Black holes and quantum gravity

Seok Kim
(Seoul National University)

Extreme Universe Colloquium
Dec 16, 2022

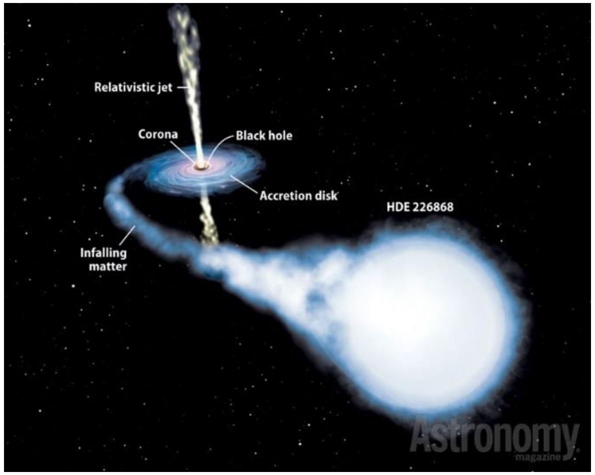
Black holes in the sky

Black holes are extreme objects in the universe.

Observed in various types/sizes:

- Early days (1950's ~ 1970's):

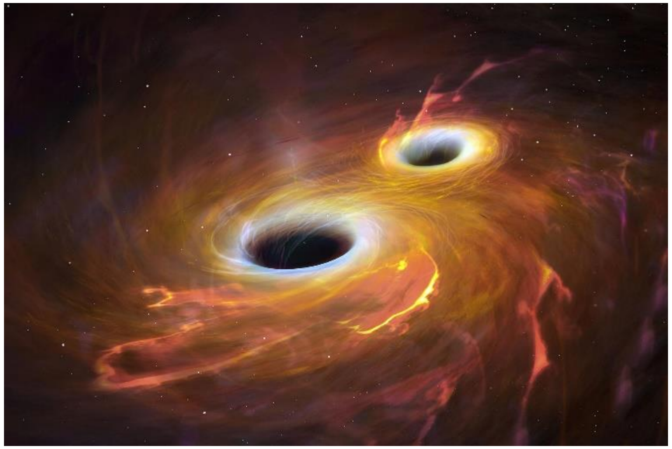
  indirect observations from radio waves or X-rays



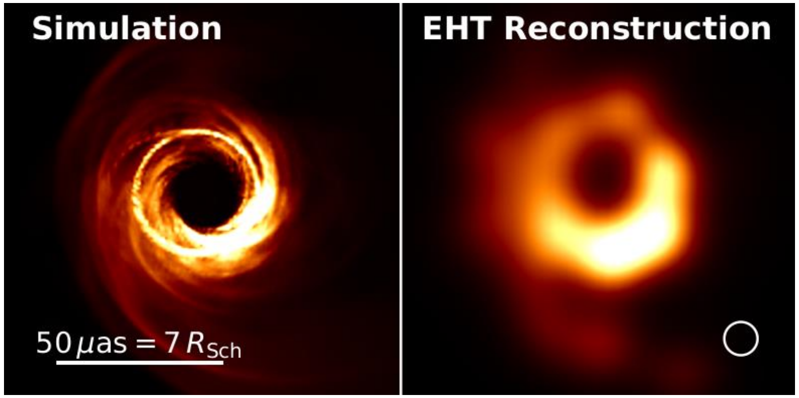
Cartoon image of Cygnus X-1 and HDE226868, a binary of a black hole at $M \approx 14.8 M_\odot$ & a star (1972)

- Black hole mergers (at $M \sim M_\odot$)

  from gravitational waves



- "Supermassive" at the galaxy centers ($M \sim 10^6 M_\odot$)

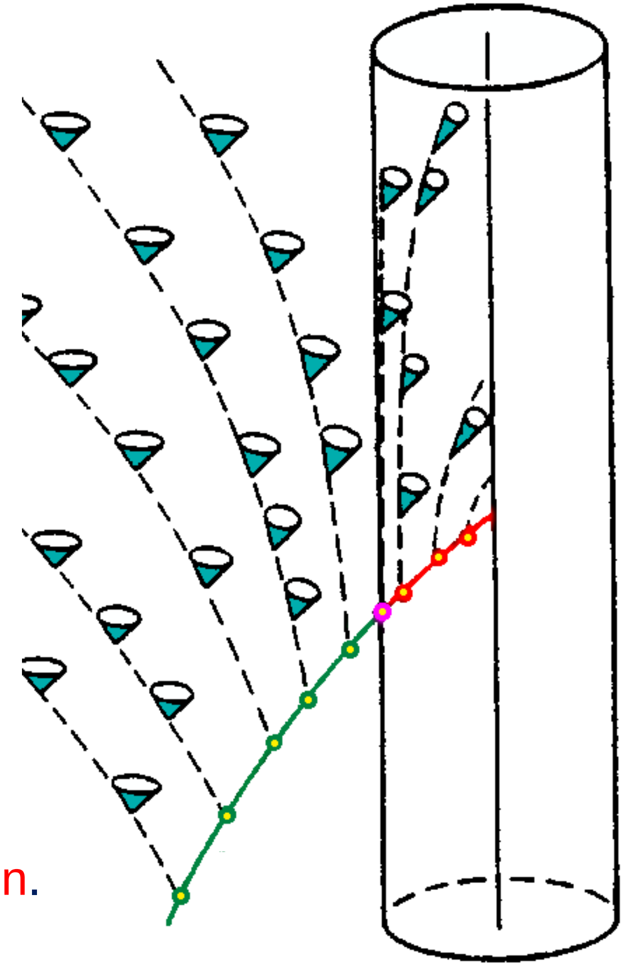


Black holes & event horizon

Today I will talk about the conceptual mysteries of black holes, especially how they can be a window to better understand foundational aspects of gravity.

Black holes are strange for many reasons, e.g.

- Have singularities
- Have “event horizons” → today’s focus



Curved spacetime bends particles & light:

- Big distortion of causal structure may forbid particles and lights from escaping a region.

We call the boundary surface if this region event horizon.

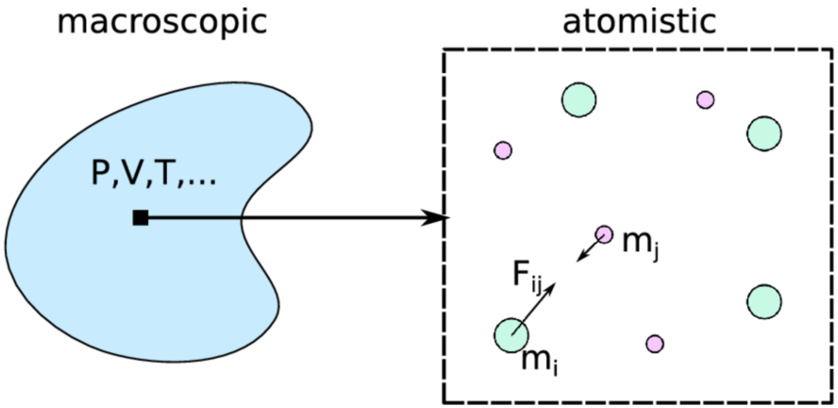
Missing information

Information behind the event horizon is unavailable to the outside observers.

- Only limited information available, such as the mass, charge, spin of the black hole

A sub-field of physics dealing with the inaccessible information: "statistical physics"

- Impractical to seek for all the atomic information of macroscopic systems.
- Limited information, like "energy" "temperature" "pressure" "volume" → thermodynamics



- The underlying "master quantity" is entropy: measures our ignorance about the system

$$S(E,V,\cdots) = \log(\#\ of\ states\ at\ fixed\ E,V,\cdots)$$

$$\frac{1}{T} = \frac{\partial S}{\partial E} \quad , \quad \text{heat} = \Delta Q = T\Delta S \quad , \quad \ldots\ldots$$

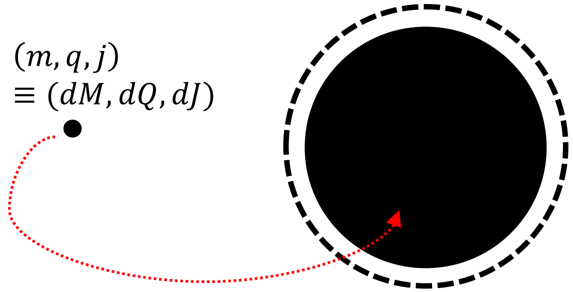
Information & thermodynamics of black holes

Surprisingly, the missing information of black holes also induces “thermal behaviors”

- 1st law: Slowly perturb BHs: behaves as if it absorbs “heat”

$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$$

κ: surface gravity at the event horizon
A: area of the even horizon

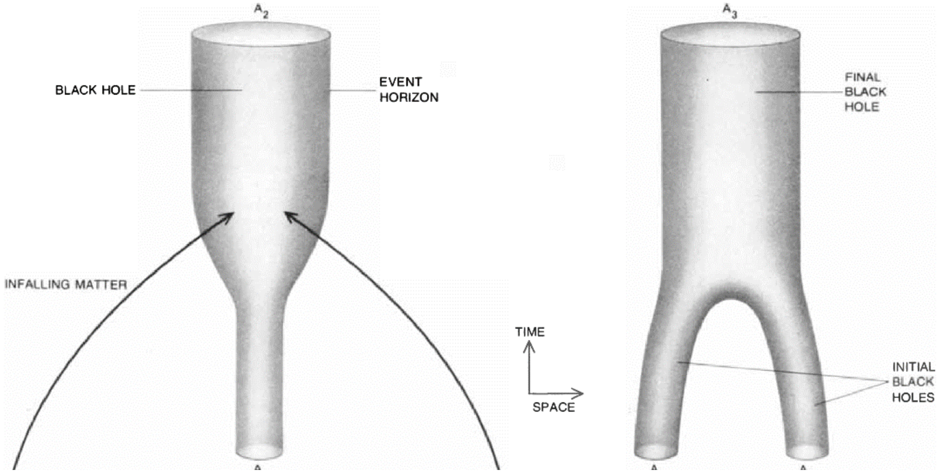


(m, q, j)
$\equiv (dM, dQ, dJ)$

$(M, Q, J) \rightarrow (M + dM, Q + dQ, J + dJ)$

- 2nd law: obeys “area law” [Hawking] (1971)

$$\frac{dA}{dt} \geq 0$$



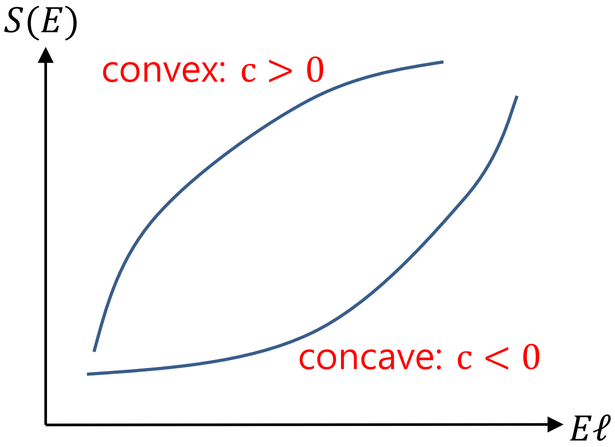
Hawking radiations strongly implies that this should be real thermodynamics, as well as precisely prescribing its entropy & temperature

$$\frac{c^2\kappa}{8\pi G} dA = TdS \quad \rightarrow \quad S = \frac{A\,c^3}{4G\,\hbar}$$ Bekenstein-Hawking entropy, $T = \frac{\kappa\,\hbar}{2\pi\,c}$ Hawking temperature

Exotic entropy of black holes

Black holes have negative specific heat. For instance, for Schwarzschild BH's,

- $D = 3+1$: $T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}} \approx 6.169 \times 10^{-8}\ \mathrm{K} \times \frac{M_\odot}{M}$
- $D \geq 3+1$: $GM \sim r^{D-3}$, $S \sim A/G \sim r^{D-2}/G \to S(M) \sim G^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}} \to \frac{dS(E)}{dE} = \frac{1}{T} \sim M^{1/(D-3)}$
- Why? Entropy grows too fast: $\frac{dS}{dE} > 0$ and $\frac{d^2S}{dE^2} > 0$.



We do occasionally encounter such systems.

- E.g. collapsing stars

But if we interpret this entropy as representing fundamental degrees of freedom of gravity behind the horizon, it raises rather puzzling implications.

- Unstable in canonical ensemble. Cannot be in equilibrium w/ a heat bath at fixed T.

Demands unusual structures of the Hilbert space of quantum gravity.

- Very fast growing entropy in the high energy regime (set by Planck scale).

Exotic entropy of quantum gravity

Fast-growing entropy at high E is unfamiliar in standard particle physics.

- Relativistic particles in D spatial dimension at high energy: On dimensional grounds ($c = 1$, $\hbar = 1$), one finds

$$S \propto VT^D \quad , \quad E \propto VT^{D+1} \quad \rightarrow \quad S(E,V) \propto V^{\frac{1}{D+1}} E^{\frac{D}{D+1}}$$

- $S \propto E^{\alpha}$ with $\alpha < 1$ is not too fast: $d^2S/dE^2 \sim \alpha(\alpha-1)E^{\alpha-2} \ < 0$

Familiar in quantum theory of gravity. For instance, string theory has many high energy degrees of freedom:

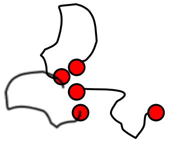
- Elementary strings: $S(E) \sim E/T_H$ at high E, where $(T_H)^2 \propto$ string tension.
- Due to ∞ tower of string oscillation modes: We call this “Hagedorn growth” [Hagedorn] (1965) [Sundborg] [Atick, Witten], which was originally found in the context of hadron physics.

- However, this perturbative entropy <u>doesn’t grow fast enough</u>: $S_{BH} \propto M^{\frac{D-2}{D-3}} \gg M$.
  - Non-perturbative degrees of freedom should play roles. (→ next slide)

Statistical entropy of extremal black holes

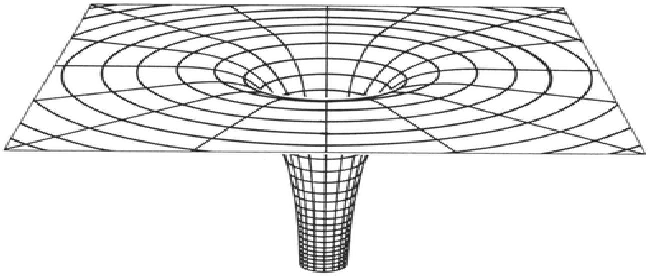
To form such black holes, non-perturbative objects like “D-branes” are needed.

- New bound states of D-branes [Polchinski] (1995) & open strings → faster growth.



Do honest statistical mechanics of
D-brane quantum mechanics or QFT

strong coupling →



Agrees w/ emergent gravitational picture?

The statistical picture of area law was justified with extremal “BPS” black holes:

- Carry electric charge & saturate the bound $M \geq \# \, Q$.
- in $D = 4 + 1$: $S = 2\pi Q^{3/2}$ [Strominger, Vafa] (1996) ……
- in $D = 3 + 1$: $S = 2\pi Q^2$ [Maldacena, Strominger, Witten] (1997) [Vafa] (1997) ……

Since Q is M, similar growth as Schwarzschild BH: $S_{BH} \propto M^{(D-2)/(D-3)}$.

- Very fast growth, due to non-perturbative bound states.
- The system is unstable in grand canonical ensemble. (negative susceptibility)

Black hole entropy & quantum gravity

The fast growth of the BH entropy is intimately related to the fundamental structure of the Hilbert space of quantum gravity, as least in string theory models.

- Historically, introducing so many extra d.o.f. to Einstein's relativity as strings was to avoid very technical inconsistencies of quantum gravity (UV divergences).
- Now we may also understand it as a constraint from the black hole thermodynamics.

It will be interesting to review how other models suggested for quantum gravity view black hole thermodynamics, focusing on the number of fundamental d.o.f. they have.

For instance,

- Higher spin gravities: Have ∞-ly many fields, but of rather different natures from strings.
- Loop quantum gravity…?
- Asymptotically free quantum gravity…? [Weinberg], etc.

Currently I don't have good enough insights to answer these questions.

Shortcomings

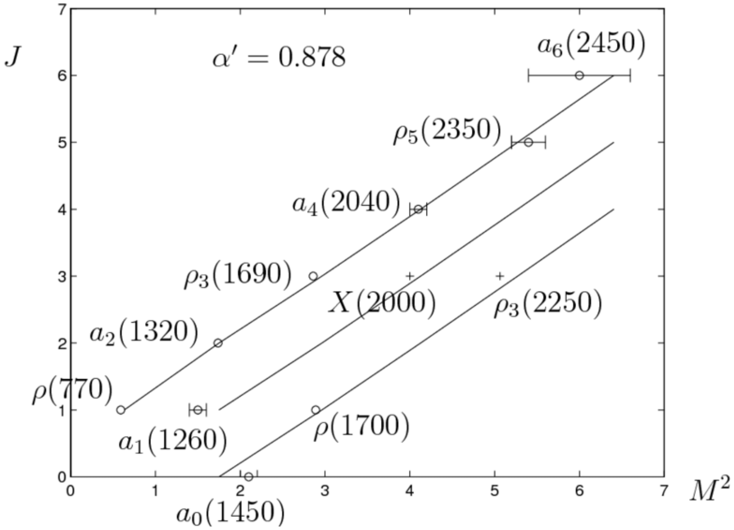
Instability of black holes (in canonical/grand canonical ensembles) makes some of the very interesting thermodynamic questions irrelevant or ill-defined.

- Sometimes related to other fundamental questions of quantum gravity.

Related to subtleties of string theory & QG.

- New high T phase of gravity after a phase transition?
- Similar to QCD: tower of hadrons vs. quark-gluon

  → QCD at high T is in "quark-gluon plasma phase"

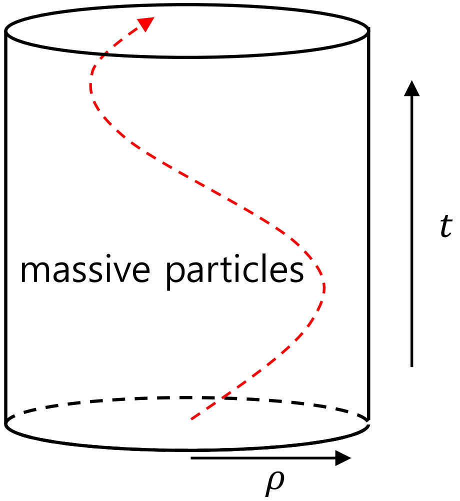


One more general subtlety of gravity at fixed T.

- Needs "finite volume" or IR regulators. (Extensive quantities ∝ "volume")
- Cannot put gravity in an artificial box: Everything is subject to equivalence principle.

Now, I will present a simple model in which gravity is put in a "box" or a "trap."

Anti de Sitter (AdS) spacetime



A theoretically consistent setup for the “finite box”

- Put gravity in AdS: [Hawking, Page] (1983)

$$ds^2_{D+1} = d\rho^2 - \cosh^2\frac{\rho}{\ell}\,dt^2 + \ell^2\sinh^2\frac{\rho}{\ell}\,ds^2(S^{D-1})$$

- Technically, the above metric is for the “global AdS” spacetime.
- Confines to $\rho = 0$: $\Phi \approx -g_{tt}(\rho)/2$. Massive particles (also black holes) cannot escape.
- Renders many thermal questions of QG well defined.

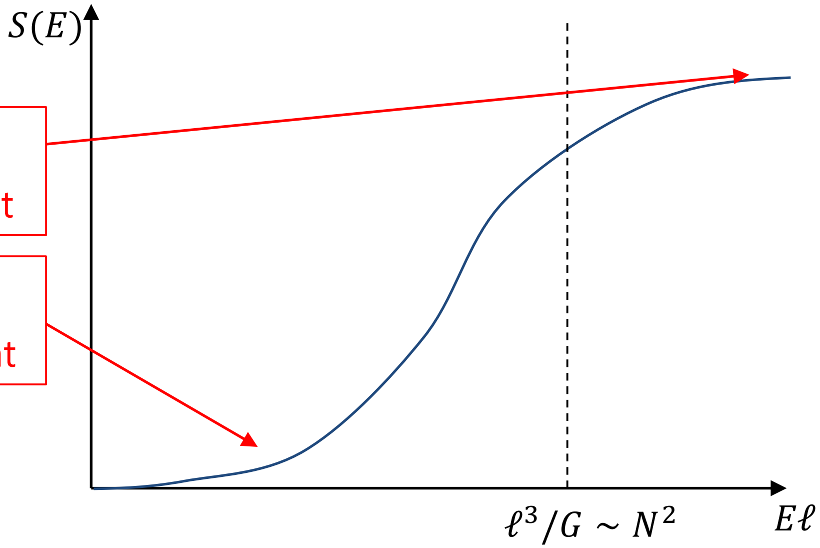
- Incidentally, we know a microscopic description of such quantum gravity.
  - We call it “AdS/CFT correspondence” [Maldacena] (1997) ……
  - QFT at the boundary $S^{D-1} \times R$ ‘holographically’ describes the QG inside.
  - In simple examples, U(N) gauge theories at $N \gg 1$: large number of “gluons”

Black holes in AdS

Since people like 4d QFTs, AdS$_5$ is best studied.

Schwarzschild black holes in AdS$_{d(=5)}$:

[Hawking, Page]



$\propto E^{\frac{d-1}{d}}$ at $E\ell \gg N^2 \leftrightarrow R_{Sch} \gg \ell$:
convex curve, positive specific heat

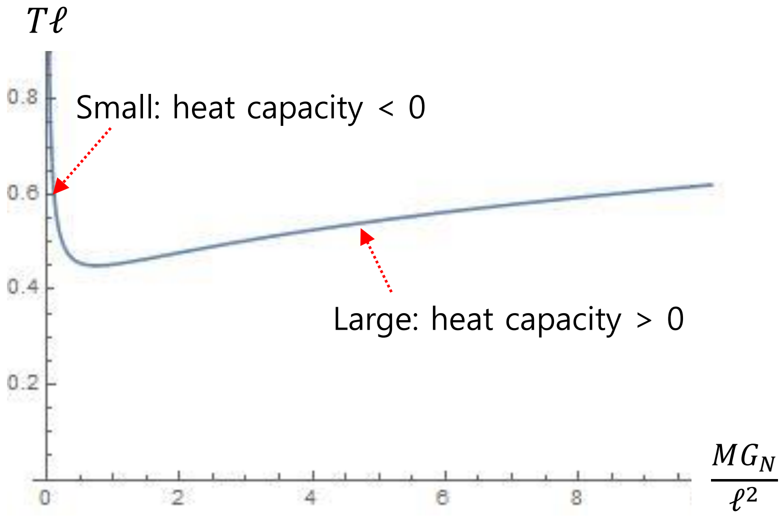
$\propto E^{\frac{d-2}{d-3}}$ at $E\ell \ll N^2 \leftrightarrow R_{Sch} \ll \ell$:
concave curve, negative specific heat

(Extensive quantities of BH's are proportional to $\ell^3/G \ \propto N^2$.)

T-E curve:

$$T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



Small: heat capacity < 0

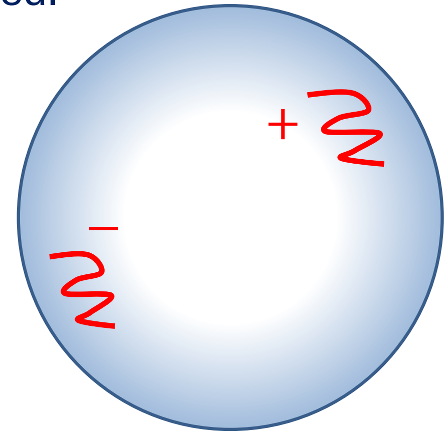
Large: heat capacity > 0

Key features:

- Small BH: IR regulated BH's in flat spacetime
- Large BH: determines thermodynamics at fixed T

Phase transition & QFT

- Large N gauge theory at finite temperature can be in different phases.
- Conformal field theory (CFT) on $S_r^3 \times R$, i.e. “compact” space:
  - low T: System confines. “Net charge 0” → forbids individual gluon excitations.
  - high T: System deconfines. Locally, individual gluons virtually liberated.



- We expect a phase transition at $T_c \sim 1/r$:

confinement-deconfinement phase transition

(Somewhat different in some details from the usual confinement of QCD)

- Hawking & Page (1983) found a similar transition for quantum gravity in AdS:
  - low T: a phase dictated by thermal graviton excitations, without black holes
  - high T: the large black hole is thermally nucleated.

- So one expects the two phase structures to be dual to each other. [Witten] (1998)

deconfined plasma of gluons ↔ “large black hole ~ high T phase of QG”

Challenges

For QFT to describe semi-classical gravity AdS, two conditions have to be met.

- Large # of degrees of freedom (large N gauge theory).
- strong coupling: Only gluons at weak coupling. Gravity emergent at strong coupling

Studying strong coupling QFT is very difficult, in general.

Need rigorously solvable sectors in simpler models, without losing physics.

“Supersymmetric” AdS/CFT models

- Supersymmetry is a hypothetical symmetry exchanging bosons and fermions.
- Has been an essential technical ingredient of the “superstring theories.”

Today, merely view them as “solvable” models which keep nontrivial physics.

- There exist black holes in AdS preserving SUSY.
- Some of their physics is very similar to that of Schwarzschild BH’s in AdS.
- Can try strong-coupling calculus: expect SUSY non-renormalization. [Seiberg]

A thermal partition function

- In strong coupling quantum mechanics or QFT, energy levels $E_n(g)$ depend on couplings, and thus are very difficult to compute in general.
- For supersymmetric models, Witten showed that certain subset of states (called BPS states) have coupling-independent energy levels.

- In our AdS gravity, such states have their energies given by

$$E\ell = \frac{3}{2}R + J_1 + J_2$$

- $J_{1,2}$ are two angular momenta, and R is an electric charge.
- This sector hosts spinning charged black holes.

- The "thermal partition function" which captures these states only:

$$Z(\omega_1, \omega_2) = \mathrm{Tr}\left[(-1)^F e^{-\omega_1(J_1+R/2)-\omega_2(J_2+R/2)}\right]$$

[Kinney, Maldacena, Minwalla, Raju] [Romelsberger] (2005)

- Very roughly speaking, $\omega_1 + \omega_2$ plays a role similar to "inverse temperature" and $\omega_1 - \omega_2$ the "angular velocity" of the black hole.

High temperature limit & black holes

This partition function has been recently studied extensively to better understand black holes. [Choi, J. Kim, SK, Nahmgoong] [Benini, Milan] [Cabo-Bizet, Cassani, Martelli, Murthy] (2018)

The “free energy” of this partition function (in the large N limit)

$$\log Z \sim \frac{2(3c-2a)(2\pi i+\omega_1+\omega_2)^3}{27\omega_1\omega_2}$$

- The “central charges” c, a represent #(d.o.f.), satisfying $c \sim a \propto N^2$ (large N gluons)

- Its Legendre transformation agrees with the entropy of the corresponding black holes.
- The general formula is complicated, but for very large charges,

$$S = \sqrt{3}\pi\left[2(3c-2a)\left(J_1+\frac{R}{2}\right)\left(J_2+\frac{R}{2}\right)\right]^{\frac{1}{3}}$$

- Further studies clarify that this describes deconfined gluons, realizing Witten’s expectation.
- The free energy scales like N^2. Liberated gluons all contribute to the free energy, signaling that the system deconfines.

Lessons for quantum gravity

We have seen that at “high T” or large charge/energy, N^2 liberated gluons all contribute to the free energy, rather than being confined.

It strongly supports that quantum gravity or string theory should undergo a phase transition at high temperature.

- Indeed one can show the existence of a 1st order phase transition.
- All the objects in the “traditional” description of (quantum) gravity at low T, such as “gravitons” and even “strings” “D-branes” etc., should lose their meanings.

In a sense, the “gluons” which form gauge-invariant strings break up into “string bits.”

Hopefully, can these shed more lights oo how our Universe should behave like in extreme conditions, such as when it was very hot at the early era…?

- I don’t have good intuitions now, but would like to better understand.

Conclusion & future directions

- Exotic black hole thermodynamics:
  - Reflects high energy d.o.f. of quantum gravity.
  - To me, it seems this almost forces QG to be something like string theory.
- “Normal” black hole thermodynamics (say in AdS):
  - Probes new phases of quantum gravity at high T.

- Studying small black holes in AdS:
  - Revisiting 2.5 decade-long studies on black hole microstates in flat space.
  - Without any ad hoc assumptions (such as D-branes, etc.)
- Large black holes in AdS:
  - General behaviors of quantum gravity at high temperature…?

- It is obvious that novel aspects of black holes provide good windows to better understand the structure of quantum gravity. It will be interesting to see in the future how “useful” they could be in better understanding the gravity around us.