Real-device quantum simulation of spin chains with integrable Trotterization

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Based on arXiv:2208.00576 (and some extra data)
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Plan of the talk

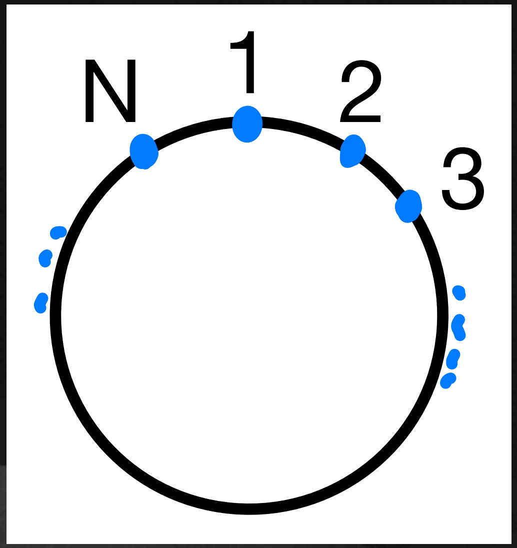
- Motivations and background
- Integrable Trotterization of XXX spin chain
- Quantum devices
- Simulation results/theoretical analysis
- Summary and conclusions
- Collaboration opportunities

Motivations and background

- With the development of quantum technologies, the quantum simulation of many-body systems (lattice quantum field theories in particular) will be an important target application.
- The abilities of the current quantum devices are limited, especially by **noise**. Q1: Can we quantify the effects of noise using a many-body system?
- Because of noise we cannot run deep quantum circuits. Errors due to discretization of time evolution are significant. Q2: Is there a way to put discretization errors under control?

Integrable Trotterization

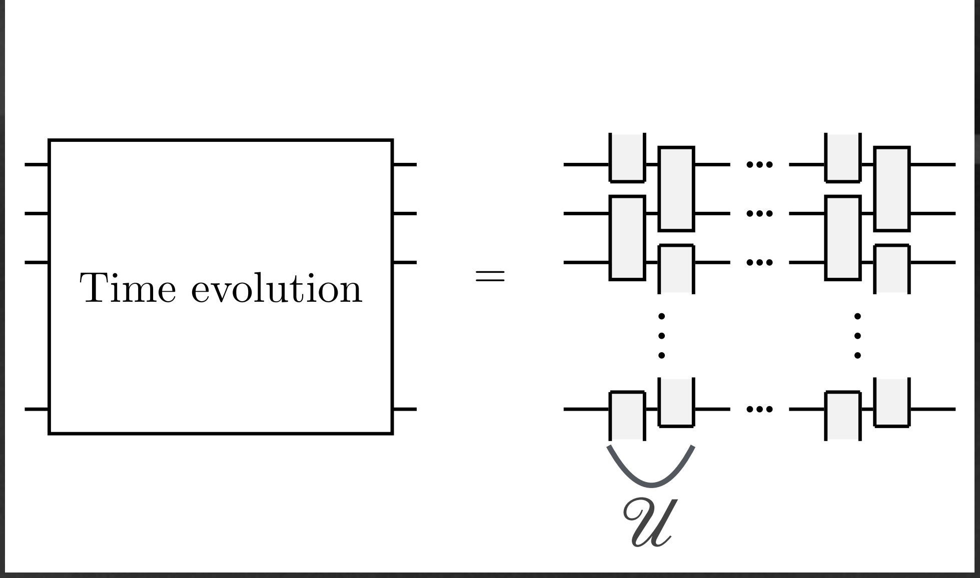
Integrable Trotterization



- XXX Hamiltonian: $H \sim \sum_{j=1}^{N} \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_{j+1}$.

$\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z) \equiv (X, Y, Z)$

- R matrix:
  $R_{ij}(\lambda) \propto 1 + i\lambda(1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$.



- Trotterized small-time evolution (even N, periodic b.c.):

$$\mathscr{U}(\delta) \sim \left(\prod_{j=1}^{N/2} R_{2j-1,2j}(\delta)\right)\left(\prod_{j=1}^{N/2} R_{2j,2j+1}(\delta)\right)$$

Conserved charges

- $\mathscr{U}(\delta)$ commutes with transfer matrix $T(\lambda) = \mathrm{tr}_0\Big(\overleftarrow{\prod_{1\le j\le N}} R_{0j}\Big(\lambda - (-1)^j\delta\Big)\Big)$ for any $\lambda \in \mathbb{C}$.

- Charges $Q_n^{\pm}(\delta) \sim \frac{d^n}{d\lambda^n}\log T(\lambda)\Big|_{\lambda=\pm\delta/2}$ are **exactly conserved** even with Trotterization.

- $Q_n^{\mathrm{dif}} \equiv [Q_n^{+}(\delta) - Q_n^{-}(\delta)]/\delta$ is also conserved.

- Higher charges can be obtained as $Q^{\pm}_{n+1} \sim [B, Q^{\pm}_{n}]$, where B is a discrete (Lorentz) boost transformation. [Vanicat et al.]
- We compute densities $q^{[n,\pm]}_{j,j+1,\ldots,j+2n}$ in

$$Q^{+}_{n}(\delta) = \sum_{j=1}^{N/2} q^{[n,+]}_{2j-2,2j-1,\ldots,2j+2n-2}(\delta)\,,$$

$$Q^{-}_{n}(\delta) = \sum_{j=1}^{N/2} q^{[n,-]}_{2j-1,2j,\ldots,2j+2n-1}(\delta)\,.$$

- We implemented the recursion in computer programs.

known

$$q^{[1,\pm]}_{1,2,3}(\delta) = \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3 \mp \delta\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3) + \delta^2\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\,,$$

$$\begin{aligned}
q^{[2,\pm]}_{1,2,3,4,5}(\delta) = {}& \mp 2\delta(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_4 + \boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_5 - \boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_5) - (1-\delta^2)\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5) - \boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4) - \delta^2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5)\\
&-\delta^2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4) - \delta^4\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5) \pm \delta\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5) \pm \delta\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4)\\
&\pm\delta^3\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5) \pm \delta^3\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5) - \delta^2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5)\,,
\end{aligned}$$

$$\begin{aligned}
q^{[3,+]}_{1,2,3,4,5,6,7} = {}& -4\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7 - 4\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
&+\delta\Big(10\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 4\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) + 8\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6) - 4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
&-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6) - 4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\Big)\\
&+\delta^2\Big(2\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7 - 10\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_6\\
&-6\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\\
&-6\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6) + 2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
&+2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\Big)\\
&+\delta^3\Big(6\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 4\boldsymbol{\sigma}_4\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 8\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\\
&-2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6) + 4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6) - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_7) + 4\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
&-2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
&-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6) - 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\Big)\\
&+\delta^4\Big(-2\boldsymbol{\sigma}_6\cdot\boldsymbol{\sigma}_7 - 8\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7 - 2\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_6 + 2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_7 - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
&+2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6)\\
&+2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) + 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big)\\
&+\delta^5\Big(4\boldsymbol{\sigma}_5\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6\times\boldsymbol{\sigma}_7)\\
&-2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7) - 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big)\\
&+\delta^6\Big(-4\boldsymbol{\sigma}_5\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_7 + 2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_7)\Big)\,.
\end{aligned}$$

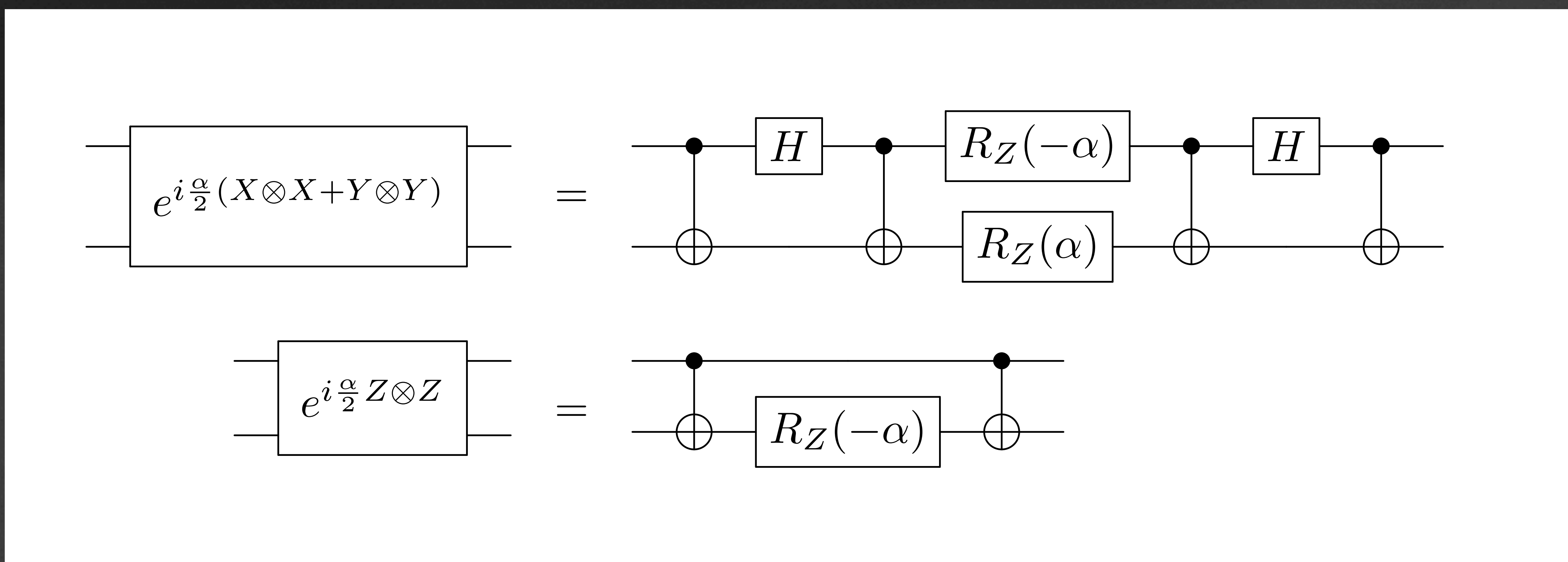
new

Densities and charges are traceless.

Here $\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\cdots\times\boldsymbol{\sigma}_{\ell-1}\times\boldsymbol{\sigma}_\ell) := \boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times(\boldsymbol{\sigma}_3\times(\cdots\times(\boldsymbol{\sigma}_{\ell-1}\times\boldsymbol{\sigma}_\ell)\cdots)))$

Quantum circuits

The actual circuits are built from the following ingredients. Here $R_Z(\alpha) = e^{-i(\alpha/2)Z}$.



Quantum devices

IBM: Superconducting devices

- IBM uses superconducting transmon qubits. These are made of materials such as niobium and aluminum placed on a silicon chip. Two energy-levels form an approximate qubit.

- We obtained access to the devices through the University of Tokyo. (Supported by UTokyo Quantum Initiative).

- We used the ibm_kawasaki and ibm_washington processors.

source: https://japan.zdnet.com/article/35174399/

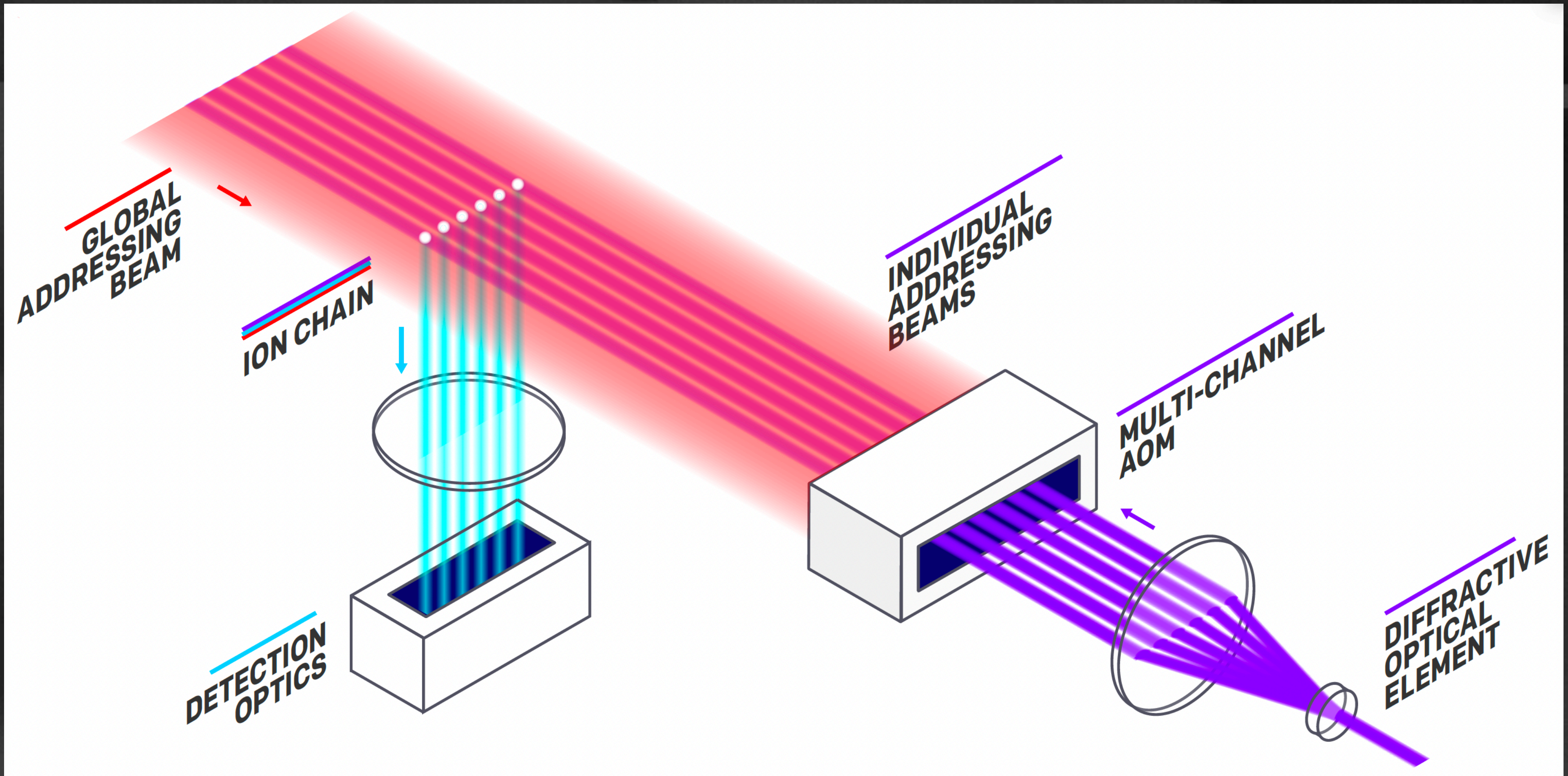
IonQ: trapped ion devices

- We mainly used IonQ's device called Harmony. (Not in the current version of the e-print.)

- A linear chain of $^{171}\mathrm{Yb}^{+}$ ions near an electrode trap.

- 11 qubits with all-to-all couplings.

- We got indirect access through Google Cloud and direct access through IonQ itself.

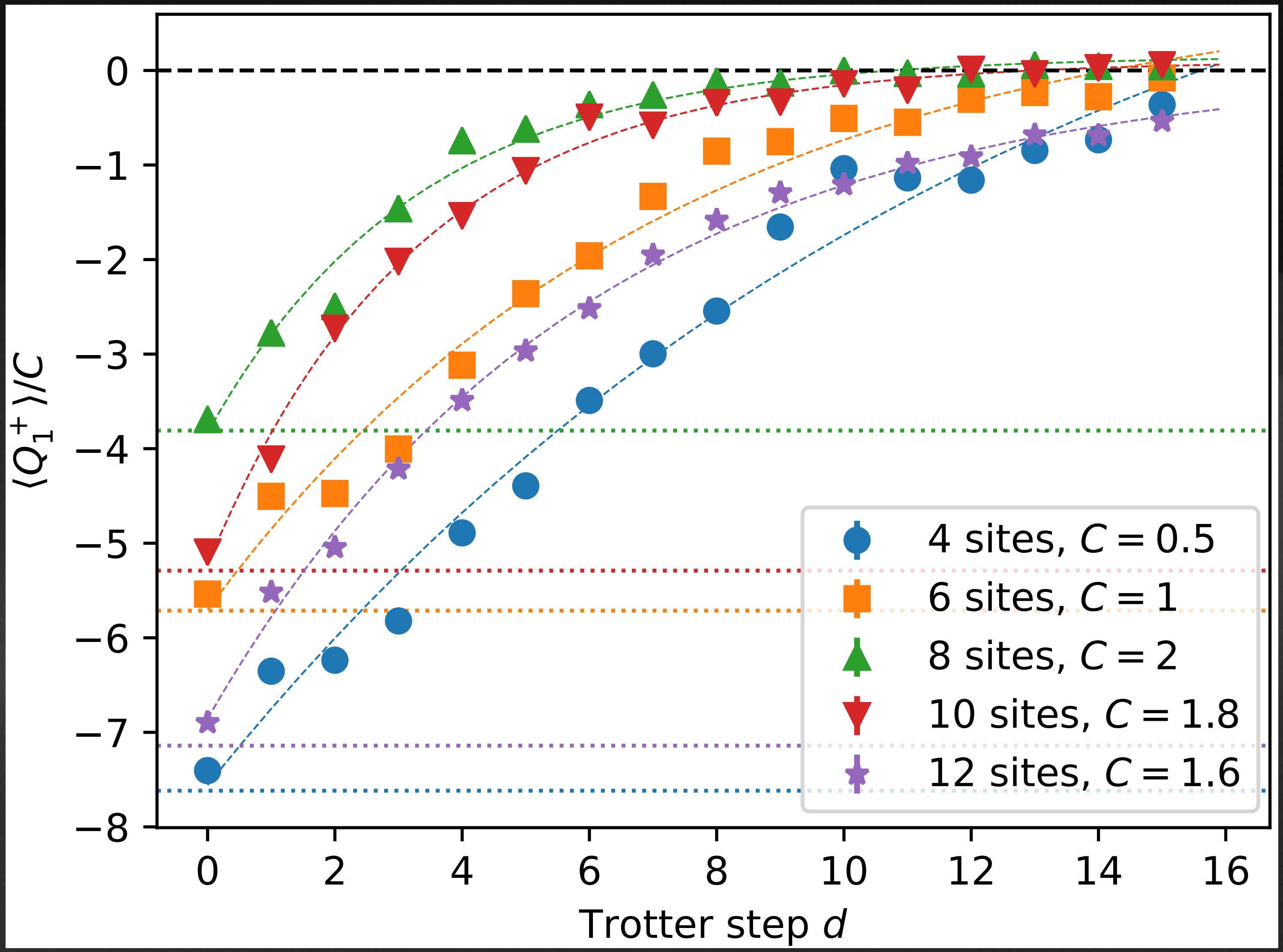


arXiv:1903.08181

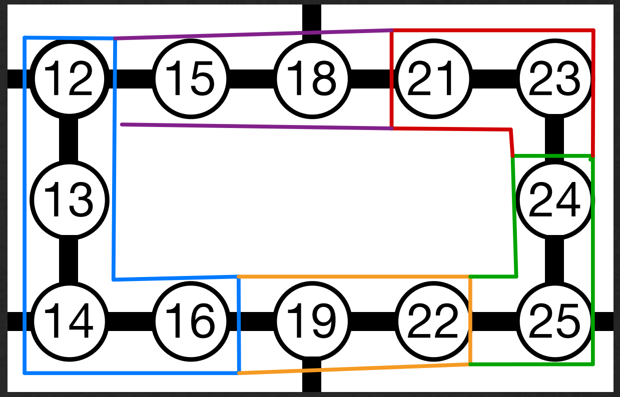
Results of real-device simulations

Simulation results for ibm_kawasaki

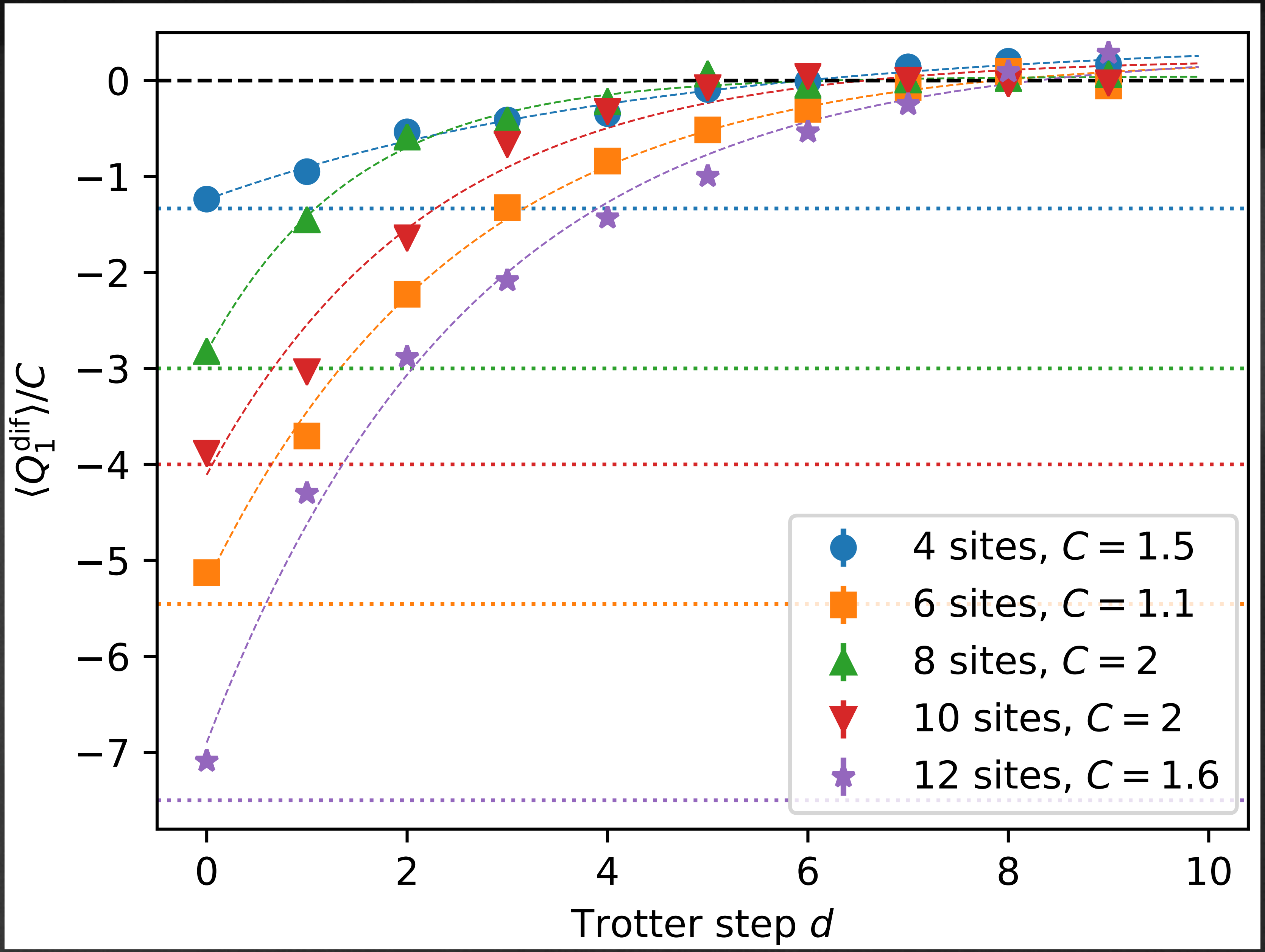
- $\langle Q_1^+ \rangle = \mathrm{tr}(\rho Q_1^+)$ **decays exponentially** to zero asymptotically, **due to noise.** No error mitigation.
- Error bars are hidden by markers. Rescaled for better visibility. The theoretical values are shown by dotted lines. Fit by $c_1 e^{-\gamma d} + c_2$.
- The initial state is $|0101\ldots01\rangle$.
- Large fluctuations from one step to the next. (Due to change in device parameters?)



Only the 12-site simulation is for a circular topology.

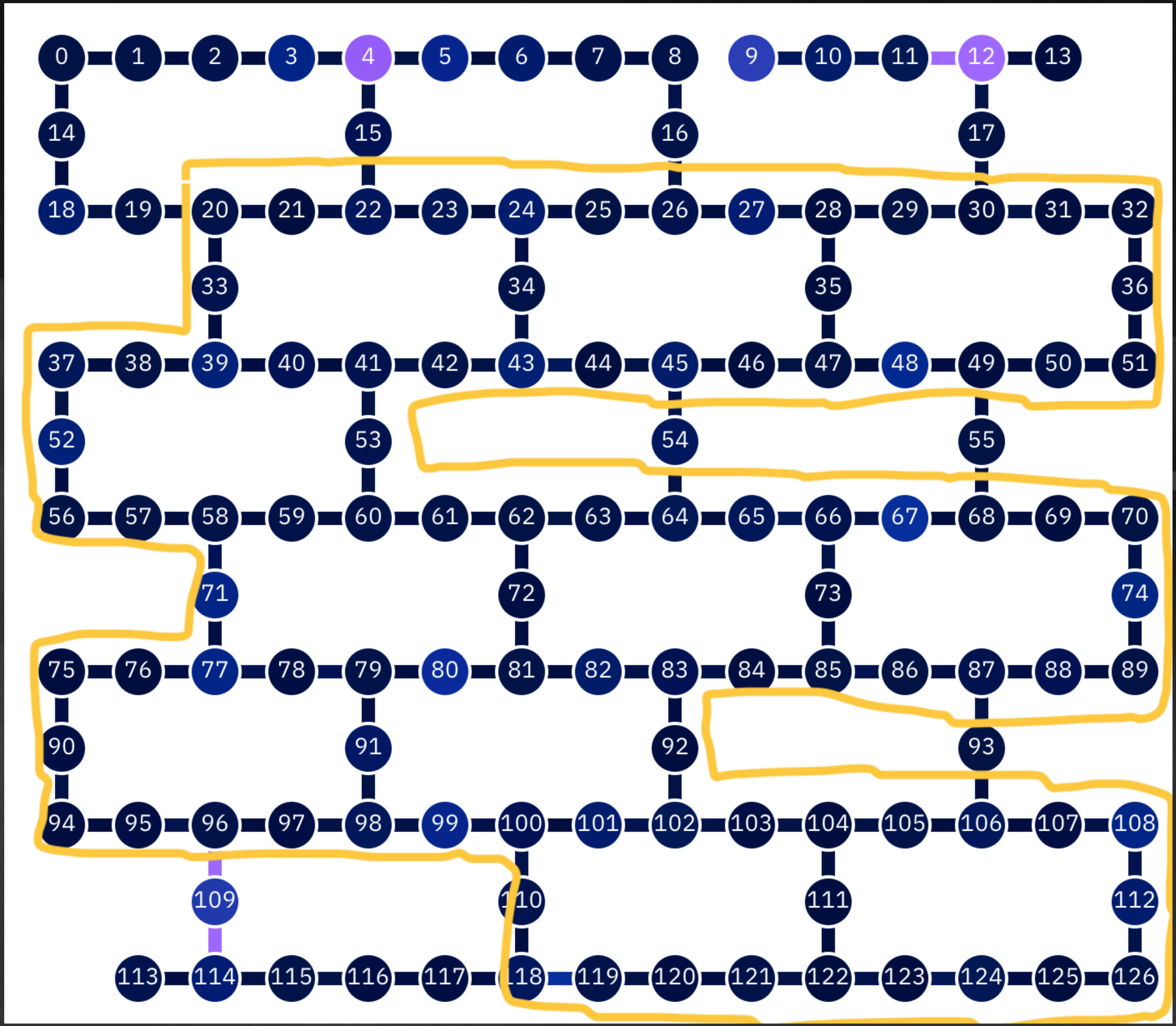


- Similar results for $Q_1^{\mathrm{dif}} = [Q_1^{+}(\delta) - Q_1^{-}(\delta)]/2$.
- The initial states are chosen appropriately to give non-zero theoretical expectation values.



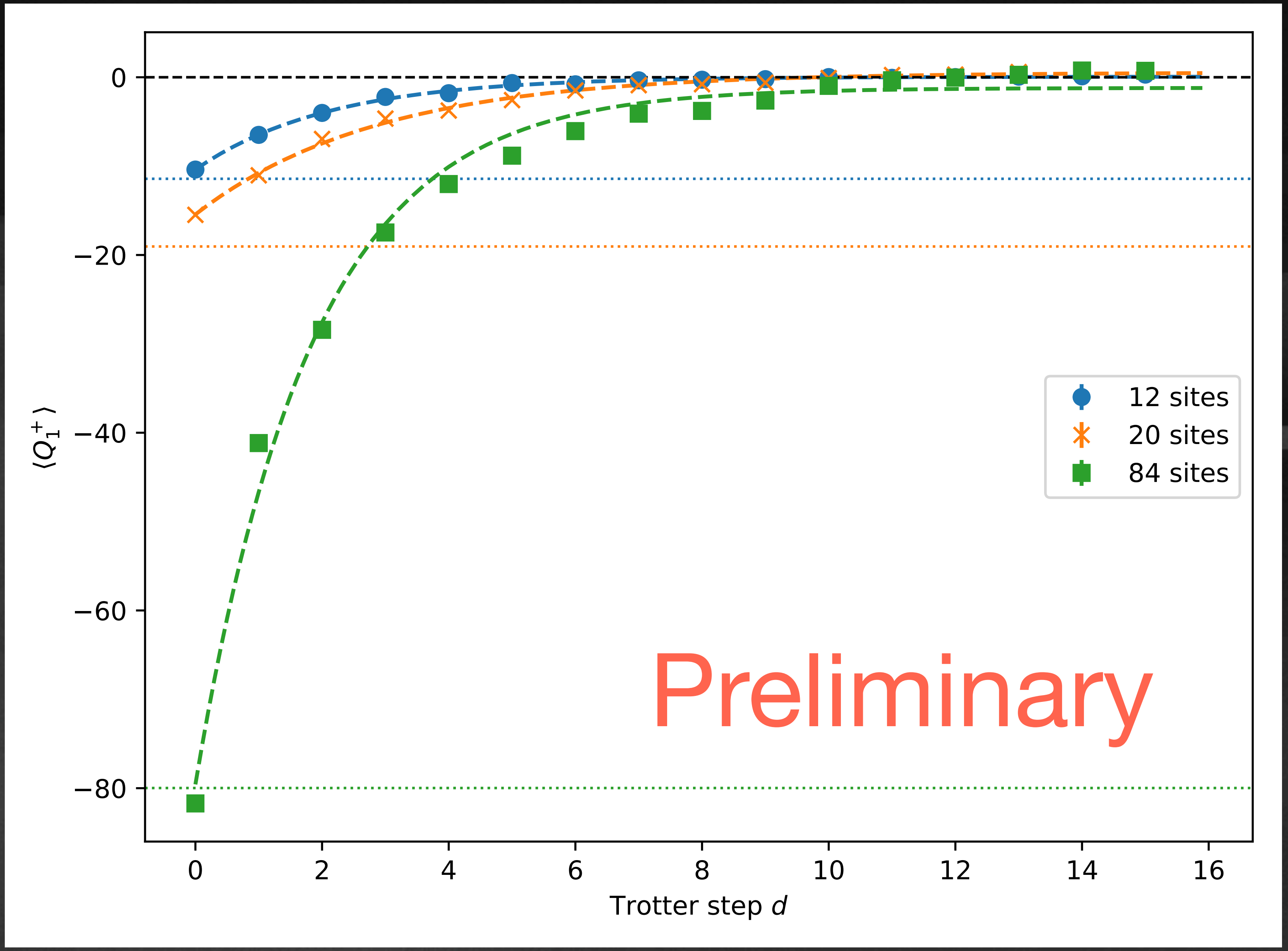
Simulations on a 127-qubit IBM device

- Quantum device ibm_washington with 127 qubits.
- We ran simulations with qubits on loops of size 12, 20, and 84. The 84-qubit loop is shown in the figure.
- To have slower decays, it is important to avoid faulty (purple) qubits and connections.



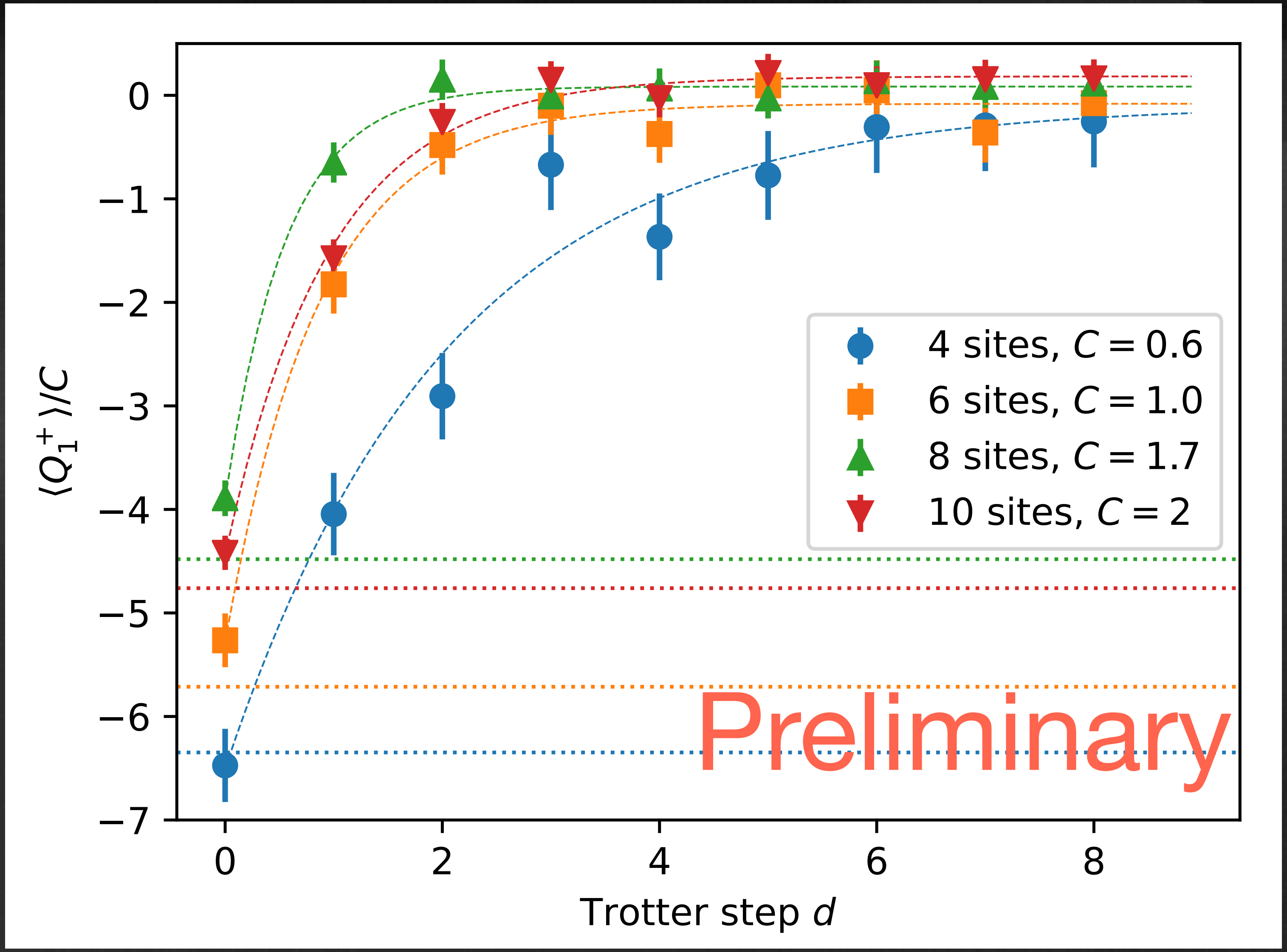
Simulation results on large chains

- Loops of size 12, 20, and 84.
- Similar exponential decays of $\langle Q_1^+ \rangle$.
- For the 84-site run, we had 10^6 shots (circuit executions) for each value of d.
- (There were significant time gaps between some data.)
- (Not in the current version of the e-print.)



Simulation results for IonQ Harmony

- Similar exponential decays.
- To have slower decays, it seems important to use the qubits (ions) in the middle of the linear chain.



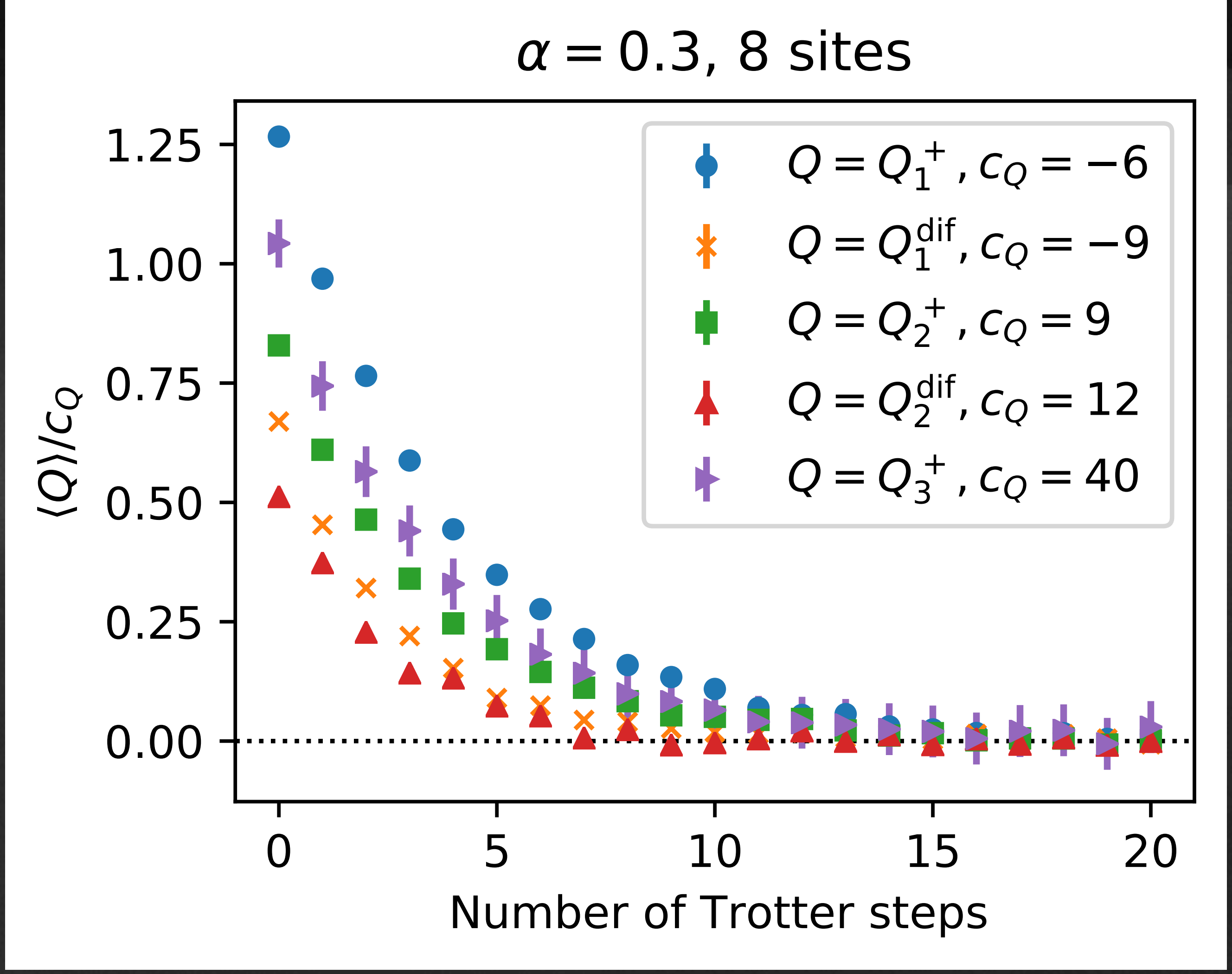
Simulator results and theoretical analysis

Numerical noise models

- We ran digital quantum simulations on the Qiskit (classical) simulator with noise models.
- We considered two noise models:
    1. (1-qubit) depolarizing error channels inserted after 1- and 2-qubit gate operations.
    2. (1-qubit) amplitude-and-phase damping error channels inserted after 1- and 2-qubit gate operations.

Classical emulation of quantum simulation with a depolarizing noise model

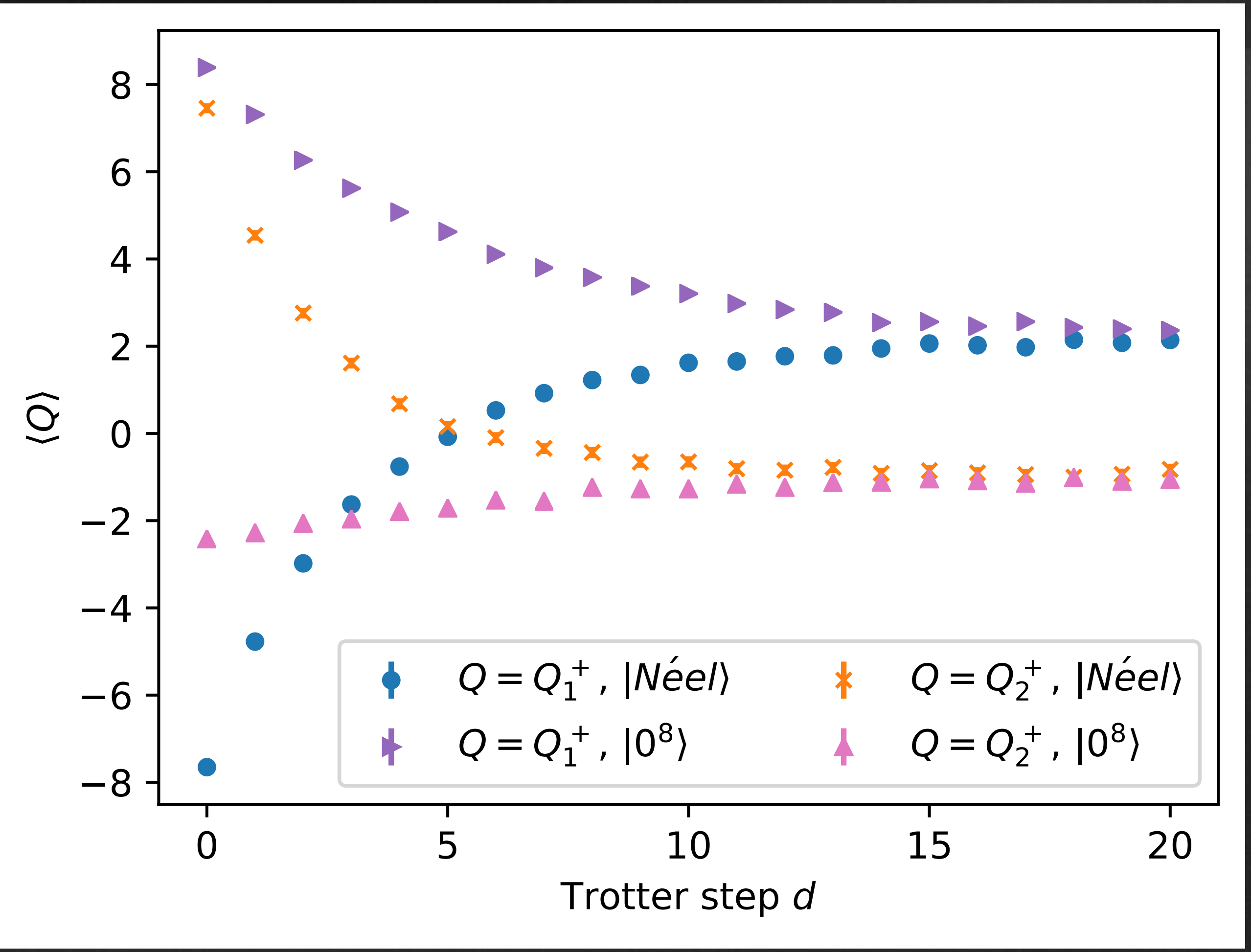
- $$\Phi_{\text{depo}}(\rho) = \sum_{j=1}^{4} D_j \rho D_j^\dagger$$ with
  $$D_1 = \sqrt{1 - \frac{3p}{4}}\, I, \quad D_2 = \sqrt{\frac{p}{4}}\, X,$$
  $$D_3 = \sqrt{\frac{p}{4}}\, Y, \quad D_4 = \sqrt{\frac{p}{4}}\, Z$$
  inserted after gate operations.



- $\langle Q_j^+ \rangle$ and $\langle Q_j^{\text{dif}} \rangle$ decay exponentially to zero. This suggests that the finite state is completely mixed.

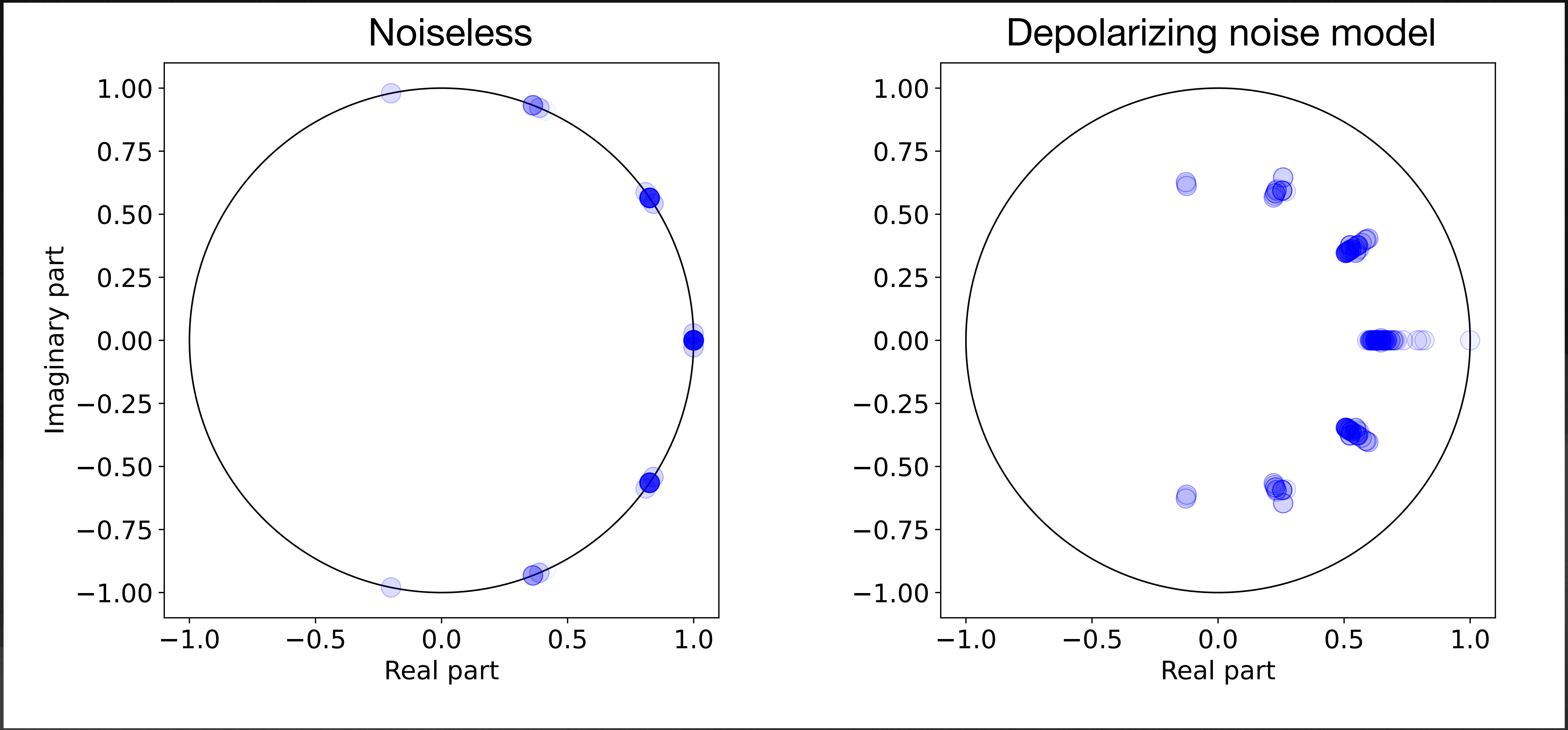
Classical emulation of quantum simulation with a amplitude-and-phase damping noise model

- $\Phi_{\text{damp}}(\rho) = \sum_{j=1}^{3} D_j \rho D_j^\dagger$ with
  $$D_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda_a-\lambda_p} \end{pmatrix},$$
  $$D_2 = \begin{pmatrix} 0 & \sqrt{\lambda_a} \\ 0 & 0 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\lambda_p} \end{pmatrix}$$
  inserted after gate operations.
- $\langle Q_j^+ \rangle$ (and $\langle Q_j^{\text{dif}} \rangle$) asymptote to finite values. The finite state is unique and is NOT completely mixed. Checked by quantum tomography.



Analysis fo quantum channels

- The initial state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ is mapped, at Trotter step d, to $\Phi^d(\rho)$, where Φ is a noisy time evolution for a single step.
- The expectation value of a conserved charge Q at step d is $\langle Q \rangle_d = \mathrm{tr}[\Phi^d(\rho)Q]$.
- We studied the eigenvalue distribution of the linear map $\rho \to \Phi(\rho)$.



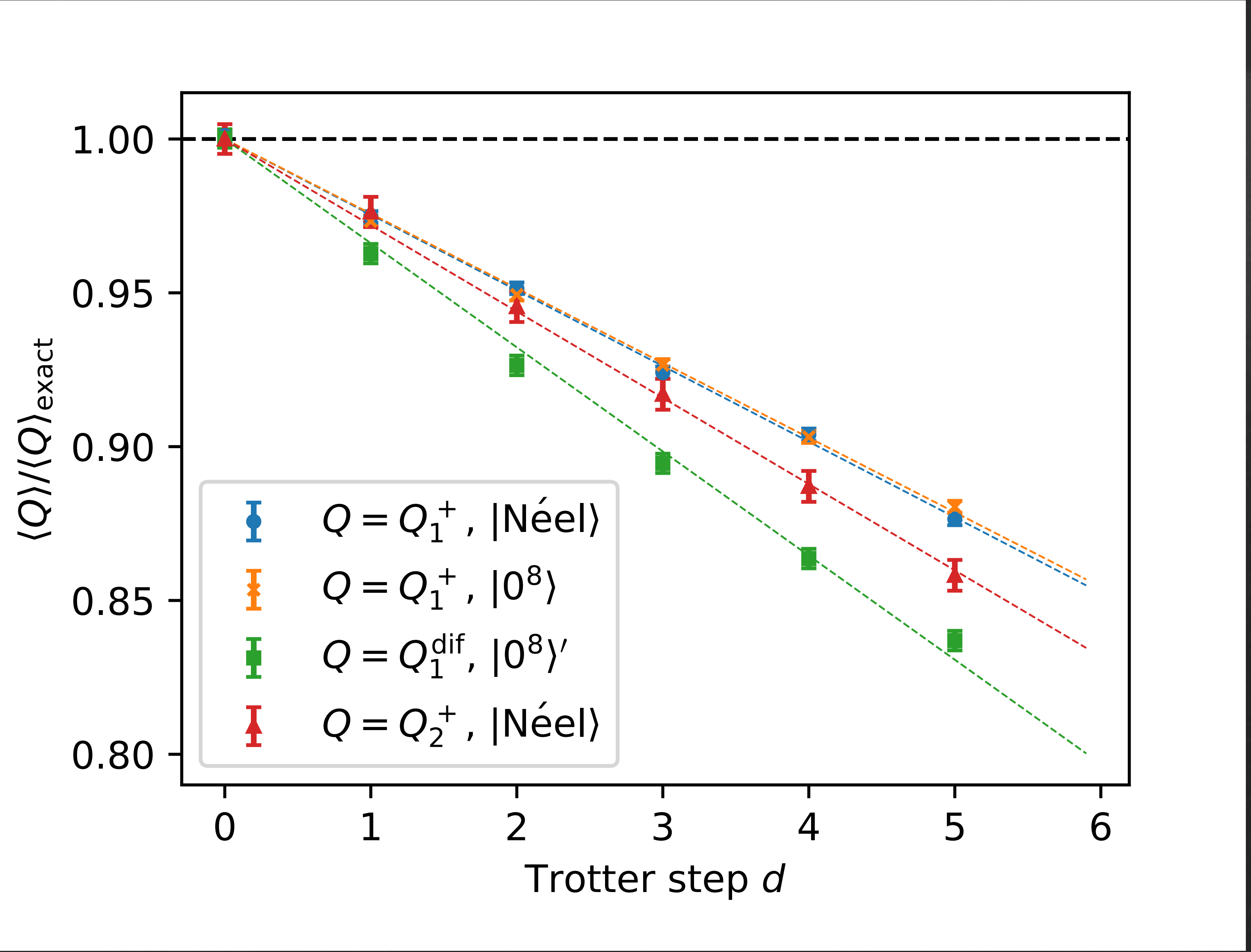
- The eigenvalues for the single time step Φ on 4 sites.
- In the noiseless case, the evolution is unitary and the eigenvalues are on a unit circle.
- In the depolarizing noise model, all the eigenvalues except one are strictly inside the unit circle. There remains a single eigenvalue 1, corresponding to the unique fixed point (completely mixed state) of Φ.

Possible use of conserved charges as benchmarks for future quantum computing

- For future quantum devices we expect smaller error rates. We propose to use the higher conserved charges of the integrable Trotterization as benchmarks.

- On a classical simulator, we numerically computed the time evolution on 8 sites.

- The slopes of early-tiime decays depend on the types and the degrees of the charges.



Summary and conclusions

- We implemented the integrable Trotterization of the Heisenberg spin 1/2 XXX spin chain on real quantum computers and on classical simulators. We used superconducting devices of IBM and trapped ion devices of IonQ.

- As expected, conserved charges decay due to noise on the current quantum devices.

- The early time decay rate seems to depend on the type and the degree of the charge. Higher charges are candidates of benchmarks for the future quantum simulation.

To members of ExU Collaboration

- Current quantum devices have particular couplings, as in the ibm_washingon case. Perhaps it would be interesting to study spin chain models with junctions analytically (∃some work in the literature), numerically, and also by quantum simulation.

- In another paper 2210.10908 with Sukeno, we proposed a measurement-based quantum simulation scheme for lattice gauge theories. Experimental implementation?

- More generally, if you have algorithms or models that you think are suitable for simulations on an NISQ device, I'd be happy to discuss them.