Random Clifford circuits for quantum coding against Pauli noise using a tensor-network decoder

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Main message: Random quantum circuits can protect qubits from noise even when they are highly constrainted (1D log-depth Clifford circuits).

Quantum computers

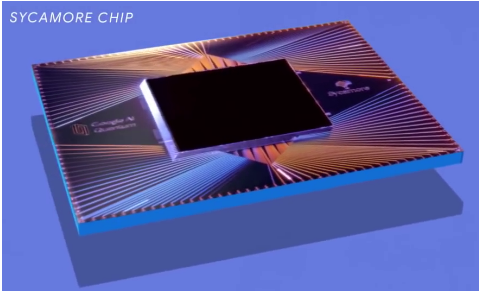


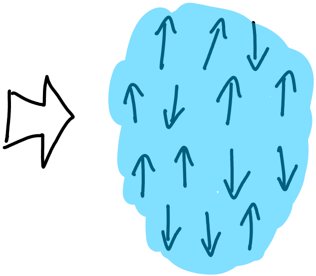
Figure: Current quantum computers: IBM's Eagle (left) and Google's Sycamore (right)

- For practical applications of quantum computers, we first need to overcome noise.

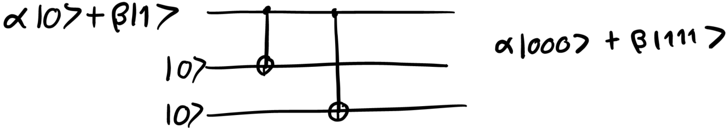
Quantum error correction (QEC)

- Purpose: protect quantum information against noise.
- **Quantum error correcting codes:** A small number of protected qubits (**logical qubits**) are encoded into the collective state of many quantum particles (**physical qubits**).

$$\alpha|0\rangle + \beta|1\rangle$$



Simple example: Repetition code



- Encode a single qubit into three physical qubits

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle \tag{1}$$

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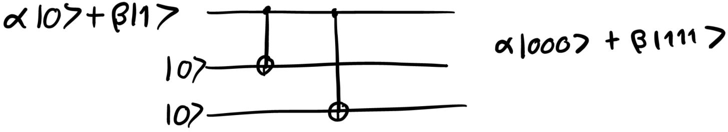

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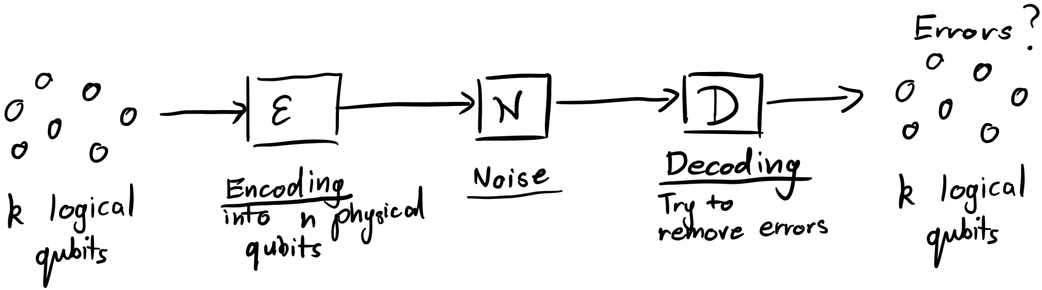
- A single X error on any qubit can be corrected. E.g. X error on first qubit:

$$\alpha|000\rangle + \beta|111\rangle \rightarrow \alpha|100\rangle + \beta|011\rangle \tag{2}$$

- Measure the parity of neighbouring qubits, $M_1 := Z_1 Z_2$, $M_2 := Z_2 Z_3$ to identify the error.

Simple picture of QEC



- What encoding gives us a good rate $r = k/n$ with logical error probability tending to 0 as $n \to \infty$?
- Highest achievable rate for a given $\mathcal{N}$ as $n \to \infty$ is called the capacity.

Random encoding

- Assume noise $\mathcal{N}$ is an i.i.d. Pauli channel:

$$\mathcal{N}(\rho) = p_I \rho + p_X X \rho X + p_Y Y \rho Y + p_Z Z \rho Z\,, \tag{3}$$

with $p_I + p_X + p_Y + p_Z = 1$. We mainly consider the case of depolarising noise $p_X = p_Y = p_Z = p/3$, $p_I = (1 - p)$.

[1] Gottesman 1997, Wilde 2013

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- It is known[1] that using random Clifford Unitaries for encoding, a rate called the hashing bound can be achieved

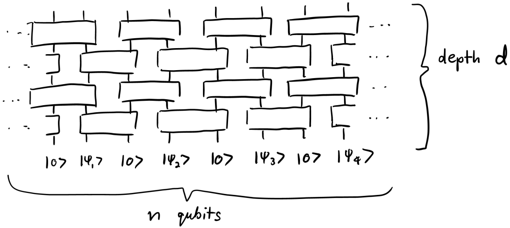
$$r = 1 - H(\vec{p}) \,, \tag{4}$$

  with $H(\vec{p})$ the Shannon entropy of $\vec{p} = (p_I, p_X, p_Y, p_Z)$.
- No constraints on locality (all-to-all connectivity), no efficient decoding procedure known.

[1] Gottesman 1997, Wilde 2013

Low-depth random encoding

- Even low-depth Clifford circuits can protect quantum information.
  - (Brown, Fawzi 2013) $\mathcal{O}(\log^3(n))$ depth Clifford circuits can achieve nearly the same as fully random Clifford unitary.
    - Requires all-to-all connectivity, no efficient decoding.
  - (Gullans et al. PRX 2021) $\mathcal{O}(\log(n))$ depth with 1D connectivity sufficient for erasure noise.
    - Erasure noise implies error locations known (unrealistic).



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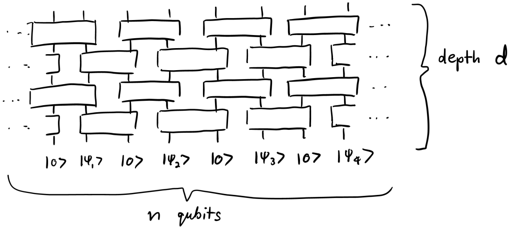
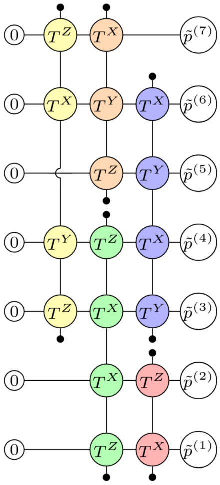


- **Our result:** Codes defined by $\mathcal{O}(\log(n))$ encoding circuits in 1D can be decoded efficiently achieve a rate close to the hashing bound for stochastic noise.

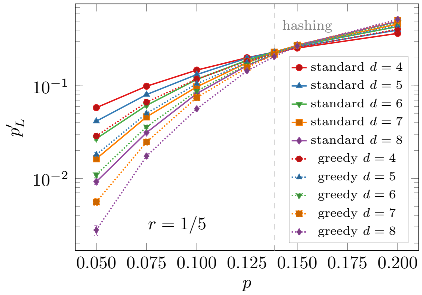
Decoding

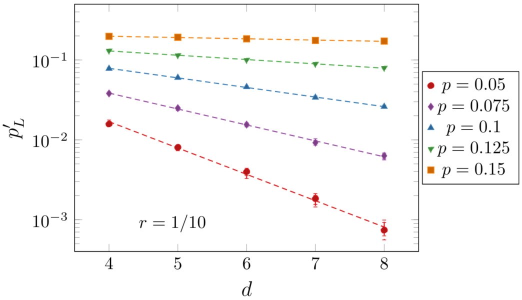
- For given encoding circuit U, measure Pauli check operators of (weight $\sim d$). Outcomes called the syndrome s.
- Classical processing of the syndrome to determine the most likely error. We do this using a tensor network contraction, that is efficient for 1D log-depth encoding circuits.



Numerics

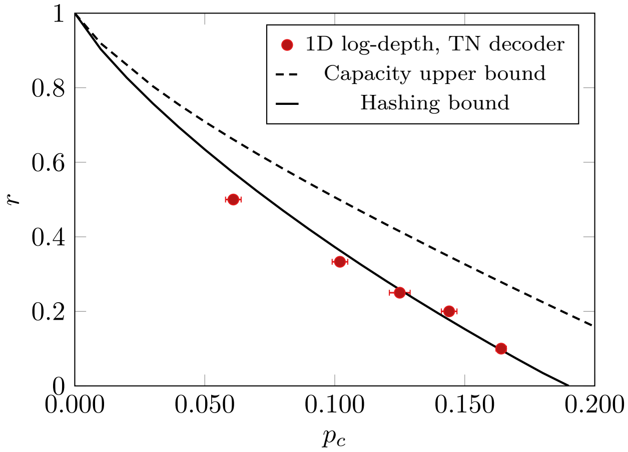
- Fix the rate $r = k/n$ and determine the maximum depolarising noise strength p that can be tolerated with this method (the threshold).
- Below the hashing bound, error rate of each logical qubit appears to decay exponentially with depth.





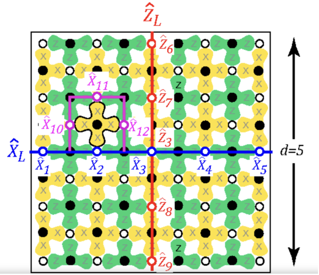
Hashing bound

- Achievable rate for 1D log-depth is close to the hashing bound for depolarising noise with a variety of noise strengths.



Potential applications

- These results show that random encoding may have practical applications:
  - High rate (hashing bound)
  - Locality in 1D
  - Efficient decoding
- Has advantages over other error correction schemes:
  - Much higher rate than the surface code.



Summary

- Quantum information can be protected against noise using quantum error correction.
- We studied the performance of log-depth Clifford circuits for quantum error correction against Pauli noise.
- We showed that with $\mathcal{O}(\log(n))$-depth Clifford circuits in 1D we can achieve the same rate as fully random Clifford encodings with efficient decoding[2].
- Future work:
  - Realistic situation when encoding and decoding are also noisy.
  - Generalisation to higher spatial dimensions.

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[2] AD, Y. Nakata, S. Tamiya, H. Yamasaki, arXiv:2212.05071

Question: Can this be regarded as a physical model for something?