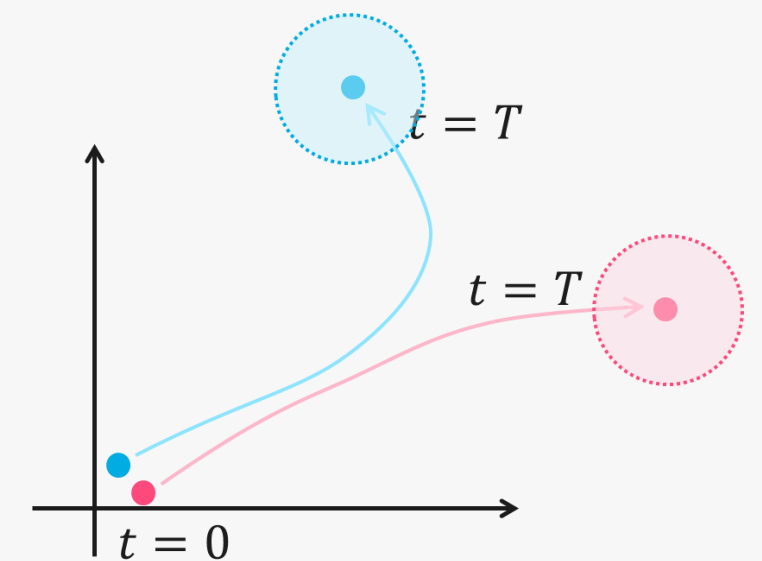
One needs to know all the details of the dynamics!

■ In physics, it is common to assume that the details of the system are **out of control**.

Hence, **chaos** is **unpredictable**.

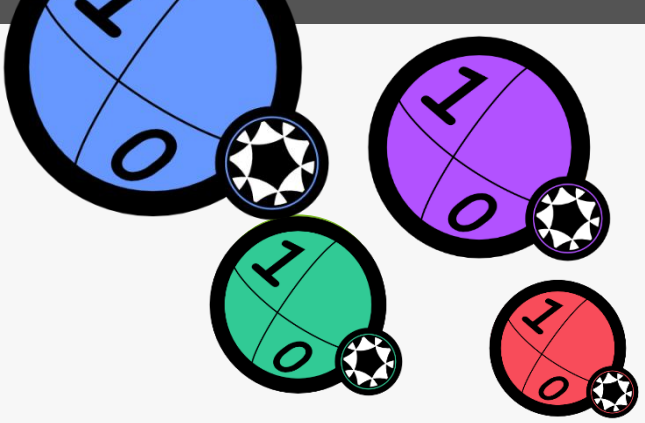
■ In quantum information, everything is **under control**.

➤ Chaotic dynamics is a powerful tool to encode information!



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

QEC in a nutshell



# 1. Information and Chaos in the classical regime.

1. Quantum Error Correction (QEC)
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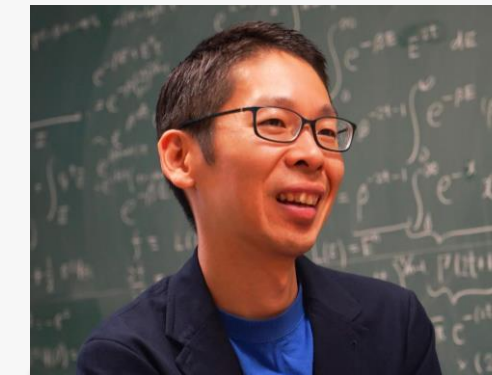
# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

## Chaos in the Quantum regime 1

□ “Quantum chaos” is **non-trivial**.

➤ (At least) two inequivalent defs of “quantum chaos”.



By Prof. Hashimoto  
3<sup>rd</sup> Colloquium of ExU  
(available on the ExU website)

### 1-4 Two definitions of quantum chaos

Def 1 “Chaos” : Quantization of classically chaotic system.

Energy level spacings follow Wigner distribution

[Muller, Heusler, Altland, Braun, Haake '09]

Def 2 “Scrambling” : Exponential growth of  
Out-of-time-order (OTO) correlator.

$$\langle x(t)p(0)x(t)p(0) \rangle \propto \exp[2\lambda t]$$

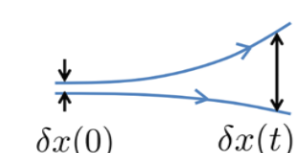
[Larkin, Ovchinnikov '69]

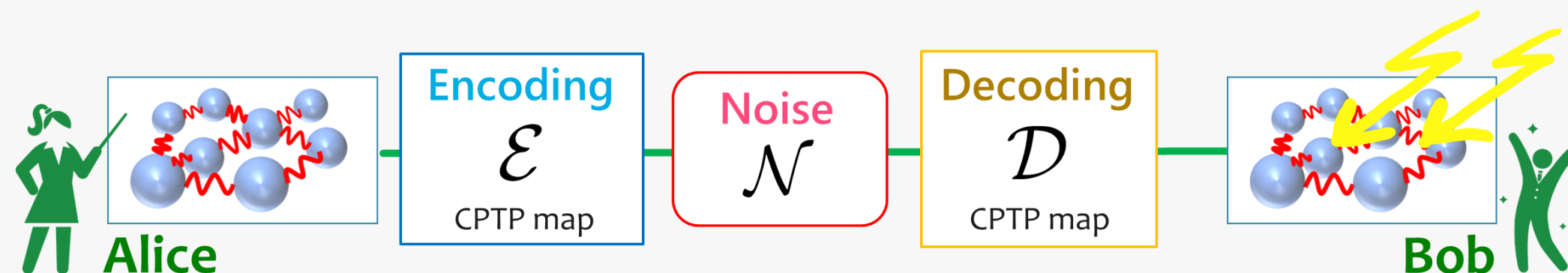
[Kitaev '14] [Maldacena,

Shenker, Stanford '15]

$\lambda$  : Quantum Lyapunov exponent

Classical analogue:

$$\left[ \langle x(t)p(0)x(t)p(0) \rangle \sim \left( \frac{\delta x(t)}{\delta x(0)} \right)^2 \right]$$




$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

QEC in a nutshell



# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

## Chaos in the Quantum regime 2

□ “Quantum chaos” is **non-trivial**.

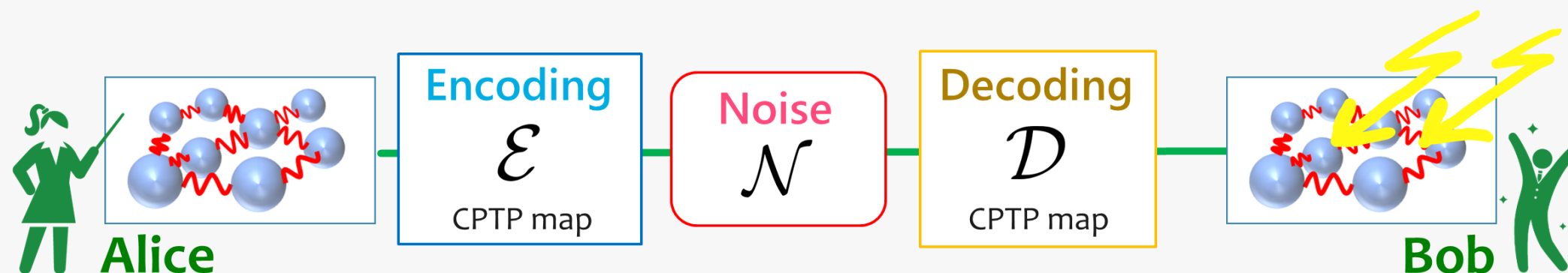
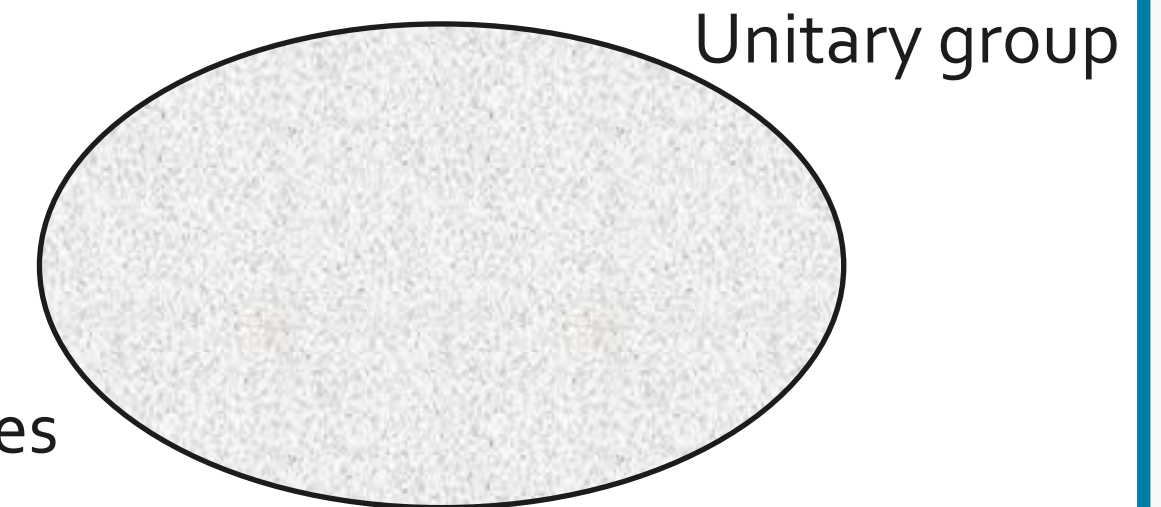
➤ We model the “chaotic dynamics” by a **Haar random unitary** (a.k.a. circular unitary ensemble).

**A Haar random unitary = Circular Unitary Ensemble (CUE)**

□ A random unitary uniformly distributed over a unitary group

➤ A typical dynamics of quantum chaos.

**A Haar random unitary**  
= uniform distribution of unitaries



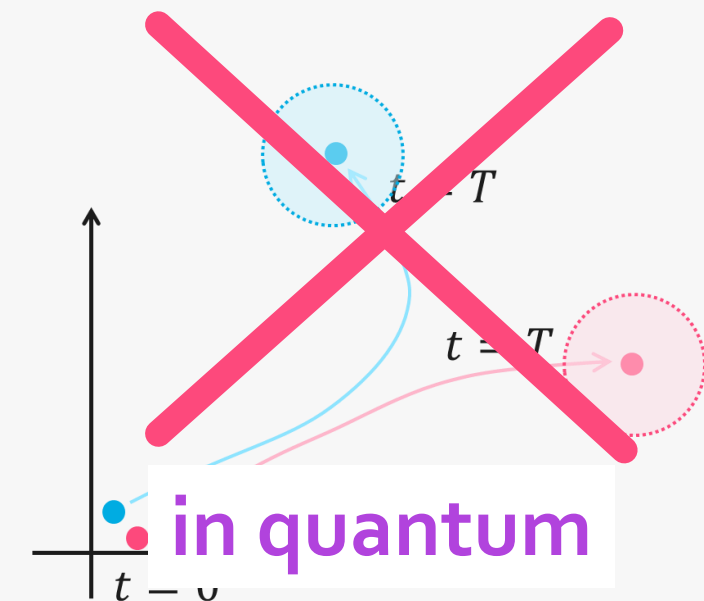
$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

QEC in a nutshell

# Chaos and Error Correction in classical

*How can we use chaotic dynamics for correcting errors?*

## Chaos and Quantum Error Correction



□ “Quantum chaos” is **non-trivial**.

➤ We model the “chaotic dynamics” by a **Haar random unitary** (a.k.a. circular unitary ensemble).

□ “QEC by **chaotic dynamics**” is **non-trivial**.

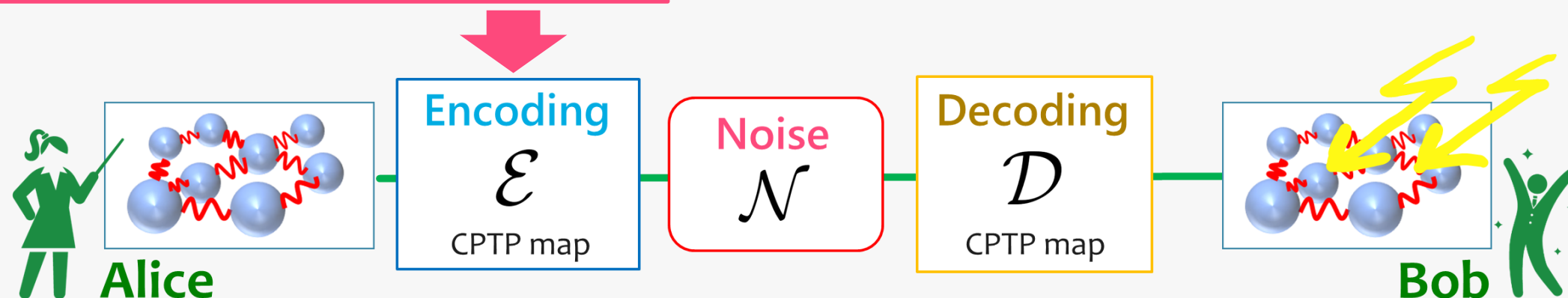
1. Unitarity preserves the distance between two states.
2. We need to recover an unknown quantum state (e.g.  $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ ), not **0** or **1**.

□ It turns out that **quantum chaotic dynamics** is useful for **QEC**. [Dupuis et al, CMP '14]

➤ Finally, established by the “**one-shot decoupling theorem**”.

A Haar random unitary

As an example, we see the **Hayden-Preskill thought experiment**.



$$\mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \approx \text{id}$$

QEC in a nutshell



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1. Information and Chaos in the classical regime.

2. The Hayden-Preskill thought experiment.

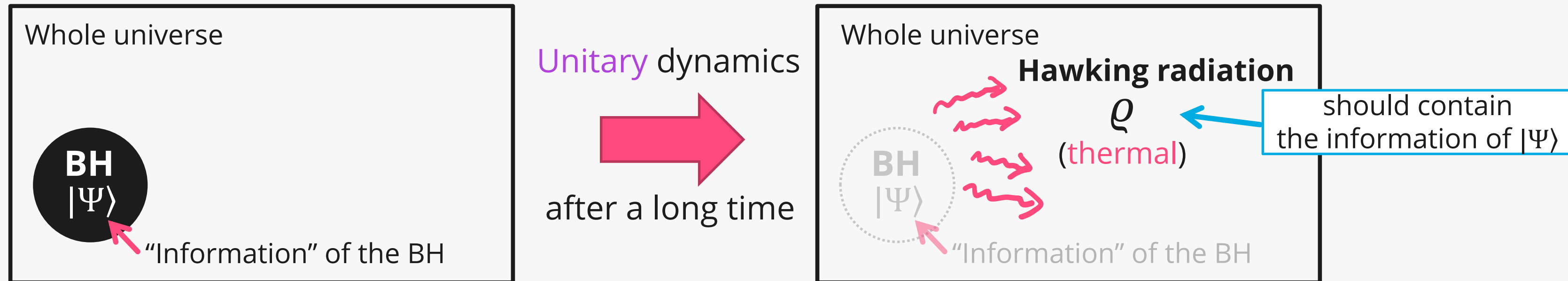
3. Beyond the Hayden-Preskill.

4. Conclusion and Outlooks.

# The Hayden Preskill thought experiment

# HOW?

Information paradox is a puzzle about the Hawking radiation.



If you look at the beginning and the end, you will notice that

a **pure** state  $|\Psi\rangle$  becomes a **mixed** state  $\rho$  by **unitary** dynamics.

**Apparent contradiction**

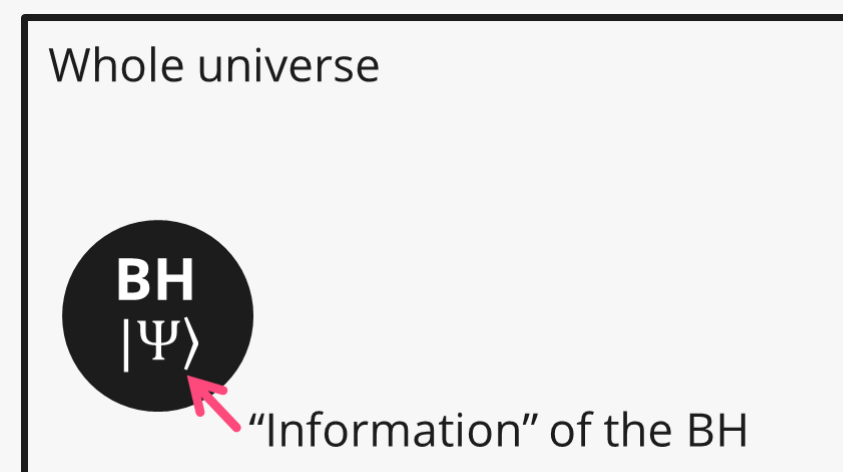
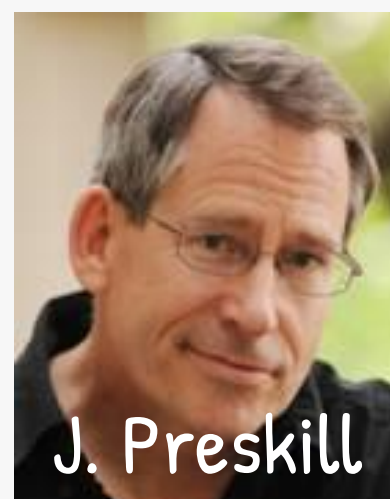
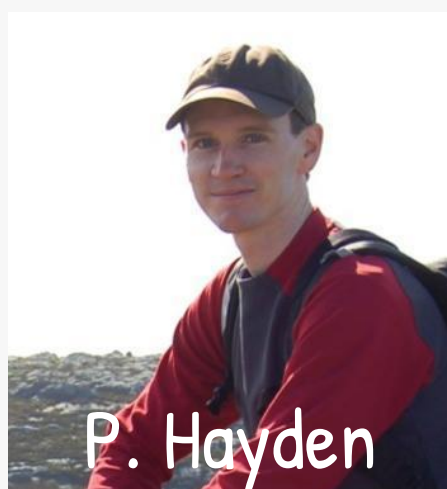
The state  $|\Psi\rangle$  should be recoverable from the radiation (if we are in the quantum side)

# The Hayden-Preskill thought experiment

*Information-theoretic toy model of quantum black holes*

How can we recover quantum information from the radiation?

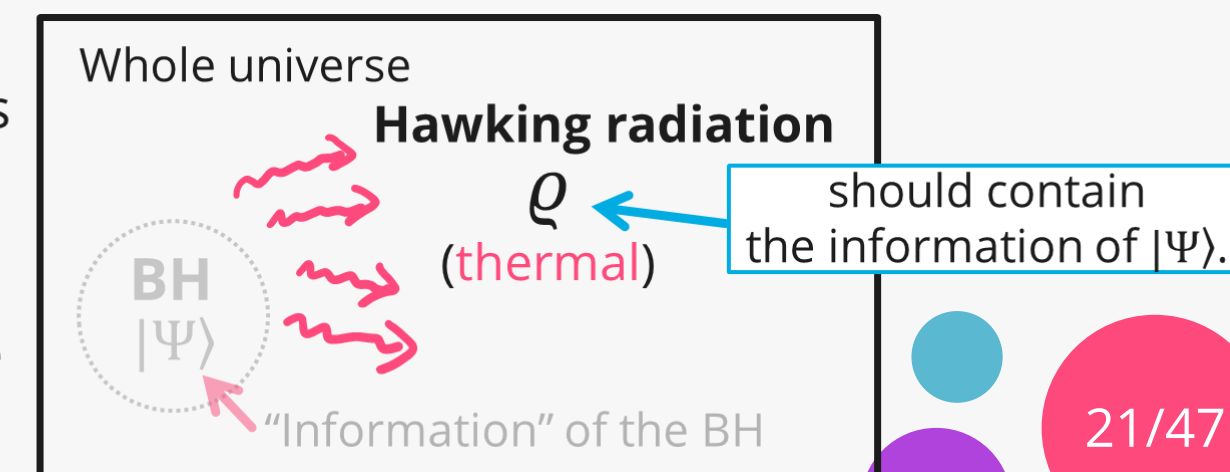
□ In 2007, Hayden and Preskill addressed this question based on a **qubit-toy model of a “BH”**.



Unitary dynamics



after a long time

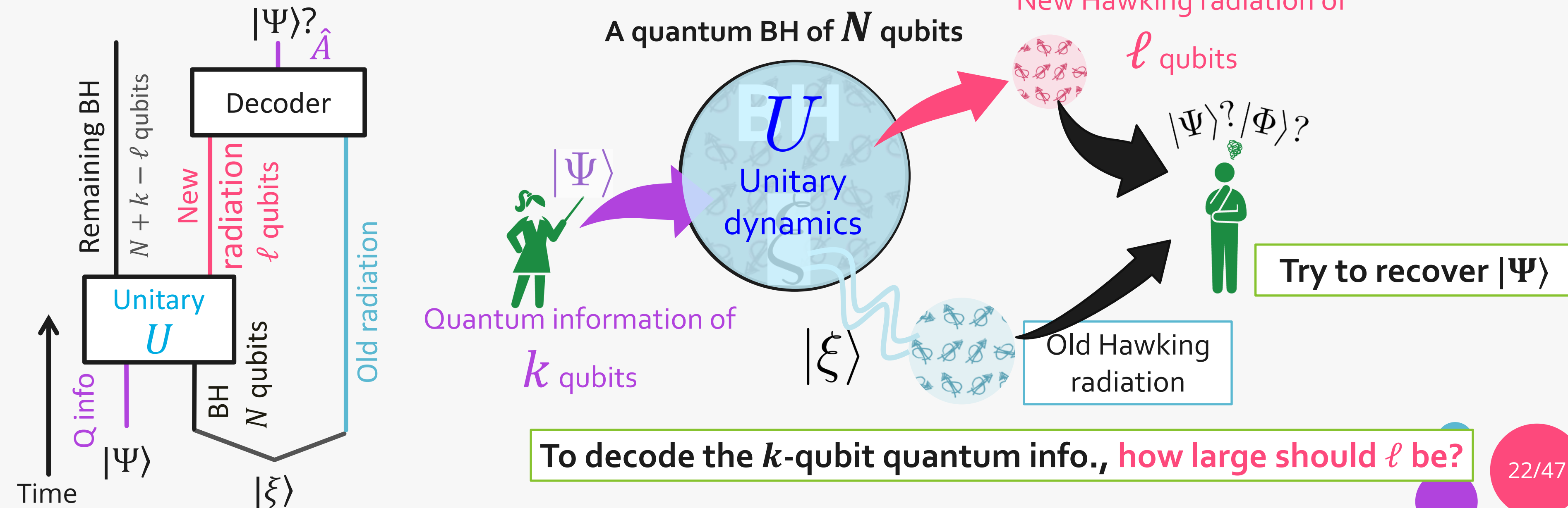


# The Hayden-Preskill thought experiment

*Information-theoretic toy model of quantum black holes*

## Information paradox and the Hayden-Preskill thought experiment

- A qubit-toy model of a quantum “BH”.





# The Hayden-Preskill thought experiment

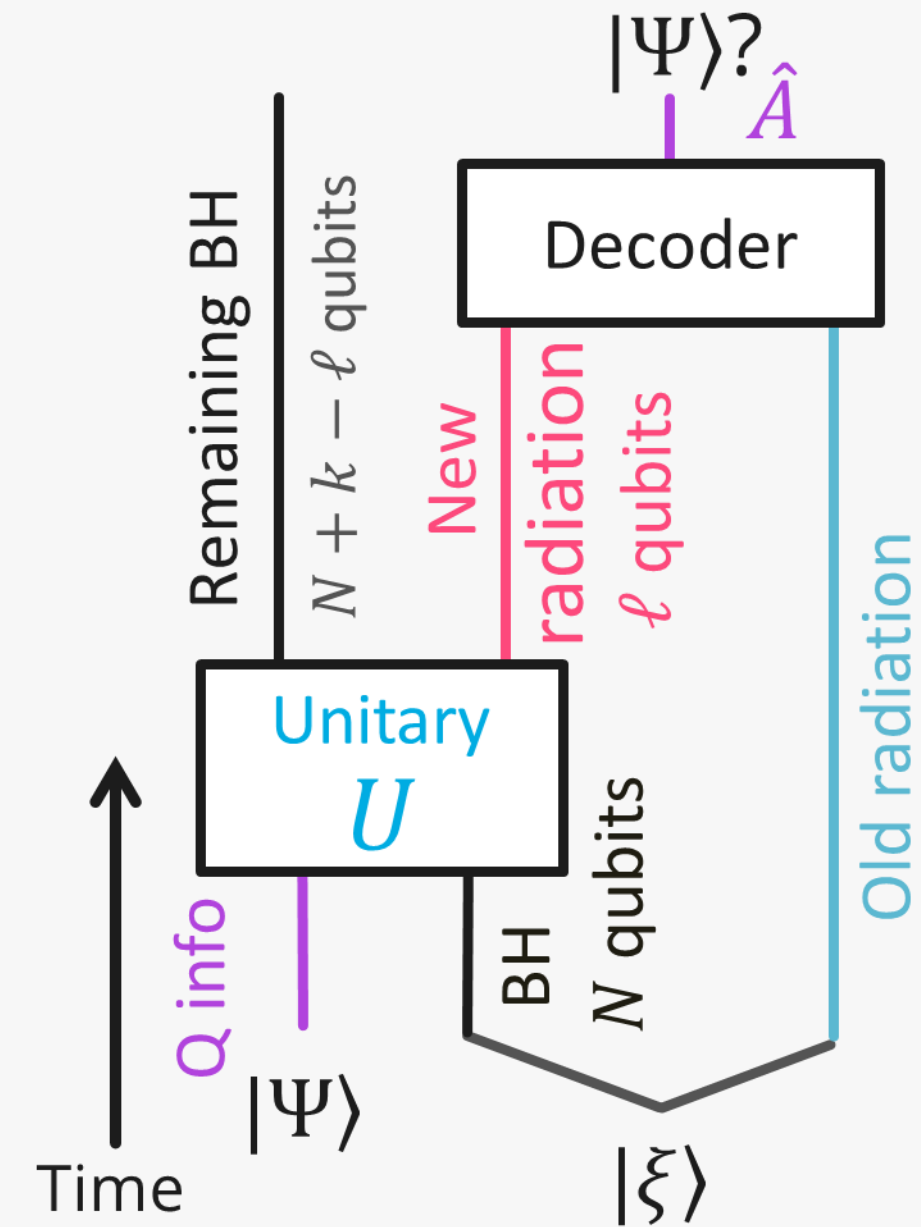
Information-theoretic toy model of quantum black holes

## Hayden-Preskill thought experiment 1

To decode the  $k$ -qubit quantum info., how large should  $\ell$  be?

1. What is quantum information?

➤ See the newsletter of ExU (available at the end of March).



### Topics Keyword

#### What is Quantum Information?

This is a slight generalization of classical information. However, in the quantum regime, a slight extension usually leads to a radical change. To see this, let us consider the following two quantum information sources A and B:

A :  $\{|\psi_i\rangle\}$  with probability  $q_i$  :  $i = 0, 1$ ,  $\langle\psi_0|\psi_1\rangle = 0$ ,  
B :  $\{|\phi_i\rangle\}$  with probability  $q_i$  :  $i = 0, 1$ ,  $\langle\phi_0|\phi_1\rangle \approx 1$ .

Since the two states in A are orthogonal and distinguishable with certainty, the source A is merely a re-labeling of a classical information source  $\{p_i, i = 0 \text{ or } 1\}$ . That is, the quantum information source A is classical though it is defined on quantum states! On the other hand, B consists of  $\{|\phi_i\rangle \approx |\phi_1\rangle\}$  and always outputs almost one type of state. Hence, the source B is very different from a classical one  $\{p_i, i = 0 \text{ or } 1\}$ . Clearly, this is due to non-orthogonality of quantum states, which can be simplified by using a trick that we are all familiar with. We can just use a density matrix to represent the information source  $\{p_i, |\psi_i\rangle\}$ , such as  $\rho = \sum_{i,j} p_i p_j |\psi_i\rangle\langle\psi_j|$ . By diagonalization  $\rho = \sum_{\alpha} \lambda_{\alpha} |\lambda_{\alpha}\rangle\langle\lambda_{\alpha}|$  ( $0 \leq K$ ), we can introduce another ensemble  $\{ \lambda_{\alpha}, |\lambda_{\alpha}\rangle \}_{\alpha=1}^K$  of pure states. It is important that this ensemble represents the quantum information source same as  $\{p_i, |\psi_i\rangle\}_{i=0}^1$ . This is due to the rule of quantum world: there is no physical operation that distinguishes ensembles of quantum states with the same density operator. Thus, we can argue quantum information based on the source  $\{ \lambda_{\alpha}, |\lambda_{\alpha}\rangle \}_{\alpha=1}^K$  rather than  $\{p_i, |\psi_i\rangle\}_{i=0}^1$ , which is a great simplification since the former consists of the pure states orthogonal to each other. The former is essentially classical!

1. Again, what is quantum information?

At this stage, one may be confused about what quantum information is. In the beginning of the previous section, it was explained that quantum information is about the output of a quantum information source  $\{p_i, |\psi_i\rangle\}$ , e.g., was the output  $|\psi_0\rangle$  or  $|\psi_1\rangle$ ? However, in the later part, the ensemble was replaced with  $\{ \lambda_{\alpha}, |\lambda_{\alpha}\rangle \}_{\alpha=1}^K$  using the principle of quantum world. Now, neither  $|\psi_0\rangle$  nor  $|\psi_1\rangle$  can you find. Well, what would you actually get to know when you obtain quantum information?

The confusion arises from the fact that we have mistakenly expected that an output state is drawn from a specific quantum source  $\{p_i, |\psi_i\rangle\}$ . This is not true since quantum tells us that any ensembles describe the same physical state if they have the same density matrix. We should never fix the ensemble when we talk about quantum information. If we do so, we will be out of quantum. For this reason, the most common definition of quantum information source is actually representing it by a density matrix  $\rho$  instead of an ensemble  $\{p_i, |\psi_i\rangle\}$ . Apart from using a density matrix, it is also possible to denote quantum information by entanglement. Recall that a density matrix  $\rho_A$  on a quantum system A can be always described by a pure state  $|\rho\rangle_{AB}$  on an extended system AB. The pure state and the system R are called a purification of  $\rho_A$  and a purifying system of A, respectively. As a consequence of the Uhlmann's theorem [1], it can be shown that any ensemble of pure states with a density matrix  $\rho_A$  can be realized in A if R of the purification  $|\rho\rangle_{AB}$  of  $\rho_A$  is measured in a proper basis. This property holds due to the entanglement between A and R. Here, a choice of the measurement in R determines a set of pure states in A, and the measurement probability defines a probability distribution.

Using this fact, we may expect that a quantum information source in A, namely a density matrix  $\rho_A$ , is kept track of by a purifying system R of  $\rho_A$ . Hence, we can say that quantum information is kept in entanglement with a purifying system. If one takes this picture, the purifying system is sometimes called a reference system. This definition is nearly equivalent to the one based on a density matrix [2], so either can be used in practice.

Before we move on, it is better to emphasize that there is no way to naturally quantify the "amount" of quantum information. In classical cases, the information content was intuitively defined from the degree of "surprise". In a quantum case, this is not possible since we should not fix the ensemble. For this reason, we cannot think of how much we will be surprised when we know the output of a quantum information source. Some may wonder what about the von Neumann entropy. In a sense, it represents quantum information content of a quantum source. However, it never follows from the definition and is a theorem established by the Schumacher's compression scheme [3].

1. Quantum information and physics

Finally, let us see what "quantum information" is in the context of physics. For instance, the Hayden-Preskill thought experiment asks how to recover quantum information thrown into a black hole (BH) from the Hawking radiation. We also sometimes hear "quantum information of a BH". How should we understand them?

Based on a density matrix, the former is interpreted as follows. First, a pure state  $|\Psi\rangle_{AB}$  probabilistically drawn from a quantum information source  $\rho_A$  is thrown into a BH. Second, one, say Bob, tries to recover  $|\Psi\rangle_A$  from the Hawking radiation. Bob is typically assumed to know what is  $\rho_A$  though he has no idea about what pure state was actually chosen. If he succeeds, then we can say that quantum information thrown into the BH is recovered.

If one prefers a description by entanglement, we should introduce a purification  $|\rho\rangle_{AB}$  of a quantum information source  $\rho_A$ . The system A is thrown into a BH, and Bob tries to recover the entanglement between A and R from the Hawking radiation. That is, he applies a good operation onto the Hawking radiation, so that resulting state is  $|\rho\rangle_{AB}$  shared between the radiation and

R, not between A and R. A success of such an entanglement transfer implies that quantum information is recovered from the radiation. In this scheme, we should be careful that no operation can be applied to the purifying system R since it is virtual.

In contrast, it is not so clear to me what "quantum information of a BH" means. Maybe, it implies the idea that a BH itself is a quantum information source, which is possible since a BH should be described by a density matrix. If so, quantum information of a BH is about the pure state a BH is actually in. In terms of entanglement, this is equivalent to say that we consider a purification of a density matrix of a BH by a reference system R and try to transfer the entanglement between R and the BH to that between R and the quantum system in our hand.

These interpretations seem natural from the viewpoint of the theory of quantum information. However, I am not sure if these are the right things. If they are, I actually wonder whether they are physically possible. Although I am not sure at all, it looks to me too ambitious since I feel that they may violate some physical principle.

When I hear talks about quantum information in physics, I often feel that I am lost in an unknown planet. This lets me wonder that nothing is certain in this world and that nature is the information source of physics. Our goal is to extract information from the information source of nature. It is extremely exciting if we can contribute to such a dream-like goal through the collaboration of extreme universe.

[1] A. Uhlmann, Rep. Math. Phys., 9:273-279 (1976).  
[2] D. Kretschmann and R. F. Werner, New J. Phys., 6, 26 (2004).  
[3] B. Schumacher, Phys. Rev. A, 51, 2738 (1995).  
[4] P. Hayden and J. Preskill, J. High Energy Phys., 0709, 120, (2007).

Author

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Current position since May 2020

# The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

## Hayden-Preskill thought experiment 1

To decode the  $k$ -qubit quantum info., how large should  $\ell$  be?

1. What is quantum information?

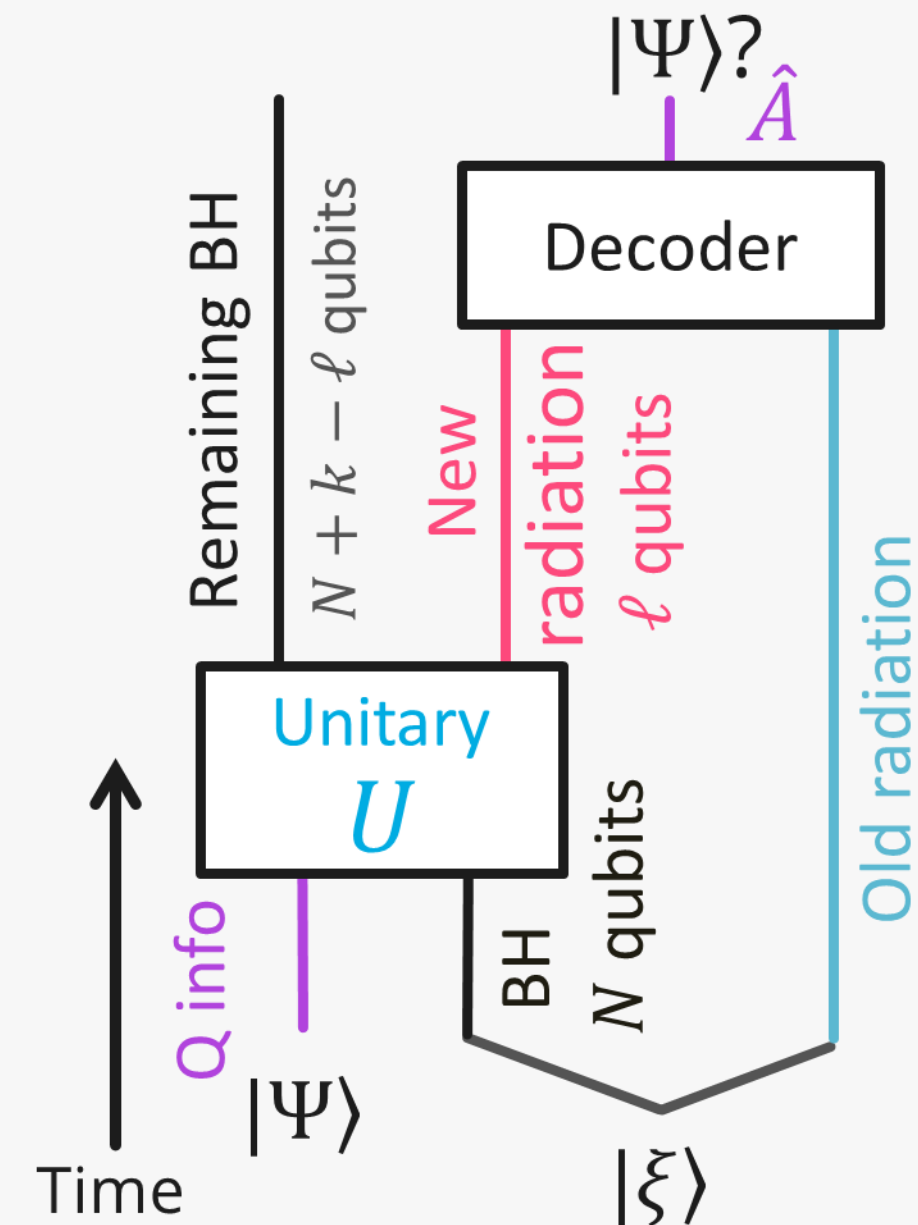
➤ See the newsletter of ExU (available at the end of March).

Recovery error:  $\Delta(\ell|\xi, U)$   $(0 \leq \Delta(\ell|\xi, U) \leq 1)$

How does this scale with  $\ell$ ?

2. How do we model the unitary dynamics  $U$  of the BH?

➤ The unitary  $U$  is assumed to be fully chaotic (a Haar random unitary).





## Information-theoretic toy model of quantum black holes

If the dynamic of the BH is **chaotic** (**Haar random**),

$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\text{th}} - \ell)}$$

Here,  $\ell_{\text{th}} = k + \frac{N - H_2(\xi)}{2}$ , and  $H_2(\xi) = -\log \text{Tr}[\xi^2]$  is the collision entropy of the initial black hole  $B_{in}$ .

- If  $\ell \gg \ell_{\text{th}} = k + \frac{N - H_2(\xi)}{2}$ ,  $k$ -qubit quantum information is recoverable.
  - Entropy of the initial BH determines the threshold  $\ell_{\text{th}}$ .



# The Hayden-Preskill thought experiment

Information-theoretic toy model of quantum black holes

## Hayden-Preskill thought experiment 3

If the dynamic of the BH is **chaotic** (**Haar random**),

[Hayden & Preskill, JHEP, '07]

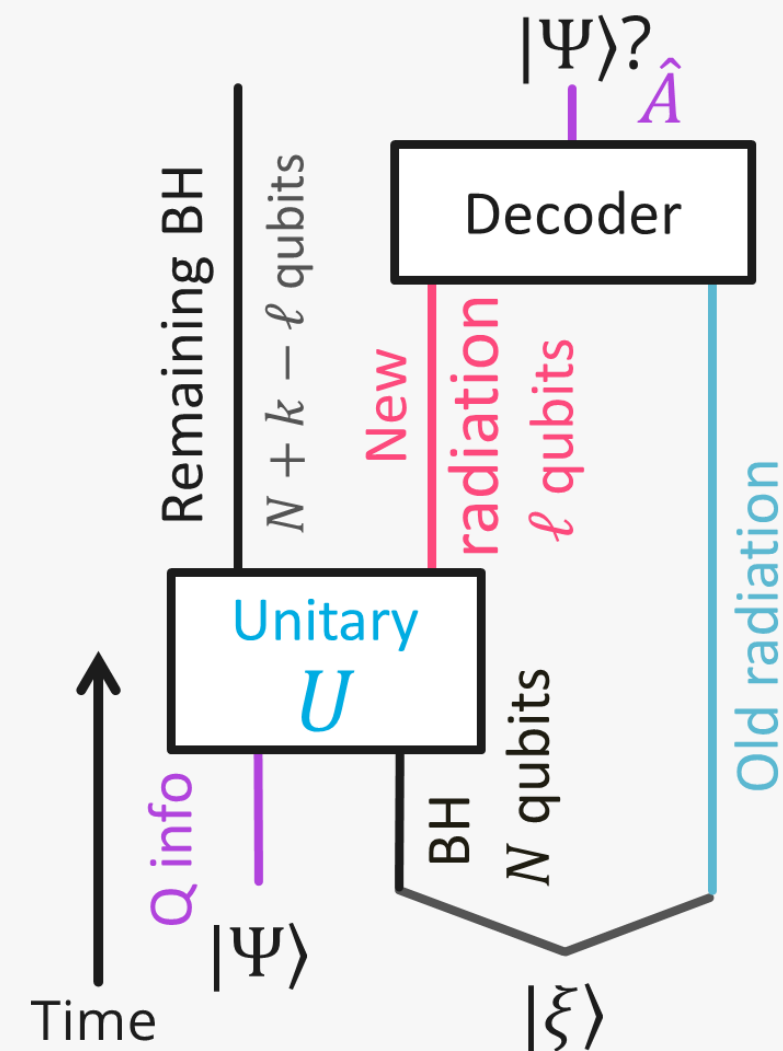
$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\text{th}} - \ell)}$$

□ If  $\ell \gg \ell_{\text{th}} = k + \frac{N - H_2(\xi)}{2}$ ,  $k$ -qubit quantum information is recoverable.

- Entropy of the initial BH determines the threshold  $\ell_{\text{th}}$ . Independent of  $N$
- If the BH is initially pure ( $T = 0$ ),  $H_2(\xi) = 0$  and  $\ell_{\text{th}} = k + N/2$ .
- If the BH is initially completely mixed ( $T = \infty$ ),  $H_2(\xi) = N$  and  $\ell_{\text{th}} = k$ .

➡ The  $k$ -qubit info is recoverable when  $O(k)$  qubits are radiated.

The Hayden-Preskill recovery



# The Hayden-Preskill thought experiment

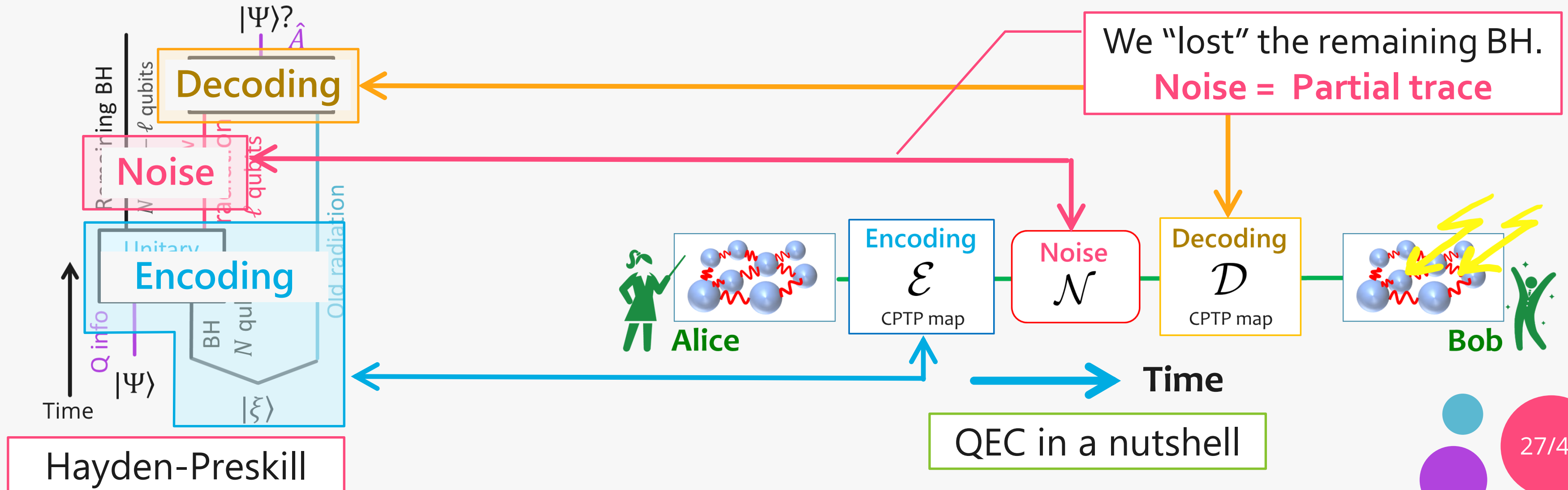
Information-theoretic toy model of quantum black holes

## Hayden-Preskill & Quantum Error Correction

□ The Hayden-Preskill recovery is a special case of QEC by chaotic dynamics.

➤ **Noise** = Partial trace over the remaining BH.

➤ **Encoding** = Adding a state  $|\xi\rangle$  and applying a chaotic (Haar random) unitary  $U$ .



# The Hayden-Preskill thought experiment

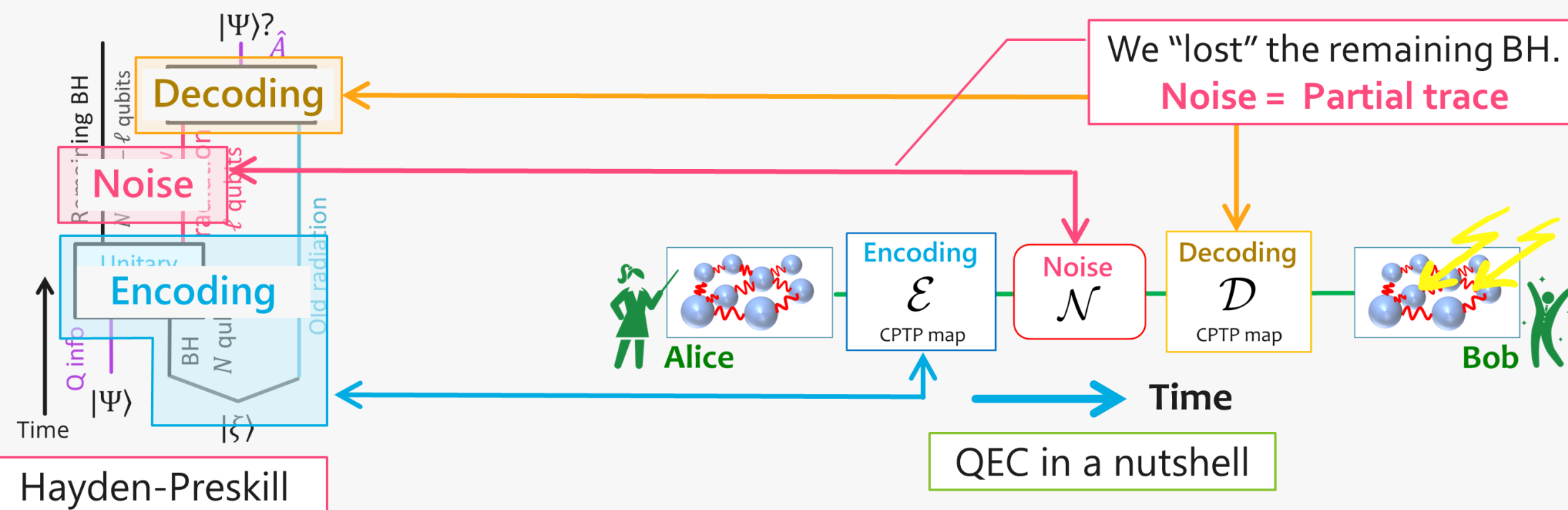
*Information-theoretic toy model of quantum black holes*

## Hayden-Preskill & Quantum Error Correction

□ The Hayden-Preskill **recovery** is a special case of **QEC** by **chaotic** dynamics.

- **Noise** = Partial trace over the remaining BH.
- **Encoding** = Adding a state  $|\xi\rangle$  and applying a **chaotic** (**Haar random**) unitary  $U$ .

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, and **a quantum black hole!!**





# Outline of this talk

*A journey of a thousand miles begins with a single step*

Information, Chaos, and Black Holes in the **Quantum** regime.

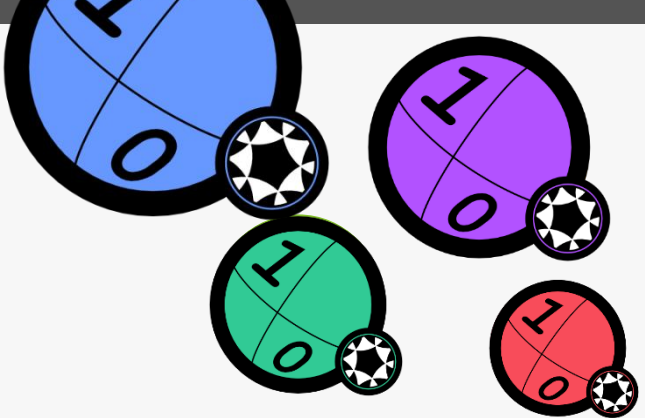


1. Information and Chaos in the classical regime.

2. The Hayden-Preskill thought experiment.

3. Beyond the Hayden-Preskill.

4. Conclusion and Outlooks.



# 3. Beyond the Hayden-Preskill.

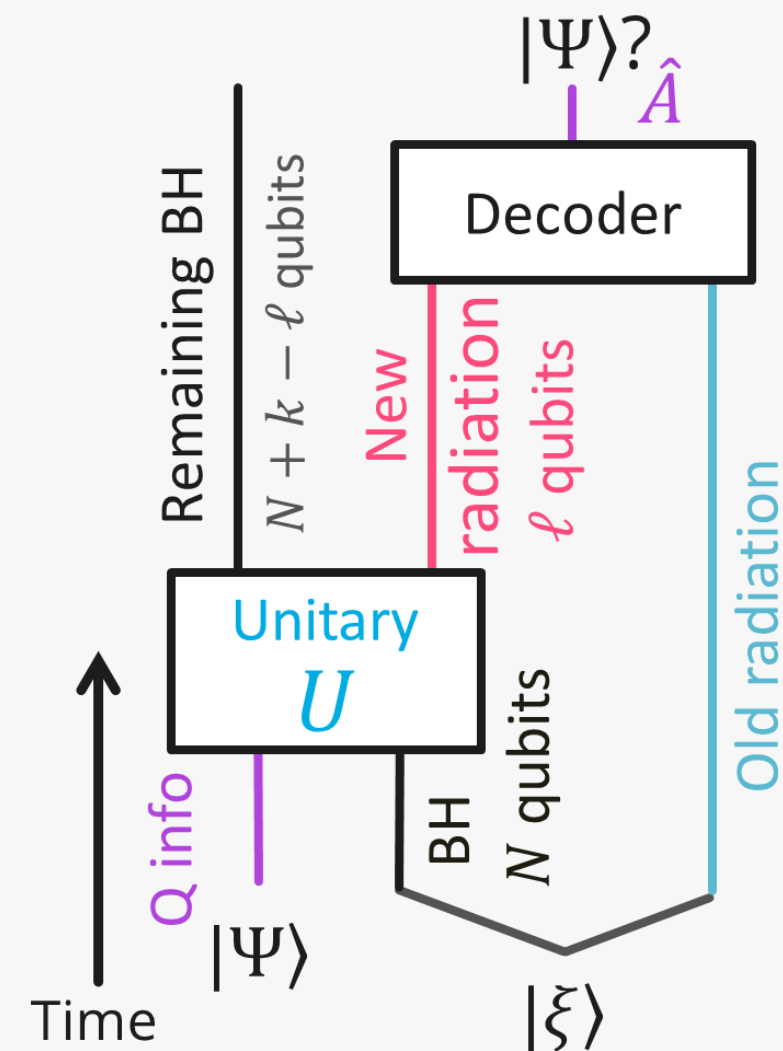


# Beyond the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

## Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, and **a quantum black hole!!**



# Beyond the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

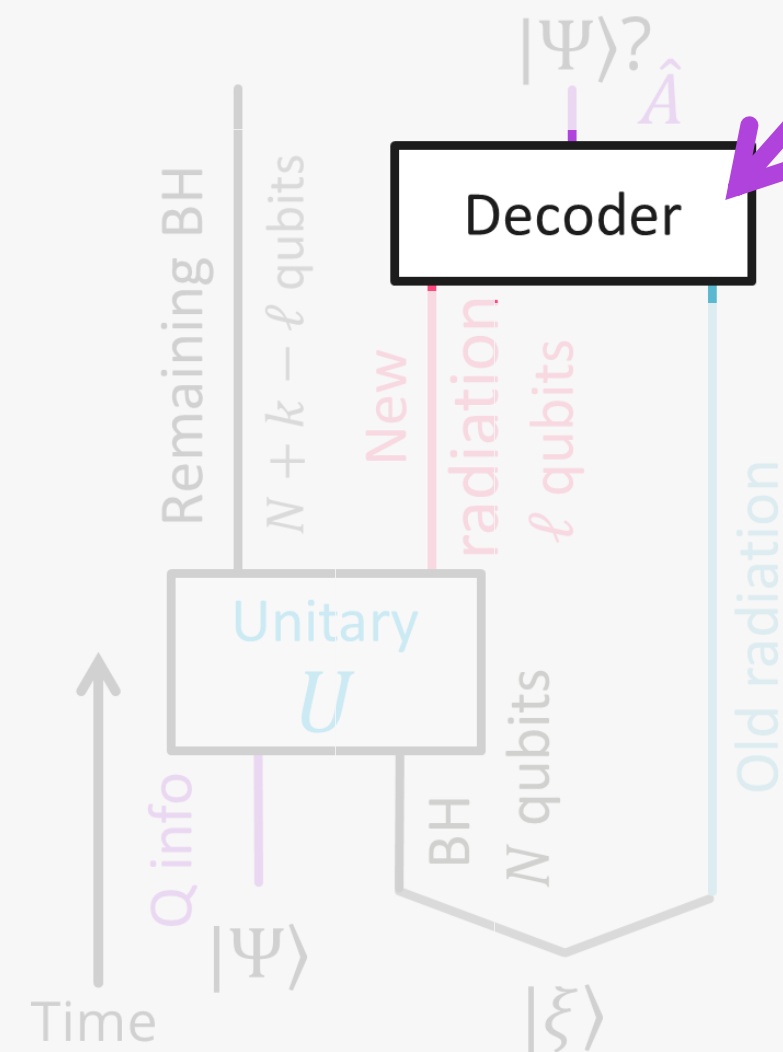
## Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, and **a quantum black hole!!**

1. No decoder is given.

➤ How can we actually decode information?

[YN & Koashi, in prep]





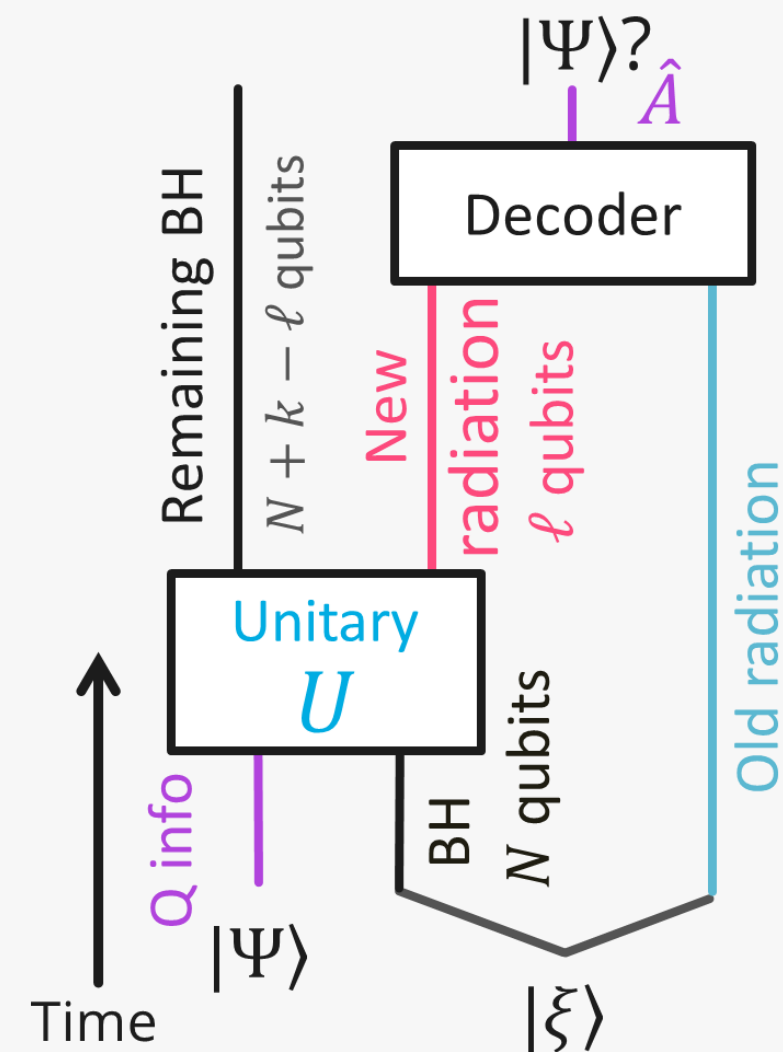
# Decoding the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

## How do we decode information?

- ❑ In the original work by Hayden&Preskill, no decoder was provided.
  - How can we explicitly construct a decoder?
- ❑ Two **decoders** in the literature, and their relations to **physics**.
  - **Teleportation-type decoder** for special cases, related to **OTOC** [Yoshida & Kitaev '17].
  - **Petz decoders**, related to **spacetime geometry** [Penington, et al'19].

A **decoder** is a key link to bridge **quantum information** to **physics**.

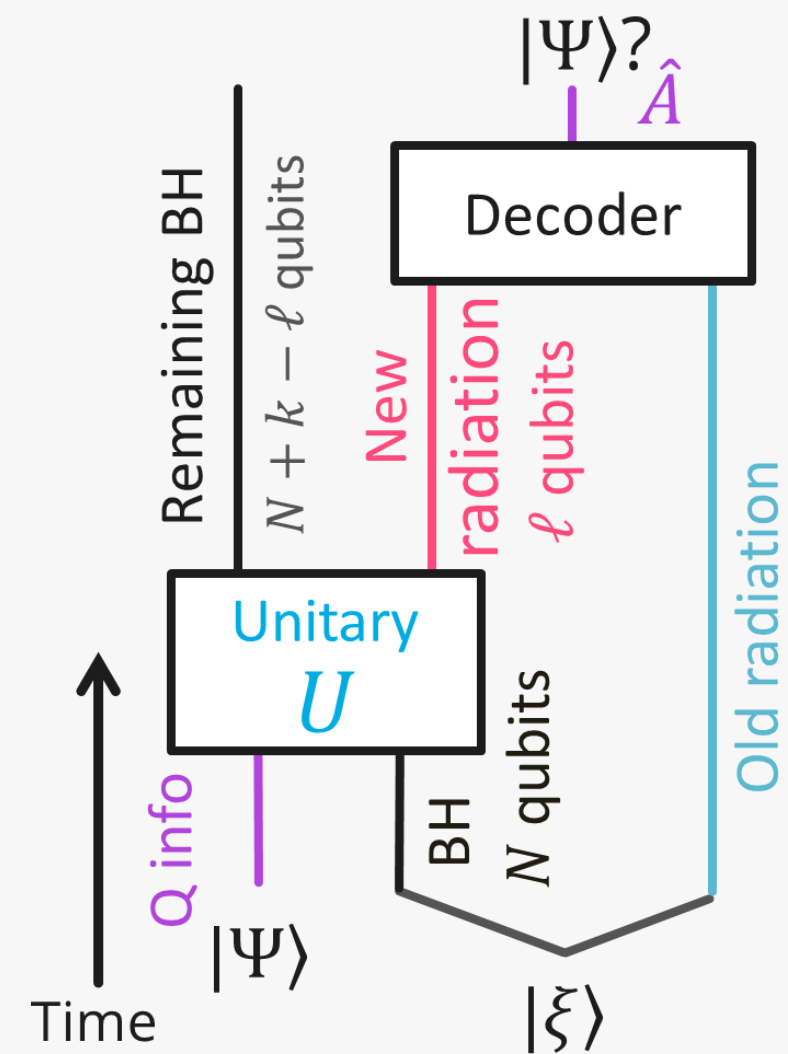


# Decoding the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

## New results by the Classical-to-Quantum decoder

- We propose an intuitive decoder, called a **Classical-to-Quantum decoder**.
  - It is universal since it works for general noise.
  - It is related to the **Renyi-2 entropy**.
- A chaotic BH **simultaneously** emits **classical** and **quantum** information.
  - There is **NO difference b/t C and Q info** in the **QEC by chaotic dynamics**.



	Recovery error for quantum information	Recovery error for classical information
Hayden & Preskill '07	$\leq 2^{\frac{1}{2}(\ell_{\text{th}} - \ell)}$	N.A.
Our results	$\leq 2^{\ell_{\text{th}} - \ell}$	$\leq 4^{\ell_{\text{th}} - \ell}$

Here,  $\ell_{\text{th}} = k + \frac{N - H_2(\xi)}{2}$

# Beyond the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

## Open questions in the Hayden-Preskill thought experiment

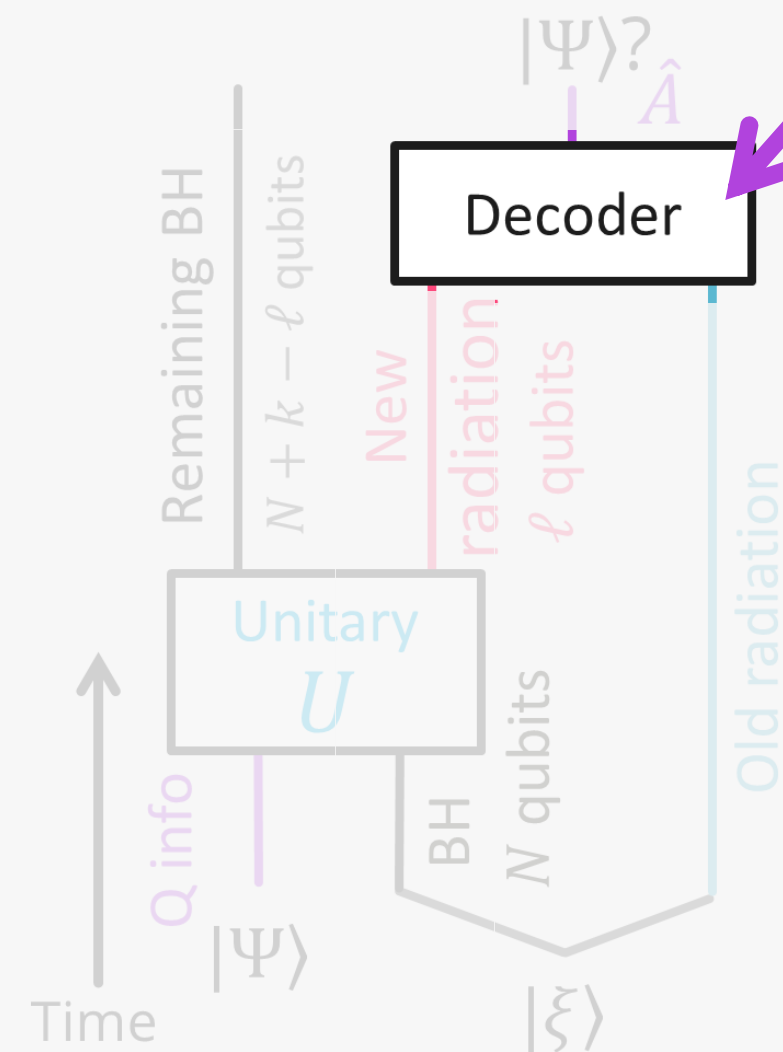
The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, and **a quantum black hole!!**

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A decoder gives a “link” b/t quantum information and physics.



# Beyond the Hayden-Preskill

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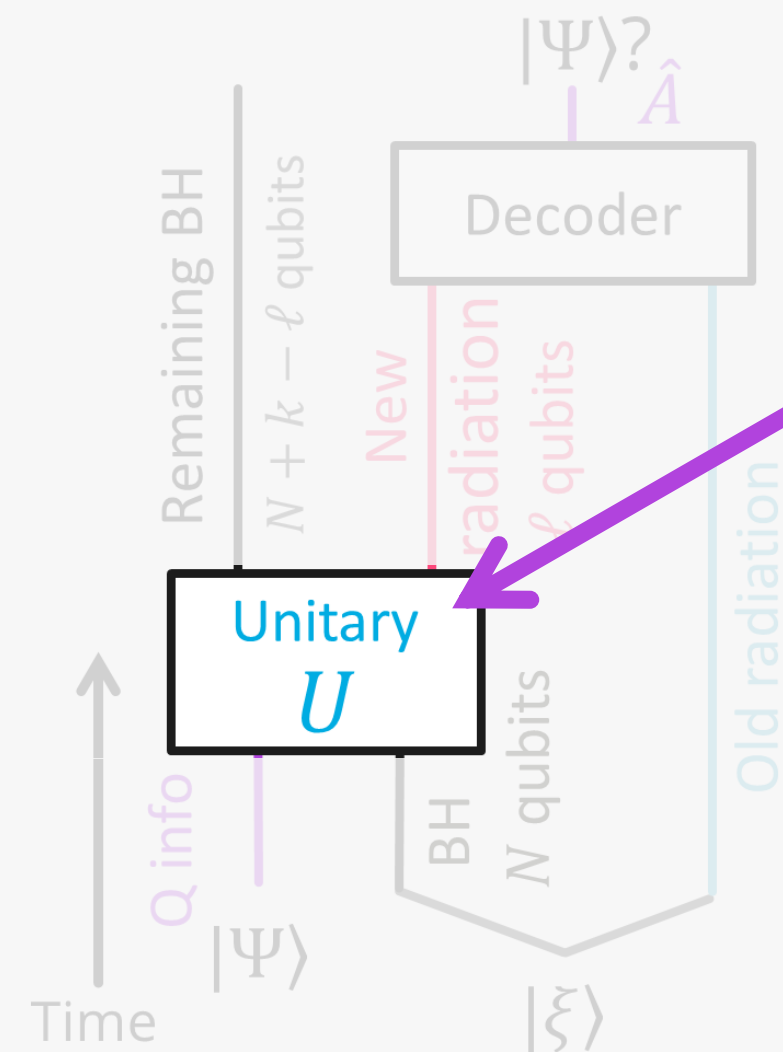
[YN & Koashi, in prep]

A decoder gives a “link” b/t quantum information and physics.

### 2. No energy conservation due to the Haar assumption.

- What if something is conserved?

[YN, Wakakuwa & Koashi, '19&'20]



# The HP with energy conservation

*Beyond the Hayden-Preskill 2*

## Haar random dynamics “scrambles” everything

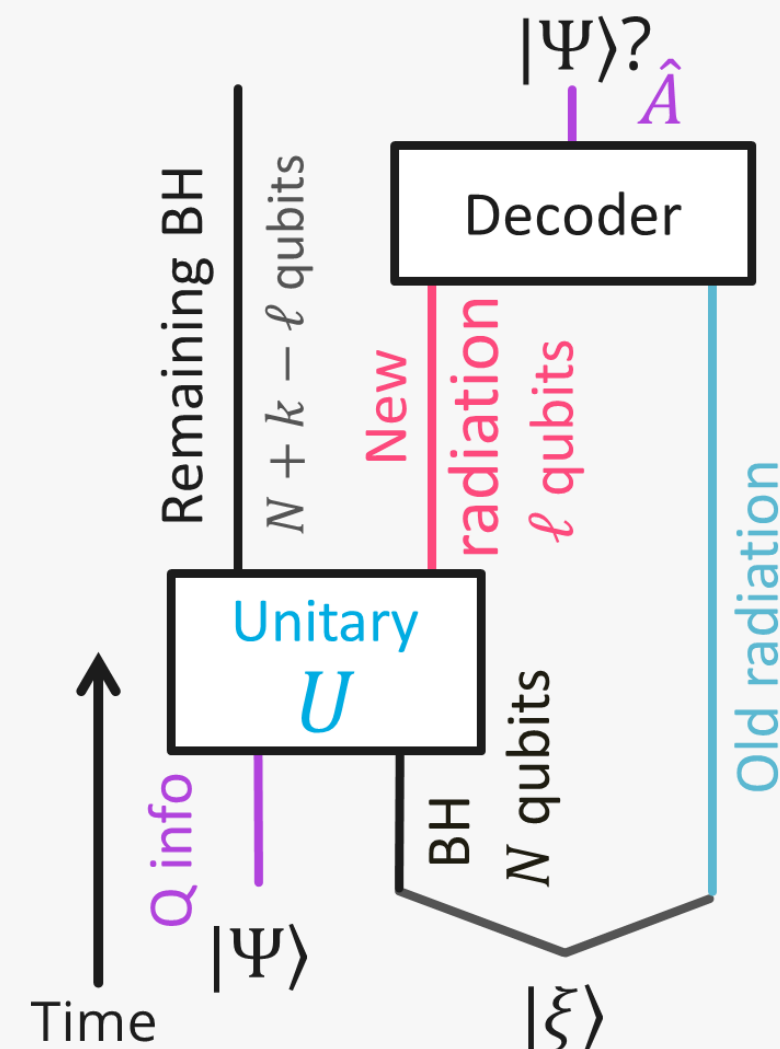
- Hayden & Preskill assume a **fully chaotic dynamics** = a **Haar random unitary**.

$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{\text{th}} - \ell)}$$

- Such a unitary  $U_{\text{Haar}}$  scrambles everything = **nothing** (e.g. energy) is conserved.

### What if a conservation law is taken into account?

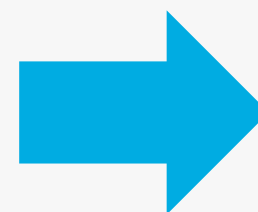
- Energy conservation (in some sense) was heuristically discussed in [Yoshida, PRD100:086001 '19] and [Liu, PRR 2:043164, '20].
- Comprehensive analysis was done in [YN, Wakakuwa, and Koashi, '19 & '20].



Hayden & Preskill

$U$

Haar random



Our model of dynamics

$\bigoplus_j U_j$

$j$ : conserved quantity

Haar random in each sector

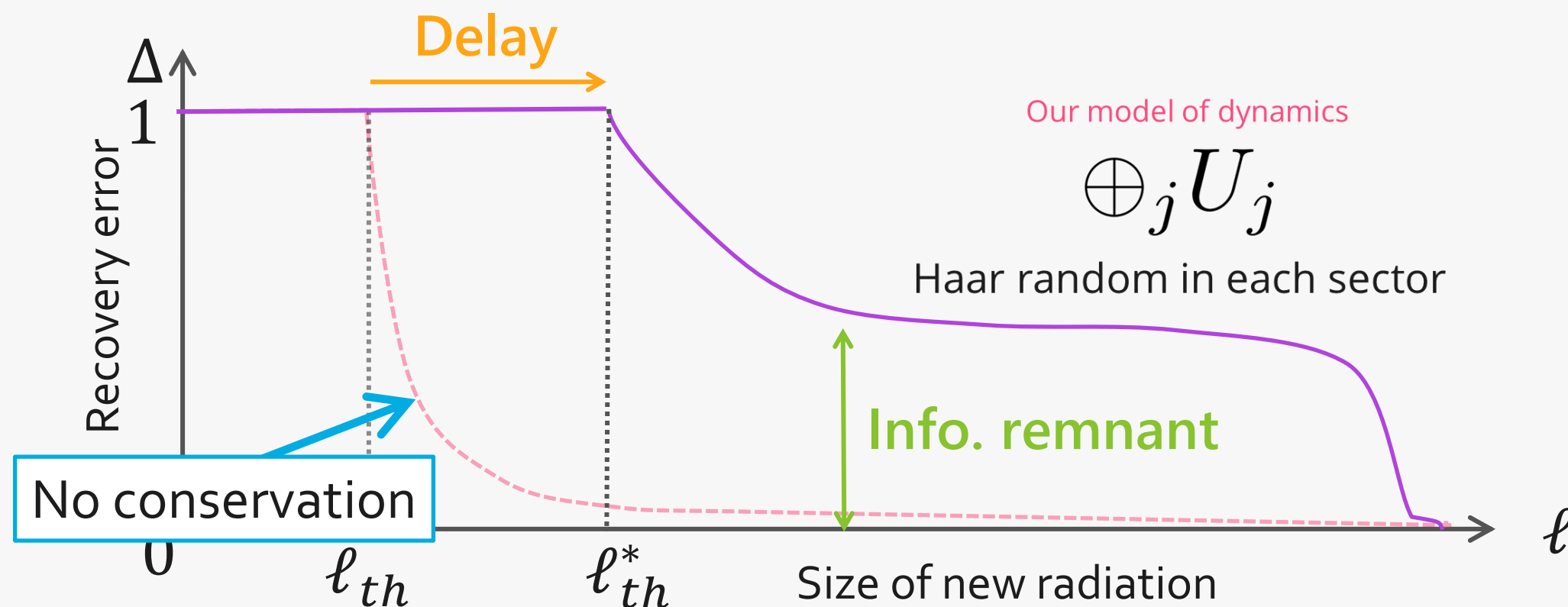
# The HP with energy conservation

*Beyond the Hayden-Preskill 2*

## What changes would occur in the HP recovery? 1

□ Two big changes by symmetry.

1. **Delay** of the threshold: the threshold  $\ell_{th}$  changes to  $\ell_{th}^*$ .
2. **Information remnant**: some information cannot be recovered from the radiation.



[Hayden & Preskill, '07]

$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{th} - \ell)}$$

$$\mathbb{E}_U [\Delta] \lesssim 2^{\frac{1}{2}(\ell_{th}^* - \ell)} + \Delta_{rem}$$

A formal expression is given by entropic quantities

# The HP with energy conservation

*Beyond the Hayden-Preskill 2*

## What changes would occur in the HP recovery? 2

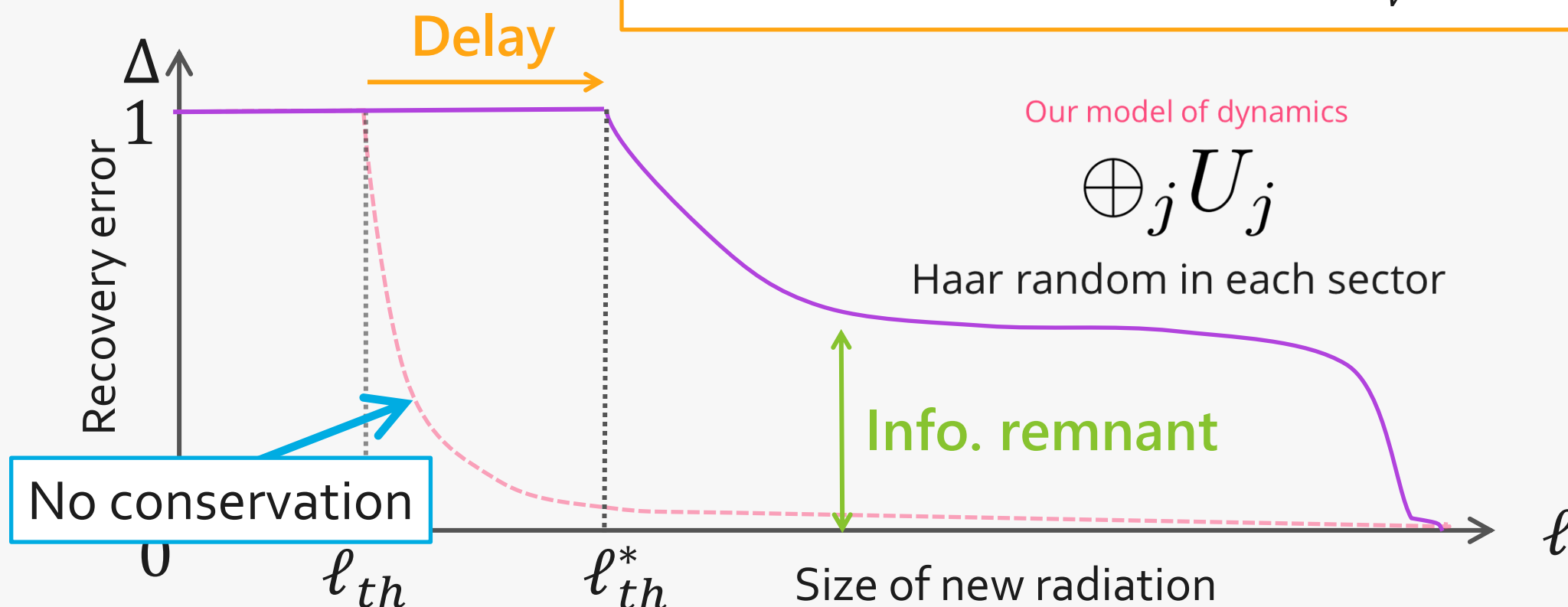
Two big changes by symmetry.

1. **Delay** of the threshold: the threshold  $\ell_{th}$  changes to  $\ell_{th}^*$ .

➤ In the case of **energy conservation**, it is characterized by the **head capacity of the BH**.

$$\text{delay} \propto \frac{E}{S(E)} \sqrt{|C_V|}$$

- $E$ : Energy of the BH
- $S(E)$ : Entropy of the initial BH
- $C_V$ : Heat capacity of the BH



$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{th} - \ell)}$$

$$\mathbb{E}_U [\Delta] \lesssim 2^{\frac{1}{2}(\ell_{th}^* - \ell)} + \Delta_{rem}$$

A formal expression is given by entropic quantities



# The HP with energy conservation

*Beyond the Hayden-Preskill 2*

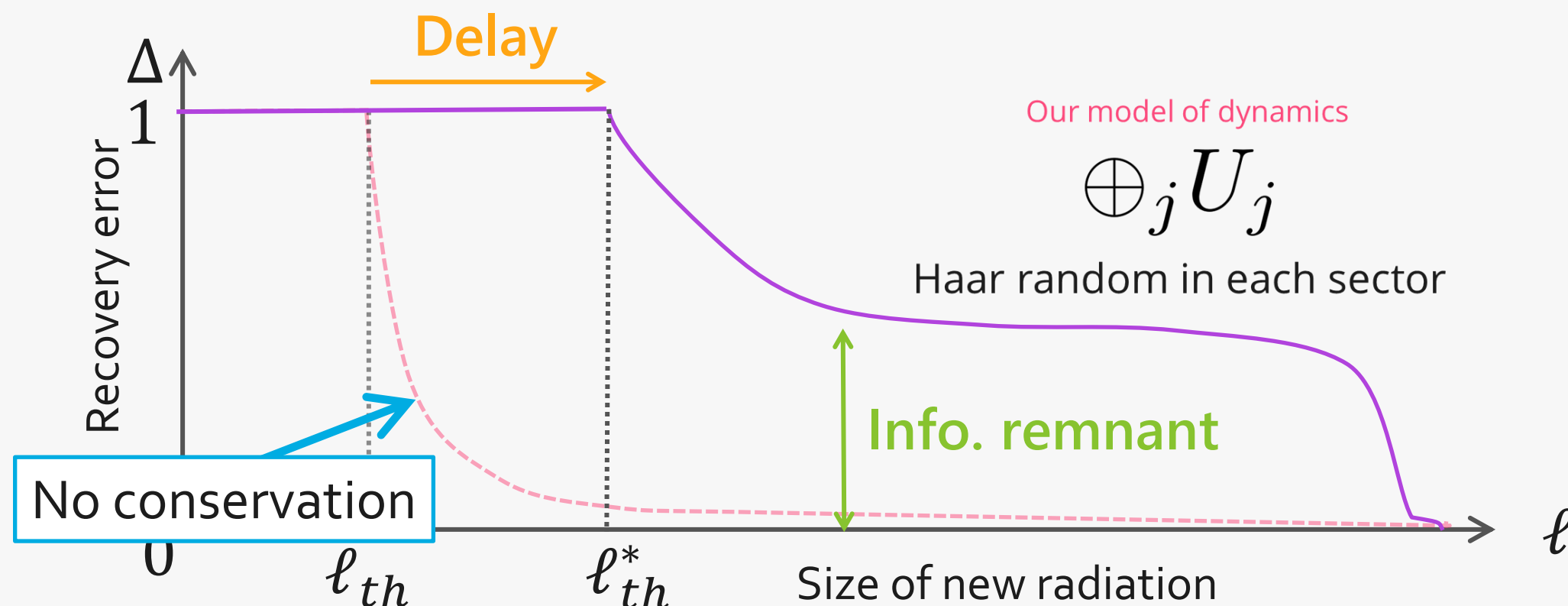
## What changes would occur in the HP recovery? 3

Two big changes by symmetry.

1. **Delay** of the threshold: the threshold  $\ell_{th}$  changes to  $\ell_{th}^*$ .
2. **Information remnant**: a certain amount of information cannot be recovered from the radiation.
  - Amount of the remnant  $\propto (\text{symmetry breaking of the initial BH})^{-1} \approx \text{Variance of energy}$

$$\text{delay} \propto \frac{E}{S(E)} \sqrt{|C_V|}$$

Interesting relations between **information** and **physics**



$$\mathbb{E}_{U_{\text{Haar}}} [\Delta(\ell|\xi, U)] \leq 2^{\frac{1}{2}(\ell_{th} - \ell)}$$

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A formal expression is given by entropic quantities



# Beyond the Hayden-Preskill

*Beyond the Hayden-Preskill 1*

## Open questions in the Hayden-Preskill thought experiment

The Hayden-Preskill thought experiment is a good playground over **quantum error correction**, **quantum chaos**, and **a quantum black hole!!**

### 1. No decoder is given.

- How can we actually decode information?

[YN & Koashi, in prep]

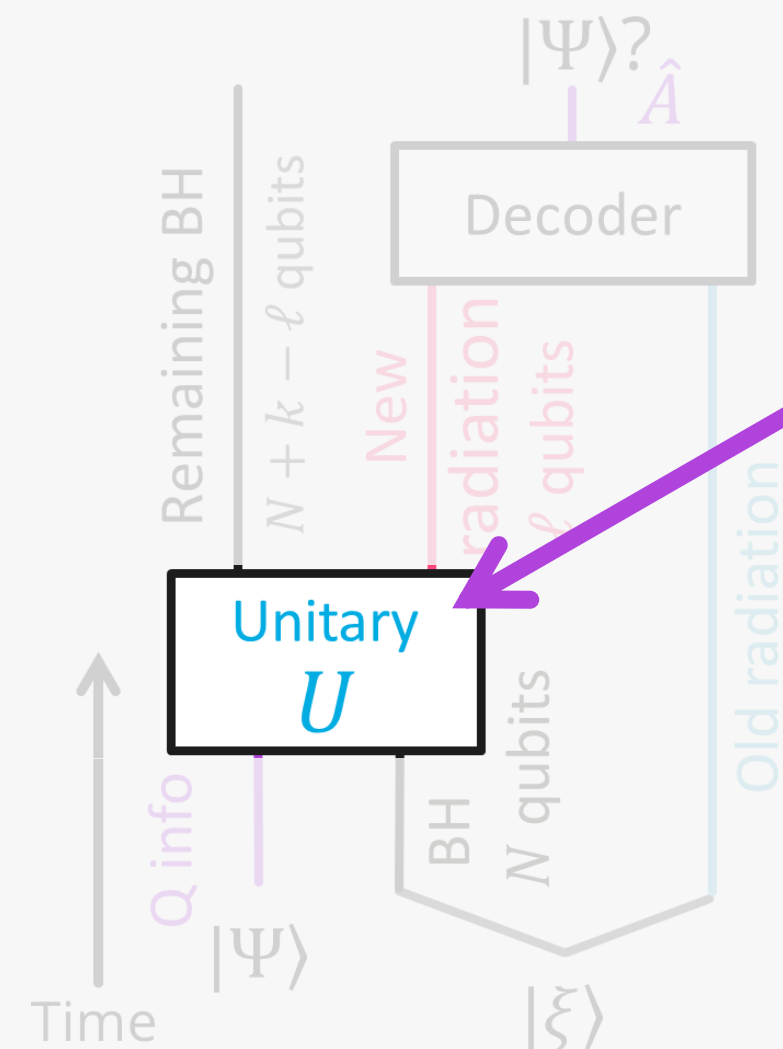
A decoder gives a “link” b/t quantum information and physics.

### 2. No energy conservation due to the Haar assumption.

- What if something is conserved?

[YN, Wakakuwa & Koashi, '19&'20]

Non-trivial “links” b/t quantum information and physics are revealed.



# Open questions in the Hayden-Preskill thought experiment

## 1. No decoder is given.

- [YN & Koashi, in prep]

## 2. No energy conservation due to the Haar assumption.

- [YN, Wakakuwa & Koashi, '19&'20]

### 3. No Hamiltonian (or Lagrangian).

- [YN & Tezuka, in prep]

# The Hayden-Preskill recovery in SYK model

Beyond the Hayden-Preskill 3

## Hayden-Preskill recovery by a Hamiltonian dynamics

### □ No Hamiltonian (or Lagrangian) is known

- The analysis is basically based on the assumption that the dynamics is given by  $\rho(t) = e^{-iHt}\rho(0)e^{iHt}$
- What if the dynamics is given by something other than  $\rho(t) = e^{-iHt}\rho(0)e^{iHt}$ ?

**STAY TUNED**

This is a non-trivial question.

### □ A Haar random unitary cannot be generated by a time-indep. Hamiltonian.

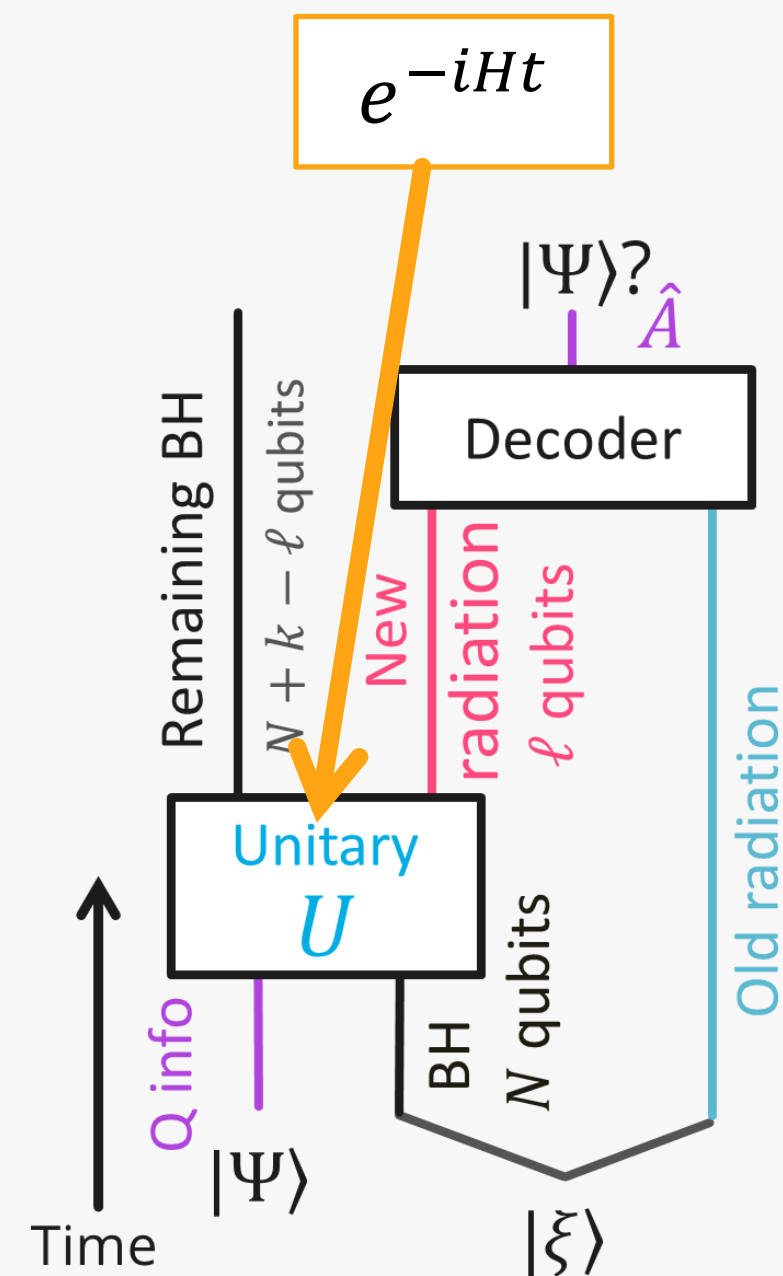
- No formal proof (to my knowledge), but highly believed.

.....however, who knows the Hamiltonian of a black hole?



In collaboration with M. Tezuka

Sachedev-Ye-Kitaev (SYK) model  
as a quantum dual!





# Outline of this talk

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Information, Chaos, and Black Holes in the **Quantum** regime.



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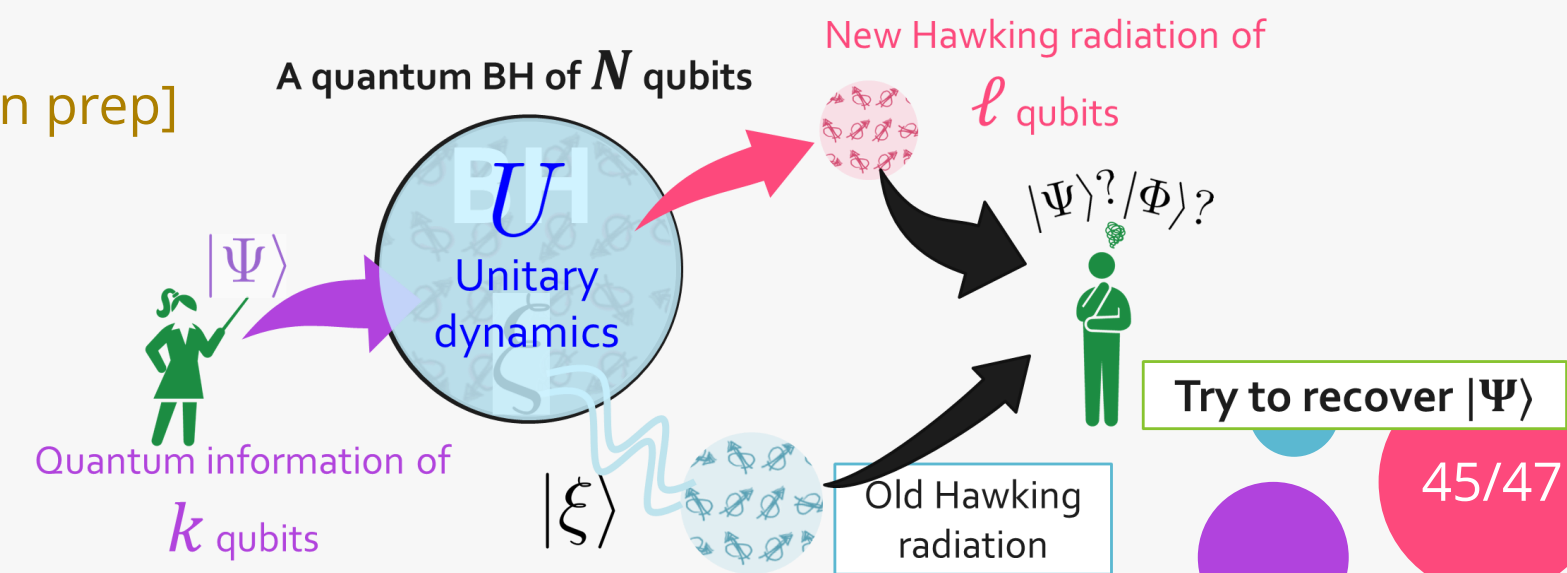
# Conclusion and outlooks

*Towards the collaboration over QI, condensed-matter, and high energy physics*

## Conclusion

### Error Correction, Chaos, and Black Holes in the Quantum regime.

- Chaotic dynamics (a Haar random unitary) is useful for correcting quantum noise.
- The Hayden-Preskill thought experiment is a canonical toy model about quantum error correction, quantum chaos, and a quantum black hole.
  1. Decoding the Hayden-Preskill [YN & Koashi, in prep]
  2. Energy conservation and the Hayden-Preskill recovery [YN, Wakakuwa & Koashi, '19&'20]
  3. The Hayden-Preskill recovery and SYK models [YN & Tezuka, in prep]



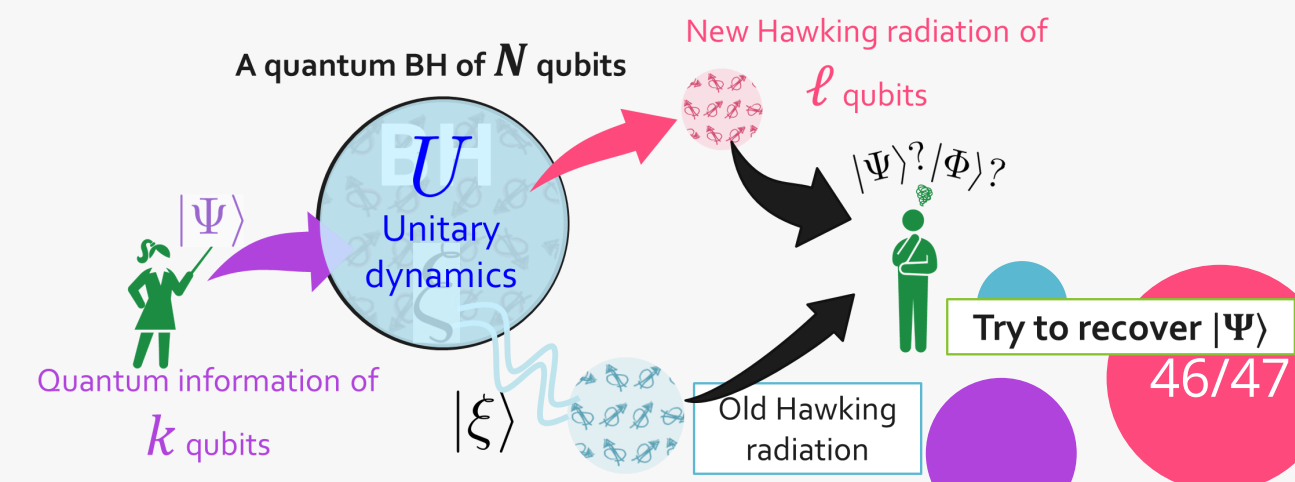
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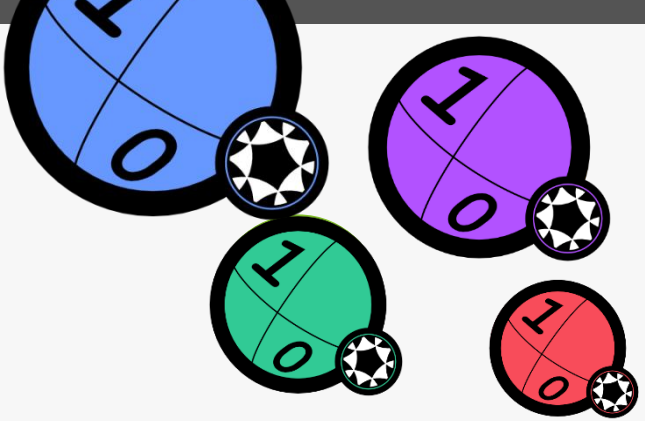
## Open questions

### Error Correction, Chaos, and Black Holes in the Quantum regime.

1. The **Hayden-Preskill thought experiment** has a little flavor of **black holes**....
  - It's about **quantum error correction**, **quantum chaos**, and a **quantum black hole**.
  - More flavor of quantum black holes?
2. **Quantum phase** vs the **Hayden-Preskill recovery**?
  - The HP recovery may succeed/fail in chaotic, MBL, and integrable phases.
  - Characterize quantum phases by the recovery?







Thank you 



for your attention







# Classical-to-Quantum decoder

*Decoding Hayden-Preskill*

## Classical-to-Quantum decoder 1

### Classical-to-Quantum decoder

