dS_3/CFT_2 correspondence

(3D de Sitter space/2D CFT correspondence)

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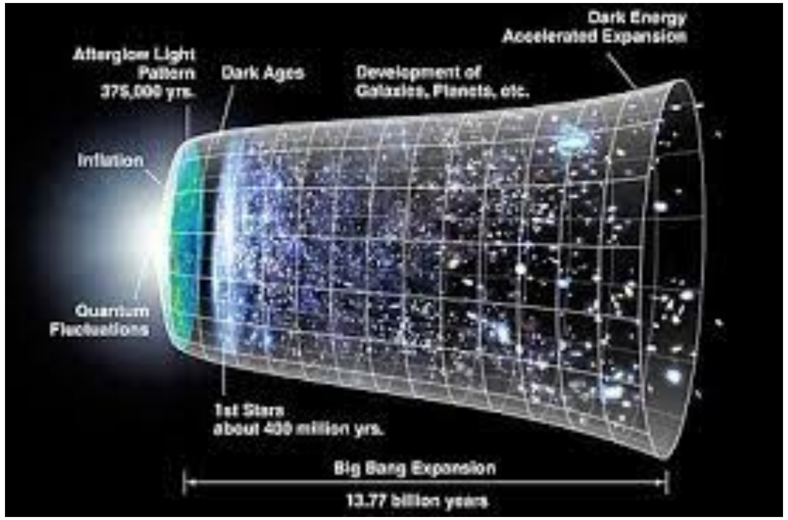
The Natural Laws of Extreme Universe

de Sitter gravity and dS/CFT correspondence

- de Sitter gravity
  - Quantum gravity in de Sitter (dS) space is important since the space approximately appears at inflation era and current universe
- dS/CFT correspondence
  - dS/CFT should play important roles on understanding quantum gravity on dS as AdS/CFT does
  - Unfortunately, dS/CFT has not been understood yet compared with AdS/CFT



From wikipedia

dS/CFT and its example

- dS/CFT is poorly understood as very few concrete examples are available
  - de Sitter space is not allowed as supersymmetric solution to supergravity [Maldacena-Nunez'00], and it is very difficult to realize it in superstring theory
  - Dual CFT is known to be exotic like with negative/imaginary central charge
- A concrete example is given by higher-spin holography

Higher-spin gravity on dS_4 at the classical limit	$\Longleftrightarrow$	3d $Sp(N)$ vector model at the large N limit

[Anninos-Hartman-Strominger'11]

  - Analytic continuation of duality between higher-spin gravity on AdS_4 and 3d $O(N)$ vector model [Klebanov-Polyakov'02]

Our proposal

- We propose a new duality involving 3d Einstein gravity

Einstein gravity on dS_3 at the classical limit (+ matters)	$\Longleftrightarrow$	2d SU(2) WZW model with large central charge (+ extra sectors)

- Evidence
  - Partition functions are computed both from gravity theory and dual CFT with large central charge $(k \to -2)$ and find perfect match at the limit
  - Our proposal can be regarded as an analytic continuation of duality with (higher-spin) gravity on AdS_3 [Gaberdiel-Gopakumar'10]

Plan of this talk

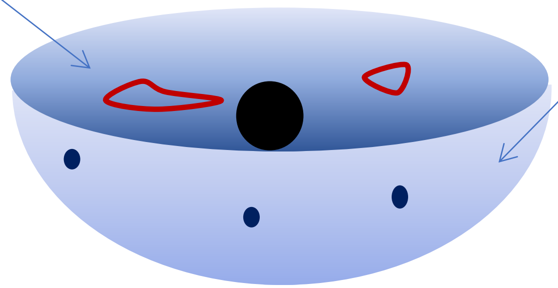
- Introduction
- dS/CFT correspondence
- Our proposal and gravity partition functions
- Analytic continuation of Gaberdiel-Gopakumar duality
- Relation to quantum information theory
- Conclusion

dS/CFT correspondence

AdS/CFT correspondence

- AdS/CFT correspondence [Maldacena'97]

(*d*+1)-dim. gravity theory on anti-de Sitter space (e.g. string theory on $AdS_5 x S^5$) $\underset{②}{\overset{①}{\rightleftarrows}}$ *d*-dim. conformal field theory (e.g. 4d N=4 U(N) gauge theory)



- Two approaches

① Strongly coupled gauge theory from classical gravity

② Quantum gravity from well-defined CFT

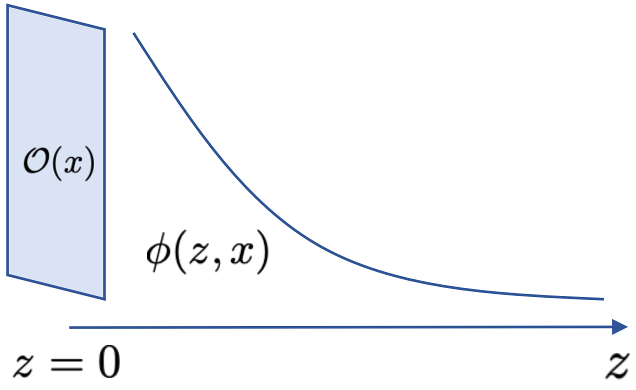
AdS/CFT duality map

- Map between CFT operators and AdS bulk fields

CFT operators $\mathcal{O}(x)$ $\Longleftrightarrow$ AdS bulk fields $\phi(z, x)$

  - Poincare coordinates for AdS space (boundary at $z = 0$)

$$ds^2 = \frac{L_{\text{AdS}}^2}{z^2}\left(dz^2 - dt^2 + \sum_{j=1}^{d-1}(dx^j)^2\right)$$



- GKP-Witten relation'98
  - CFT correlation functions ⇔ gravity scattering amplitudes

$$\left\langle \exp\left(\int d^d x \phi_0(x)\mathcal{O}(x)\right)\right\rangle = Z_{\text{Gravity}}[\phi(z,x)|_{\text{boundary}} = \phi_0(x)]$$

$$\left(\left\langle \prod_{i=1}^{n} \mathcal{O}(x_i)\right\rangle = \prod_{i=1}^{n} \frac{\delta}{\delta\phi_0(x_i)}\left\langle \exp\left(\int d^d x \phi_0(x)\mathcal{O}(x)\right)\right\rangle\Bigg|_{\phi_0=0}\right)$$

Wave functional of de Sitter universe

- A way to describe gravity theory on dS space is utilized wave functional of universe

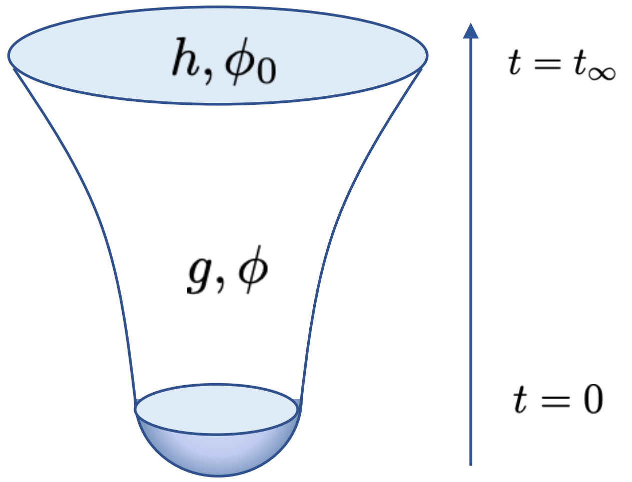
$$\Psi_{\mathrm{dS}}[h,\phi_0] = \int \mathcal{D}g\mathcal{D}\phi \exp iS[g,\phi]$$
$$\text{with } g = h, \phi = \phi_0 \text{ at } t = t_\infty$$

- Hartle-Hawking wave functional is used as a particularly useful choice
  - Universe starts from the hemisphere

$$ds^2 = L_{\mathrm{dS}}^2(d\tau^2 + \cos^2\tau d\Omega_d^2)\ (-\pi/2 \le \tau < 0)$$

  - continues to Lorentzian dS space

$$ds^2 = L_{\mathrm{dS}}^2(-dt^2 + \cosh^2 t d\Omega_d^2)\ (t \ge 0)$$



dS/CFT correspondence

[Maldacena'02]

- Wave functional of universe is proposed to be the same as generating function of correlation functions in dual CFT

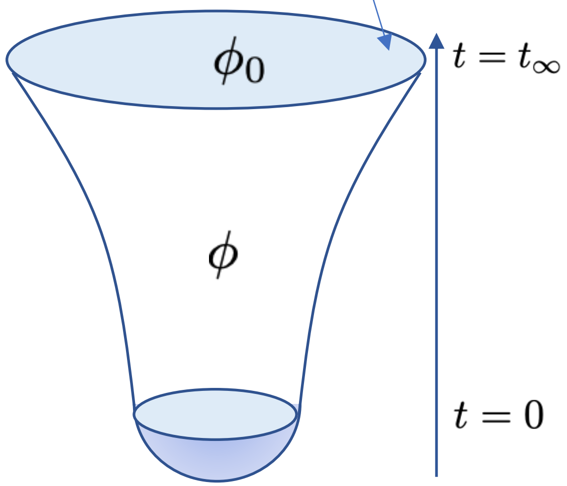
$$\Psi_{\mathrm{dS}}[\phi_0] = \left\langle \exp\left(\int d^dx \phi_0(x)\mathcal{O}(x)\right)\right\rangle$$
$$= \exp\left[\frac{1}{2}\int d^dx d^dy \langle \mathcal{O}(x)\mathcal{O}(y)\rangle \phi_0(x)\phi_0(y) + \cdots\right]$$

- Late time correlators can be computed as expectation values

$$\langle \phi_0(\vec{k})\phi_0(-\vec{k})\rangle = \int \mathcal{D}\phi_0 |\Psi_{\mathrm{dS}}|^2 \phi_0(\vec{k})\phi_0(-\vec{k}) = -\frac{1}{2\mathrm{Re}\langle \mathcal{O}(\vec{k})\mathcal{O}(-\vec{k})\rangle}$$

$$\langle \phi_0(\vec{k}_1)\phi_0(\vec{k}_2)\phi_0(\vec{k}_3)\rangle = \int \mathcal{D}\phi_0 |\Psi_{\mathrm{dS}}|^2 \phi_0(\vec{k}_1)\phi_0(\vec{k}_2)\phi_0(\vec{k}_3) = \frac{2\mathrm{Re}\langle \prod_i \mathcal{O}(\vec{k}_i)\rangle}{\prod_i(-2\mathrm{Re}\langle \mathcal{O}(\vec{k}_i)\mathcal{O}(-\vec{k}_i)\rangle)}$$

Correlators are computed by dual Euclidean CFT

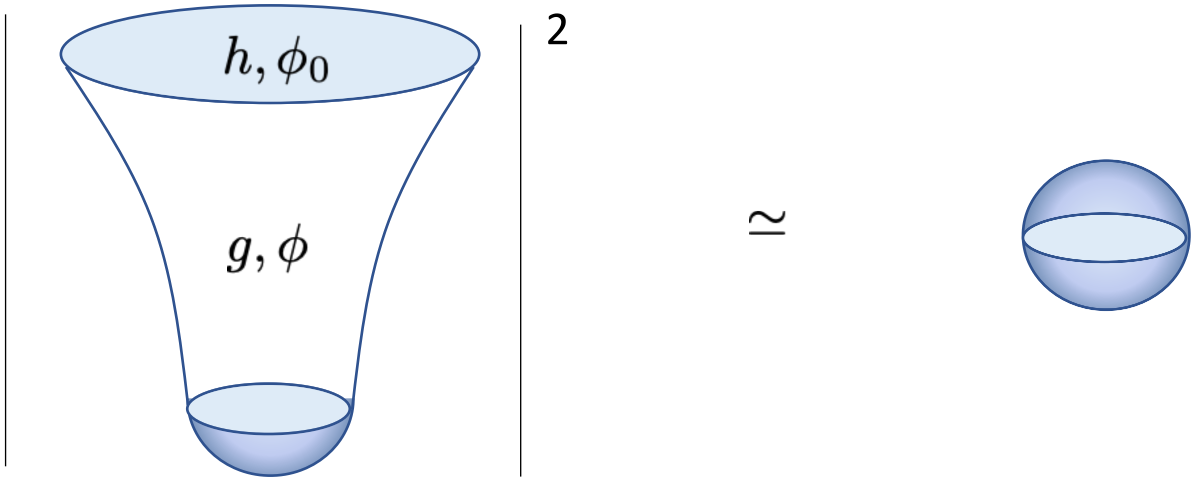


Gravity partition function on sphere

- Gravity partition function is computed from square of wave functional

$$Z_{\rm G} = \int \mathcal{D}h\mathcal{D}\phi_0 |\Psi_{\rm dS}[h,\phi_0]|^2 \qquad \left(\Psi_{\rm dS}[h,\phi_0] = \int \mathcal{D}\phi\mathcal{D}\phi \exp iS[g,\phi]\right)$$

- Sphere partition function from Hartle-Hawking wave functional
  - Lorentzian dS part leads to only a pure phase which cancels out



Our proposal and gravity partition functions

Our proposal

- We propose a new duality involving 3d Einstein gravity

Einstein gravity on dS_3 at the classical limit (+ matters)	$\Longleftrightarrow$	2d SU(2) WZW model with large central charge (+ extra sectors)

- Gravity partition functions
  - Partition functions on $\mathbb{S}^3$ computed from Einstein gravity are reproduced from 2d SU(2) WZW model large central charge $(k \to -2)$
  - We utilize Witten's method to compute partition functions on $\mathbb{S}^3$ from 2d SU(2) WZW model [Witten'89]

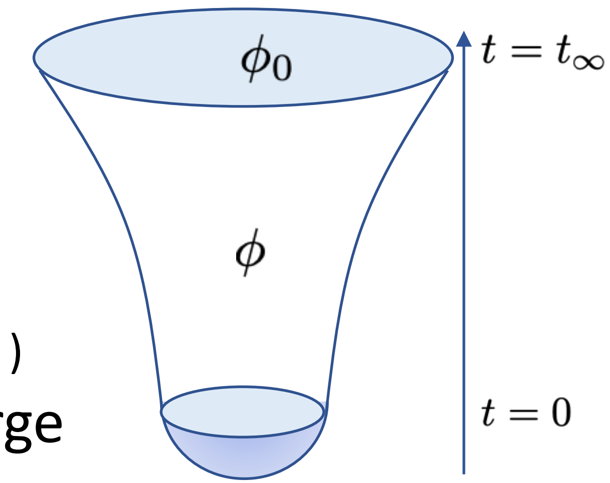
Central charge and the level of WZW model

- Virasoro symmetry appears near the future infinity with central charge [Strominger'01]

$$c = i\frac{3L_{\mathrm{dS}}}{2G_N} \equiv ic^{(g)}$$

Radius of de Sitter space

Newton constant (classical limit $G_N \to 0$)



  - Dual CFT is quite strange as it has pure imaginary central charge

- 2d SU(2) WZW model with level *k*

$$S = \frac{k}{2\pi}\int d^2z[g^{-1}dg \cdot g^{-1}dg] + k\,\Gamma_{\mathrm{WZ}}, \;\; g \in \mathrm{SU}(2) \;\; \text{with} \;\; c = \frac{3k}{k+2}$$

  - To reproduce the requirement from dual gravity, we consider a strange limit

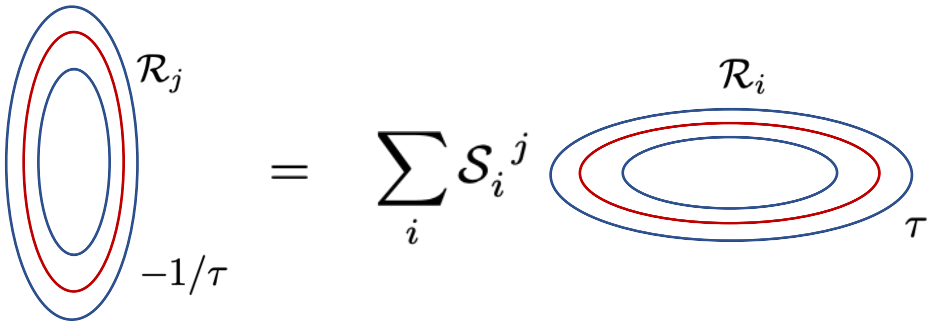
$$\boxed{k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})}$$

Sketch of Witten's method

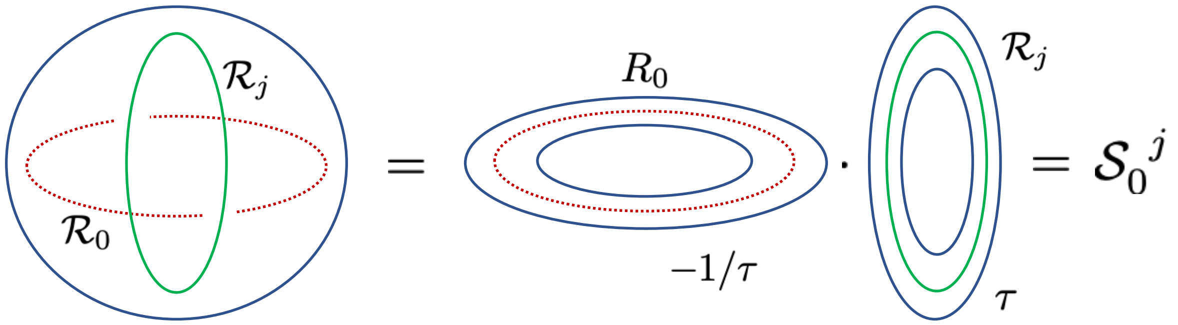
[Witten'89]

- We use Witten's method to compute partition functions on $\mathbb{S}^3$ via modular *S*-matrix of 2d SU(2) WZW model
  - Wilson loop in spin-*j* rep $\mathcal{R}_j$ can be inserted
- Modular *S*-matrix relates two amplitudes of solid tori related by *S*-transformation

$$S : \tau \to -1/\tau$$



- Sphere can be obtained by gluing two tori related by *S*-transformation via surgery



Partition function on 3-sphere

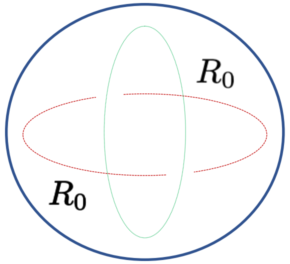
[YH-Nishioka-Takayanagi-Taki'21;'22]

- CFT computation
  - Modular *S*-matrix of 2d SU(2) WZW model

$$\mathcal{S}_j{}^l = \sqrt{\frac{2}{k+2}} \sin\left[\frac{\pi}{k+2}(2j+1)(2l+1)\right]$$

  - Vacuum partition function at the leading order in $1/c^{(g)}$

$$\left(k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2}) \right)$$



$$\left| \quad \right|^2 = |\mathcal{S}_0{}^0|^2 \simeq e^{\frac{\pi c^{(g)}}{3}}$$

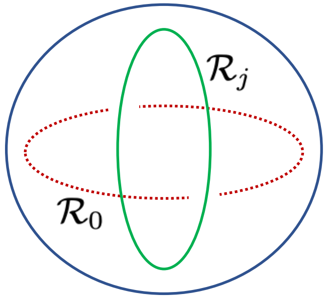
- Gravity computation
  - Definition of classical partition function

$$Z_{\mathrm{G}} = e^{-I_{\mathrm{G}}}, \quad I_{\mathrm{G}} = -\frac{1}{16\pi G_N}\int d^3x\sqrt{g}(R - 2L_{\mathrm{dS}}^{-2})$$

  - Classical action on $\mathbb{S}^3$: $I_{\mathrm{G}} = -\frac{\pi L_{\mathrm{dS}}}{2G_N} = -\frac{\pi c^{(g)}}{3}$ ⟵ Reproduces CFT computation!!

Wilson loop and bulk excitation

- Partition function on $\mathbb{S}^3$ with Wilson loop in rep. $\mathcal{R}_j$



$$= \mathcal{S}_0^{\ j}$$

  - The Wilson line on $\mathbb{S}^3$ ⇔ Operator in 2d WZW model [Witten'89]
  - Conformal dimension of CFT operator

$$\Delta_j = \frac{2j(j+1)}{k+2} \equiv i\Delta^{(g)}$$

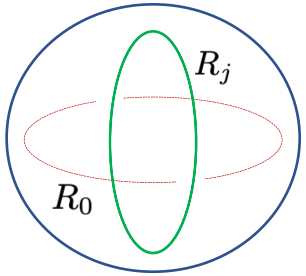
- dS/CFT map to bulk excitation energy

$$\Delta^{(g)} = L_{\mathrm{dS}} E_j$$

Partition function on Euclidean dS$_3$ black hole

[YH-Nishioka-Takayanagi-Taki'21;'22]

- CFT computation
  - The modular *S*-matrix leads at the leading order in $1/c^{(g)}$ $\left(k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2}) \right)$

$\left|\right.$  $\left.\right|^2$

$$= |\mathcal{S}_0{}^j|^2 \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}}$$

- Gravity computation
  - Bulk excitation creates Euclidean dS$_3$ black hole

$$ds^2 = L_{\rm dS}^2 \left[(1 - 8G_N E_j - r^2) d\tau^2 + \frac{dr^2}{1 - 8G_N E_j - r^2} + r^2 d\phi^2 \right]$$

  - Classical action on the geometry

$$I_{\rm G} = -\frac{\pi}{3} c^{(g)} \sqrt{1 - 8G_N E_j}$$ $\Longleftarrow$ Reproduces CFT computation!!

Analytic continuation of Gaberdiel-Gopakumar duality

Gaberdiel-Gopakumar duality for AdS_3

[Castro-Gopakumar-Gutperle-Raeymaekers'11;Gaberdiel-Gopakumar'12]
(see [Gaberdiel-Gopakumar'10] for original proposal)

- A version of Gaberdiel-Gopakumar duality

Higher-spin gravity on AdS_3 at the classical limit ⟺ 2d coset model at large central charge $\frac{\mathrm{SU}(N)_k \times \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$

Spins of gauge fields

$$s = 2, 3, \ldots, N$$

The coset describes analytic continuation of W_N-minimal model, which was shown to reduce to Toda theory [Creutzig-YH'21]

- The simplest case with N=2

Einstein gravity on AdS_3 with matter at the classical limit ⟺ 2d coset model at large central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

Central charge and the level of coset model

- A version of Gaberdiel-Gopakumar duality

Einstein gravity on AdS_3 with matter at the classical limit $\Longleftrightarrow$ 2d coset model at large central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

- Comparison of central charge

  - Near the boundary of AdS_3 there appears Virasoro symmetry with central charge [Brown-Henneaux'86]

$$c = \frac{3L_{\mathrm{AdS}}}{2G_N} \to \infty$$

  - The central charge of the coset is

$$c = 1 - \frac{6}{(k+2)(k+3)}$$

  - To have large central charge, we have to set

$$k = -2 - \frac{6}{c} + \mathcal{O}(c^{-2})$$

Analytic continuation from AdS_3 to dS_3

[YH-Nishioka-Takayanagi-Taki'21;'22] (see [Ouyang'11] for previous attempt)

- Formally we can move from AdS_3 to dS_3 by replacing $L_{\mathrm{AdS}} \to i\, L_{\mathrm{dS}}$
  - Note that this is possible only for (higher-spin) holography
- Gaberdiel-Gopakumar duality becomes

Einstein gravity on dS_3 with matter at the classical limit	$\Longleftrightarrow$	2d coset model with imaginary central charge $\frac{SU(2)_k \times SU(2)_1}{SU(2)_{k+1}}$

- Comparison of central charge

$$c = 1 - \frac{6}{(k+2)(k+3)} = ic^{(g)}, \quad c^{(g)} = \frac{3L_{\mathrm{dS}}}{2G_N} \to \infty \quad \Longleftrightarrow \quad k \to -2 + i\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})$$

At the leading order in $1/c^{(g)}$ only $SU(2)_k$ part dominates and the duality reduces to our proposal of dS_3/CFT_2 correspondence

Relation to quantum information theory

Holographic entanglement entropy

[YH-Nishioka-Takayanagi-Taki'22]
(see also [Narayan'15;Sato'15;Miyaji-Takayanagi'15] for previous works)

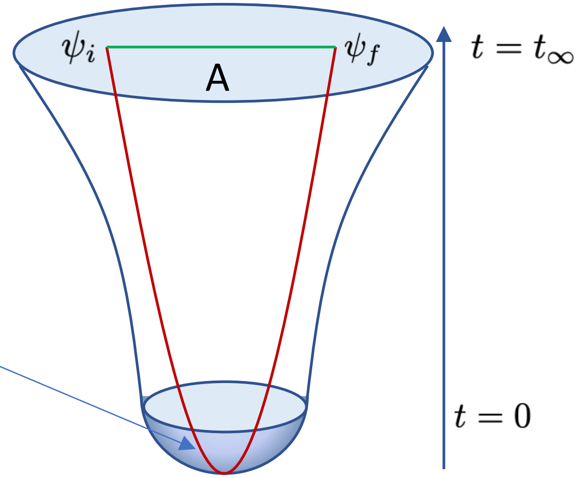
- Entanglement entropy of dual CFT at $t = t_\infty$
  - Metric of dS_3: $ds^2 = -dt^2 + \cosh^2 t(d\psi^2 + \sin^2\psi d\phi^2)$
  - Subsystem A: $\psi_i \leq \psi \leq \psi_f, \phi = 0$
- Holographic computation
  - Geodesic distance

$$D(\psi_i, \psi_f) = 2it_\infty + i \log \sin^2[(\psi_f - \psi_i)/2] + \pi$$

Real part

  - Holographic entanglement entropy [Ryu-Takayanagi'06]

$$S_A = \frac{D(\psi_i, \psi_f)}{4G_N} = \frac{ic^{(g)} t_\infty}{3} + \frac{ic^{(g)}}{6} \log \sin^2[(\psi_f - \psi_i)/2] + \frac{c^{(g)}\pi}{6}$$



  - Reproduce the standard CFT result [Calabrese-Cardy'04]

$$S_A = \frac{c}{6} \log \left[\frac{4 \sin^2[(\psi_f - \psi_i)/2]}{\epsilon^2} \right] \quad \left(c = ic^{(g)}, \epsilon = i2e^{-t_\infty} \right)$$

Analytic continuation from AdS to dS

de Sitter entropy

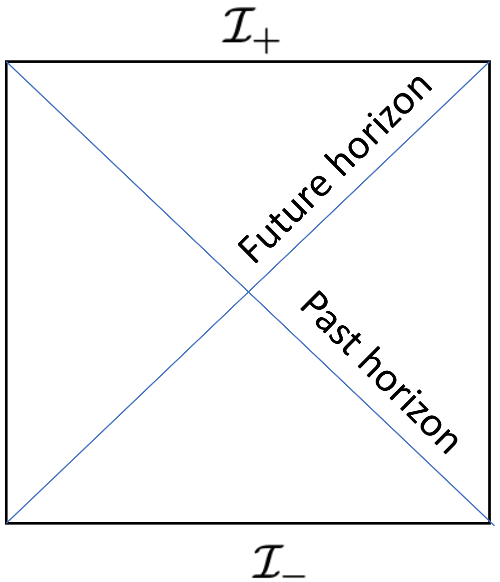
[YH-Nishioka-Takayanagi-Taki'21;'22]
(see also [McGough-Verlinde'13] for AdS_3)

- Quantum gravity should explain the origin of black hole entropy $S_{\mathrm{BH}} = \dfrac{A}{4G_N}$
- Similarly dS gravity should explain dS entropy
  - The metric of dS_3 black hole

$$ds^2 = L_{\mathrm{dS}}^2 \left[-(1 - 8G_N E_j - r^2)dt^2 + \frac{dr^2}{1 - 8G_N E_j - r^2} + r^2 d\phi^2 \right]$$

  - dS entropy associated with horizon $r = \sqrt{1 - 8G_N E_j}$ is

$$S_{\mathrm{dS}} = \frac{A}{4G_N} = \frac{\pi}{3} c^{(g)} \sqrt{1 - 8G_N E_j}$$



- Topological entanglement entropy for our system is

$$S_{\mathrm{top}} = \log |S_0{}^j|^2 \simeq \frac{\pi}{3} c^{(g)} \sqrt{1 - 8G_N E_j} \Longrightarrow$$ reproduces the gravity result

[Kitaev-Preskill'05;Levin-Wen'05]

Conclusion

Summary & future problems

- Summary
  - dS/CFT correspondence should play important roles on understanding quantum gravity on dS space
  - New dS/CFT is proposed between 3d classical Einstein gravity and 2d SU(2) WZW model with $k \to -2$ limit
  - Evidence is provided by computing gravity partition functions and relating to (higher-spin) AdS_3 holography
  - Lorentzian contributions and quantum corrections in our dS/CFT were also analyzed [YH-Nishioka-Takayanagi-Taki'22]
- Future problems
  - Pursue on relation to quantum information theory furthermore
  - Bulk correlation functions at late time and relation to in-in formulation are now investigated [Chen-YH-Nishioka in progress] (see also [Sleight-Taronna'20;'21])