Superradiant instability of rotating black strings

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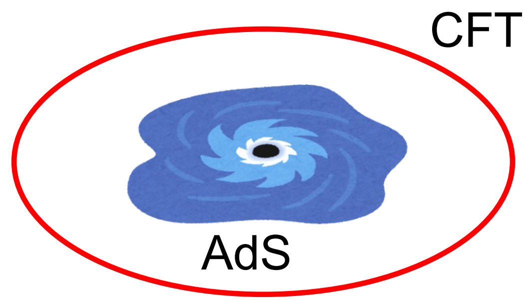
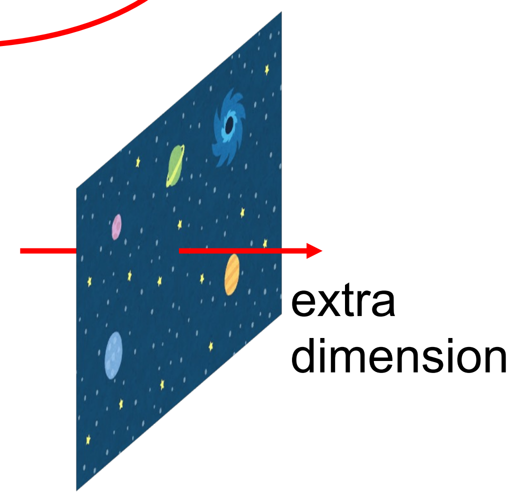
“Gregory-Laflamme encounters Superradiance” arxiv:2211.02672
“Gregory-Laflamme and Superradiance encounter Black Resonator Strings” arxiv:2212.01400
“Superradiance and black resonator strings encounter helical black strings” arxiv:2301.????

Why general relativety (GR) in higher dimensitonal spacetimes?

- String theory
- AdS/CFT
- Brane world





- Better understanding of general relativity.

Plan of this talk

- What is the black string?
- Gregory-Laflamme instability
- Superradiance and superradiant instability
- Superradiant instability of rotating black strings

Review part

What is black string?

Schwarzshild black hole in (3+1)-d

Spherically symmetric static black hole in (3+1)-d.

$$ds^2 = -\left(1 - \frac{r_h}{r}\right) dt^2 + \left(1 - \frac{r_h}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Schwarzshild black hole

r=rh : event horizon

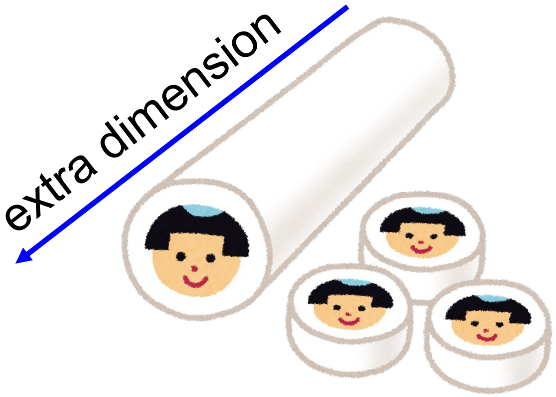
Solution of Einstein equation

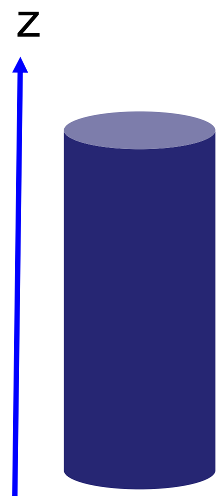
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \qquad \mu, \nu = 0, 1, 2, 3$$

In this talk, we only consider spacetime with cosmological constant = 0.

Schwarzchild black string in 5d

$$ds^2 = ds^2_{\rm Sch} + dz^2$$





Exact solution of
the Einstein equation in 5-d.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \qquad \mu, \nu = 0, 1, 2, 3, 4$$

z-constant slice of black string
= Sch black hole

Gregory-Laflamme instability

Stability analysis

$$ds^2 = ds^2_{\mathrm{Sch}} + dz^2 + \underline{h_{\mu\nu} dx^\mu dx^\nu}$$

Perturbation

$$h_{\mu\nu} = h_{\mu\nu}(r) e^{-i\omega t + ikz}$$

Spherically symmetric perturbation

Taking the first order in h

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

→ Perturbation equation

Perturbation equations reduce to "Schrodinger equation”

Hovdebo & Myers 06

$$\left[-\frac{d^2}{dx^2}+V(r)\right]\Psi=\omega^2\Psi \qquad \begin{cases}\Psi(r)\equiv\left(k^2r^2+\frac{r_h}{r}\right)h_{zz}(r)\\ dx=\left(1-\frac{r_h}{r}\right)^{-1}dr\end{cases}$$

$$V(r)=\frac{(r-1)(k^6r^9+6k^4r^7-9k^4r^6-12k^2r^4+9k^2r^3+1)}{r^4(k^2r^3+1)^2} \qquad (r_h=1)$$

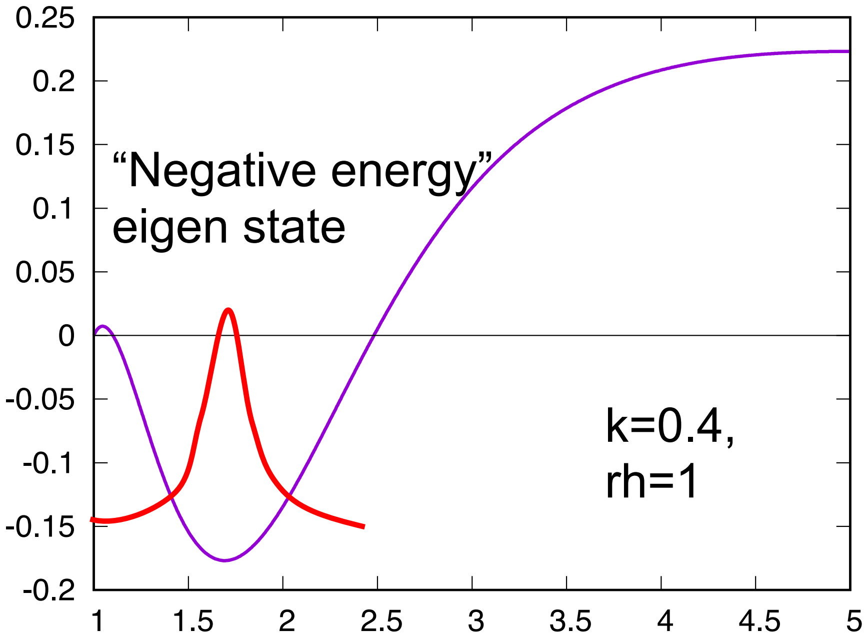
ω^2 = Energy eigenvalue

All other components of $h_{\mu\nu}$ are determined from the single master variable Ψ .

Gregory–Laflamme instability

Gregory&Laflamme 93

$$\left[-\frac{d^2}{dx^2}+V(r)\right]\Psi=\omega^2\Psi$$



“Negative energy”

↓

ω is pure imaginary.

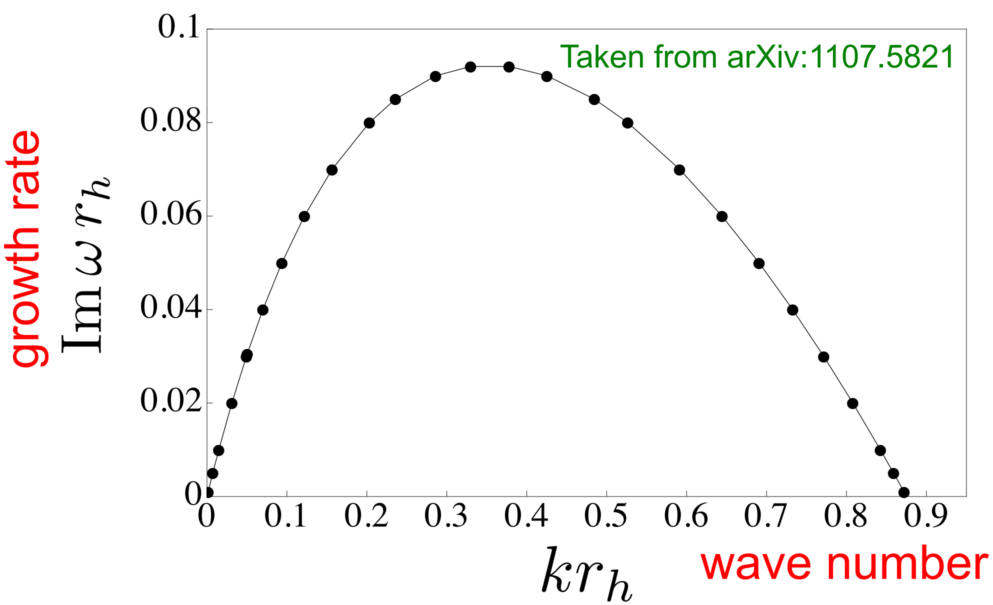
↓

Instability

$$h_{\mu\nu}=h_{\mu\nu}(r)e^{-i\omega t+ikz}$$

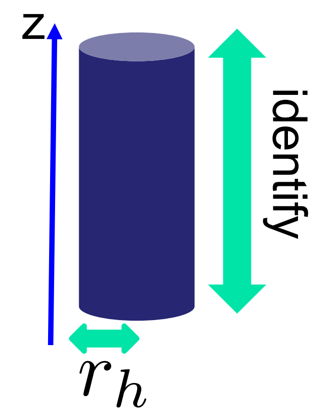
Growth rate

$$h_{\mu\nu} = h_{\mu\nu}(r)e^{-i\omega t+ikz}$$



When z is compactified with scale L

$$z \sim z + L$$



$$k = \frac{2\pi}{L} n \quad (n = 0, \pm 1, \pm 2, \cdots)$$

Black string is unstable for

$$k\, r_h < 0.88$$

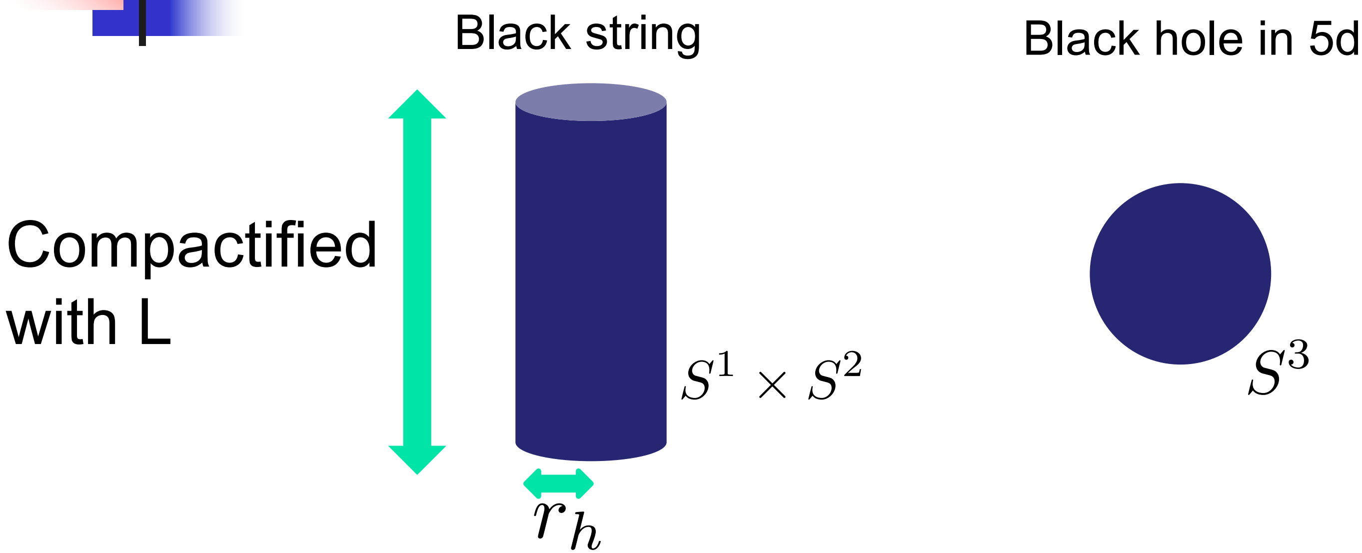
Long wavelength instability

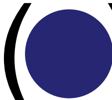
Unstable for

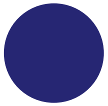
$$r_h < \frac{0.88}{2\pi} L$$

Thin black string is unstable.

Entropic argument



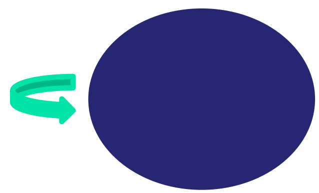
S() < S() for $r_h \lesssim L$

Phase transition from  to  . → Gregory-Laflamme instability

Superradiance and Superradiant instability

Rotating black holes

Kerr black hole (4-dim)

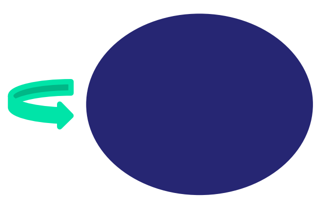


M : Mass
J : Angular momentum

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

Kerr 63

Myers-Perry black hole (D-dim)



M : Mass
J1, J2, …, Jn : Angular momenta

$n = [(D-1)/2]$ ← Gauss symbol

$$ds^2 = -dt^2 + \frac{\mu r}{\Pi F}\left(dt + \sum_{i=1}^{n} a_i\,\mu_i^2\,d\phi_i\right)^2 + \frac{\Pi F}{\Pi - \mu r}dr^2 + \sum_{i=1}^{n}(r^2+a_i^2)\left(d\mu_i^2 + \mu_i^2\,d\phi_i{}^2\right) + r^2\,d\alpha^2$$

Myers-Perry 86

Wave in Kerr

$$\Box \Phi = 0$$ in Kerr spacetime.

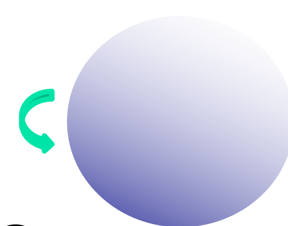
Separation of variable: $\Phi(t,r,\theta,\phi) \sim \Psi(r) S(\theta) e^{-i\omega t + im\phi}$

$$\Rightarrow \left[-\frac{d^2}{dr_*^2} + V(r) \right] \Psi = (\omega - m\Omega(r))^2 \Psi$$

Tortoise coordinate:

$$dr_* = \frac{r^2 + a^2}{\Delta} dr$$

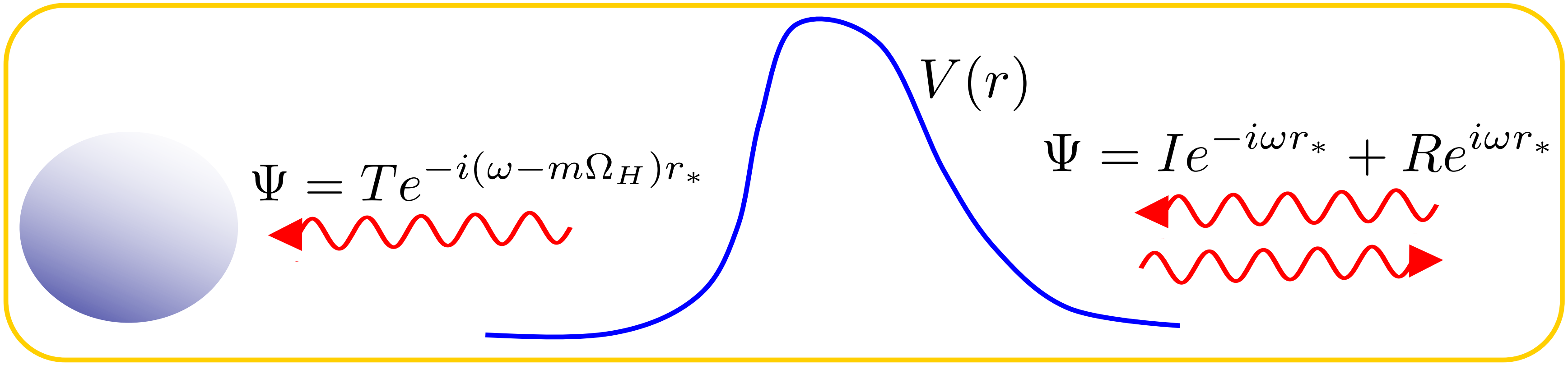
$$\begin{cases} V(r) \to 0 & \text{at horizon and infinity} \\ \Omega(r) \to 0 & \text{at infinity} \\ \Omega(r) \to \Omega_H & \text{at horizon} \end{cases}$$



Ω_H : angular velocity

Superradiance

$$\left[-\frac{d^2}{dr_*^2}+V(r)\right]\Psi=(\omega-m\Omega(r))^2\Psi$$



Wronskian is conserved

$$W=\Psi\frac{d\Psi^*}{dr_*}-\Psi^*\frac{d\Psi}{dr_*} \Rightarrow \frac{dW}{dr_*}=0$$

Estimate at horizon: $W=2i(\omega-m\Omega_H)|T|^2$

Estimate at infinity : $W=2i\omega(|I|^2-|R|^2)$

$$\Rightarrow \left(1-\frac{m\Omega}{\omega}\right)|T|^2=|I|^2-|R|^2$$

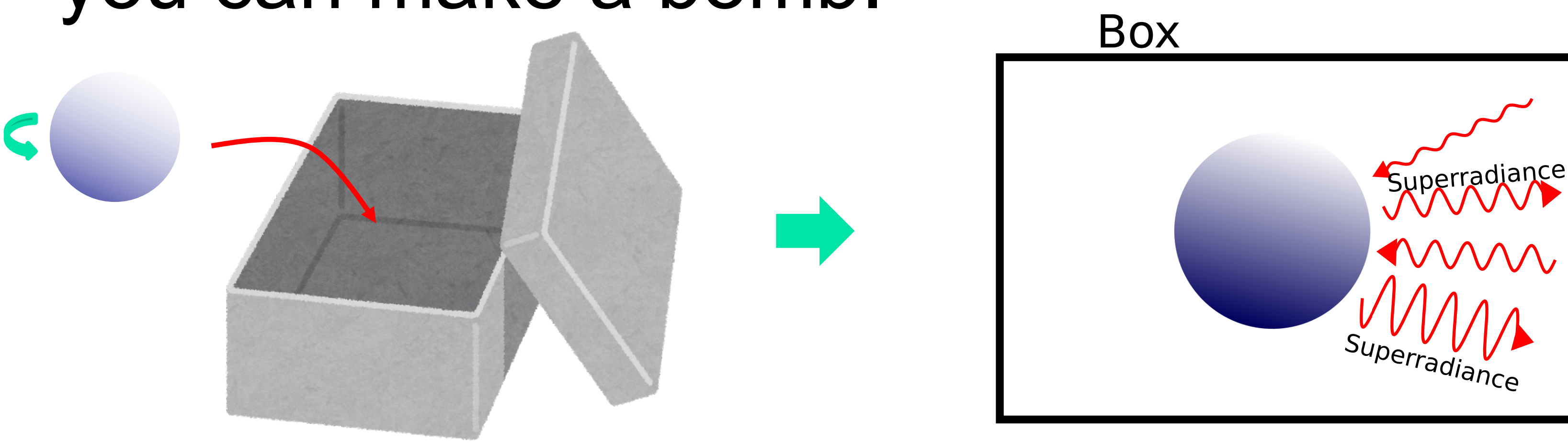
Reflected wave is amplified for

$$0<\omega<m\Omega_H$$

Black hole bomb (ブラックホール爆弾)

Press&Teukolsky 72, Cardoso, Dias, Lemos &Yoshida 04

If you have a Kerr black hole,
you can make a bomb.



Instability caused by the repetition of superradiance
= Black hole bomb or Superradiant instability

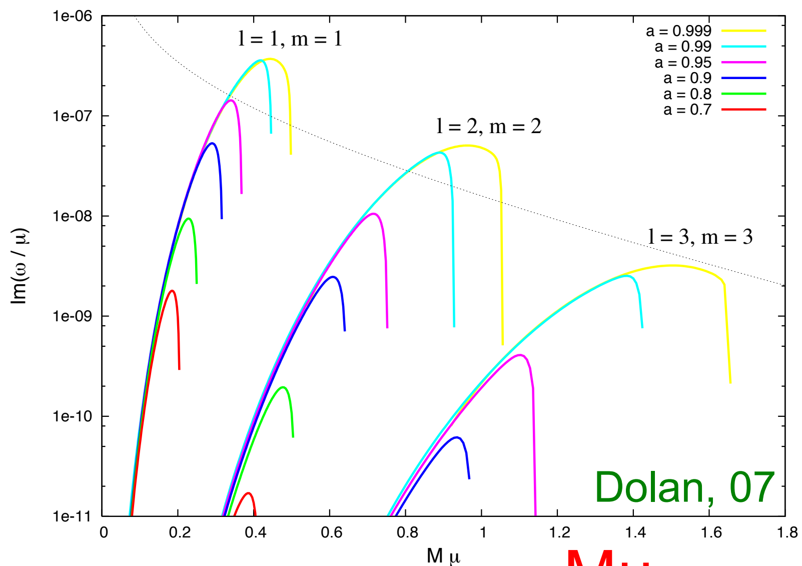
Massive field in Kerr spacetime

$$\Box\Phi = \mu^2\Phi$$

The mass gives
an effective potential barrier at infinity.

➡ Superradiant instability Detweiler, 80

growth rate



Mμ

$$\mathrm{Im}(\omega/\mu) \approx 10^{-7} e^{-1.84M\mu}/(M\mu),$$

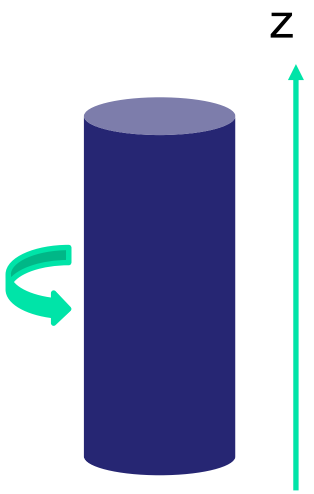
M: Mass of Kerr

Perturbation of rotating black strings

Rotating black string

For (3+1+1)-d

$$ds^2 = ds^2_{\text{Kerr}} + dz^2$$



For (d+1+1)-d

$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

Superradiant instability of rotating black string

$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

Perturbation in this geometry

➡ Massive field in $ds^2_{\text{Myers-Perry}}$ from KK-mode.

➡ Superradinat instability?

Scalar field perturbation

For the **scalar field** perturbation, $\Box\phi = 0$

4d Kerr + 1d: Unstable

Myers-Perry + 1d: No evidence of instability

Cardoso, Lemos 05, Cardoso, Yoshida 05

See also Marolf Palmer 04.

Centrifugal force potential $\dfrac{1}{r^2}$

Gravitational potential $-\dfrac{1}{r^{D-3}}$

There were no study of gravitational and Maxwell superradiant instability of rotating black string.

Gravitational perturbation of Myers-Perry black string

"Gregory-Laflamme encounters Superradiance" arxiv:2211.02672
"Gregory-Laflamme and Superradiance encounter Black Resonator Strings" arxiv:2212.01400
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Myers-Perry-string with equal angular momenta in 6D

$$ds^2 = ds^2_{\text{Myers-Perry}} + dz^2$$

$$ds^2_{\text{Myers-Perry}} = -f(r)d\tau^2 + \frac{dr^2}{g(r)} + \frac{r^2}{4}\{\sigma_1^2 + \sigma_2^2 + \beta(r)(\sigma_3 + 2\Omega(r)d\tau)^2\}$$

Invariant 1-forms of SU(2)

$$\sigma_1 = -\sin\chi d\theta + \cos\chi \sin\theta d\phi \ ,$$
$$\sigma_2 = \cos\chi d\theta + \sin\chi \sin\theta d\phi \ ,$$
$$\sigma_3 = d\chi + \cos\theta d\phi \ .$$

$$g(r) = 1 - \frac{2\mu}{r^2} + \frac{2a^2\mu}{r^4} \ , \quad \beta(r) = 1 + \frac{2a^2\mu}{r^4} \ ,$$
$$\Omega(r) = \Omega_H - \frac{2\mu a}{r^4 + 2a^2\mu} \ , \quad f(r) = \frac{g(r)}{\beta(r)} \ .$$

Symmetry of this spacetime:

$$SU(2) \times U(1) \times R_z \times R_t$$

χ-translation z-translation

➡ Stability analysis is simpler than that of Kerr-black string.

$$\sigma_1 = -\sin\chi \mathrm{d}\theta + \cos\chi \sin\theta \mathrm{d}\phi \ ,$$
$$\sigma_2 = \cos\chi \mathrm{d}\theta + \sin\chi \sin\theta \mathrm{d}\phi \ ,$$
$$\sigma_3 = \mathrm{d}\chi + \cos\theta \mathrm{d}\phi \ .$$

Decoupled gravitational perturbation

$$h_{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t + ikz} r^2 \delta\alpha(r) \sigma_+^2 \ .$$

where $\sigma_\pm = \frac{1}{2}(\sigma_1 \mp i\sigma_2)$

This perturbation is decoupled from other perturbations thanks to SU(2) and U(1) isometry.

KM&J.Soda 07

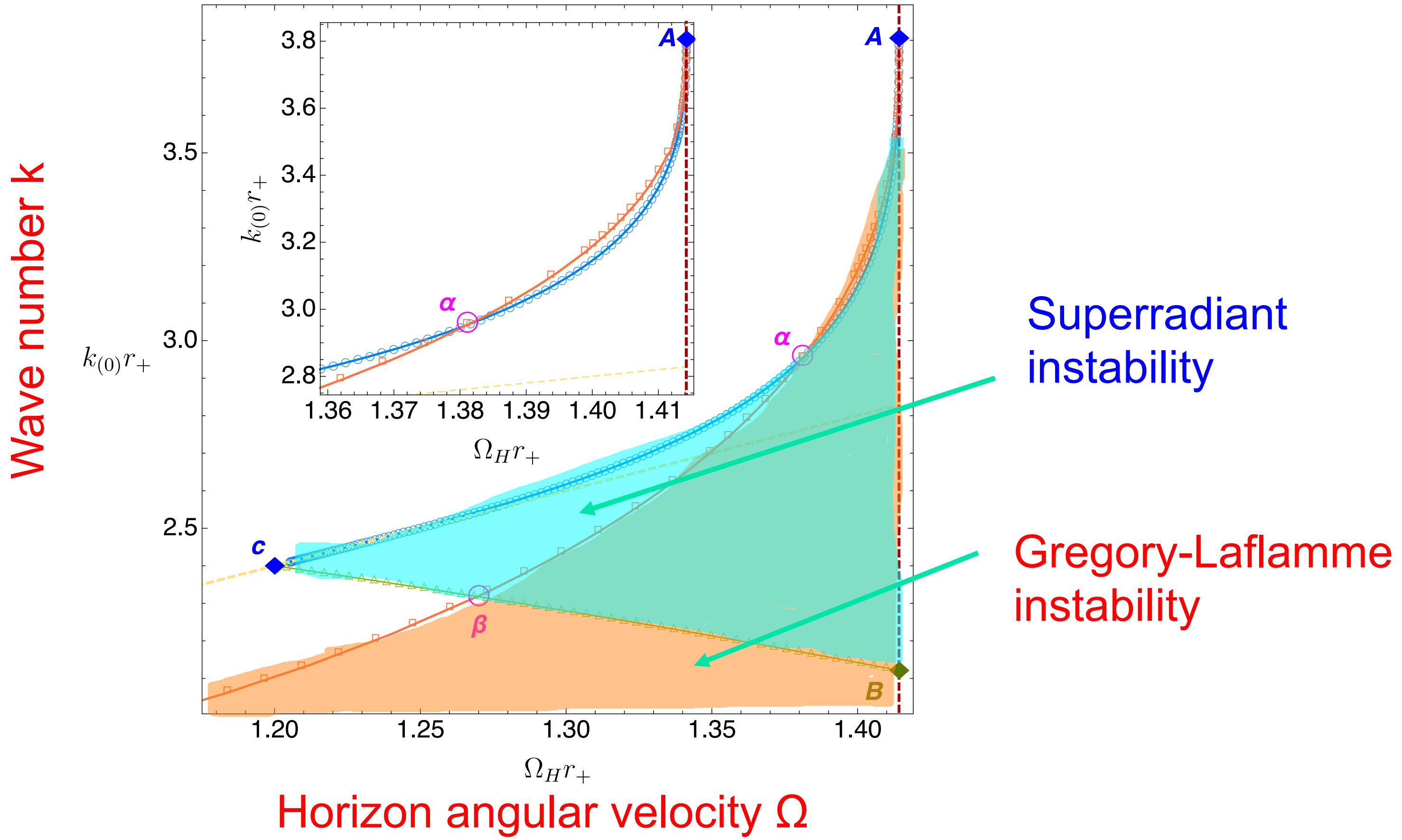
Master equation

$$h_{\mu\nu}dx^\mu dx^\nu = e^{-i\omega t+ikz} r^2 \delta\alpha(r)\sigma_+^2 \ .$$

Single ODE

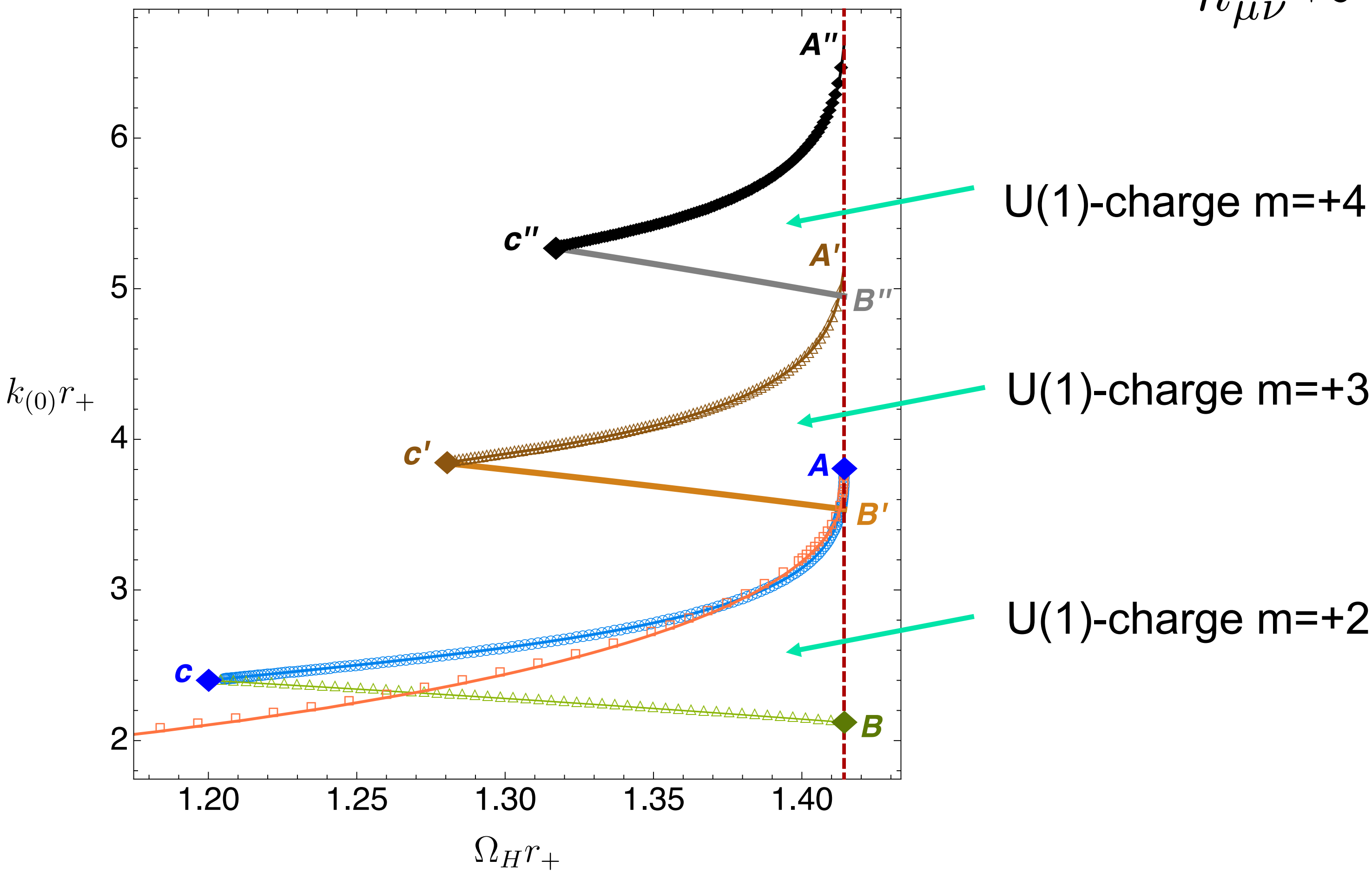
$$\delta\alpha'' + \left\{ \frac{(fg\beta)'}{2fg\beta} + \frac{3}{r} \right\} \delta\alpha' + \left\{ \frac{(fg\beta)'}{rfg\beta} + \frac{4}{r^2}\left(1 + \frac{\beta}{g} - \frac{4}{g\beta}\right) + \frac{(\omega - 4\Omega)^2 - fk^2}{fg} \right\} \delta\alpha = 0 \ .$$

Unstable region

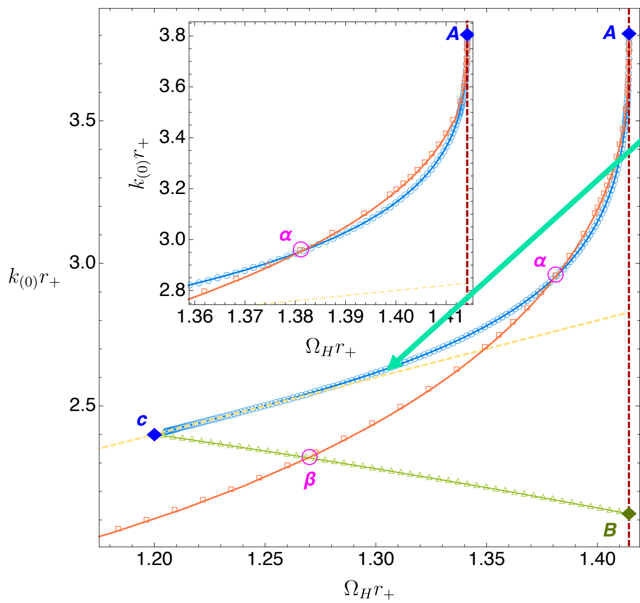


Results of higher modes

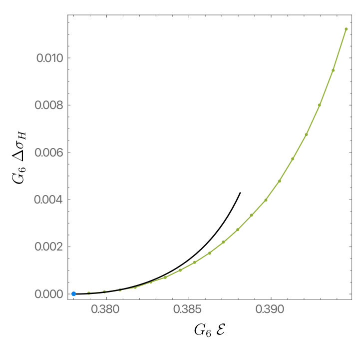
$$h_{\mu\nu} \sim e^{im\chi}$$



Final fate of instability?



We can extend nonlinear solution from the onset of the superradiant instability.



Higher entropy than the MP black string.

“Gregory-Laflamme and Superradiance encounter Black Resonator Strings” arxiv:2212.01400
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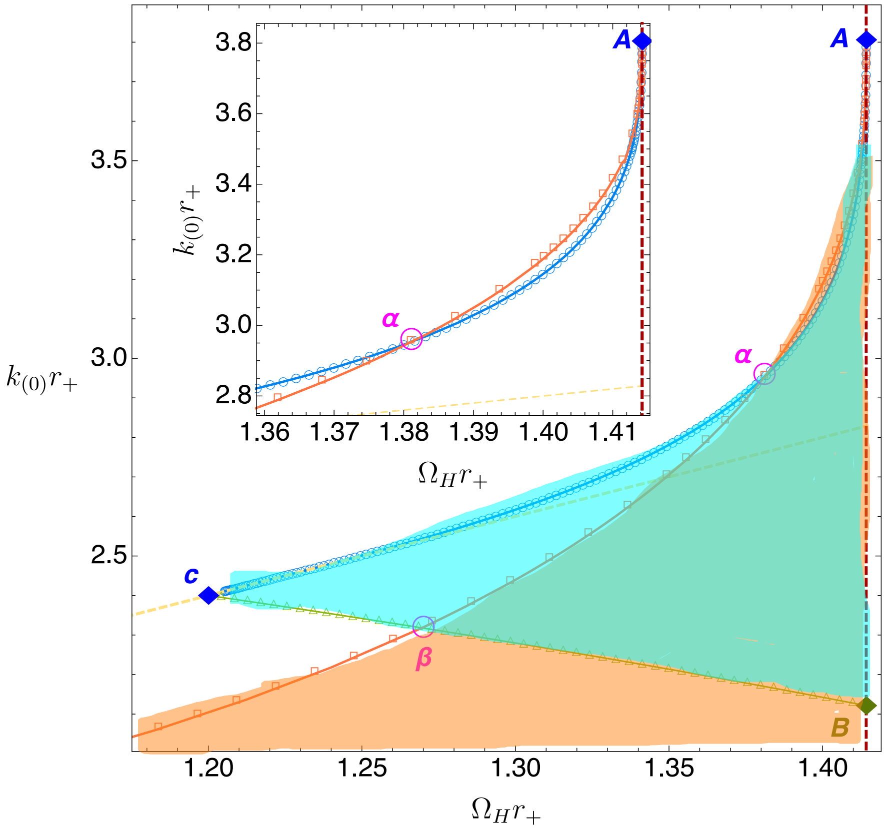
Summary

Superradiant instability of Myers-Perry black string was studied.



In a wider region in the parameter space,
the black string becomes unstable.

Collaboration with other research area?

GL-instability is described by
1d SYM (or 2d SYM by T-duality).

Aharony et al 04

GL-instability ⟺ Transition from uniform to localized distribution of the eigen value of X^1.

Dual picture of the superradiant instability?

Superradinat instability of rotating black string is subtle.

	4d Kerr+1d D=5	MP+1d D=6	MP+1d D>=7
Scalar	unstable	stable?	stable?
Maxwell	unstable*	stable?	stable?
Gravitational perturbation	unstable*	unstable	stable?

* : No proof.

This instability is subtle.

D=6 (5d Myers-Perry + 1d)
- Scalar field perturbation
- Maxwell field perturbation

D>=6
- Scalar field perturbation
- Maxwell field perturbation
- Gravitational perturbation

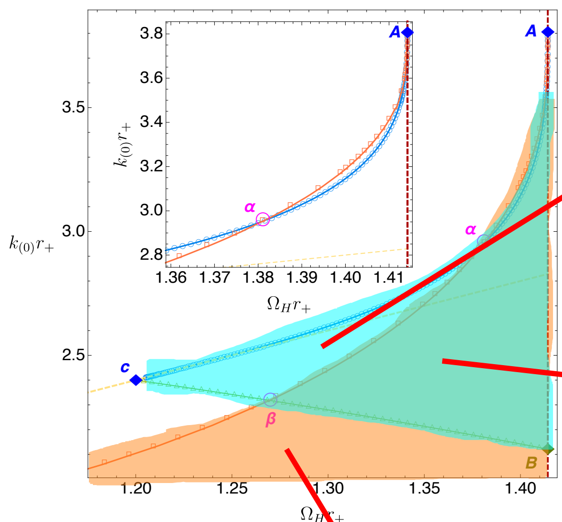
→ No evidence of superradiant instability.

D=5 (4d Kerr + 1d)
- Scalar field perturbation
- Maxwell field perturbation
- Gravitational perturbation

D=6 (5d Myers-Perry + 1d)
- Gravitational perturbation

→ Unstable to superradiance

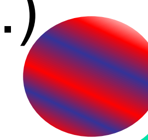
Possible end point?



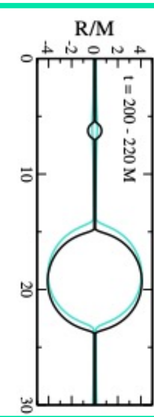
Helical black string?
(Both of U(1) and R_z broken.)

Ishii's Talk

"Helical" localized BH?
(Localized BH with broken U(1).)

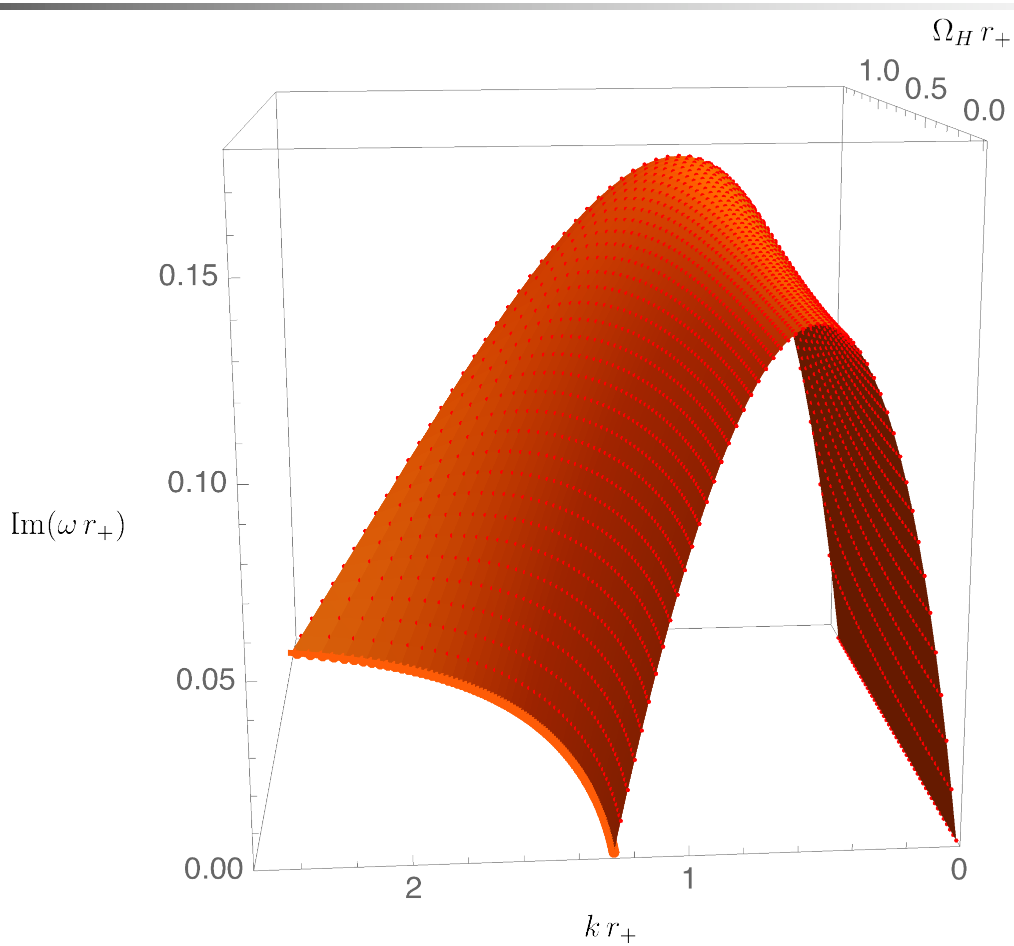


"Localized" BH?

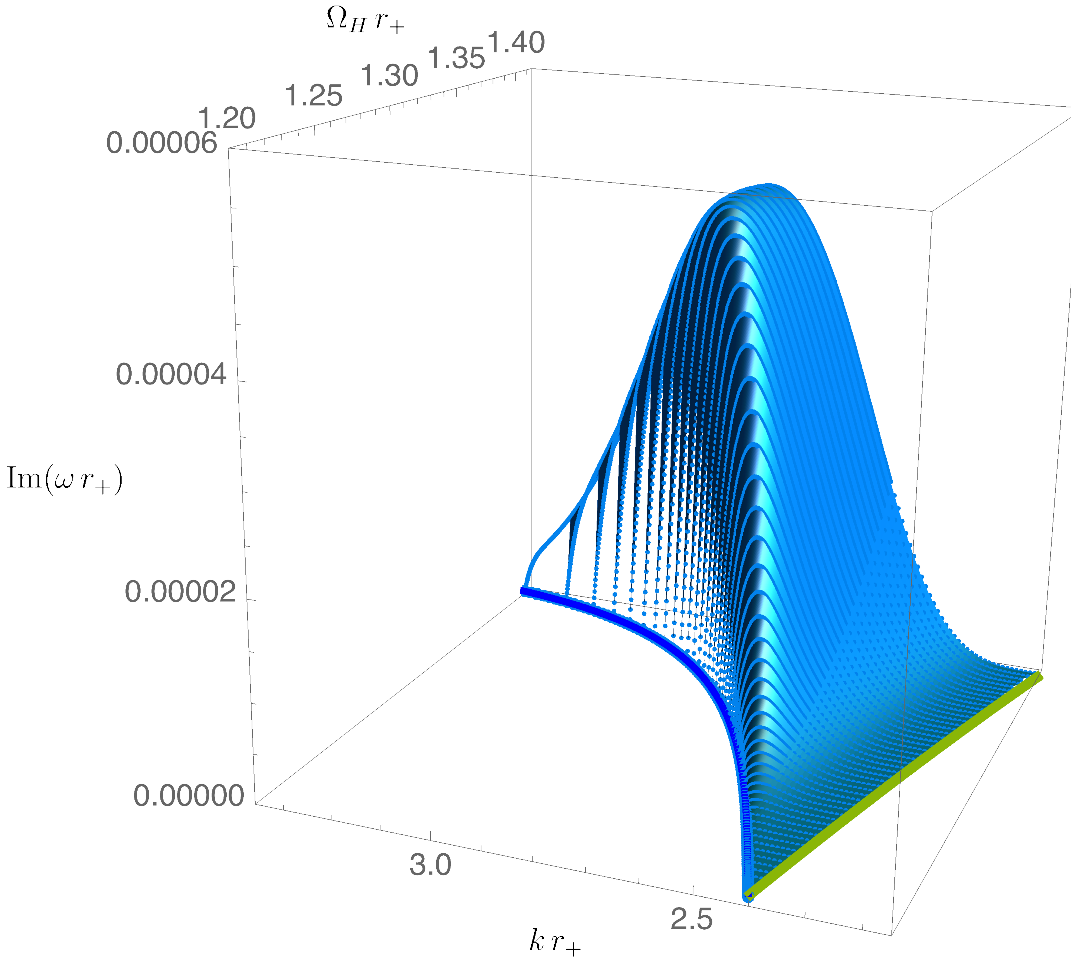


Taken from
Lehner & Pretorius
PRL 105, 101102 (2010)

GL

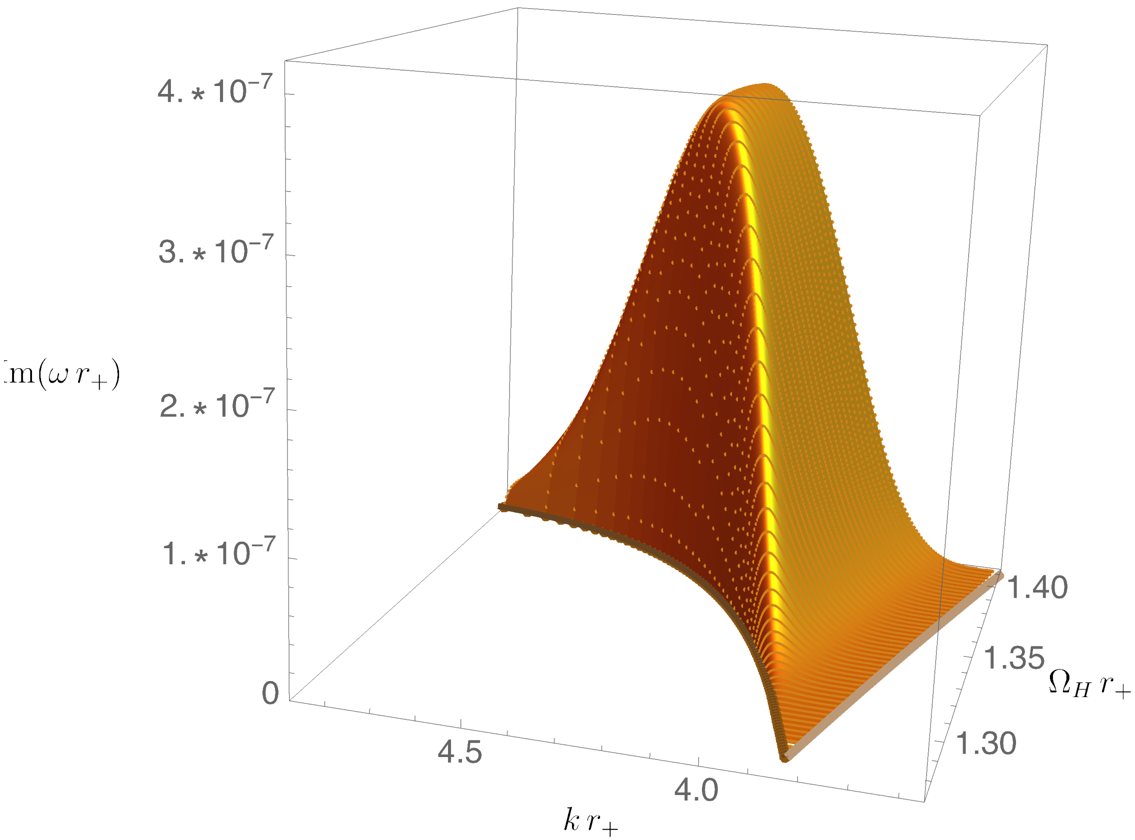


SR m=2

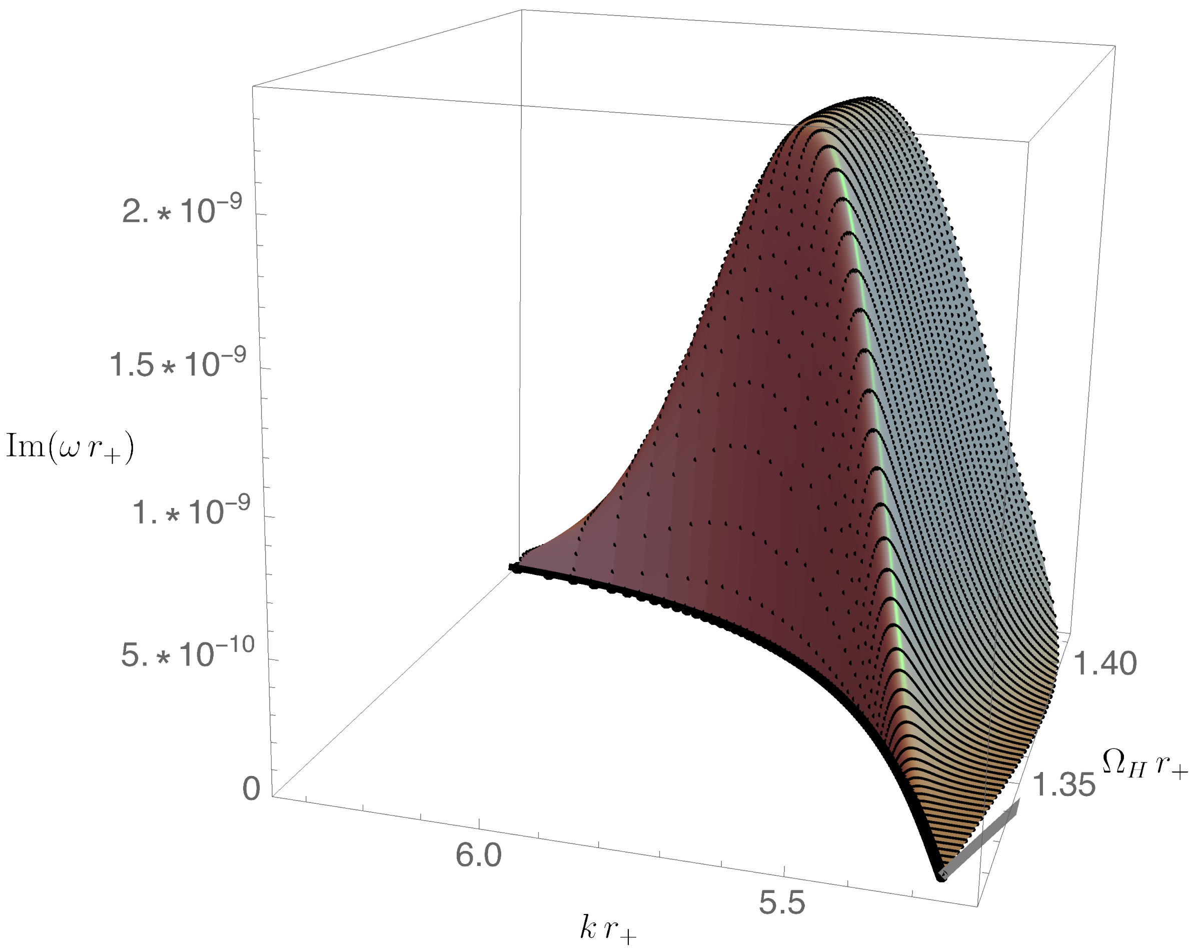


$\Omega_H r_+$
1.20
1.25
1.30
1.35
1.40
0.00006
0.00004
0.00002
0.00000
$\mathrm{Im}(\omega r_+)$
3.0
2.5
$k r_+$

m=3

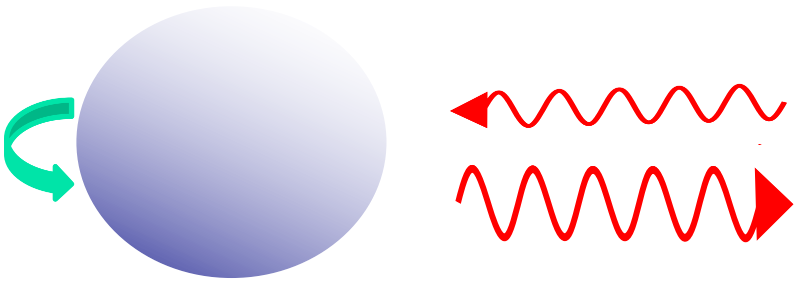


m=4



Superradiance

Wave amplification by the rotating black hole.



“Wave version of the Penrose process”

Decoupled gravitational perturbation

$$\sigma_1 = -\sin\chi \mathrm{d}\theta + \cos\chi \sin\theta \mathrm{d}\phi \ ,$$
$$\sigma_2 = \cos\chi \mathrm{d}\theta + \sin\chi \sin\theta \mathrm{d}\phi \ ,$$
$$\sigma_3 = \mathrm{d}\chi + \cos\theta \mathrm{d}\phi \ .$$

$$h_{\mu\nu} dx^\mu dx^\nu = e^{-i\omega t + ikz} r^2 \delta\alpha(r) \sigma_+^2 \ .$$

where $\sigma_\pm = \frac{1}{2}(\sigma_1 \mp i\sigma_2)$

Under χ-translation,

$$\chi \to \chi + \lambda \quad \Longrightarrow \quad \sigma_\pm \to e^{\mp i\lambda} \sigma_\pm$$

$\sigma_\pm$ has U(1)-charge m=±1.

Above is unique perturbation
which is SU(2)-symmetric and has U(1)-charge m=+2.

⟹ Decouple

KM&J.Soda 07