Workshop “String theory, gravity and cosmology (SGC2021), APCTP Korea (online), Nov 17 2021
Workshop “Application of Quantum Information in QFT and Cosmology”, U.Lethbridge (online), Nov 22 2021
Workshop “Theoretical Physics at CQUeST: Past, Present, and Future, CQUeST, Korea (online), Feb 10 2022
The 3rd Colloquium of Extreme Universe collaboration (online), Feb 28 2022

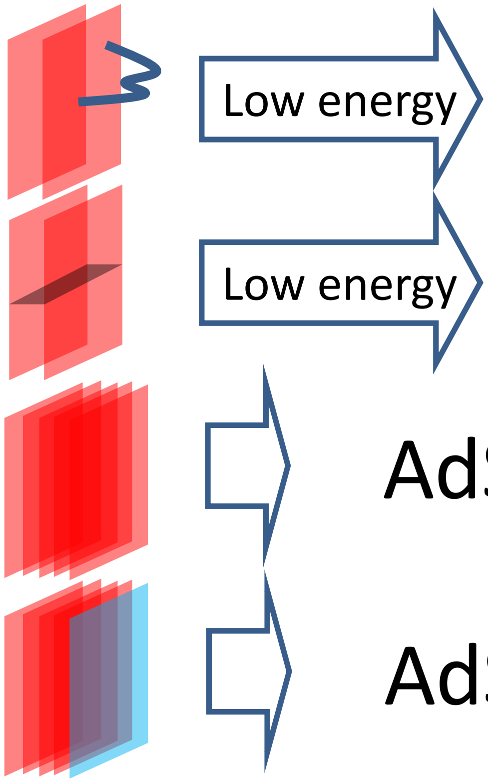
Chaos and Holography

Koji Hashimoto
(Kyoto U)

2112.11163 w/ K. Murata, N. Tanahashi, R. Watanabe

Based also on
2007.04746, 1903.04718, 2004.04381, 1903.04718, 1804.01737, 1803.06756, 1703.09435, 1610.06070, 1605.08124 w/ T. Akutagawa, K. B. Huh, K. Y. Kim, T. Miyazaki, K. Murata, T. Ota, T. Sasaki, N. Tanahashi, K. Yoshida, R. Yoshii, R. Watanabe)

24-year history of AdS/CFT



[Polchinski] 1995 — SUSY QFT

[Douglas] 1996 — Soliton

[Maldacena] 1998 — AdS = CFT

[Sakai Sugimoto] 2004 — AdS = QCD

[Ryu Takayanagi] 2006 — AdS = Entanglement

[Kitaev] 2014 — Horizon = Chaos

[Susskind] 2014 — Inside BH = Complexity

[Penington] 2019 — Worm hole = Information recovery

22-year history of AdS/CFT

What kind of QFT allows a gravity dual?

[Ryu Takayanagi] 2006	AdS = Entanglement
[Kitaev] 2014	Horizon = Chaos
[Susskind] 2014	Inside BH = Complexity
[Penington] 2019	Worm hole = Information recovery

Information Preservation and Weather Forecasting for Black Holes*

S. W. Hawking[1]

[1] *DAMTP, University of Cambridge, UK*

Abstract

It has been suggested [1] that the resolution of the information paradox for evaporating black holes is that the holes are surrounded by firewalls, bolts of outgoing radiation that would destroy any infalling observer. Such firewalls would break the CPT invariance of quantum gravity and seem to be ruled out on other grounds. A different resolution of the paradox is proposed, namely that gravitational collapse produces apparent horizons but no event horizons behind which information is lost. This proposal is supported by ADS-CFT and is the only resolution of the paradox compatible with CPT. The collapse to form a black hole will in general be chaotic and the dual CFT on the boundary of ADS will be turbulent. Thus, like weather forecasting on Earth, information will effectively be lost, although there would be no loss of unitarity.

Chaos and Holography

1. **Quantum chaos, redundantly defined**
2. **BH horizon is universally chaotic**
3. **How to look for “BH-like” QM**
4. **Conjecture: Chaos energy bound**

1 Quantum chaos, redundantly defined

1-1 Chaos : sensitivity to initial conditions

1-2 Example : Stadium billiard

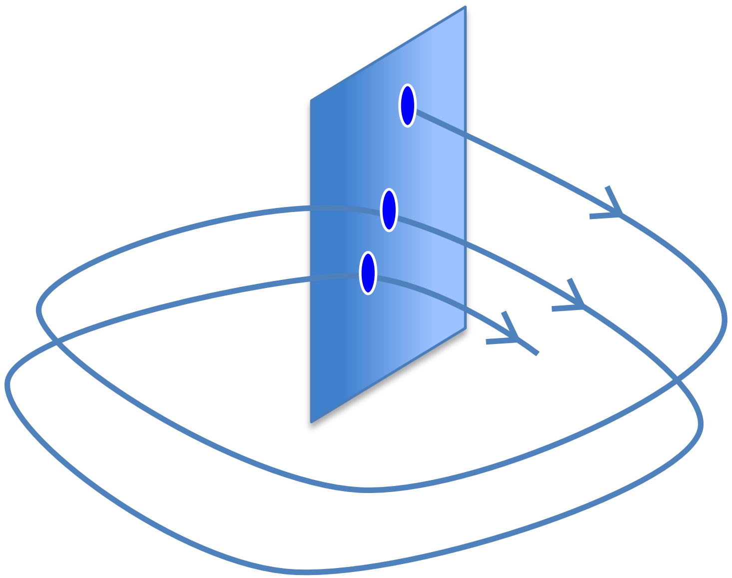
1-3 No quantum chaos?

1-4 Two definitions of quantum chaos

1-1 Chaos : sensitivity to initial conditions

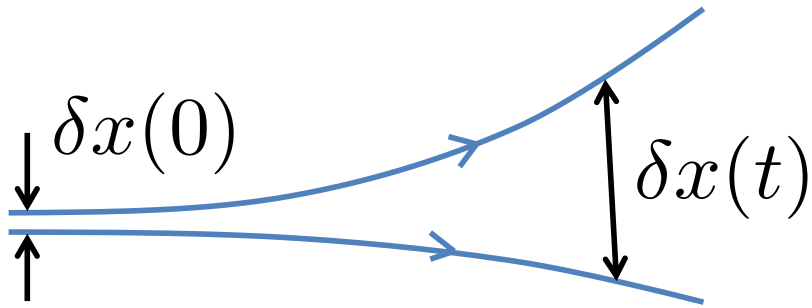
Classical chaos
= Non-periodic bounded orbits
sensitive to initial conditions
in non-linear deterministic dynamical systems

Poincare section,
scattered

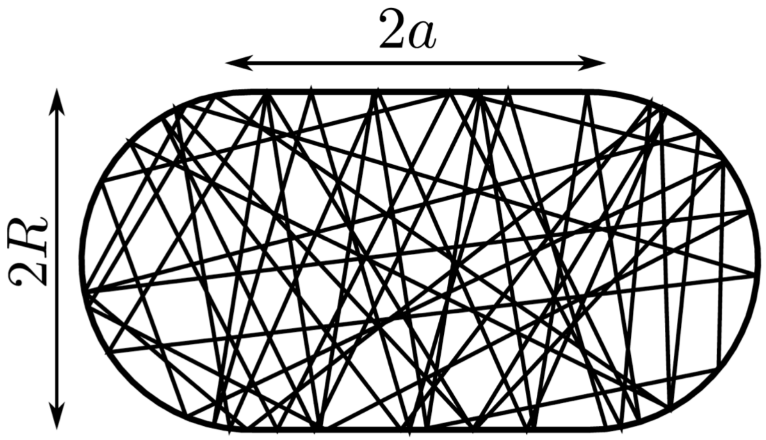


Lyapunov exponent λ ,
positive

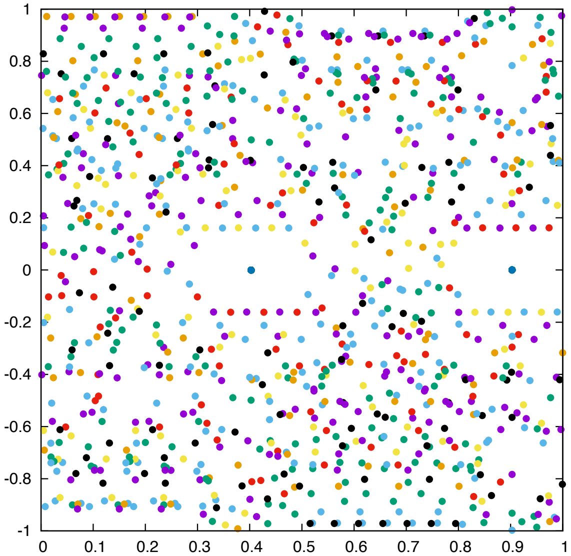
$$\delta x(t) \sim \delta x(0) \exp[\lambda t]$$



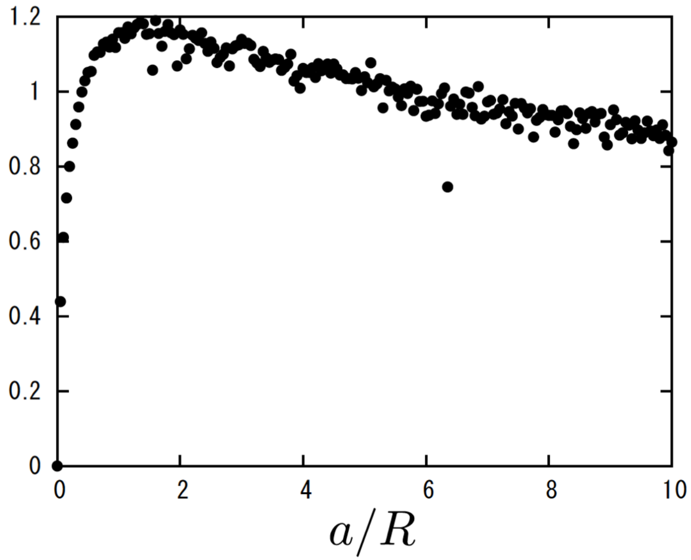
1-2 Example : Stadium billiard



Poincare section, scattered



Lyapunov exponent L, positive

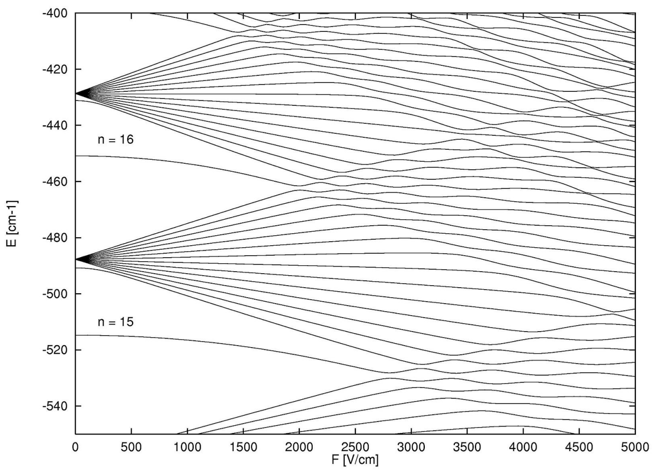


1-3 No quantum chaos?

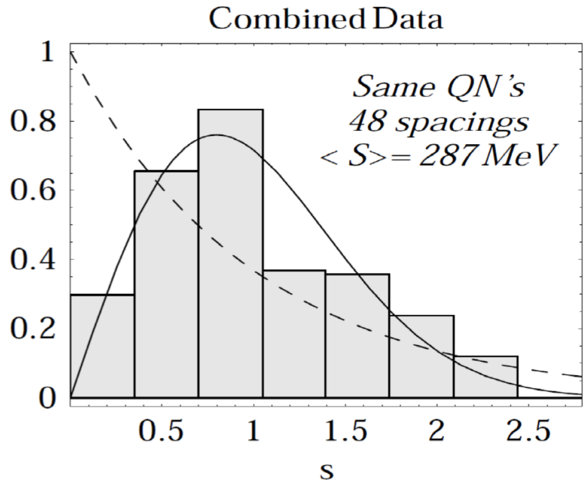
Reason 1 : Schroedinger eq. is linear.

Reason 2 : Infinitesimal change in initial condition
does not exist in quantum systems.

Quantum chaos, seen by energy level spacings



Atomic spectra of Lithium in electric field
[Courtney, Spellmeyer, Jiao, Kleppner, 95]



Hadron spectra spacings
[Pando-Zayas, 00]

1-4 Two definitions of quantum chaos

Def 1 "Chaos" : Quantization of classically chaotic system.

Energy level spacings follow Wigner distribution

[Muller, Heusler, Altland, Braun, Haake `09]

Def 2 "Scrambling" : Exponential growth of Out-of-time-order (OTO) correlator.

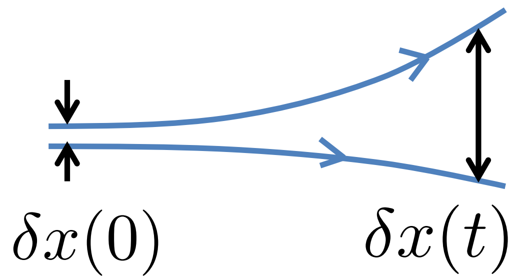
$$\langle x(t)p(0)x(t)p(0)\rangle \propto \exp[2\lambda t]$$

[Larkin, Ovchinnikov `69] [Kitaev `14] [Maldacena, Shenker, Stanford `15]

λ : Quantum Lyapunov exponent

Classical analogue:

$$\langle x(t)p(0)x(t)p(0)\rangle \sim \left(\frac{\delta x(t)}{\delta x(0)}\right)^2$$



2 BH horizon is universally chaotic

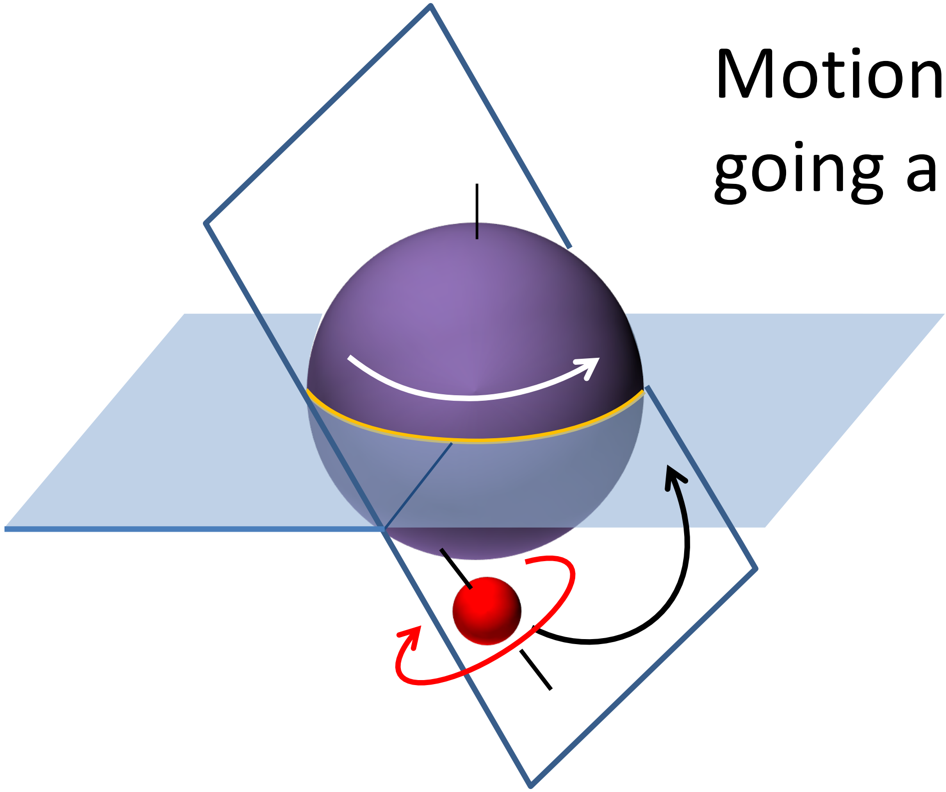
2-1 Chaotic motion around BH, known, but…

2-2 Lyapunov is Hawking temperature

2-3 Quantum Lyapunov upper bound

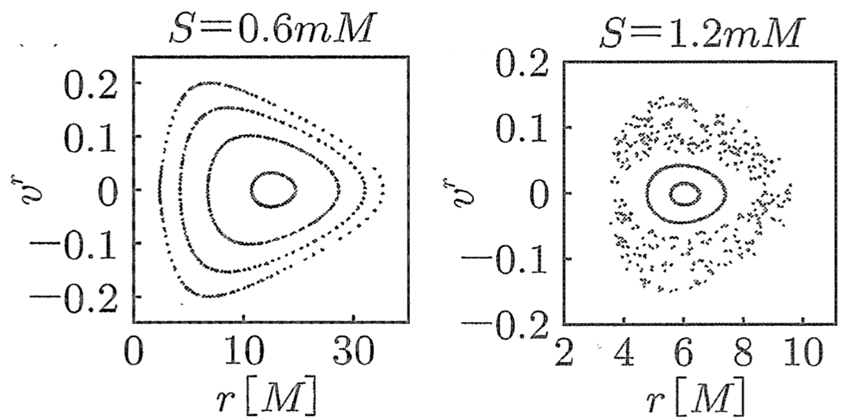
2-4 Chaotic motion near horizon is universal

2-1 Chaotic motion around BH, known, but…



Motion of a spinning particle going around Kerr BH

[Suzuki, Maeda '96]



But 1) the motion is far from horizon

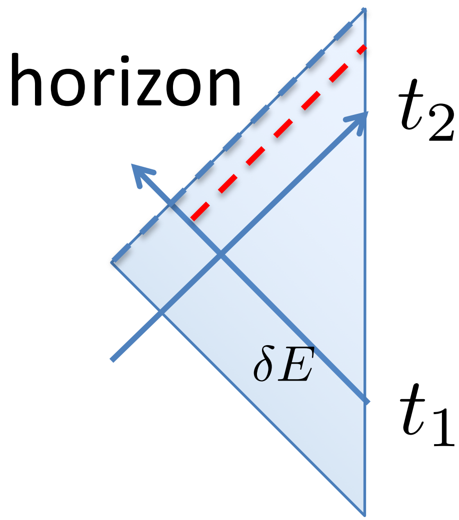
But 2) Geodesic motion around Schwarzschild or Kerr is integrable

2-2 Lyapunov is Hawking temperature

Black hole is the fastest scrambler? [Sekino, Susskind `08]

Shock wave delay : Lyapunov = Hawking temperature

[Shenker, Stanford `13,`14]



2d dilaton gravity dual to SYK

[Almheiri, Polchinski `14] [Engelsoy, Martens, Verlinde `16]

$$\delta t_2 = \frac{\delta E}{8\pi T M} e^{2\pi T (t_2 - t_1)}$$

2-3 Quantum Lyapunov upper bound

Quantum Lyapunov upper bound for thermal OTOC

$$\lambda \leq 2\pi T$$ [Maldacena, Shenker, Stanford `15]

Suggested from AdS/CFT with black holes

SYK model saturates the bound [Kitaev `15] [Maldacena, Stanford `16]

SYK (Sachdev-Ye-Kitaev) model [Kitaev `15][Sachdev,Ye `95]

(1+0 dim., *N* Majorana fermions, disordered interaction)

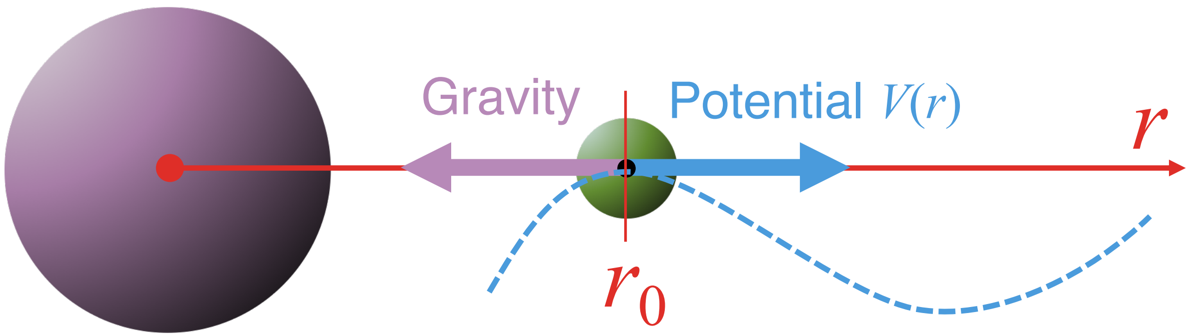
$$H = \frac{-1}{4!} \sum_{i,j,k,l=1}^{N} j_{[ijkl]} \psi_i \psi_j \psi_k \psi_l \qquad \left(\sum_{j,k,l=1}^{N} \langle j_{ijkl} j_{ijkl} \rangle = 6J^2 \right)$$

Generalizations: [Gross, Rosenhaus `16] [Witten `16][Murugan Stanford Witten `17]

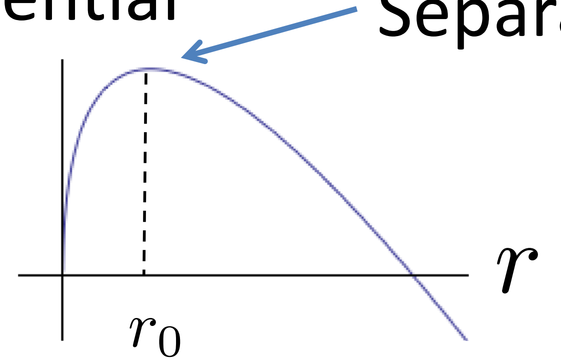
Solvable at Large *N* and strong coupling $\beta J \to \infty$

2-4 Chaotic motion near horizon is universal

[Tanahashi, KH `16]



Universal Lyapunov exponent = Hawking temperature



$$\mathcal{L} = -m\sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu} - V(X)$$

$$\sim C\left[\dot{r}^2 + \frac{1}{(2\pi T)^2}(r-r_0)^2\right]$$

22-year history of AdS/CFT

What kind of QFT allows a gravity dual?

Look for a quantum system with $\lambda = 2\pi T$

[Ryu Takayanagi] 2006	AdS = Entanglement
[Kitaev] 2014	Horizon = Chaos
[Susskind] 2014	Inside BH = Complexity
[Penington] 2019	Worm hole = Information recovery

3 How to look for "BH-like" QM

3-1 Computing OTOC is difficult

OTOC in N=4 Supersymmetric Yang-Mills theory?

-- Difficult, because…

1) Energy eigenstate wave function, unknown

2) Perturbation theory does not give long time behavior

3) It's out-of-time order, so path integral does not work

Ex) OTOC of ϕ^4 matrix theory. [Stanford `15]

Ex) OTOC of IP model and IOP model.

[Michel, Polchinski, Rosenhaus, Suh `16]

3-2 How to look for "BH-like" QM

1. Prepare your Hamiltonian and solve energy eigenstates

$$H|E\rangle = E|E\rangle$$

2. Calculate microcanonical OTOC

$$f(t;E) \equiv \langle E|x(t)p(0)x(t)p(0)|E\rangle$$

3. Convert it to thermal OTOC

$$f(t;T) \equiv \int dE\,\rho(E) f(t;E) e^{-\beta E}$$

4. Read your thermal Lyapunov and check!

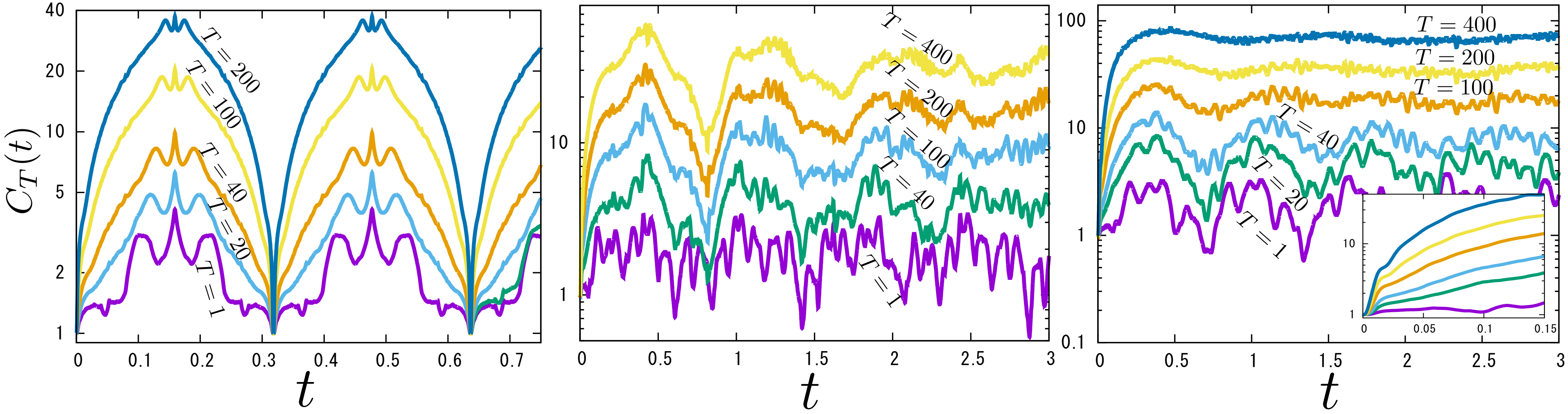
$$\lambda_{\text{thermal}}(T) = \lim_{t:\text{large}} \frac{1}{t} \log f(t;T)$$

3-3 Quantum billiards : no scrambling

[Murata, Yoshii, KH `17]

Particle in a box	Circle billiard	Stadium billiard

OTOC



Harmonic oscillator $H = p^2 + \frac{\omega^2}{4} x^2$

OTOC : $C_T(t) = \cos^2 \omega t$ Periodic, no scrambling.

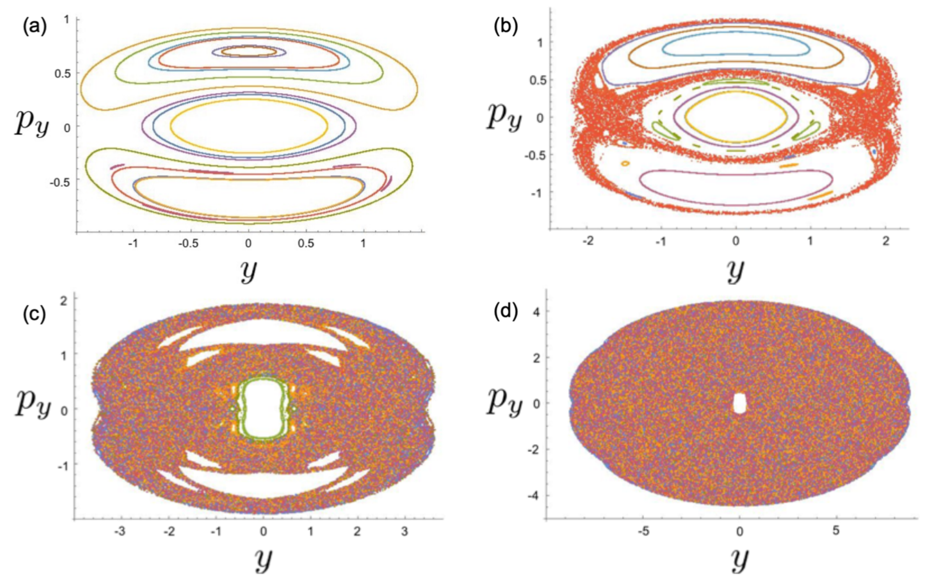
3-4 Coupled harmonic oscil. is scrambled

[Akutagawa, Sasaki, Watanabe, KH `20]

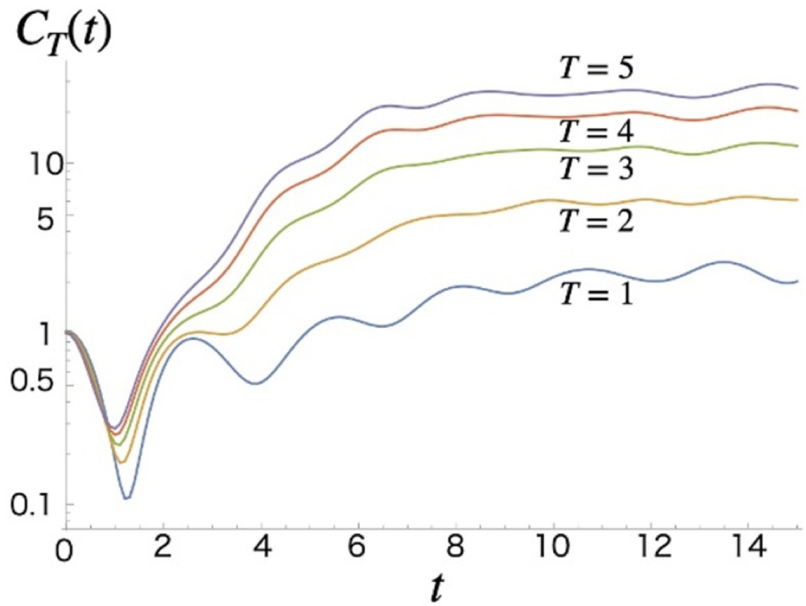
Hamiltonian is essence of BFSS matrix model

$$H = p_x^2 + p_y^2 + \frac{1}{4}(x^2 + y^2) + \frac{1}{10}x^2y^2$$

Classical Poincare sections



OTOC



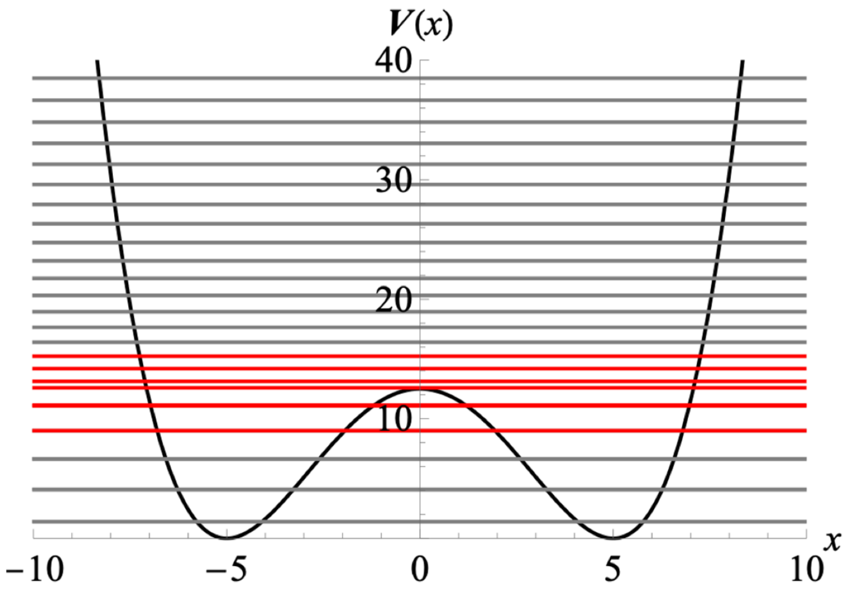
3-5 Inverted harm. oscil., scrambled w/o chaos

[Huh, Kim, Watanabe, KH `20]

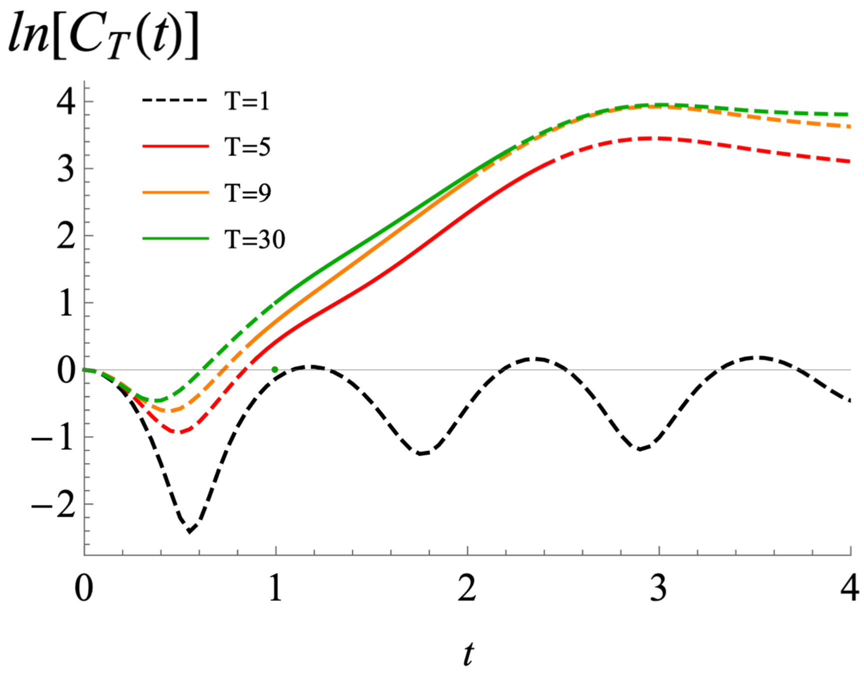
Classical one-dim. system does not give chaos

$$H \equiv p^2 + g\left(x^2 - \frac{\lambda^2}{8g}\right)^2 \qquad \lambda = 2, g = 1/50$$

Quantum energy levels



OTOC



Cf) Analyses of pure IHO [Bhattacharyya, Chemissany, Haque, Murugan, Yan `20]

3-6 Caution! Scrambling $\neq$ Chaos

	Def. 1 "Chaos" (Classical $\lambda > 0$)	Def. 2 "Scrambling" (Quantum $\lambda > 0$)
Harm. oscil.		
Billiard	✓	
Coupled harm. oscil.	✓	✓
Inverted harm. oscil.		✓

4 Conjecture: Chaos energy bound

4-1 Conjecture

4-2 General billiards satisfy the bound

4-3 General 2d mechanics satisfies the bound

4-4 Saturation necessary for black holes

4-5 Discrimination of BSM models?!

1-4 How to look for "BH-like" QM

1. Prepare your Hamiltonian and solve energy eigenstates

$$H|E\rangle = E|E\rangle$$

2. Calculate microcanonical OTOC

$$f(t;E) \equiv \langle E|x(t)p(0)x(t)p(0)|E\rangle$$

3. Convert it to thermal OTOC

$$f(t;T) \equiv \int dE\, \rho(E) f(t;E) e^{-\beta E}$$

4. Read your thermal Lyapunov and check!

$$\lambda_{\text{thermal}}(T) = \lim_{t:\text{large}} \frac{1}{t} \log f(t;T)$$

4-1 Conjecture

[Murata, Tanahashi, Watanabe, KH `21]

Chaos energy bound

$$c \leq 1 \quad \text{for} \quad \lambda(E) \propto E^c \quad (E \to \infty)$$

$\lambda(E)$: Lyapunov exponent for energy E

3. Convert it to thermal OTOC

$$f(t;T) \equiv \int dE\, \rho(E) f(t;E) e^{-\beta E}$$

When the microcanonical Lyapunov grows faster than linearly in energy, you cannot go to canonical!

4-1 Conjecture

Chaos energy bound

$$c \leq 1 \quad \text{for} \quad \lambda(E) \propto E^c \quad (E \to \infty)$$

$\lambda(E)$: Lyapunov exponent for energy E

"Lyapunov does not grow faster than linearly in energy in the high energy limit"

4-1 Conjecture

Chaos energy bound

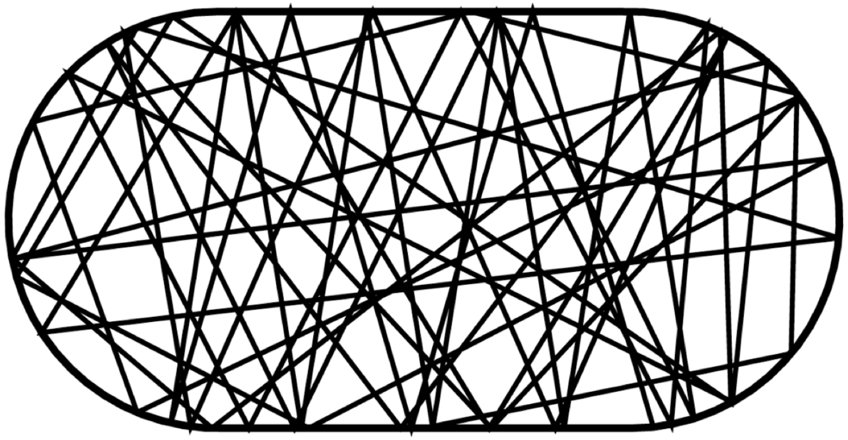
$$c \leq 1 \quad \text{for} \quad \lambda(E) \propto E^c \quad (E \to \infty)$$

$\lambda(E)$: Lyapunov exponent for energy E

Satisfied by :

Double penduram, Henon-Heiles system,
Particle in black hole geometries, Nonlinear sigma models,
Fermi-Pasta-Uram β-model, large N coupled rotors,
Chaotic string in AdS soliton geometry,
Homogeneous Yang-Mills mechanics,
Yang-Mills theories on a lattice, ...

4-2 General billiards satisfy the bound



$$H = p^\gamma$$

$$(\gamma > 0)$$

$$\lambda(E) \propto \dot{x} \equiv \frac{\partial H}{\partial p} = \gamma p^{\gamma-1} \propto E^{(\gamma-1)/\gamma}$$

$$\Rightarrow \quad c = 1 - 1/\gamma < 1$$

4-3 General 2d mechanics satisfies the bound

General 2d Hamiltonian :

$$H = y^p \dot{x}^a + x^q \dot{y}^b + x^m y^n$$

- Consistent motion needs $a > 1\,, \quad b > 1$
- Bounded orbits need $\dfrac{a(bm-nq)}{ab-pq} > 0\,, \; \dfrac{b(an-mp)}{ab-pq} > 0$

$\Rightarrow$ Scaling symmetry determines energy exponent *c*,

$$c = -\frac{(ab-pq)-(an-pm)-(bm-nq)}{b(an-pm)+a(bm-nq)} < 1$$

4-4 Saturation necessary for black holes

Chaos energy bound
(quantum / classical,
finite / large N)

$$\lambda \propto E^c, c \leq 1$$

The chaos bound
(quantum large N)
[Maldacena, Shenker, Stanford `15]

$$\lambda_T \leq \frac{2\pi T}{\hbar}$$

Saturation

$$\lambda \propto E$$

Large N ⇒

$$\lambda_T \propto T$$

$$\lambda_T = \lambda(E = \gamma T)$$

(Density of states $\rho(E) = E^\gamma$)

4-5 Discrimination of BSM models?!

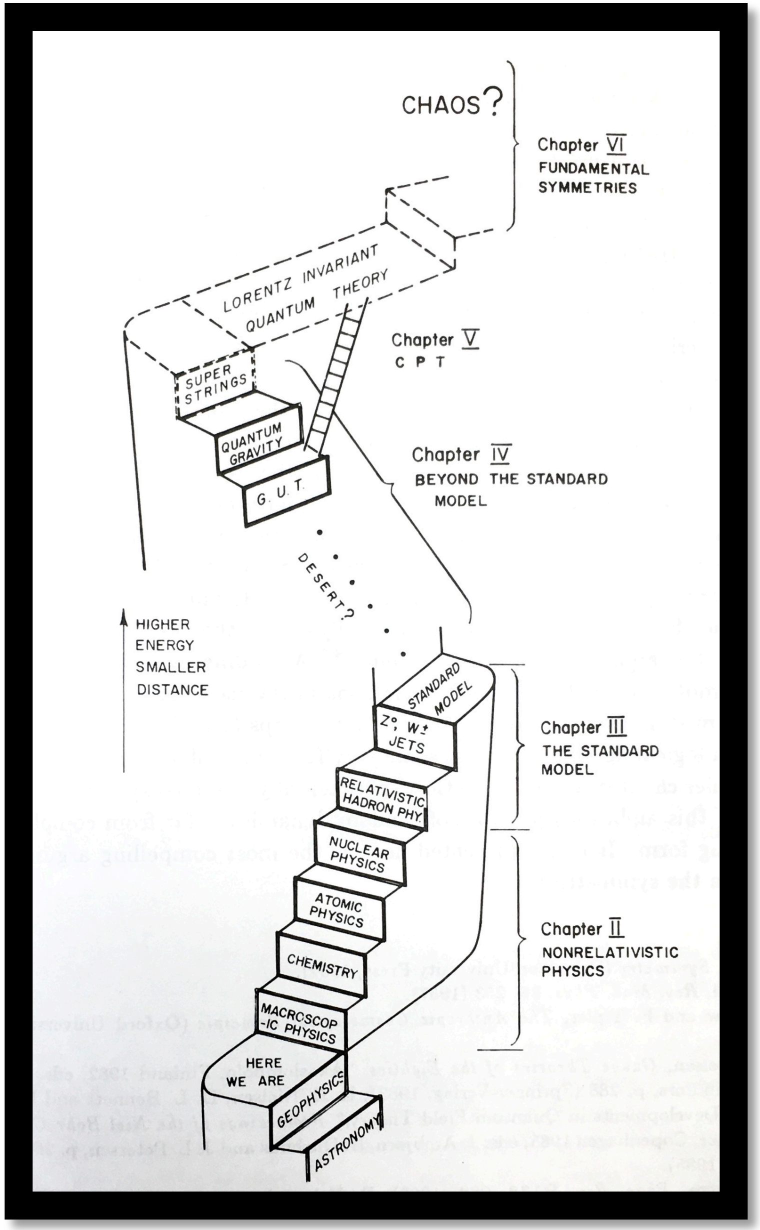
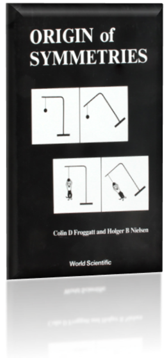


Fig.1.1 of “Origin of Symmetries” by Froggatt and Nielsen



Lyapunov is Kolmogorov-Sinai entropy, and saturation means the fastest entropy production at high energy.

QFT saturating the chaos energy bound is selected by entropic principle, as the theory of extreme universe?!

Chaos and Holography

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