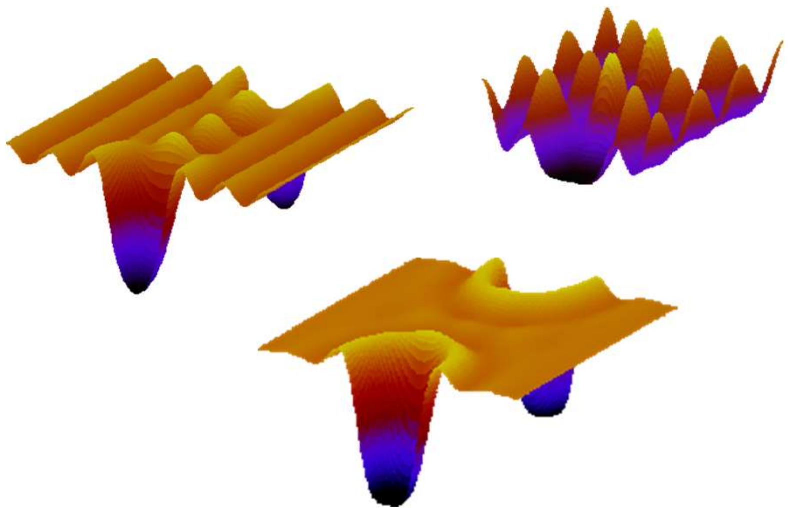
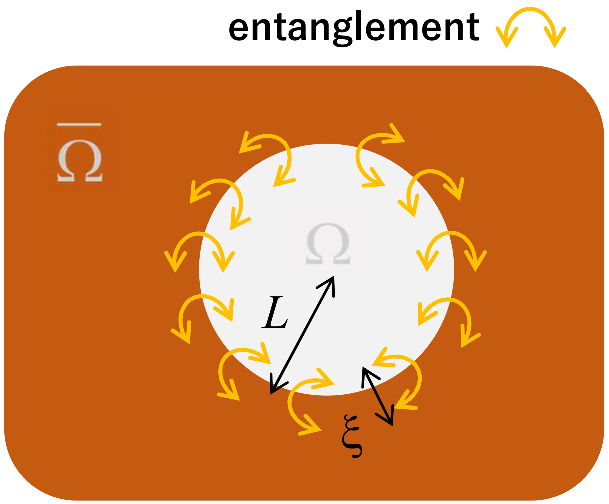
Entanglement entropy of quantum Hall systems in torus and spherical geometry

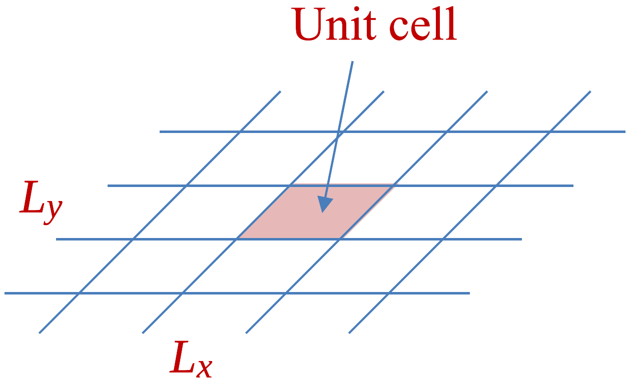
Tohoku Univ.   N. Shibata

- Density matrix renormalization group method
- Ground state of quantum Hall system
- Topological entanglement entropy of fractional quantum Hall states





Application of DMRG to 2D systems



Periodic boundary conditions
for both x and y directions

$k_y = 2\pi n/L_y = X_n / l^2$

Initial basis states

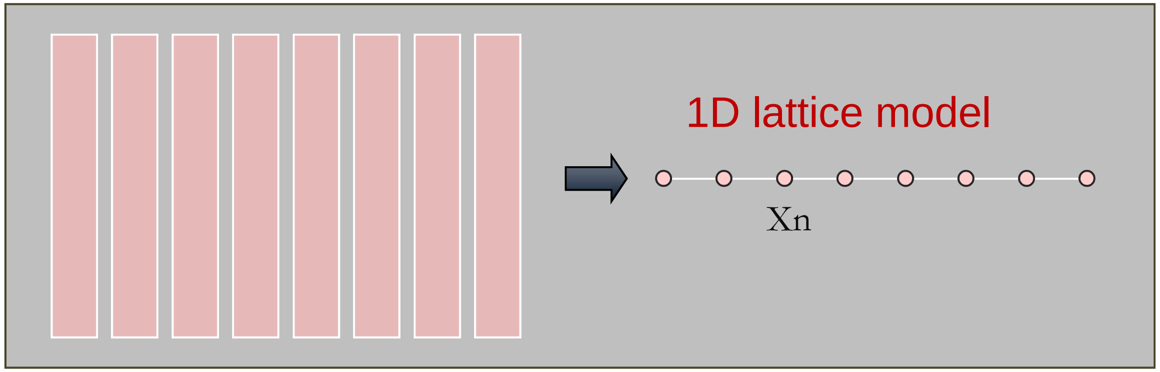
$$\varphi_{XN}(\mathbf{r}) = \exp\left[\, i\,\frac{X_n\, y}{l^2} - \frac{(x-X_n)^2}{2l^2}\right] H_N\left(\frac{x-X_n}{l}\right)$$

H_N : Hermite polynomials

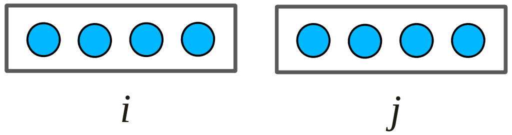
One particle states are uniquely
specified by X_n and N

X_n: guiding center
N : Landau level index

Mapping on to effective 1D lattice model



Density matrix renormalization



Ground state $| \Psi > = \Psi_{ij} \, |i > | \, j >$

Density matrix $\rho_{ii'} = \sum_j \Psi^*_{ij} \Psi_{i'j}$

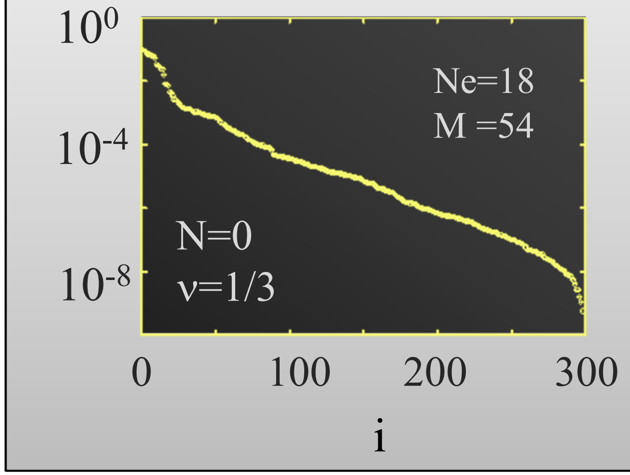
Norm $< \Psi \, | \Psi > = \mathrm{Tr} \, \rho$

Ground state energy (Ne=10)

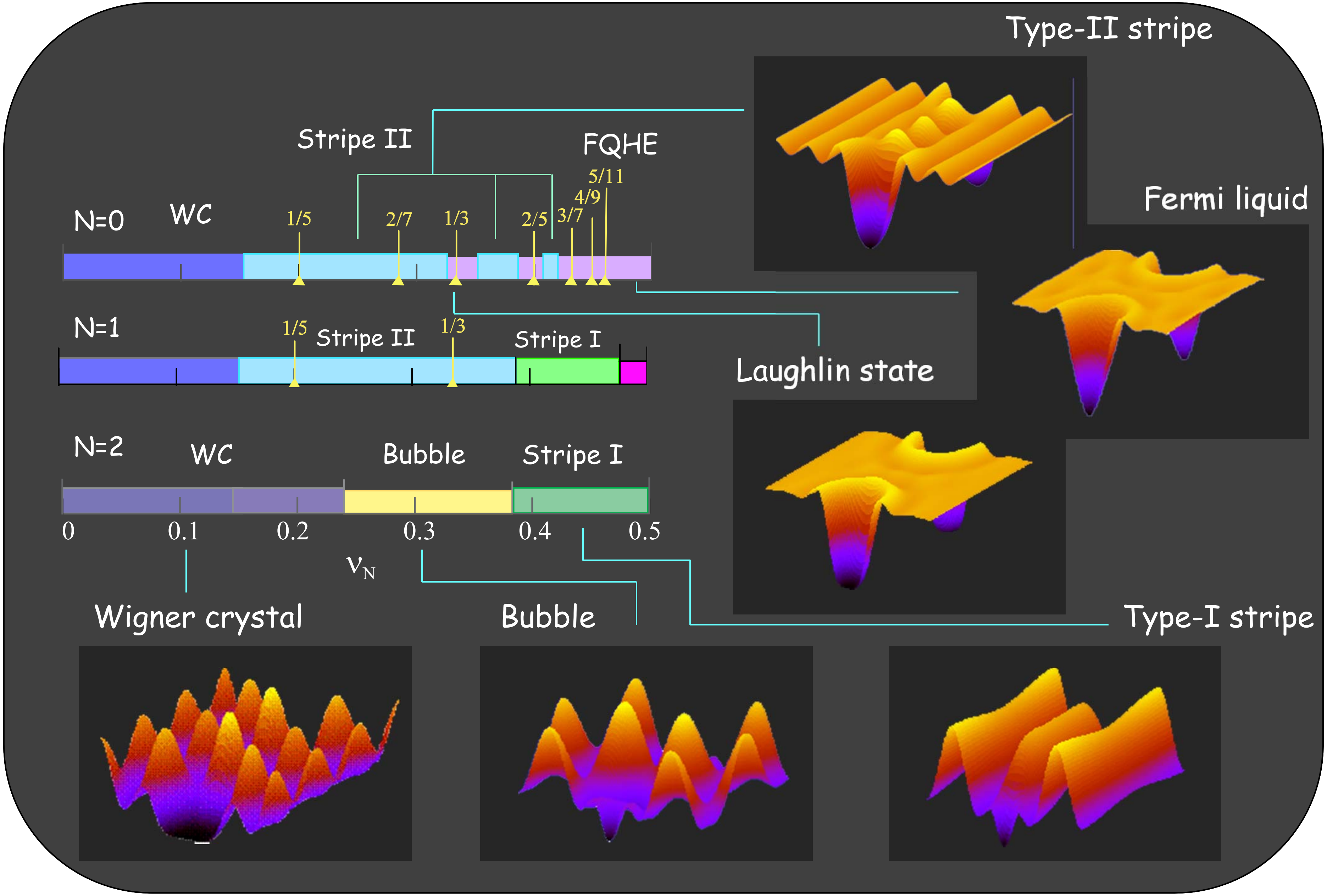
DMRG	m=100	-3.239340
	m=200	-3.239686
	m=300	-3.239981
	m=400	-3.239993
Exact		-3.239995

N=2
ν=1/2

Eigenvalues of ρ

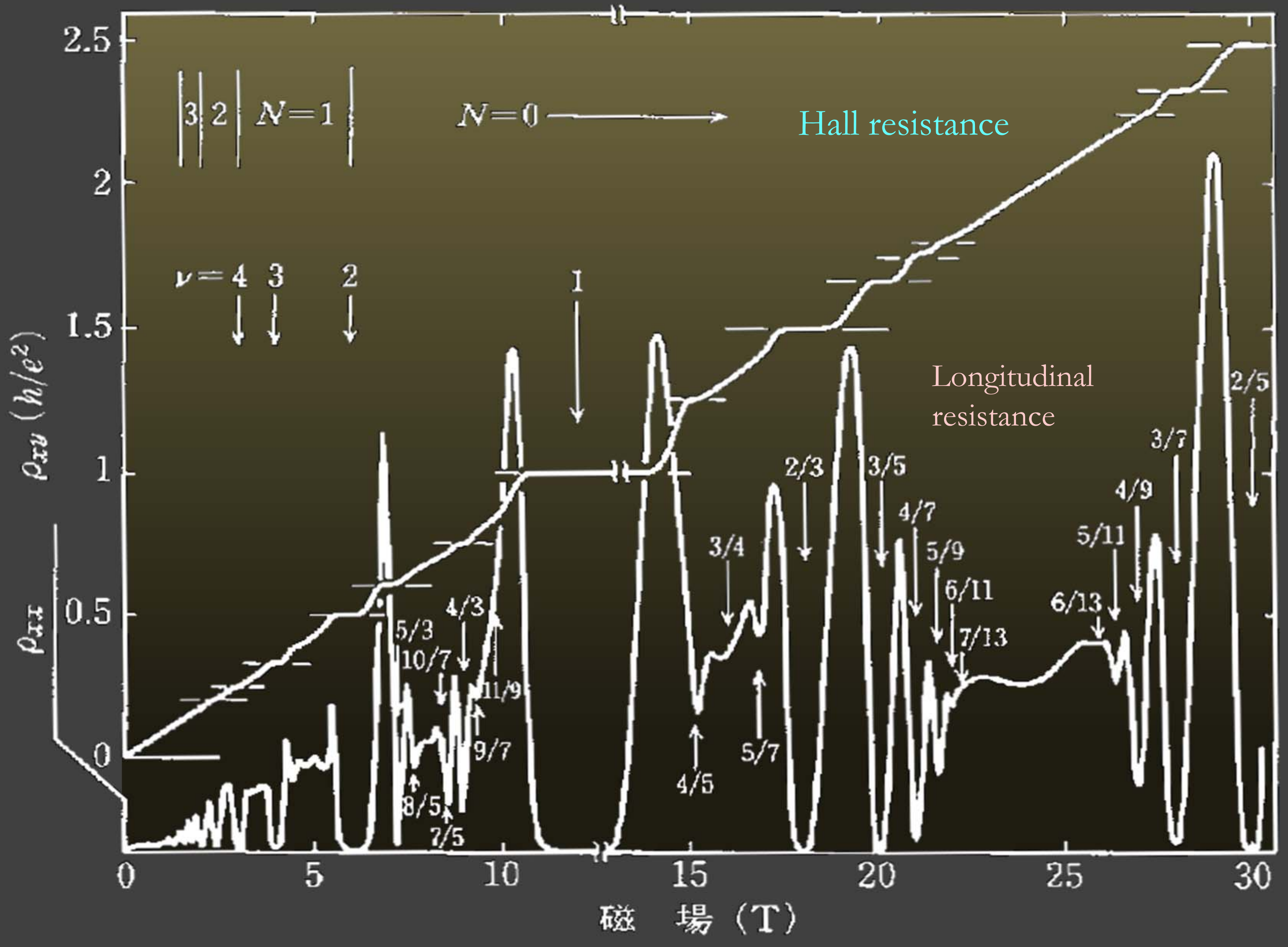


Ground state of quantum Hall systems



Fractional quantum Hall effect

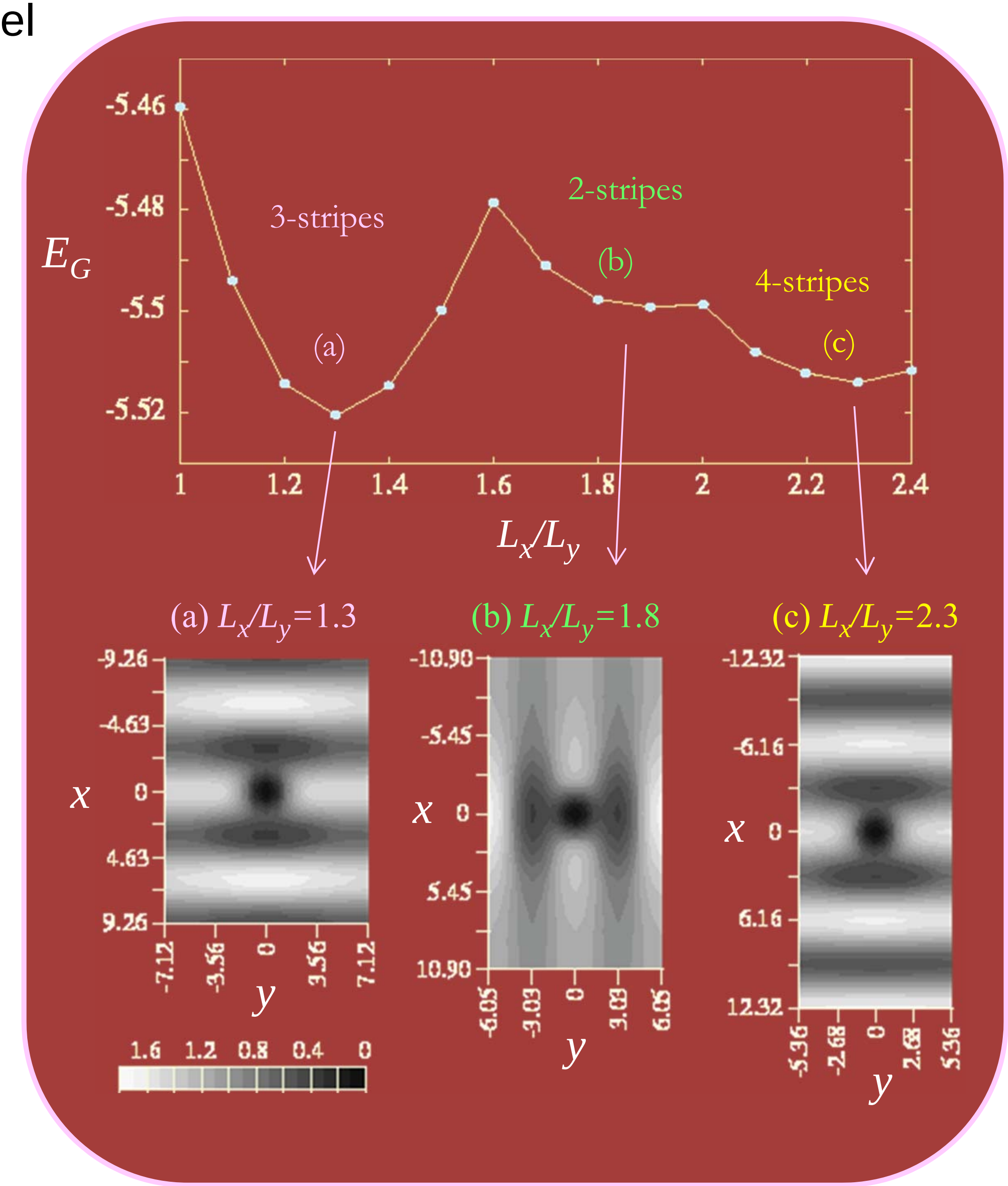
R. Willett *et al* (1987)



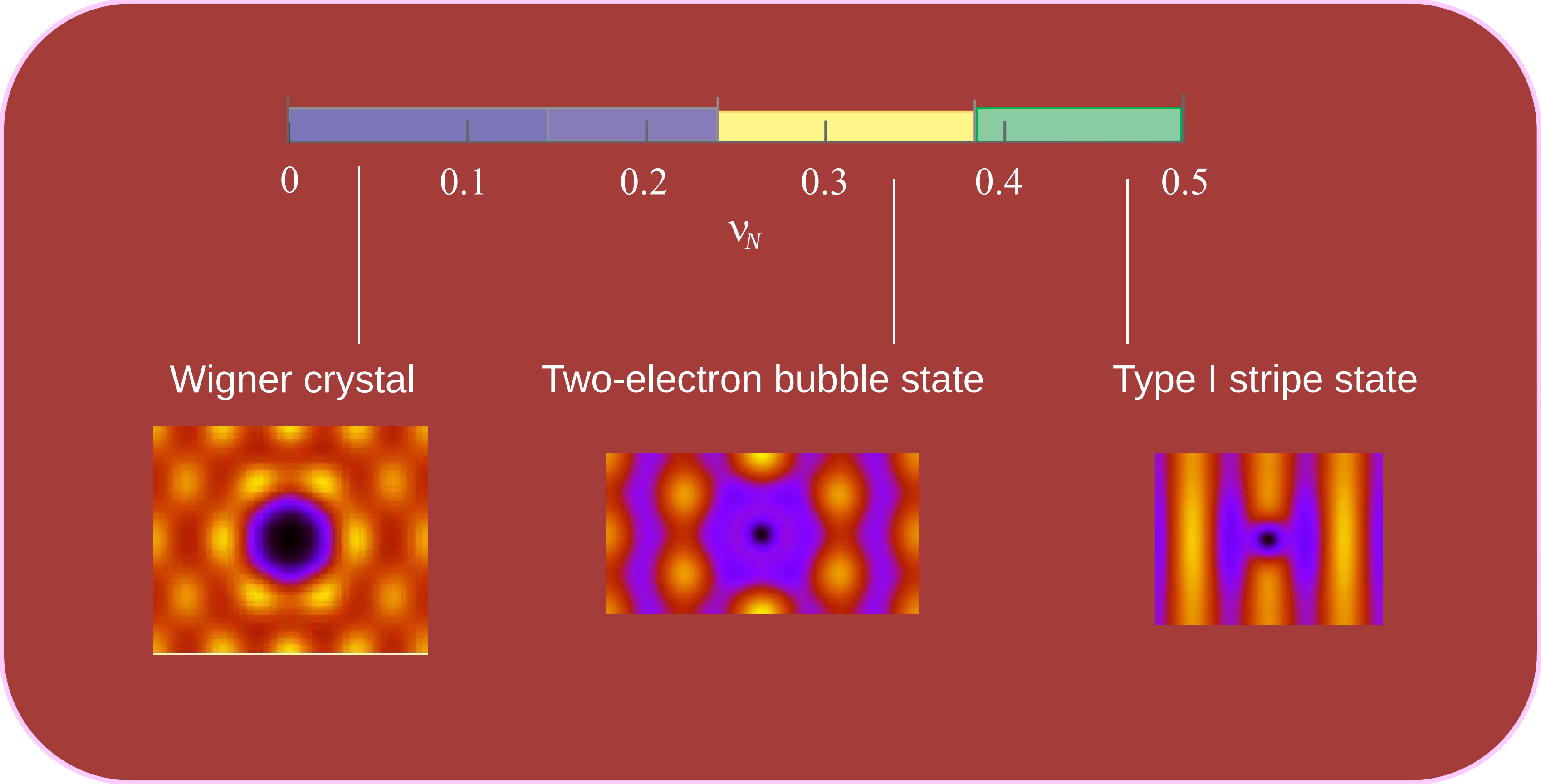
Stripe state

N=2 Landau level

$\nu=3/7$



Ground state of N=2 Landau level



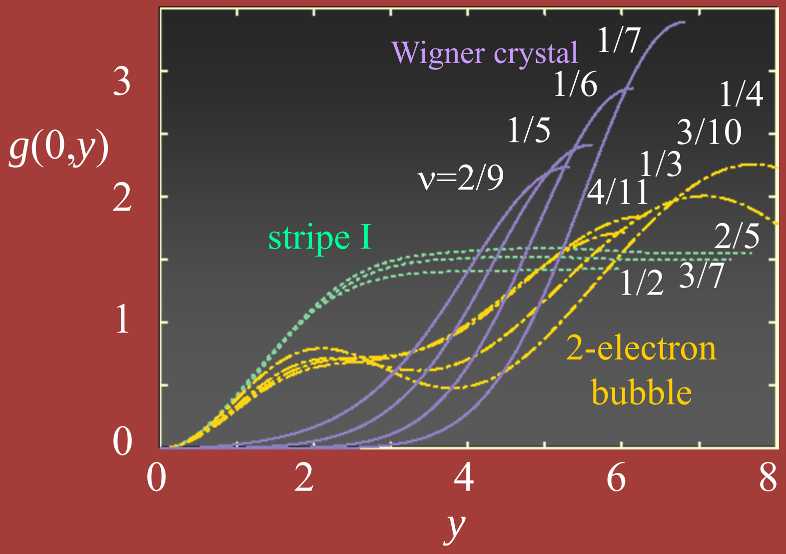
Wigner crystal

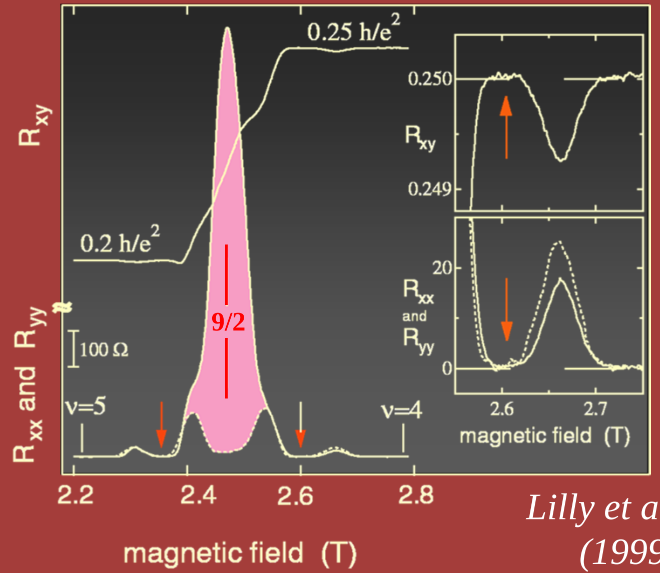
Two-electron bubble state

Type I stripe state

Shibata and Yoshioka: Phys. Rev. Lett. **86** 5755 (2001)

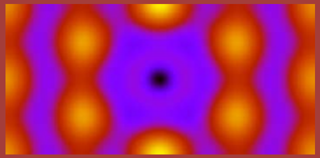
Stripe and bubble states



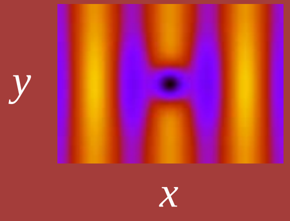


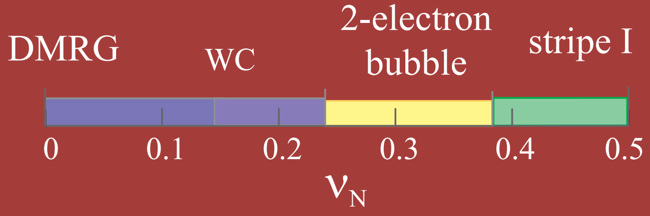
Lilly et al.
(1999)

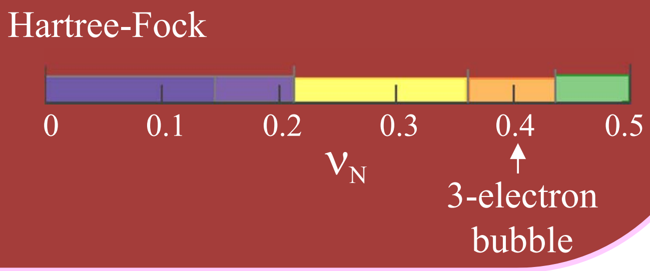
Two-electron bubble state



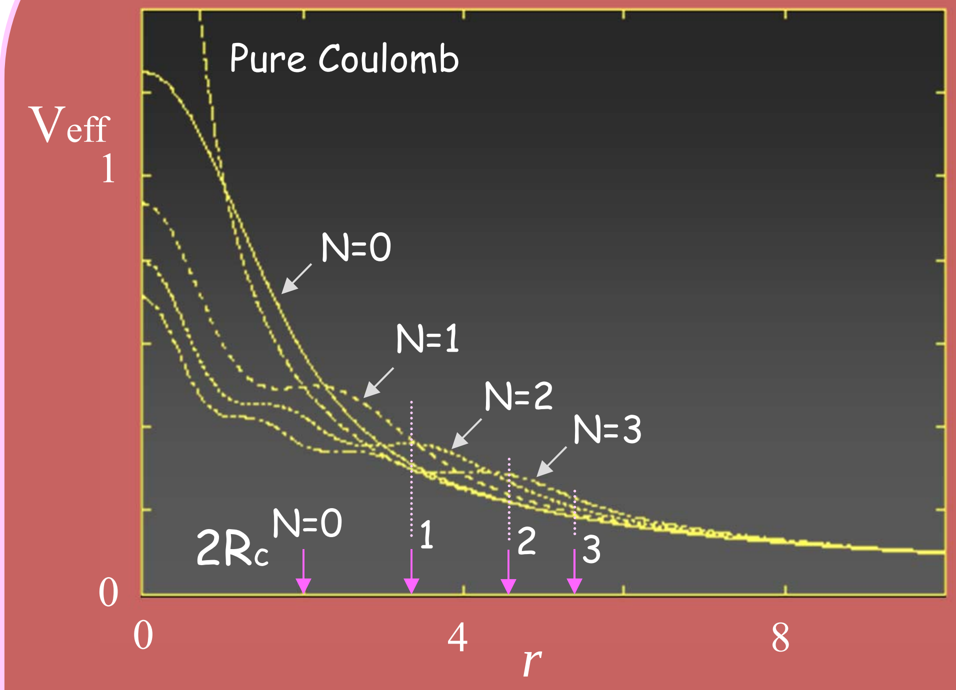
Type I stripe state



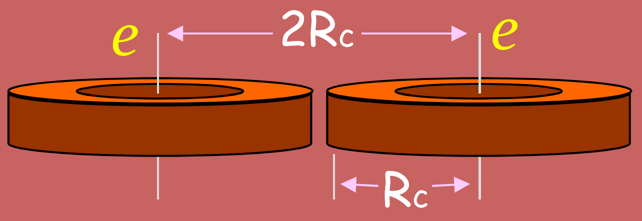




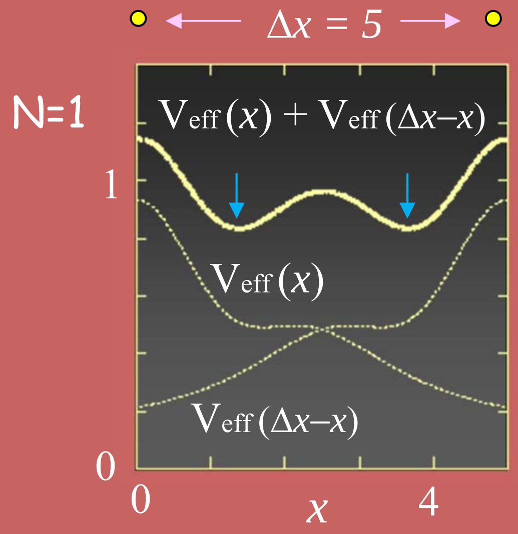
Effective interaction in higher Landau levels

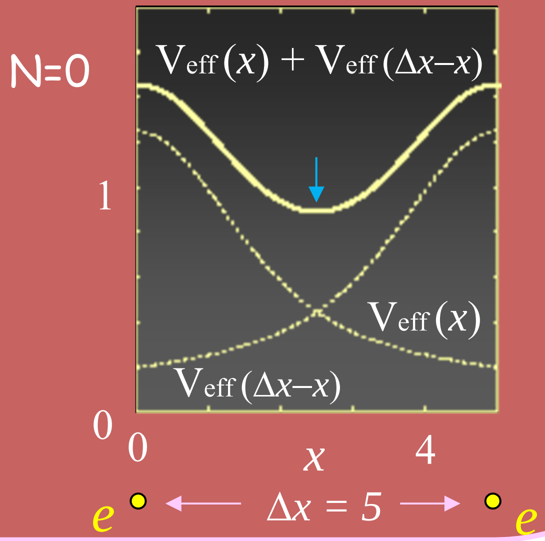


R_c : cyclotron radius



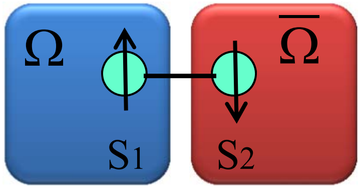
Potential energy generated by the two electrons separated by Δx





Entanglement entropy

S=1/2 2-Spin system



$S_1 = \uparrow$ or $\downarrow$

$S_2 = \uparrow$ or $\downarrow$

Wave function

$$|\Psi> = \sum_{S_1S_2} \Psi_{S_1S_2}\ |S_1>|S_2>$$

$$\Psi_{S1\ S2} : \begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\downarrow\uparrow} & \Psi_{\downarrow\downarrow} \end{pmatrix} = \Psi$$

Reduced density matrix

$$\rho_\Omega = \Psi^* \Psi^t$$

$$(\rho_\Omega)_{ii'} = \sum_j \Psi^*_{ij} \Psi_{i'j}$$

Entanglement entropy

$$S_\Omega = -Tr\ \rho_\Omega \ln \rho_\Omega$$

S_1 is independent of S_2

- Not correlated -

$$\frac{1}{\sqrt{2}}(\underset{S_1S_2}{|\uparrow\uparrow>} + |\uparrow\downarrow>) \quad ➡ \quad \Psi = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \underset{S_1}{|\uparrow>} \otimes \underset{S_2}{(|\uparrow> + |\downarrow>)}$$

$$\rho_\Omega = \overset{\Psi^*}{\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}} \overset{\Psi^t}{\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$S_\Omega = -\ln 1 = 0$$

disentangled

S_1 depends on S_2

- correlated -

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow> - |\downarrow\uparrow>) \quad ➡ \quad \Psi = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Singlet state

$$\rho_\Omega = \overset{\Psi^*}{\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}} \overset{\Psi^t}{\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

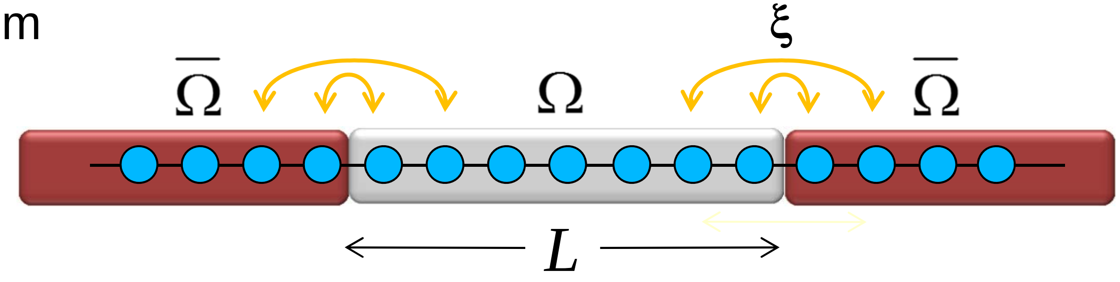
$$S_\Omega = 2\left(\frac{1}{2}\ln 2\right) = \ln 2$$

entangled

Scaling of entanglement entropy

$$S_{\Omega(L)} = -Tr\,\rho_{\Omega(L)} \ln \rho_{\Omega(L)}$$

1D system



Short range correlation :

$$S_{\Omega(L)} \approx const.$$

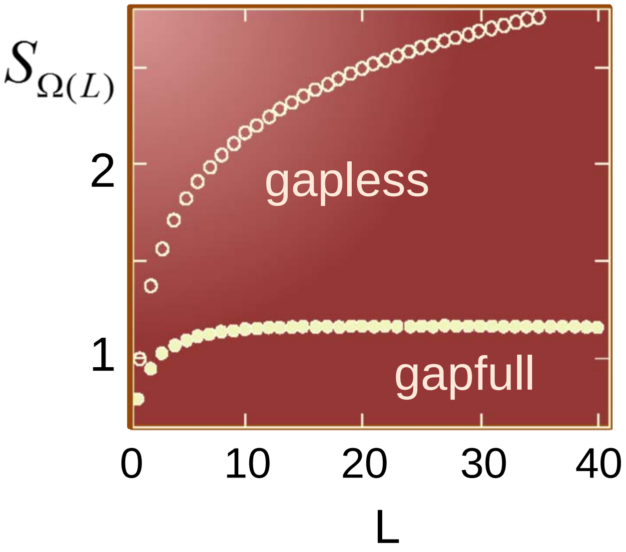
$(\, L \;>>\;$ **correlation length** $\xi\,)$

Power law correlation : (1D critical system)

$$S_{\Omega(L)} \approx \frac{c}{3} \ln L + s_0$$

$$L \rightarrow \infty$$

1D-XXZ model



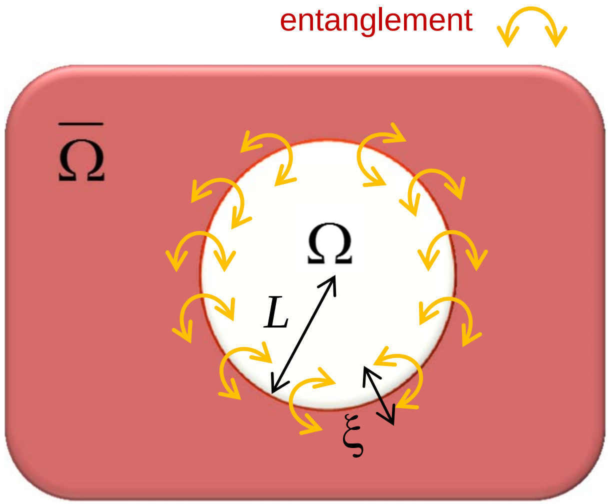
Scaling of entanglement entropy

Short range correlation $(\boldsymbol{L} >> \xi)$

D-dimensional system

Area law $S_{\Omega(L)} \propto L^{D-1}$

L^{D-1} : boundary size (length)



Topological order in 2D

$$S_{\Omega(L)} = \underbrace{\alpha L}_{\text{Boundary term}} - \underbrace{\ln D}_{\text{Topological term}}$$

Non-trivial universal correction

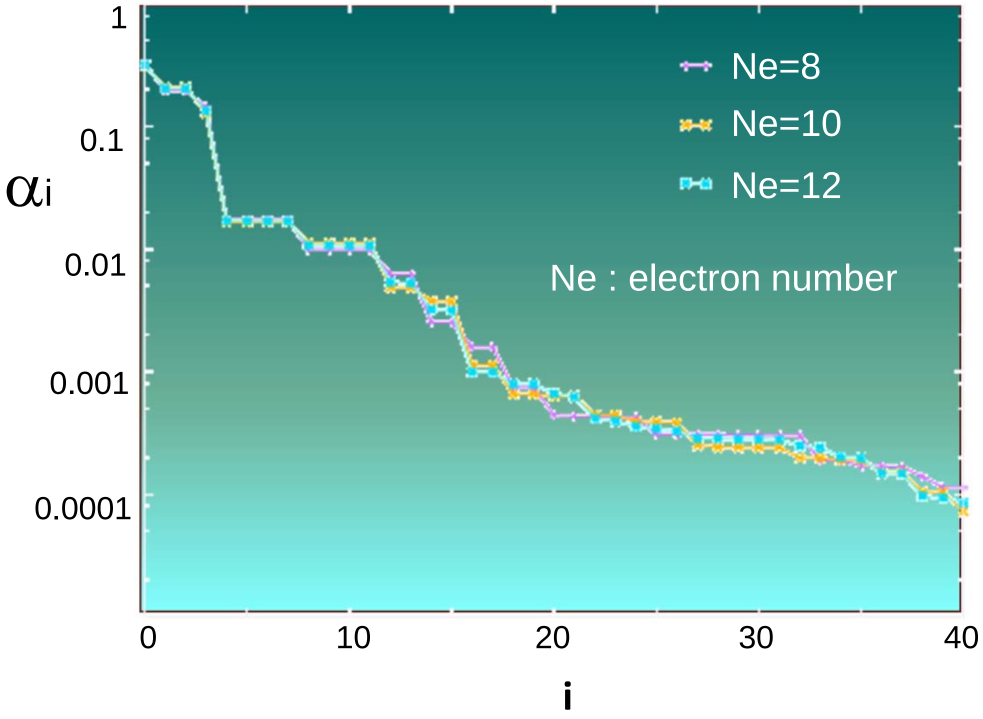
Fractional quantum Hall state

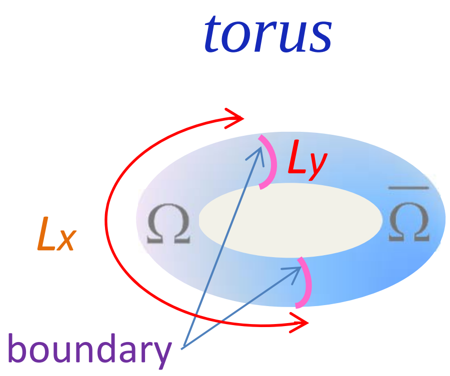
$\nu = 1/m$ Laughlin state : $D = m^{1/2}$

Density matrix eigenvalues α_i

Torus geometry

Fractional quantum Hall state $\nu = 1/3$

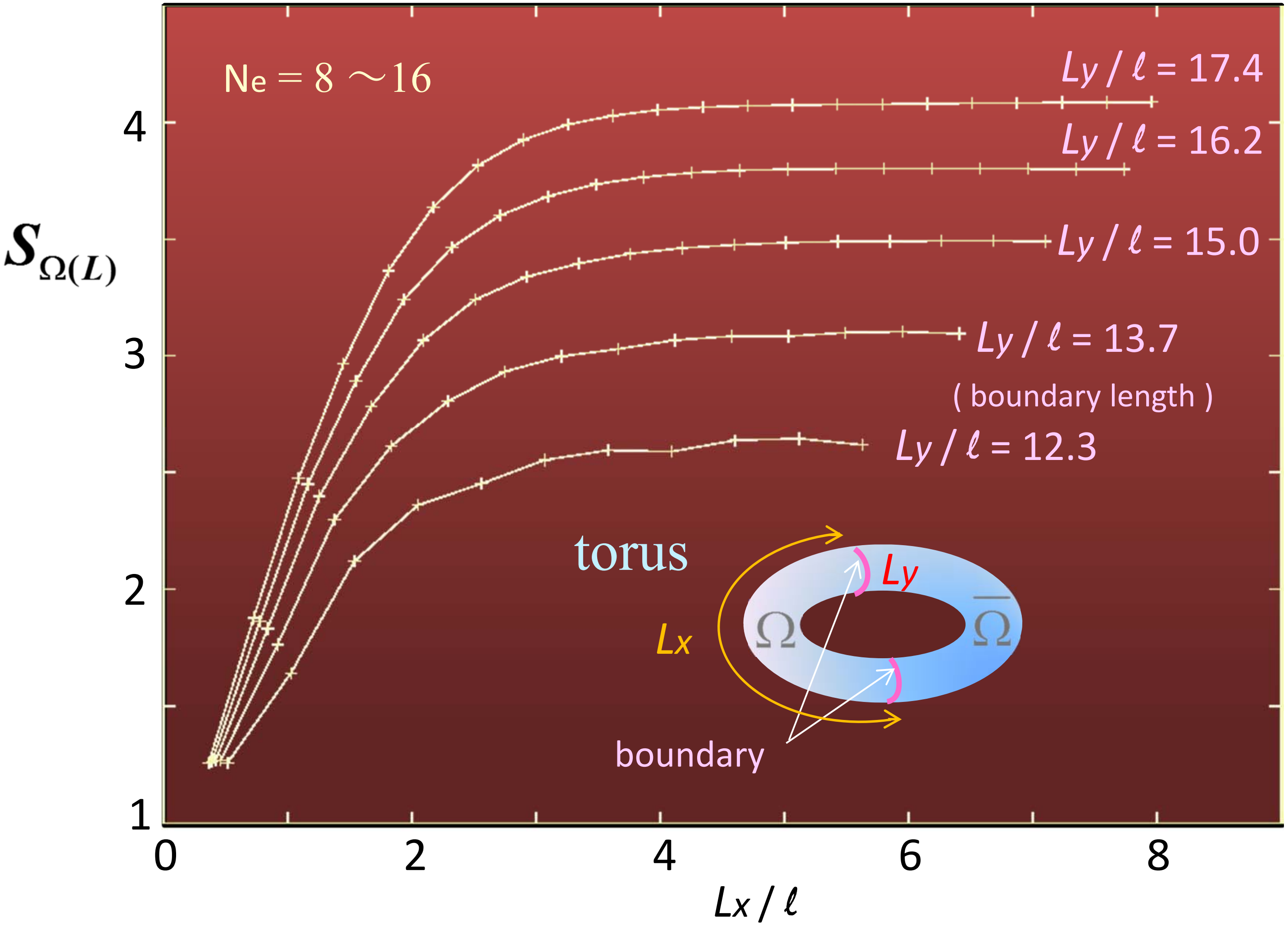




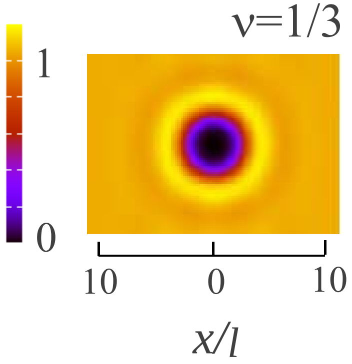
Entanglement entropy

Torus geometry

Fractional quantum Hall state $\nu = 1/3$

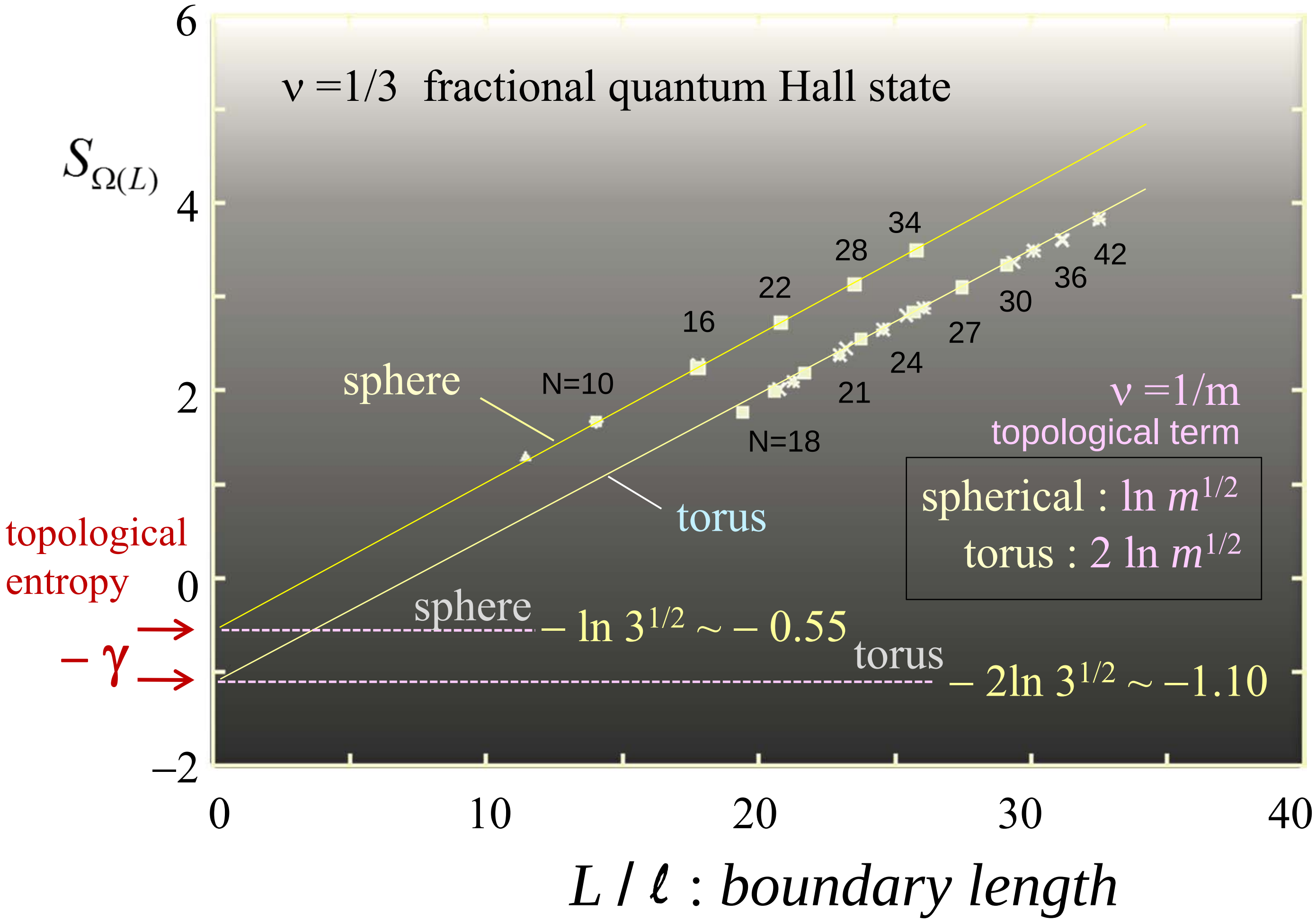


Pair correlation functions

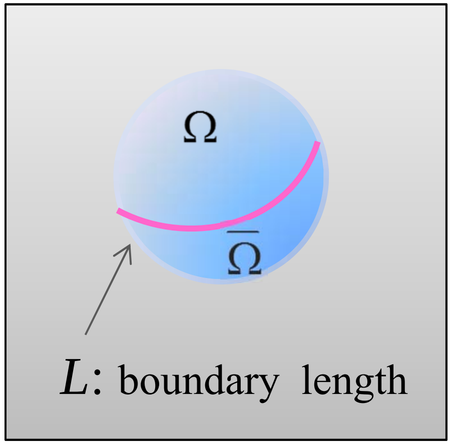


Entanglement entropy

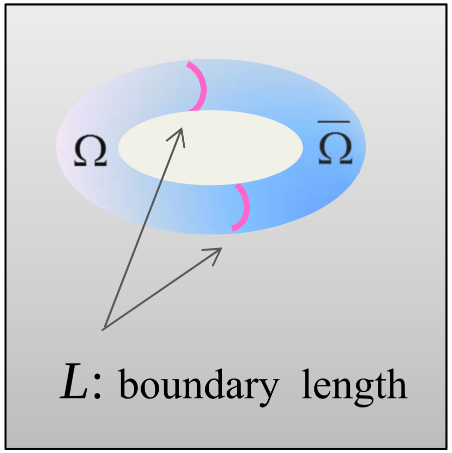
Entanglement entropy $S_{\Omega(L)} = \underbrace{\alpha L}_{\text{boundary term}} - \underbrace{\gamma}_{\text{topological term}}$



sphere



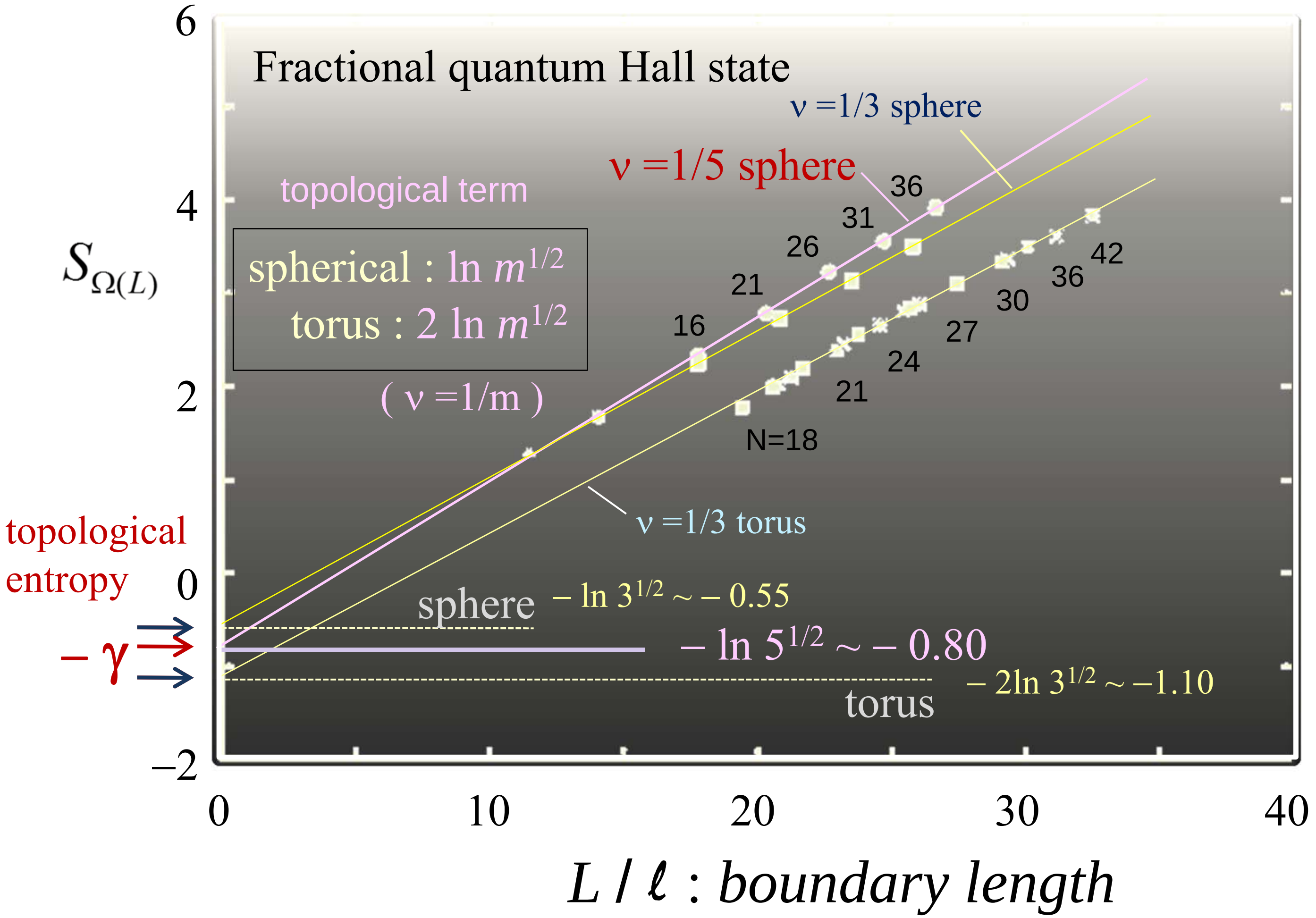
torus



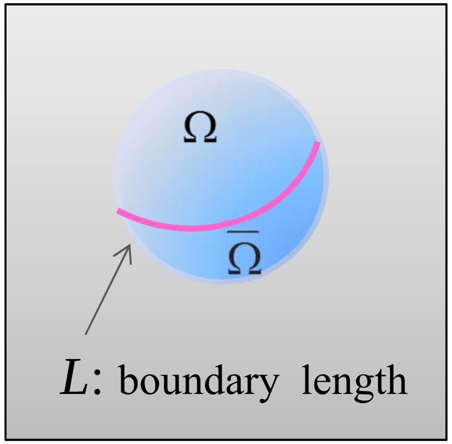
Entanglement entropy

Entanglement entropy $S_{\Omega(L)} = \underline{\alpha L} - \underline{\gamma}$

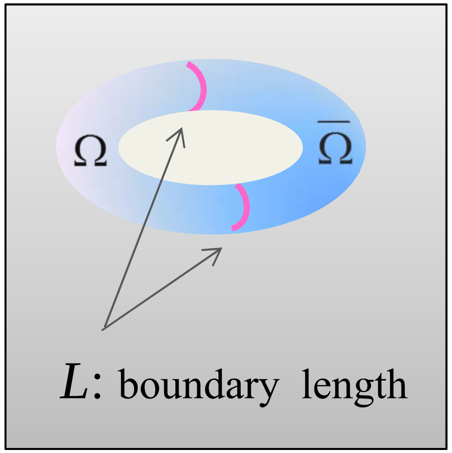
boundary term topological term



sphere



torus



Summary and future collaboration

Topological order in FQHS is confirmed by DMRG

- **Searching nontrivial topological order in 2D quantum systems**
- **Finding a method for experimental confirmation**