Spectral form factor of JT gravity

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based on work (to appear) with Kazuhiro Sakai (Meiji Gakuin U.)

JT gravity

- JT gravity is a simple example of 2d dilaton gravity

$$I = -\frac{1}{2}\int d^2x\sqrt{g}\phi(R+2)$$

- JT gravity is a useful toy model for studying important questions in quantum gravity

Various problems in quantum gravity

- Using JT gravity, one can study various important problems in quantum gravity
  - Black hole and quantum chaos
    [Cotler et al, Kitaev, Maldacena-Stanford-Shenker]
  - Entropy of Hawking radiation and Page curve
    [Almheiri et al, Penington et al]
  - Sum over topologies in gravitational path integral
    [Saad-Shenker-Stanford, Stanford-Witten]

JT gravity as a random matrix model

- In 2019 [Saad-Shenker-Stanford] showed that JT gravity is equivalent to a random matrix model

$$Z_{\text{JT gravity}} = \left\langle Z(\beta) \right\rangle = \left\langle \operatorname{Tr} e^{-\beta H} \right\rangle_{\text{random matrix } H}$$

Holography with ensemble average

- This is an example of holography involving ensemble average
- JT gravity is holographically dual to the SYK model at low energy
- Average over the random couplings in SYK model is replaced by the average over the random matrix H

Spectral form factor

- Spectral form factor (SFF) is a useful diagnostics of quantum chaos

$$\mathrm{SFF} = \Big\langle Z(\beta + it) Z(\beta - it) \Big\rangle, \quad Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$$

- Spectrum of H in quantum chaotic system is expected to obey random matrix statistics [Bohigas-Giannoni-Schmit]

- Indeed, JT gravity is dual to a random matrix model!

SFF in GUE

- In Gaussian Unitary Ensemble (GUE), SFF can be computed in a closed form

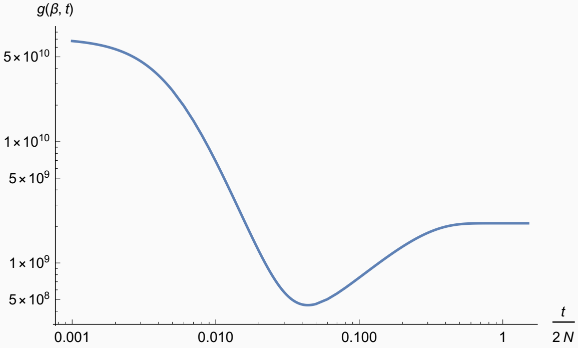
$$\text{GUE} = \int_{N\times N \text{ hermitian}} dH\, e^{-\frac{1}{2}\operatorname{Tr} H^2}$$

- SFF in GUE is defined by

$$\text{SFF} = \left\langle \operatorname{Tr} e^{-(\beta+it)H} \operatorname{Tr} e^{-(\beta-it)H} \right\rangle$$

Slope, ramp, and plateau

- SFF in GUE exhibits the characteristic feature of slope, ramp, and plateau as a function of time



Disconnected and connected parts

- Matrix model correlator is decomposed into disconnected and connected parts

$$\mathrm{SFF} = \langle \mathrm{Tr}\, e^{-(\beta+it)H} \rangle \langle \mathrm{Tr}\, e^{-(\beta-it)H} \rangle + \langle \mathrm{Tr}\, e^{-(\beta+it)H}\, \mathrm{Tr}\, e^{-(\beta-it)H} \rangle_c$$

- Slope, ramp and plateau originate from

disconnected	$\Rightarrow$	slope
connected	$\Rightarrow$	ramp and plateau

Slope

- Slope is the early time power-law decay of SFF
- Slope comes from the spectral edge of the eigenvalue distribution $\rho(E) \sim \sqrt{E - E_0}$

$$\text{SFF} \sim \left| \int_{E_0} dE \sqrt{E - E_0} e^{-itE} \right|^2 \sim t^{-3}$$

Plateau

- At late times, due to the cancellation of random phases only the diagonal part $E_1 = E_2$ survives

$$\mathrm{SFF} = \int dE_1 dE_2 \rho(E_1)\rho(E_2) e^{-\beta(E_1+E_2)-it(E_1-E_2)}$$

$$\xrightarrow[t\to\infty]{} \int dE \rho(E) e^{-2\beta E} = \langle Z(2\beta)\rangle$$

- This constant behavior at late time is called plateau

Ramp

- Linear growth of SFF is called ramp

$$\mathrm{SFF} \sim t$$

- Ramp arises from the eigenvalue correlation known as the sine kernel formula

- Time scale for the ramp-plateau transition is called Heisenberg time t_H determined by level spacing

$$t_H \sim (\Delta E)^{-1} \sim N$$

Bulk gravity interpretation

- Via holographic duality, we expect that SFF has a bulk gravity interpretation

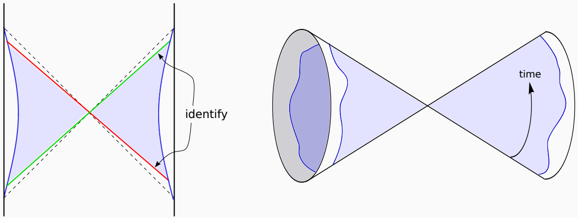
- Disconnected/connected part of SFF corresponds to disconnected/connected spacetime

$$\langle Z(\beta_1)Z(\beta_2)\rangle = \langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle + \langle Z(\beta_1)Z(\beta_2)\rangle_{\text{conn}}$$

- Connected part of SFF corresponds to a wormhole

Double cone

- Bulk dual of $\langle Z(\beta + it) Z(\beta - it) \rangle_{\text{conn}}$ is known as double cone [Saad-Shenker-Stanford]



- Ramp($\sim t$) comes from the relative rotation of two boundaries

Bulk dual of plateau?

- Bulk dual of plateau is not well understood
- It is suggested that non-perturbative effect of quantum gravity is responsible for the plateau
  - Spacetime D-brane, Andreev-Altshuler instanton [Saad-Shenker-Stanford]
  - Eigenvalue instanton [KO]

Perturbative road to plateau?

- Recently it is realized that plateau can be explained by summing over the perturbative genus expansion of SFF

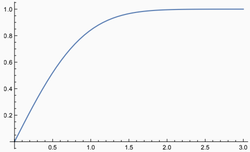
  [Saad-Stanford-Yang-Yao, Blommaert-Kruthoff-Yao]

- This is based on our earlier observation for the so-called Airy case $\rho(E) = \sqrt{E}$ [KO-Sakai]

SFF of Airy case

- Connected part of SFF for the Airy case is given by the error function

$$\mathrm{SFF}_{\mathrm{conn}} = \mathrm{Erf}(\sqrt{\beta} t \hbar), \qquad (\hbar \sim N^{-1})$$



- Small $\hbar$ expansion of the error function has a finite radius of convergence, hence "the perturbative plateau"

τ-scaling limit

- This computation of the Airy case can be generalized to the matrix model of 2d gravity, including JT gravity
- To this end, we can focus on the ramp-plateau transition regime $t_H \sim \hbar^{-1}$ by taking the τ-scaling limit

$$t \to \infty, \hbar \to 0, \quad \tau = t\hbar = \text{fixed}$$

- In this limit, SFF is exactly given by the Fourier transform of the sine kernel [Saad-Stanford-Yang-Yao, Blommaert-Kruthoff-Yao]

Sine kernel

- SFF is written by the Christoffel-Darboux kernel $K(E_1, E_2)$

$$\begin{aligned}&\langle Z(\beta + it)Z(\beta - it)\rangle_{\text{conn}}\\ = \ &\langle Z(2\beta)\rangle - \int dE_1 dE_2 e^{-\beta(E_1+E_2)-it(E_1-E_2)} K(E_1, E_2)^2\end{aligned}$$

- In the τ-scaling limit, CD kernel can be replaced by the sine kernel

$$K(E_1, E_2) = \frac{\sin\left[\pi\rho(E)(E_1 - E_2)\right]}{\pi(E_1 - E_2)}, \quad E = \frac{E_1 + E_2}{2}$$

Leading term of SFF in the τ-scaling limit

- From the Fourier transformation of the sine kernel, leading term of SFF is written as

$$\mathrm{SFF}_0 = \int_0^{\tau} d\tau' \frac{e^{-2\beta E(\tau')}}{2\beta}, \quad \rho(E(\tau)) = \tau$$

- One can show that this reduces to the error function in the Airy case

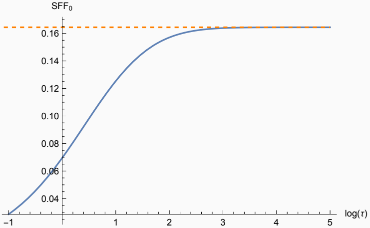
Higher order corrections to SFF

- In our recent work, we have developed a method to compute higher order corrections to SFF in the τ-scaling limit [KO-Sakai]

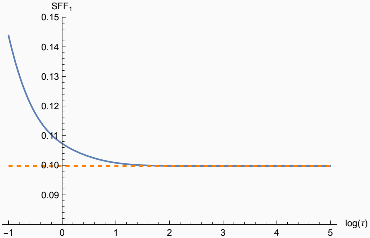
$$\mathsf{SFF}_{\mathsf{conn}} = \sum_{n=0}^{\infty} \hbar^{2n-1} \mathsf{SFF}_n(\beta, \tau)$$

- Our method is based on the KdV equation underlying the matrix model of 2d gravity [Witten]

- In particular, we have computed the higher order corrections of SFF in JT gravity in the τ-scaling limit



SFF_0



SFF_1

- SFF_0 exhibits the ramp-plateau behavior
- Late time value of SFF_1 gives a correction to the plateau
- Early time divergence of SFF_1 is an artifact of τ-scaling

Early time divergence of SFF_1

- Genus-zero part of matrix model correlator has a universal form

$$\langle Z(\beta_1) Z(\beta_2) \rangle_{g=0} = \frac{\sqrt{\beta_1 \beta_2}}{\beta_1 + \beta_2}$$

- Negative power of τ appears in the small $\hbar$ expansion

$$\begin{aligned} &\langle Z(\beta + i\tau\hbar^{-1}) Z(\beta - i\tau\hbar^{-1}) \rangle_{g=0} \\ &= \frac{\sqrt{\beta^2\hbar^2 + \tau^2}}{2\beta\hbar} = \frac{1}{2\beta\hbar}\left[\tau + \frac{\beta^2\hbar^2}{2\tau} - \frac{\beta^4\hbar^4}{8\tau^3} + \cdots\right] \end{aligned}$$

- Total SFF is finite at $\tau = 0$

Summary

- SFF is a useful diagnostics of random matrix statistics (quantum chaos) of energy eigenvelues
- Ramp of SFF corresponds to a wormhole in bulk gravity
- Plateau has a perturbative explanation by taking the τ-scaling limit
- We have computed the higher order corrections to SFF in the τ-scaling limit [KO-Sakai, to appear in January 2023?]
- We are also working on the multi-point functions of $Z(\beta)$'s in the τ-scaling limit [Anegawa-Iizuka-KO-Sakai, work in progress]

Questions/comments to ExU members

- Perturbative explanation of plateau is nice, but the τ-scaling limit throws away most of the contributions of moduli space integral
- We still do not understand non-perturbative effects which might contribute to the plateau
- What is the non-perturbative definition of (2d) quantum gravity?