Information dynamics in the long-range interacting systems

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In collaboration with
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Tomotaka Kuwahara (Riken) : Part 2

Content

- Short-range versus Long-range interactions in many-body systems

Part 1

- Long-range effect in information propagation

Lieb-Robinson physics

Part 2

- Suppression of information propagation due to measurement

Measurement-induced phase transition

- Conclusion

Short-range versus Long-range interactions in many-body systems

- Short-range interacting many-body systems

e.g., Heisenberg model

$$H = \sum_{i=1}^{n} [J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + BS_i^z]$$

- Long-range interacting many-body systems

Power-law interaction $V(r) \propto r^{-\alpha}$ in our context

e.g., Heisenberg model with the power-law interaction

$$H = \sum_{\boldsymbol{r},\boldsymbol{r}'} \frac{J}{|\boldsymbol{r}-\boldsymbol{r}'|^{\alpha}} \boldsymbol{S}_{\boldsymbol{r}} \cdot \boldsymbol{S}_{\boldsymbol{r}'} + \sum_{\boldsymbol{r}} BS_{\boldsymbol{r}}^z$$

- Change of thermodynamics due to long-range interaction

Nonadditivity : Entropy can be nonconcave

A. Campa, T. Dauxois, and S. Ruffo, Phys. Rep. (2009)

Phase transition : A. Campa, A. Giansanti, and D. Moroni, Phys. Rev. E. (2000)

Quasi-stationary state: V. Latora, Rapisarda, C. Tsallis, Phys. Rev. E. (2001)

Suppression of chaos: D. Baguchi and C. Tsallis, Phys. Rev. E. (2016)

Long-range interactions are ubiquitous

- Long-range interaction in nature $V(r) \propto r^{-\alpha}$

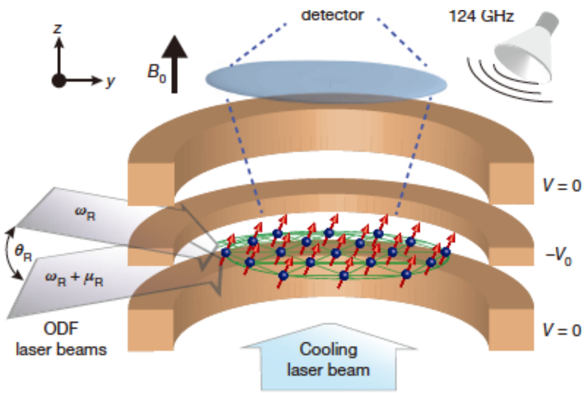
  $\alpha = 1$ Gravitational field

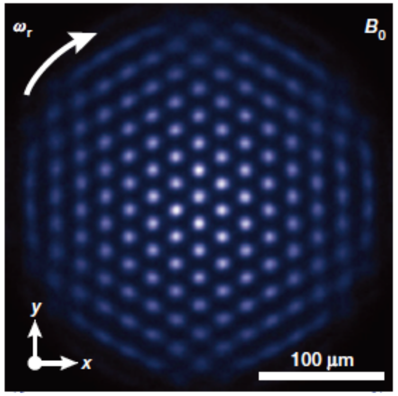
  $\alpha = 3$ Magnetic dipole interaction

  $\alpha = 6$ Lenard-Jones potential

- One can tailor interaction range in recent cold atomic gases , Rydberg atoms, and in ion trapped systems

$$H = \sum_{\boldsymbol{r}} \sum_{\boldsymbol{r}'} \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|^{\alpha}} \sigma^z_{\boldsymbol{r}} \sigma^z_{\boldsymbol{r}'}$$





Even global coupling !

J. W.Britton, et.al., Nature (2012)

They may provide Black-Hole-like physics !?

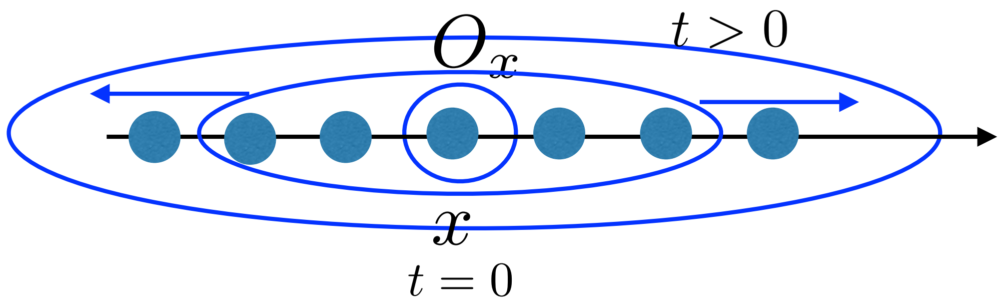
The main theme in this talk

What is the effect of Long-range interaction in information dynamics ?

Information ? and information dynamics ?

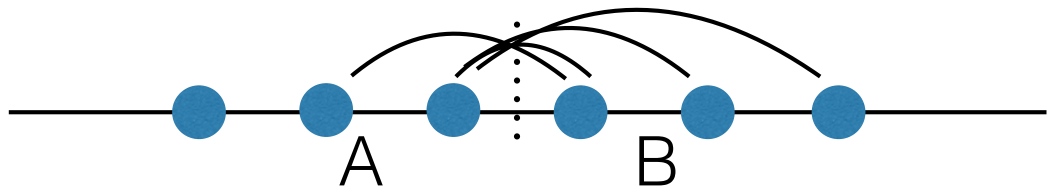
(1) In wider context, information dynamics means operator spreading.

$$e^{iHt}O_x e^{-iHt} = O_x + it\,[H, O_x] + \cdots$$



(2) More specifically, in quantum information, it usually means Shannon entropy. Information dynamics is the dynamics of entropy, or dynamics of entanglement.

$$S(t) = -\mathrm{Tr}_A\, \rho_A(t) \ln \rho_A(t)$$



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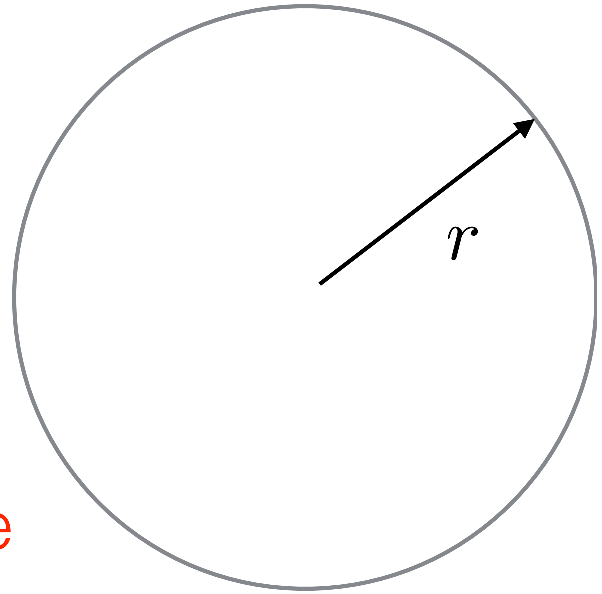
Part I: Information propagation

- Information propagation as operator spreading

  It has connection to Black-hole physics, especially in the context of scrambling, giving the time scale releasing information from BH

- Time that information reaches



$t_{\mathrm{reach}} \sim r$: indicating linear light cone

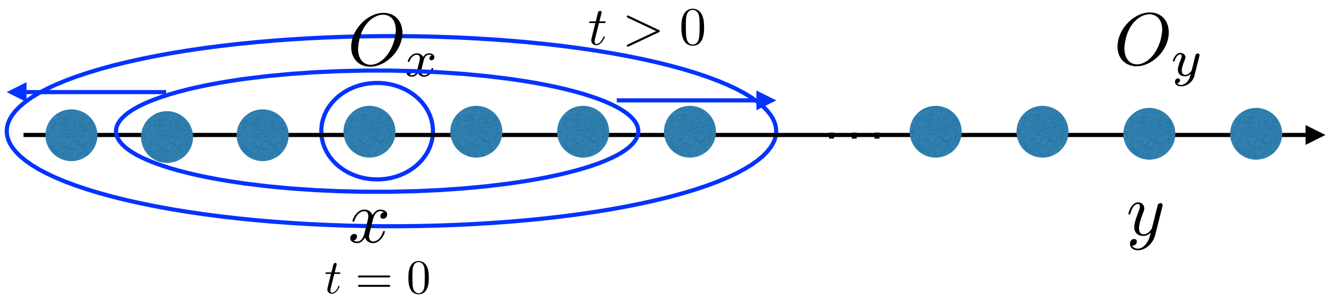
$t_{\mathrm{reach}} \sim r^z$: still light cone, but polynomial light cone

$t_{\mathrm{reach}} \sim \ln r$ or faster : fast propagation

How to characterize ?

- Important function

$$\| [O_x(t), O_y] \|_p \qquad \|O\|_p := \left[\mathrm{Tr}(O^\dagger O)^{\frac{p}{2}} \right]^{\frac{1}{p}}$$



$p = \infty$ Operator norm: gives the fastest propagation

$p = 2$ Frobenius norm: gives typical propagation at $\beta = 0$

This is the Out-of-time order correlator (OTOC)

J. Maldacena, SH. Shenker, and D. Stanford, JHEP (2016).

Lieb-Robinson bound for short-range interaction

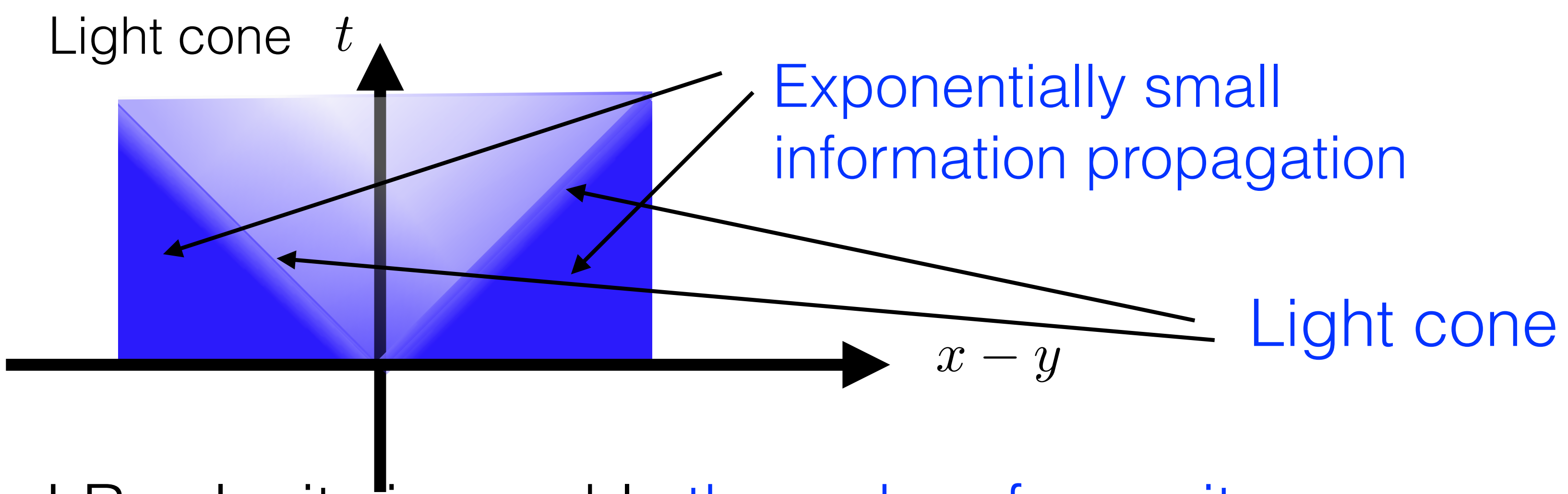
E. Lieb and DW Robinson, CMP (1972)

- Information propagation in interacting systems

$$H = \sum_i h_{i,i+1} + h_i \qquad \text{e.g., } H = \sum_{i=1}^{n} [J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + BS_i^z]$$

- Rigorous bound for $\| [O_x(t), O_y] \|_\infty$: Lieb-Robinson bound

$$\| [O_x(t), O_y] \|_\infty < \text{const.} \|O_x\| \|O_y\| e^{vt-|x-y|} \qquad v \propto \sum_j \|h_{i,j}\|$$



- LR velocity is roughly the order of one site interacting operators $\quad v \propto \sum_j \|h_{i,j}\|$

Lieb-Robinson bound for short-range interaction

- LR bound is important !

1. Causality: fundamental

2. Many Applications: leading to crucial physics

(2-1) Exponential decay in ground state in gapped system

Hasting, Koma, J.Stat.Phys.

(2-2) No-go theorem of long-range entanglement at finite temperatures

T. Kuwahara and KS, PRX (2022)

(2-3) Estimation of computation time

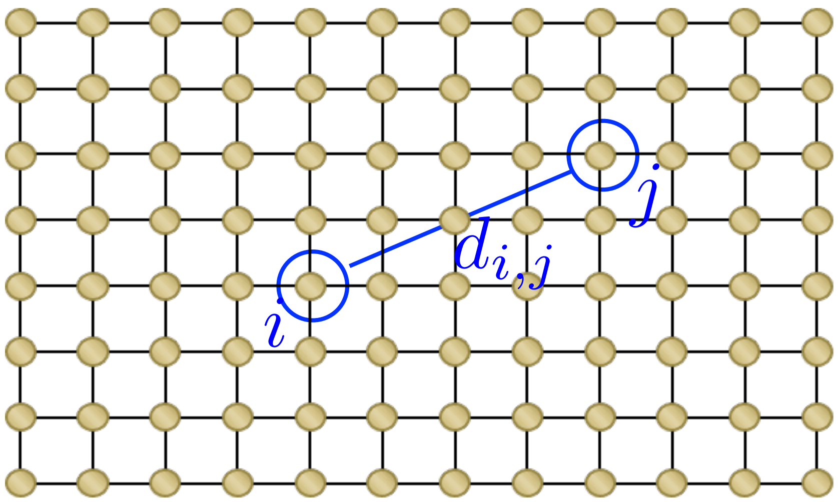
…

Question for information propagation

- Long-range interacting system

$$H = \sum_{i,j \in \Lambda} h_{i,j} \qquad \|h_{i,j}\| \le \frac{g_0}{d_{i,j}^{\alpha}}$$

$$\text{e.g.,} \quad H = \sum_{i,j} \frac{J}{d_{i,j}^{\alpha}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$$V(r) \propto r^{-\alpha}$$

- Question

Long-range interaction instantly transfers the information even to distant locations.

But the amplitude of Long-range potential is small for large r.

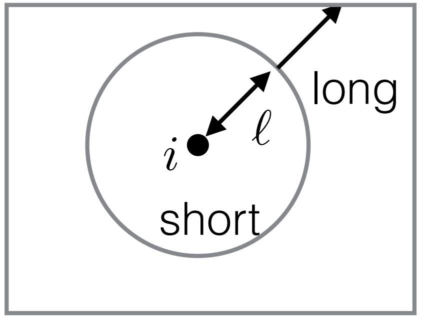
What is the condition of the linear light cone ?

Hand-waving argument

$H = \sum_{i,j \in \Lambda} h_{i,j} \qquad \|h_{i,j}\| \le \frac{g_0}{d_{i,j}^{\alpha}}$

- To get the feeling, divide H into $H_{\rm short}$ and $H_{\rm long}$

$$H_{\rm short} = \sum_{i,j: d_{i,j} \le \ell} h_{i,j} \qquad H_{\rm long} = \sum_{i,j: \ell < d_{i,j}} h_{i,j}$$



- We want to know the operator spreading of $e^{-i(H_{\rm short}+H_{\rm long})t}$

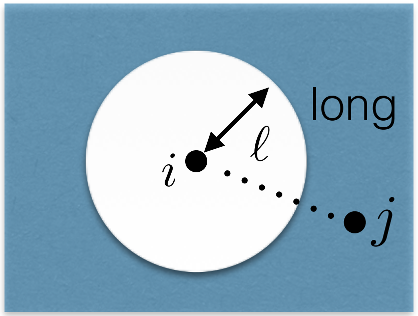
(1) Easier problem to get the feeling: $e^{-iH_{\rm short}t}e^{-iH_{\rm long}t}$

$e^{-iH_{\rm short}t}$: gives the LR linear light cone e^{vt-r}

$e^{-iH_{\rm long}t}$: $e^{v'\ell t - r}$ $\qquad v'\ell \sim \ell^{d-\alpha+1}$

Amplitude of one-site interacting operators

$$v' \sim \sum_{j: \ell < d_{i,j}} \|h_{i,j}\| \sim g_0 \int_{\ell}^{\infty} dr r^{d-1} r^{-\alpha} \sim g_0 \ell^{d-\alpha}$$



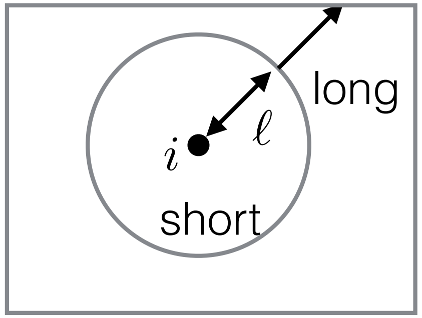
If $\alpha > d+1$, $v'\ell \sim \ell^{d-\alpha+1} \to 0 \ (\ell \to \infty)$

linear light cone is expected

Hand-waving argument $H = \sum_{i,j\in\Lambda} h_{i,j} \quad \|h_{i,j}\| \le \frac{g_0}{d_{i,j}^\alpha}$

- To get the feeling, divide H into $H_{\rm short}$ and $H_{\rm long}$

$$H_{\rm short} = \sum_{i,j:d_{i,j}\le \ell} h_{i,j} \qquad H_{\rm long} = \sum_{i,j:\ell< d_{i,j}<2\ell} h_{i,j}$$



- We want to know the operator spreading of $e^{-i(H_{\rm short}+H_{\rm long})t}$

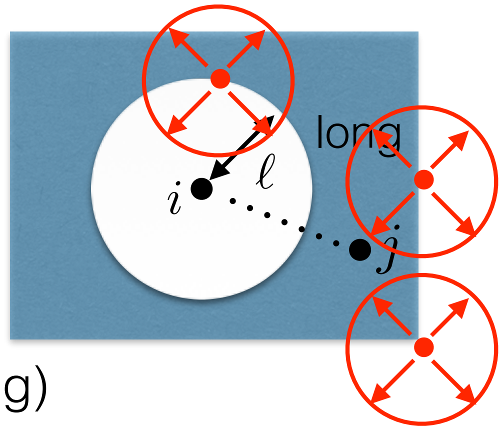
(2) How is $e^{-i(H_{\rm short}+H_{\rm long})t}$? $= e^{-iH_{\rm short}t} e_{\leftarrow}^{-i\int_0^t ds H_{\rm long}(s)}$

$$H_{\rm long}(t) = e^{iH_{\rm short}t} H_{\rm long} e^{-iH_{\rm short}t}$$

Amplitude of one-site interacting operators

$$t=0 \quad \to \quad \sum_{j:\ell<d_{i,j}} \|h_{i,j}\| \sim g_0 \ell^{d-\alpha}$$

$$t>0 \quad \to \quad g_0 \ell^{d-\alpha} \times t^d$$ (Ballistic operator spreading)

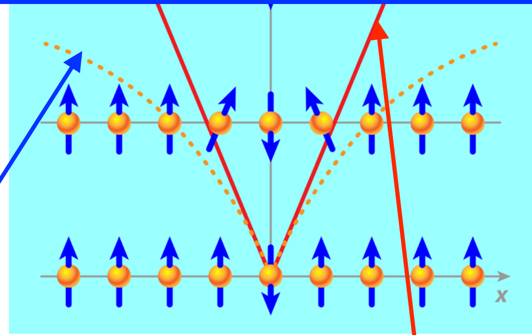


$$e_{\leftarrow}^{-i\int_0^t ds H_{\rm long}(s)} : e^{v'\ell t - r} \qquad v'\ell \sim \ell^{d-\alpha+1} t^d \sim \ell^{d-\alpha+1}\ell^d = \ell^{2d-\alpha+1}$$

If $\alpha > 2d+1$, light cone is expected

(Rigorously proven) State-of-the-art diagram

- State-of-the-art classification



1. Operator norm $\| [O_X(t), O_Y] \|_\infty$

Fast propagation
No light cone

Light-cone

α

d $2d$ $2d+1$

LinearLight cone

$\sim t^0$ $\quad \dfrac{\sim e^{ct}}{r^{\alpha-d}}$

Polinomial
Light cone
$\sim \dfrac{t}{r^{\min(1,\alpha-2d)}}$

$\sim e^{vt-r}$

M. Turan et al., PRL (2021) T. Kuwahara and KS, PRX (2020)

2. OTOC $\| [O_X(t), O_Y] \|_2$

(SYK model)

Light-cone (No fast scrambling regime)

T. Kuwahara and KS, PRL (2021)

α

0 d

Details in light-cone regime are not very clear…

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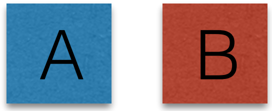
Part II : Suppression of information propagation

- So far, we have seen operator spreading
  Here, let's see on the entanglement spreading

- Quantified by the Von-Neumann entropy



$$S = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\} \qquad \rho_A = \mathrm{Tr}_B(|\varphi\rangle\langle\varphi|)$$

Product state: $S = 0$

Above singlet state: $S = \log 2 > 0$ $\qquad |\varphi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Entanglement usually grows up to the system size in pure dynamics

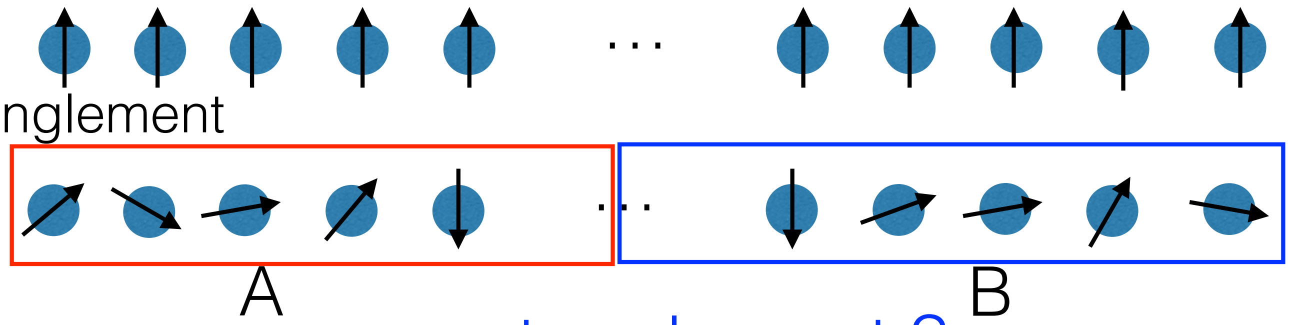
- Quantum entanglement in many-body systems

$$H = \sum_{i=1}^{L} h_{i,i+1} \qquad \text{e.g.,} \quad h_{i,i+1} = -\sigma_i^z \sigma_{i+1}^z + h\sigma_i^z + g\sigma_i^x$$

$|\varphi(0)\rangle = |\uparrow\uparrow \cdots \uparrow\uparrow\rangle$

Product state: no entanglement

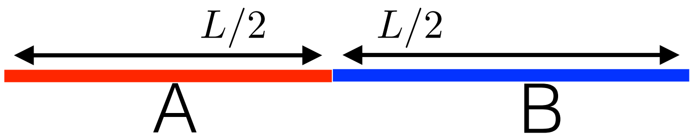
$|\varphi(t)\rangle = e^{-itH}|\varphi(0)\rangle$

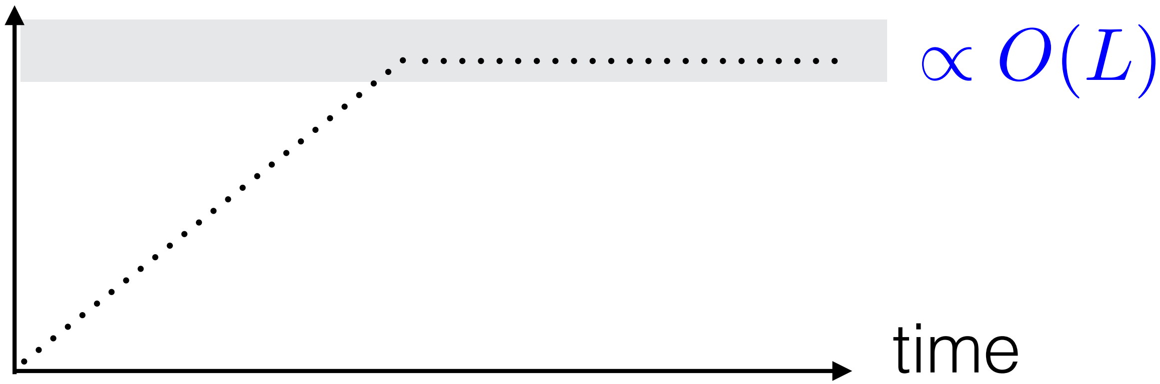


entanglement ?

- Entropy linearly grows in time up to the order of size

$S = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\}$

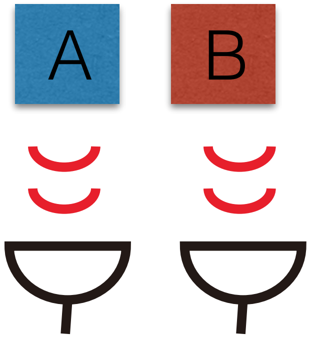




Suppression of entanglement due to quantum measurement

- In general, measurement kills the quantum entanglement

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Device — Local spin measurement

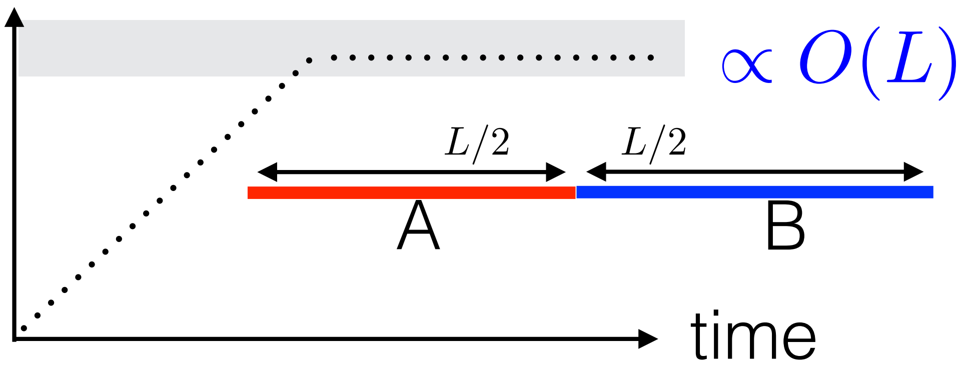
Outcome: $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$ — Product state (no entangled)

- Local measurement: Entangled states → Product states

Phase transition of entanglement

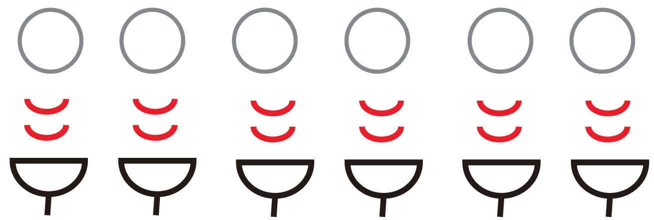
- Pure dynamics

$S = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\}$



Enhances entanglement

- Measurement

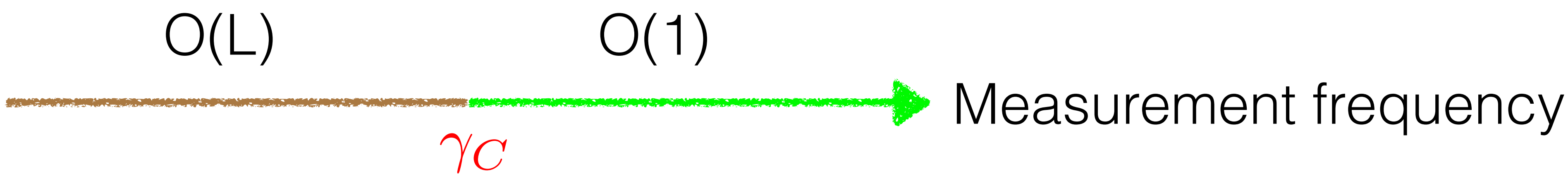


Kills entanglement

Competition

- Phase transition as a result of the competition

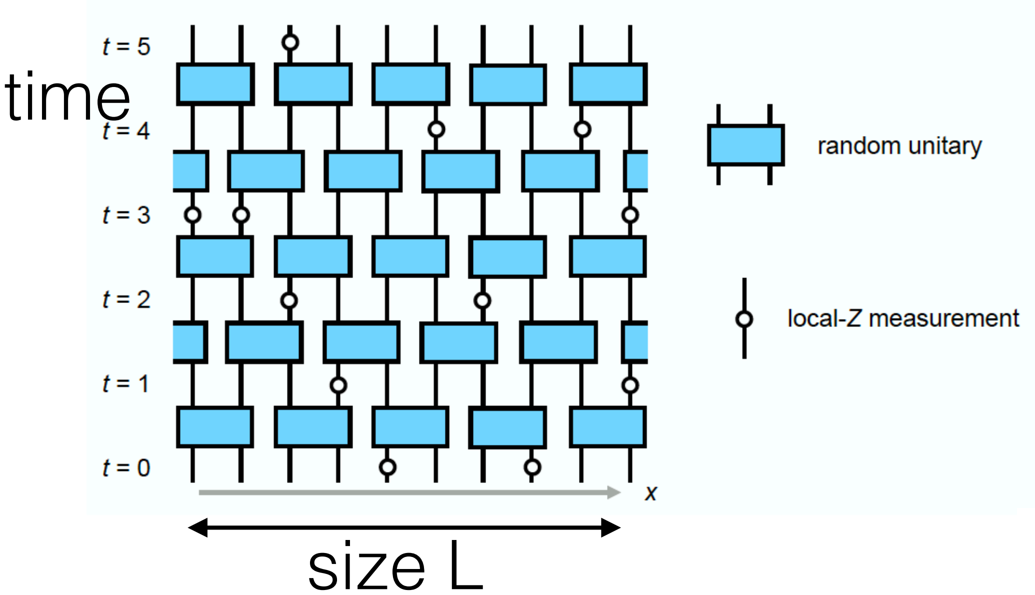
Measurement-induced phase transition (MIP)



Measurement-induced Phase Transition (MIP)

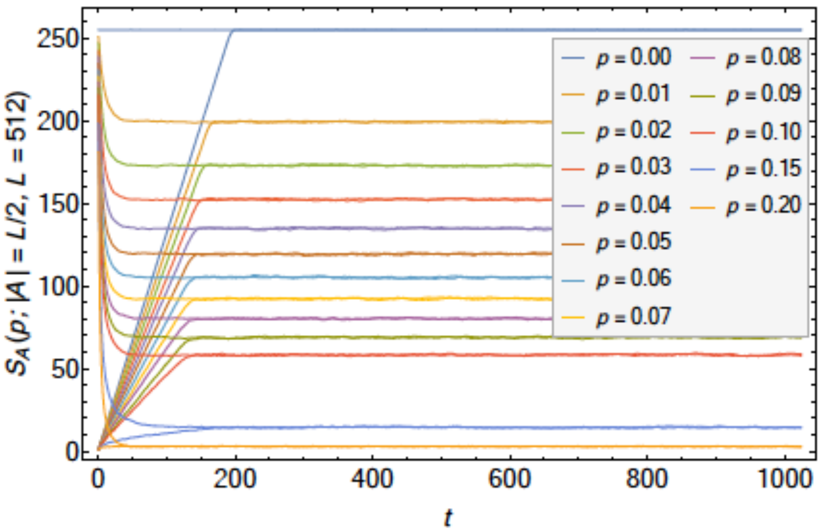
Li, et al., PRB (2019) Skinner, PRX (2019)

- Circuit model: Unitary gate + projective measurement



with probability p

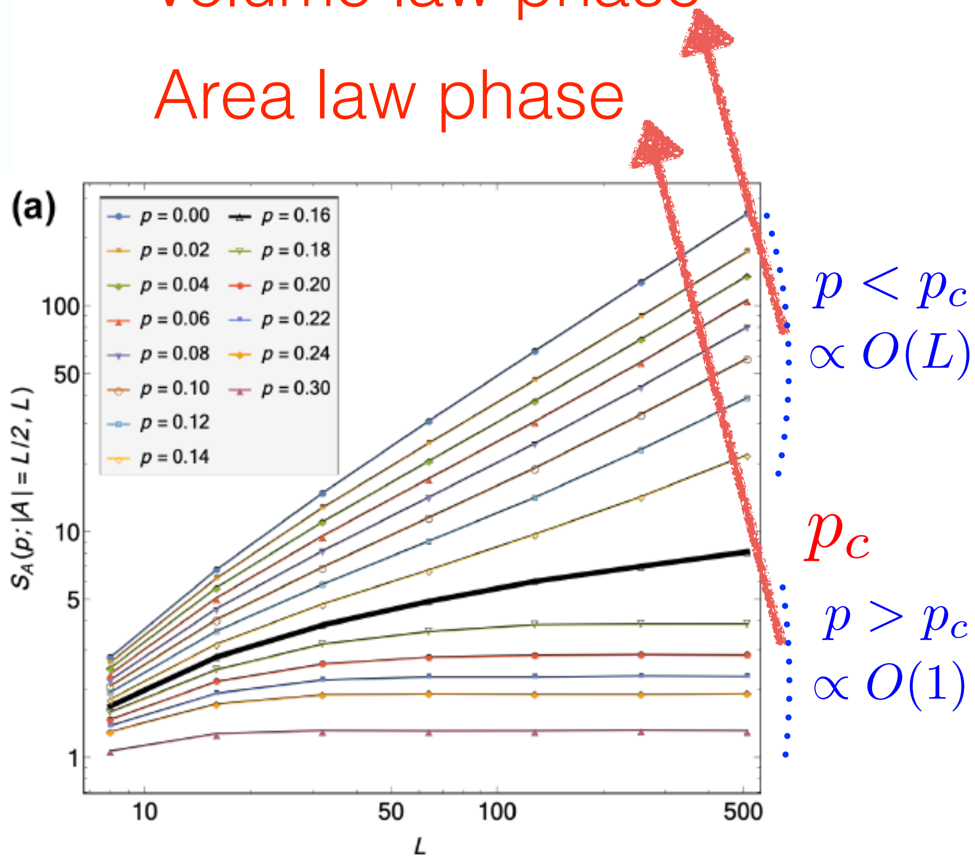
- Measurement-induced phase tr.



Time-evolution of S: realization of steady state

Volume law phase

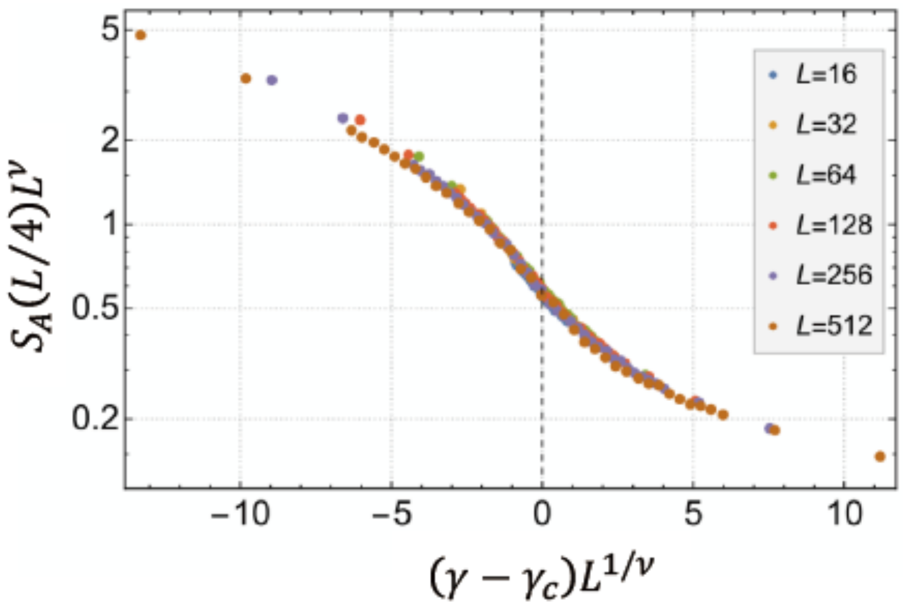
Area law phase



Size-dependence of S at steady state

Measurement-induce phase transition (MIP)

- Finite-size scaling works well →Quantum dynamical phase tran.

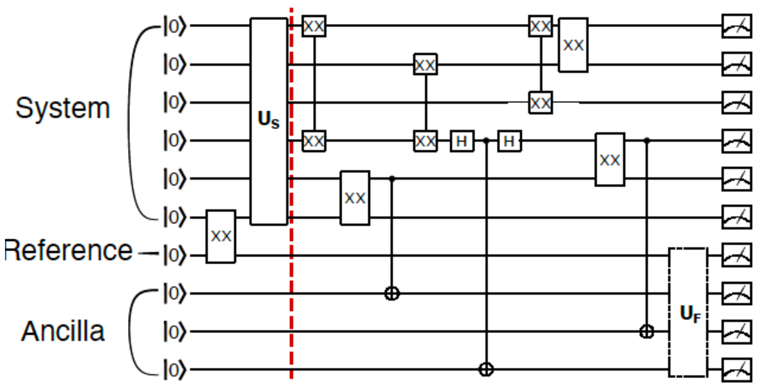


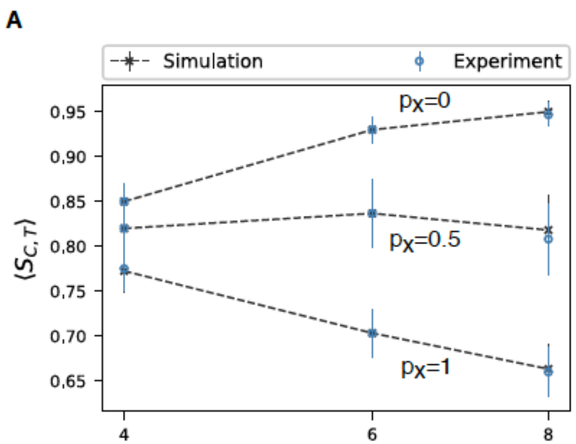
Numerical simulation

Li, et al., PRB (2019) Skinner, PRX (2019)

- First experiment recently exists !: Noel et al., arXiv:2106.05881

  Ion-trapped system 4-8 qubits

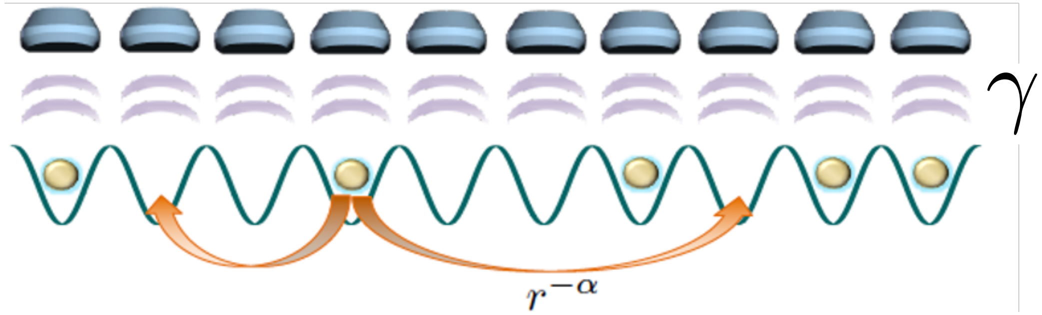




Question

Minato, Sugimoto, Kuwahara, KS, PRL 2022

- Effect of long-range interaction with $r^{-\alpha}$?



1d example

$$H = \sum_{j=1}^{L} \sum_{r=1}^{L/2} \frac{1}{r^{\alpha}} \left[-c_{j+r}^{\dagger} c_j - c_j^{\dagger} c_{j+r} + V n_{j+r} n_j \right]$$

- Background of this motivation

As we have seen in Part I,
long-range interaction enhances operator spreading

This indicates that
the entanglement growth should be affected by long-range int.

What is the effect in the MIP ?

Main statement

- Main statement

  Sufficient condition to observe the MIP

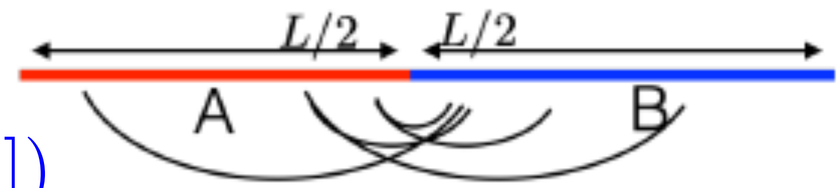
  $\alpha > \frac{d}{2} + 1$ Bilinear Fermion systems

  $\alpha > d + 1$ General non-integrable systems

- Background of this statement $H = H_A + H_B + H_{AB}$

  Formal expression of entanglement growth rate

$$\dot{S}_A = i\|H_{AB}\|\lambda(\rho)$$



$$\lambda(\rho) := \mathrm{Tr}_{AB}(h_{AB}\,[\rho_{AB}, \ln\rho_A \otimes \mathbf{1}_B])$$

$$H_{AB} := \sum_{i\in A}\sum_{j\in B} h_{i,j}$$

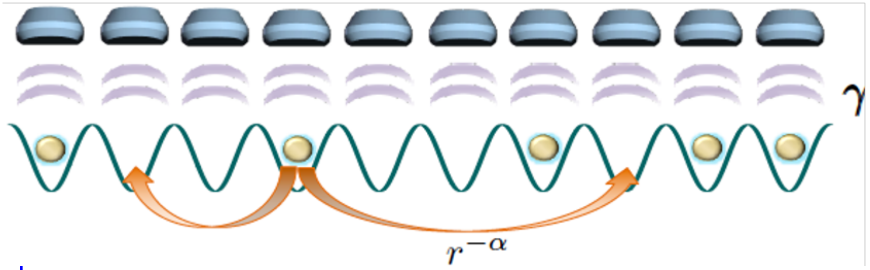
$$h_{AB} = \frac{H_{AB}}{\|H_{AB}\|}$$

$\|H_{AB}\| < \infty$ →strong measurement beats entanglement growth
→MIP should exist

$\|H_{AB}\| \to \infty$ →measurement cannot entanglement growth
→No MIP

Numerical check

- 1D Setup



$$H = \sum_{j=1}^{L} \sum_{r=1}^{L/2} \frac{1}{r^{\alpha}} \left[-c_{j+r}^{\dagger} c_j - c_j^{\dagger} c_{j+r} + V n_{j+r} n_j \right] ,$$

- Quantum jump process with the quantum measurement

Measures the onsite particle

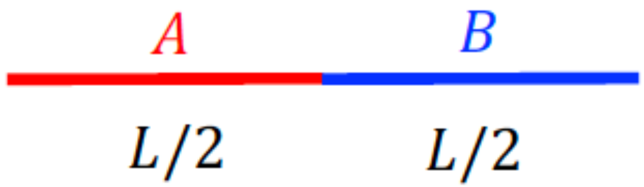
$$d|\psi(t)\rangle = -iH|\psi(t)\rangle + \sum_{j=1}^{L} \Big[\frac{n_j|\psi(t)\rangle}{\sqrt{\langle\psi(t)|n_j|\psi(t)\rangle}} \Big] dw_j(t)$$

$$dw_j(t) = 0 \text{ or } 1 \qquad \overline{dw_j(t)} = \gamma\langle n_j\rangle dt$$

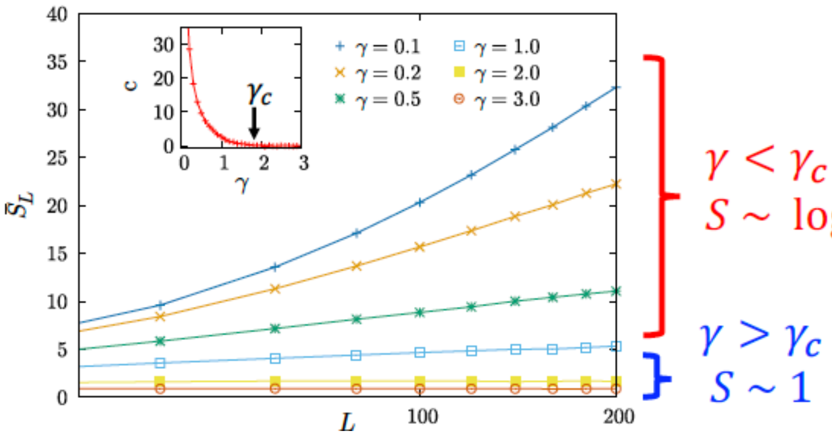
Demonstration for free fermion system V=0

- Stationary state entanglement entropy from the Neel state

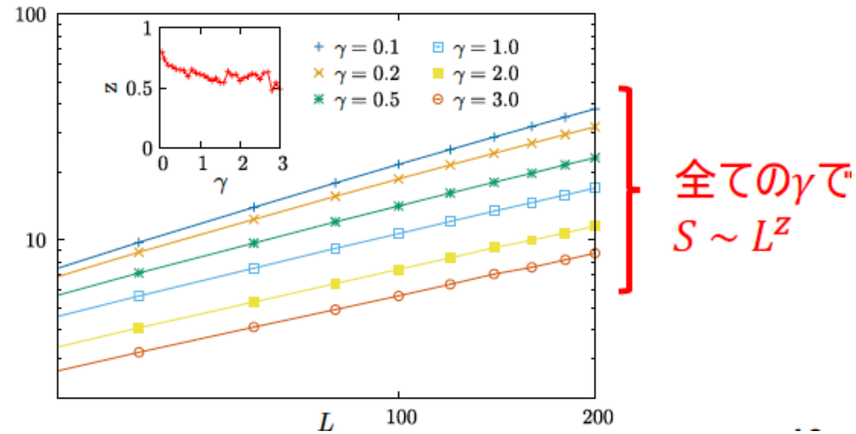
$$S_A = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\}$$



- $\alpha = 2.3$: the MIP exists

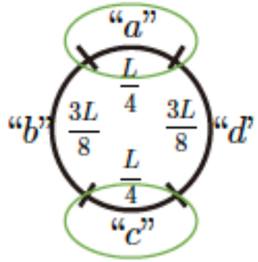


- $\alpha = 0.8$: No MIP exists

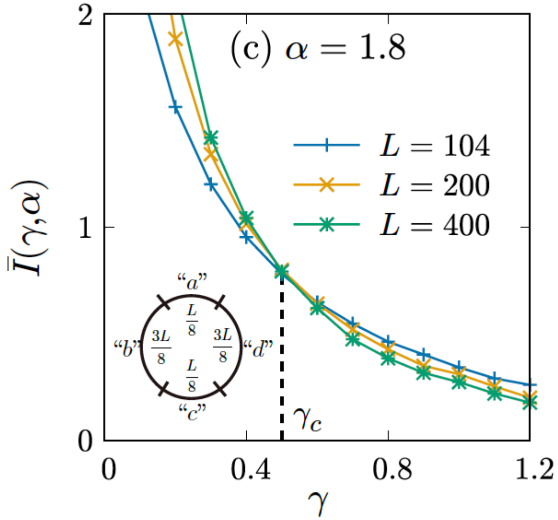


α dependence of the critical point

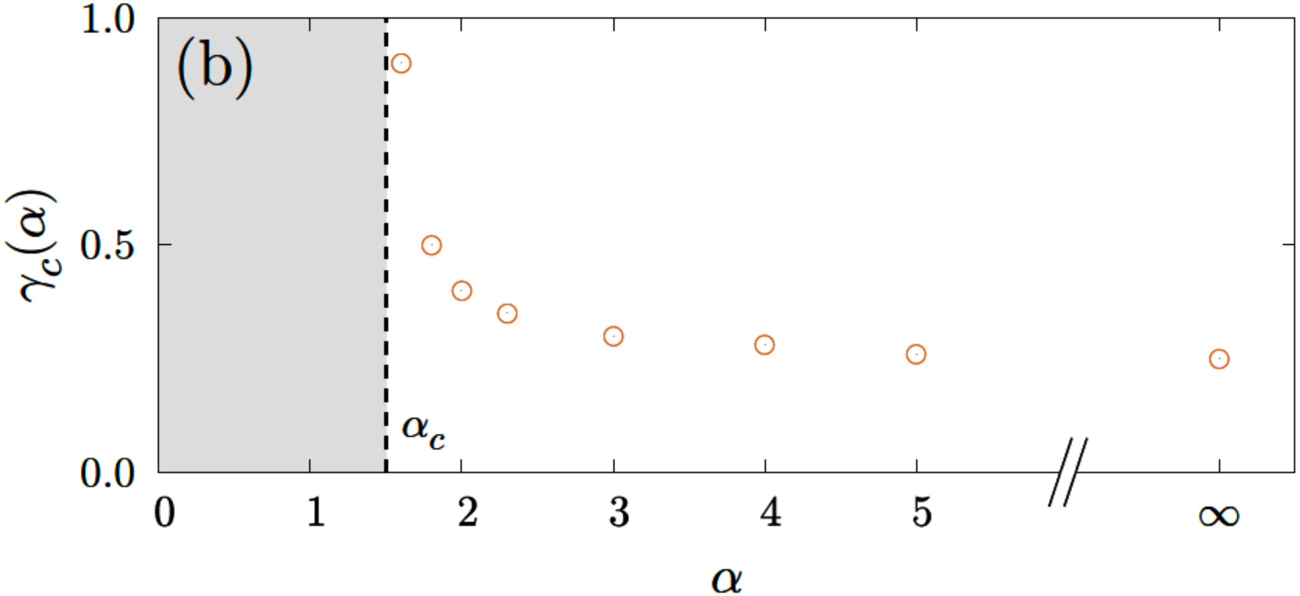
- Calculate: mutual information $\bar{I} := \overline{S_a} + \overline{S_c} - \overline{S_{ac}}$



- Cross point
  indicates the critical point



- Consistent result in the theory ($\alpha > 3/2$)



Summary Part I and II

- Information propagation in the long-range systems is considered.

- In the first part, the operator spreading is considered

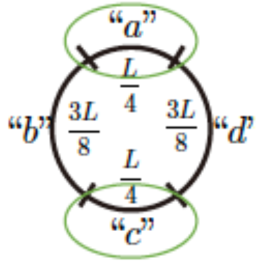
  Operator norm version is classified

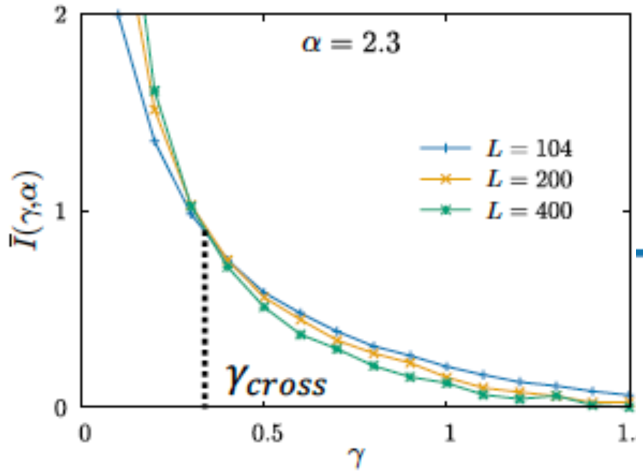
- In the second part, growth and suppression of entanglement is considered.

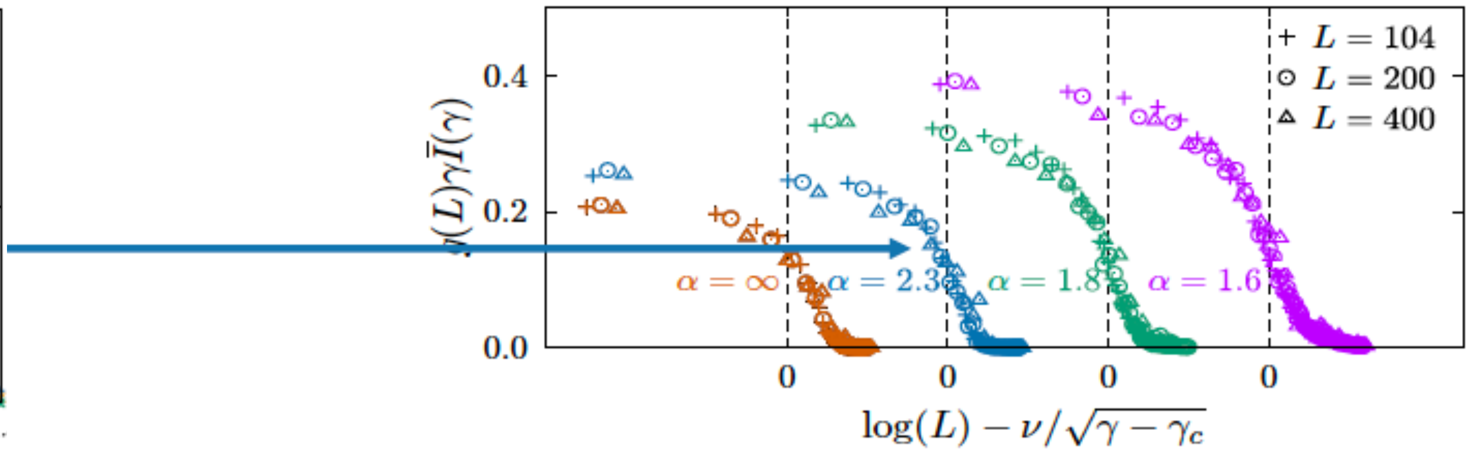
  General criterion to observe the measurement-induced phase transition is provided.

Free Fermion systems show BKT transition

- Calculate: mutual information $\bar{I} := \overline{S_a} + \overline{S_c} - \overline{S_{ac}}$







Finite-size scaling of BKT transition

- For noninteracting Fermion case: BKT transition.

Noninteracting case

- Nonintegrable systems shows the 2nd order PT.