# When does a system affect another? Causal influence vs signalling

## Extreme Universe - Colloquium

Yukawa Institute for Theoretical Physics, Kyoto University

Paolo Perinotti - July 14th 2022

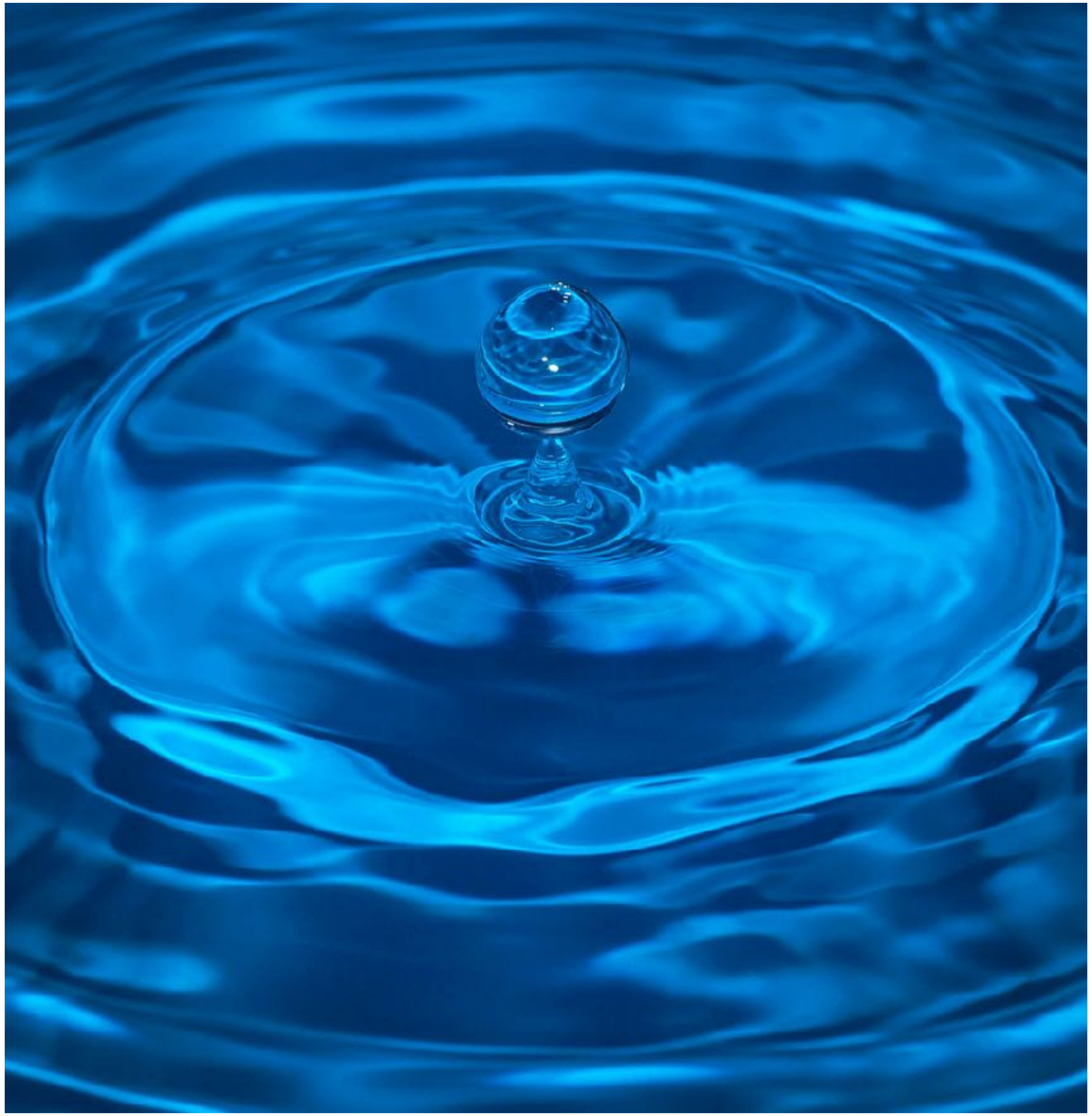
UNIVERSITÀ DI PAVIA

INFN Istituto Nazionale di Fisica Nucleare

QUit quantum information theory group

# Summary

- OPTs
- Networks and causal cones
- Signalling
- Propagation of interventions
- Classical and Quantum theory
- No interaction without disturbance



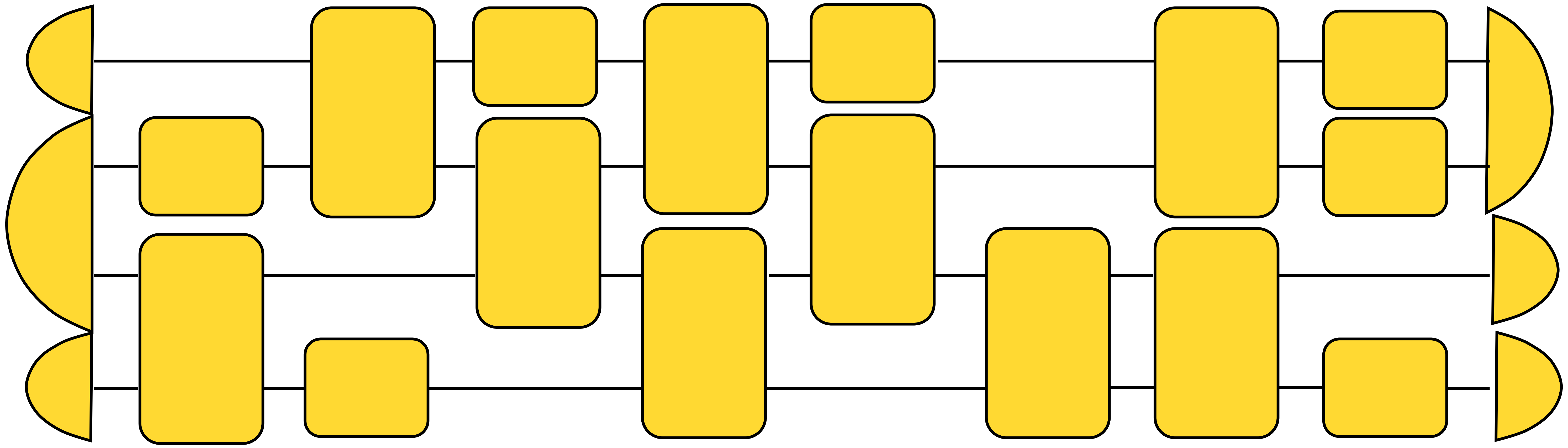
- Propagation of interventions


• Propagation of interventions
• Signalling

# Chapter I: Prologue

The framework: Operational Probabilistic Theories

# Operational Language



- Operational theory: tests with composition rules

G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A **81**, 062348 (2010)

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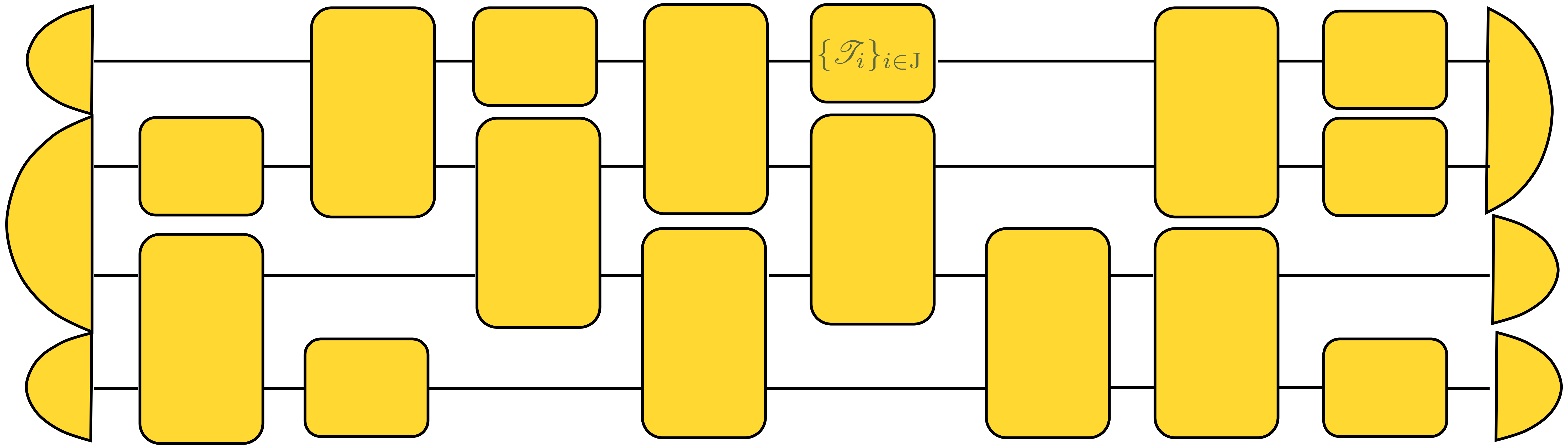


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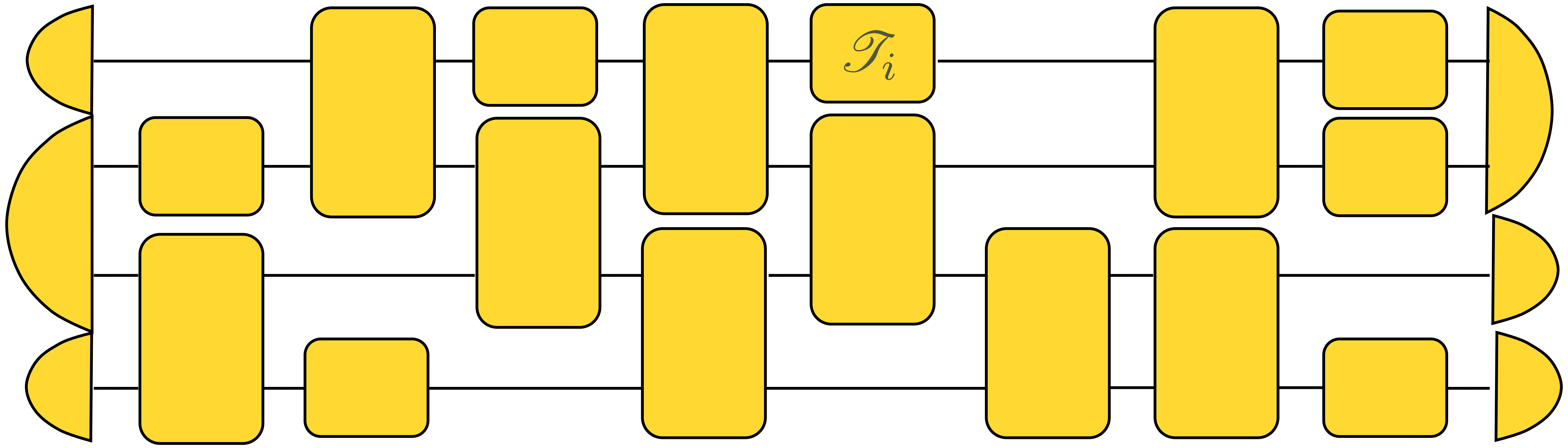


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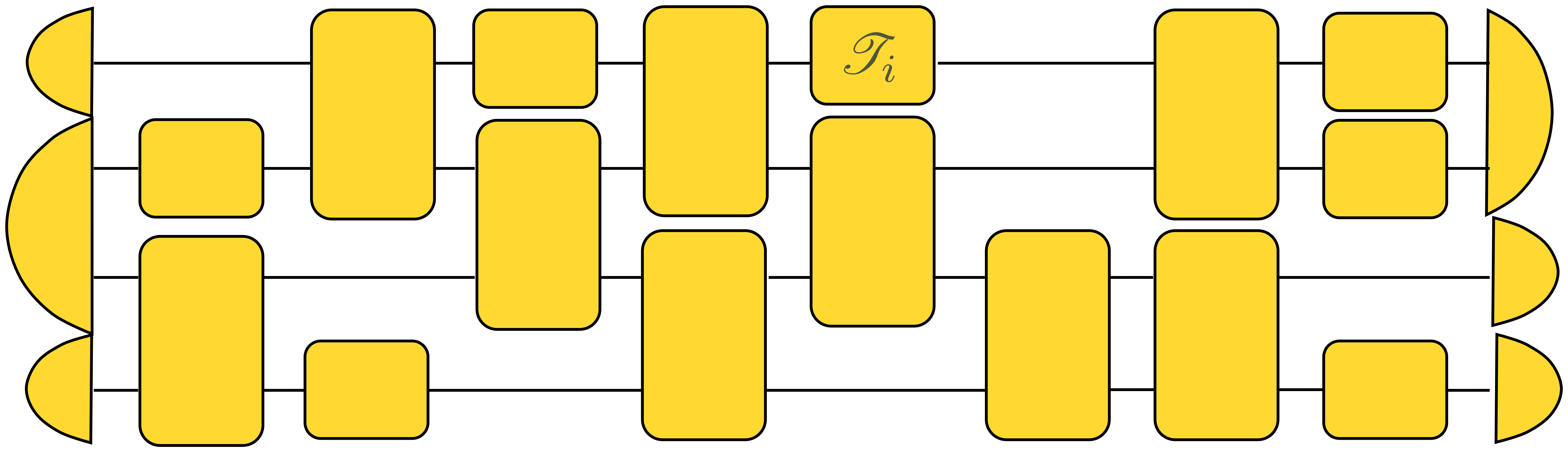


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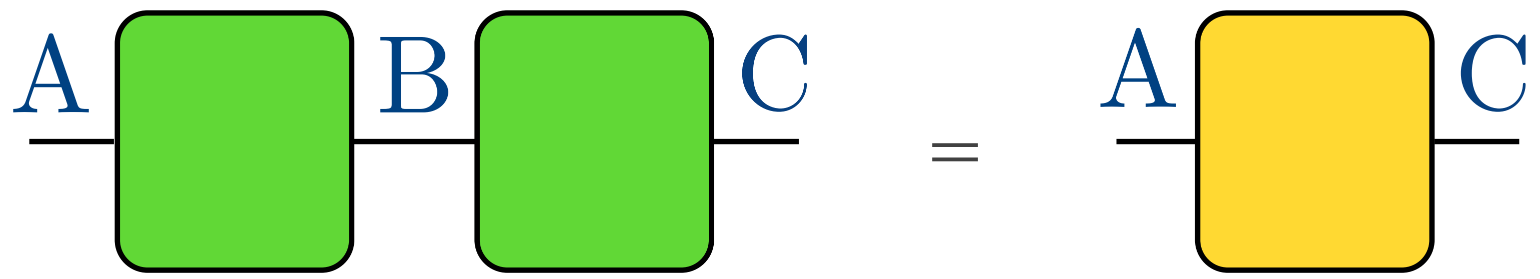
Sequential

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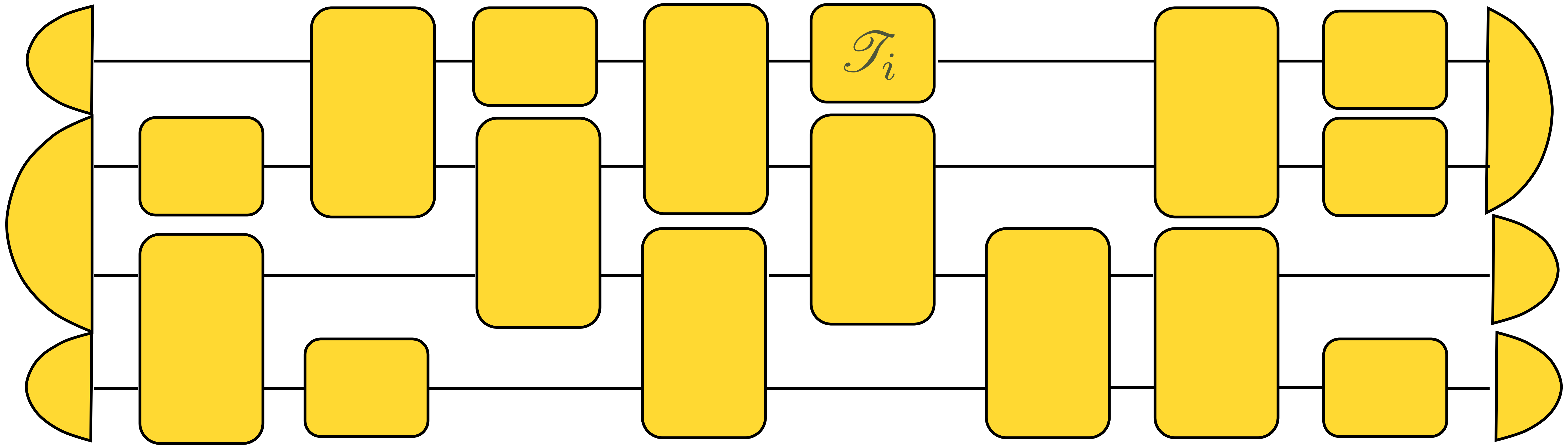


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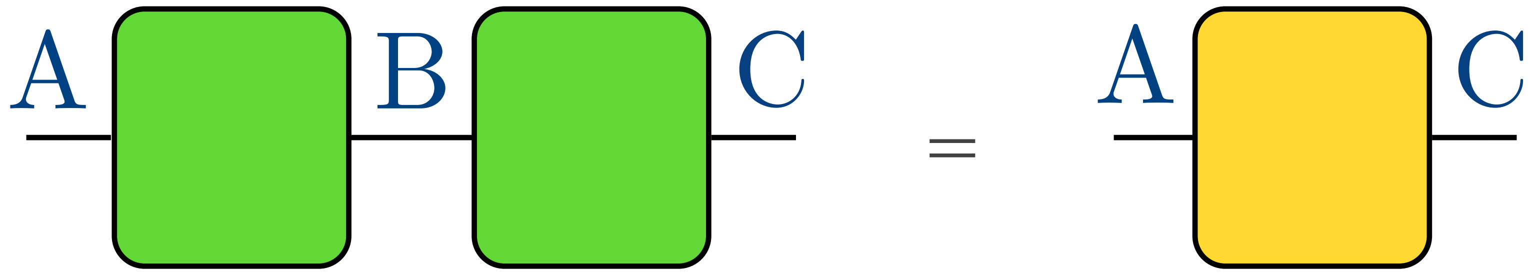
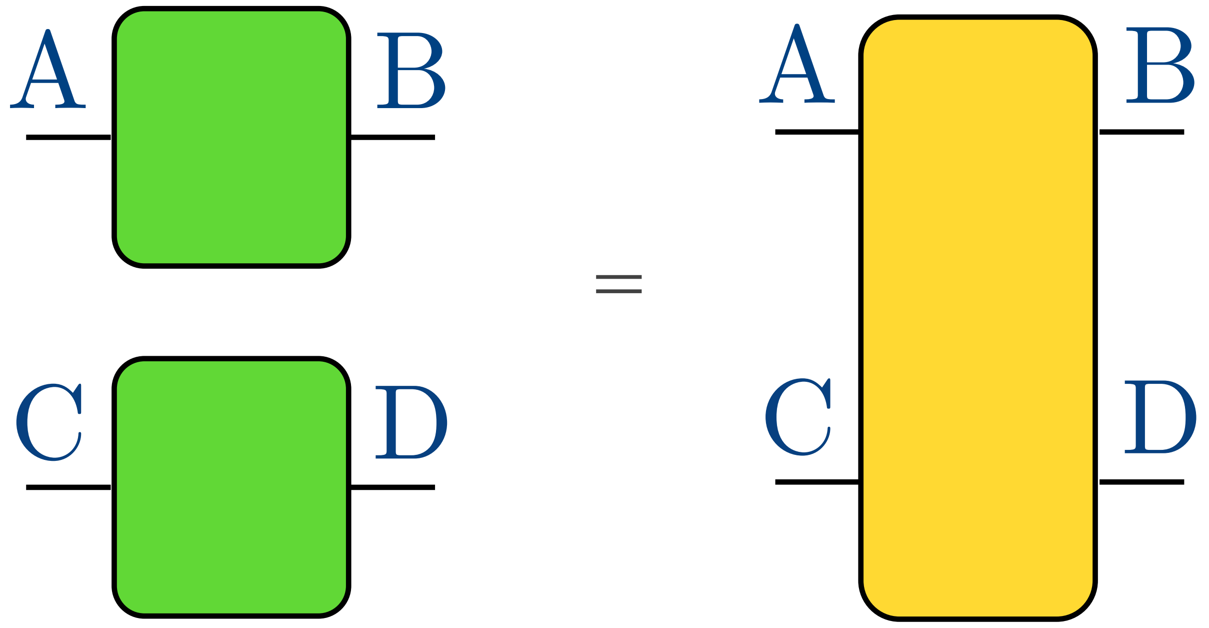



Parallel

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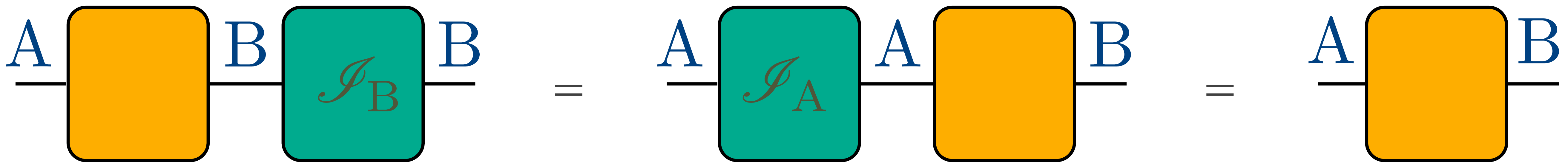
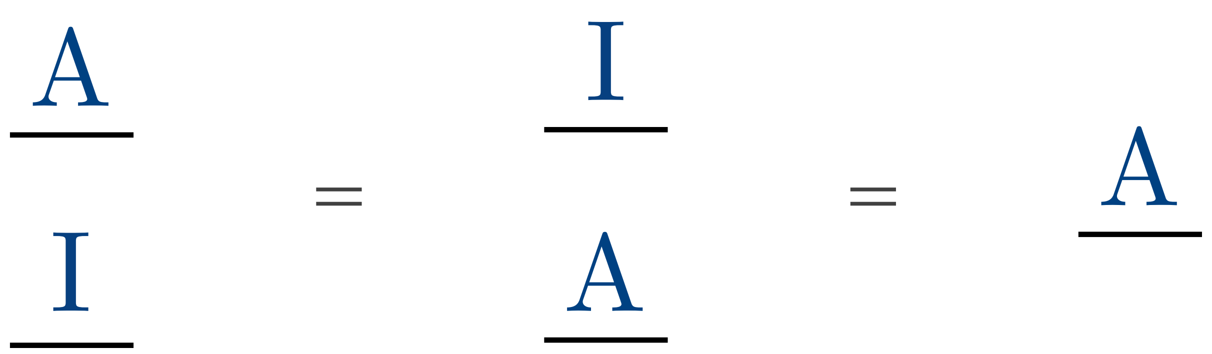
# Operational Language

- Properties of composition rules:
  - Associativity
  - Unit

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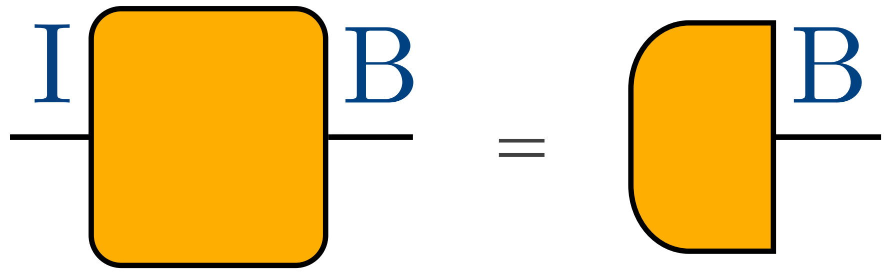


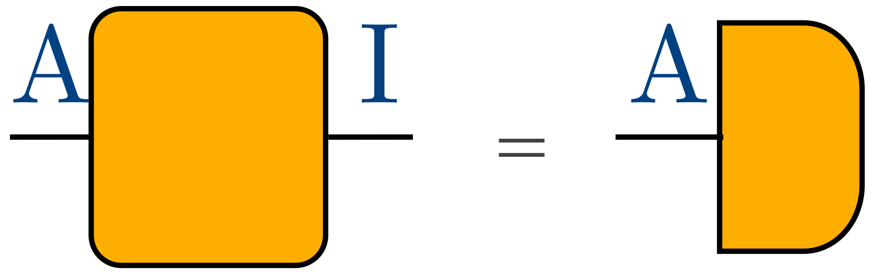

# Reversible events and swap

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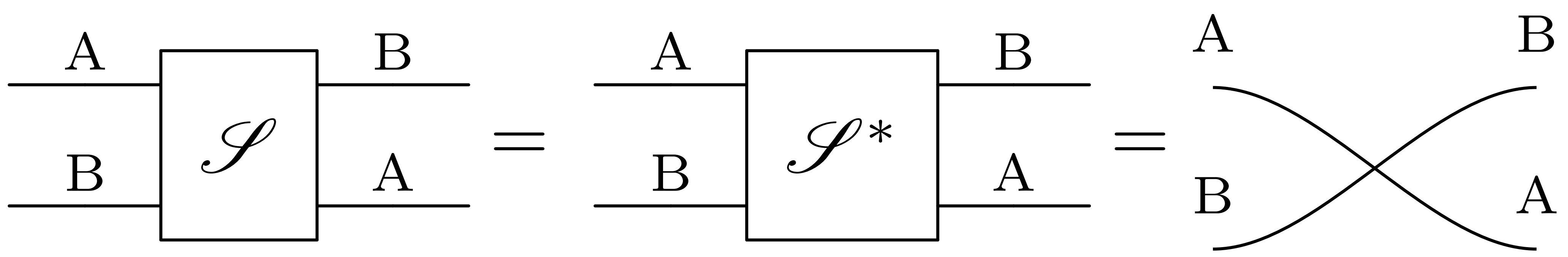
Swap (symmetric theories):



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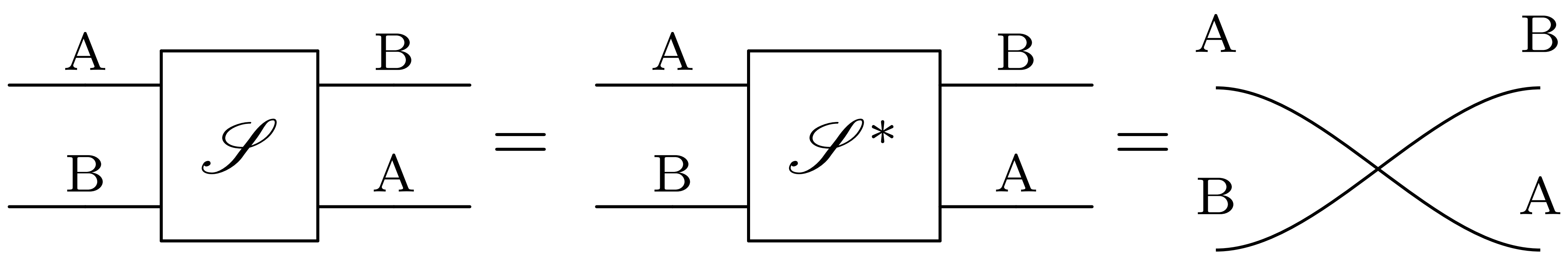
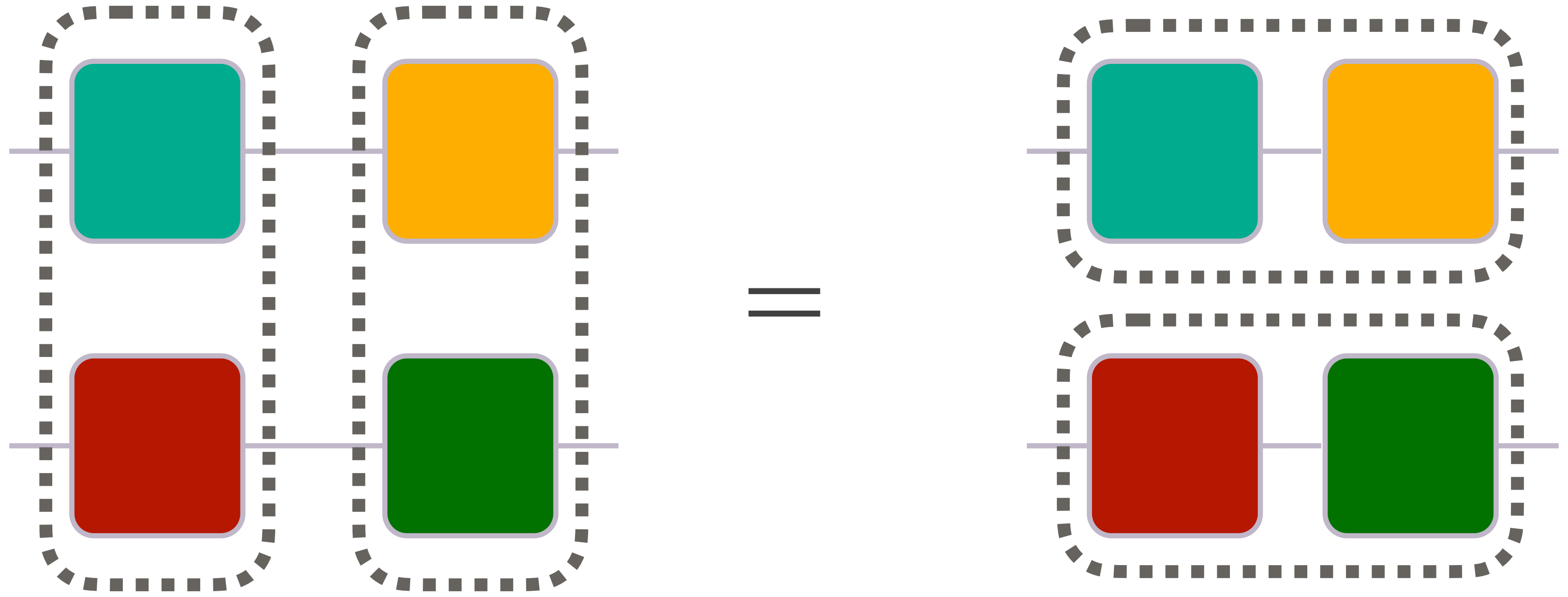



# Monoidal structure

- Most important rule:



$$(\mathscr{A} \otimes \mathscr{B})(\mathscr{C} \otimes \mathscr{D}) = (\mathscr{A}\mathscr{C}) \otimes (\mathscr{B}\mathscr{D})$$

B. Coecke, AIP Conf. Proc. **810**, 81 (2006)

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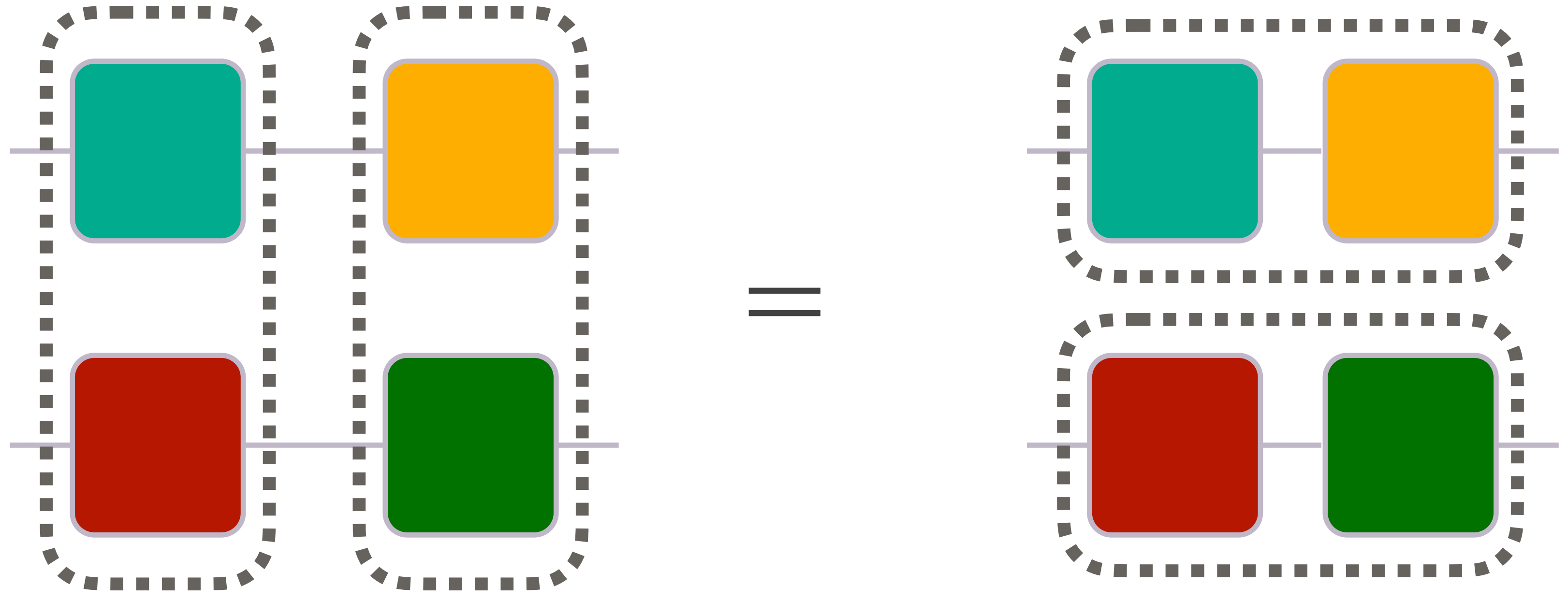
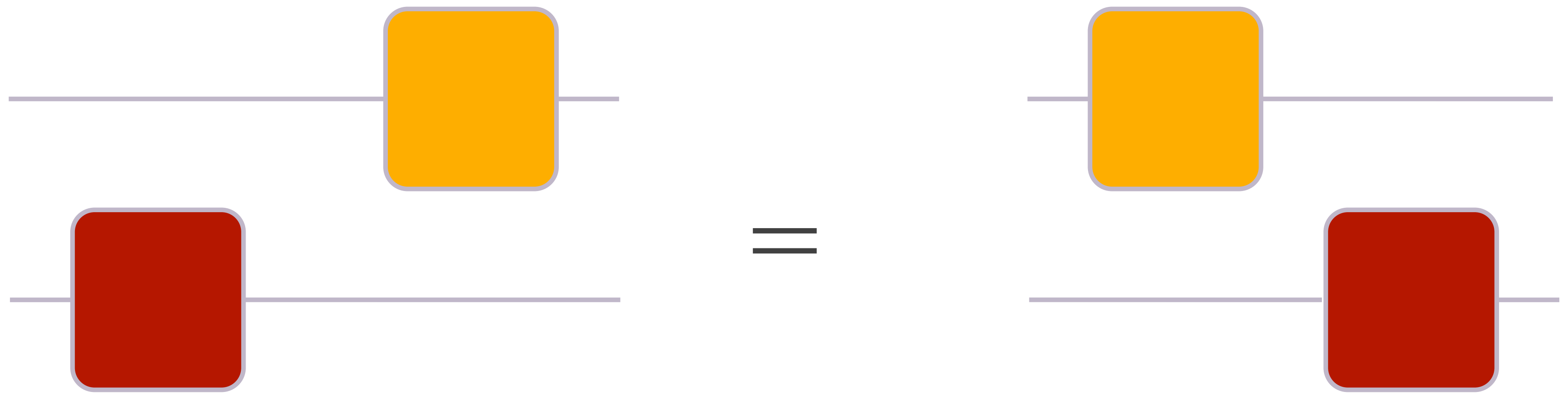


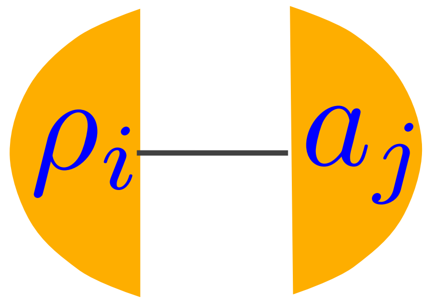
$$(\mathscr{A} \otimes \mathscr{B})(\mathscr{C} \otimes \mathscr{D}) = (\mathscr{A}\mathscr{C}) \otimes (\mathscr{B}\mathscr{D})$$

- Consequence:



B. Coecke, AIP Conf. Proc. **810**, 81 (2006)

# Probabilistic theories

Every test of type I→I is a probability distribution 

$= \Pr(a_j, \rho_i)$

States are functionals on effects and vice-versa

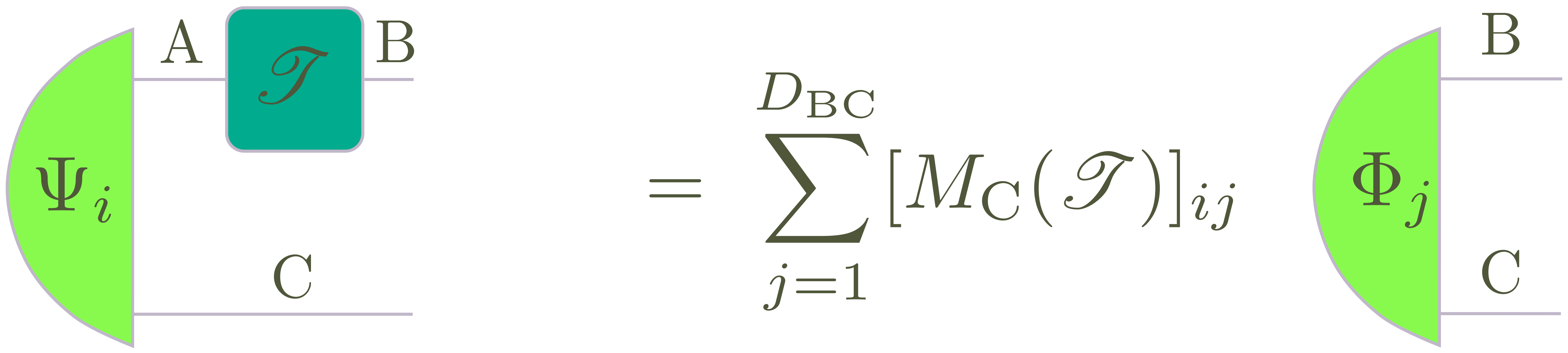
Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A **81**, 062348 (2010)

# Transformations

A transformation $\mathscr{T} \in \mathsf{Transf}(\mathrm{A} \to \mathrm{B})$ induces a family of linear maps:

$\{M_{\mathrm{C}}(\mathscr{T})\}_{\mathrm{C}}$ representing $\mathscr{T} \otimes \mathscr{I}_{\mathrm{C}}$ on $\mathsf{St}(\mathrm{AC})_{\mathbb{R}}$

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# Transformations

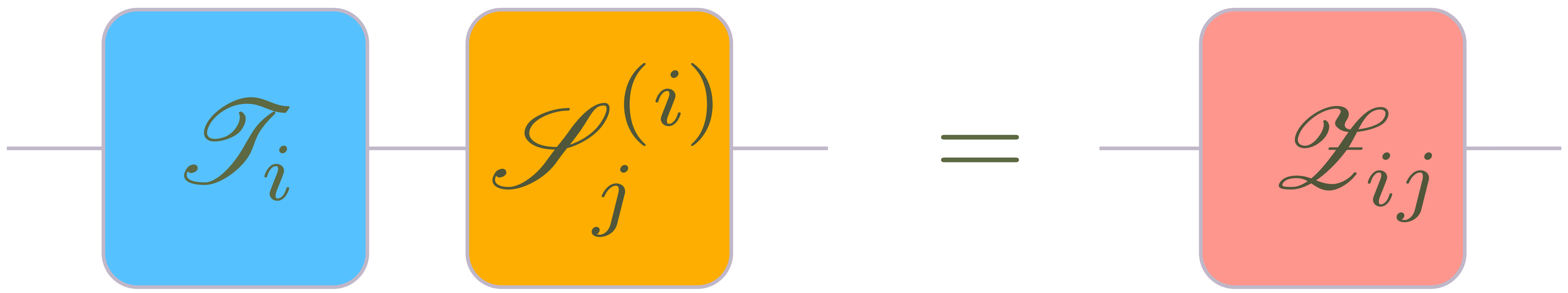
Indeed, it is not sufficient to know the linear map induced by $\mathscr{T}$ on $\mathsf{St}(\mathrm{A})_{\mathbb{R}}$

E.g.: transpose map in real quantum theory

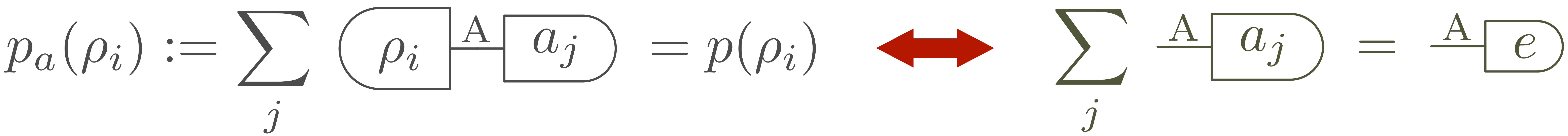
- $\rho^T = \rho$ → $\mathscr{T}(\rho) = \mathscr{I}(\rho) \quad \forall \rho$
- $\sigma_y \otimes \sigma_y \in \mathsf{St}(\mathrm{AC})_{\mathbb{R}}$ $\qquad (\mathscr{T} \otimes \mathscr{I}_{\mathrm{C}})(\sigma_y \otimes \sigma_y) = -\sigma_y \otimes \sigma_y$

# Causal theories

- Possibility of arbitrary conditional tests



- Causality implies no “backward” signalling



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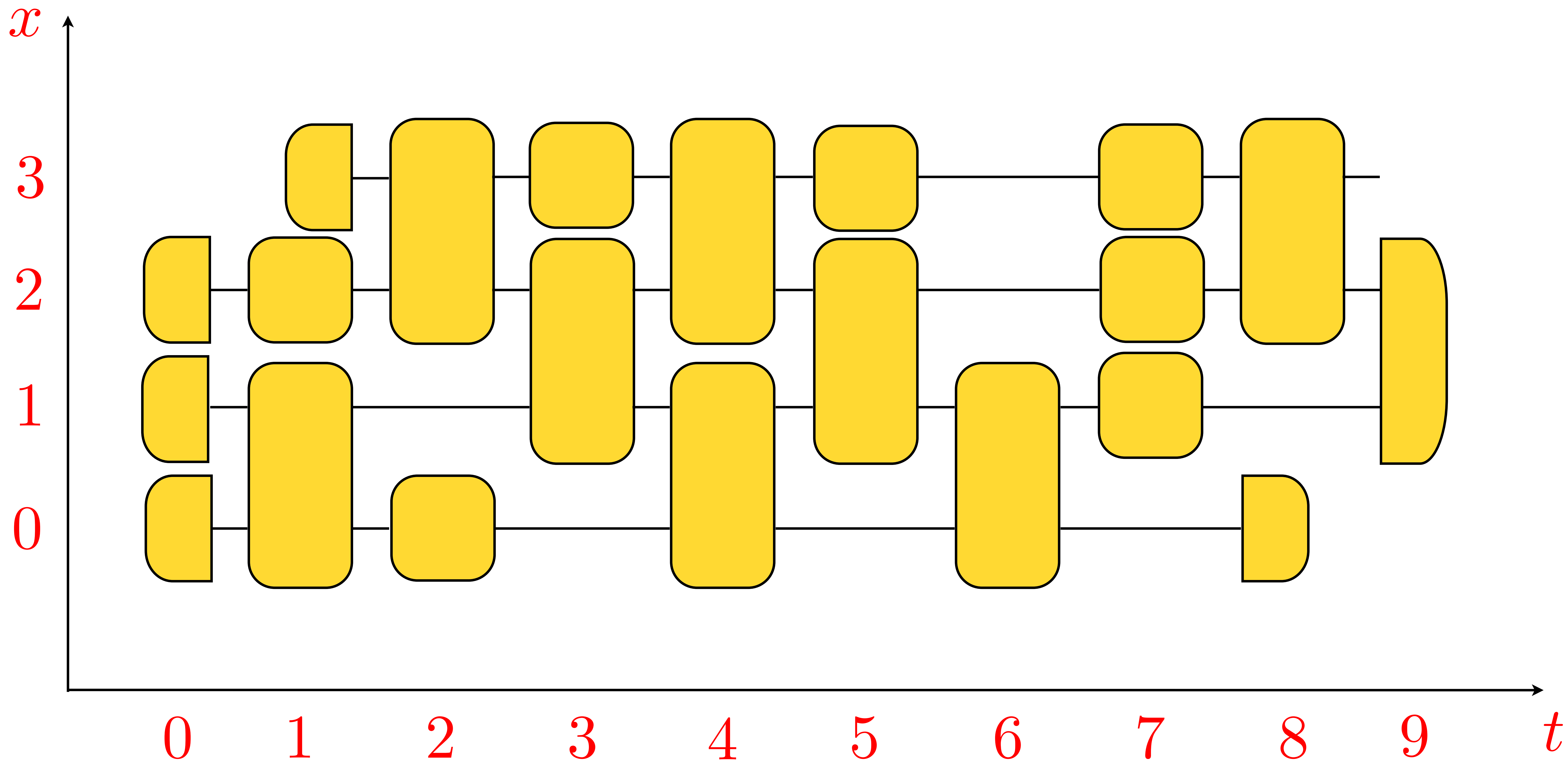
PP, Quantum **4**, 294 (2020)

# Circuits and causal chains

## Logical space-time




G. M. D'Ariano, G. Chiribella, and PP, *Quantum Theory from first principles. An informational approach.* (CUP 2017)

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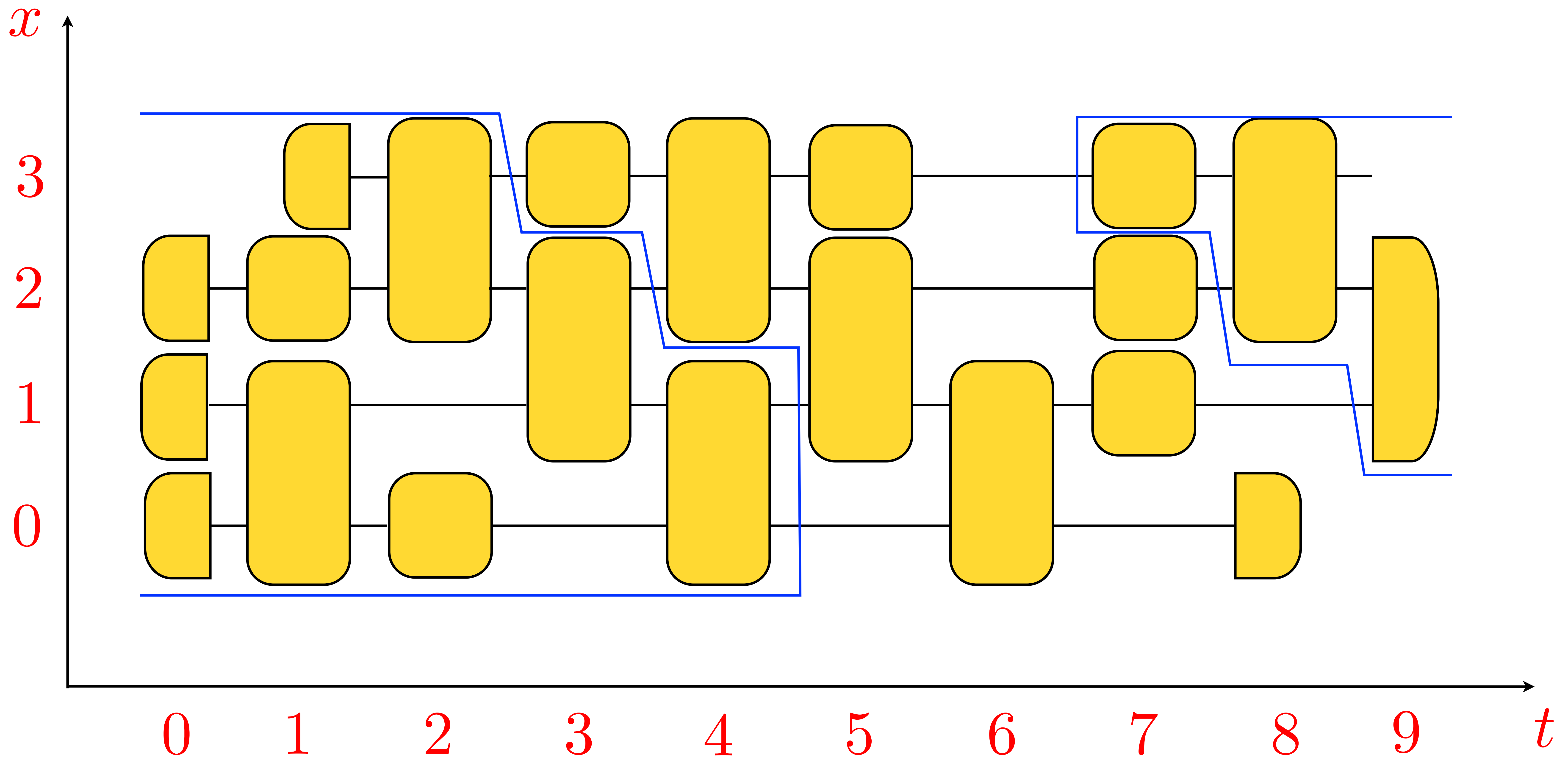


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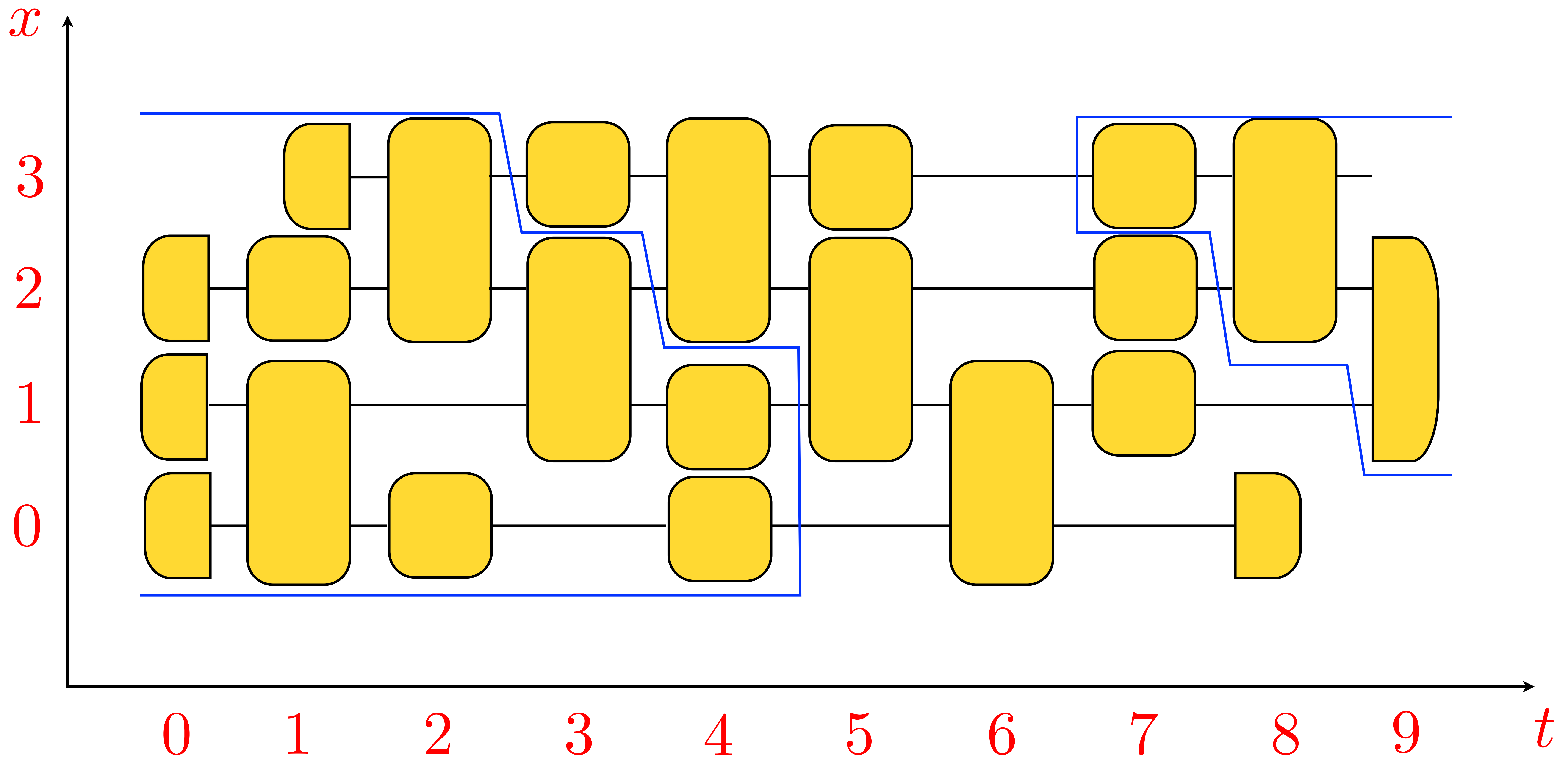


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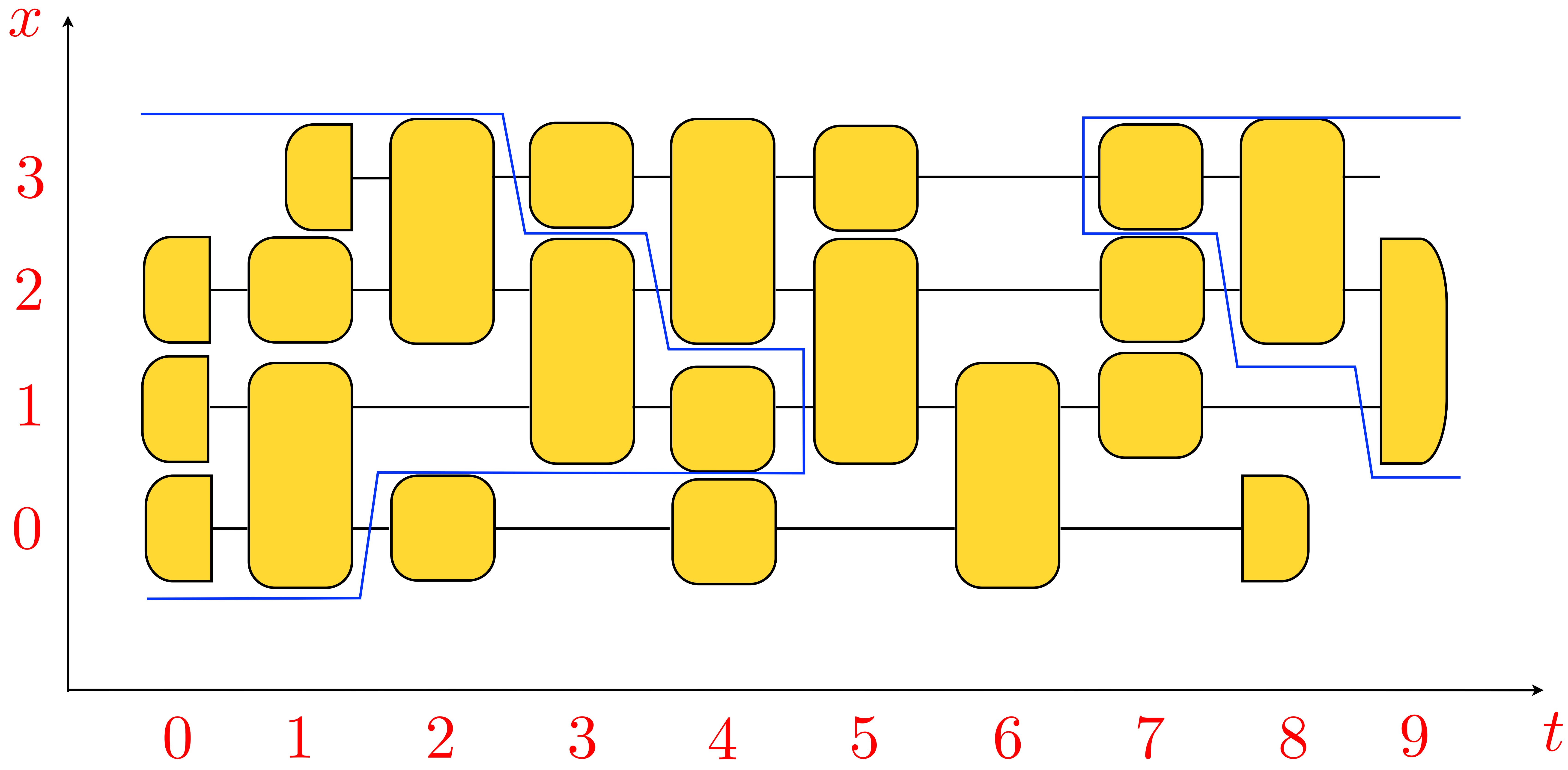


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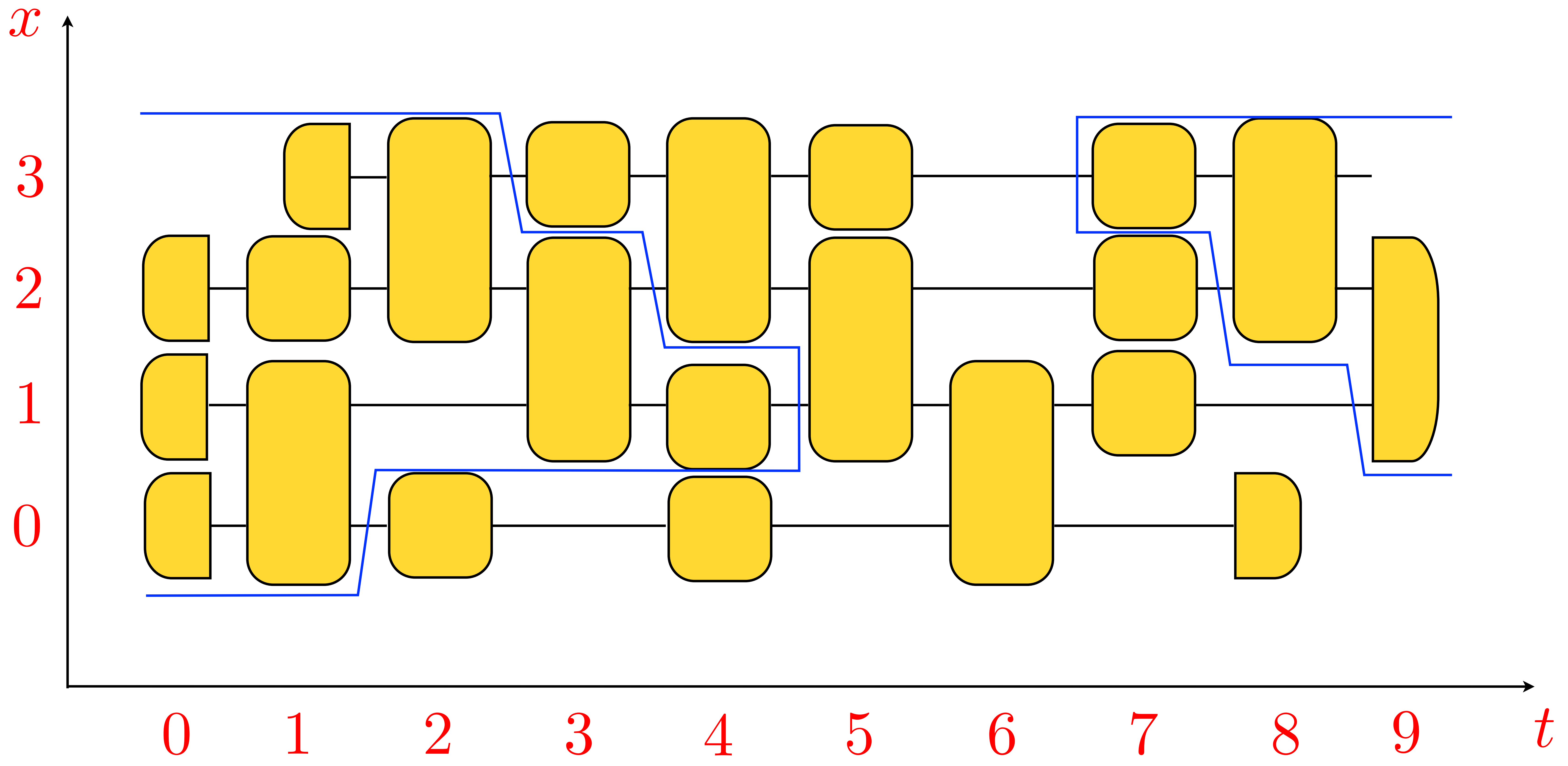


Can we make the causal order relation sharper?

G. M. D'Ariano, G. Chiribella, and PP, *Quantum Theory from first principles. An informational approach.* (CUP 2017)

# Chapter II: The main characters

Signalling vs causal influence

# Sending signals

## When is it (im)possible?

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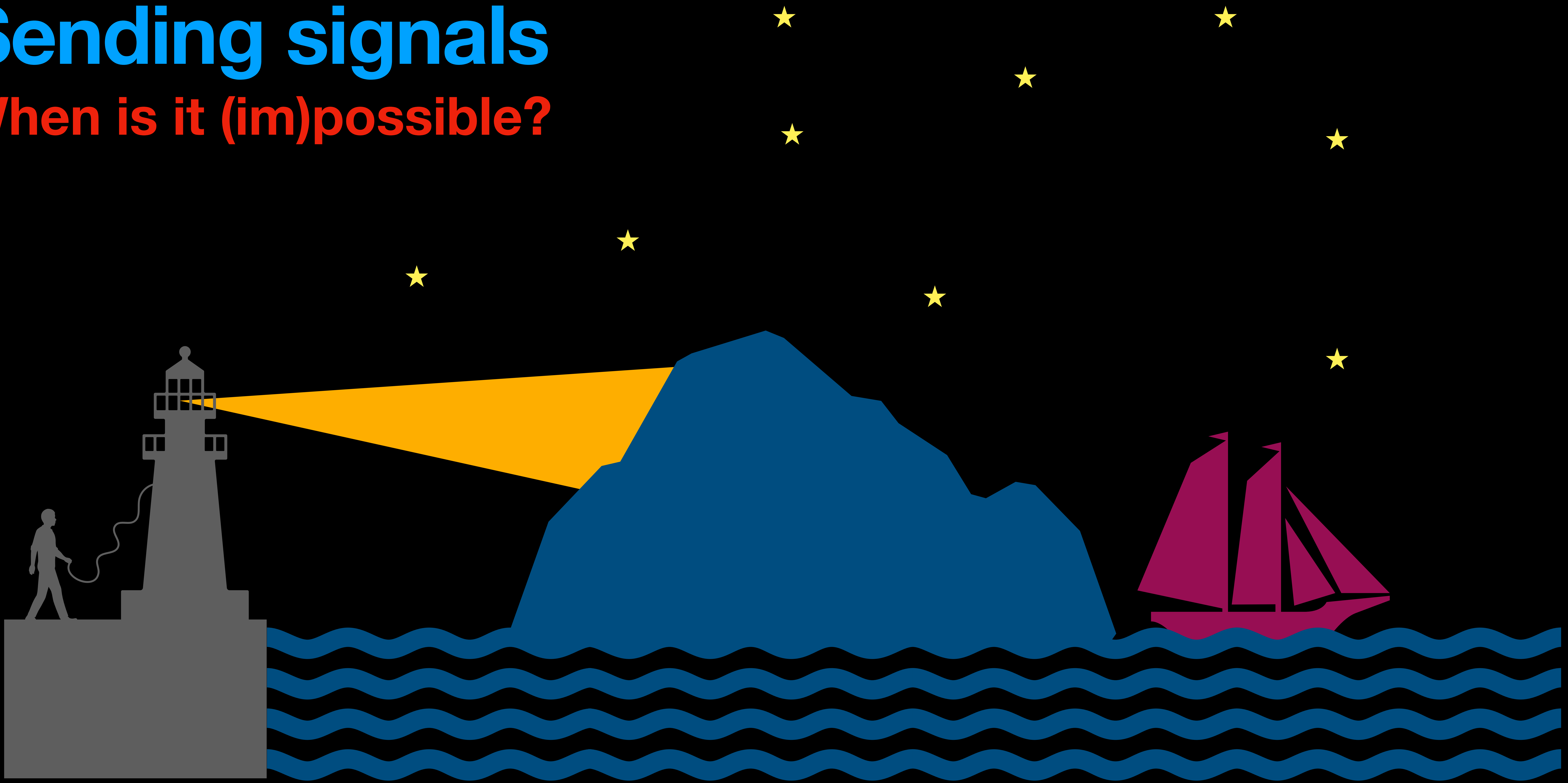

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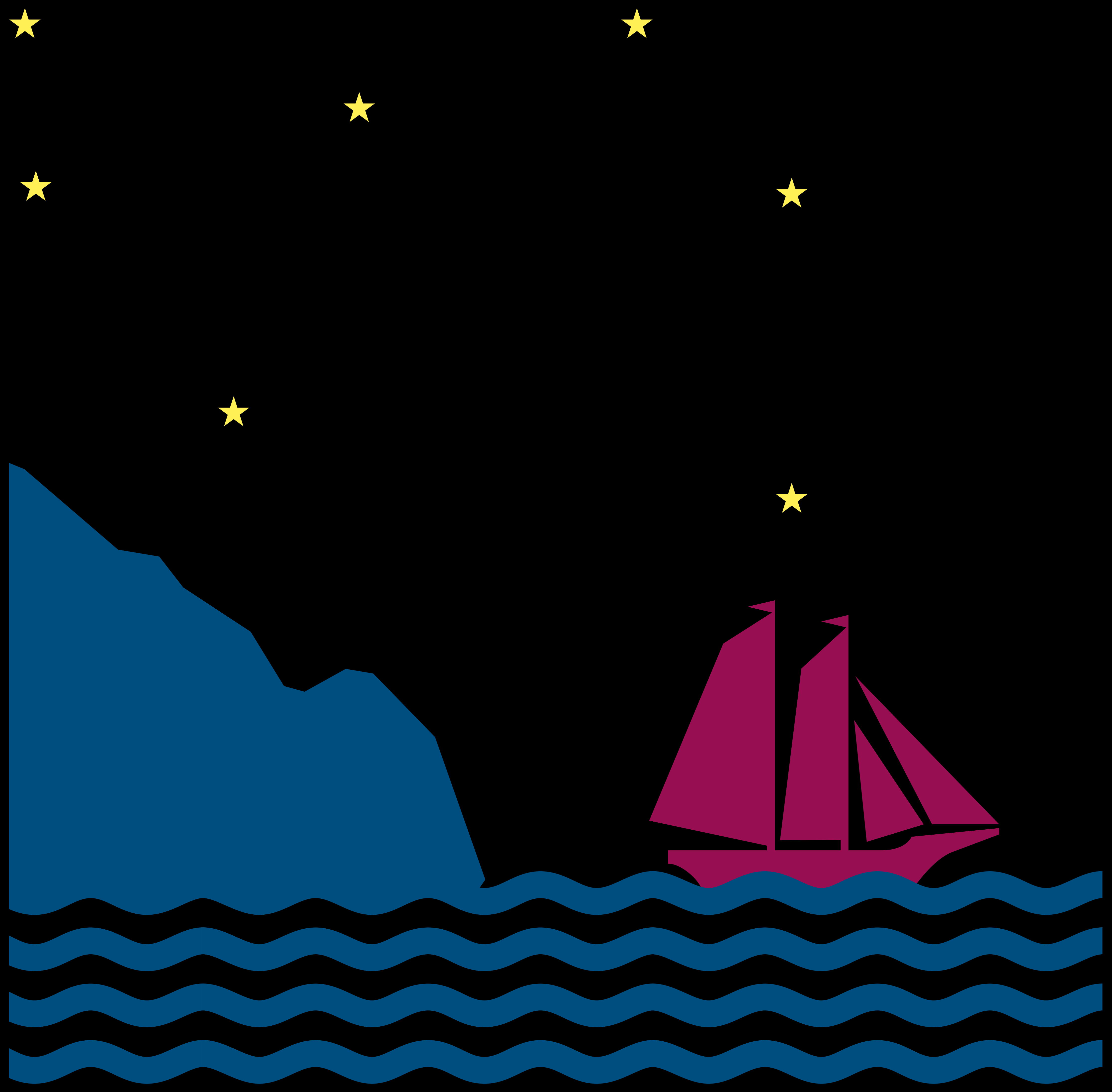

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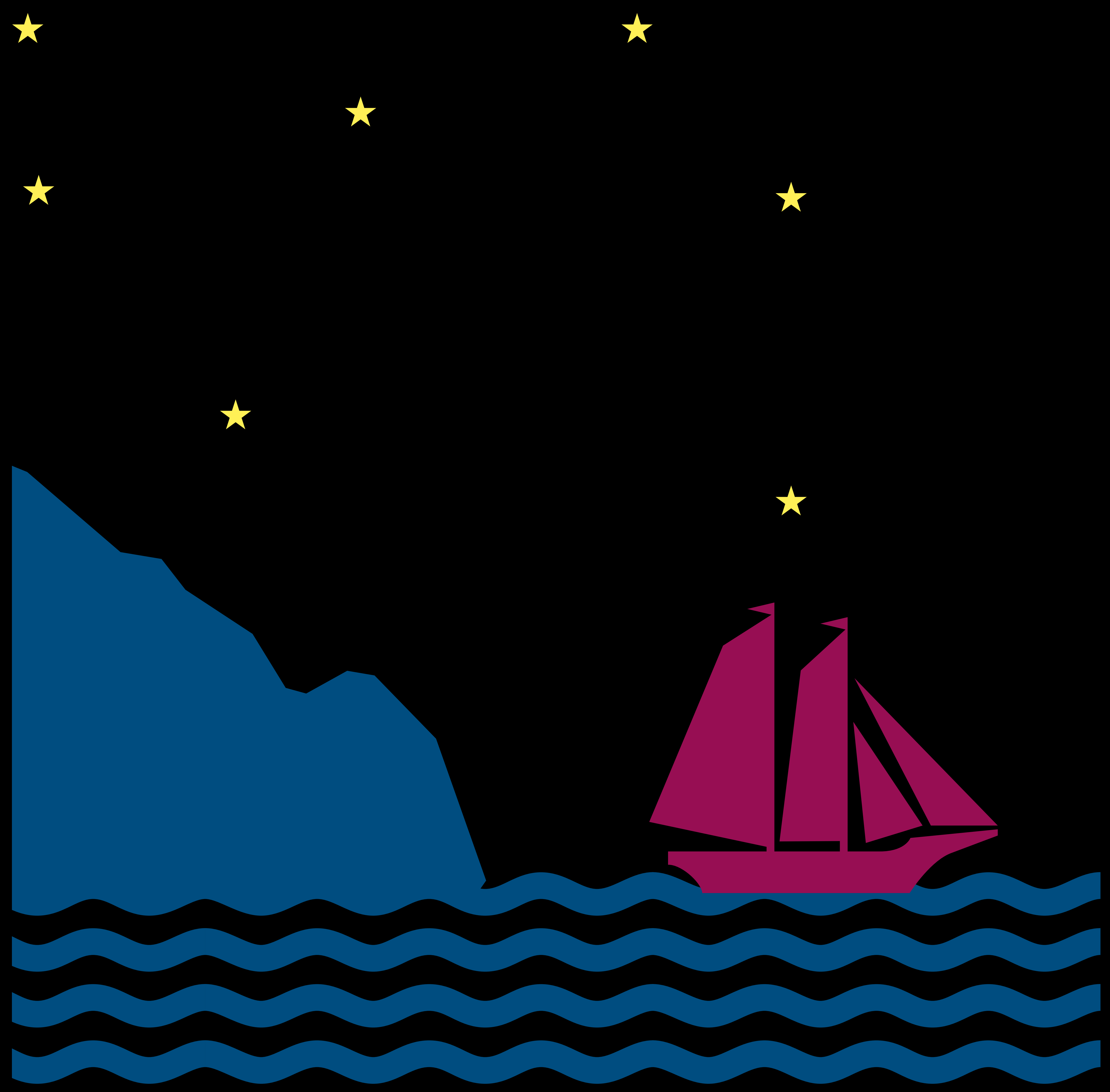
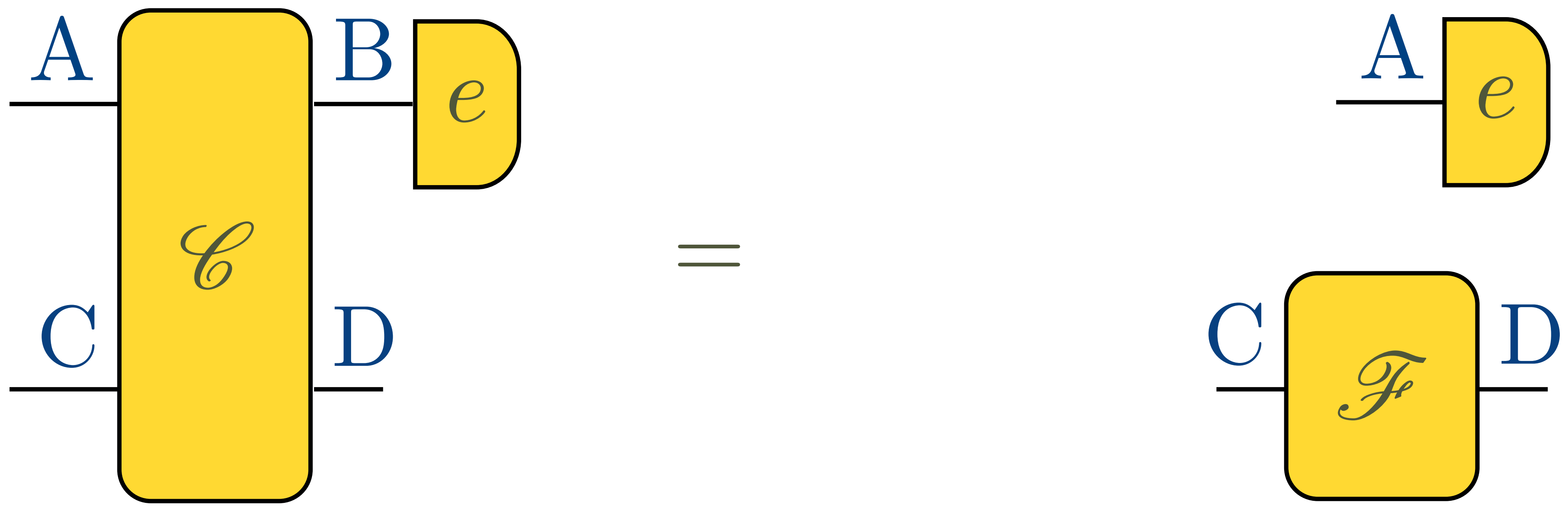


# Non-signalling channels

## The usual approach

$A \not\rightsquigarrow D$




No intervention on the state of A can influence the state of C

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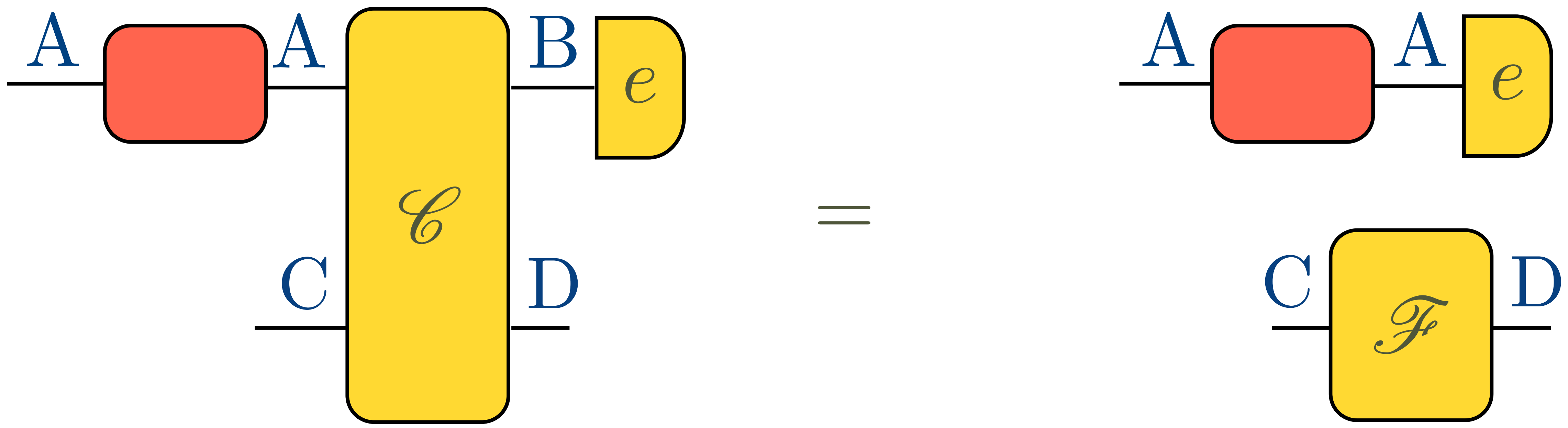


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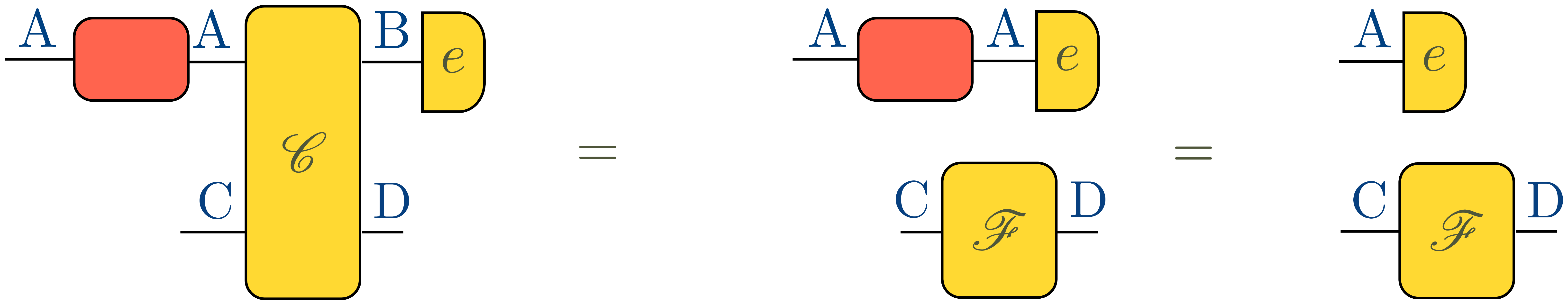


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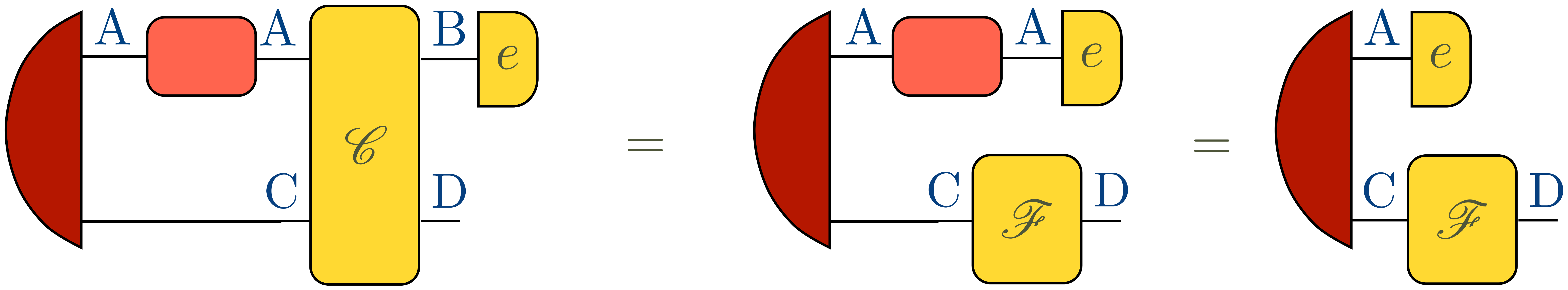


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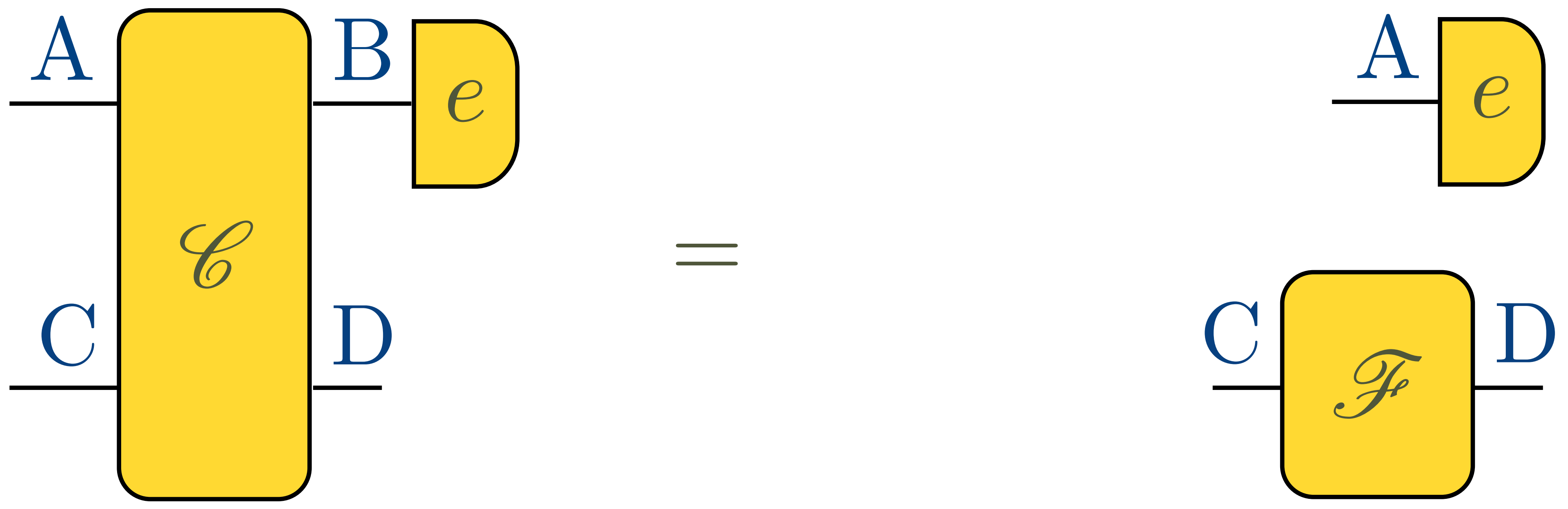


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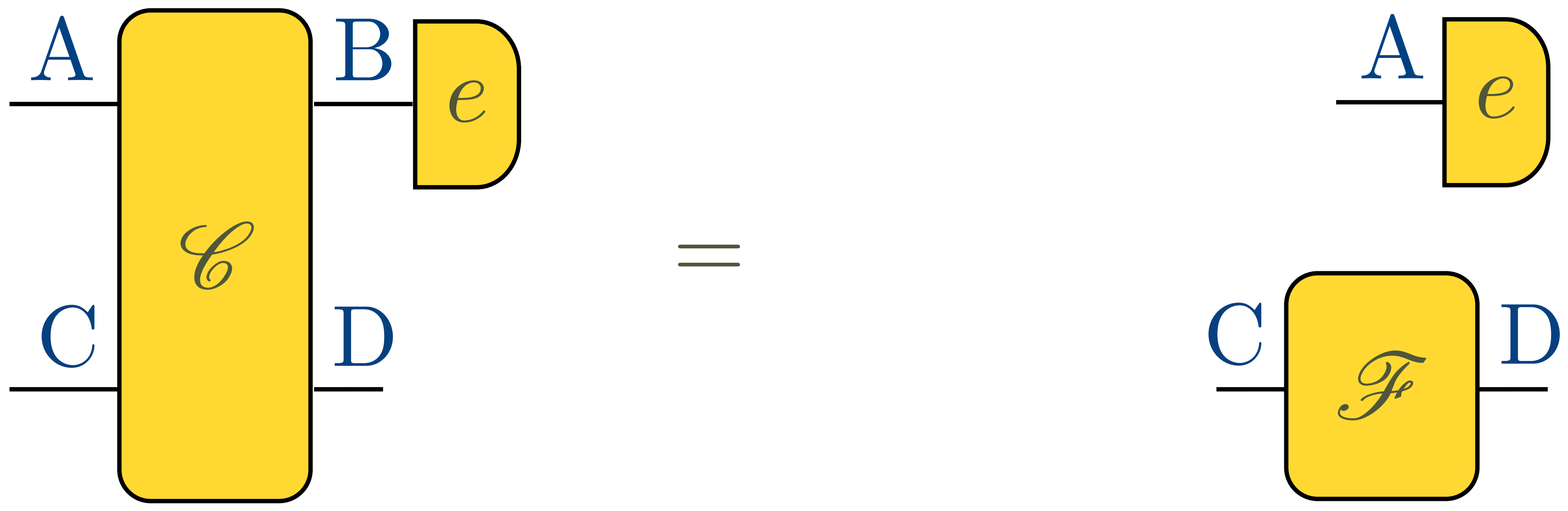


No intervention on the state of A can influence the state of C

In quantum theory

$$\mathrm{Tr}_{\mathrm{B}}[R_{\mathscr{C}}] = I_{\mathrm{A}} \otimes R_{\mathscr{F}}$$

$R_{\mathscr{E}}$ denoting the Choi operator corresponding to $\mathscr{E}$


D. Beckman, D. Gottesman, M. A. Nielsen, and J. Preskill, Phys Rev. A **64**, 052309 (2001)
T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

# Propagation of an intervention

## When does(n't) it happen?

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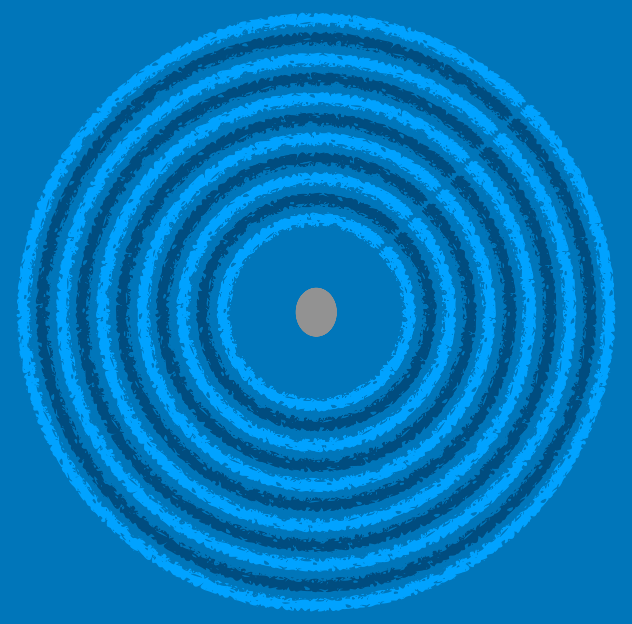

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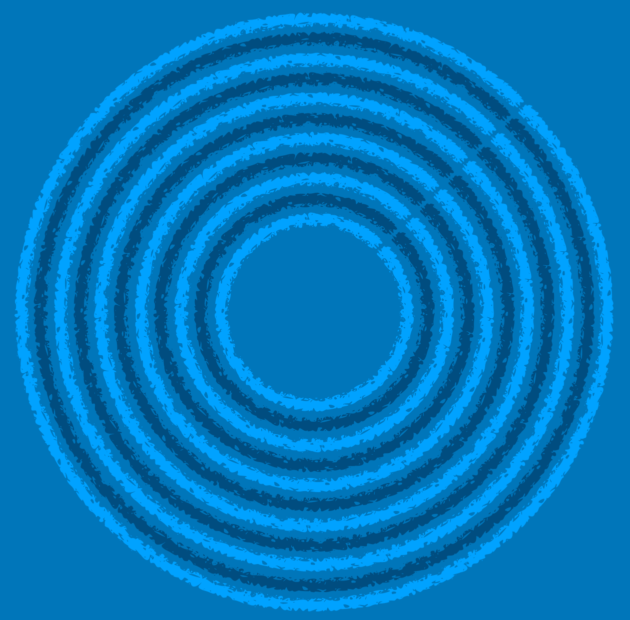

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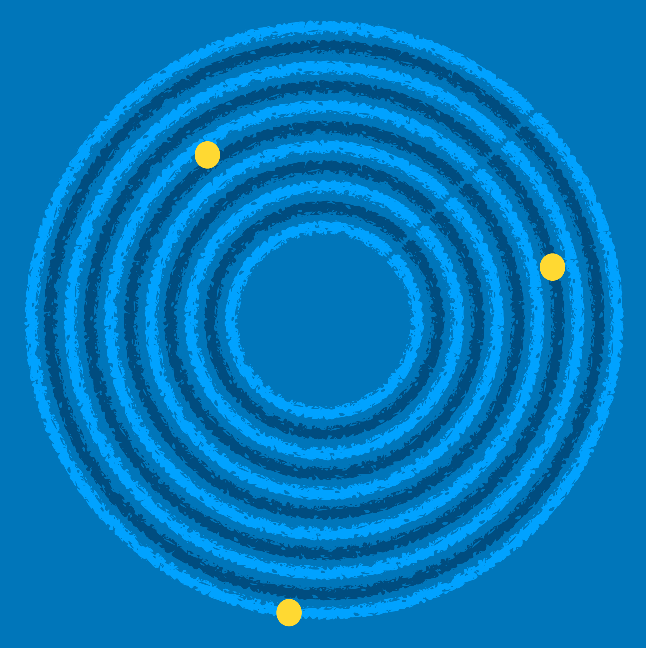

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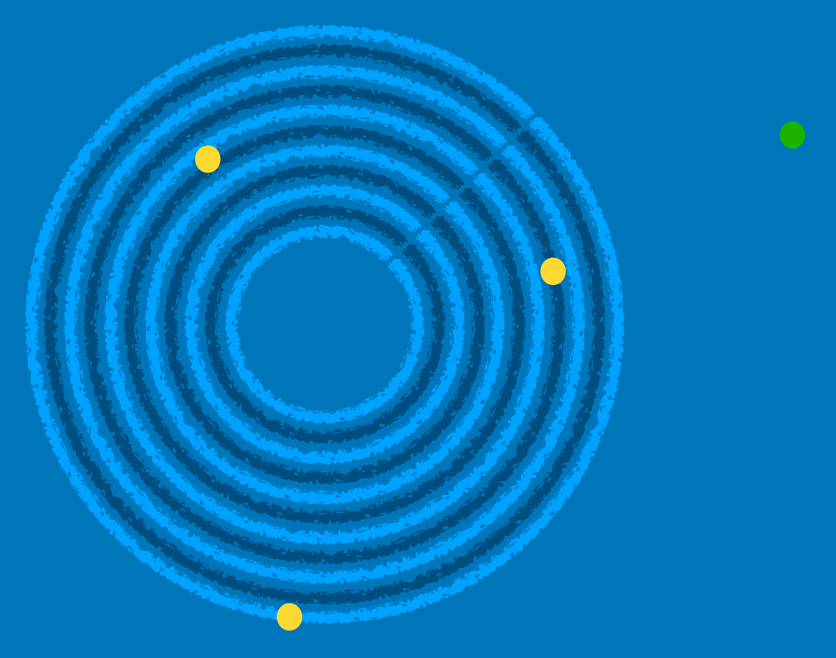

# Propagation of interventions

## The raw idea

- The definition is inspired by the notion of neighbourhood in QCAs
- It holds for reversible transformations

PP, Quantum **4**, 294 (2020)
PP, Quantum **5**, 515 (2021)

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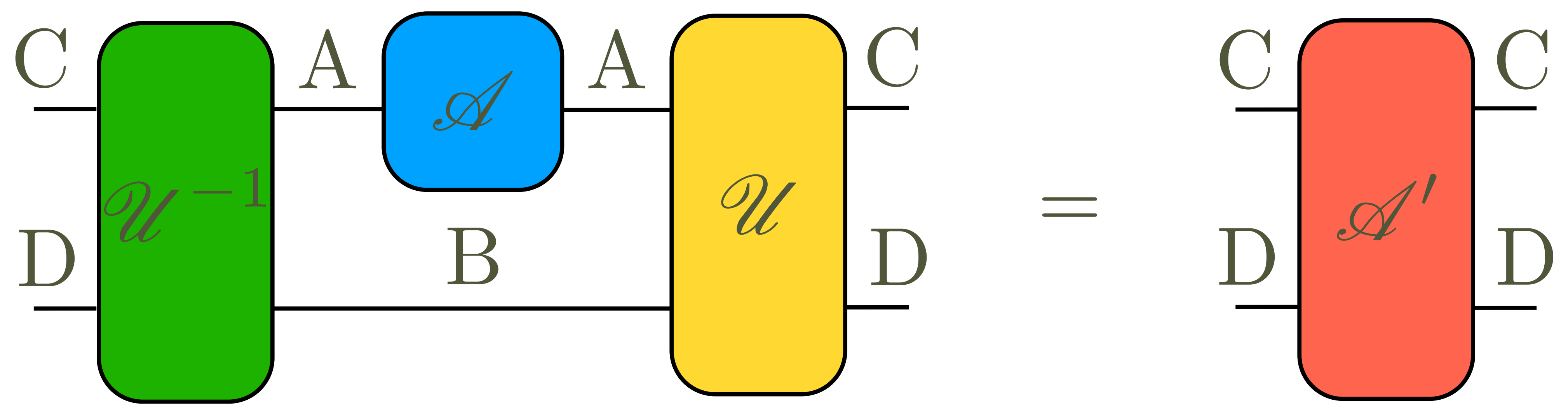


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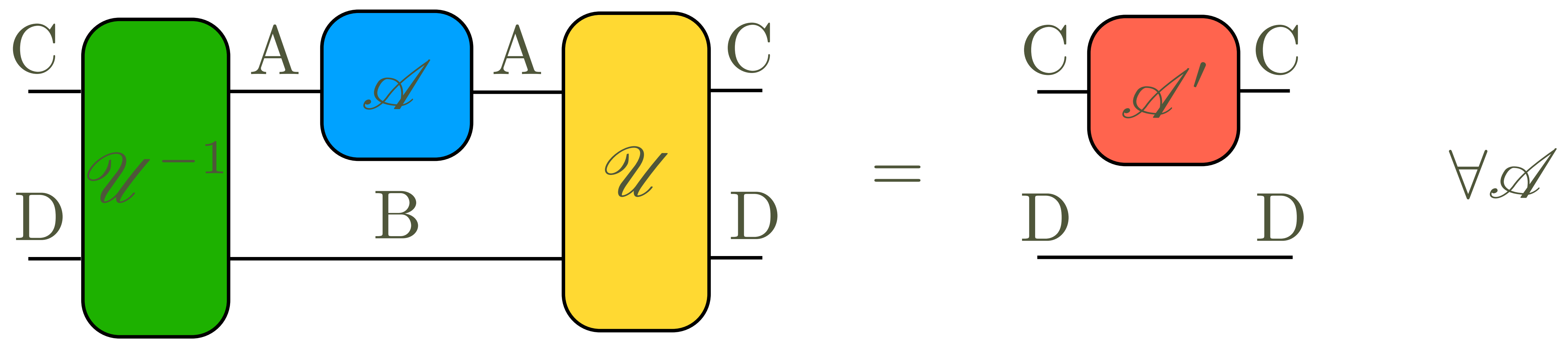


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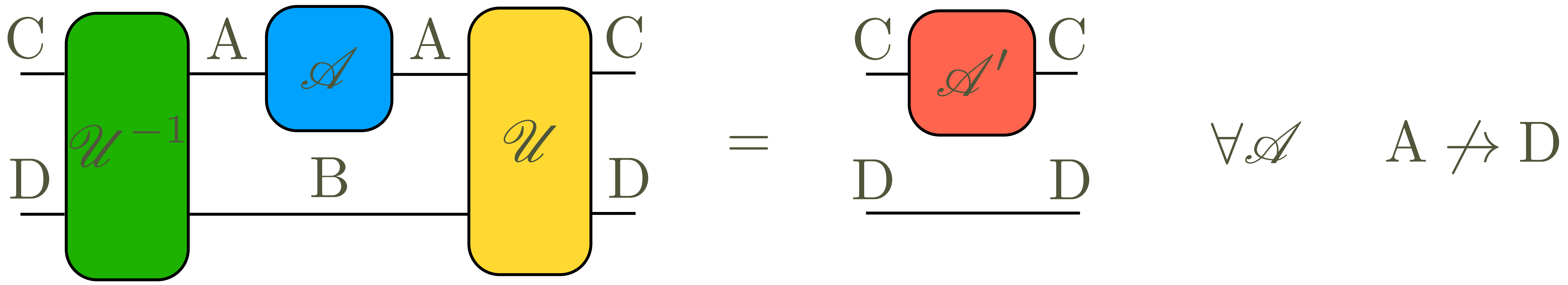


PP, Quantum **4**, 294 (2020)
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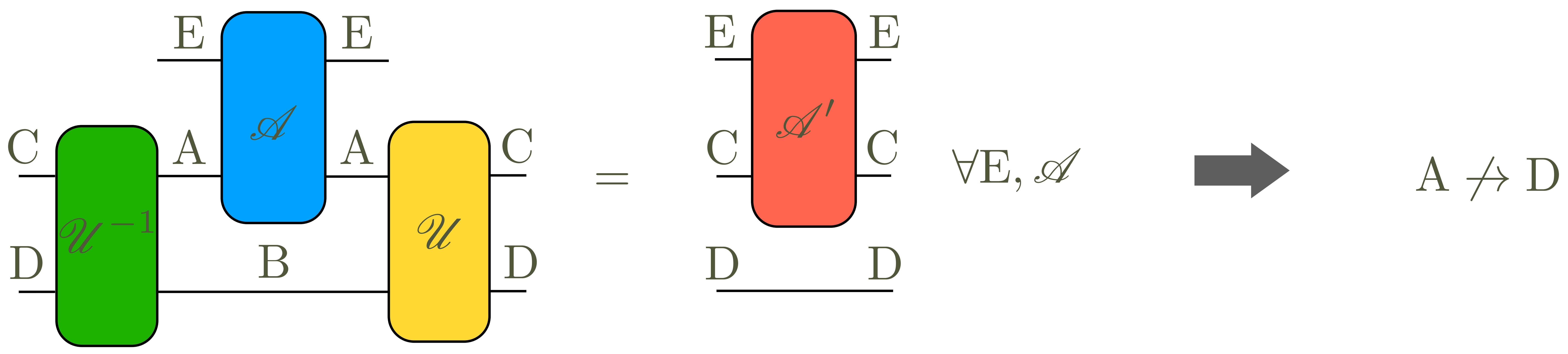
# Defining (no) causal influence in OPTs

## The precise notion

- Without local discriminability (local tomography/tomographic locality) we need to take into account interventions involving ancillary systems




PP, Quantum **4**, 294 (2020)
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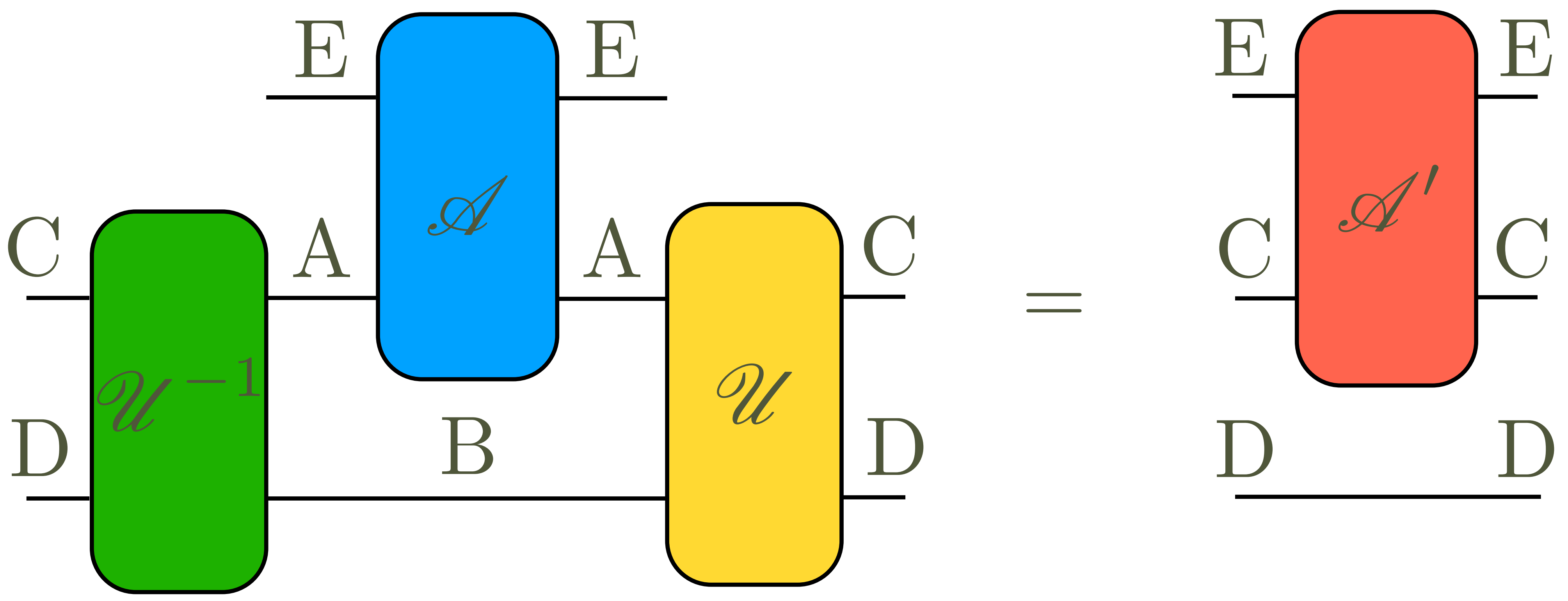
# Explanation of the definition

Suppose that under $\mathscr{U}$ one has $\mathrm{A} \not\to \mathrm{D}$



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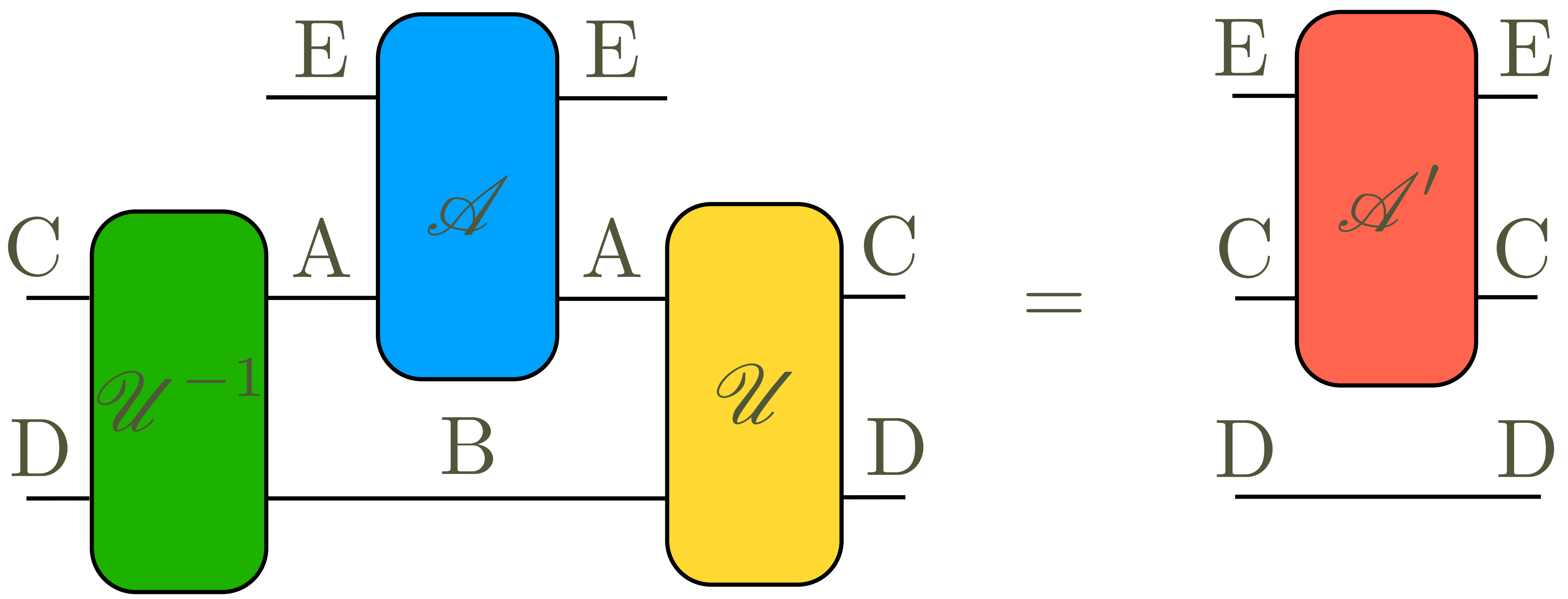
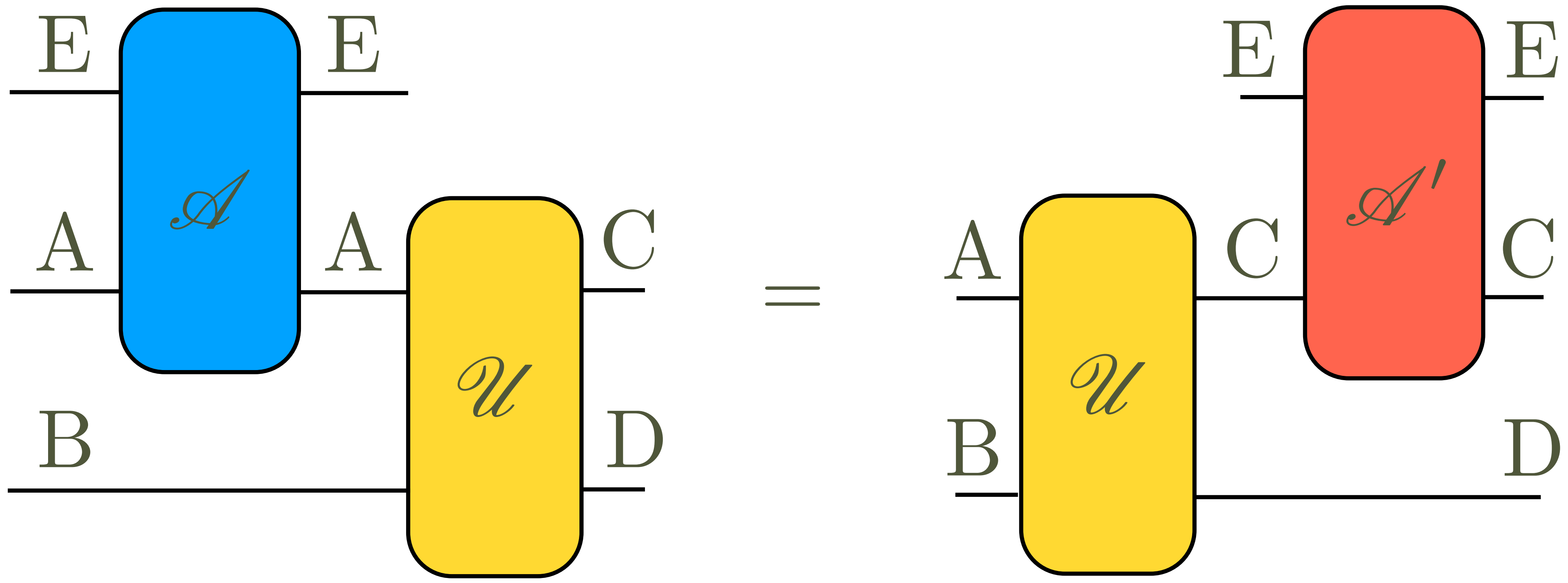



Equivalently:



# Chapter III: The story

Results

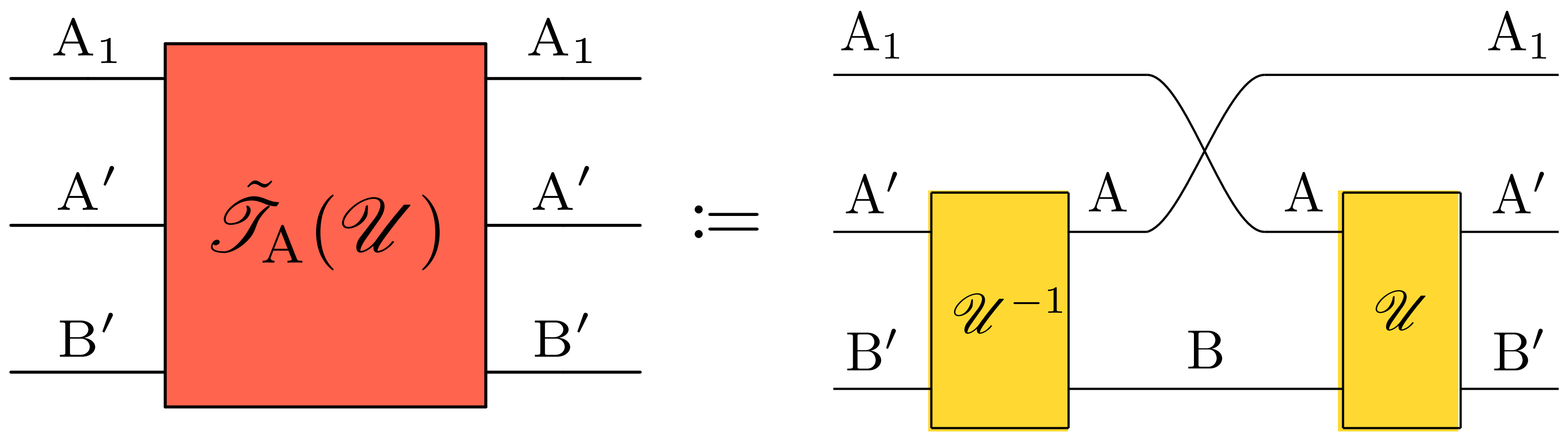
# Signalling: sufficient for causal influence

Signalling: sufficient for causal influence

But not necessary!

# Necessary and sufficient condition

- Definition:

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# Necessary and sufficient condition

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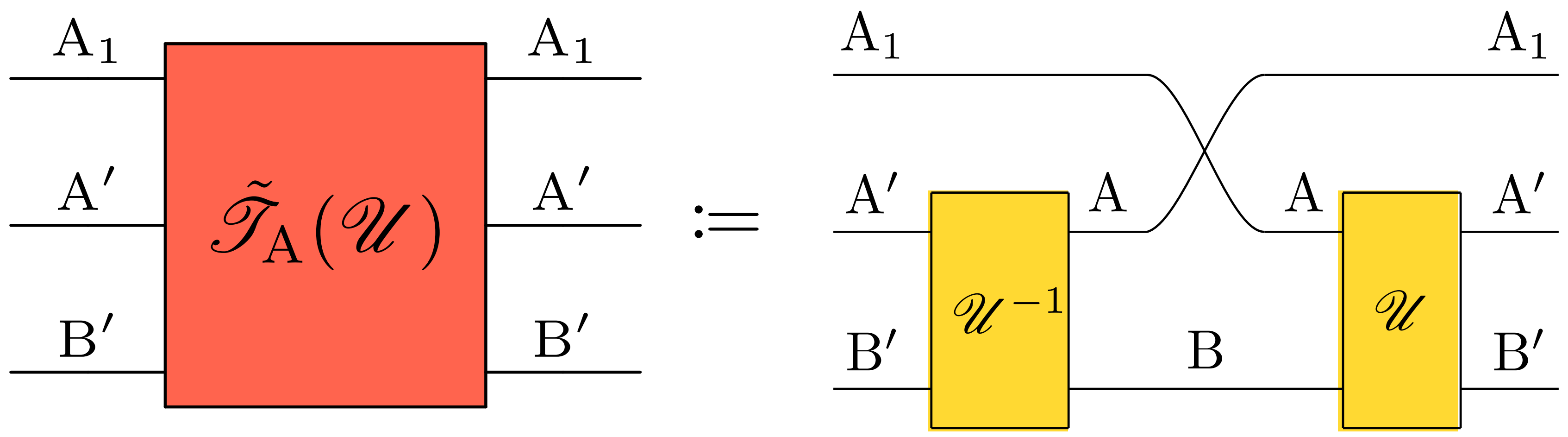
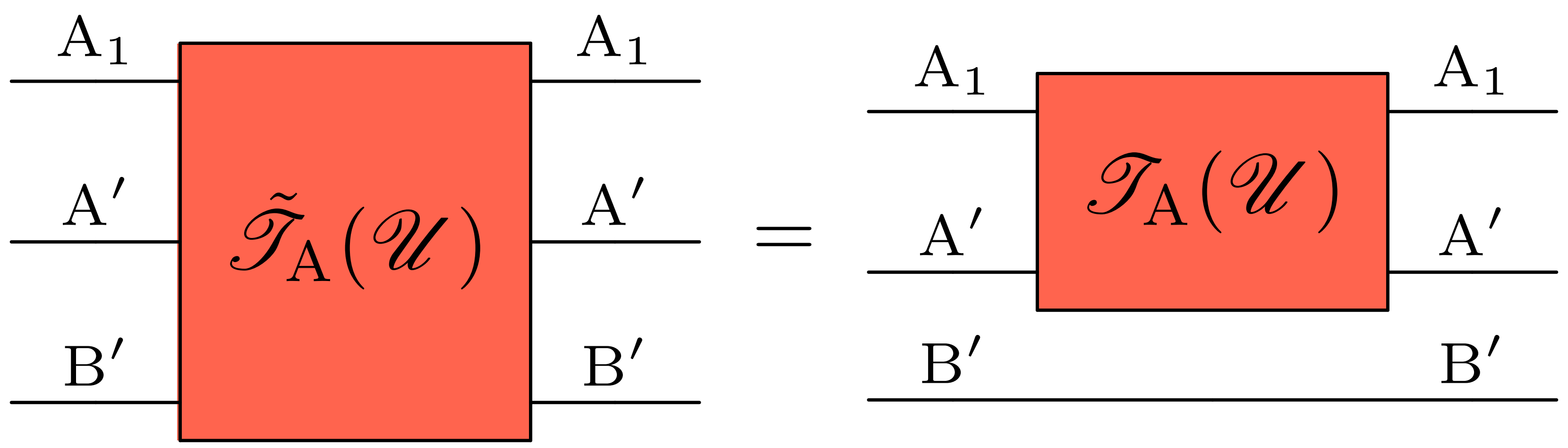



- Condition:

$A \not\to B'$ iff





PP, Quantum **4**, 294 (2020)
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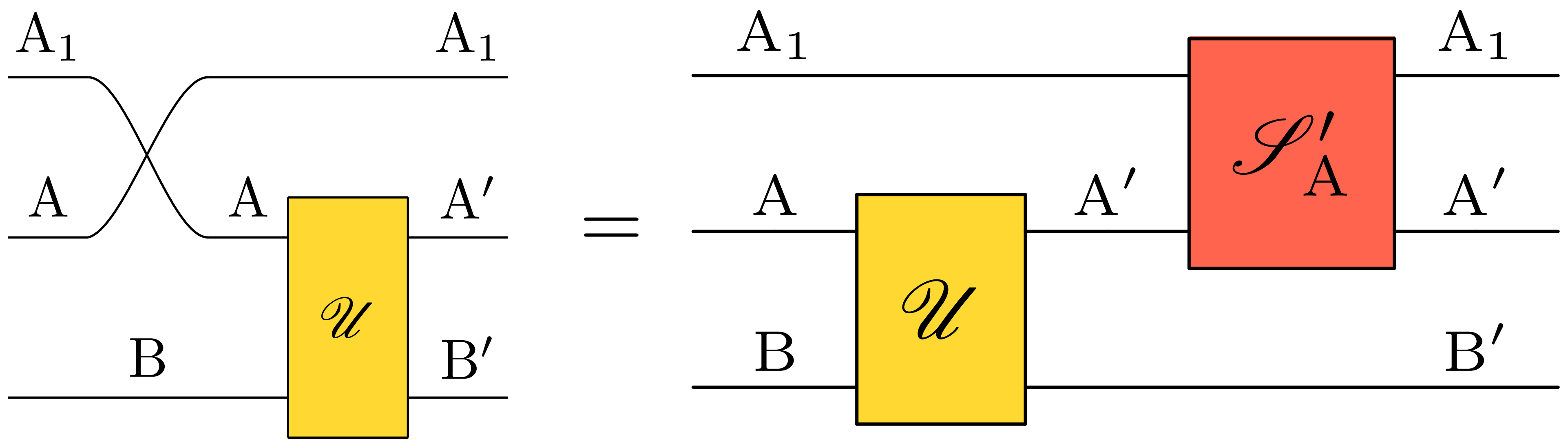
# Necessary condition: comb structure

- Suppose that $\mathrm{A} \not\to \mathrm{B}'$ . Then it must be

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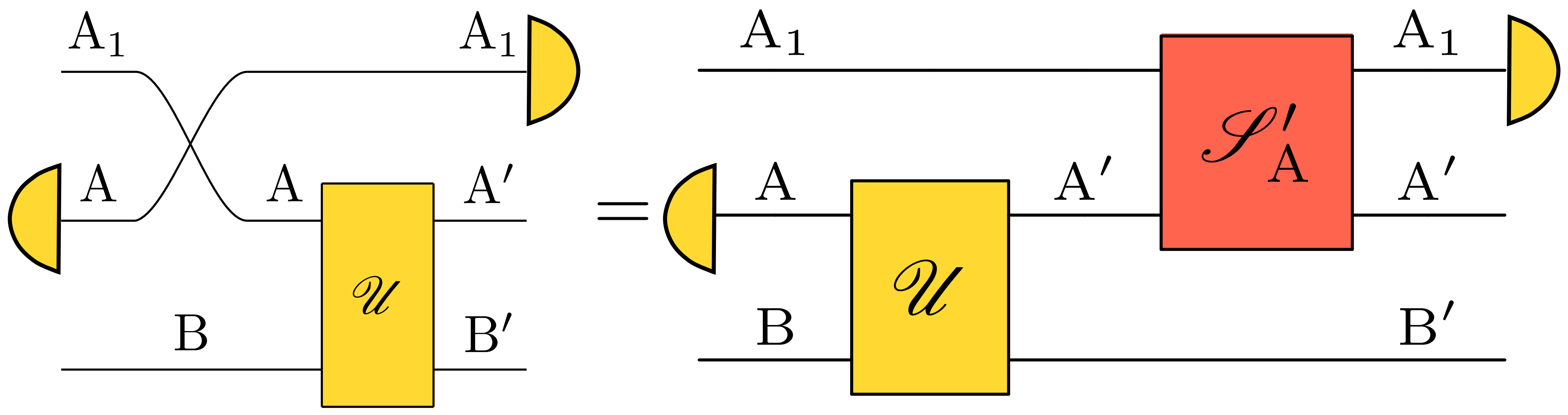
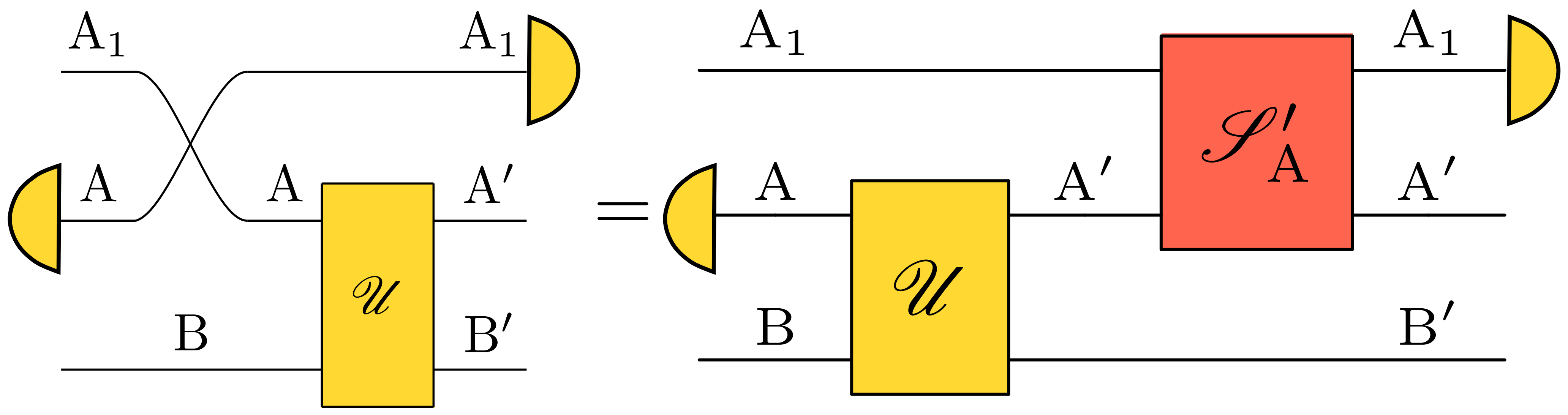



- Preparing a state of $\mathrm{A}$ and discarding $\mathrm{A}_1$

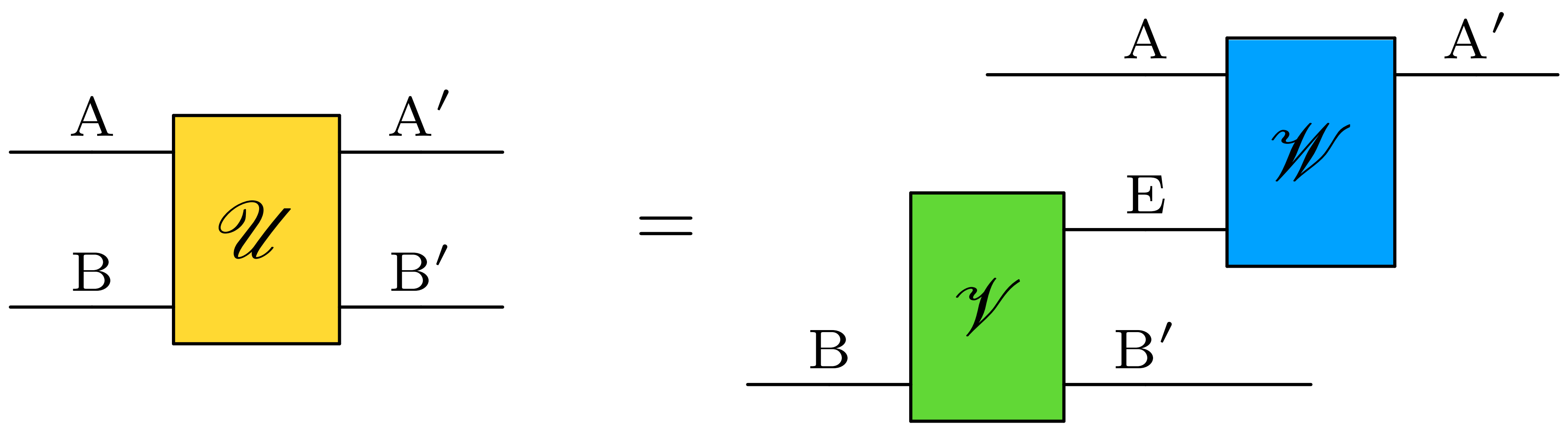
PP, Quantum **5**, 515 (2021).

# Necessary condition: comb structure

- Suppose that $\mathrm{A} \nrightarrow \mathrm{B}'$ . Then it must be

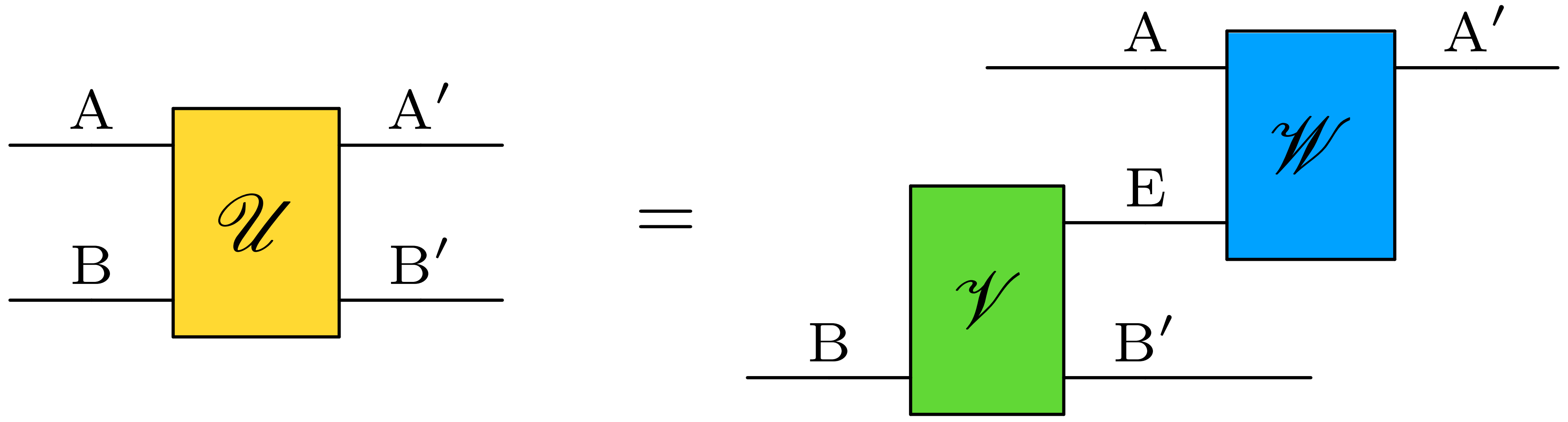



- Preparing a state of $\mathrm{A}$ and discarding $\mathrm{A}_1$




PP, Quantum **5**, 515 (2021).

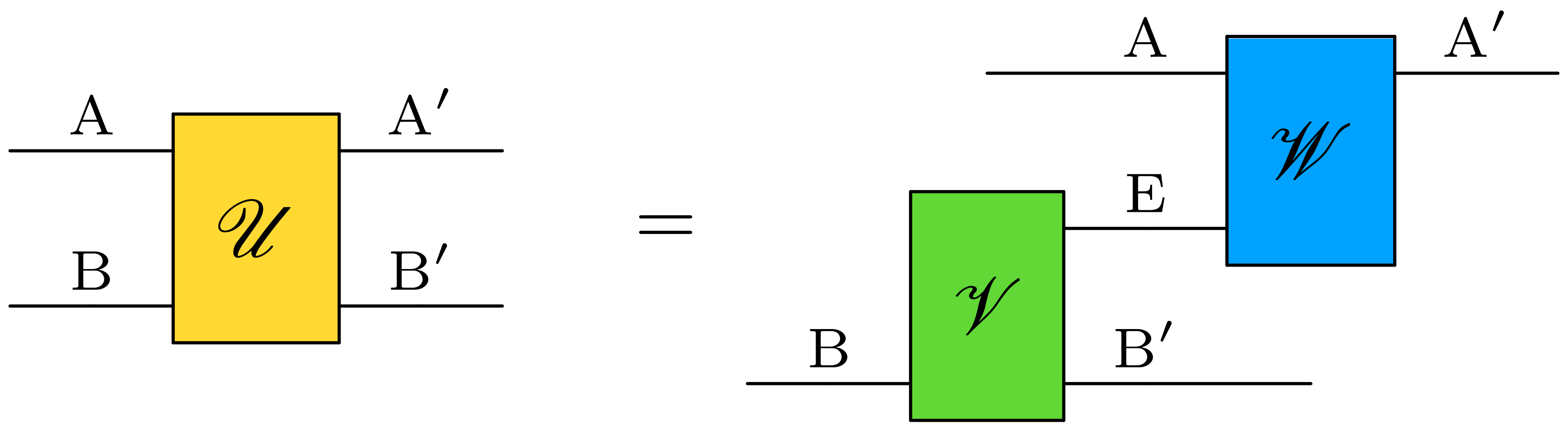
# Necessary condition: no-signalling




PP, Quantum **5**, 515 (2021).

# Necessary condition: no-signalling

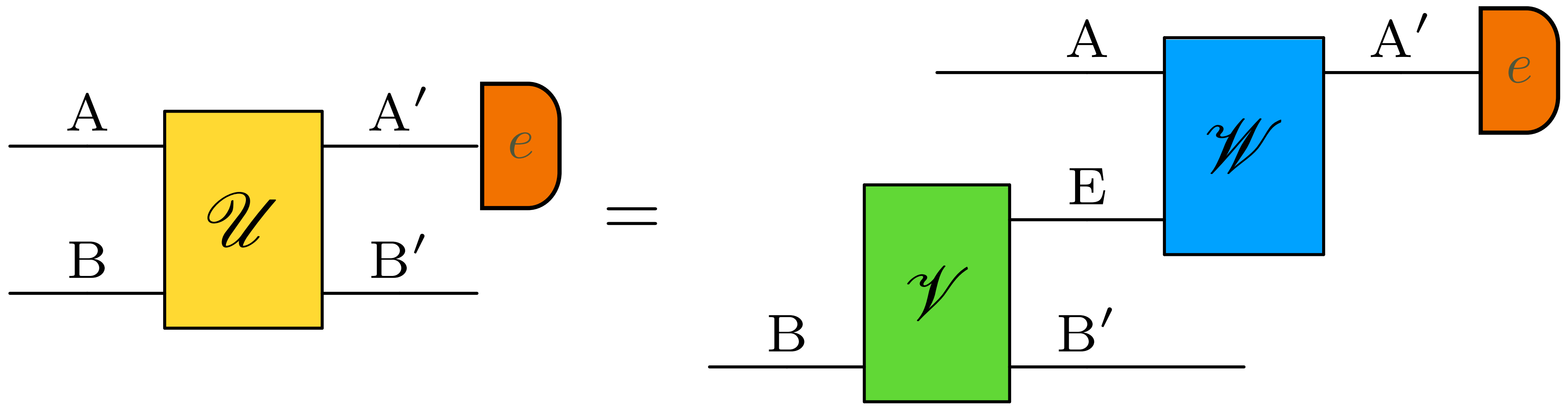
- Discarding $A'$

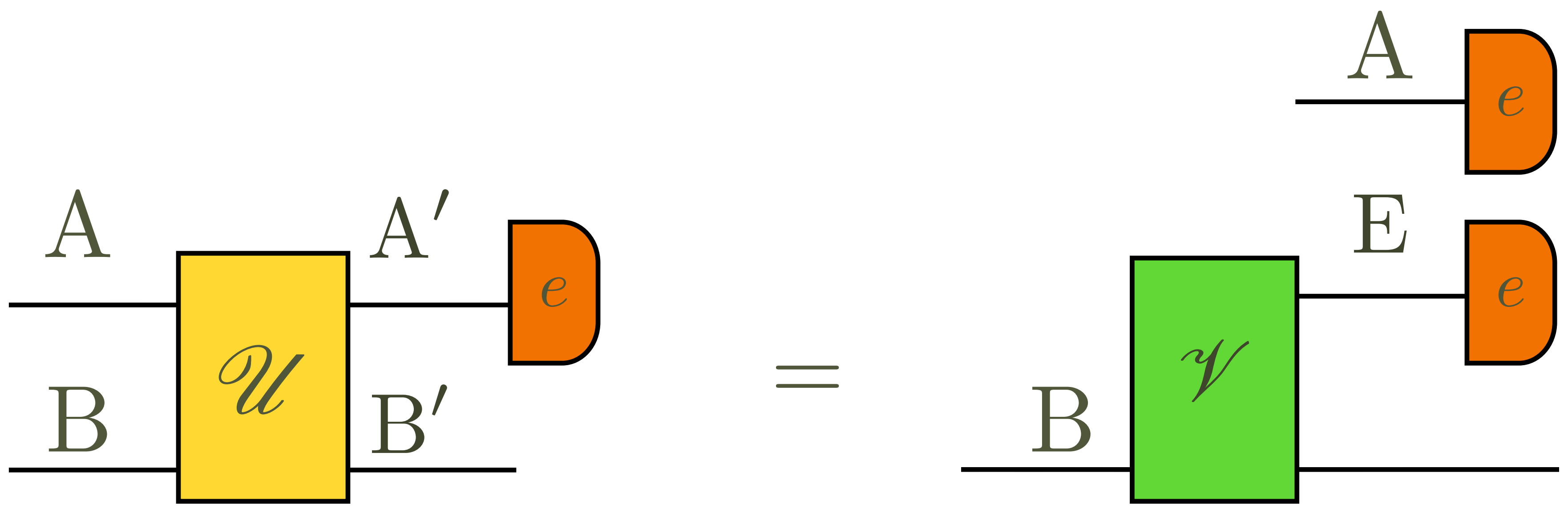



PP, Quantum **5**, 515 (2021).

# Necessary condition: no-signalling

- Discarding $\mathrm{A}'$

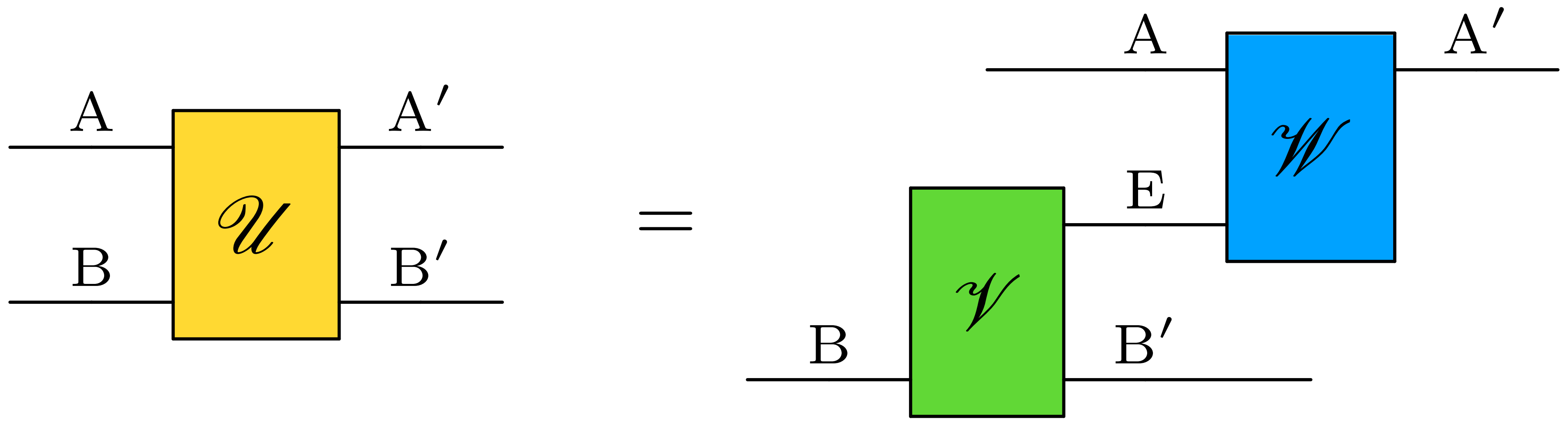



PP, Quantum **5**, 515 (2021).

# Chain of conditions

$\mathrm{A} \nrightarrow \mathrm{B}'$ ⟹ 



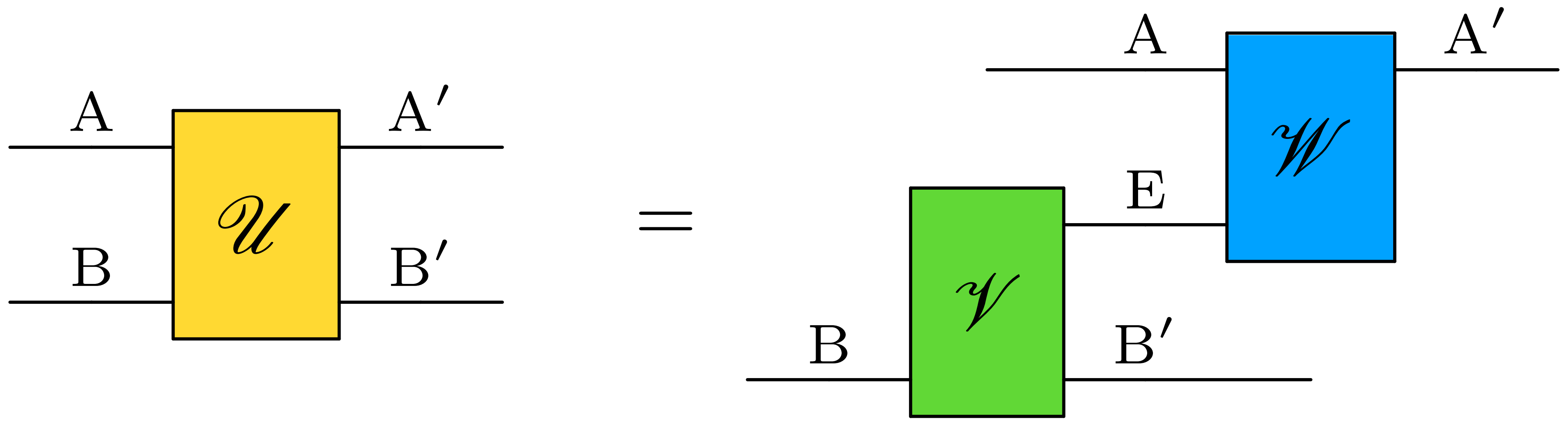

⟹ $\mathrm{A} \not\rightsquigarrow \mathrm{B}'$

PP, Quantum **5**, 515 (2021).

# Classical theory

## Example 1

$A \not\to B'$

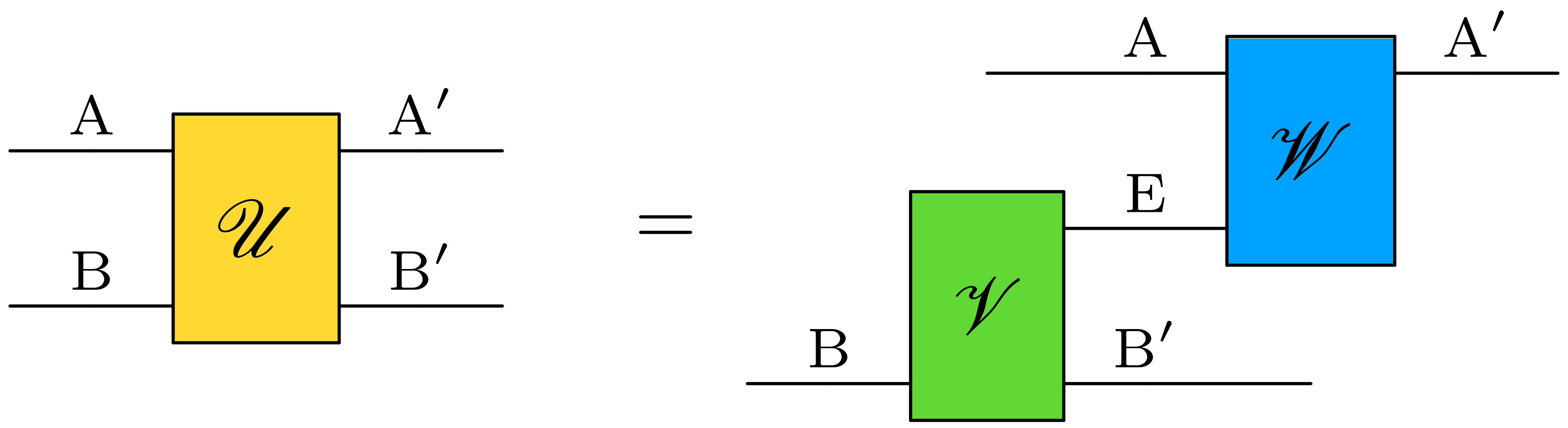



$A \not\rightsquigarrow B'$

# Classical theory

## Example 1

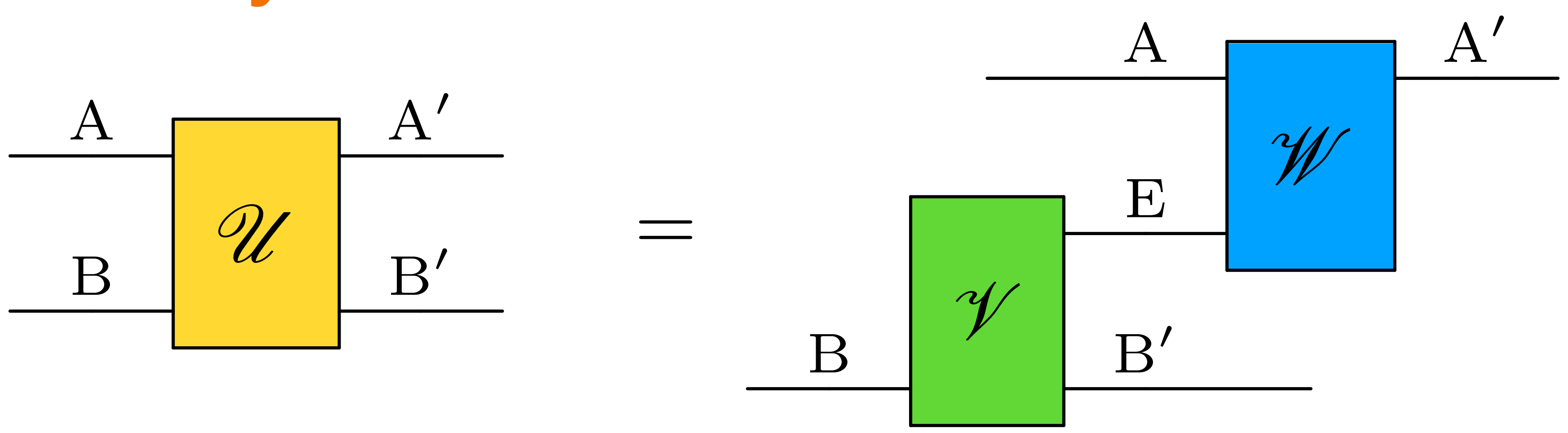
$A \not\to B'$ ➡




➡ $A \not\rightsquigarrow B'$

- One can prove that in classical theory

$A \not\rightsquigarrow B'$ ➡

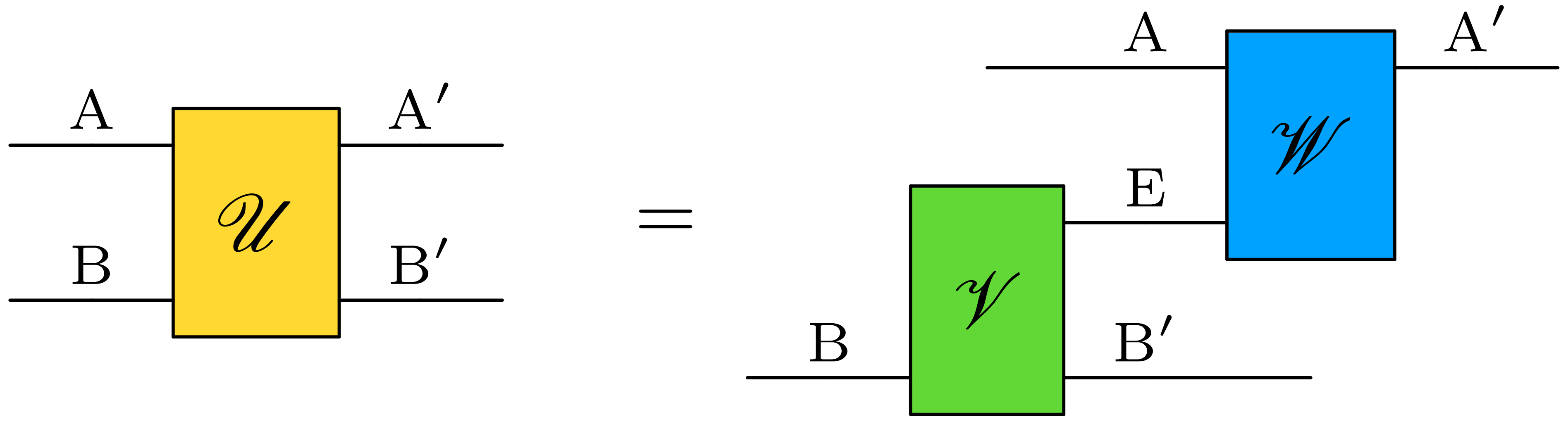



PP, Quantum **5**, 515 (2021).

# Classical theory

## Example 1

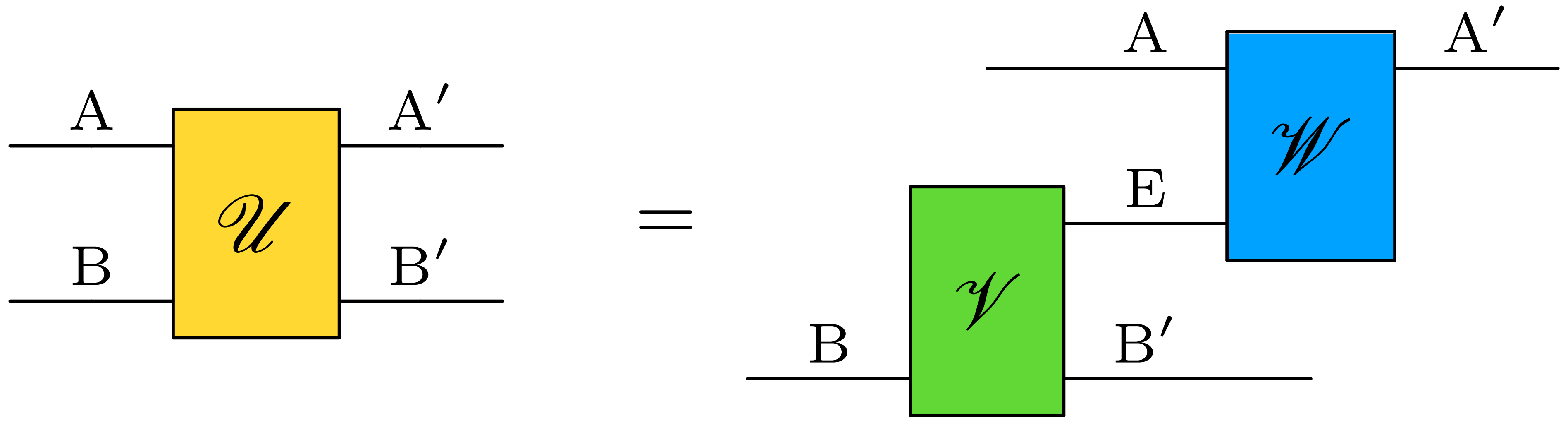
$A \not\to B'$ ⇒




⇔ $A \not\rightsquigarrow B'$

- One can prove that in classical theory

$A \not\rightsquigarrow B'$ ⇒




PP, Quantum **5**, 515 (2021).

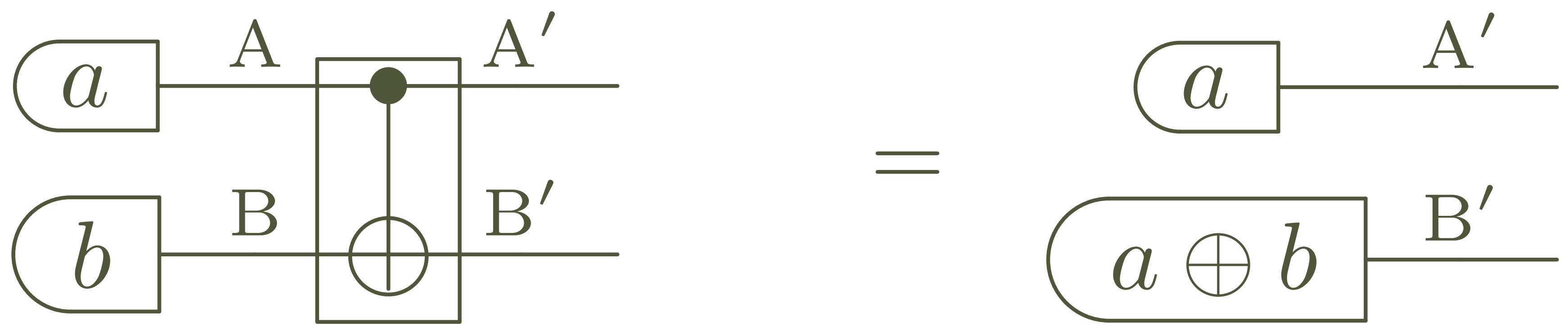
# Classical theory
## Example 1

- Classical C-not

# Classical theory

## Example 1

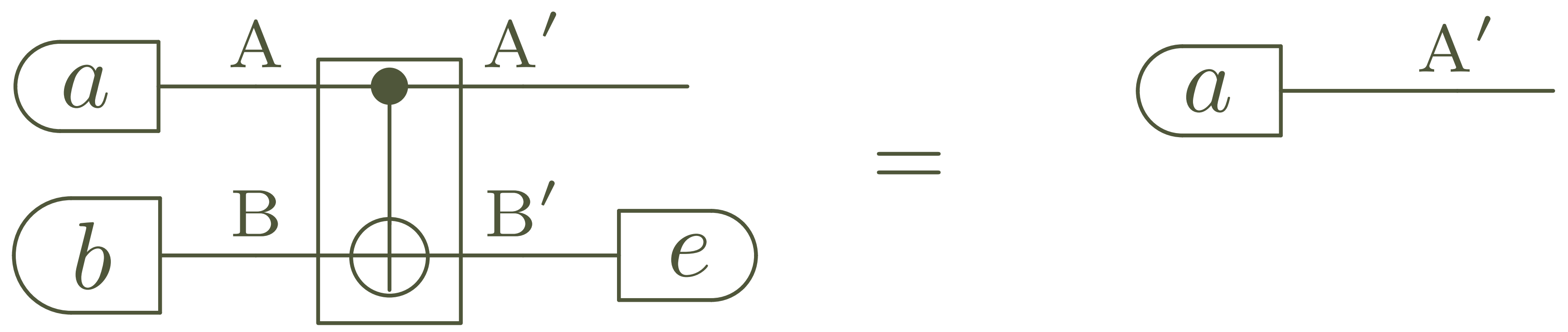
- Classical C-not



# Classical theory

## Example 1

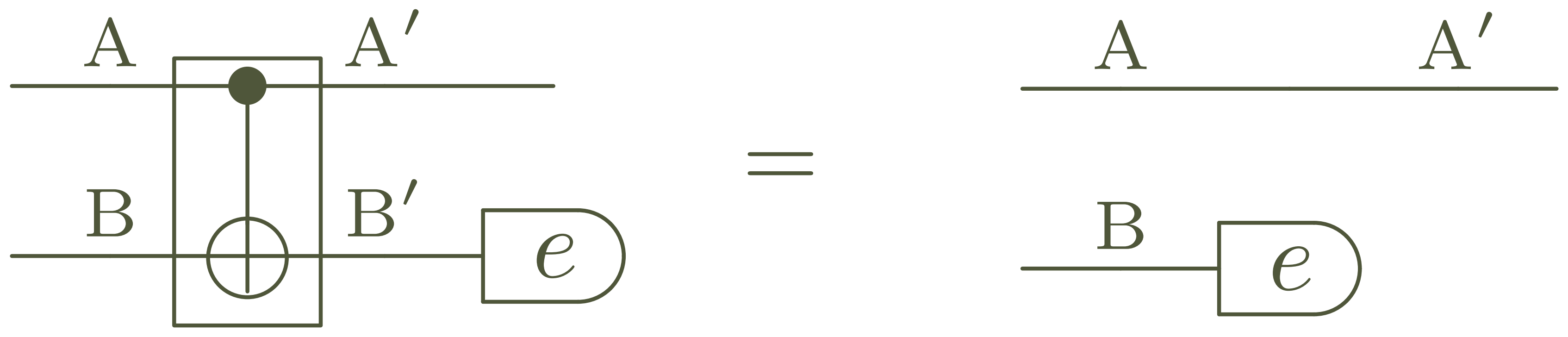
- Classical C-not



# Classical theory

## Example 1

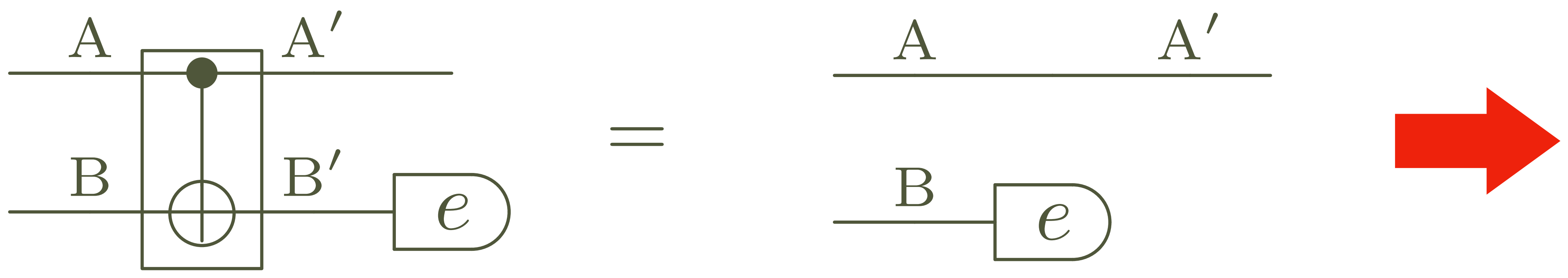
- Classical C-not



# Classical theory

## Example 1

- Classical C-not

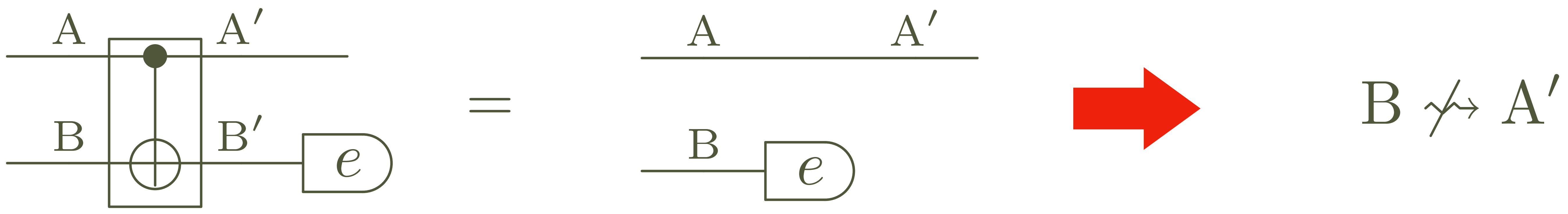



$$\mathrm{B} \not\rightsquigarrow \mathrm{A}'$$

# Classical theory

## Example 1

- Classical C-not

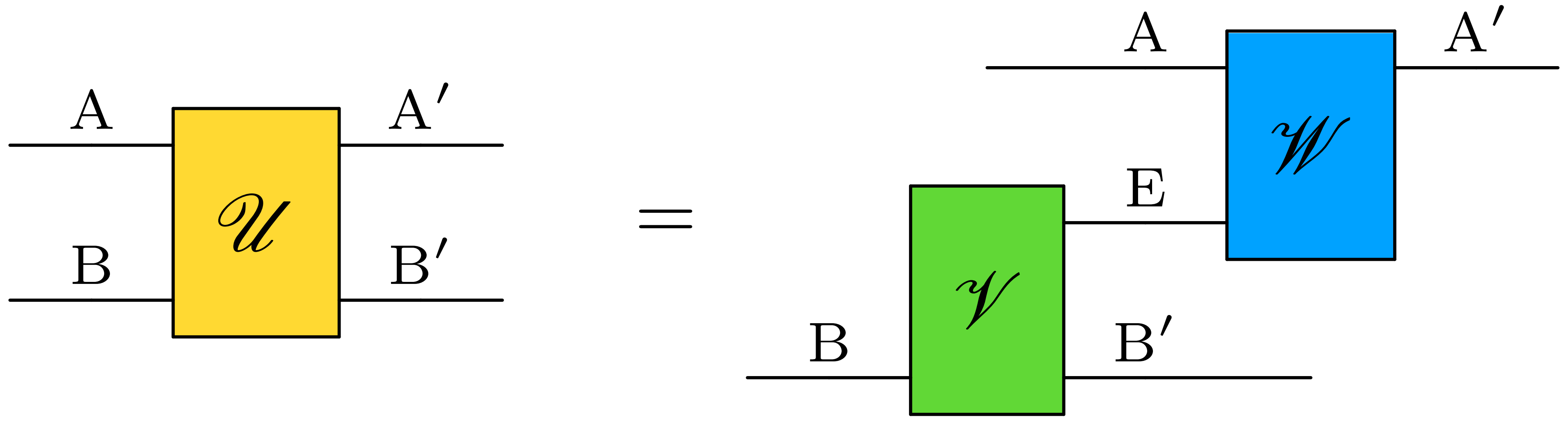



- No causal influence: violated, thus $\mathrm{B} \to \mathrm{A}'$

# Chain of conditions

## In classical theory

$A \not\to B'$ $\Rightarrow$

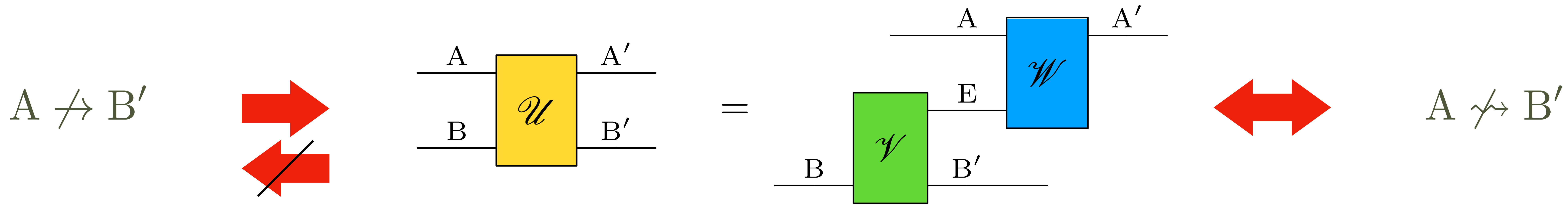



$\Leftrightarrow$ $A \not\rightsquigarrow B'$

PP, Quantum **5**, 515 (2021).

# Chain of conditions

## In classical theory



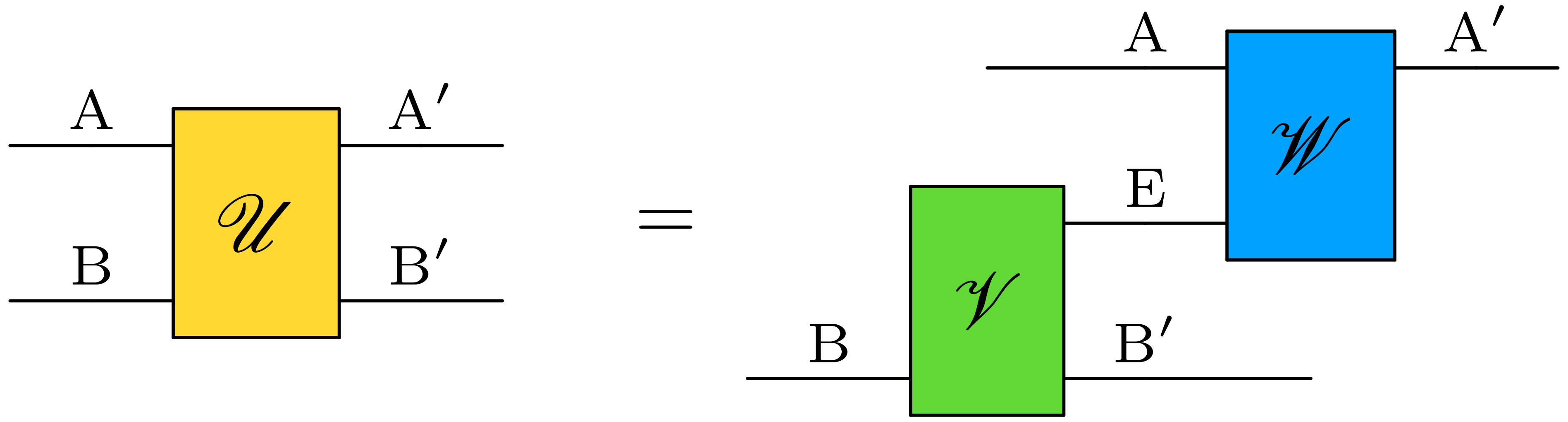

PP, Quantum **5**, 515 (2021).

# Quantum theory

## Example 2

- Also in quantum theory

$$A \not\rightsquigarrow B'$$



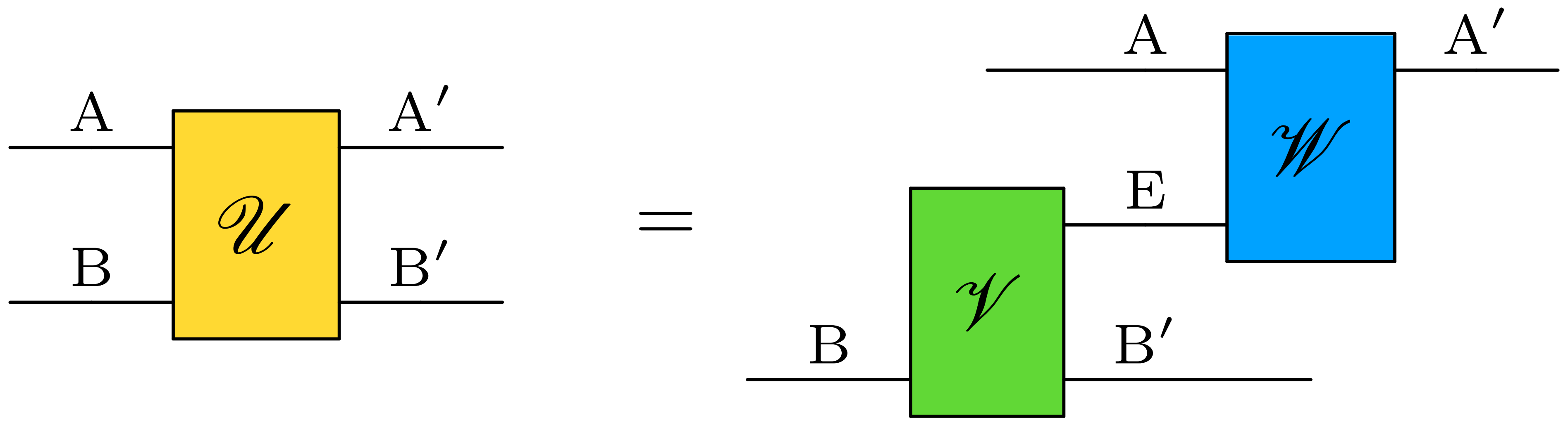

T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

# Quantum theory

## Example 2

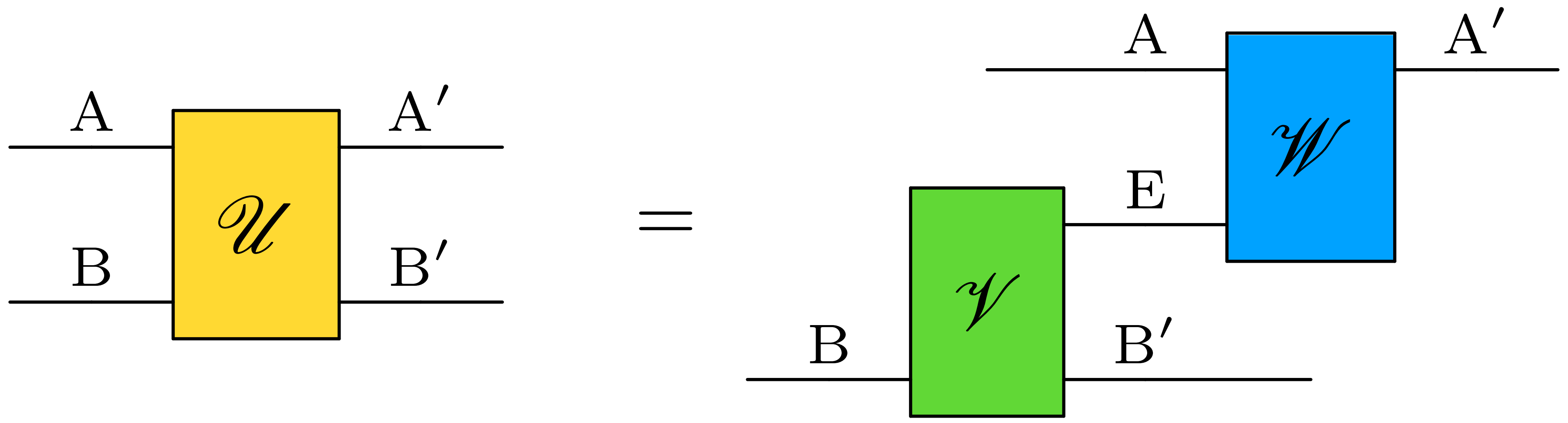
- Also in quantum theory

$A \not\rightsquigarrow B'$ ➡




- Thus

$A \not\rightarrow B'$ ➡

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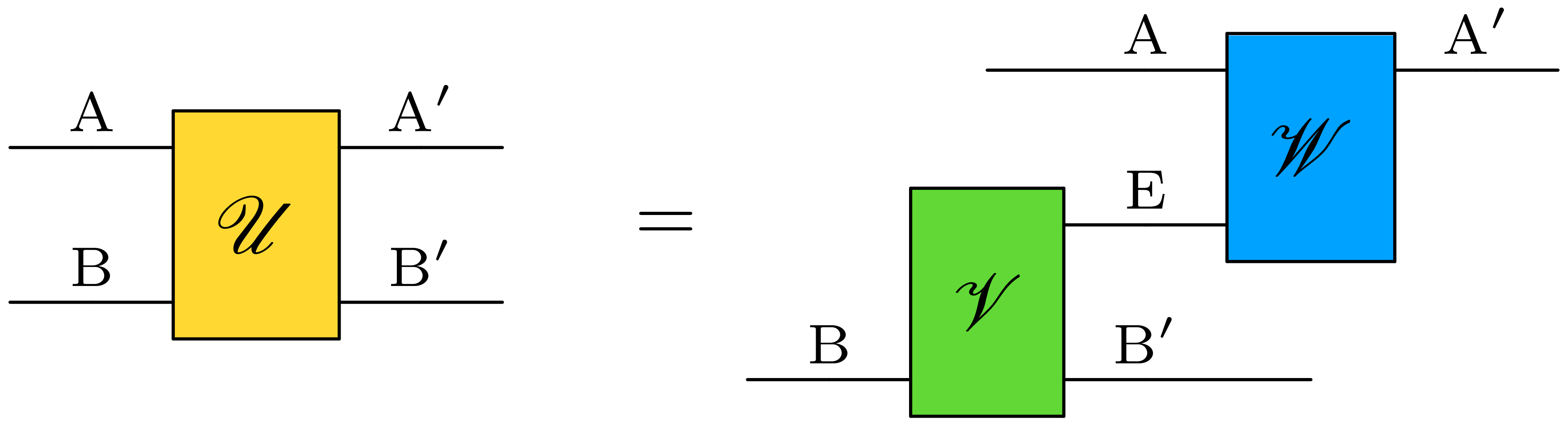
T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

# Quantum theory

## Example 2

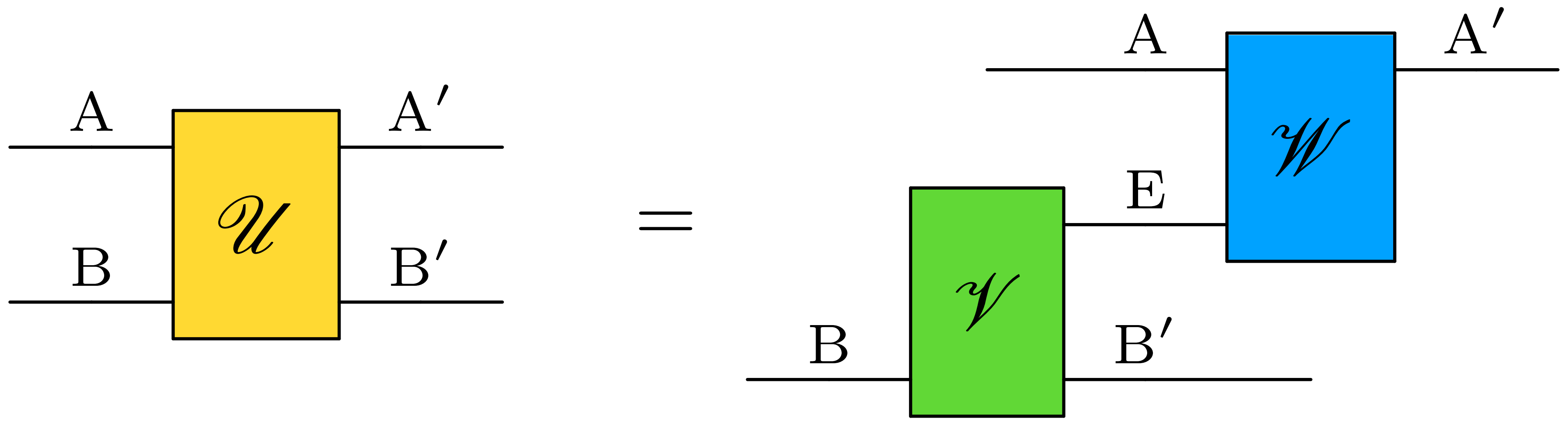
- Also in quantum theory

$A \not\rightsquigarrow B'$ ⇒




- Thus

$A \not\to B'$ ⇒



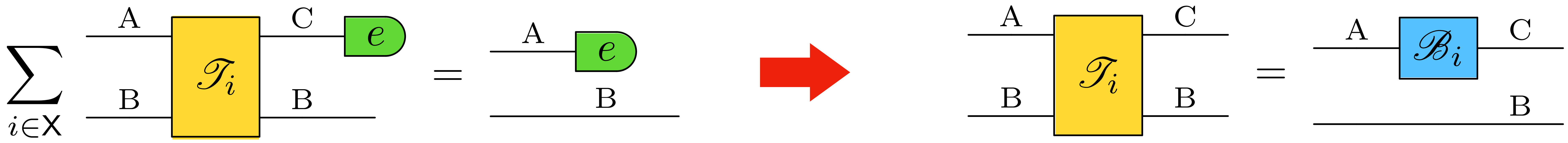

⇔ $A \not\rightsquigarrow B'$

- What about the first implication?

T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

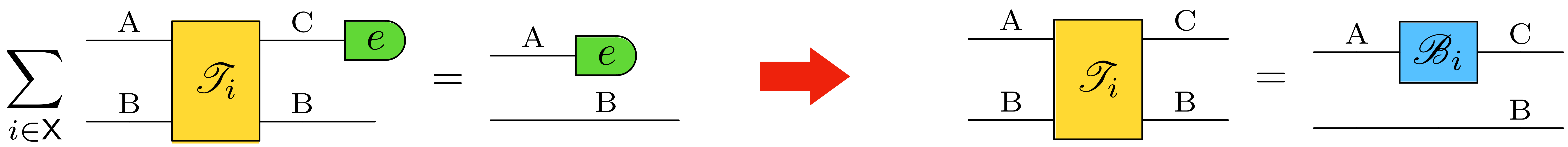
# Quantum theory

- From the characterisation of Kraus decompositions of a given channel



# Quantum theory

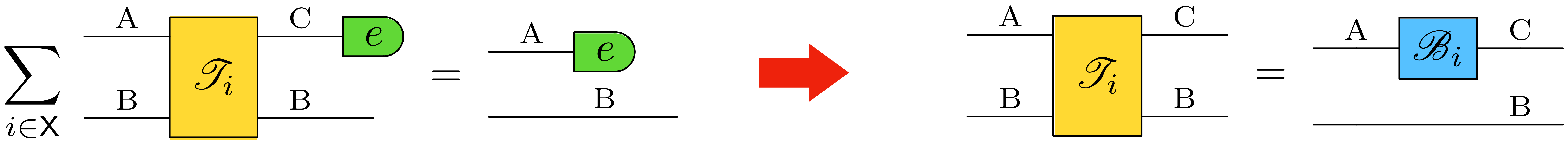
- From the characterisation of Kraus decompositions of a given channel




- Also from purification

# Quantum theory

- From the characterisation of Kraus decompositions of a given channel

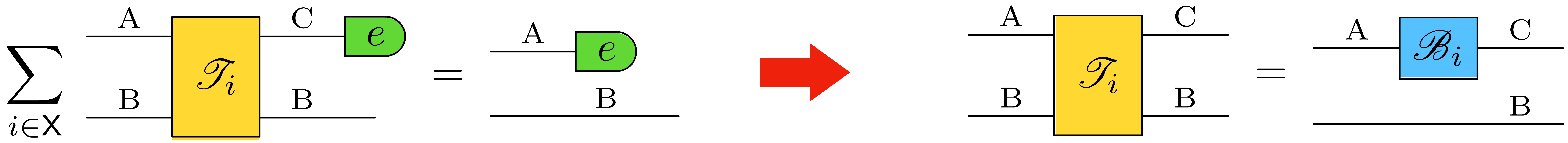



- Also from purification

➡ The above result holds also in Fermionic theory and Real Quantum theory

# Quantum theory

- From the characterisation of Kraus decompositions of a given channel

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➡ The above result holds also in Fermionic theory and Real Quantum theory

**No interaction without disturbance**

# Quantum theory

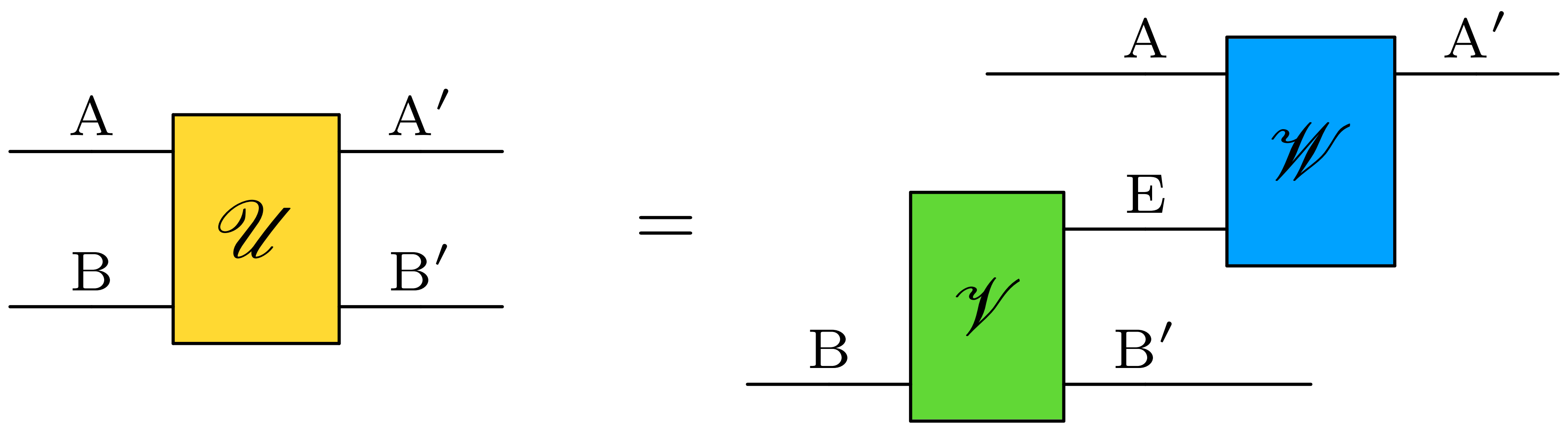
- From no interaction without disturbance one has $\mathrm{A} \not\rightsquigarrow \mathrm{B} \Rightarrow \mathrm{A} \not\to \mathrm{B}$

PP, Quantum **5**, 515 (2021).

# Quantum theory

- From no interaction without disturbance one has $\mathrm{A} \not\rightsquigarrow \mathrm{B}$ $\Rightarrow$ $\mathrm{A} \not\to \mathrm{B}$
- Thus

$\mathrm{A} \not\to \mathrm{B}'$ $\Leftrightarrow$



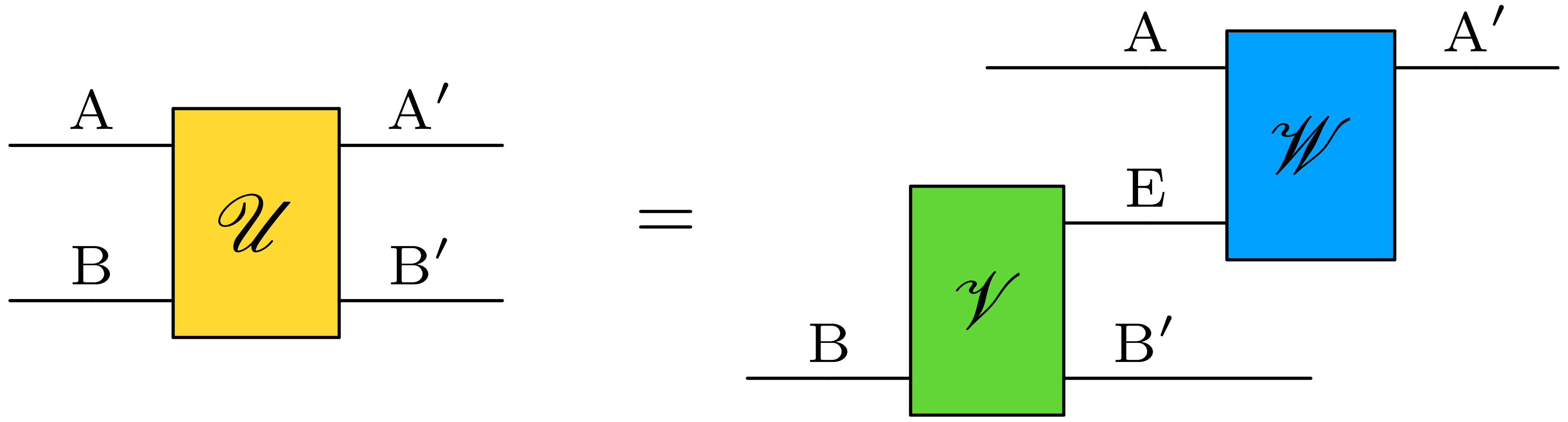

$\Leftrightarrow$ $\mathrm{A} \not\rightsquigarrow \mathrm{B}'$

PP, Quantum **5**, 515 (2021).

# Quantum theory

- From no interaction without disturbance one has $\mathrm{A} \not\rightsquigarrow \mathrm{B} \Rightarrow \mathrm{A} \not\to \mathrm{B}$
- Thus

$\mathrm{A} \not\to \mathrm{B}' \Leftrightarrow$



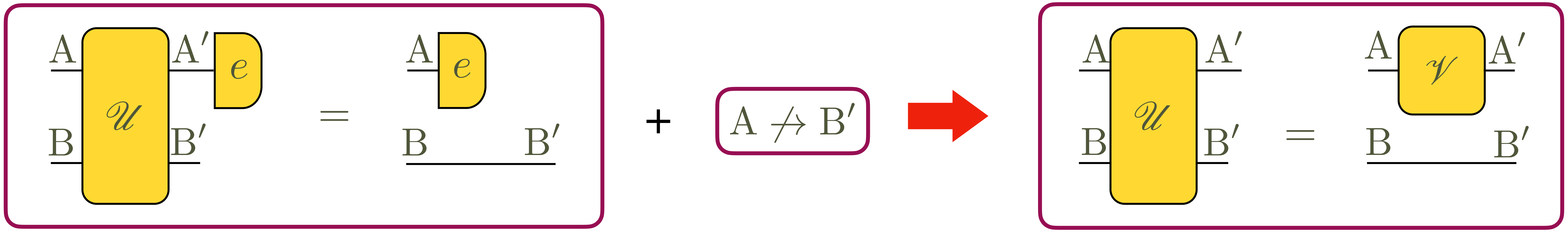

$\Leftrightarrow \mathrm{A} \not\rightsquigarrow \mathrm{B}'$

- True in every theory with purification or just no interaction without disturbance

PP, Quantum **5**, 515 (2021).

# Interaction without disturbance

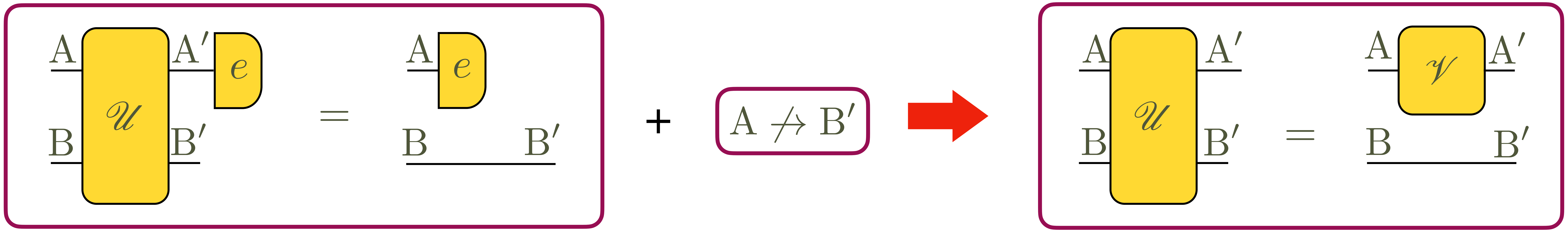
- What about a theory featuring interactions without disturbance?



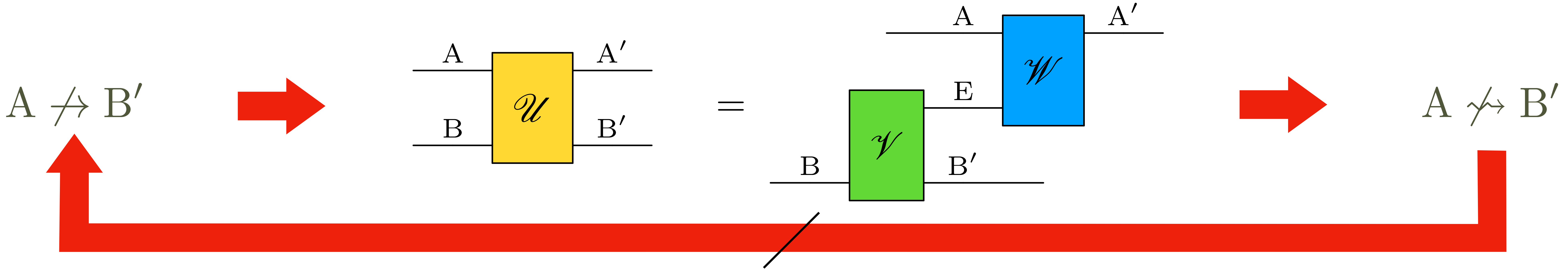

PP, Quantum **5**, 515 (2021).

# Interaction without disturbance

- What about a theory featuring interactions without disturbance?




- Thus, if the special interaction without disturbance is reversible, one has




PP, Quantum **5**, 515 (2021).

# Chapter IIII: Epilogue

Open problems and conclusions

# Open question

## The quest for counterexamples

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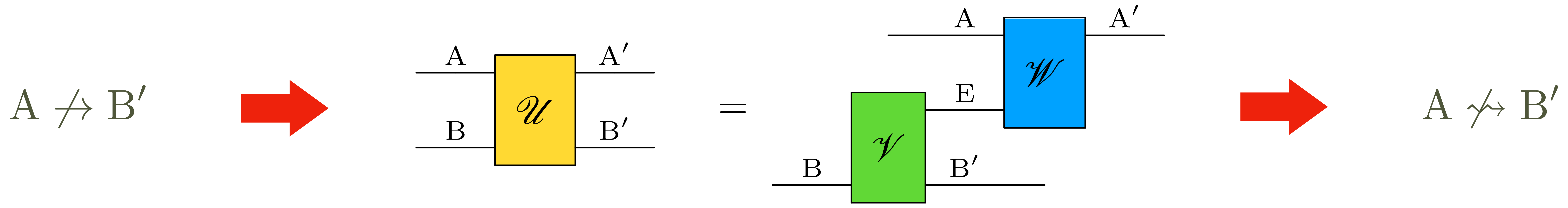

# Open question

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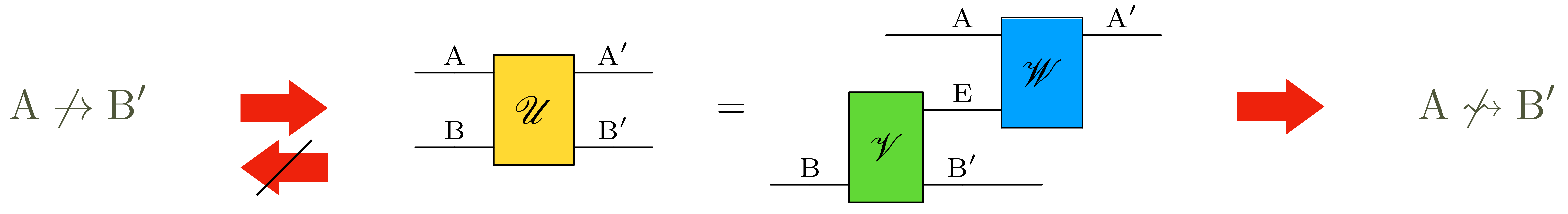

# Open question

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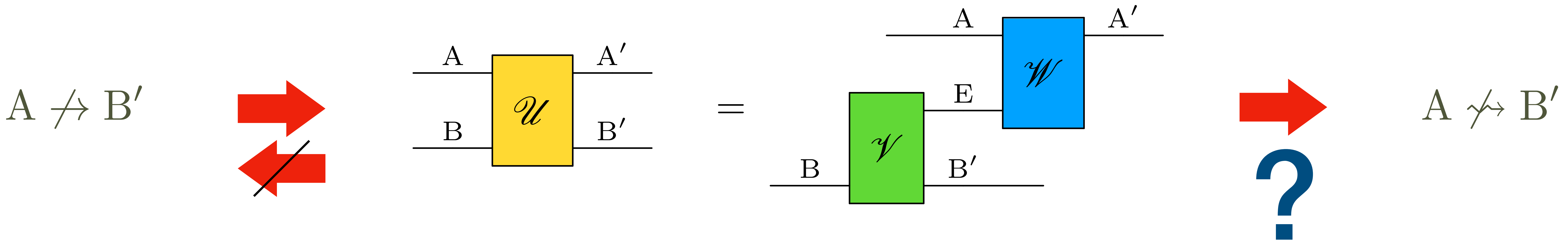
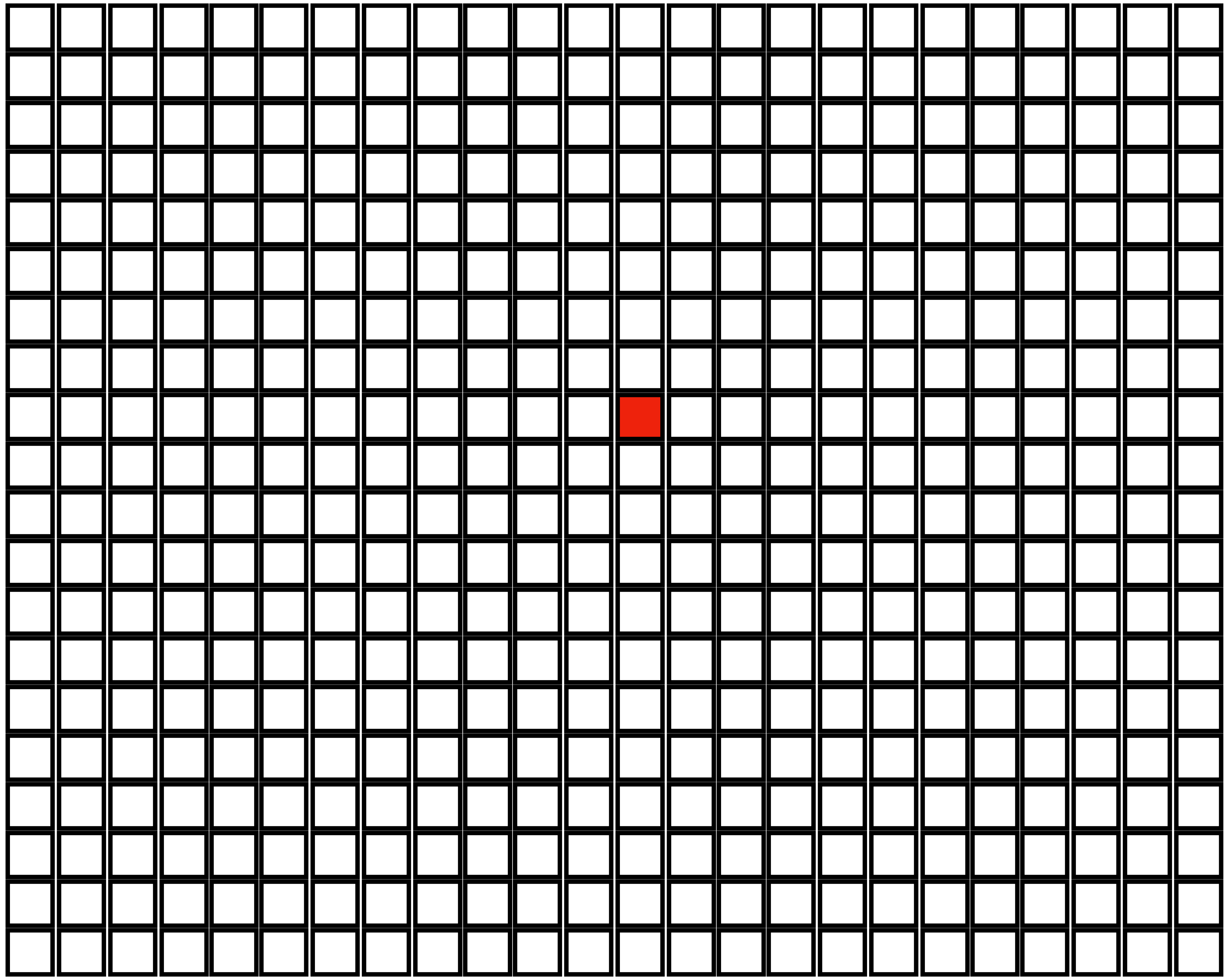


# Why bother?

## Neighbourhood of a cell in a QCA

$\square = \mathcal{H}_x \leftrightarrow \mathsf{A}_x$

C.A.: $U : \bigotimes_x \mathcal{H}_x \to \bigotimes_x \mathcal{H}_x$



B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

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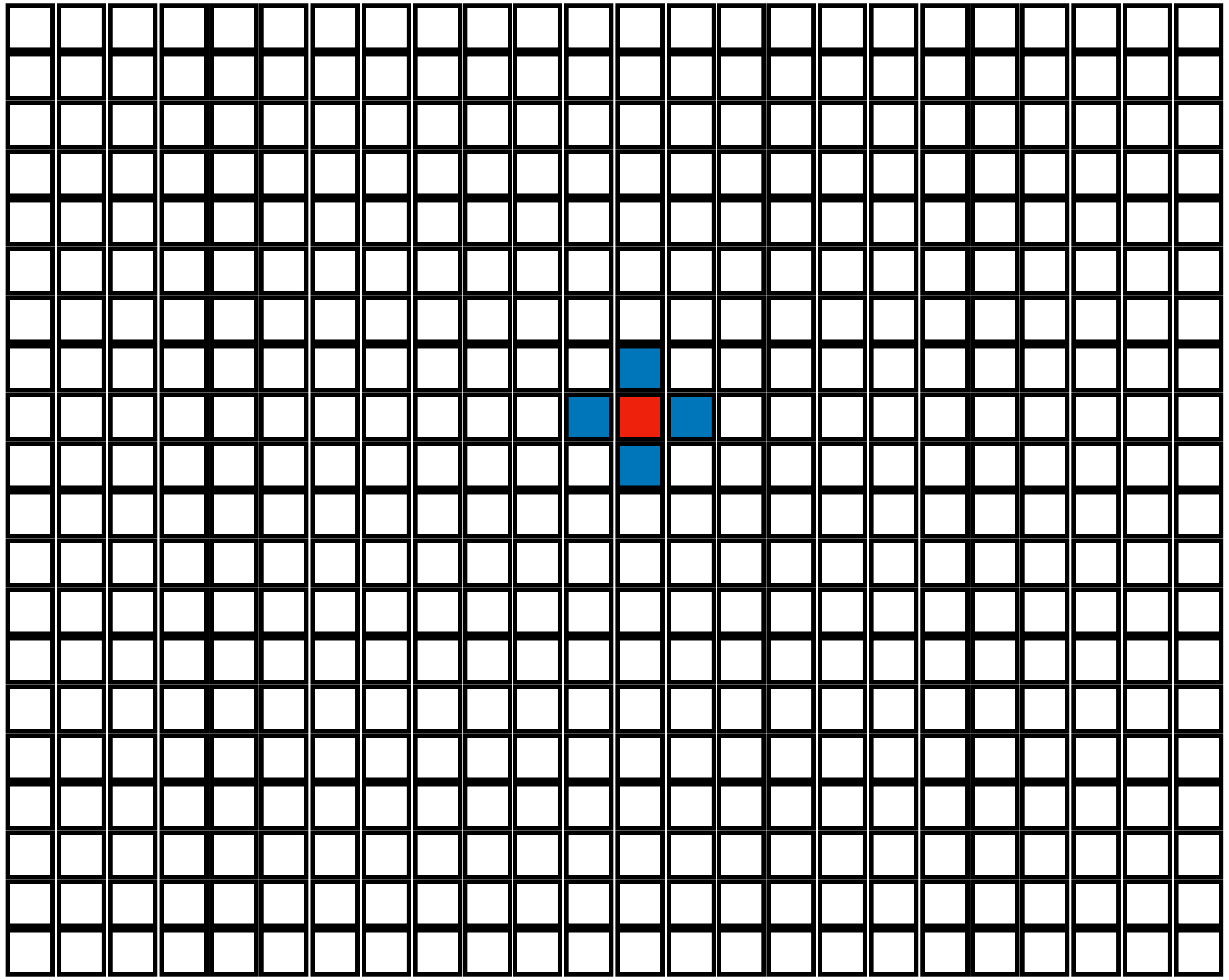
Neighbourhood of the cell $x_0$

$U \mathsf{A}_{x_0} U^{-1} \subseteq \mathsf{A}_{\mathsf{N}(\mathsf{x}_0)} \otimes I_{\bar{x}_0}$



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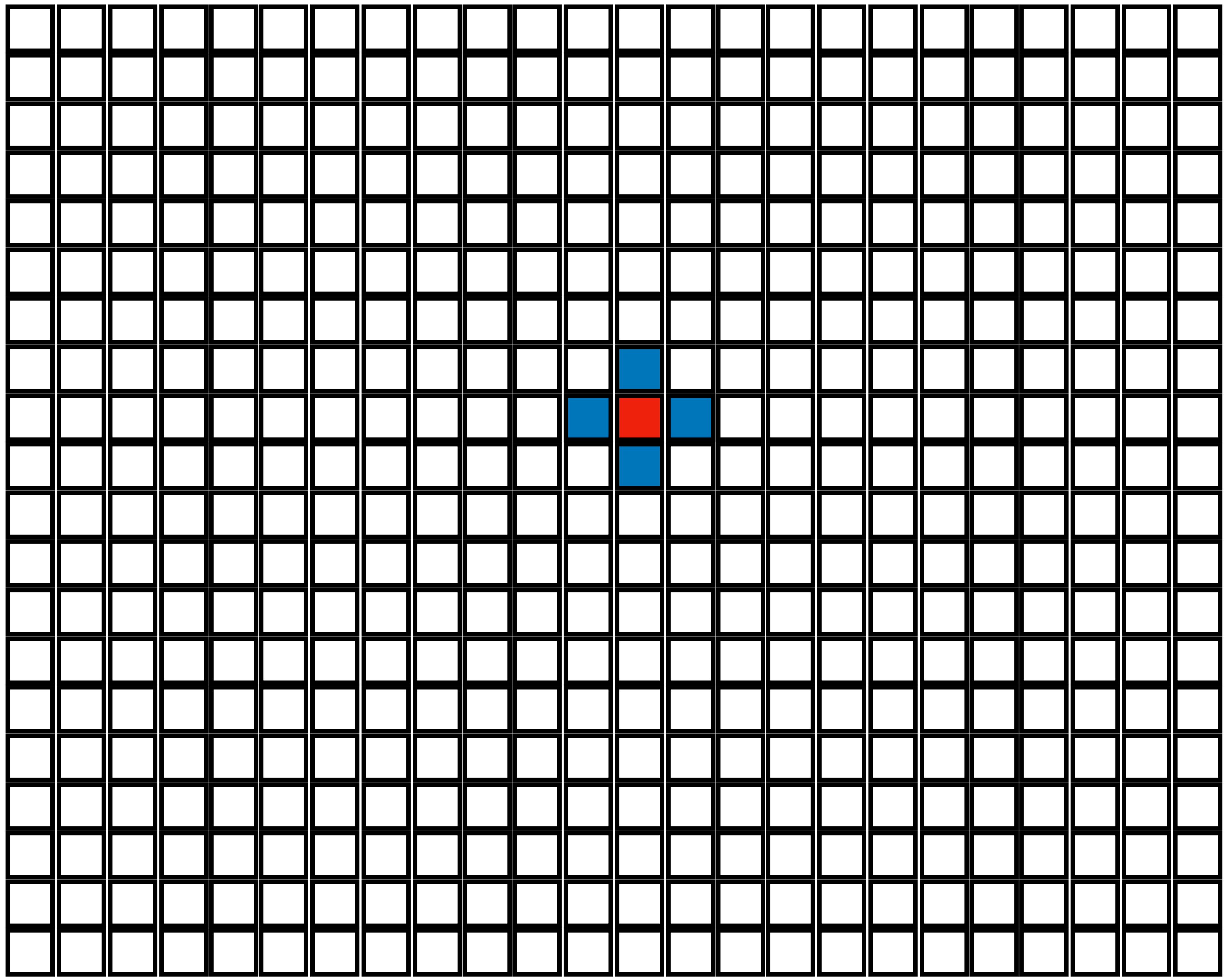


$x_0 \to y \quad \forall y \in N(x_0)$

B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

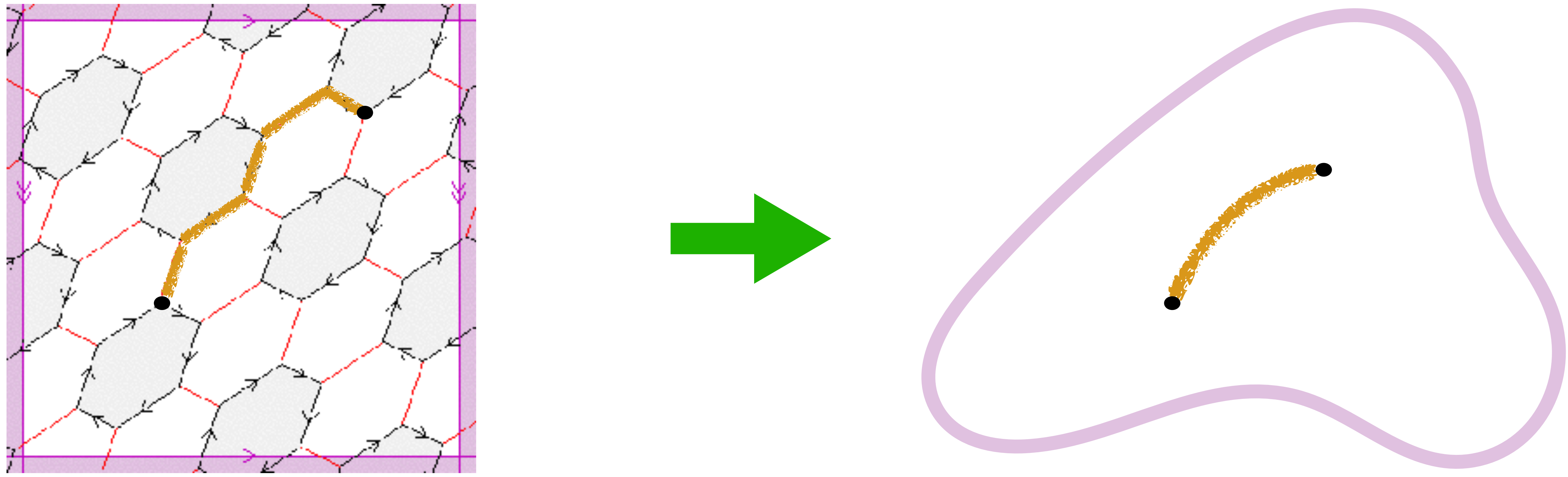
# Emergent space-time

Properties of the evolution $W = \sum_{h \in S} T_h \otimes A_h$

Dynamical laws

$$i\partial_t \psi(\mathbf{k},t) = \mathbf{k}\cdot\boldsymbol{\sigma}^{\pm}\psi(\mathbf{k},t)$$

Essentially unique geometry



G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014).

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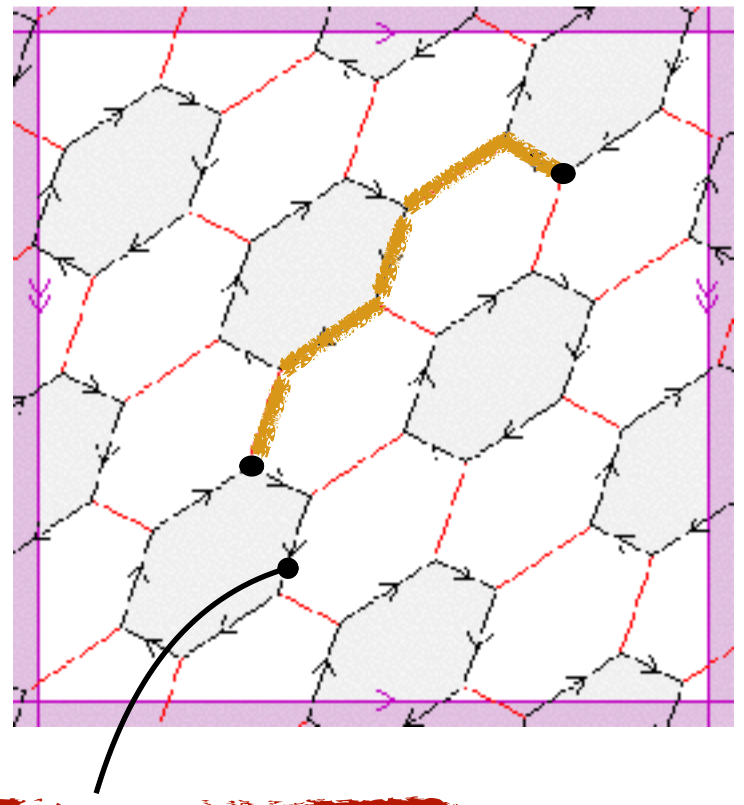

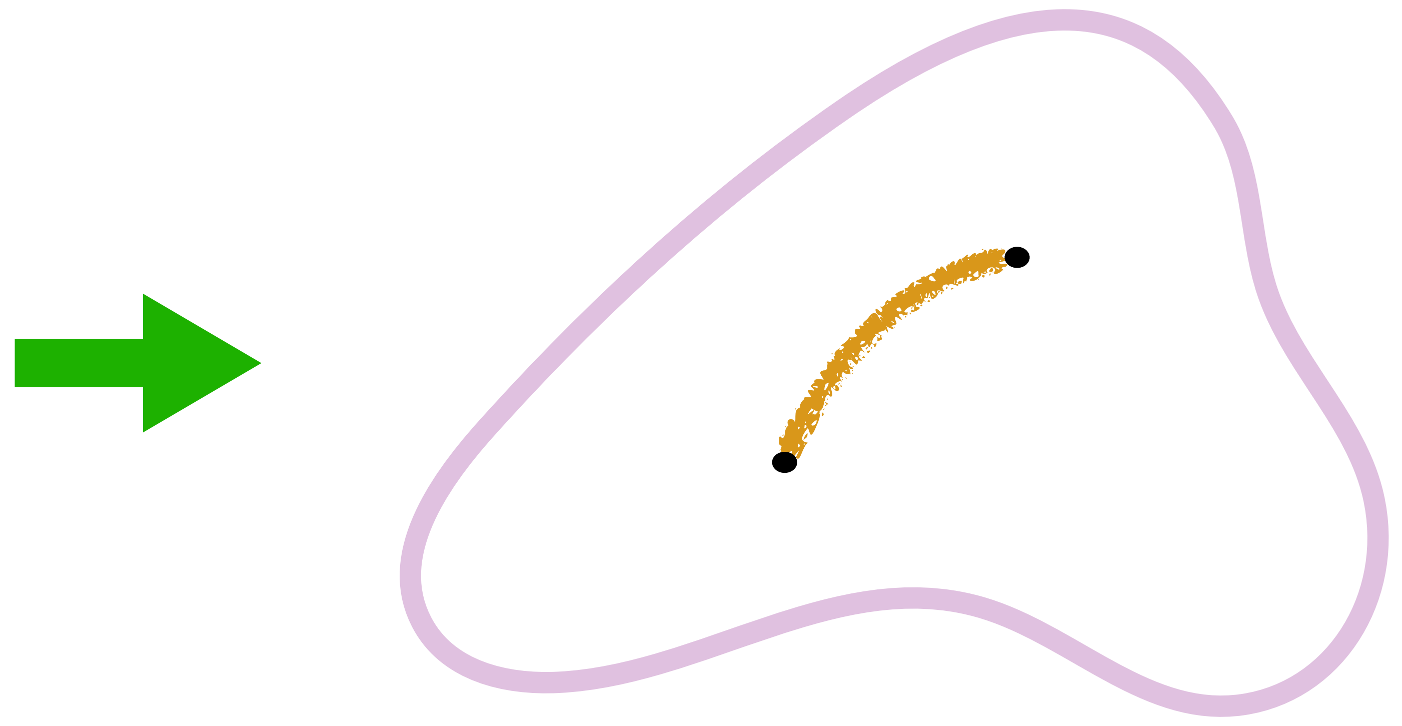

Causal influence

G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014).

# Action equals reaction

- In CT: accidental consequence of translation invariance of physical laws
- It is universal in QT or in theories with no interaction without disturbance

# Conclusion

- Proposal: (no) causal influence
- Relation with comb structure and (no) signalling
- Classical and Quantum theory
- No interaction without disturbance
- Cellular automata and causal relations