Wormholes, Sachdev-Ye-Kitaev model and universal dynamics of dissipative quantum chaotic matter

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PRX 12, 021040 (2022)
PRL128, 081601 (2022)
arXiv:2210.01695

Summary:

Part I: Introduction

1. Universality and quantum chaos
2. Sachdev-Ye-Kitaev model and quantum chaos
3. Why SYK is interesting in quantum gravity?

Part II: SYK and wormholes

1. Traversable wormholes and SYK (Maldacena and Qi). Why is it relevant beyond quantm gravity?
2. Euclidean wormholes and SYK (Godet, AGG)
3. Kelydish wormholes and real time evolution of SYK (AGG, Sa, Verbaarschot, Zheng)

Interacting quantum many-body problems are very difficult to solve

QCD

Nuclei

Strongly correlated electrons

Quantum gravity?

Solution

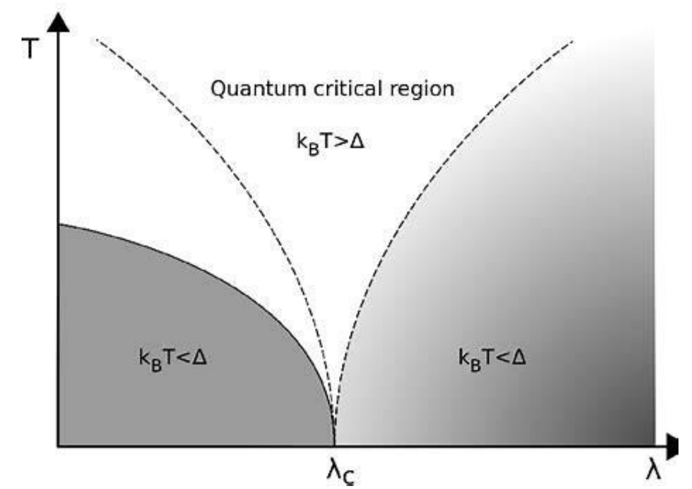
Search for universal features of the quantum dynamics

What universality means?

Universality can be a quite misleading name!

For some, it is related to the behavior of systems close to thermal second order phase transitions controlled by critical exponents.

For others, it is related to systems *close* to quantum phase transitions. More boldly, it is claimed that one can use CFT to describe strongly interacting phases of quantum matter close to these critical points.



This talk is not about this!

What universality means?

Details of the Hamiltonian are not important. For sufficiently long times, the system relax to a state which is only controlled by the global symmetries of the system

What dynamical feature is related to universality?

Maybe easier in simpler systems:

Non-interacting quantum chaotic systems

Non-interacting quantum disordered systems

Quantum Chaos

What is quantum chaos?

Quantum mechanics of classically chaotic systems

Why is quantum chaos important?

Universality

All happy families are alike; each unhappy family is unhappy in its own way.

Anna Karenina, Toltstoy

Butterfly effect

Classical chaos

Hadamard 1898

Lyapunov 1892

$$\|\delta x(t)\| = e^{\lambda t}\|\delta x(0)\|$$

$$\lambda > 0$$

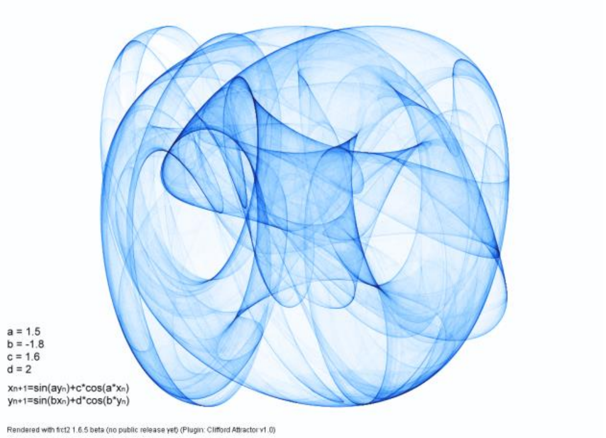
$$h_{KS} > 0$$

Pesin formula

Difficult to compute!

Lorenz 60's

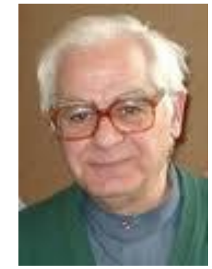
Meteorology



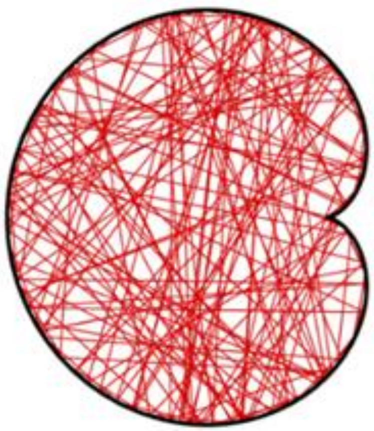
Random Matrix Theory characterizes universality of Quantum Chaos

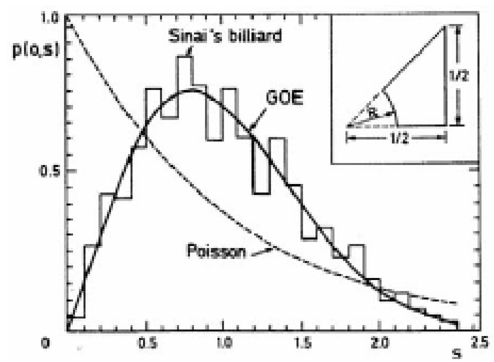
Bohigas-Giannoni-Schmit conjecture PRL 52, 1 (1984)

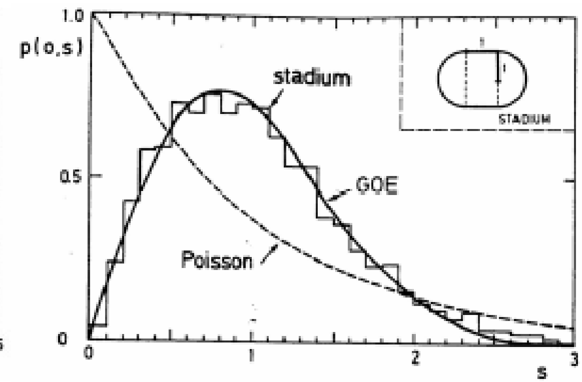
Proof Periodic Orbit Theory Sieber, Richter, Altland, Haake, ~2000



Oriol Bohigas







Quantum Chaos ⟷ Level statistics is Random Matrix

Universality $t \sim t_H$ Heisenbserg time

Similar results for a particle in a random potential (d>2)

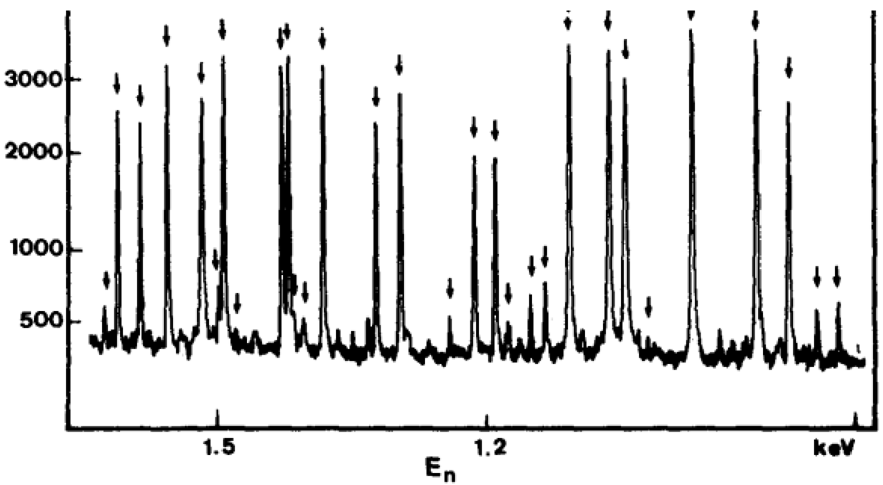
Efetov 1984

Nuclear Physics 60's:

Wigner

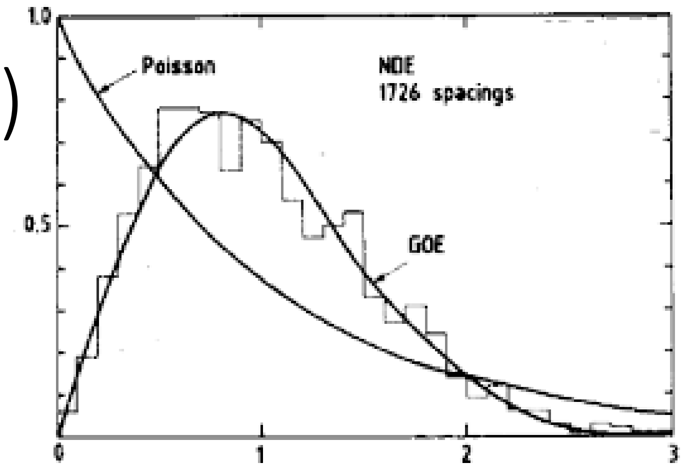
How to model the atomic nucleus?

The ultimate approximation "A random matrix as an effective nuclear Hamiltonian"



Coceva and Stefanon, Nuclear Physics A, 1979

P(s)



O. Bohigas, R.U. Haq, and A. Pandey, in Nuclear Data for Science and Technology, (1983)

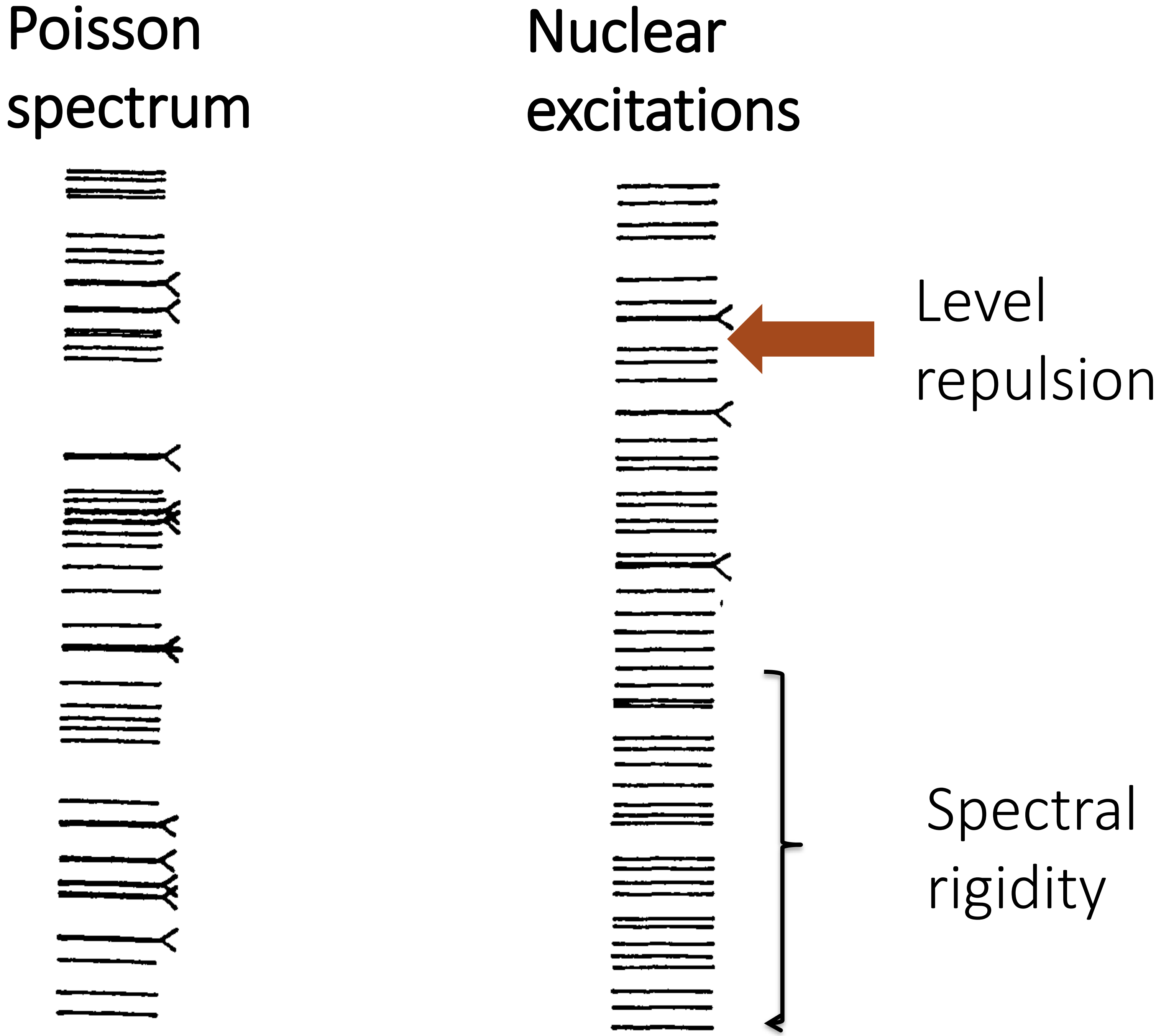
Flores, Bohigas, French 1970

When is it valid?

$$t_H \sim 1/\Delta$$

Heisenberg time

$\Delta \equiv$ Mean level spacing


Poisson spectrum
Nuclear excitations
Level repulsion
Spectral rigidity

Random Matrix

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}$$

$a_{ij} = a_{ji} \in \mathbb{R}$ GOE

$a_{ij} = a_{ji}^* \in \mathbb{C}$ GUE

Dyson-Mehta

Semicircle law $\rho(E) \sim \sqrt{E_0^2 - E^2}$ Only for Gaussian variables. No universal

Two level universal correlations: $R_2(s) \sim 1 - \frac{\sin^2 \pi s}{\pi^2 s^2}$ GUE

Level Repulsion $P(s) \sim s^\beta e^{-As^2}$ $s = (E_{i+1} - E_i)/\Delta$

$\Sigma^2(N)$ Number variance related to $\int R_2$

Spectral rigidity

Long range correlations

$$\Sigma^2(N) = \langle n(N)^2 \rangle - \langle n(N) \rangle^2$$

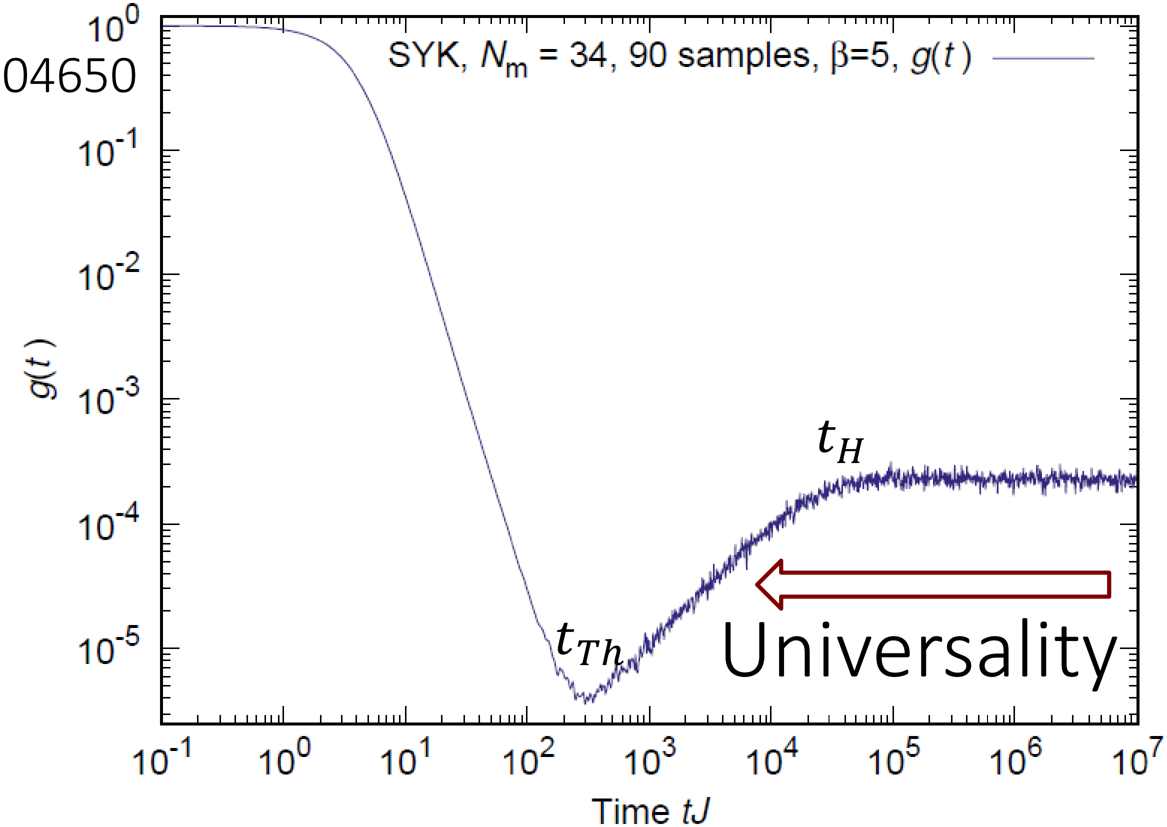
$$\sim \log(N) \quad N \gg 1$$

$$R_2(s) \sim \frac{1}{s^2} \quad s \gg 1$$

Universality

Spectral form factor ≡ Fourier transform of $\Sigma^2(N)$

1611.04650



Universal ramp related to spectra rigidity

$R_2(s) \sim \frac{1}{s^2} \quad s \gg 1$

$t_{Th} \equiv$ Thouless time. Minimum time scale for which the system feels the spectrum is discrete

Multiple applications: Quantum chaos-RMT

Mesoscopic physics: Interactions, disorder Efetov

Deterministic chaotic systems Bohigas

k-body random ensembles French, Bohigas, Ma, Flores 70's Before Sachdev-Ye!

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\leq l, p\leq q} \langle pq| V |kl\rangle \, a_p^\dagger a_q^\dagger a_l a_k$$

QCD: Spectrum Dirac operator Verbaarschot, Shuryak

Chiral ensembles, lattice simulations

Quantum gravity?

Another window of universality

t_E (exponential) growth of quantum uncertainty

Kicked Rotors: Zaslavsky, Berman, Physica 91A 450 (1978)

Disorder: Larkin, Ovchinnikov, Soviet Physics JETP 28, 1200 (1969)

Quantum Chaos

$$\left\langle \left[\hat{O}(t), \hat{O}(0)\right]^2 \right\rangle^{t_E \sim \log \hbar^{-1}/\lambda} \approx \propto \hbar^2 \exp(\boldsymbol{\lambda} t)$$

$\boldsymbol{\lambda}$ Classical Lyapunov exponent

The exponential growth at Ehrenfest time t_E is Universal for quantum chaotic systems

Non-quantum chaotic motion

$$\left\langle \left[\hat{O}(t), \hat{O}(0)\right]^2 \right\rangle \approx \propto \hbar^2\, t^{\beta}$$

$t_E \propto \hbar^{\alpha}, \alpha > 0$ Non-universal

Quantum effects develop faster in quantum chaos!

What quantum chaos has to do with (quantum) gravity?

Chaos in black-hole physics

Quantum Black Holes are fastest scramblers in nature

P. Hayden, J. Preskill, JHEP 0709 (2007) 120 Sekino, Susskind,JHEP 0810:065,2008

Membrane paradigm → Handwaving GR + Heisenberg principle ….

$$t_S \sim t_E \sim \log(\hbar^{-1}) \sim \log(N)$$

Rindler geometry is important

AdS/CFT

Field theory dual also fastest scramblers

A bound on chaos

arXiv:1503.01409

Maldacena, Shenker, Stanford

OTOC

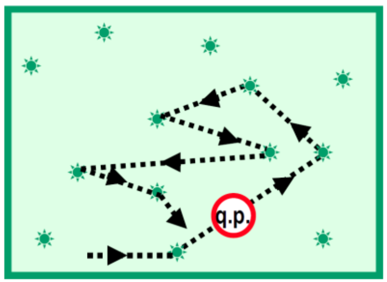
$$y^4 = \frac{1}{Z} e^{-\beta H}$$

$$t_* = \frac{\beta}{2\pi} \log N^2$$

$$F(t) = \mathrm{tr}[yVyW(t)yVyW(t)]$$

$$t_d \ll t < t_*$$

$$F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$$



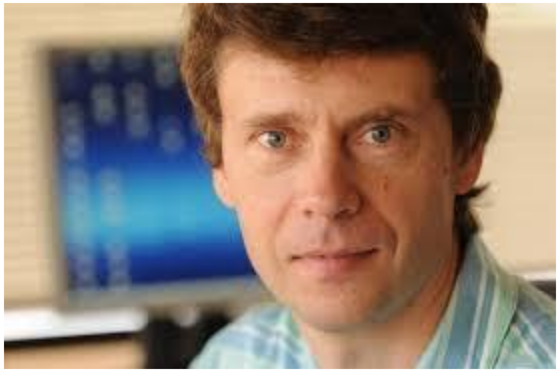
$$\lambda \leq 2\pi k_B T/\hbar$$

$$\langle [p_z(t), p_z(0)]^2 \rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2 \rangle \propto \hbar^2 \exp(\lambda t)$$

Black holes and its field theory dual saturate the bound

Holographic dualities (AdS/CFT): Certain (quantum) asymptotically AdS gravity theories in d+1 are dual to field theories in d dimensions. N=4 Super-Yang Mills theory

Maldacena et. al, proposes that $\lambda = \frac{2\pi k_B T}{\hbar}$ in these theories but no example is given!!!



Kitaev,
KITP 2015
talks

"I have got one example"

Kitaev: "A simple model of quantum holography"

http://online.kitp.ucsb.edu/online/entangled15/kitaev/

$$H = \sum_{ijkl} J_{ijkl}\psi_i\,\psi_j\,\psi_k\,\psi_l \qquad \text{q=4}$$

Majoranas $\{\psi_i, \psi_j\} = \delta_{ij}$

Gaussian $\langle J_{ijkl}^2 \rangle \sim \mathrm{J}^2/\mathrm{N}^3$

Strong coupling $\beta J \gg 1 \quad \tau J \gg 1$

A **solvable field theoy** with a quantum gravity dual

SYK = Sachdev-Ye-**Kitaev**

Motivation: Test Maldacena bound on chaos

Before Kitaev .... only Dirac fermions, technically more challenging... Kitaev idea of Majoranas very clever

N fermions
m levels
2-body

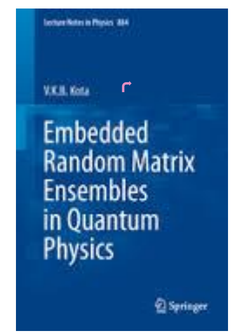
French, **Bohigas**,
Flores, Wong,70's

k-random body ensembles

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\le l, p\le q} \langle pq| V |kl\rangle\, a_p^\dagger a_q^\dagger a_l a_k$$

French, Mon, Annals of Physics 95, 90 (1975).

$$m \gg N \quad \langle H^p \rangle \quad \rightarrow \quad \rho(E) \propto e^{-E^2/\sigma^2}$$



Kota

Very popular in 1980-2000's!

Thermalization

Metal-insulator transitions

Level statistics

Transport, quantum dots

Heisenberg Spin-Chain

Sachdev, Ye, PRL. 70, 3339 (1993)

Holography dual?

S. Sachdev PRL 83, 74408 (2010)

Why is SYK interesting?

Conformal (reparametrization) symmetry in the IR limit… like gravity

$$J\tau, J\beta \gg 1 \quad \partial_\tau \to 0$$

$$\Delta = 1/q$$

$$G \longrightarrow G_f(\tau,\tau') = [f'(\tau)f'(\tau')]^\Delta G(f(\tau), f(\tau'))$$

Finite Zero Temperature entropy

$$\frac{S_0}{N} = \frac{1}{2}\log 2 - \int_0^\Delta dx\pi\left(\frac{1}{2} - x\right)\tan \pi x$$

Georges, Parcollet 90's Kitaev 2015

Conformal symmetry spontaneously and explicitly broken to SL(2,R) due to finite T, N effects

Conformal symmetry broken to SL(2,R) in the $\beta J \gg 1$ limit

$$S = -N\frac{\alpha_S}{\mathcal{J}}\int d\tau\{f,\tau\} \qquad \{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

Schwarzian action

$$\rho(E) \propto \sinh(\gamma\sqrt{|E-E_0|})$$

Quantum black holes

$$\frac{\langle\psi_i(0)\psi_j(\tau)\psi_i(0)\psi_j(\tau)\rangle}{\langle\psi_i(0)\psi_i(0)\rangle\langle\psi_j(\tau)\psi_j(\tau)\rangle} \propto 1 + i\frac{\beta J}{N}e^{\frac{2\pi\tau}{\beta}}$$

τ around $t_E \ll t_H$

SYK is quantum chaotic and analytically tractable

SYK saturates Maldacena bound

SYK in the low T is dual to NCFT/NAdS2, why?

Jackiw-Teitelboim gravity

1606.01857

1809.08647

Simple non-trivial model in (near) AdS_2

$$I = \underbrace{-\frac{\phi_0}{2}\left(\int R + 2\int_{\partial M} K\right)}_{\text{Einstein-Hilbert Action}} \underbrace{-\frac{1}{2}\left(\int_M \phi(R+2) + 2\int_{\partial M} \phi_b K\right)}_{\text{Jackiw-Teitelboim action}} + S_{matter}(g,\psi)$$

K extrinsic curvature

$\phi \equiv$ dilaton

Classical EOM:

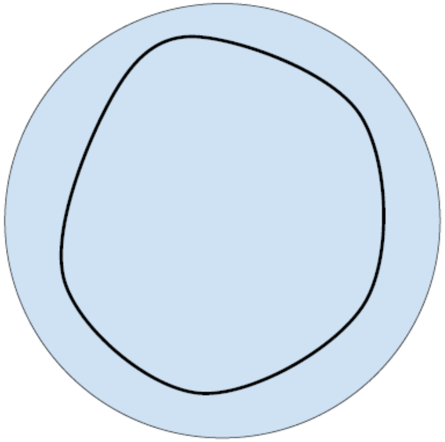
$R = -2$

Poincare disk $d = 2$

Boundary action

$$I = -2\pi\phi_0\chi(M) - \phi_b \int_{\partial M} K + S_{matter}(g,\psi)$$

Schwarzian low energy action



$$\epsilon \to 0 \qquad I = -\int du Sch(\tan\frac{\varphi(u)}{2}, u)$$

Diffs invariance $\longrightarrow$ SL(2,R)

The physics is in the motion of the physical boundary of AdS$_2$ inside a **rigid** AdS$_2$ space.

Quantum Chaotic

$$\langle V(a) W_3(b+\hat{u}) V(0) W(\hat{u})\rangle \sim \frac{\beta\Delta^2}{C} e^{\frac{2\pi\hat{u}}{\beta}}$$

SYK dual* to a quantum JT gravity

SYK and RMT

RMT in SYK?

Is q=4 SYK spectral density consistent with that of a black hole ?

Yes

AGG, Verbaarschot Phys. Rev. D 94, 126010 (2016)
(a few weeks later: 1611.04650)

Yes

AGG, Verbaarschot, Phys. Rev. D 96, 066012 (2017)

Universality classes in SYK?

Yes

AGG, Jia, Verbaarschot, PRD 97, 106003 (2018) Before: Wettig et al., Ludwig et al.,

Is RMT a feature of JT quantum gravity?

Yes

Saad et al. 1903.11115

AGG, S. Zacarias Phys. Rev. Res. 2 (2020) 4, 043310

Symmetries of SYK depend on N,q

$$C_1 = \gamma_1 \prod_{i=2}^{N} \gamma_{2i} K,$$

$$C_2 = \gamma_2 \prod_{i=2}^{N-1} \gamma_{2i+1} K$$

$$[C_1 K, H] = 0, \qquad [C_2 K, H] = 0$$

N	$(C_1K)^2$	$(C_2K)^2$	C_1KC_2K	RMT
2	1	-1	$-i\Gamma_5$	GUE
4	-1	-1	$-\Gamma_5$	GSE
6	-1	1	$-i\Gamma_5$	GUE
8	1	1	Γ_5	GOE
10	1	-1	$-i\Gamma_5$	GUE
12	-1	-1	Γ_5	GSE

$K \equiv$ Charge conjugation. C_1, C_2 anti-unitary symmetries

Why? Clifford algebra representations in N dimensions

You, Ludwig, Xu 1604.06964

Why relevant?

If SYK is quantum ergodic for long times, then it can be a toy model for different systems that share the same global symmetries: Tenfold way of RMT

SYK spectral density is the same as that of quantum black holes

$$\rho_{\rm edge}(E) \approx \; 2c_N \exp\left[\frac{\pi^2}{2\log\eta}\right] \sinh\left[\frac{2\pi\sqrt{2}\sqrt{1-(E/E_0)}}{-\log\eta}\right]$$

q= $4, \eta(N)$ J. Riordan, Mathematics of Computation 29, 215 (1975)

$q \propto N^{1/2}$ Erdos, et. al, 1407.1552

AGG, Verbaarschot, PRD 96, 066012 (2017)

Density = Q-Hermite polynomials

Agreement with the exact Schwarzian path integral

Bagrets, et al., 1702.08902

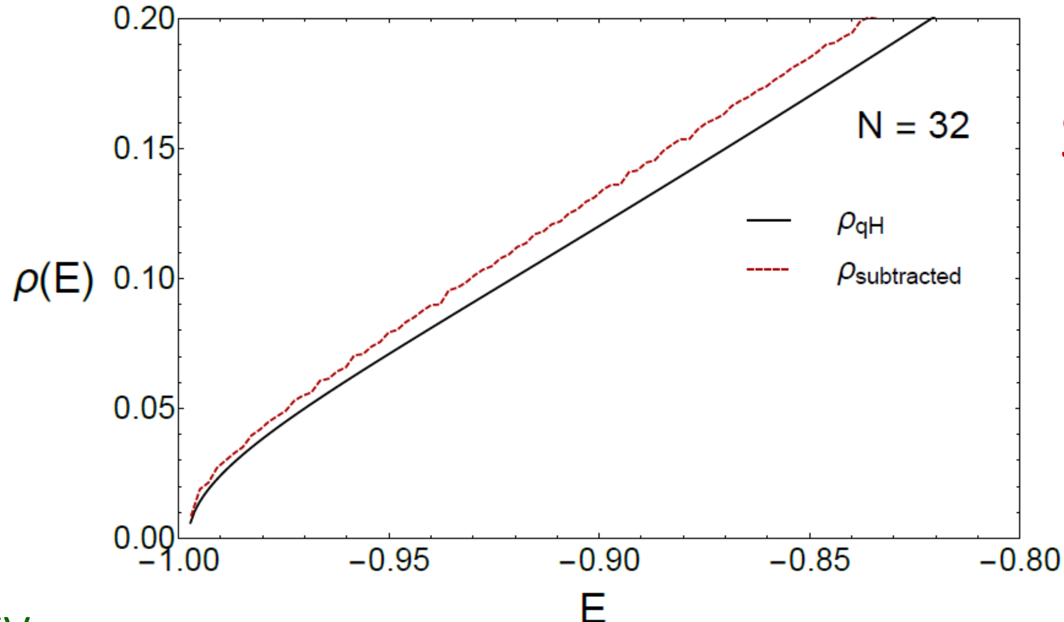
Stanford, et al. 1703.04612

Exponential increase

Cardy's formula

Bethe's formula

Black holes density

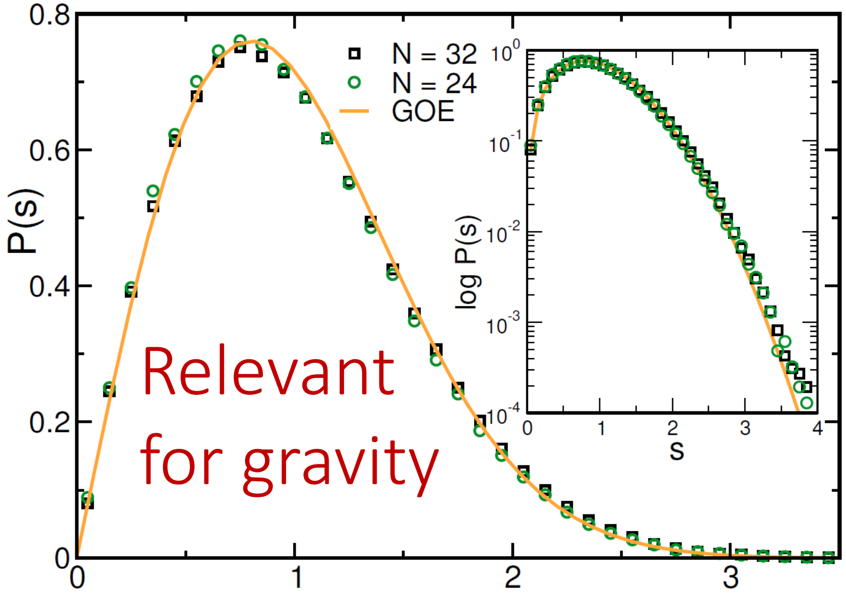


Sqrt edge

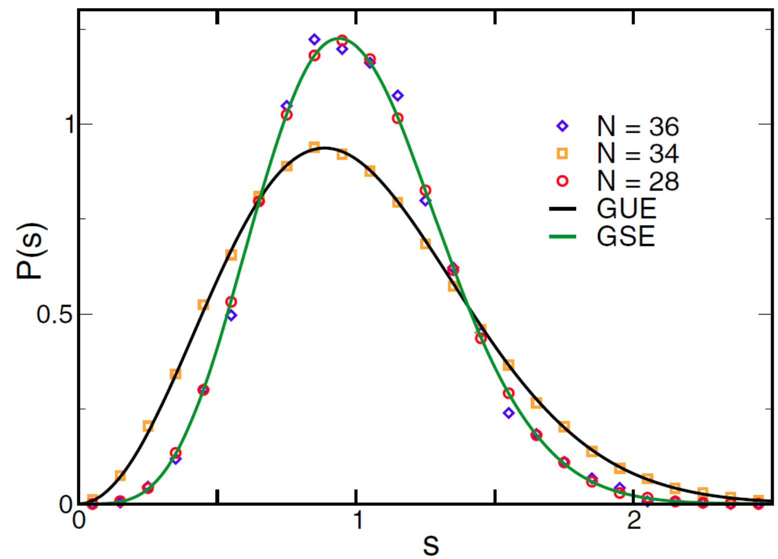
Typical of random matrices

Level statistics RMT $P(s) \approx a_\beta s^\beta \exp(-b_\beta s^2)$

Close ground state



High excitations



AGG, Verbaarschot Phys. Rev. D 94, 126010 (2016), later Cotler et al. 1611.04650

Random matrix correlations characterize quantum black holes?

Tenfold way in black hole physics?

Yes! Supersymmetric SYK Model

1610.08917
Fu, Gaiotto, Maldacena, Sachdev

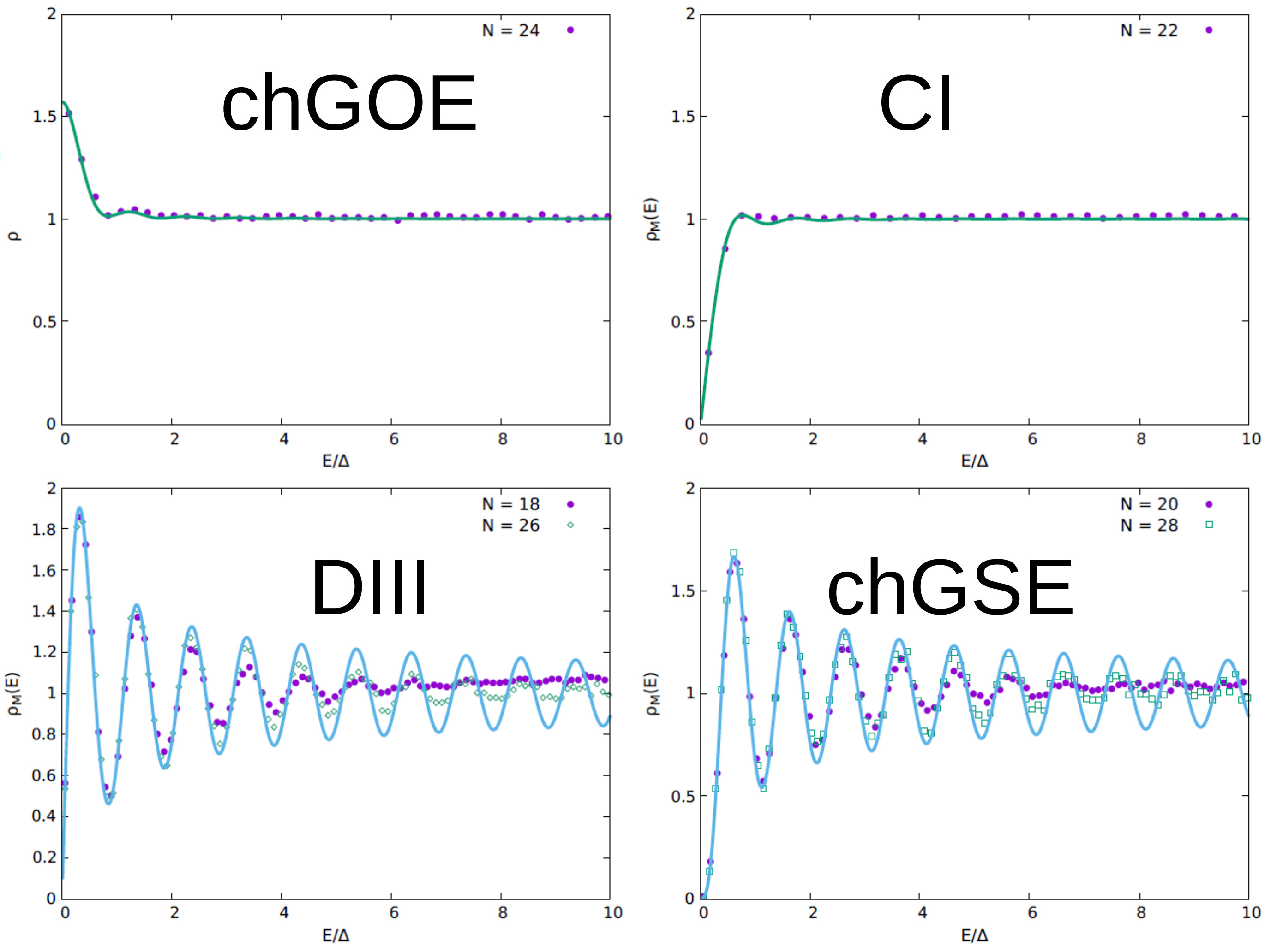
$$H = Q^2 \qquad Q = i \sum_{i,j,k=1}^{N} J_{ijk}\gamma_i\gamma_j\gamma_k$$

$$\rho_M(E) = \Delta\rho\left(\frac{E}{\Delta}\right)$$

AGG, Jia, Verbaarschot,
PRD 97, 106003 (2018)

Li, Liu, Xin, Zhou
1702.01738

Wettig, Kanazawa
1706.03044



Even more, the non-Hermitian SYK is also a toy model for strongly interacting open and/or dissipative quantum chaotic matter

$$H_{NH-SYK} = \sum_{ijkl} J_{ijkl}\,\psi_i\psi_j\psi_k\psi_l$$

$$J_{ijkl} \to J_{ijkl} + iM_{ijkl}$$

Depending on q and N, we identify 19 out of the 38 Phys. Rev. X 9, 041015 (2019) non-Hermitian universality classes in the nHSYK model,

AMGG, L Sa, JJM Verbaarschot, PRX 12, 021040 (2022)

Global symmetries

$$
\begin{array}{llll}
\mathcal{T}_+ H \mathcal{T}_+^{-1} = H, & \mathcal{T}_+^2 = \pm 1, & \mathcal{T}_+ \text{ anti-unitary} & \text{(Time-Reversal Symmetry)}, \quad (3) \\
\mathcal{T}_- H \mathcal{T}_-^{-1} = -H, & \mathcal{T}_-^2 = \pm 1, & \mathcal{T}_- \text{ anti-unitary} & \text{(Particle-Hole Symmetry)}, \quad (4) \\
\mathcal{C}_+ H^\dagger \mathcal{C}_+^{-1} = H, & \mathcal{C}_+^2 = \pm 1, & \mathcal{C}_+ \text{ anti-unitary} & \text{(Time-Reversal Symmetry)}, \\
\mathcal{C}_- H^\dagger \mathcal{C}_-^{-1} = -H, & \mathcal{C}_-^2 = \pm 1, & \mathcal{C}_- \text{ anti-unitary} & \text{(Particle-Hole Symmetry)}, \\
\Pi H \Pi^{-1} = -H, & \Pi^2 = 1, & \Pi \text{ unitary} & \text{(Chiral Symmetry)}, \\
\eta H^\dagger \eta^{-1} = H, & \eta^2 = 1, & \eta \text{ unitary} & \text{(Pseudo-Hermiticity)}.
\end{array}
$$

SYK Operators

$$
\mathcal{P} = K \prod_{i=1}^{N/2} \gamma_{2i-1} \qquad \mathcal{R} = K \prod_{i=1}^{N/2} \mathrm{i}\gamma_{2i} \qquad \mathcal{S} = \mathcal{P}\mathcal{R} = \mathrm{i}^{N^2/4} \prod_{i=1}^{N} \gamma_i
$$

K charge conjugation

$$\mathcal{P}^2 = (-1)^{\frac{1}{2}N/2(N/2-1)} \quad \text{and} \quad \mathcal{R}^2 = (-1)^{\frac{1}{2}N/2(N/2+1)}$$

$$S^2 = \mathbb{I} \qquad S \sim \Gamma_5$$

Action of symmetry operators on the SYK

$$\mathcal{P}H^\dagger\mathcal{P}^{-1} = (-1)^{q(q+1)/2}(-1)^{qN/2}H$$

$$\mathcal{R}H^\dagger\mathcal{R}^{-1} = (-1)^{q(q-1)/2}(-1)^{qN/2}H$$

$$\mathcal{S}H\mathcal{S}^{-1} = (-1)^q H.$$

$\mathcal{C}_+^2$	$\mathcal{C}_-^2$	$\mathcal{S}^2$	Matrix realization	Class	Hermitian corresp.
0	0	0	A	A	GUE (A)
0	0	1	$\begin{pmatrix} & A \\ B & \end{pmatrix}$	AIII†	chGUE (AIII)
+1	0	0	$A = A^\top$	AI†	GOE (AI)
−1	0	0	$\begin{pmatrix} A & B \\ C & A^\top \end{pmatrix}, \begin{cases} B = -B^\top \\ C = -C^\top \end{cases}$	AII†	GSE (AII)
0	+1	0	$A = -A^\top$	D	BdG-S (D)
0	−1	0	$\begin{pmatrix} A & B \\ C & -A^\top \end{pmatrix}, \begin{cases} B = B^\top \\ C = C^\top \end{cases}$	C	BdG-A (C)

$+1$	$+1$	1	$\begin{pmatrix} & A \\ A^\top & \end{pmatrix}$	$\mathrm{AI}^\dagger_+$	chGOE (BDI)
-1	-1	1	$\begin{pmatrix} & & A & B \\ & & C & D \\ D^\top & -B^\top & & \\ -C^\top & A^\top & & \end{pmatrix}$	$\mathrm{AII}^\dagger_+$	chGSE (CII)
$+1$	-1	1	$\begin{pmatrix} & A \\ B & \end{pmatrix}, \begin{cases} A = A^\top \\ B = B^\top \end{cases}$	$\mathrm{AI}^\dagger_-$	chBdG-S (CI)
-1	$+1$	1	$\begin{pmatrix} & A \\ B & \end{pmatrix}, \begin{cases} A = -A^\top \\ B = -B^\top \end{cases}$	$\mathrm{AII}^\dagger_-$	chBdG-A (DIII)

Symmetry Classification for nhSYK

$N \bmod 8$	0	2	4	6
$q \bmod 4 = 0$	$\mathrm{AI}^\dagger$	A	$\mathrm{AII}^\dagger$	A
$q \bmod 4 = 1$	$\mathrm{AI}^\dagger_+$	$\mathrm{AI}^\dagger_-$	$\mathrm{AII}^\dagger_+$	$\mathrm{AII}^\dagger_-$
$q \bmod 4 = 2$	D	A	C	A
$q \bmod 4 = 3$	$\mathrm{AI}^\dagger_+$	$\mathrm{AII}^\dagger_-$	$\mathrm{AII}^\dagger_+$	$\mathrm{AI}^\dagger_-$

This completes the tenfold way and add a few more universality classes

$$\mathcal{H} = \mathrm{antidiag}(H, H') := \begin{pmatrix} & H \\ H' & \end{pmatrix}$$

Symmetry Classification for *chiral* nhSYK

$N \bmod 8$	0	2	4	6
$q \bmod 4 = 0$	$\mathrm{BDI}^\dagger_{-+}$	AIII_-	$\mathrm{CII}^\dagger_{-+}$	AIII_-
$q \bmod 4 = 2$	BDI_{+-}	AIII_-	CII_{+-}	AIII_-

H, H' Hermitian

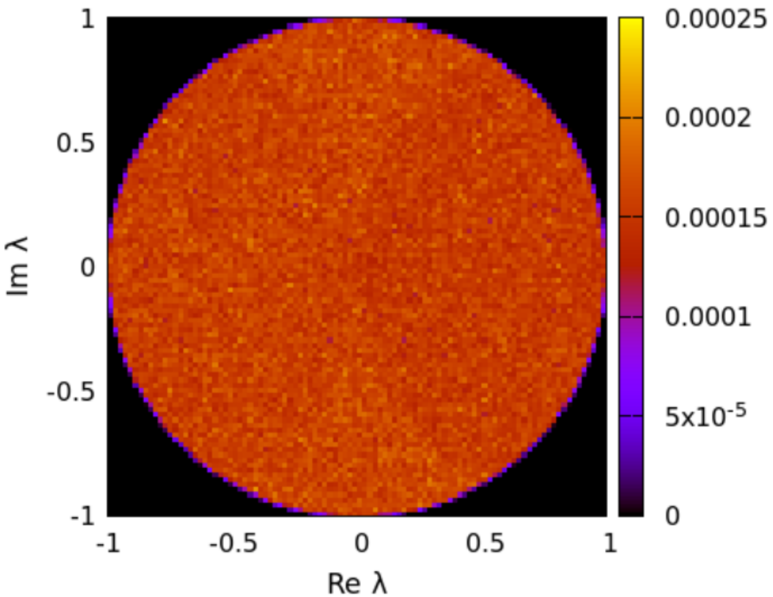
$N \bmod 8$	0	2	4	6
$q \bmod 4 = 0$	$\mathrm{AI}^\dagger_-$	$\mathrm{AIII}^\dagger$	$\mathrm{AII}^\dagger_-$	$\mathrm{AIII}^\dagger$
$q \bmod 4 = 2$	$\mathrm{AII}^\dagger_-$	$\mathrm{AIII}^\dagger$	$\mathrm{AI}^\dagger_-$	$\mathrm{AIII}^\dagger$

H, H' non-Hermitian

Complex spacing ratios

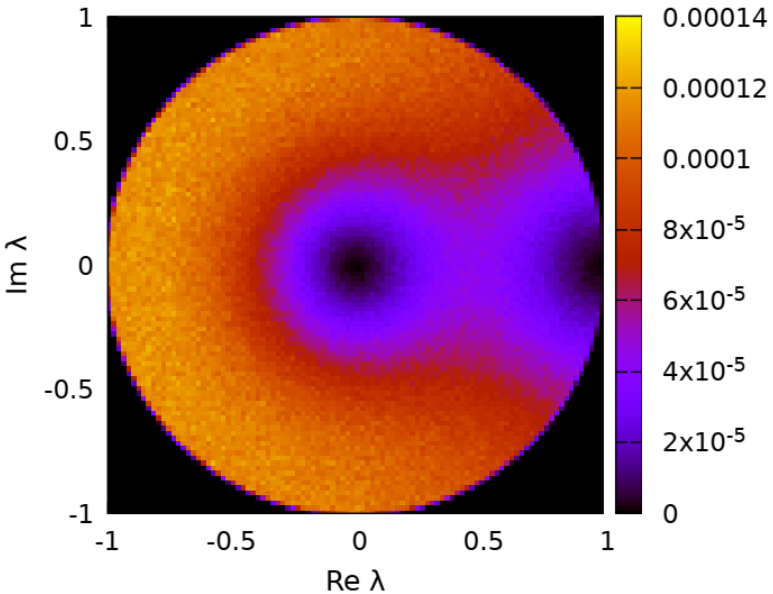
$$\lambda_k = \frac{E_k^{\mathrm{NN}} - E_k}{E_k^{\mathrm{NNN}} - E_k}.$$

q = 2



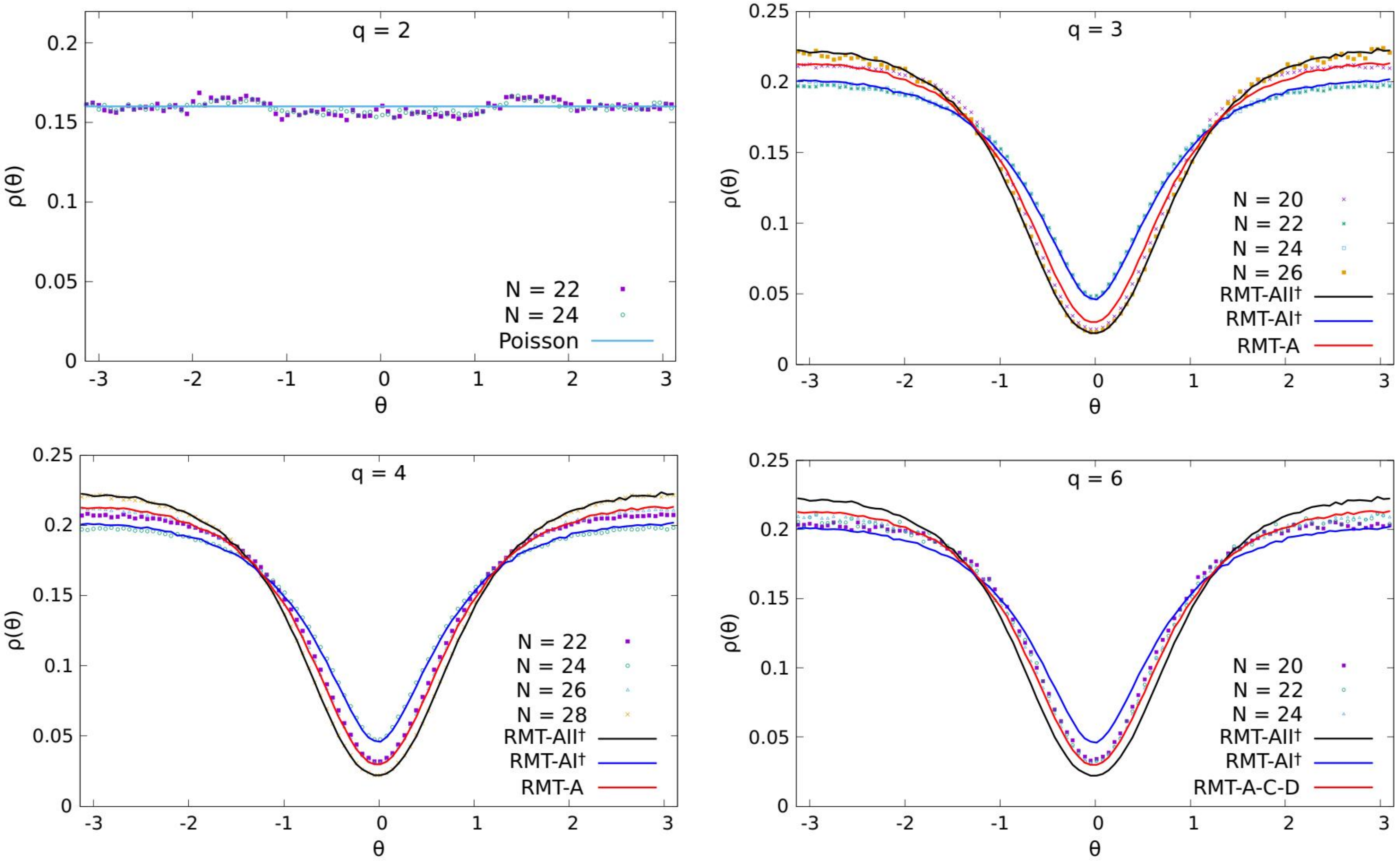
Integrability

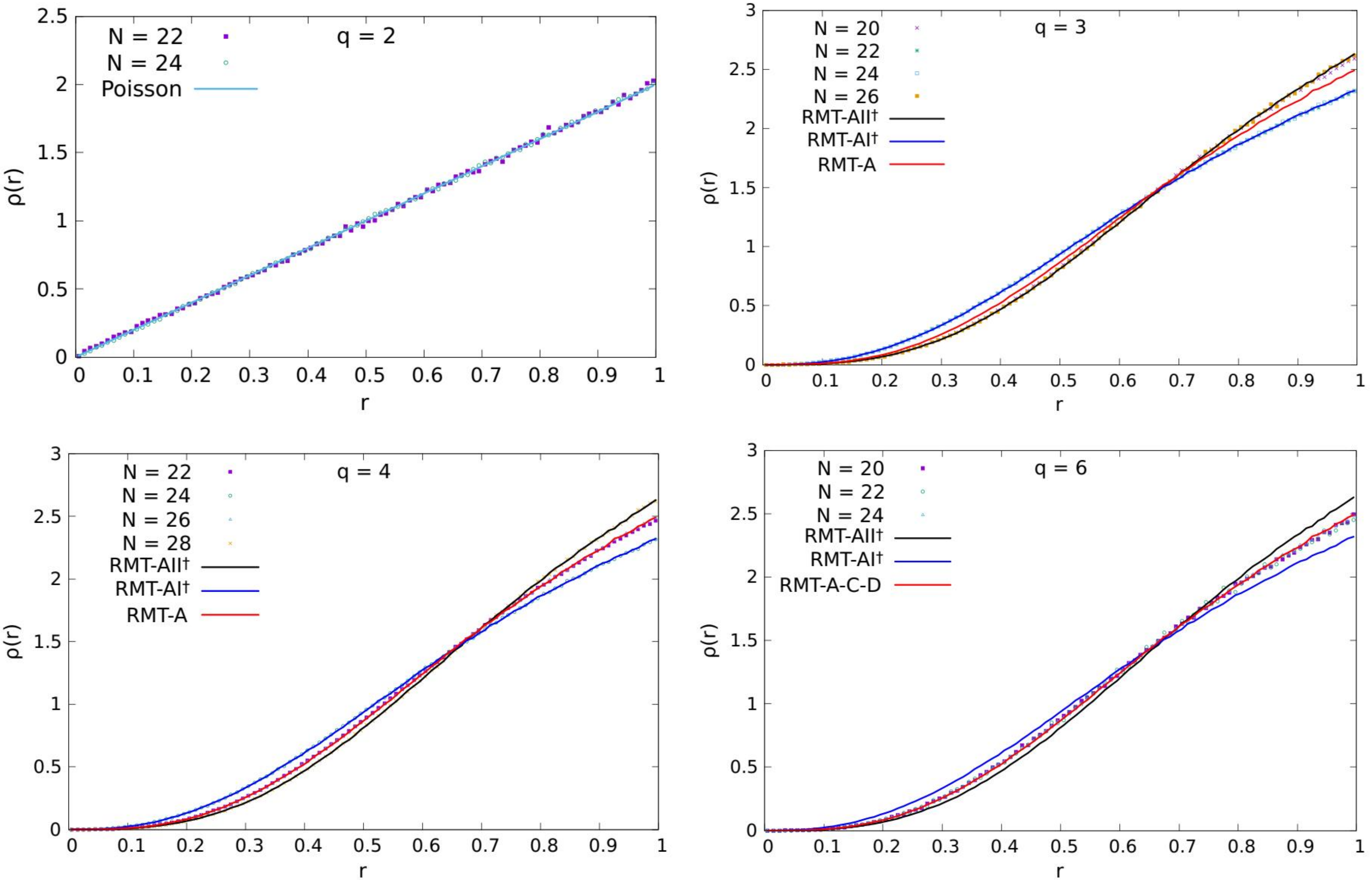
q > 2



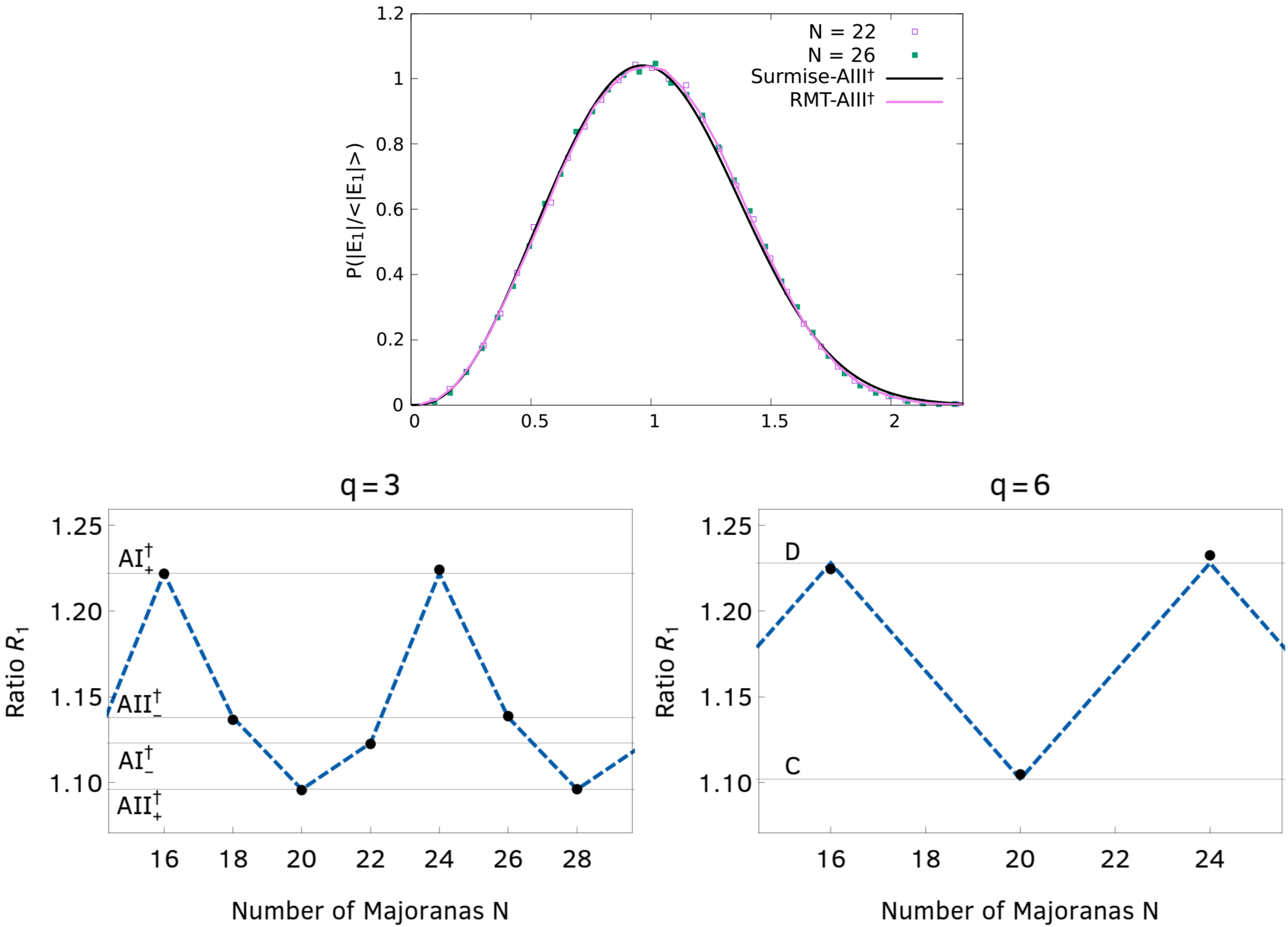
Quantum Chaos

Radial $\rho(r)$ and angular $\rho(\theta)$ distributions of the complex spacing ratio

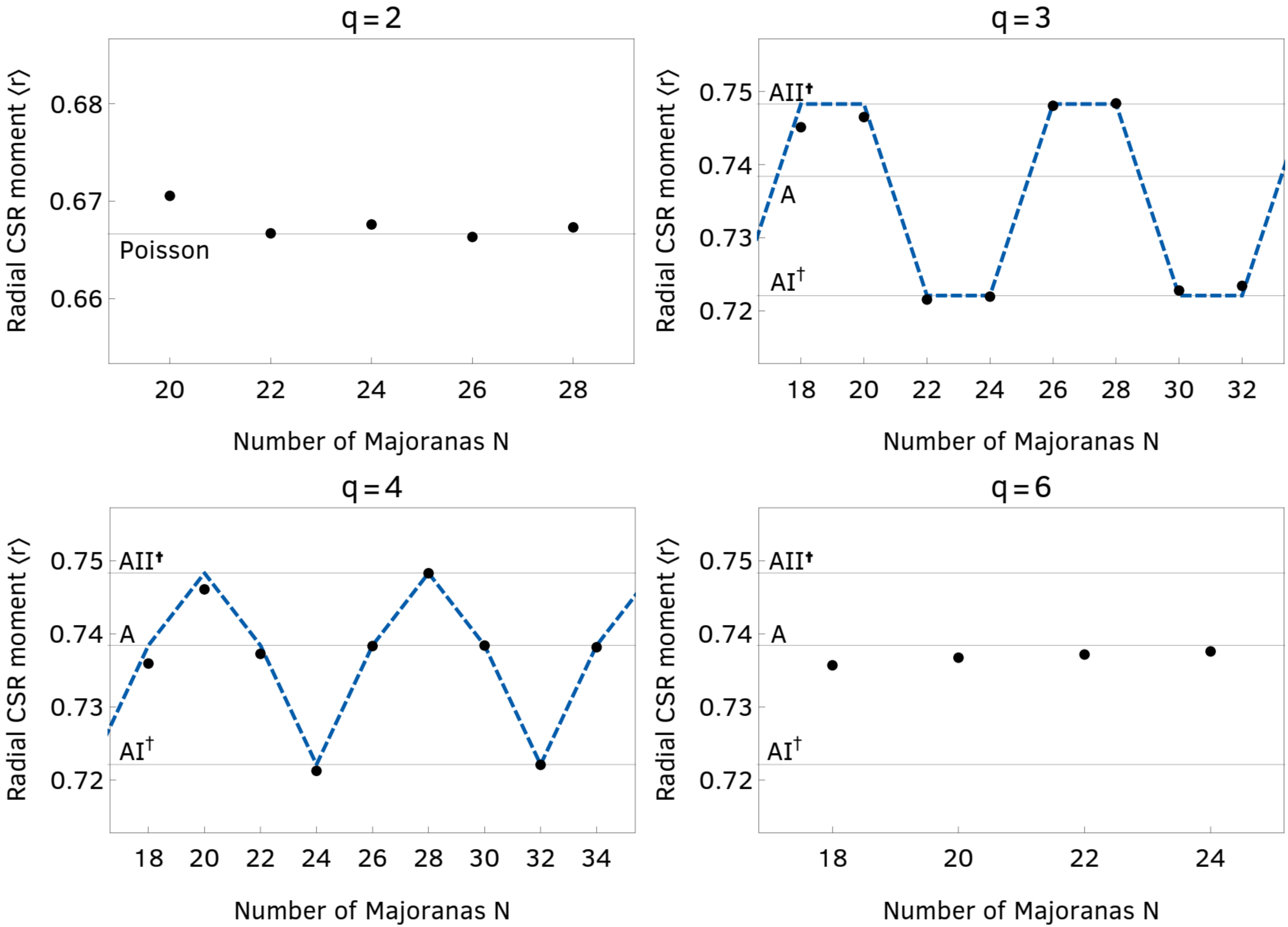




No free parameters!!



N = 22
N = 26
Surmise-AIII†
RMT-AIII†
P(|E1|/<|E1|>)
q = 3
q = 6
Ratio R_1
$AI_+^\dagger$
$AII_-^\dagger$
$AI_-^\dagger$
$AII_+^\dagger$
D
C
Number of Majoranas N



q = 2
Radial CSR moment ⟨r⟩
0.68
0.67
0.66
Poisson
20
22
24
26
28
Number of Majoranas N
q = 3
Radial CSR moment ⟨r⟩
0.75
0.74
0.73
0.72
AII†
A
AI†
18
20
22
24
26
28
30
32
Number of Majoranas N
q = 4
Radial CSR moment ⟨r⟩
0.75
0.74
0.73
0.72
AII†
A
AI†
18
20
22
24
26
28
30
32
34
Number of Majoranas N
q = 6
Radial CSR moment ⟨r⟩
0.75
0.74
0.73
0.72
AII†
A
AI†
18
20
22
24
Number of Majoranas N

Excellent quantitative agreement with random matrix theory

Assuming that BGS conjecture applies, it means that random matrix theory can be used to describe many-body dissipative quantum chaos

SYK, a toy model for many-body dissipative quantum chaos. Can all universality classes be identified?

Relation to quantum gravity? See next

Long range spectral correlations and limits of dissipative quantum chaos? See AMG, Sa, Verbaarschot 2211.01650

SYK beyond quantum black holes: traversable/ Euclidean/Keldysh(?) wormholes

SYK and traversable wormholes

Gao,Jafferis,Wall
arXiv:1608.05687

$$H_W = H_L^{SYK} + H_R^{SYK} + i\lambda \sum_i \psi_i^L \psi_i^R$$

Maldacena and Qi 1804.00491

A two-site SYK model with a weak $\lambda \ll 1$ coupling is dual, in the low T limit, to a traversable wormhole in a near AdS_2 background

Why?

Both models share the same Schwarzian effective action

$$S = N \int d\tilde{u} \left\{ - \left(\left\{ \tan \frac{t_l(\tilde{u})}{2}, \tilde{u} \right\} + \left\{ \tan \frac{t_r(\tilde{u})}{2}, \tilde{u} \right\} \right) + \eta \left[\frac{t_l'(\tilde{u}) t_r'(\tilde{u})}{\cos^2 \frac{t_l(\tilde{u}) - t_r(\tilde{u})}{2}} \right]^\Delta \right\}$$

Where this gravity action comes from?

Global AdS_2

$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}, \sigma \in [0, \pi]$$

It covers the full spacetime (two boundaries) which has a $SL(2,R)$group of isometries

The idea of MQ paper is to find solutions where the boundaries corresponds to lines of constant $\boldsymbol{\sigma}$ in these coordinates. In this setting, a t-translational invariant dilaton would grow towards both boundaries This is not possible in pure JTgravity. It is also not possible if matter obeys the integrated null energy condition in the bulk. However, it becomes possible (Gao, Jafferis 1608.05687) if we introduce a double trace deformation that (weakly) couple the two boundaries explicitly.

$$S_{int} = g \sum_i^N \int du\, O_L^i(u) O_R^i(u)$$

N operators of conformal weight Δ corresponding to N bulk matter fields. Large N is needed to enhance its effect.

For technical reasons we need $g \ll 1$ so

$$\left\langle e^{ig \sum_i du O_L^i(u) O_R^i(u)} \right\rangle \sim e^{ig \sum_i \int du \langle O_L^i(u) O_R^i(u) \rangle}$$

This is equivalent to a perturbative re-summation of ladder diagrams, see next figure, that is dominant in the large N, small g limit with Ng fixed.

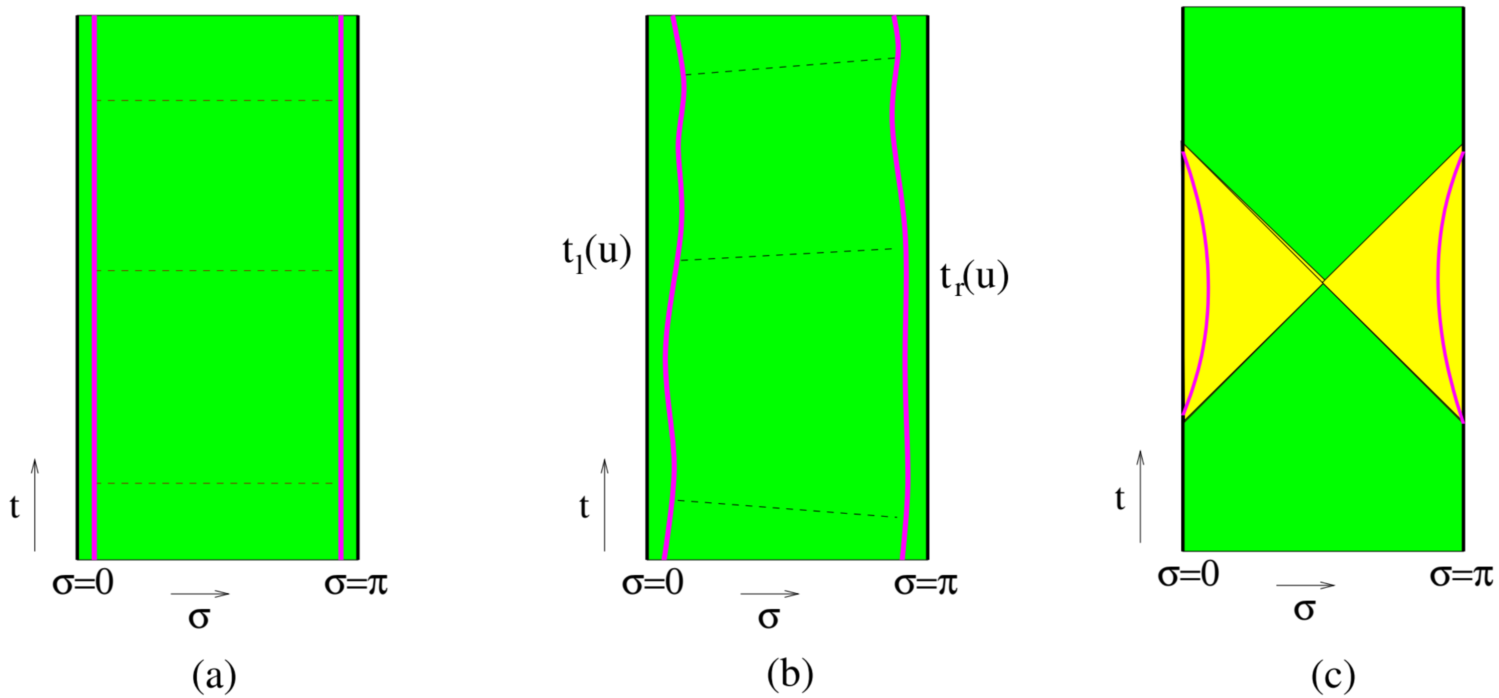


Figure 2: (a) Trajectories of the physical boundaries (in magenta) for the Nearly-AdS_2 geometry with a global time isometry. These trajectories are the lines where the dilaton acquires its boundary value. It can be obtained by introducing an interaction between the two boundaries. (b) We can describe the fluctuations of the boundary trajectories in terms of a pair of functions, $t_l(u)$ and $t_r(u)$, mapping (rescaled) proper time u along the trajectory to the global AdS_2 time coordinate t. The dotted lines can be viewed as insertions of the interaction Hamiltonian. They join points with the same value of u on both boundaries. (c) The physical boundaries for a Nearly-AdS_2 geometry with thermal isometry. Here the two boundary trajectories cover only a finite range of global time and we cannot send a signal between the two trajectories.

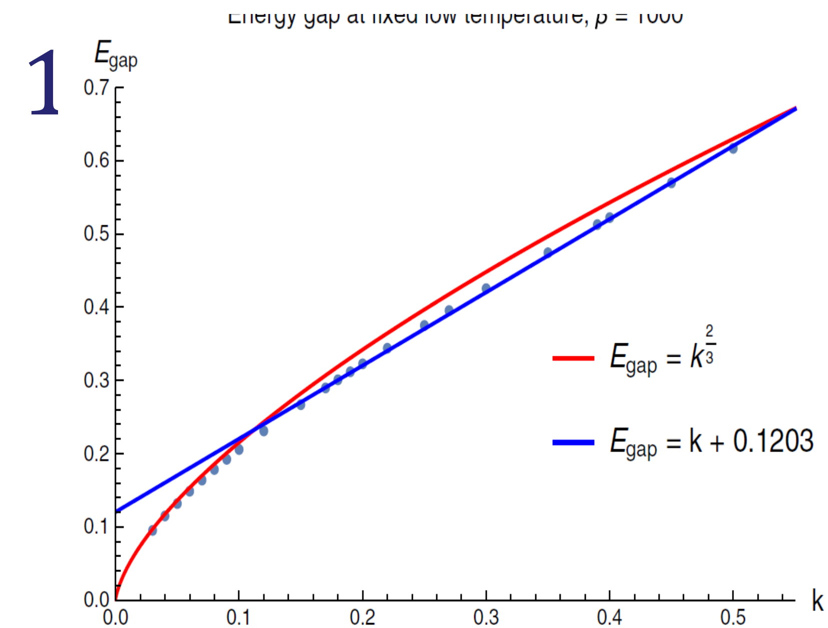
After coupling this to the gravity modes by performing a reparametrization of the left and right times. More specifically, this is a map between the boundary time u and the global times $t_l(u), t_R(u)$ at the two boundaries.

$$S = N \int d\tilde{u} \left\{ - \left(\{\tan \frac{t_l(\tilde{u})}{2}, \tilde{u}\} + \{\tan \frac{t_r(\tilde{u})}{2}, \tilde{u}\} \right) + \eta \left[\frac{t_l'(\tilde{u}) t_r'(\tilde{u})}{\cos^2 \frac{t_l(\tilde{u}) - t_r(\tilde{u})}{2}} \right]^{\Delta} \right\}$$

In this way, we describe the theory describes the position of the physical boundary.

Why is this interesting?

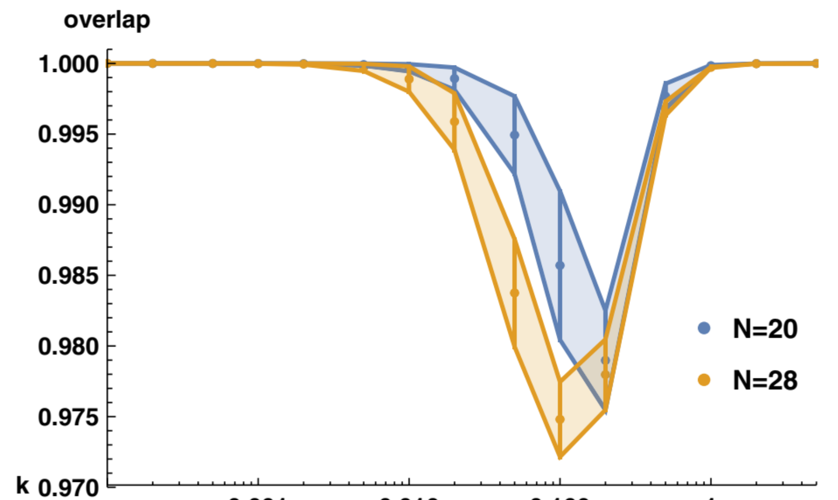
Interesting features $\lambda \ll 1$



Gapped system $E_G \sim \lambda^{2/3}$

Not $E_G \sim \lambda$ as expected
from a explicit coupling
Important!

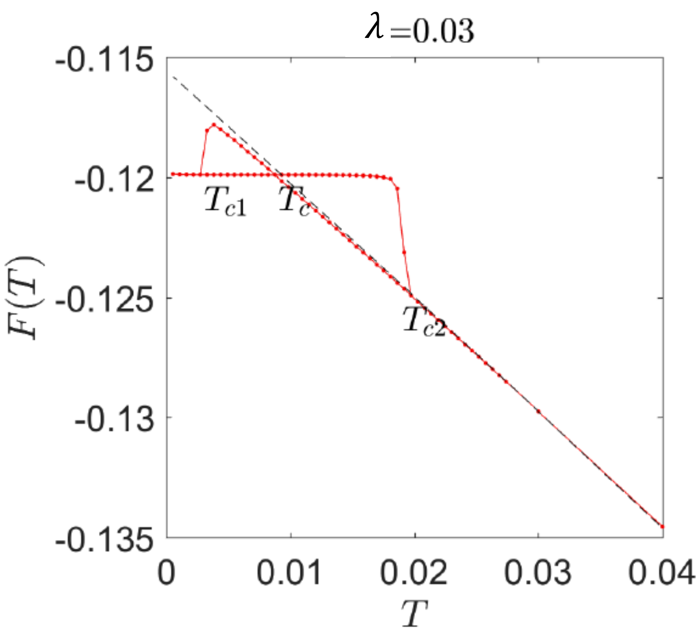
Non trivial interplay of interactions in each site and intersite-coupling leads to a novel form of interaction-enhanced tunneling

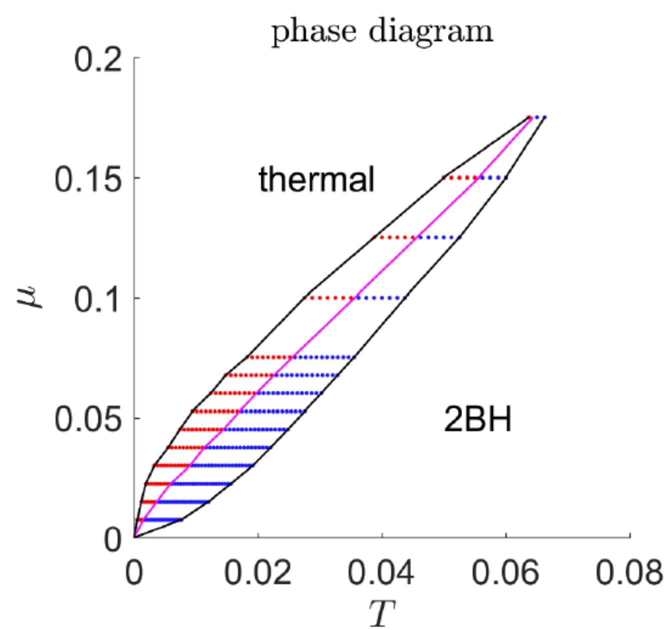


Ground state close to a TFD state

$$\langle TFD_{\beta(k)}|GS\rangle$$

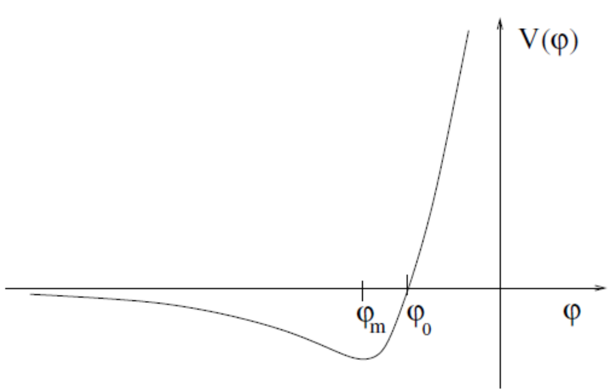
Hawking-Page transition between wormhole and two blackhole configurations





Mapping to a Liouville like QM problem with bound states related to wormhole excitations

$$S = N \int d\tilde{u}[\dot{\varphi}^2 - e^{2\varphi} + \eta e^{2\Delta\varphi}]$$

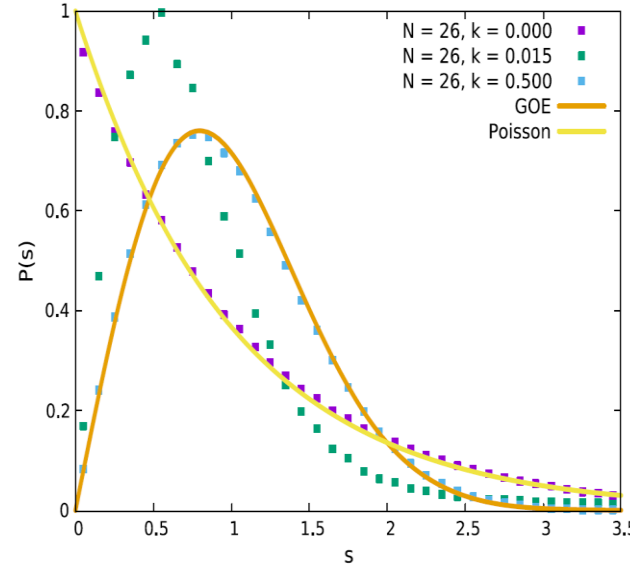


Quantum chaos and traversable wormholes

$$H_{\text{total}} = H_L + \alpha H_R + H_{\text{int}} \qquad H_{\text{int}} = ik \sum_{j=1}^{N/2} \psi_{L,j}\psi_{R,j},$$

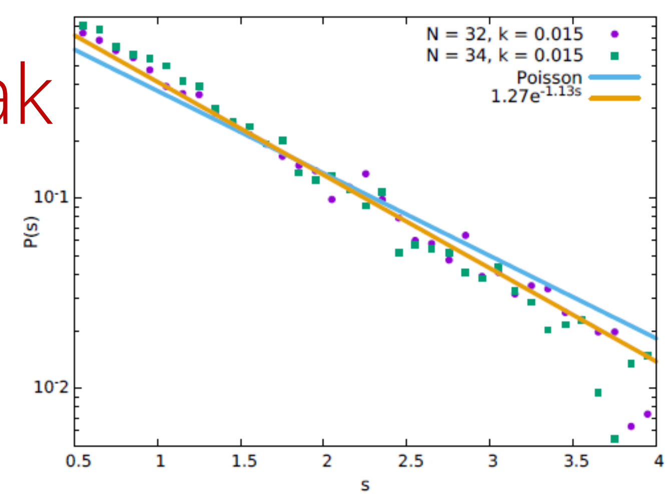
Two site SYK dual to a transversable wormhole

Maldacena, Qi 1804.00491



Hawking Page transition (weak coupling) is also a quantum-chaos integrable transition

AGG, Rosa, Nosaka, Verbaarschot, PRD 100, 026002 (2019), **28**



Non-Hermitian two-site SYK and Euclidean wormholes

$$H_{EW} = H_L^{SYK} + H_R^{SYK} + \cancel{i\lambda \sum_i \psi_i^L \psi_i^R}$$

$$\lambda = 0$$

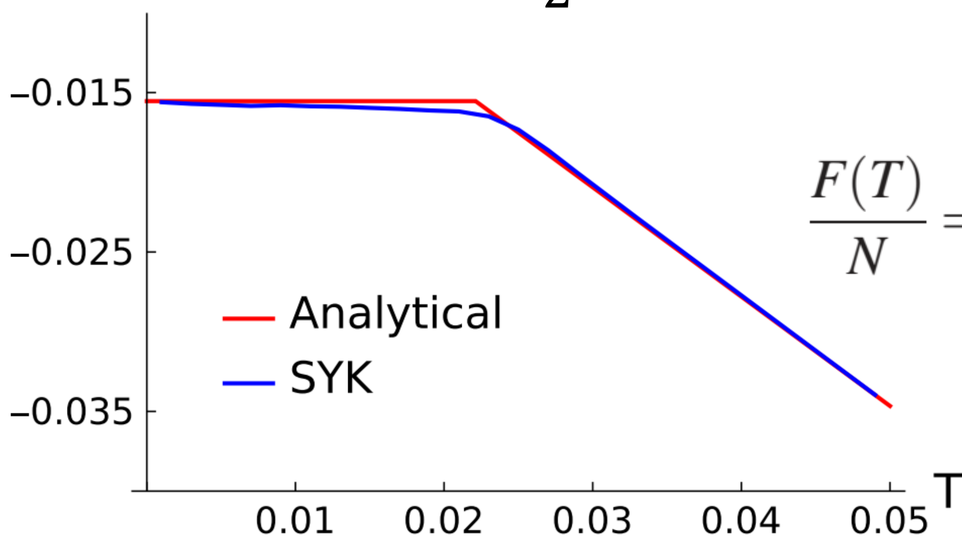
$$H_{SYK} = \sum_{ijkl} J_{ijkl}\, \psi_i \psi_j \psi_k \psi_l$$

$$J_{ijkl} \to J_{ijkl} + iM_{ijkl}$$

Complex couplings

Because of dominance of replica off diagonal configurations, the properties of the SYK system after ensemble average are qualitatively different.

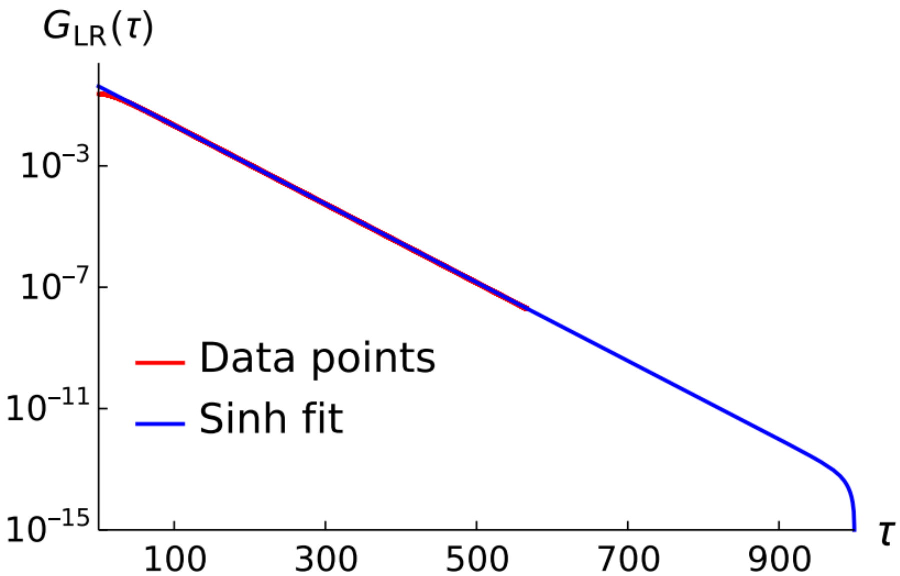
SYK free energy undergoes a first order transition that in the gravity dual is interpreted as wormhole/black hole phase transition in near AdS_2

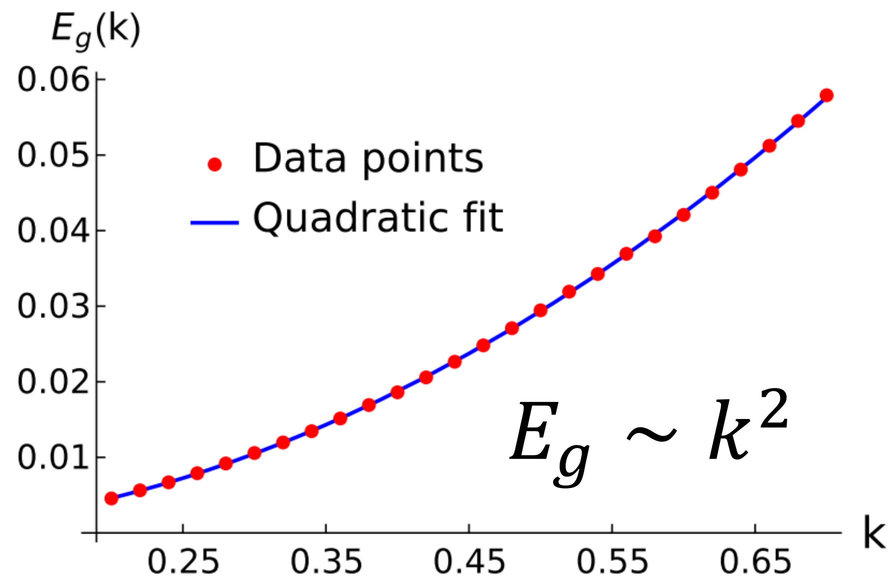


$$\frac{F(T)}{N} = \frac{-2E_0}{N}\theta(T_c - T) - \frac{T\ \log 2}{2}\theta(T - T_c).$$

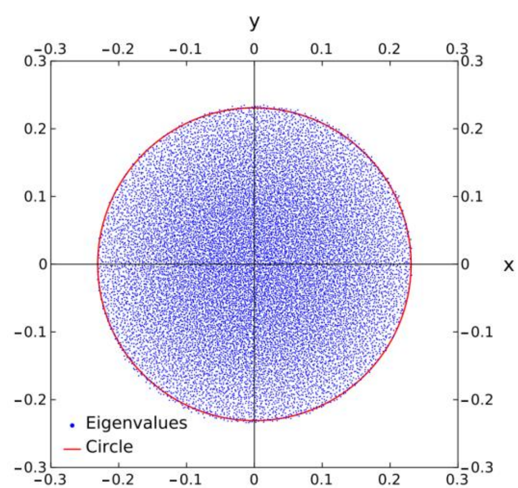
AMG, Jia, Rosa, Verbaarschot,
PRL, 128, 081601 (2022)

Spectrum of the **averaged** system is gapped



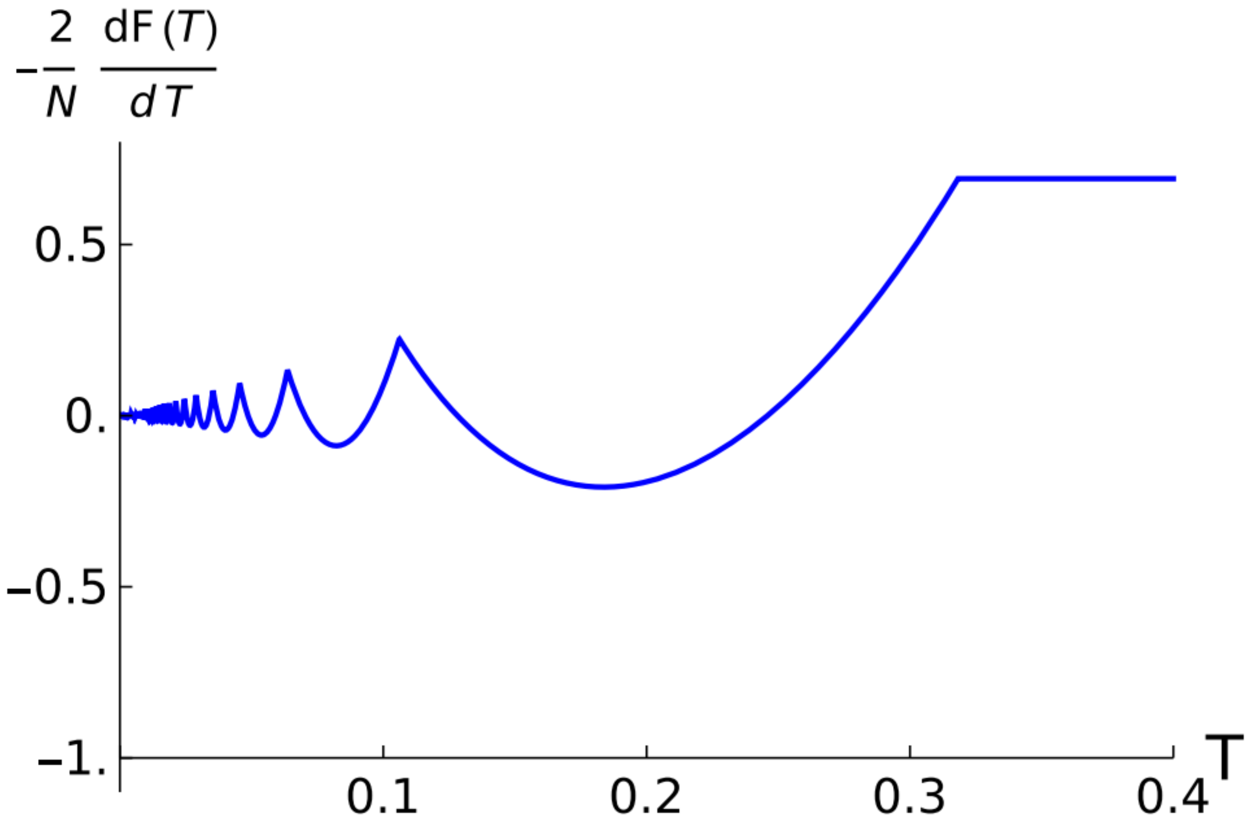


But there is no gap for a single disorder realization!



Interactions $q = 4$ are important

The free energy for a $q = 2$ two site SYK is qualitatively different



Gravity dual?

Euclidean Wormholes

We find solutions in near AdS2 background (JT gravity) with additional matter

Relevant for the problem of factorization in holography and, in a different context, for the information paradox

V. Godet, AGG, Phys. Rev. D 103, 046014 (2021)

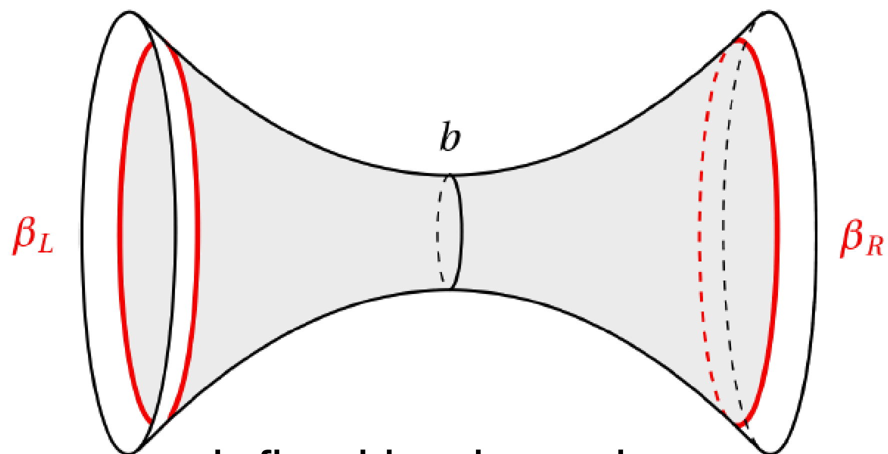
$$S = S_{\rm JT} + S_{\rm matter}\,, \tag{5}$$

where we have

$$S_{\rm JT} = -\frac{S_0}{2\pi}\left[\frac{1}{2}\int d^2x\,\sqrt{g}\,R + \int d\tau\,\sqrt{h}\,K\right] - \frac{1}{2}\int d^2x\,\sqrt{g}\,\Phi(R+2) - \int d\tau\,\sqrt{h}\,\Phi(K-1)\,, \tag{6}$$

$$S_{\rm matter} = \frac{1}{2}\int d^2x\,\sqrt{g}\,(\partial\chi)^2\,. \tag{7}$$

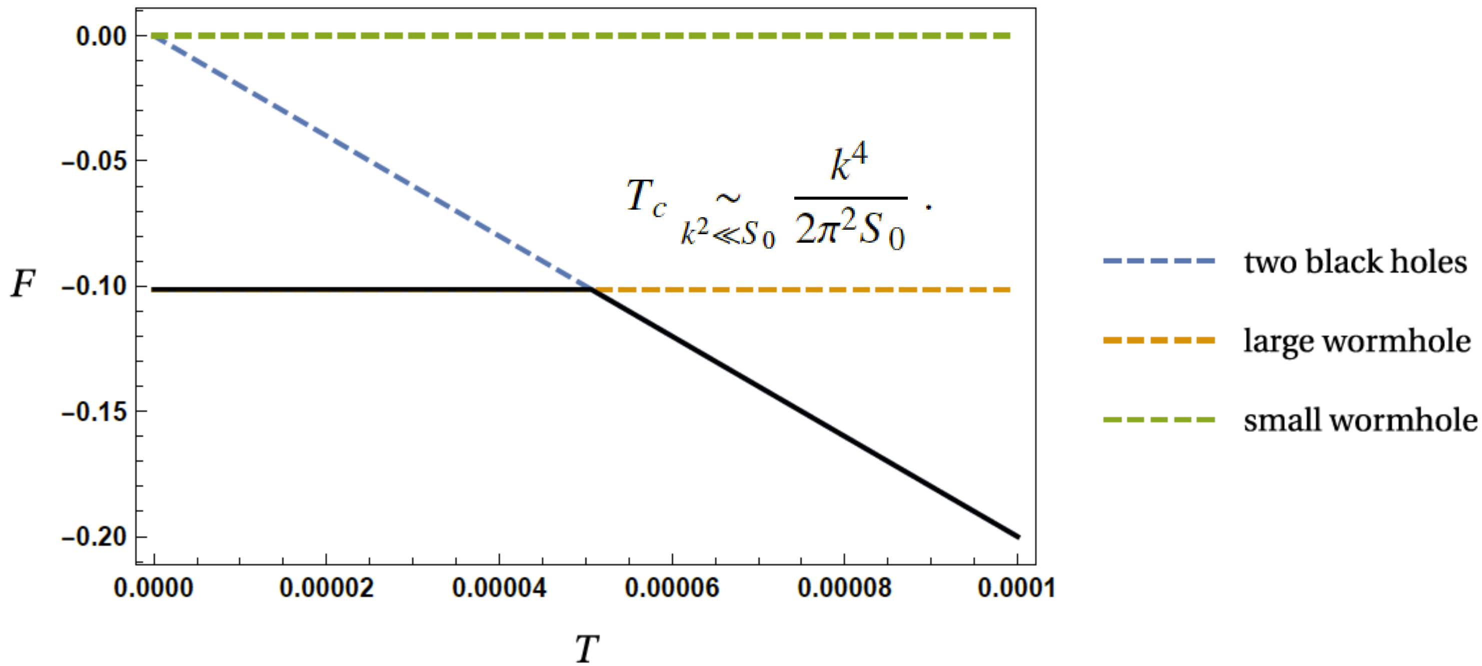
$$\lim_{\rho\to\pi/2}\chi = ik, \qquad \lim_{\rho\to-\pi/2}\chi = -ik\,,$$



b fixed by dynamics

Complex sources

Euclidean wormhole solutions of JT gravity



$$F_{\mathrm{BH}} = -T \log Z = -2S_0 T - 4\pi^2 T^2 \ .$$

$$F_{\mathrm{WH}} = -T \log Z = -\frac{k^4}{\pi^2}$$

How is the physics of wormholes related to the out of equilibrium dynamics of dissipative quantum chaotic matter?

Keldysh Wormholes and Anomalous Relaxation in the Dissipative Sachdev-Ye-Kitaev Model

Antonio M. García-García,[1,*] Lucas Sá,[2,†] Jacobus J. M. Verbaarschot,[3,‡] and Jie Ping Zheng (郑杰平)[1,§]

arXiv:2210.01695

AdS/CFT Takayanagi 1911.07861

Overlap with Ryu. et al. 2210.04093 that came out a few days later

The real time dynamics of a SYK at $T \to \infty$ weakly perturbed by a Markovian environment is equal to the Euclidean dynamics of a two-site non-Hermitian SYK in the $T \to 0$ limit

The late time dynamics is characterized by the decay rate/gap to a steady state which is computed by the exponential decay of certain Green's functions.

Quantum chaos is important. Only for $q > 2$, the decay rate is finite even if there is no coupling with the bath.

The free energy of the Euclidean problem undergoes a first order transition as in wormholes settings.

We speculate that our SYK is dual to a bra-ket wormhole: a perturbed double cone in near dS_2 2007.16091

Real time evolution of a SYK coupled to a Markovian bath

$$H^{\mathrm{SYK}} = i^{q/2} \sum_{i_1<\cdots<i_q} J_{i_1\cdots i_q} \psi^{i_1} \cdots \psi^{i_q},$$ Single site SYK

$$L_i = \sqrt{\mu}\psi^i$$ Jump operators that characterize the bath

$$\mathcal{L}(\rho) = -i[H^{\mathrm{SYK}}, \rho] + \sum_\alpha \left[L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right].$$

We study the dynamics after a weak perturbation (linear response) of a TFD state at $\mathbf{T} = \infty$

$$\rho_\infty = \frac{1}{2^{N/2}} \sum_k |k\rangle\langle k|,$$

Keldysh formalism

We are interested in the following retarded Green's function

$$iG^{\mathrm{R}}(t)\delta_{ij} = \Theta(t)\left\langle \mathrm{Tr}\left[\rho_\infty\{\psi^i(t),\psi^j(0)\}\right]\right\rangle.$$

In order to proceed, it is useful to employ the Keldysh path integral that involves the doubling of degrees of freedom and the vectorization of the Liouvillian

The partition function related to the path integral is given by,

$$Z = \int \mathcal{D}\psi_L \mathcal{D}\psi_R \, e^{iS[\psi_L,\psi_R]},$$

The degree of freedom are doubled in the Keldysh action S_K

$$S_K = \int_{\infty}^{\infty} dt\left(-\frac{1}{2}\sum_i \psi_L^i \partial_t \psi_L^i - \frac{1}{2}\sum_i \psi_R^i \partial_R^i + \mathcal{L}\right)$$

Vectorized Liouvillian $H = H_L \otimes H_R$

$$\mathcal{L} = -iH_L^{SYK} + i(-1)^{\frac{q}{2}} H_R^{SYK} - i\mu \sum_i \psi_L^i \psi_R^i - \frac{1}{2}\mu N$$

We assume the system is sufficiently close to the steady state so we neglect initial conditions. We assume $\beta = 0$

Note the similarities are differences with the previous SYK related to wormholes

Solving numerically the saddle point equations for $q = 4$ and analytically in the large q limit and for $q = 2$, we have found that for $q > 2,$ the retarded Green's function decays exponentially even if there is no coupling ($\mu = 0$) to the environment.

$$iG^R(t) \propto e^{-\Gamma(\mu)t} \cos \Omega(\mu)t$$

Γ is the typical (inverse) relaxation time to the steady state.

We interpret Ω as an imaginary part of Γ indicating metastability though it also occurs at $\mu = 0$!

The Euclidean problem

$$H = iH_L^{\mathrm{SYK}} - i(-1)^{q/2} H_R^{\mathrm{SYK}} + i\mu \sum_k \psi_L^k \psi_R^k,$$

Note that the SYK's are anti-Hermitian

The ground state

$$|0\rangle = \sum_k |k\rangle \otimes UK|k\rangle,$$

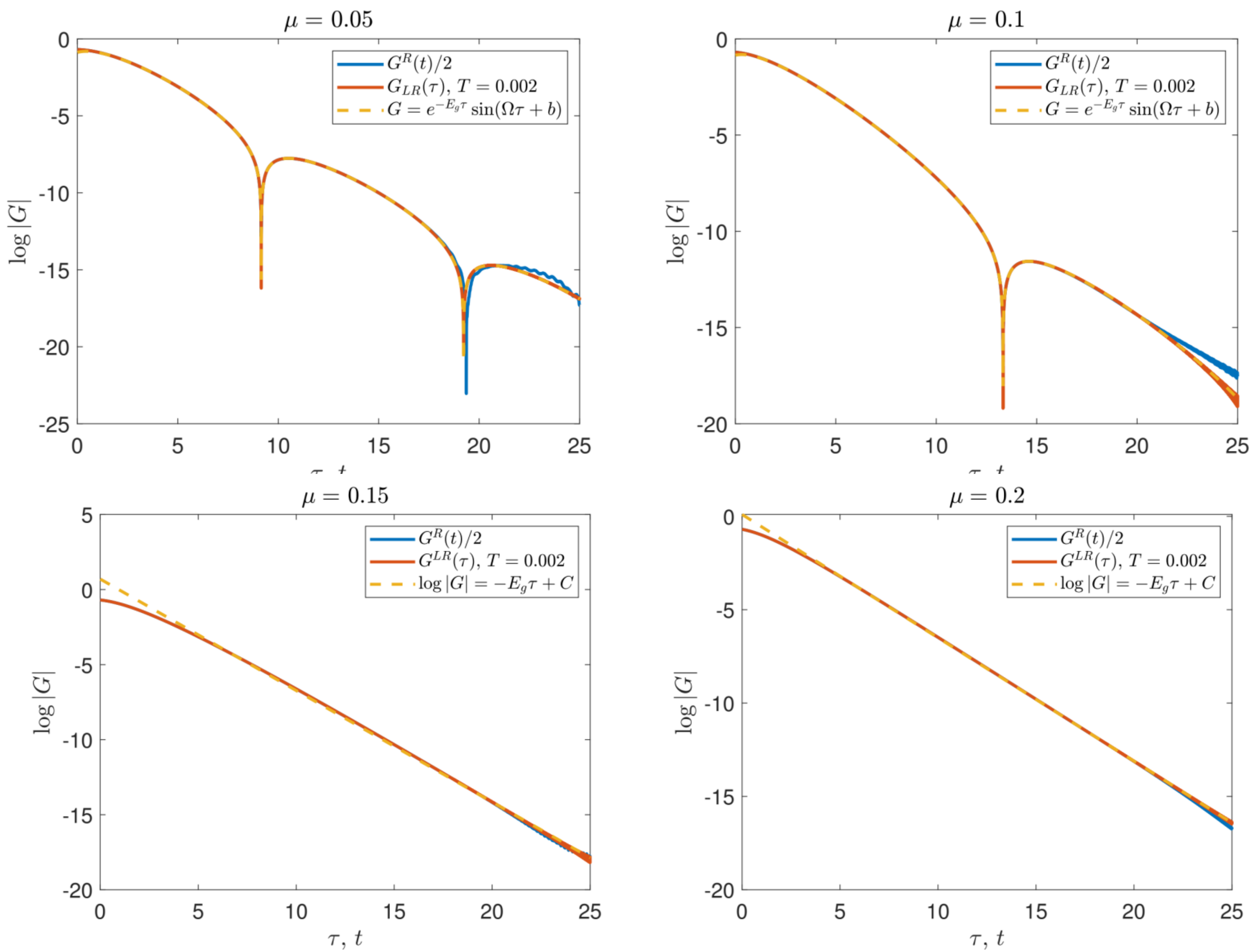
of this Hamiltonian is the thermofield-double (TFD) state at $\beta = 0$

$$iG_{LR}\,(\tau) \propto e^{-\Gamma(\mu)t} \cos\Omega(\tau)t$$

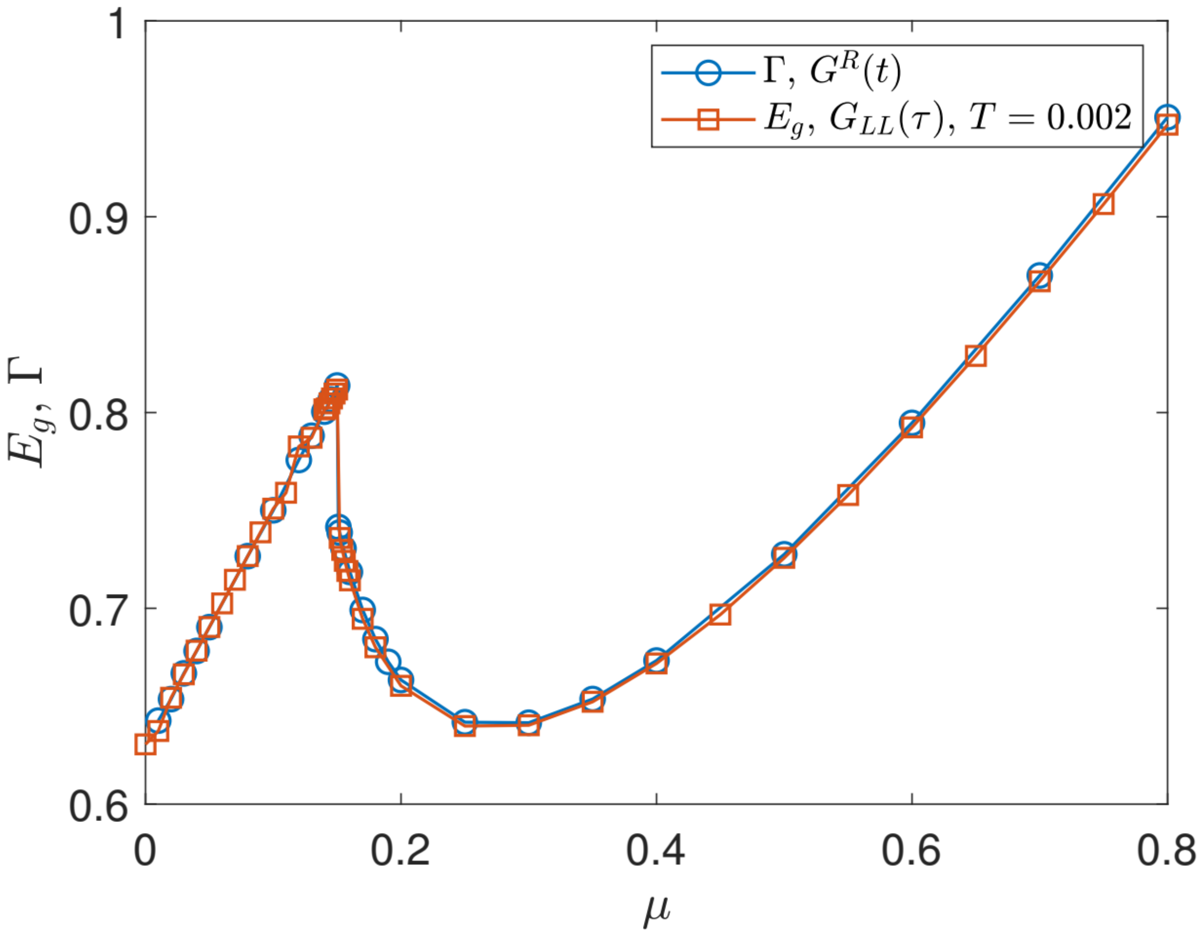
This is identical to the real time results with $\tau = t$ provided that the two Green's functions $G_R(\mathrm{t})$ and $G_{LR}(\tau)$ are identified

Therefore the original one-site SYK coupled to an environment is mapped to a two-site non Hermitian SYK at zero temperature

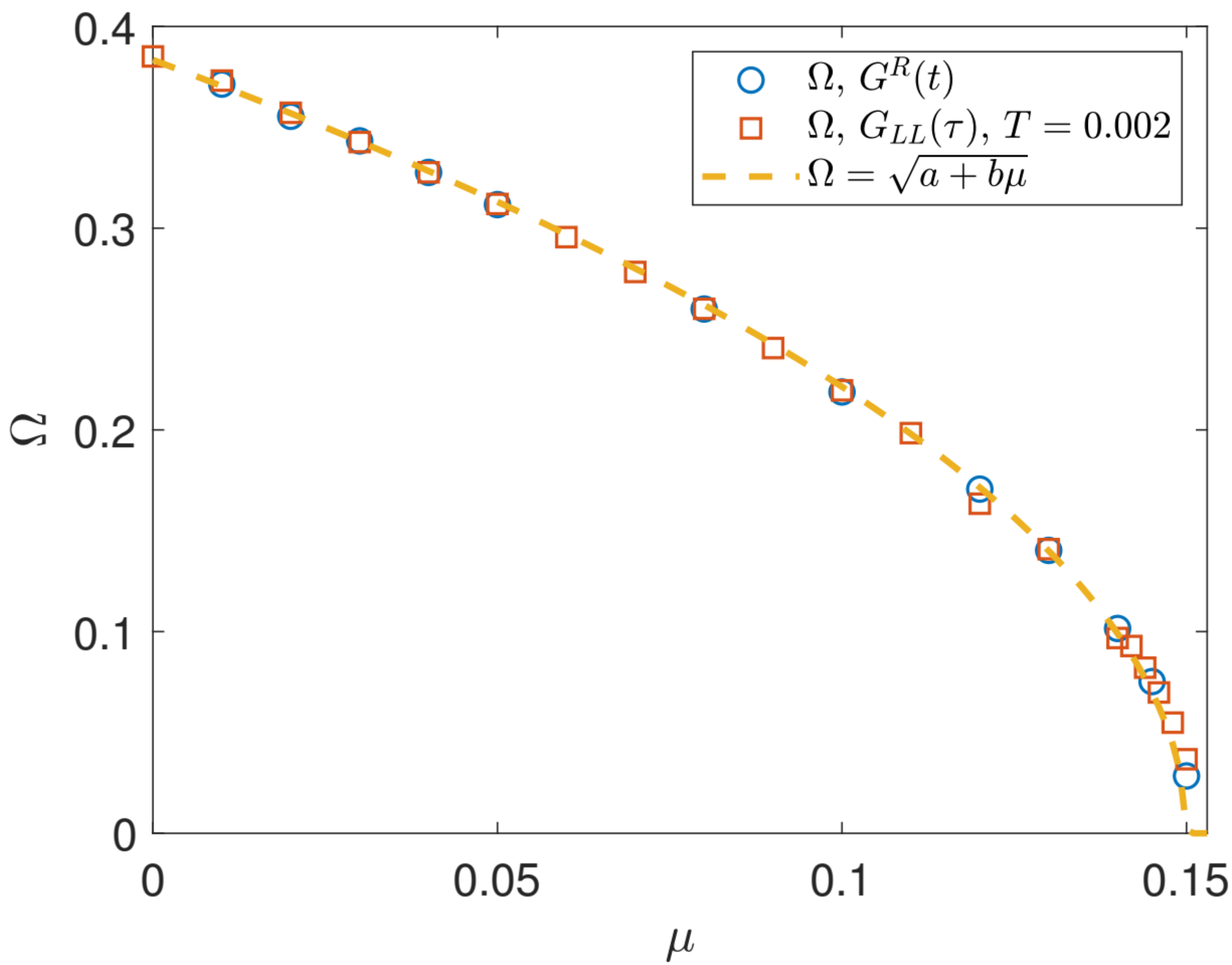
Equivalence of $G_{LR}(\tau)$ and $G_R(t)$



$q = 4$



The existence of a gap/decay rate (E_g, Γ) does not require the coupling to a bath. Only for μ large, we recover the expected $\Gamma \sim \mu$ dissipation driven decay. Transition at $\mu \sim 0.14$



The frequency $\mathbf{\Omega}$ vanishes in what looks a second order phase transition for sufficiently strong coupling to the environment.

Analytical large q calculation

$$G_{LL} = \frac{1}{2}\mathrm{sign}(\tau)e^{\frac{1}{q}g_{LL}} = \frac{1}{2}\mathrm{sign}(\tau)(1 + \frac{1}{q}g_{LL} + \dots),$$
$$G_{LR} = \frac{i}{2}e^{\frac{1}{q}g_{LR}} = \frac{i}{2}(1 + \frac{1}{q}g_{LR} + \dots).$$

It is important to impose $G_{LL}(\tau) = -i\,\mathrm{sign}(\tau)\,G_{LR}(\tau),$

$$e^{g_{LL}} = e^{g_{LR}} = \frac{\alpha^2}{J^2\cosh^2(\alpha|\tau| + \gamma)}$$

$$\alpha^2/J^2\cosh^2\gamma = 1, 2\alpha\tanh y = \hat{\mu}$$

$$\boxed{E_g = \frac{2\alpha}{q} = \frac{2J}{q}\sqrt{\left(\frac{\hat{\mu}}{2J}\right)^2 + 1}}$$

This result is only about 20% off the numerical one for $q = 4$.
It agrees with $E_g \sim \mu$ for $\mu \gg 1$ but it does predict oscillations Ω.

Non-chaotic case $q = 2$

The previous results require that the dynamics is quantum chaotic.

For $q = 2$, where the dynamics is not quantum chaotic, $\Gamma = 0$ if no coupling to the bath $\mu = 0$. For $\mu > 0$, $\Gamma = \mu$ so the relaxation is always dissipation driven.

$\mu = 0$

$$1 = (-i\omega + J^2 G_{LL})G_{LL} - J^2 G_{LR} G_{LR},$$

$$0 = (-i\omega + J^2 G_{LL})G_{LR} + J^2 G_{LR} G_{LL}.$$

$$G^{\mathrm{R}}(\omega) = -\frac{\omega}{2J^2} + \frac{\mathrm{sign}(\omega)}{2J^2}\sqrt{(\omega + i\epsilon)^2 - 4J^2}$$

with $\epsilon \to 0^+$, and the advanced Green's function is equal to

$$G^{\mathrm{A}}(\omega) = -\frac{\omega}{2J^2} + \frac{\mathrm{sign}(\omega)}{2J^2}\sqrt{(\omega - i\epsilon)^2 - 4J^2}.$$

$$G_{LR}(\omega) = i\frac{1}{J}\sqrt{1-\frac{\omega^2}{4J^2}}, \qquad G_{LL}(\omega) = \frac{i\omega}{2J^2}, \qquad \text{if } |\omega| < 2J, \tag{48}$$

$$G_{LR}(\omega) = 0, \qquad G_{LL}(\omega) = i\frac{1}{J}\left(\frac{\omega}{2J} - \operatorname{sign}(\omega)\sqrt{\frac{\omega^2}{4J^2}-1}\right), \qquad \text{if } |\omega| > 2J. \tag{49}$$

In the time domain, the retarded and advanced Green's functions are given by

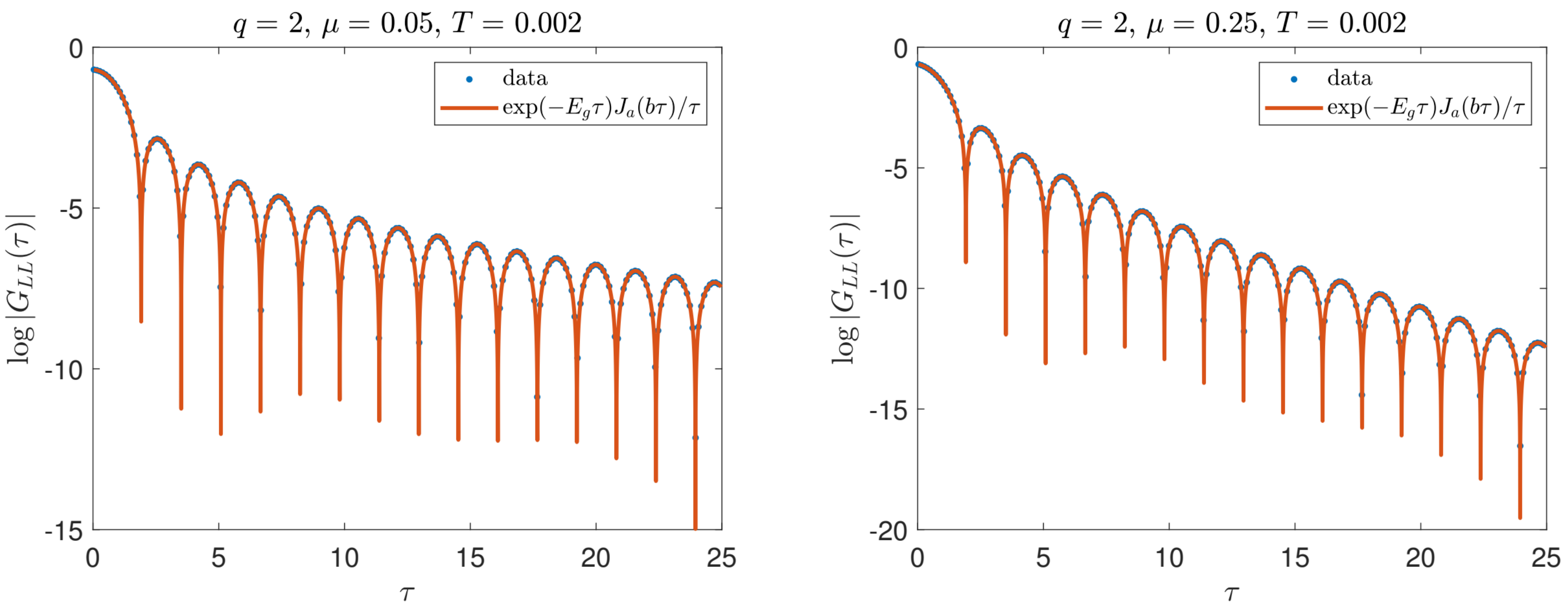
$$G^{\mathrm{R}}(\tau) = -i\Theta(\tau)e^{-\epsilon\tau}\frac{J_1(2J\tau)}{J\tau}, \qquad G^{\mathrm{A}}(\tau) = i\Theta(-\tau)e^{\epsilon\tau}\frac{J_1(2J\tau)}{J\tau}, \tag{50}$$

$\mu > 0$

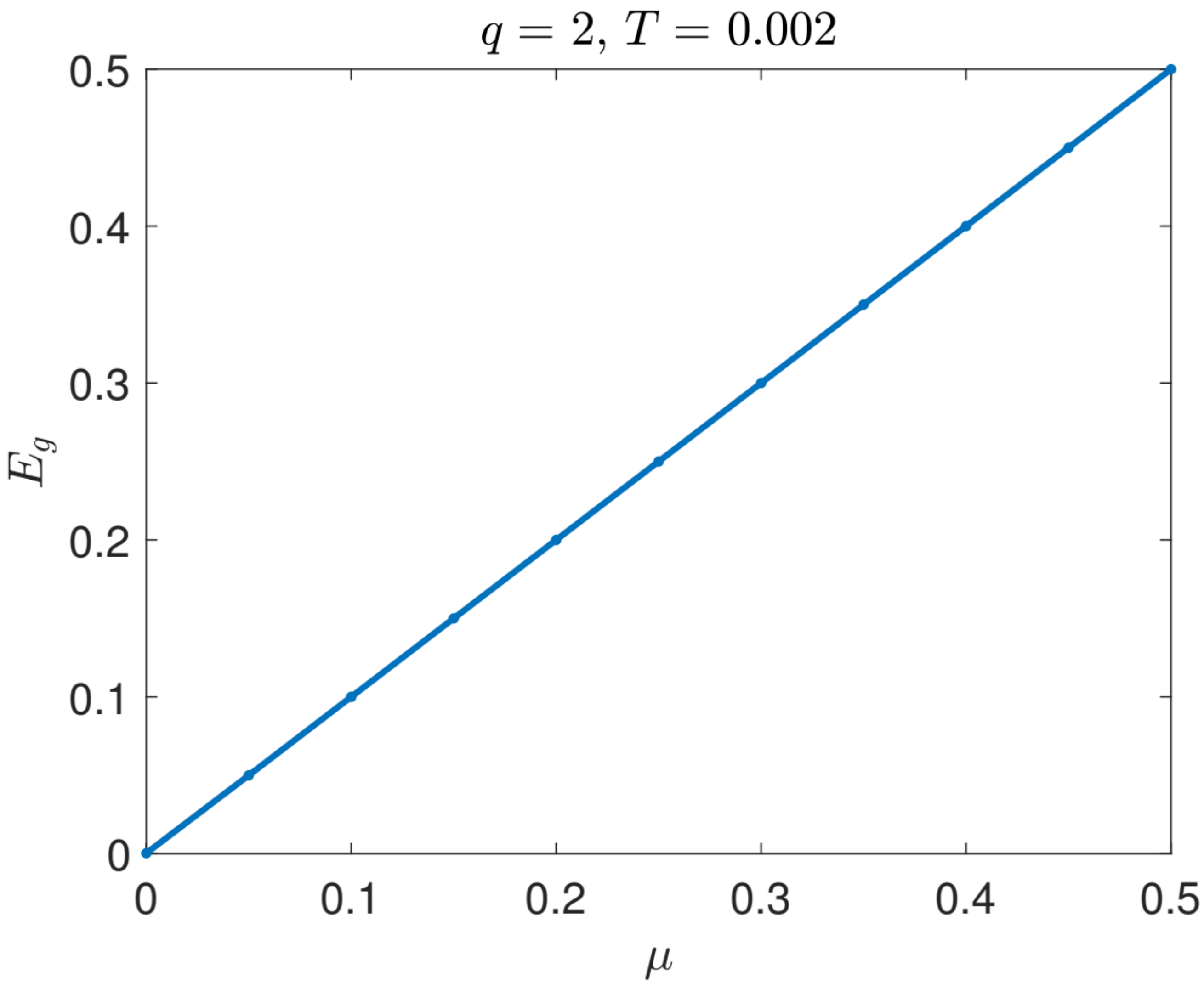
$$G^{\mathrm{R}}(\omega) = \frac{(\omega+i\mu)}{2J^2} - \frac{\operatorname{sign}(\omega)}{2J^2}\sqrt{(\omega+i\mu)^2-4J^2},$$
$$G^{\mathrm{A}}(\omega) = \frac{(\omega-i\mu)}{2J^2} - \frac{\operatorname{sign}(\omega)}{2J^2}\sqrt{(\omega-i\mu)^2-4J^2}.$$

Fourier transforming to the time domain, we find

$$\boxed{\begin{aligned} G^{\mathrm{R}}(\tau) &= -i\Theta(\tau)\, e^{-\mu\tau}\,\frac{J_1(2J\tau)}{J\tau}, \\ G^{\mathrm{A}}(\tau) &= i\Theta(-\tau)\, e^{\mu\tau}\,\frac{J_1(2J\tau)}{J\tau}. \end{aligned}}$$

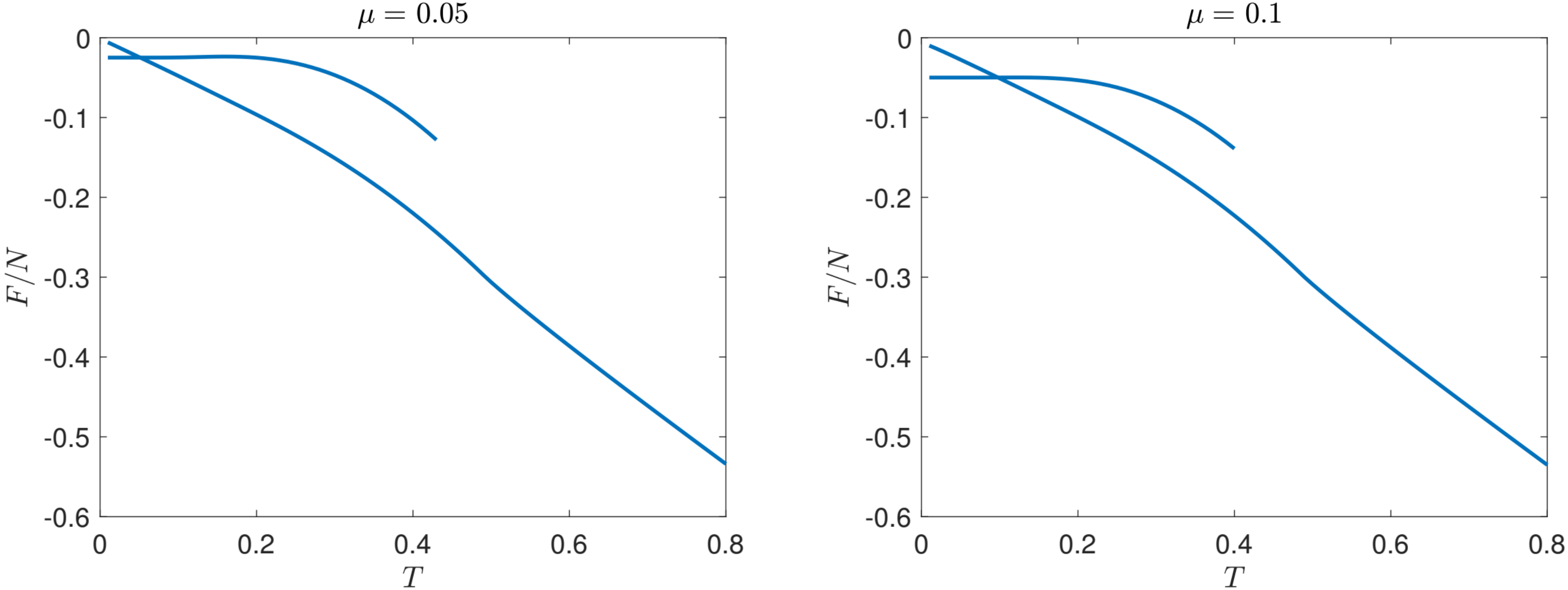


Excellent agreement with analytical results. $\boldsymbol{E_g(0) = 0}$ so quantum chaos ($\boldsymbol{q > 2}$), especially for $\boldsymbol{\mu \ll 1}$**,** alters qualitatively the way the system approaches the steady state.



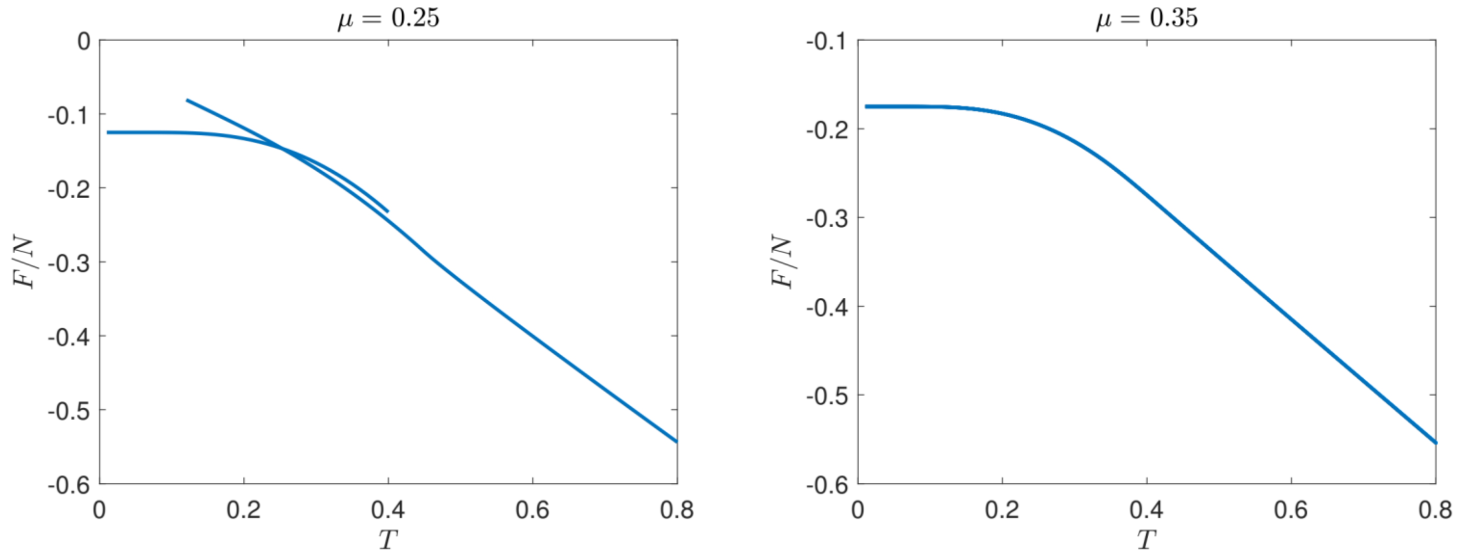
Dissipation driven relaxation $E_g = \mu$

Gravity dual of a dissipative SYK: Keldysh dS wormholes

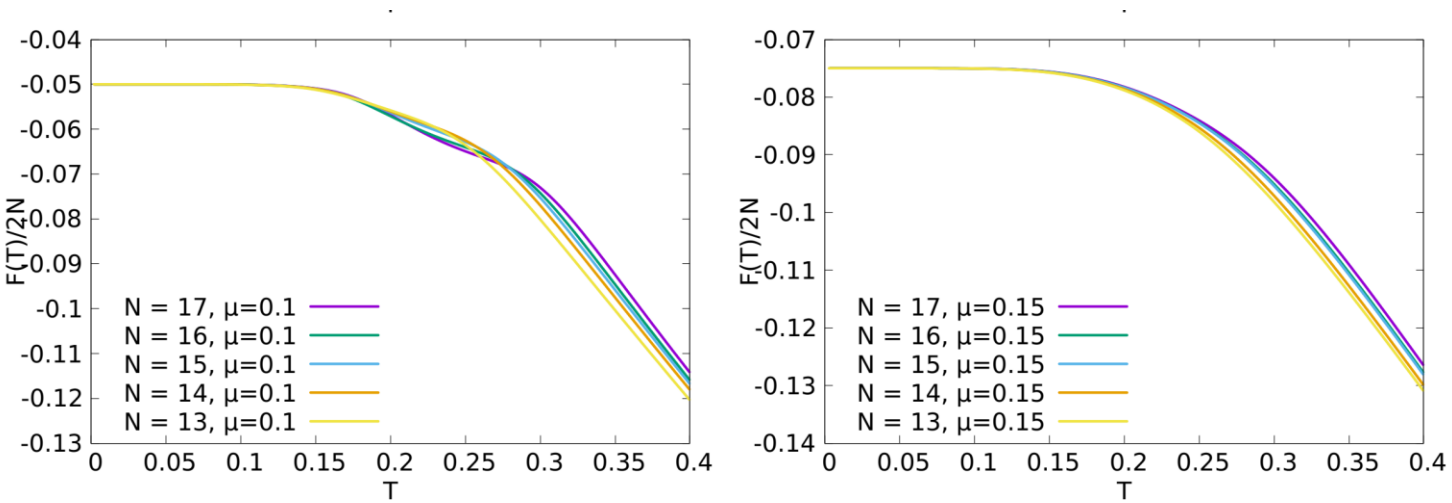


Free energy of the Euclidean SYK setting at finite T is similar to that of wormholes in near AdS_2

SD eqs.



Exact diagonalization



Good agreement with numerical results

Gravity dual cannot be traversable or Euclidean wormhole in near AdS_2 because the Schwarzian action is different

However, the difference is just the complexification of the couplings

$J \rightarrow iJ_L$, $J \rightarrow -iJ_R$

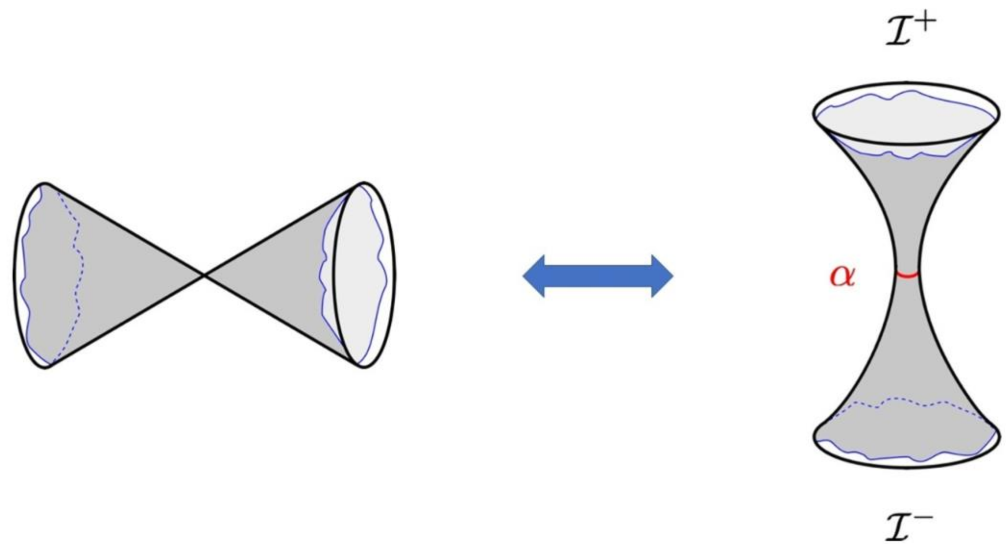


Figure 2. Under the analytic continuation $b \to i\alpha$, $J'_R \to -iJ$, $J'_L \to iJ$, the double trumpet of [21] is mapped to nearly global dS_2.

Cotler et al. JHEP 2020, 6, (2020), Maldacena et al., JHEP 2021, 1 (2021)

Double-trumpet configuration in a near de Sitter background in two dimensions (dS_2) in Lorentzian time

After a matter perturbation (μ in our case) we speculate that dS_2 is unstable to the formation of bra-ket wormholes

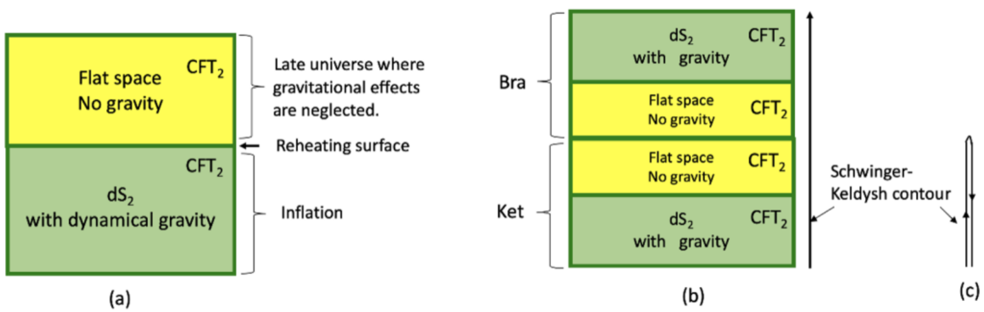


Figure 17. (a) We study a model which consists of de Sitter gravitational evolution followed by an evolution in flat space with no gravity. (b) When we consider expectation values, we join the bra and the ket following the Schwinger-Keldysh contour. In (c) we see a more traditional depiction of the Schwinger-Keldysh evolution. We go forwards in time to prepare the bra, then go backwards in time to prepare the ket.

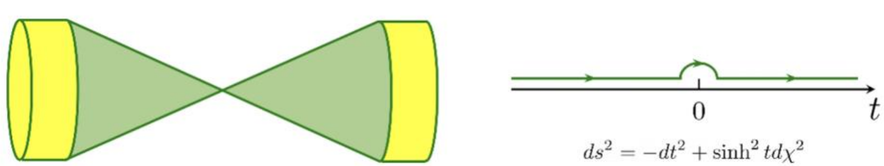


Figure 23. The geometry that appear in the π contour is very similar to the double cone considered in [31], except for an overall sign in the metric, which exchanges space and time and changes the sign of the curvature, taking the locally AdS_2 space to a locally dS_2 one geometry.

Maldacena et al, 2007.16091

Looking for gravity specialists to test this hypothesis!

Conclusions

Quantum ergodicity (quantum chaos) and random matrix behaviour seems to be distinctive features of quantum black holes and SYK models

SYK models reproduce most, if not all, universality classes of strongly interacting systems

Both traversable and Euclidean wormhole have SYK duals which facilitates the study or wormhole physics in condensed matter systems

Wormhole physics seems to play a role in the process of equilibration of strongly interacting quantum matter both isolated or in contact with an environment.

Thanks!