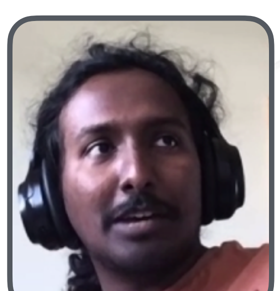
ON THE LINEAR GROWTH OF

QUANTUM CIRCUIT COMPLEXITY

JENS EISERT, FU BERLIN

With J. Haferkamp, P. Faist, N. Kothakonda, A. Munson, N. Yunger Halpern





Nature Physics 18, 528 (2022), Phys Rev A 106, 062417 (2022), Phys Rev Lett 127, 020501 (2021), Commun Math Phys (2022)

QUANTUM CIRCUIT COMPLEXITY

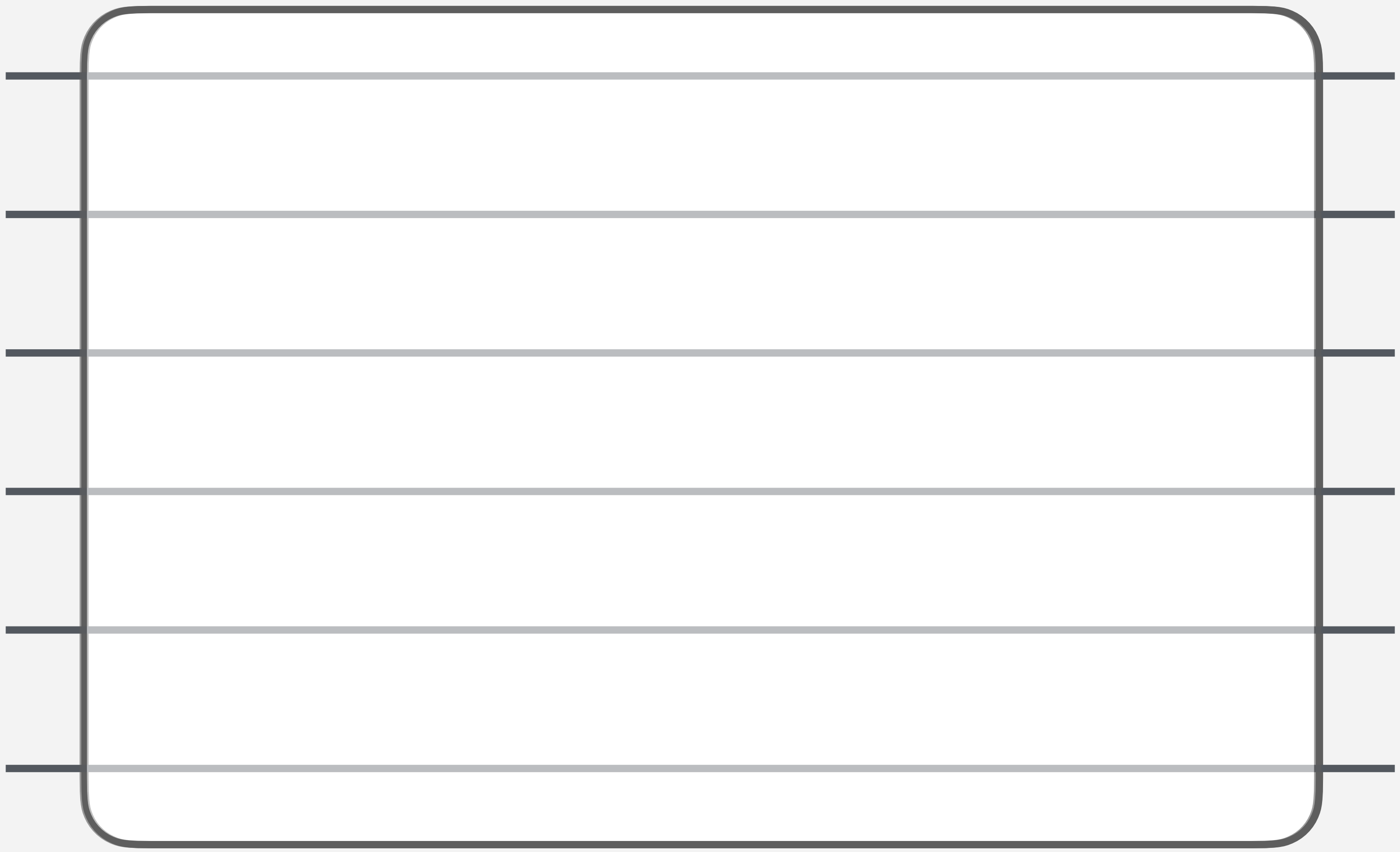
- High energy physics, holography and the wormhole growth paradox
- Quantum computing
- Statistical physics and thermodynamics

QUANTUM CIRCUIT COMPLEXITY

- High energy physics, holography and the wormhole growth paradox
- Quantum computing
- Statistical physics and thermodynamics
- This talk

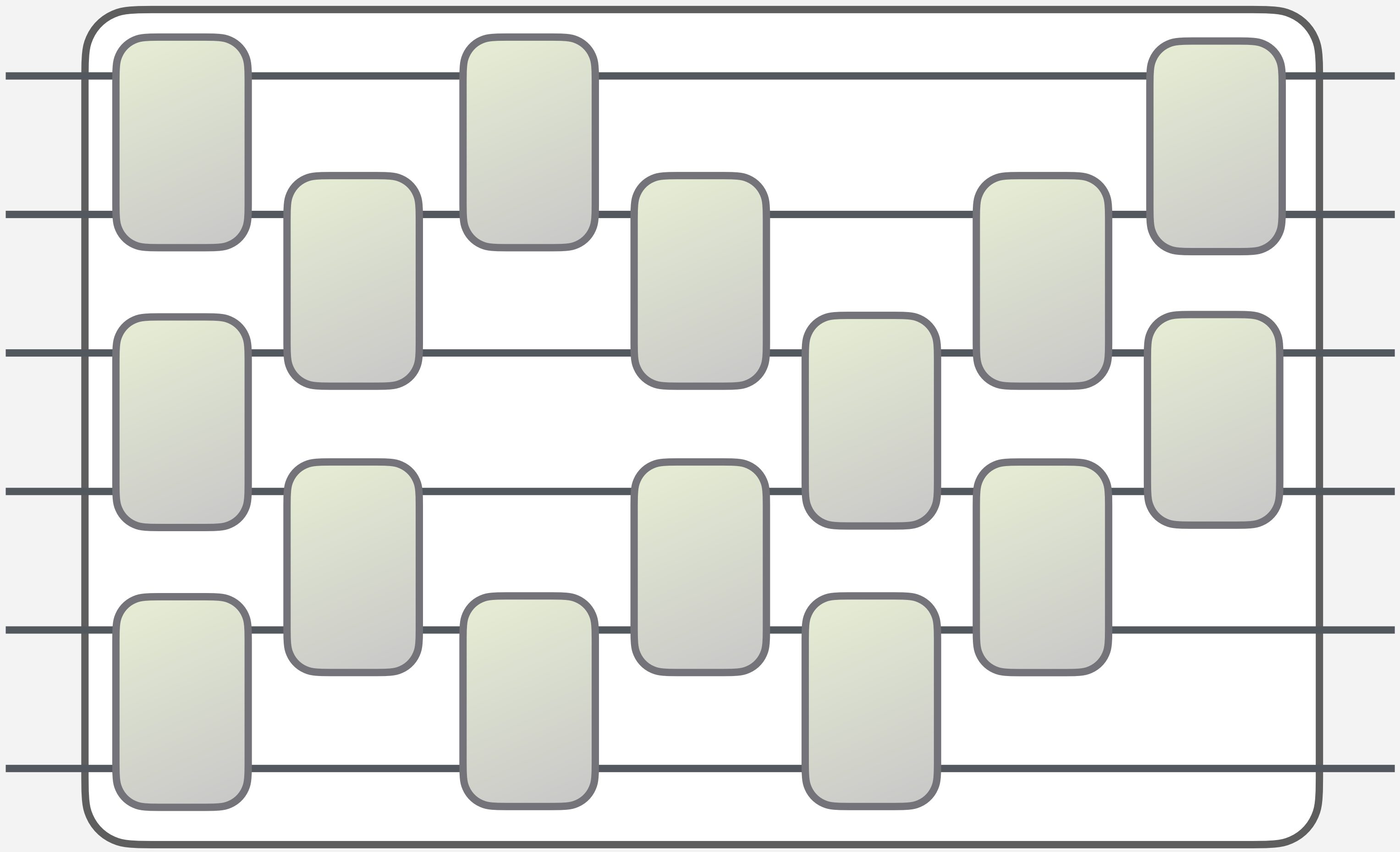
- **Complexity** is studied to make sense of the enormity of Hilbert space…

- **Circuit complexity** of a computation captures the number of elementary steps it minimally takes to determine its outcome



- Separate computational tasks into ‘**easy**’ and ‘**hard**’

- **Circuit complexity** of a computation captures the number of elementary steps it minimally takes to determine its outcome



- Separate computational tasks into ‘**easy**’ and ‘**hard**’
- In **quantum** setting relevant for phases of matter

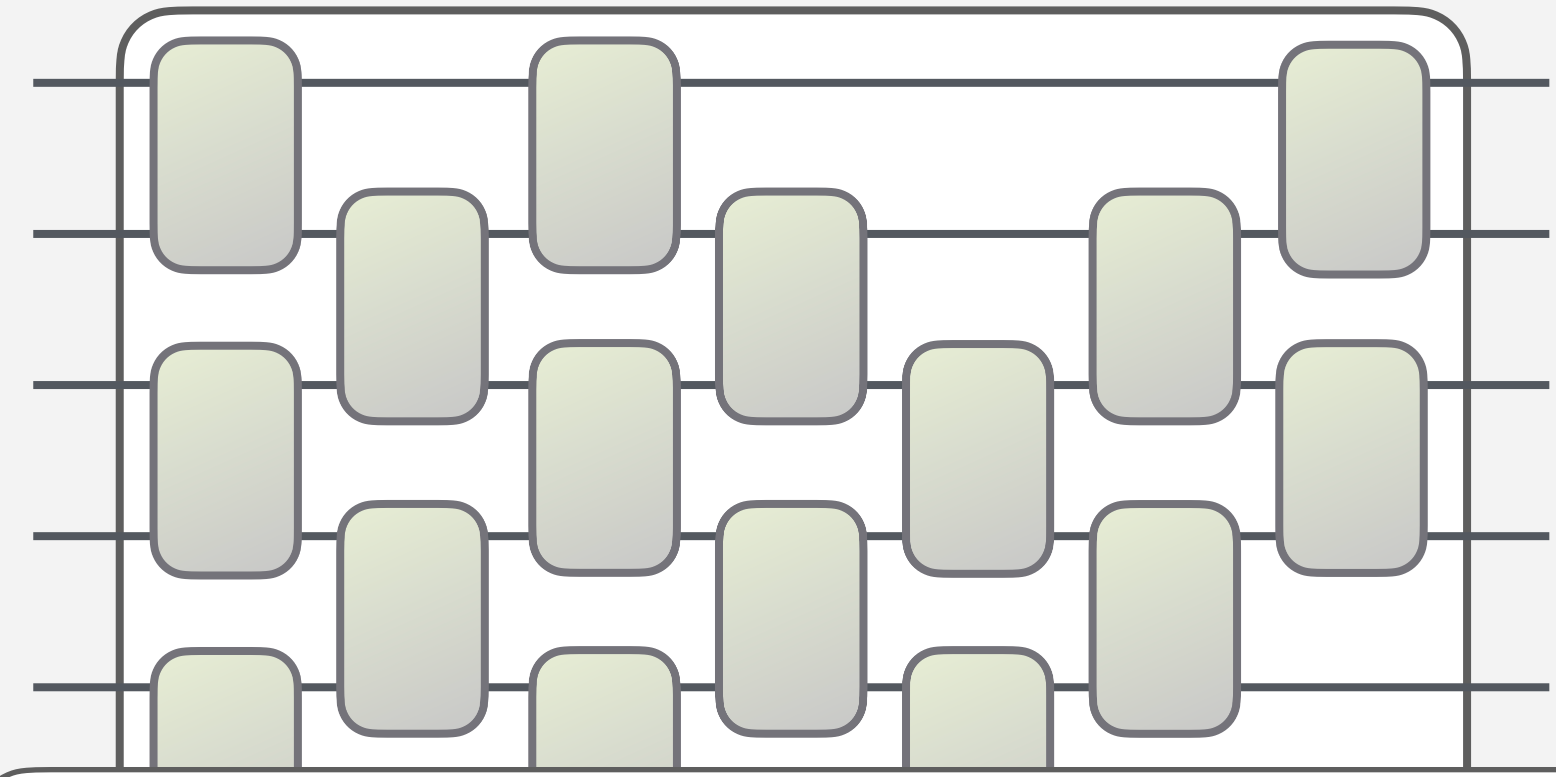
CIRCUIT COMPLEXITY

- **Circuit complexity** of a computation captures the number of elementary steps it minimally takes to determine its outcome

- **Definition 1** (Exact circuit complexities). *Let* $U \in \mathrm{SU}(2^n)$ *denote an* n*-qubit unitary. The (exact) circuit complexity* $C_{\mathrm{u}}(U)$ *is the least number of two-qubit gates in any circuit that implements* U*. Similarly, let* $|\psi\rangle$ *denote a pure quantum state vector. The (exact) state complexity* $C_{\mathrm{state}}(|\psi\rangle)$ *is the least number* r *of two-qubit gates* $U_1, U_2, \ldots, U_r$*, arranged in any architecture, such that* $U_1 U_2 \ldots U_r |0^n\rangle = |\psi\rangle$.

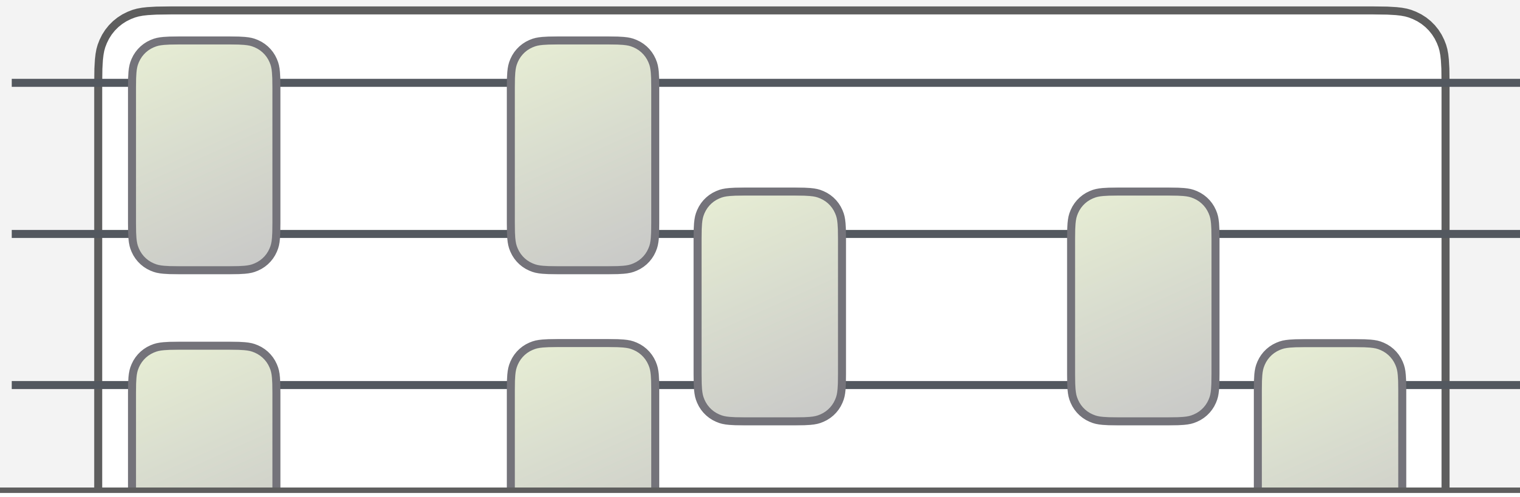
CIRCUIT COMPLEXITY

- **Circuit complexity** of a computation captures the number of elementary steps it minimally takes to determine its outcome



NOTORIOUSLY HARD TO COMPUTE

- **Circuit complexity** of a computation captures the number of elementary steps it minimally takes to determine its outcome



- The run-time of the best known algorithms for the T-count—deciding whether the optimal gate decomposition of a circuit that is given as a sequence of Clifford and T gates on n qubits involves fewer than or equal to m T gates or more—is $O(N^m \text{poly}(m, N))$, with $N := 2^n$.

Gosset, Gosset, Kliuchnikov, Mosca, Russo, Quant Inf Comp 14, 1277 (2014)
Aaronson, Gottesman, Phys Rev A 70, 02328 (2004)

- **Circuit lower bounds**

- **Circuit and state complexities** organize unitaries and quantum states

Topological phases

Thermalization

Long time quantum chaotic dynamics

$O(1)$ $O(n)$ $\mathrm{poly}(n)$ $\exp(n)$

Quantum states

Product states

Matrix product states, GHZ states

Multi-scale renormalization

Unitaries

Variational quantum circuits, QAOA

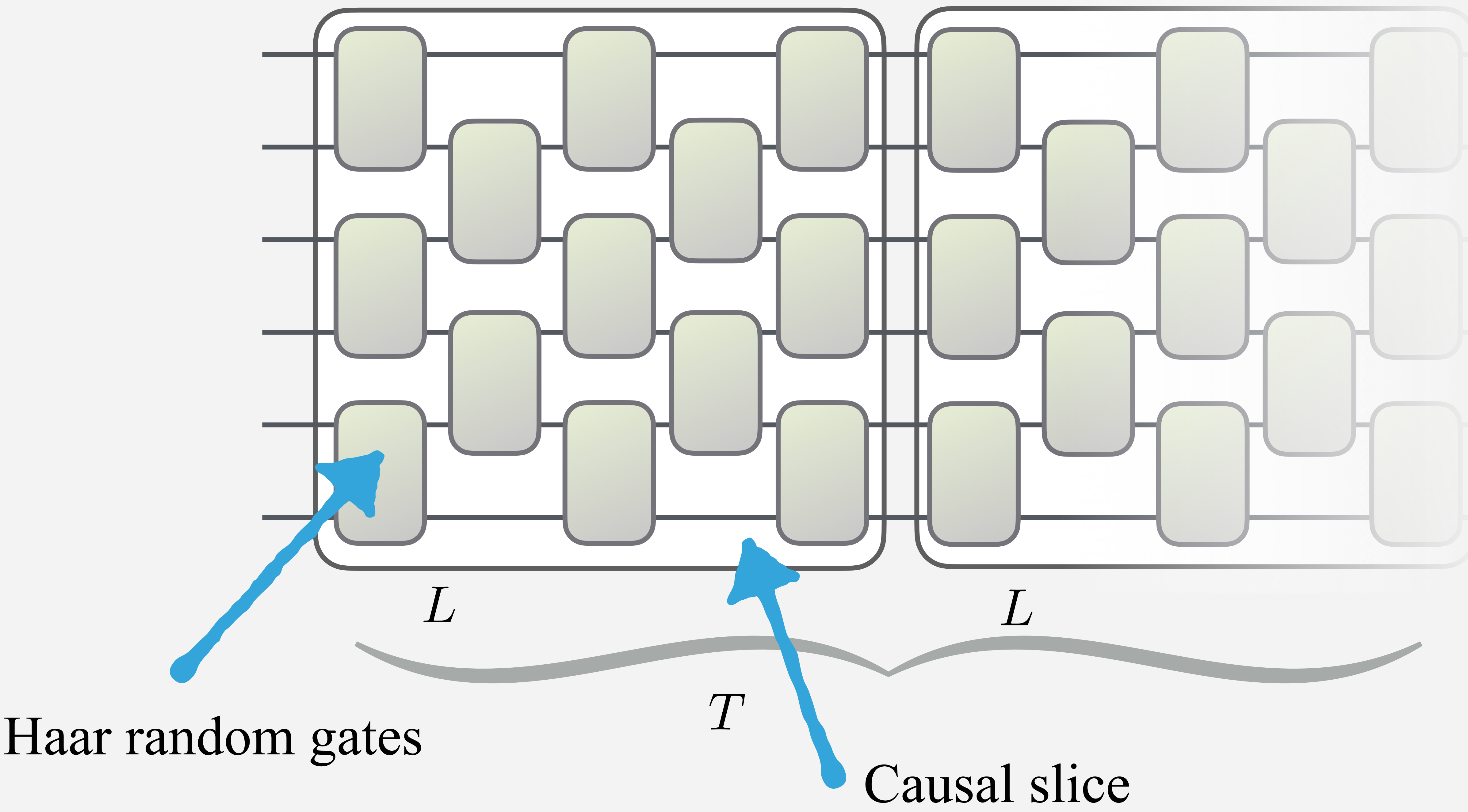
Circuits for classical shadows

Unitary k-designs

Polynomial random circuits

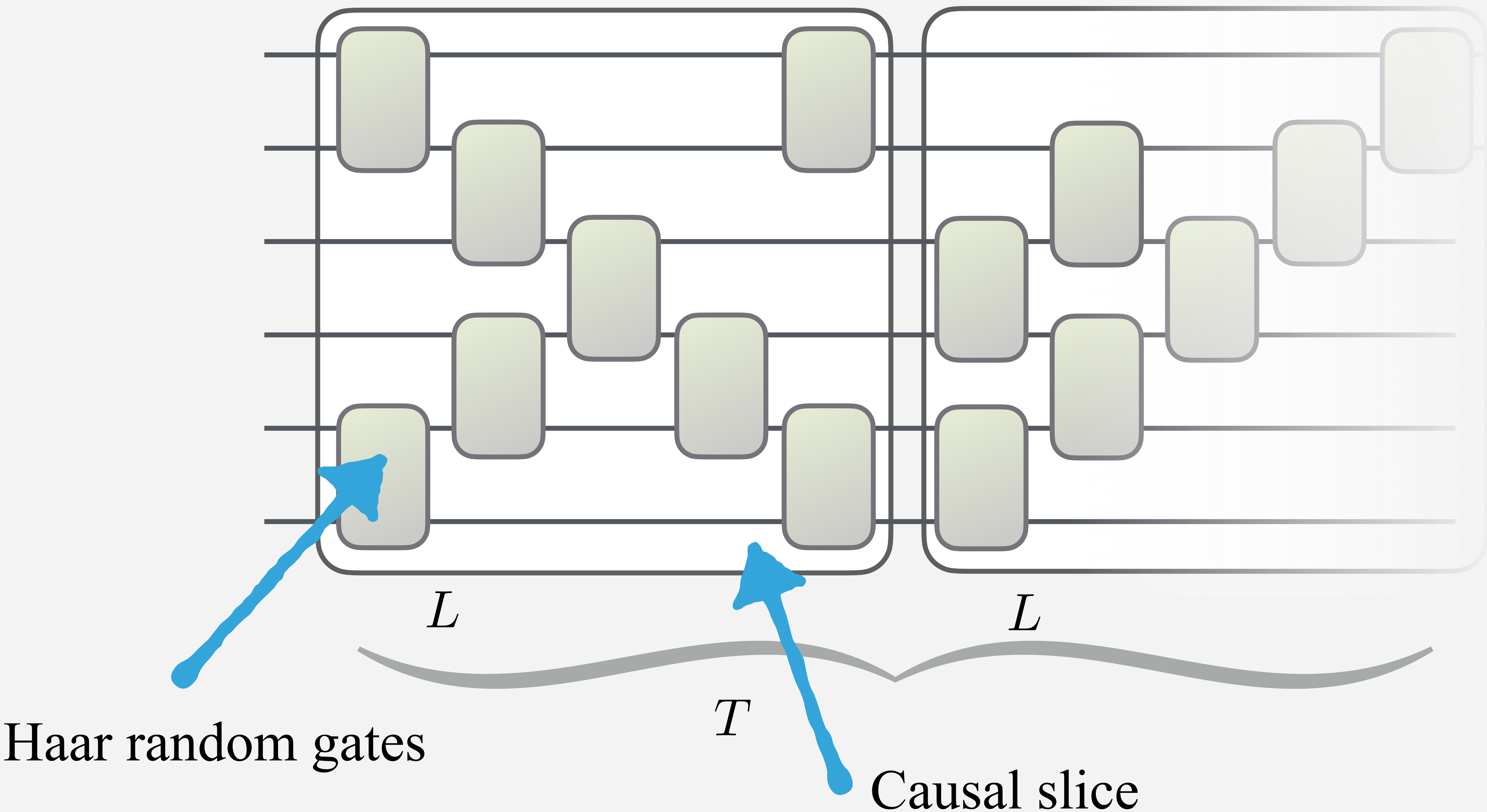
Haar-random unitaries

- **Random circuits:** Proxies for quantum chaotic dynamics
- Bricklayer circuits



RANDOM QUANTUM CIRCUITS

- **Random circuits:** Proxies for quantum chaotic dynamics
- Bricklayer circuits, random arrangements



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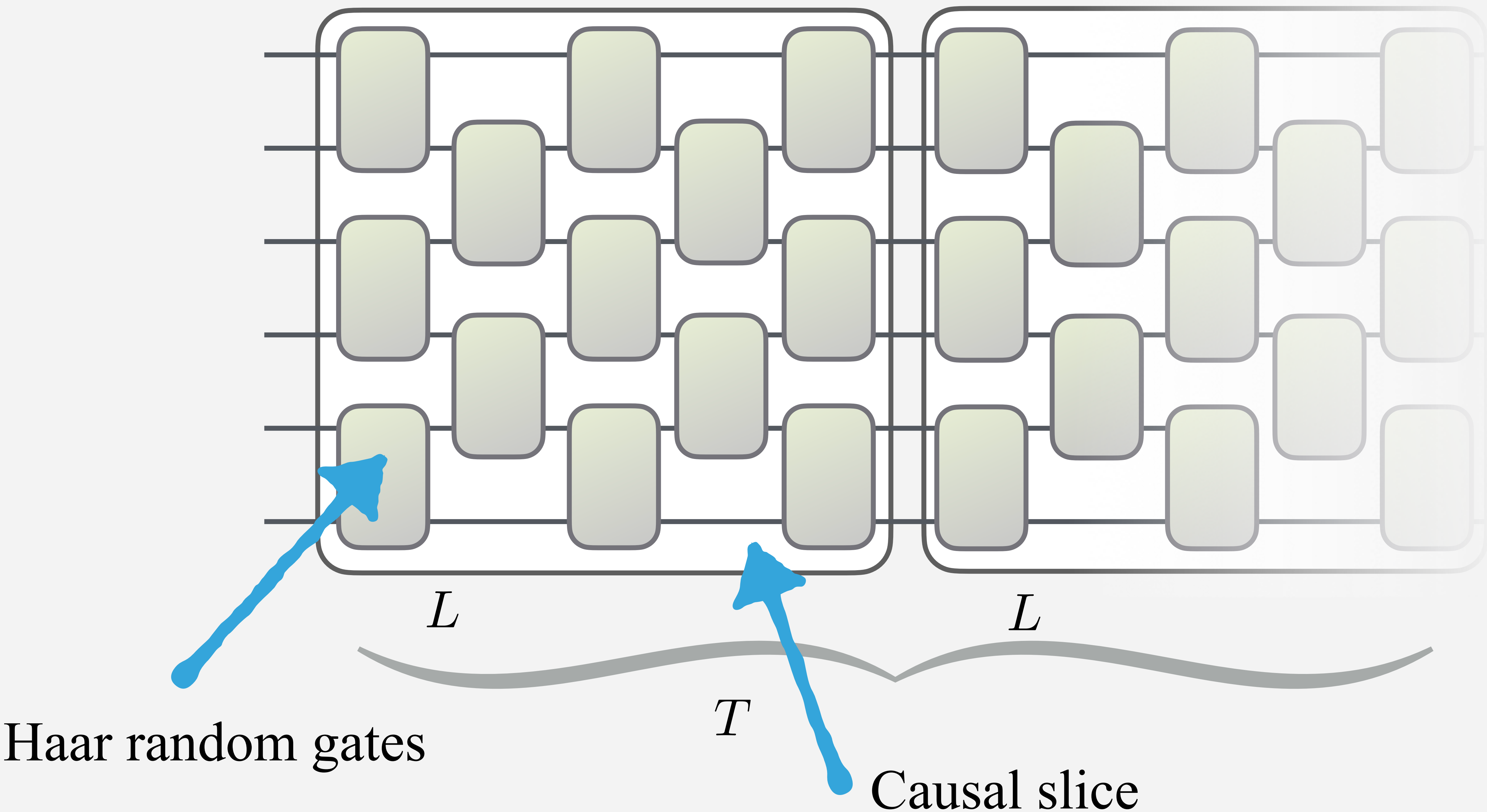

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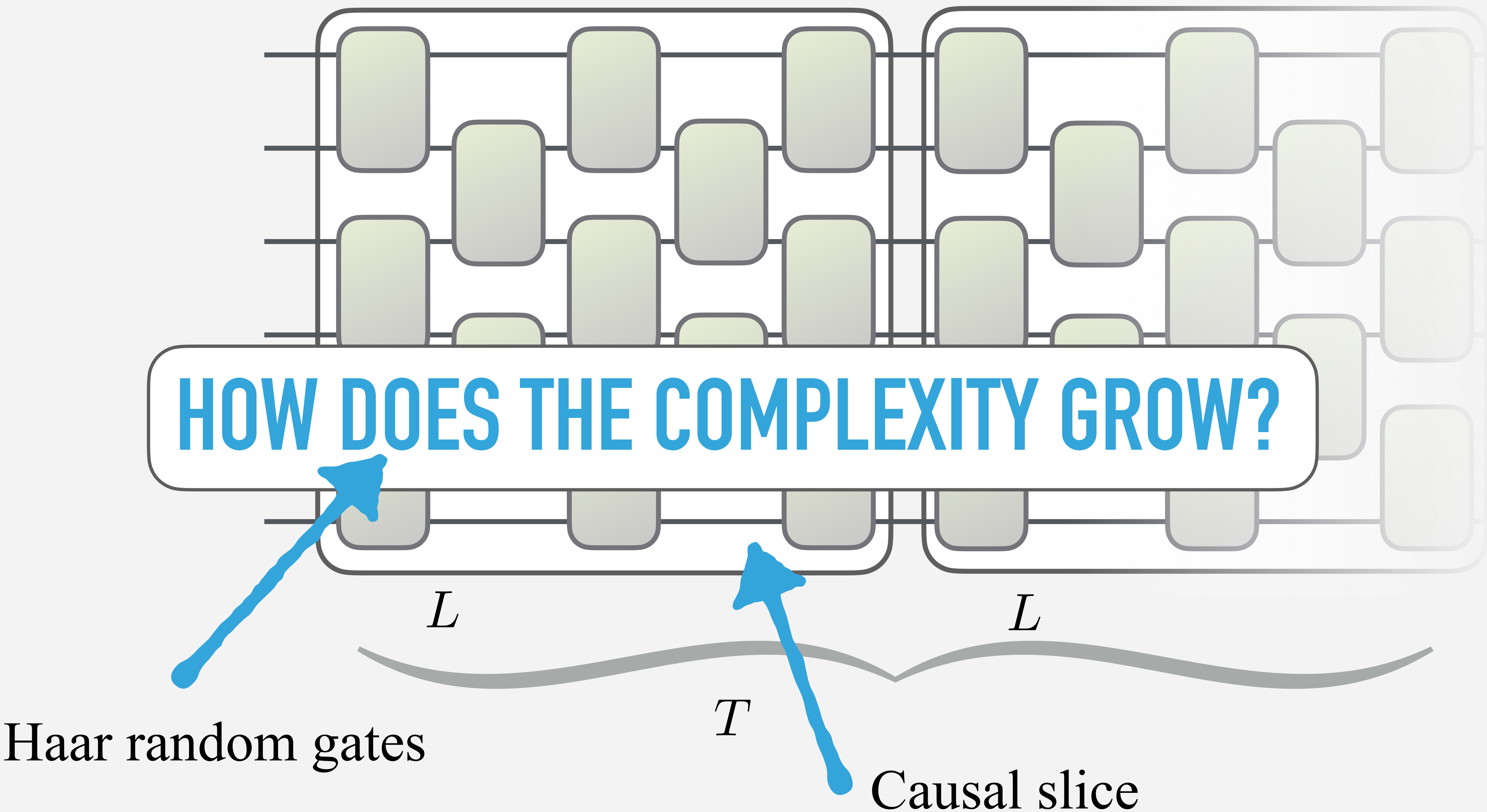
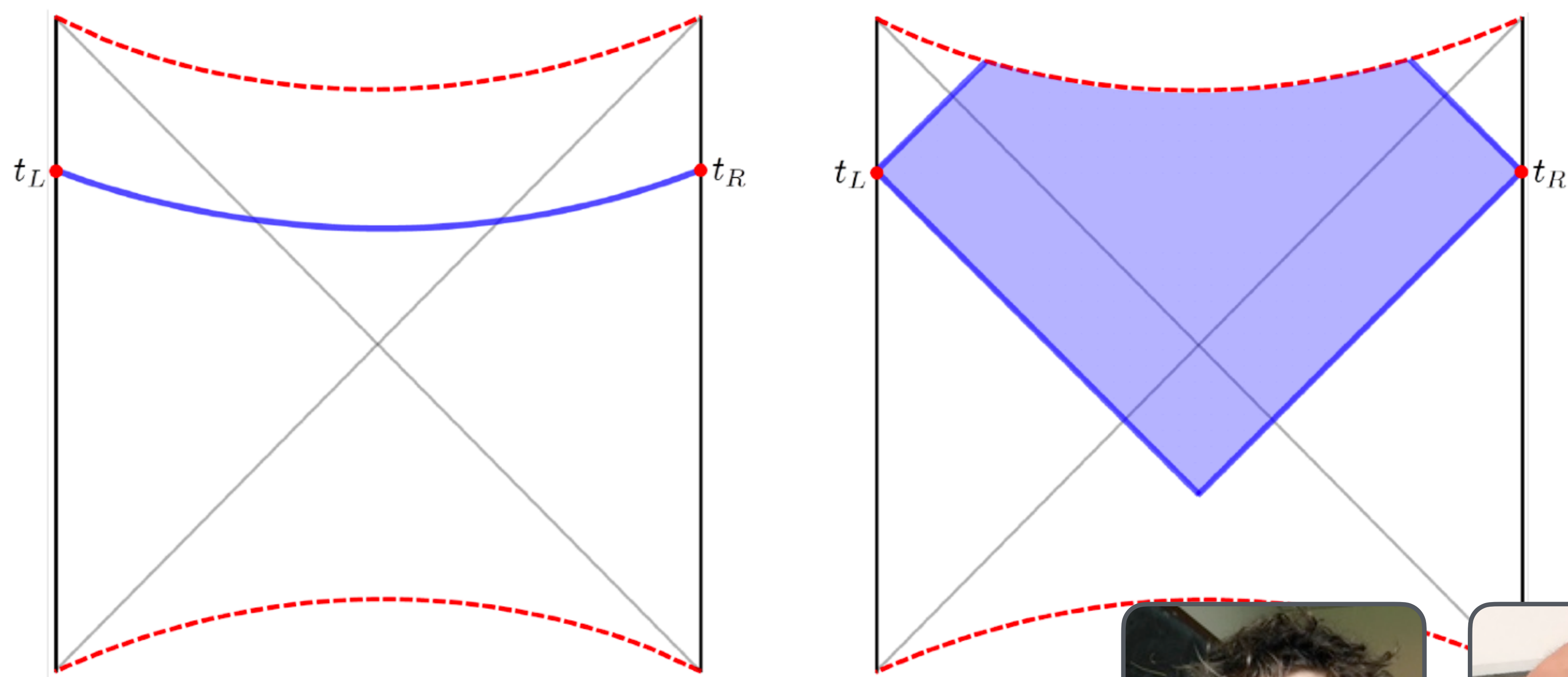
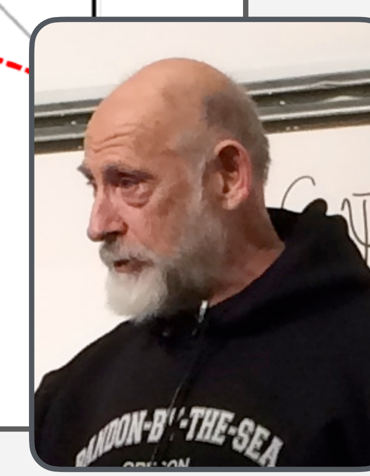


- **Holographic context:** Complexity growth of thermofield doubles

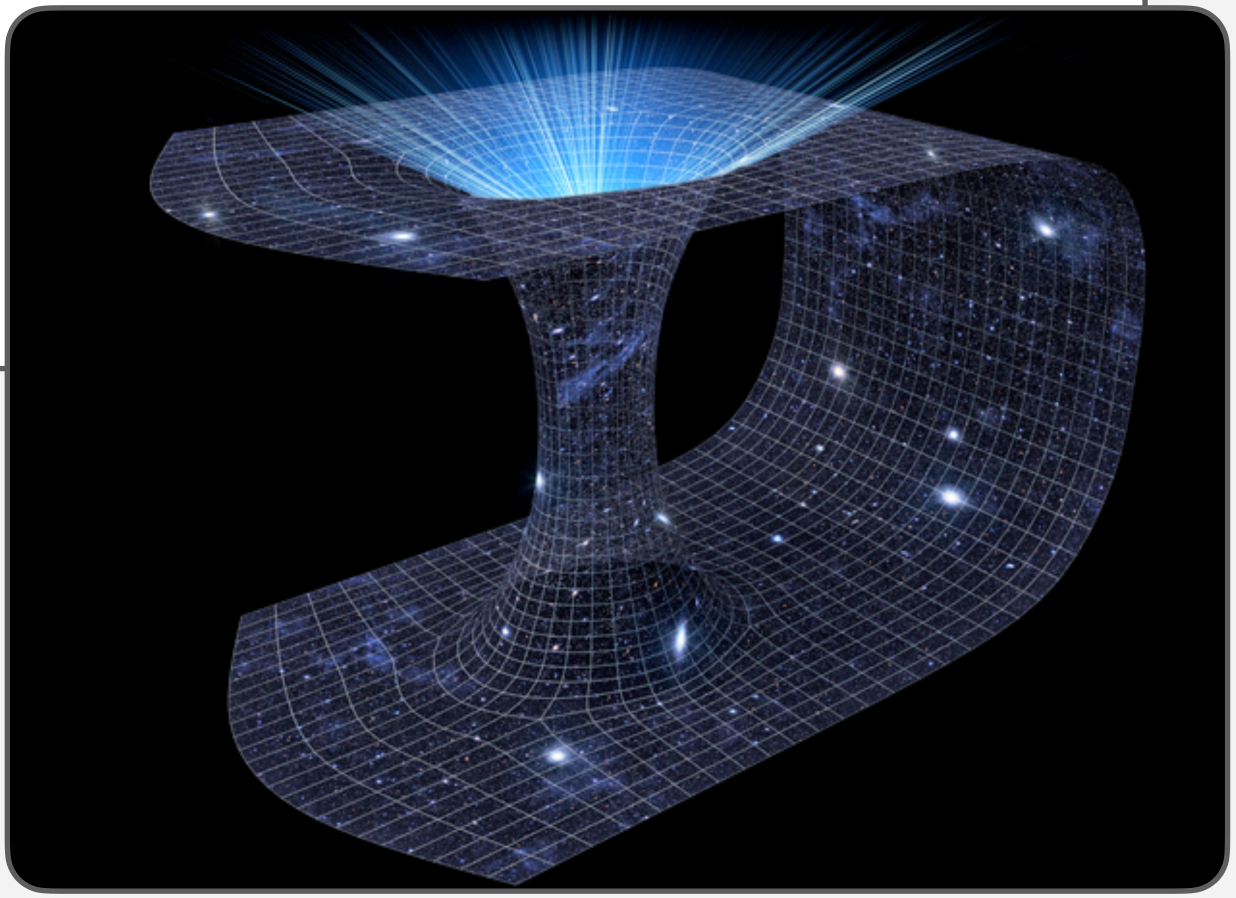




Brown, Roberts, Susskind, Swingle, Zhao, Phys Rev Lett 116, 191301 (2016)
Brown, Roberts, Susskind, Swingle, Zhao, Phys Rev D 93, 086006 (2016)
Chapman, Eisert, Hackl, Heller, Jefferson, Marrochio, Myers, SciPost Phys 6, 034 (2019)
Brown, Susskind, Phys Rev D 97, 086015 (2018)

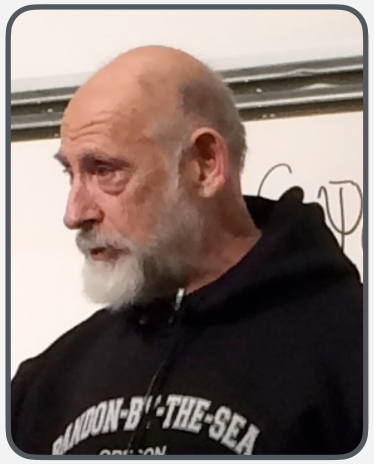
- **Holographic context:** Wormhole-growth paradox

- **AdS**: Volume grows for exponentially long time



- **CFT**: Local observables equilibrating?

$|\psi\rangle$



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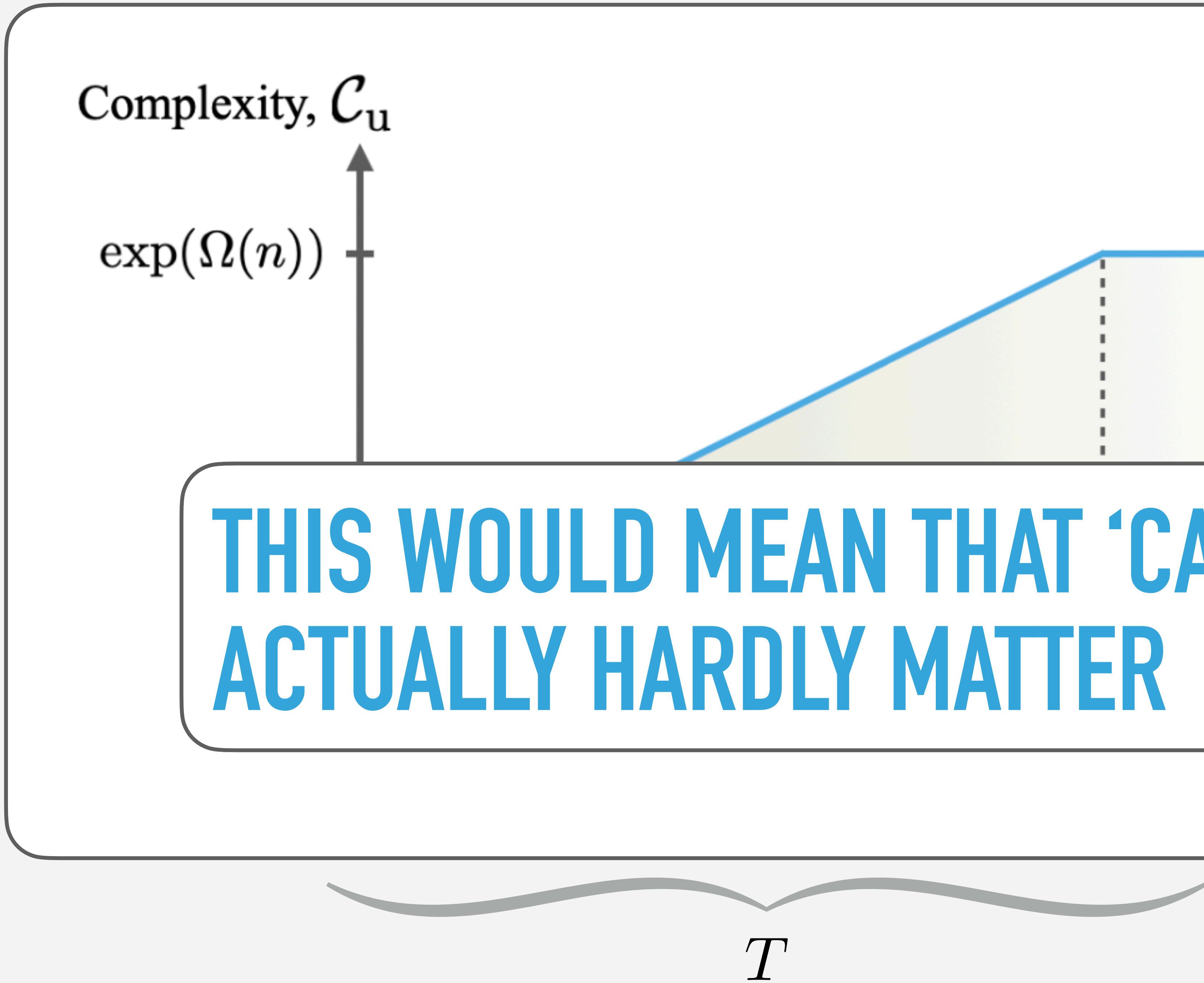
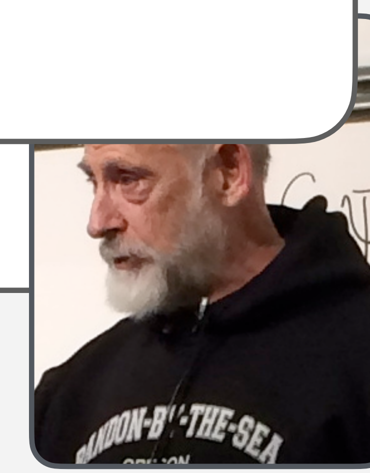
BROWN SUSSKIND CONJECTURE FOR RANDOM QUANTUM CIRCUITS

- **Brown-Susskind conjecture:** Linear growth until exponential time



THIS WOULD MEAN THAT 'CANCELLATIONS' ACTUALLY HARDLY MATTER



Brown, Susskind, Phys Rev D 97, 086015 (2018)

BUT HOW CAN THIS BE JUDGED?


Complexity
Dimension
BUT HOW CAN THIS BE JUDGED?

SKETCH OF THE PROOF

Contraction map

$F^A : \mathrm{SU}(4)^{\times R} \to \mathrm{SU}(2^n)$

→

Accessible dimension

$d_A = \dim(\mathcal{U}(A))$

Haferkamp, Faist, Kothakonda, Eisert, Yunger-Halpern, Nature Physics 18, 528 (2022)

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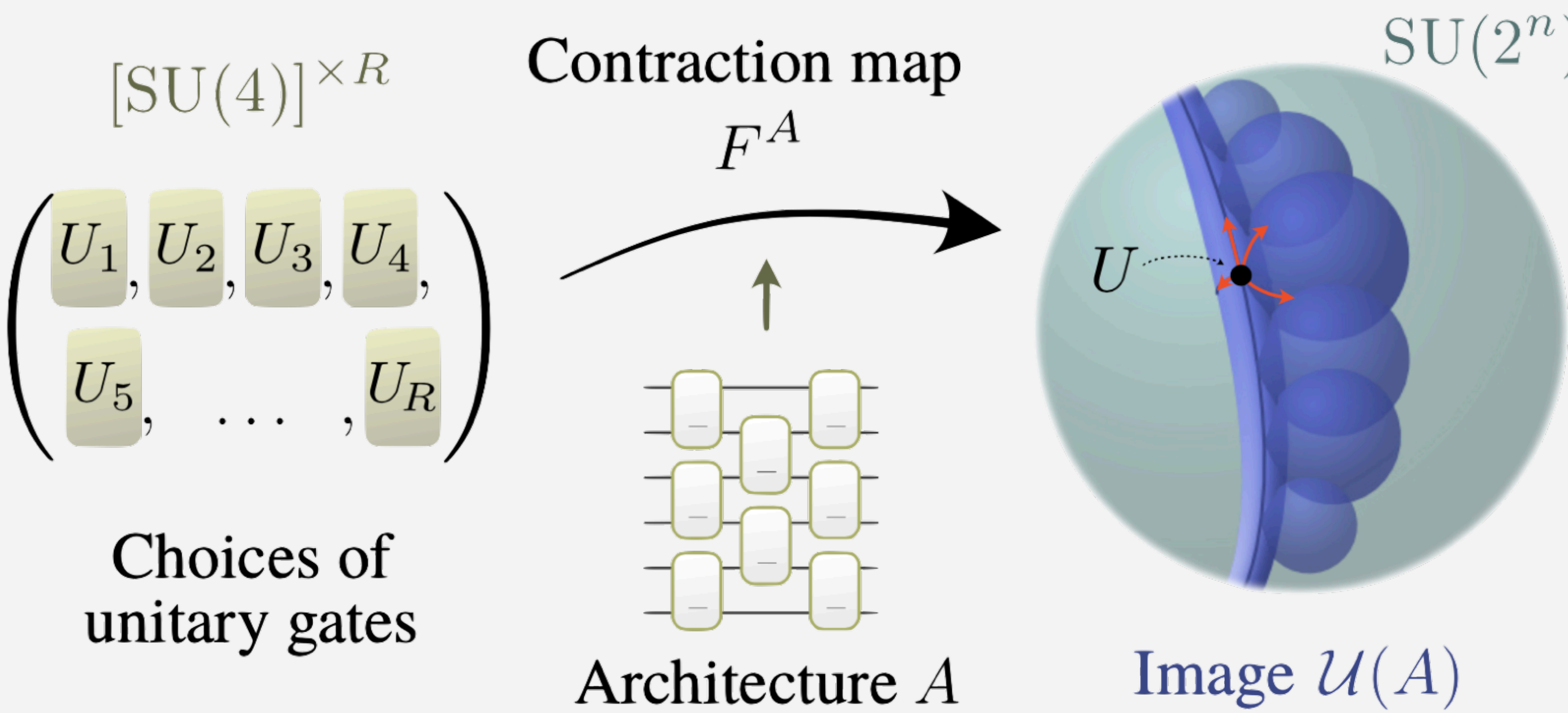

“Reachable set”

Haferkamp, Faist, Kothakonda, Eisert, Yunger-Halpern, Nature Physics 18, 528 (2022)

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- **Rank** of F^A in x important: Rank of a matrix that approximates F^A linearly around x
- Prove that F^A has the **same rank** throughout the domain, except on a measure-zero set

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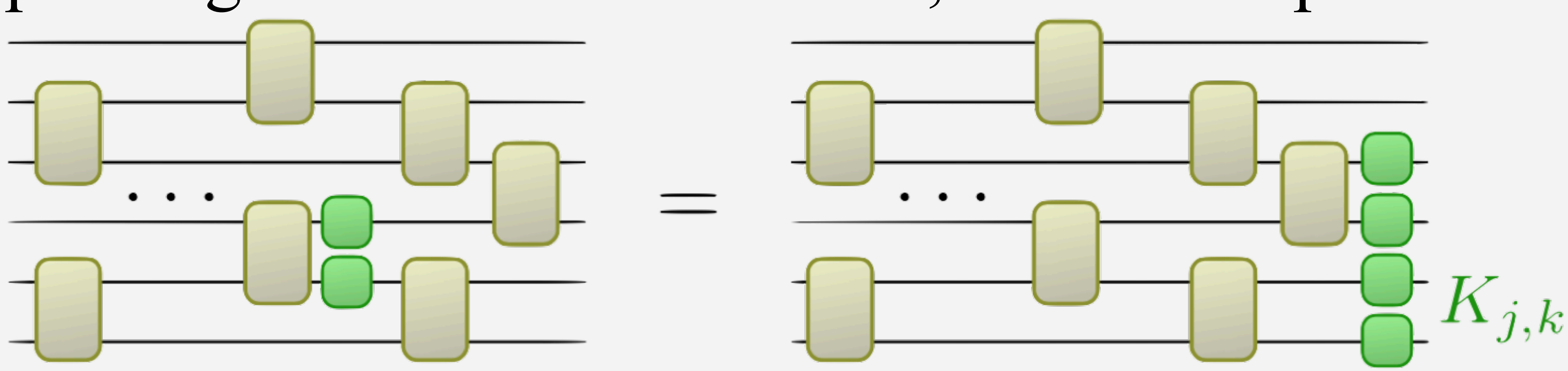
- Identify an x where r grows **linearly** with R
- Demonstrate the point's existence by perturbing **Clifford circuits**, 'appending infinitesimal unitaries', 'count independent directions'



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Proposition 1 (Lower bound of accessible dimension). *Let A_T denote an architecture with $R = TL$ gates. Assume that A_T consists of causal slices of $\leq L$ gates each. The architecture's accessible dimension is lower-bounded as*

$$d_{A_T} \geq T = \frac{R}{L} .$$

Haferkamp, Faist, Kothakonda, Eisert, Yunger-Halpern, Nature Physics 18, 528 (2022)

SKETCH OF THE PROOF

- Elaborate counting

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Haferkamp, Faist, Kothakonda, Eisert, Yunger-Halpern, Nature Physics 18, 528 (2022)

$d_A \leq 9R + 3n$

- Let R' be less than a linear fraction of R: $9R' + 3n < T = R/L$
- For every R' gate architecture A', $d_{A'} < d_{A_T}$ holds
- Almost every $U \in \mathcal{U}(A_T)$ has complexity greater than greatest possible R'

- Elaborate counting (last step has been much simplified) Li, arXiv:2205.05668

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SKETCH O

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$$\mathcal{C}_{\mathrm{u}}(U) \geq \frac{R}{9L} - \frac{n}{3},$$

with unit probability, until the number of gates grows to $T = R/L \geq 4^n - 1$. The same bound holds for $\mathcal{C}_{\mathrm{state}}(U|0^n\rangle)$, until $T = R/L \geq 2^{n+1} - 1$.

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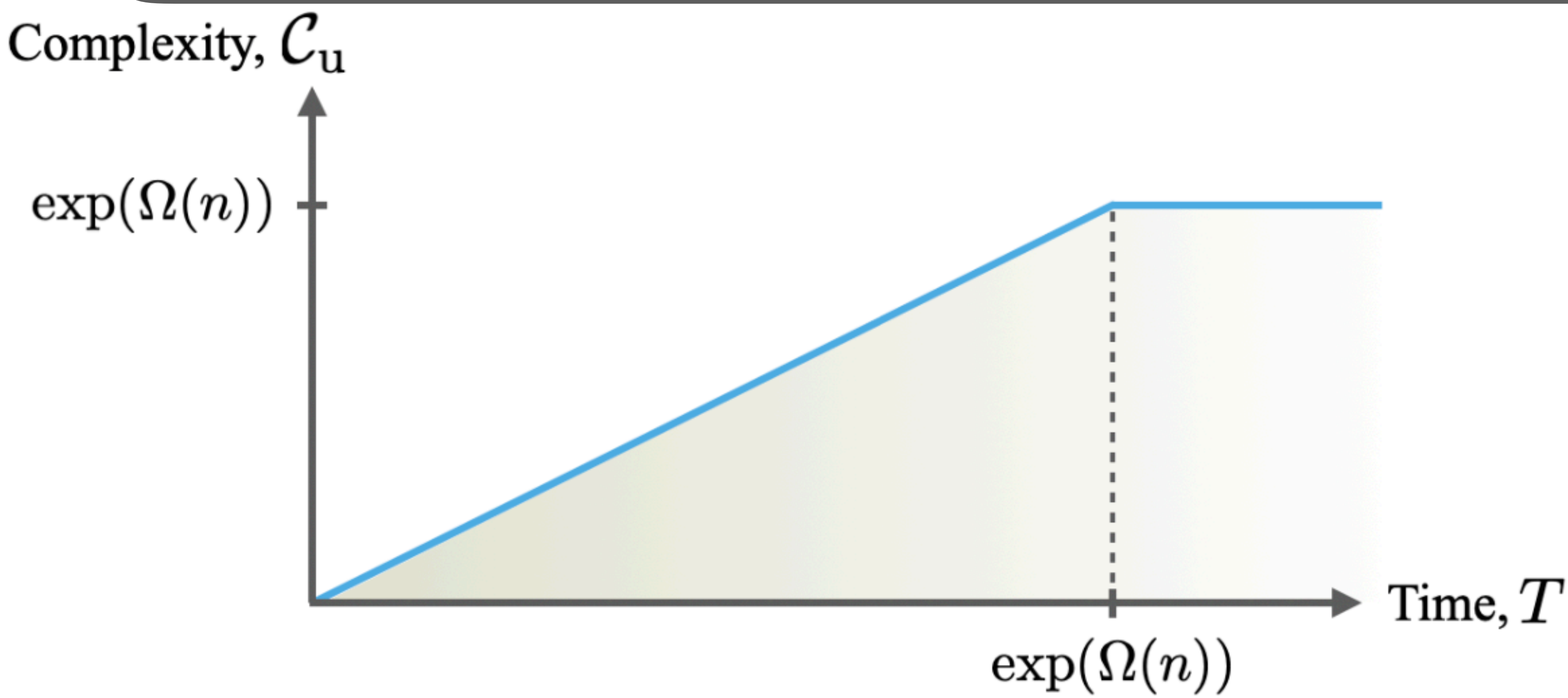

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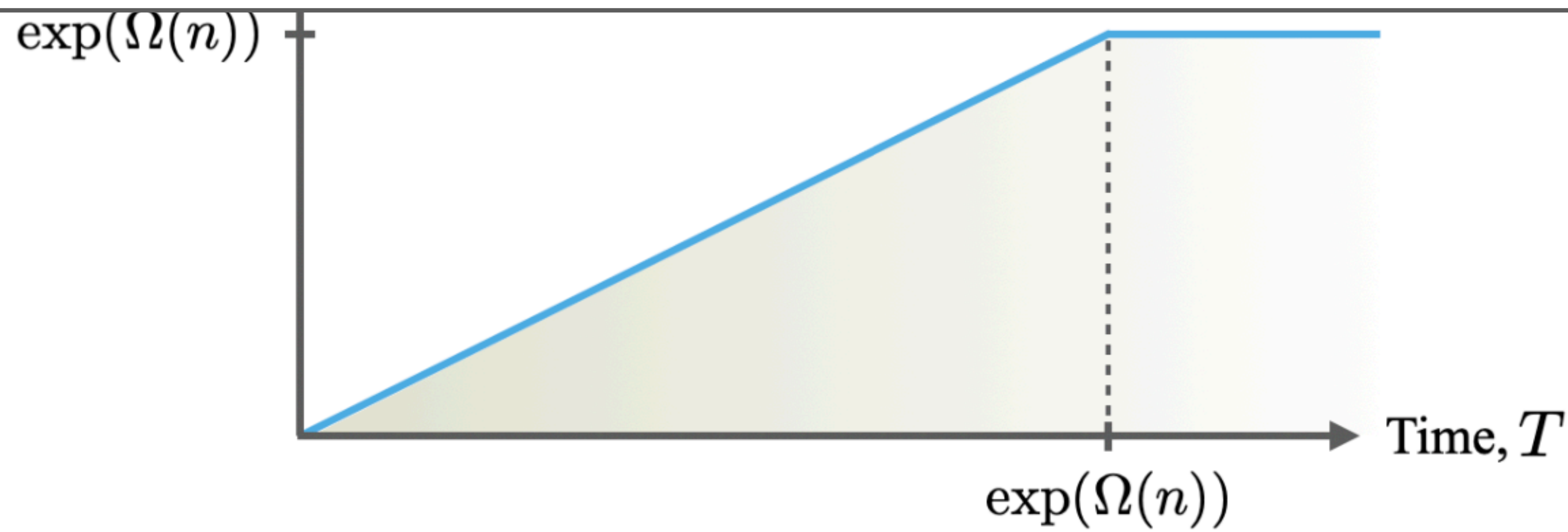
SO THERE IS A LINEAR COMPLEXITY GROWTH



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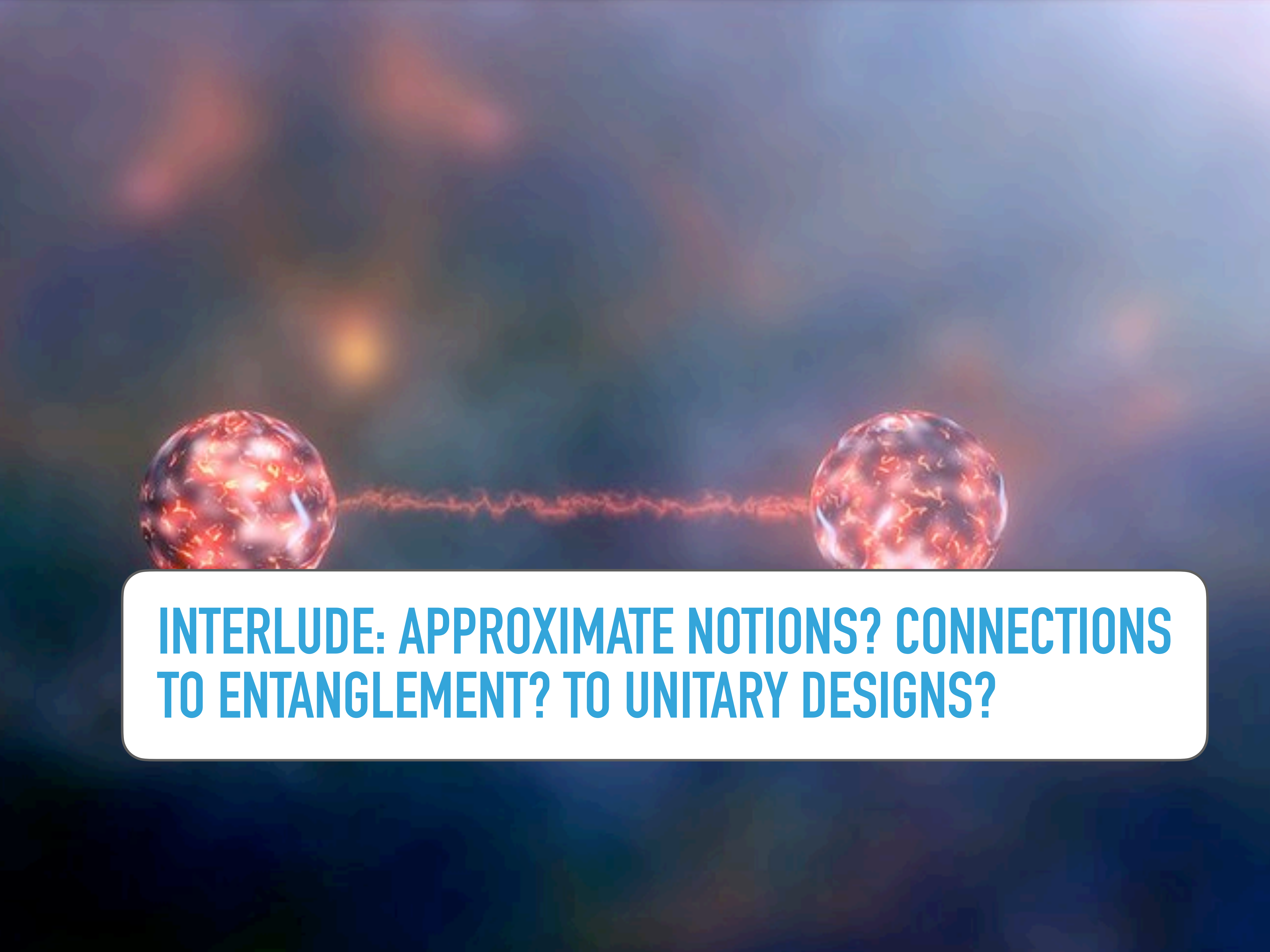


INTERLUDE: APPROXIMATE NOTIONS? CONNECTIONS TO ENTANGLEMENT? TO UNITARY DESIGNS?

CONNECTION TO ENTANGLEMENT?

- Fair resolution of the **Brown-Susskind conjecture**
- **Approximate notions** desirable: Approximate in $\|.\|_\infty$ -norm

Haferkamp, Faist, Kothakonda, Eisert, Yunger-Halpern, Nature Physics 18, 528 (2022)
Bouland, Fefferman, Vazirani, arXiv:1910.14646
Brandão, Chemissany, Hunter-Jones, Kueng, Preskill, PRX Quantum 2, 030316 (2021)
Dowling, Nielsen, Quant Inf Comp 8, 861–899 (2008)

CONNECTION TO ENTANGLEMENT?

- Fair resolution of the **Brown-Susskind conjecture**
- **Approximate notions** desirable: Approximate in $\|.\|_\infty$ -norm
- **Nielsen cost** and **entanglement**

- Write circuits as integral

$$U = \mathcal{P}\exp\left(-i\int_0^1 dt \sum_I y_I(t) M_I\right)$$

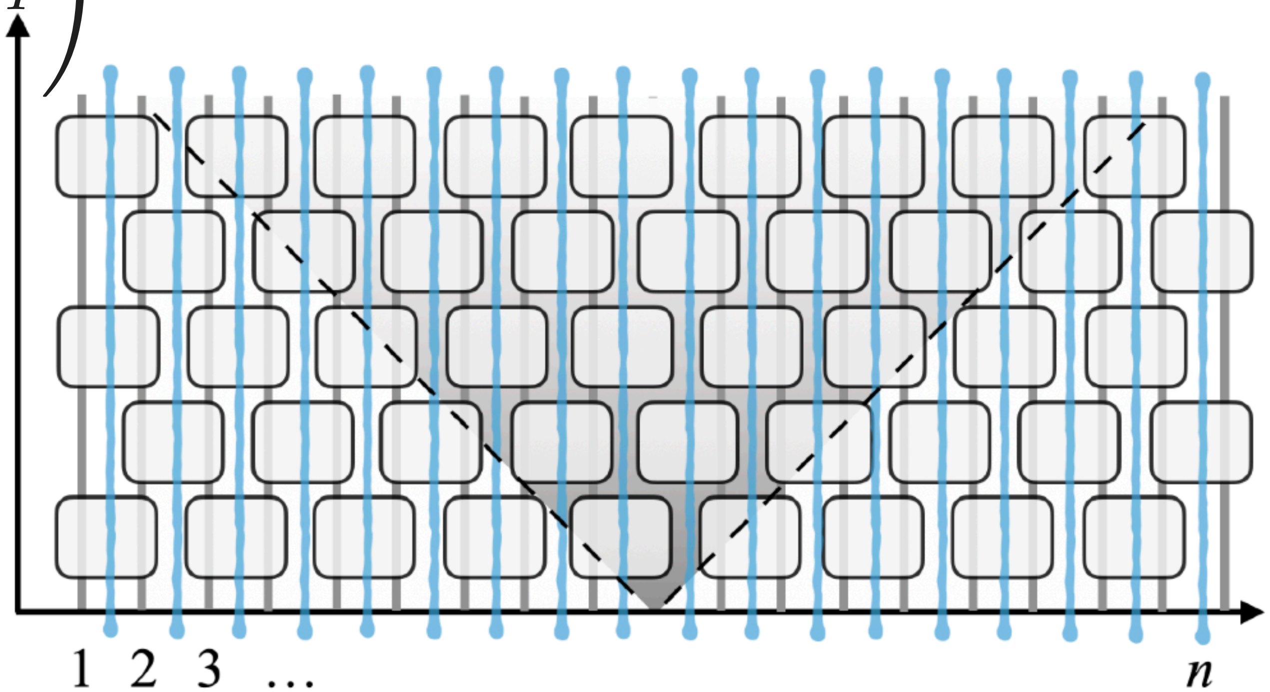
- **Cost** (common l_1 choice)

$$C := \inf \int_0^1 \sum_I |y_I(t)| ds$$

- **Theorem 2:**

$$C \geq cE,\ c > 0$$

Eisert, Phys Rev Lett 127, 020501 (2021)



Compare Balasubramanian, DeCross, Kar, Parrikar, arXiv:1811.04085 (2018)
Susskind, arXiv:1411.0690 (2014)

UNITARY DESIGNS FROM CLIFFORD AND T-GATES

- Fair resolution of the **Brown-Susskind conjecture**
- **Approximate notions** desirable: Approximate in $\|.\|_\infty$ -norm
- **Unitary designs:** The generation of unitary t-designs at a depth $O(nt)$ would imply the approximate Brown-Susskind conjecture

HOW CAN UNITARY DESIGNS BE IMPLEMENTED?

Brandão, Chemissany, Hunter-Jones, Kueng, Preskill, PRX Quantum 2, 030316 (2021)
Brandao, Harrow, Horodecki, Phys Rev Lett 116, 170502 (2016)

- Random **Clifford circuits** are unitary 3-designs
- T**-gates** uplift then to arbitrary order designs*

- Fair resoluti

- **Approxima**

- **Unitary designs:** The generation of unitary t-designs at a depth $O(nt)$ would imply the approximate Brown-Susskind conjecture

*In approximate sense

Brandão, Chemissany, Hunter-Jones, Kueng, Preskill, PRX Quantum 2, 030316 (2021)
Brandao, Harrow, Horodecki, Phys Rev Lett 116, 170502 (2016)

- Random **Clifford circuits** are unitary 3-designs
- T**-gates** uplift then to arbitrary order designs*

- **Theorem 3:** *A* ***constant*** *(!) number of T-gates is sufficient*

Haferkamp, Montaelegre-Mora, Heinrich, Eisert, Gross, Roth, Comm Math Phys (2022)
arXiv:2002.09524 (2022)

Compare also Zhou, Yang, Hamma, Chamon, arXiv:1906.01079

- Fair resoluti
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Brandão, Chemissany, Hunter-Jones, Kueng, Preskill, PRX Quantum 2, 030316 (2021)
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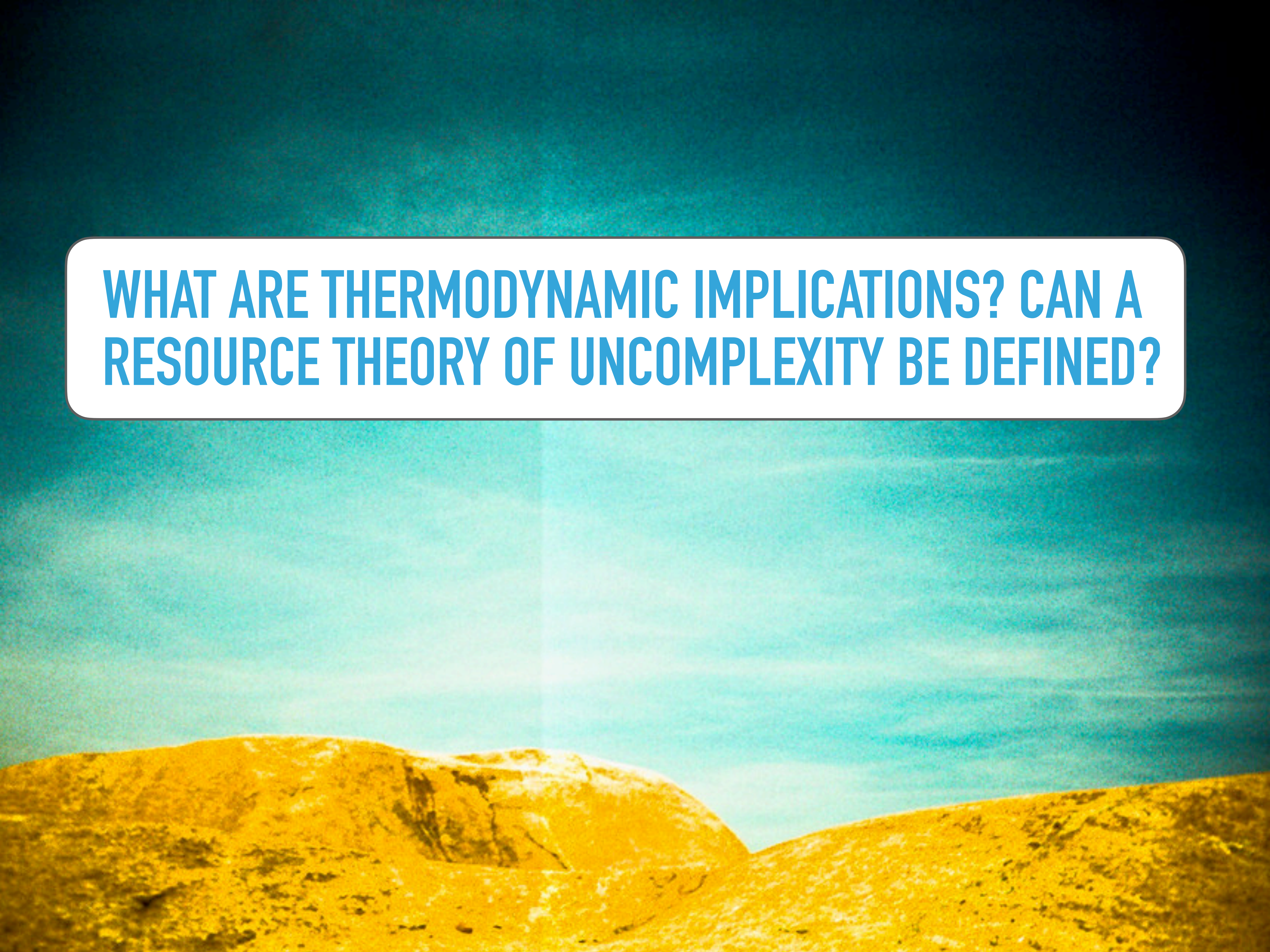
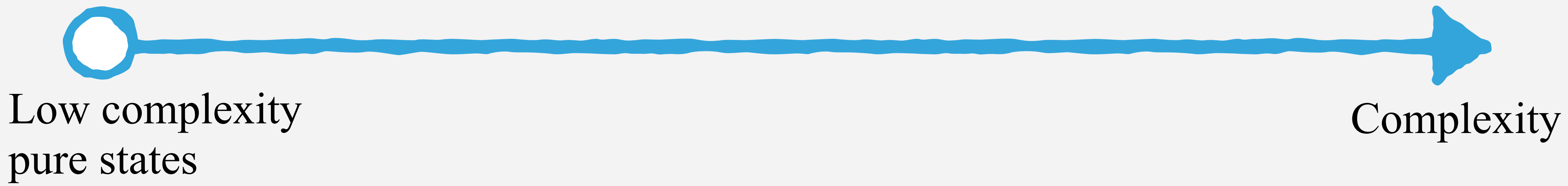

WHAT ARE THERMODYNAMIC IMPLICATIONS? CAN A RESOURCE THEORY OF UNCOMPLEXITY BE DEFINED?

HOW CAN COMPLEXITY CAPTURE WHAT WE CAN DO OPERATIONALLY TO A SYSTEM?

Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)

THINKING IN OPERATIONAL TERMS



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Brown, Susskind, Phys Rev D 97, 086015 (2018)

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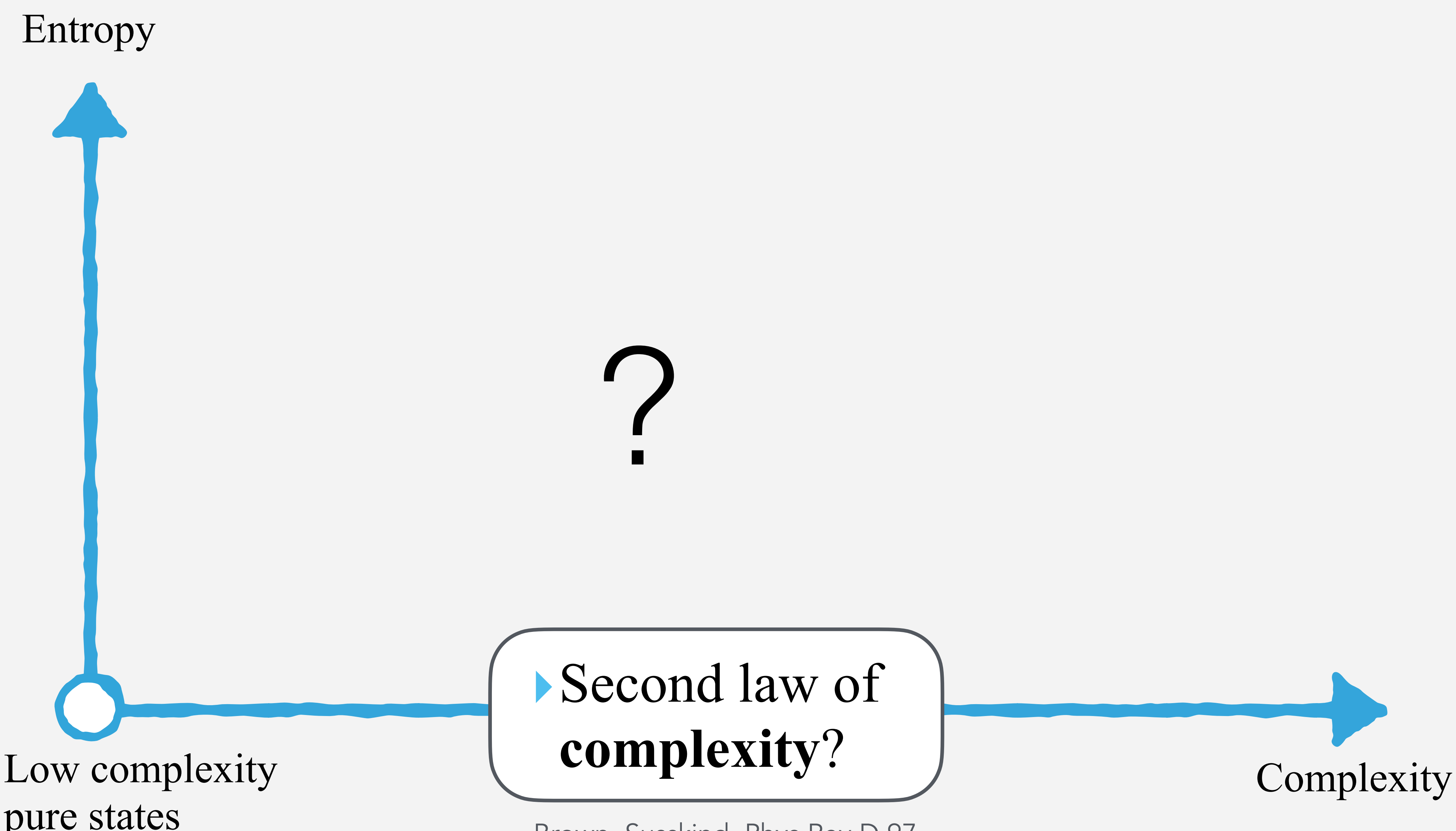
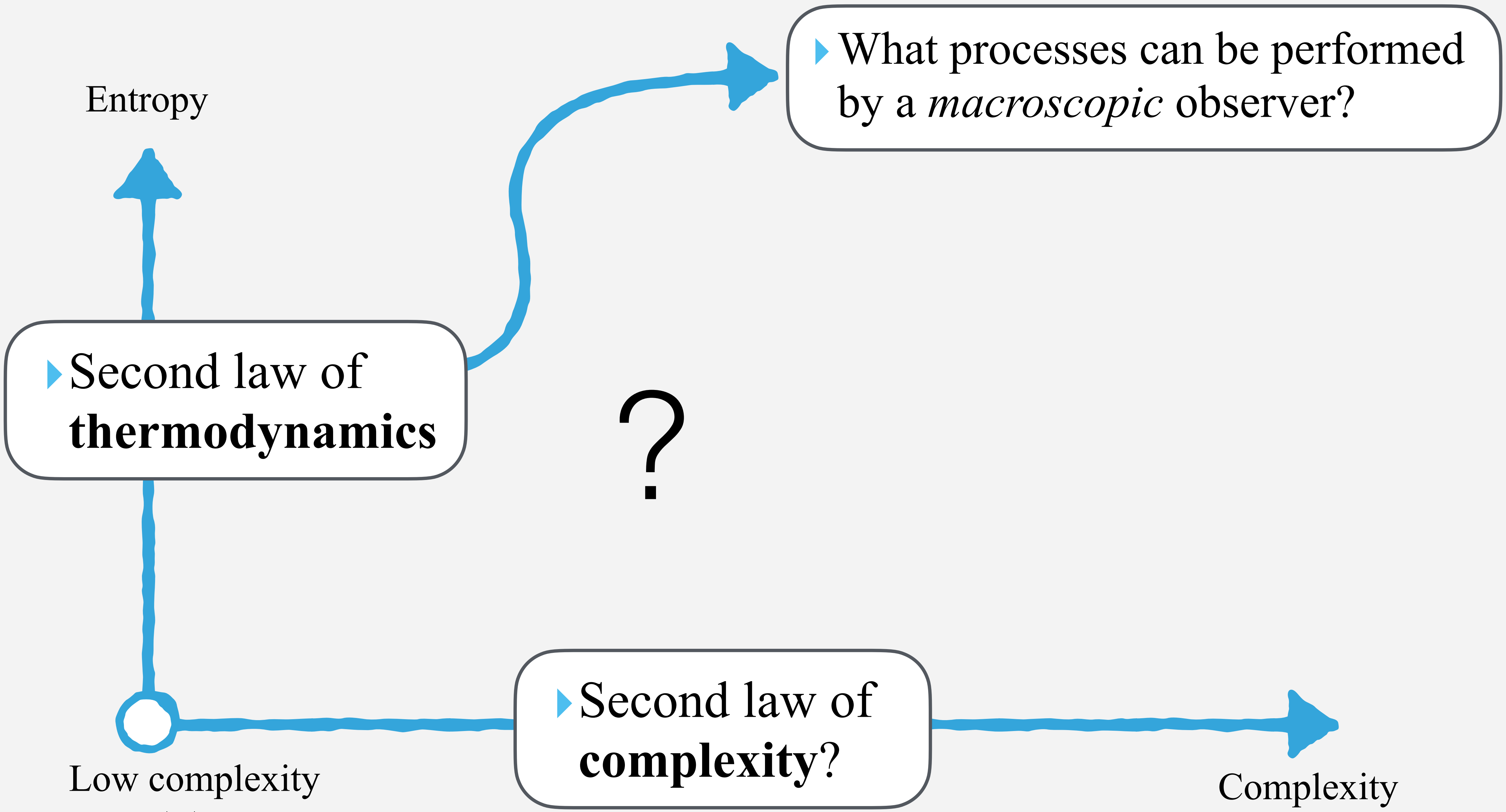


- Second law of **complexity**?

Brown, Susskind, Phys Rev D 97, 086015 (2018)



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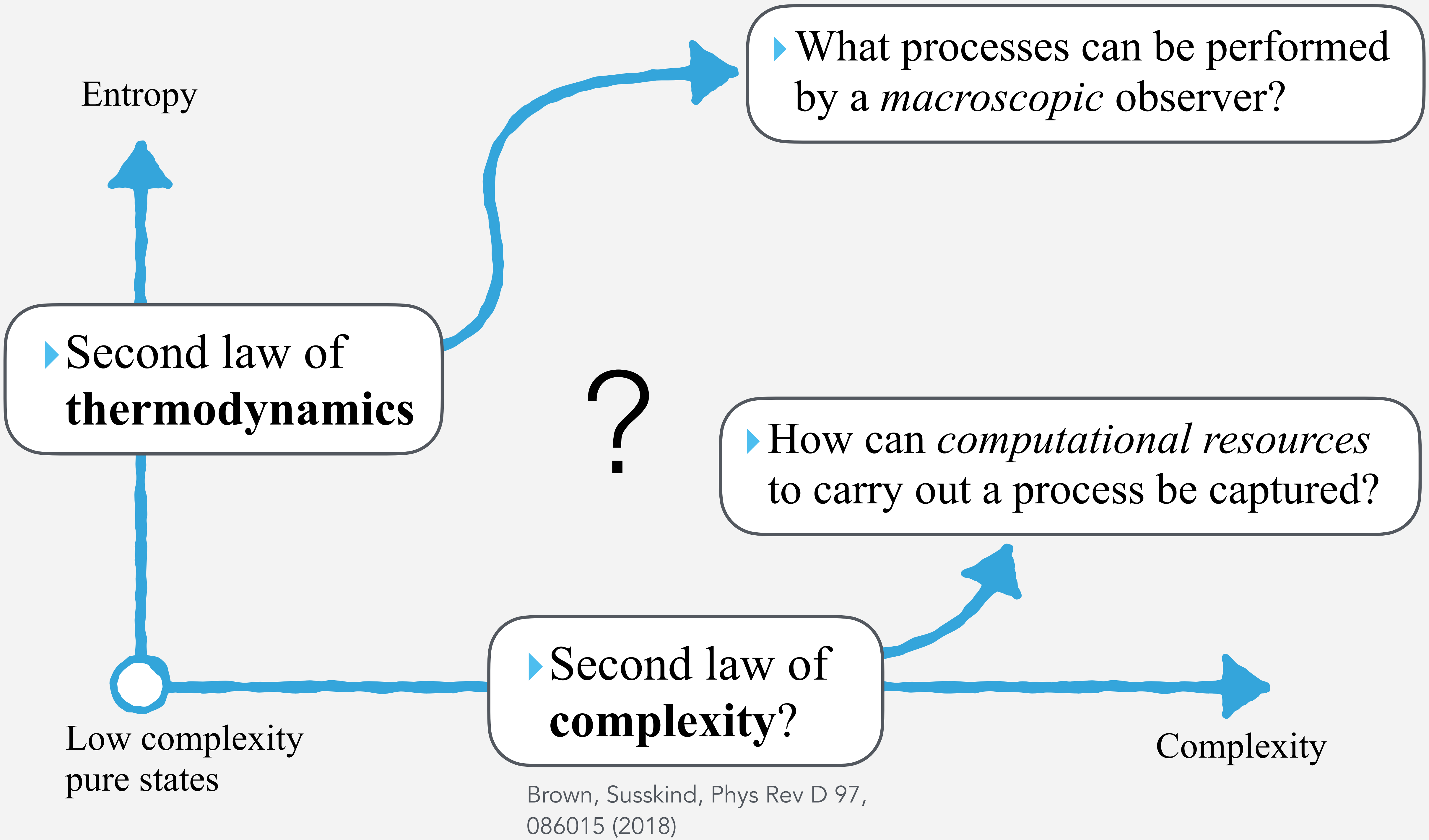
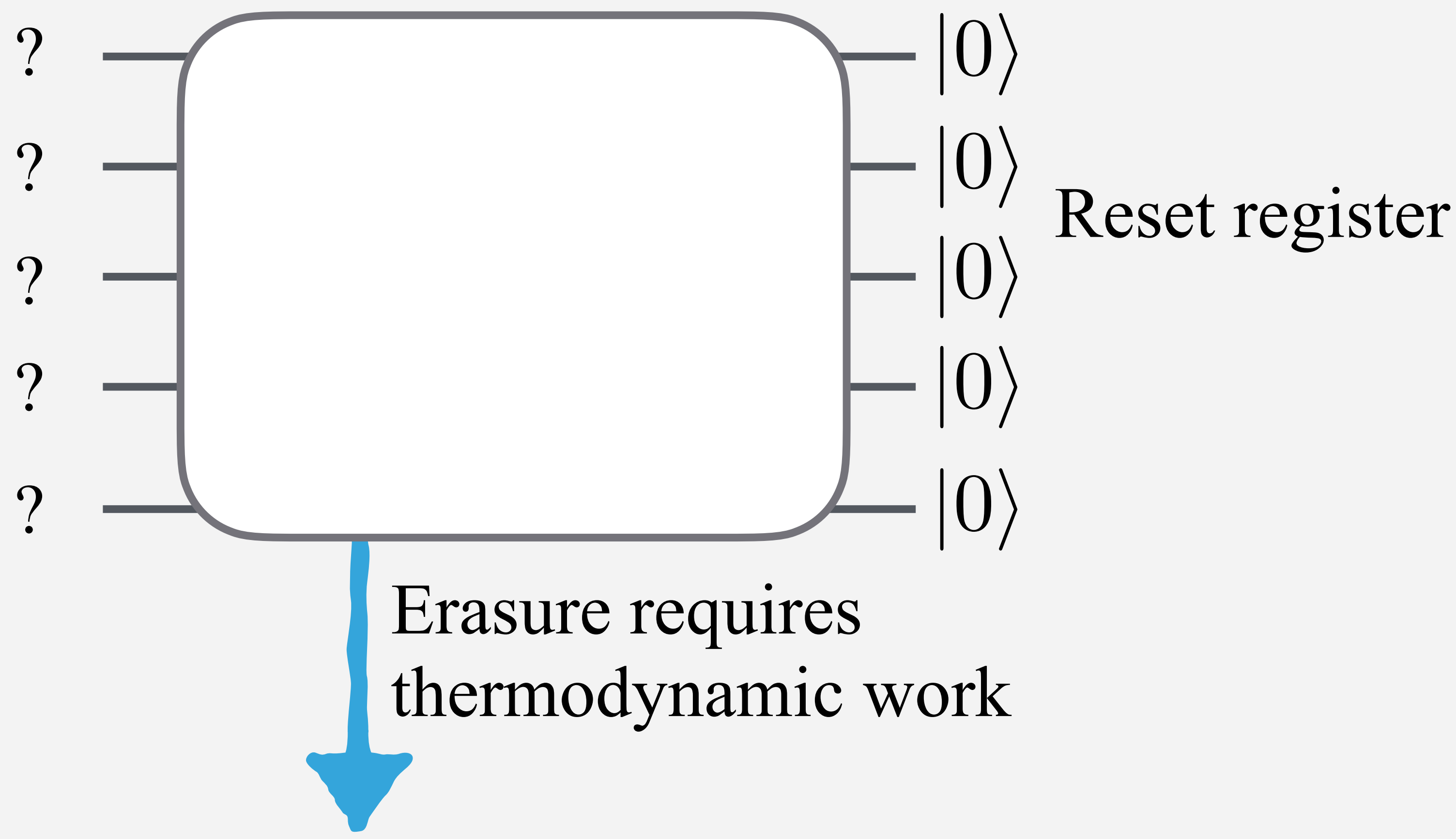
THINKING IN OPERATIONAL TERMS



- **Landauer erasure** resetting a collection of quantum systems



$kT\log(2)$ per bit discarded in environment at temperature T

Landauer, IBM J Res Dev 5, 183 (2015)
Bennett, Stud Hist Phil Mod Phys 34, 501 (2003)
Reeb, Wolf, New J Phys 16, 103011 (2014)

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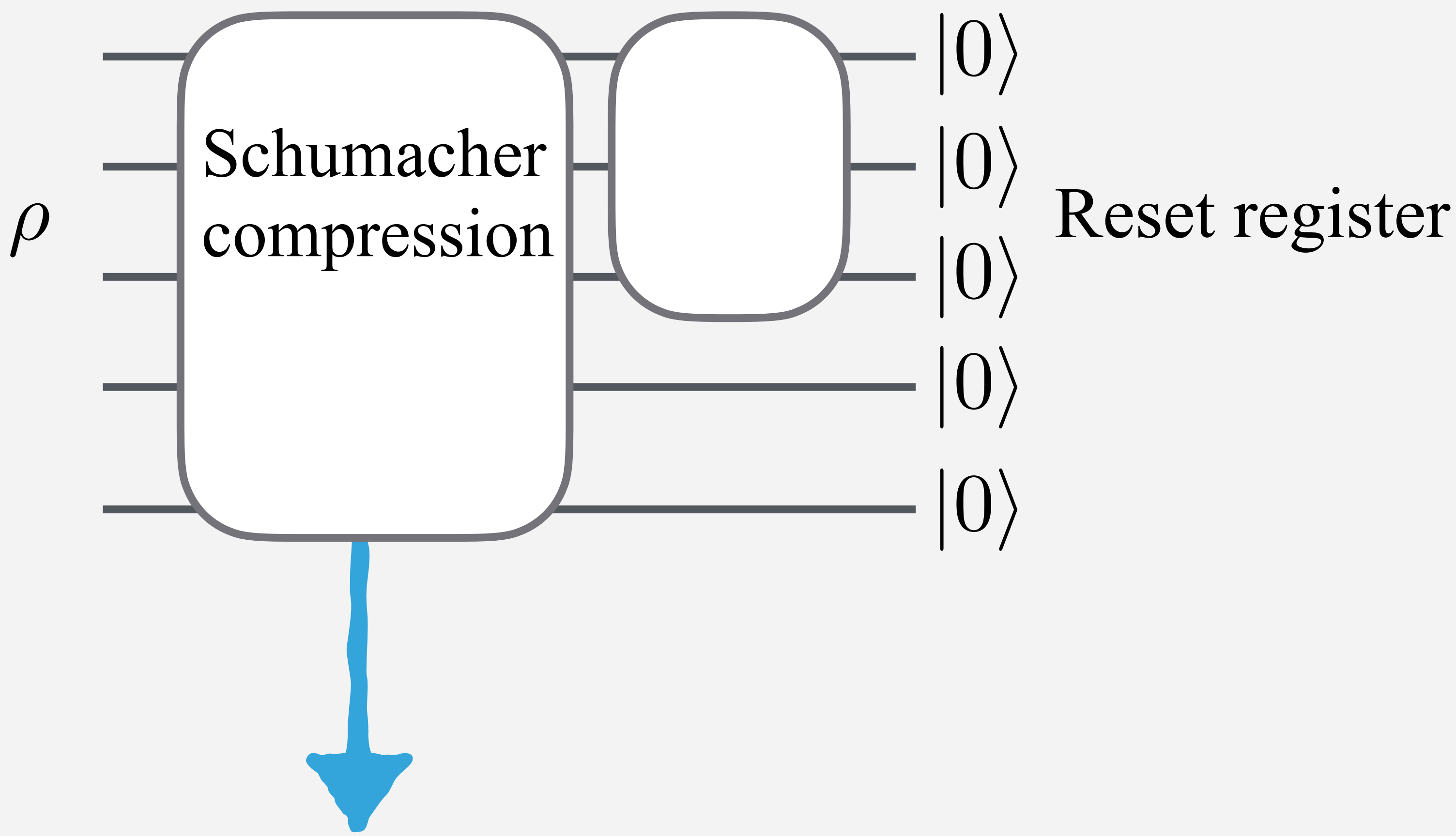
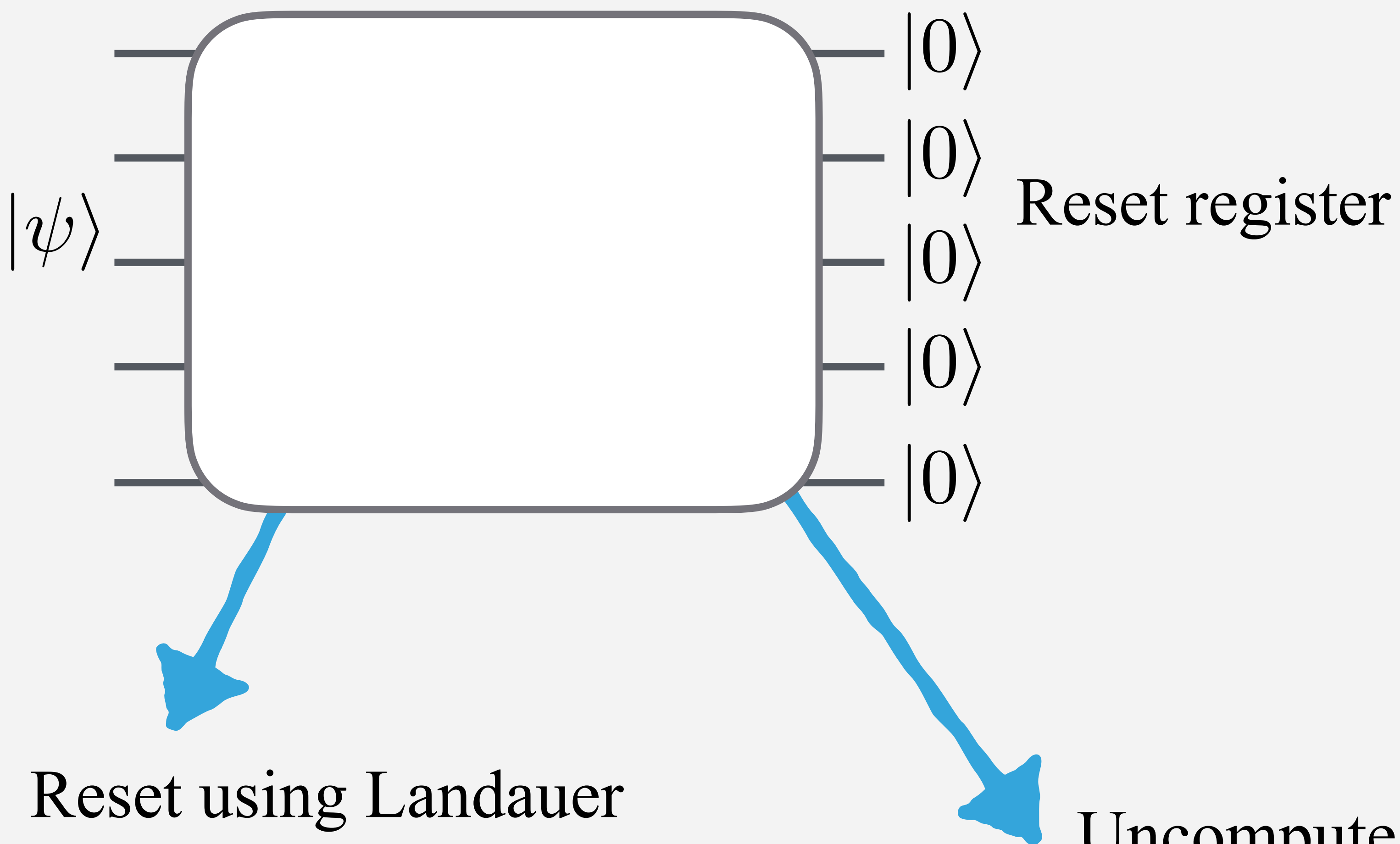


$kT \log(2) H^{1-\epsilon}(\rho)$ total work cost

Brandao, Horodecki, Oppenheim, Renes, Spekkens, Phys Rev Lett 111, 250404 (2013)
Faist, Dupuis, Oppenheim Renner, Nature Comm 6, 7669 (2015)

LANDAUER ERASURE

- Start from **high complexity** pure state



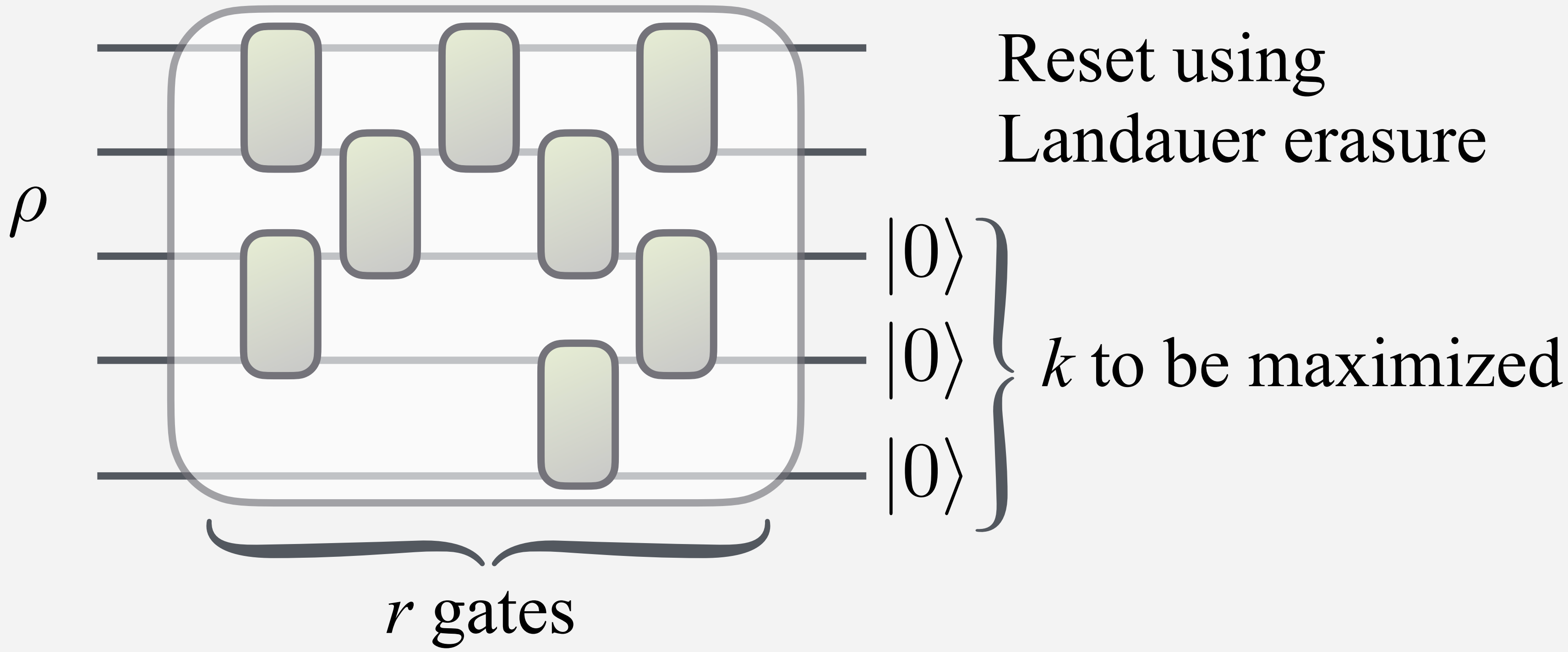
Reset using Landauer erasure at little complexity

Uncompute at no work cost, but at high complexity

SUGGESTS TRADE-OFF

Brandao, Horodecki, Oppenheim, Renes, Spekkens, Phys Rev Lett 111, 250404 (2013)
Faist, Dupuis, Oppenheim Renner, Nature Comm 6, 7669 (2015)

- Allow for at most r quantum gates

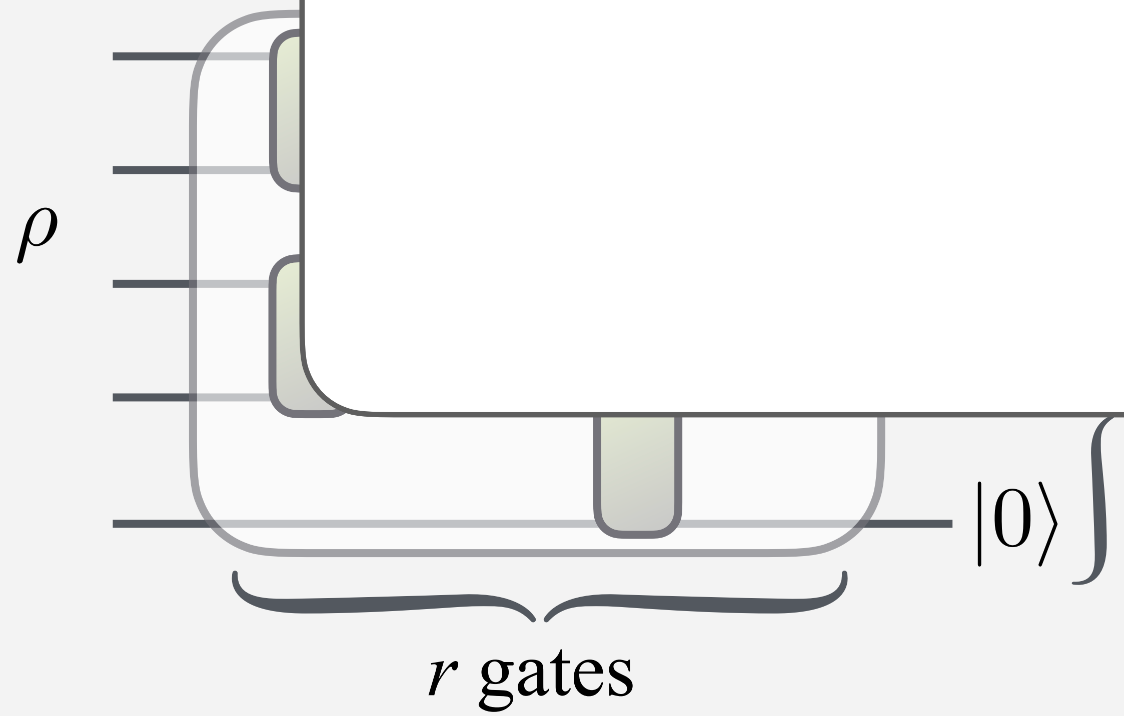


OPERATIONAL INTERPRETATION: AMOUNT OF WORK REQUIRED TO RESET A STATE FOR AN AGENT THAT CAN DO AT MOST R GATES

- **Theorem 4:** $k_{\mathrm{opt}} = n - H_h^{r,1-\epsilon}(\rho)$

Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)
Brandao, Datta, IEEE Trans Inf Th 57,1754 (2011)

- Allow fo



- A new **entropy quantity** that accounts for complexity

$$H_h^{r,\eta}(\rho) = \log \min_{\substack{\mathrm{tr}(Q\rho)\geq\eta \\ Q\in M_r}} \mathrm{tr}(Q)$$

How mixed must Q be?

POVMs with at most complexity r captures most of ρ

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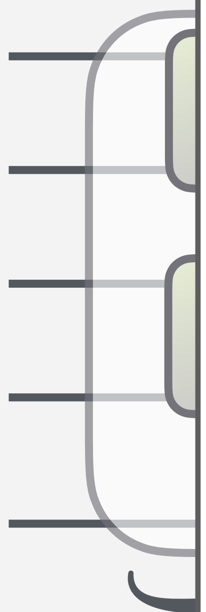

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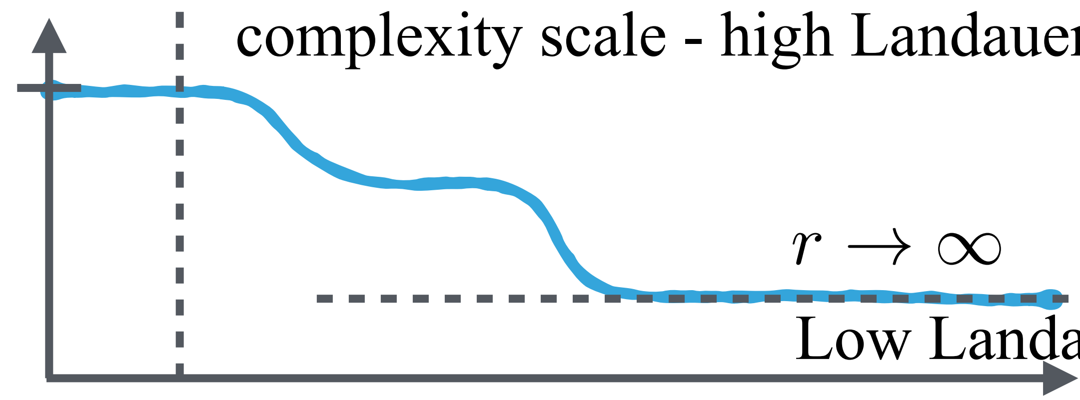
- Variant of **hypothesis testing entropy**
- *Monotonous* in r, η

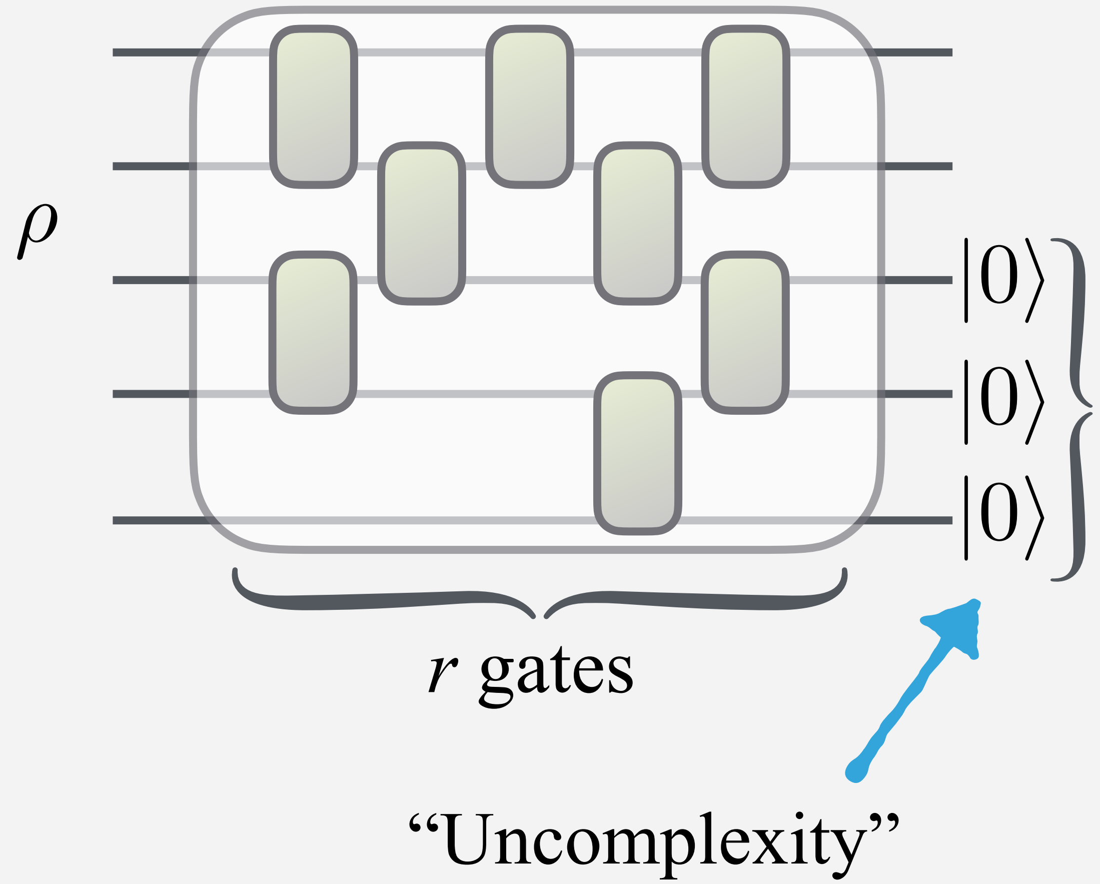


- Not *unitarily invariant*

- **Theorem 4:** $k_{\mathrm{opt}} = n - H_h^{r,1-\epsilon}(\rho)$

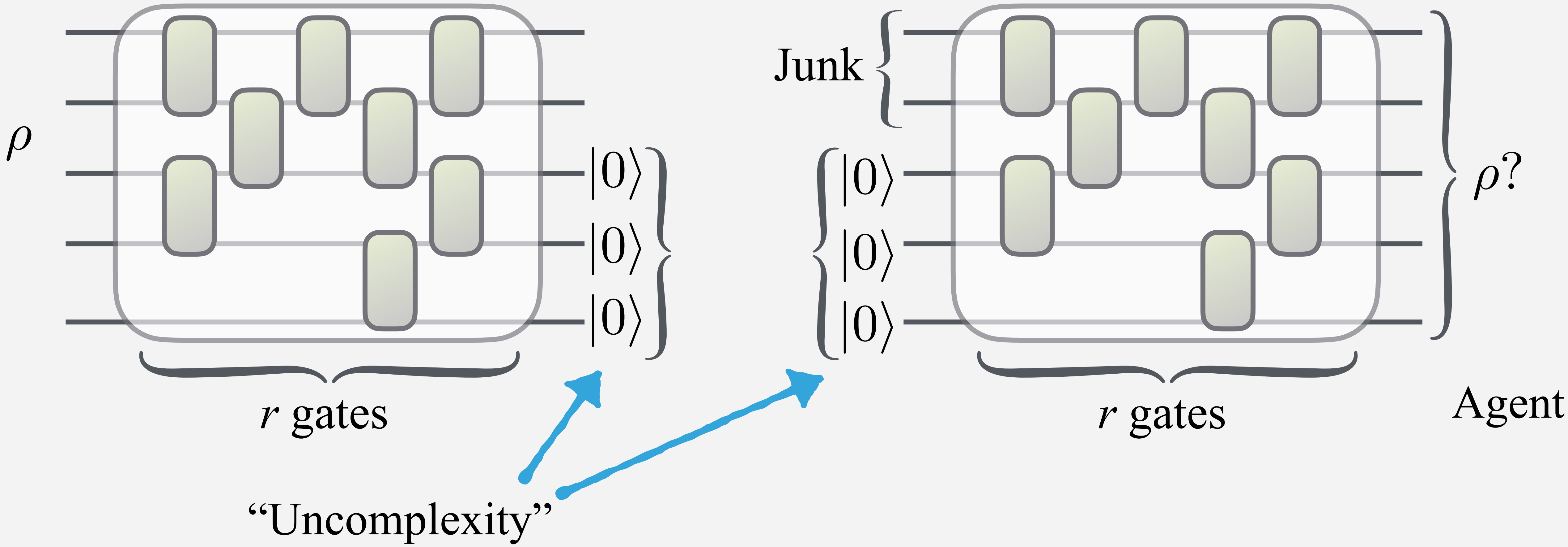
Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)
Brandao, Datta, IEEE Trans Inf Th 57,1754 (2011)

- **Uncomplexity extraction:**
Distills pure qubits from a state



Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)

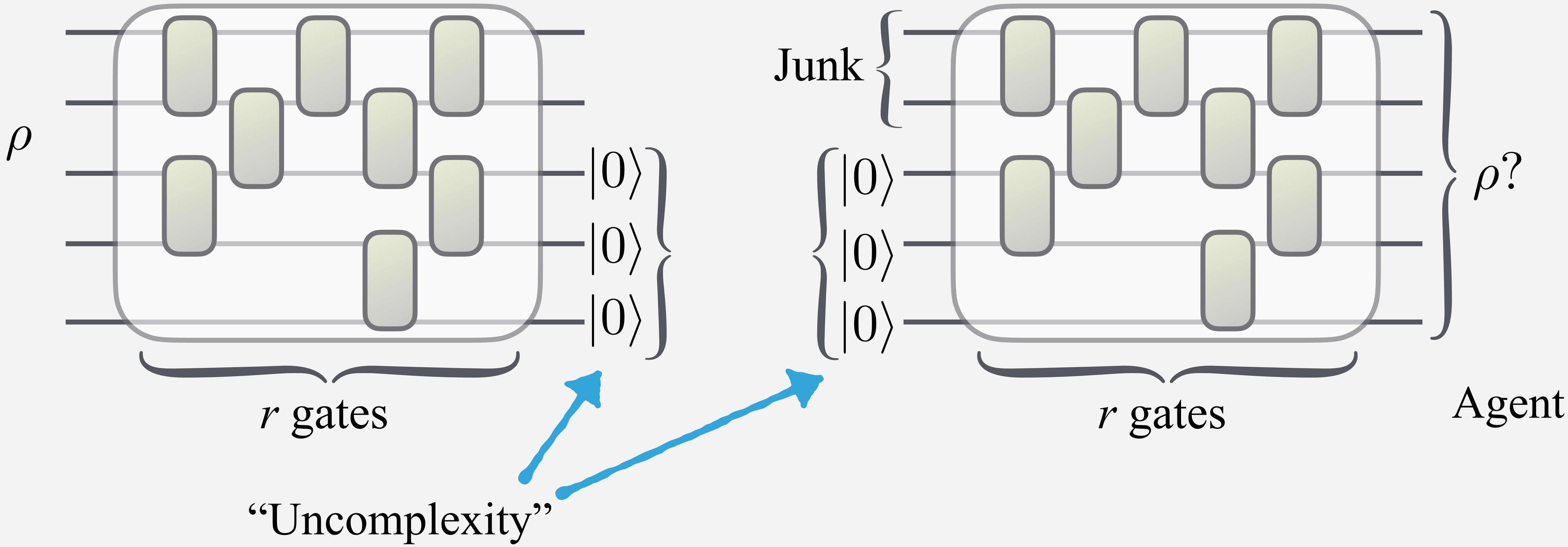
- **Uncomplexity extraction:** Distills pure qubits from a state
- **Uncomplexity expenditure:** Imitates a state



Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)

STEPS TOWARDS A RESOURCE THEORY OF UNCOMPLEXITY

- **Uncomplexity extraction:** Distills pure qubits from a state
- **Uncomplexity expenditure:** Imitates a state



CAN YET ANOTHER BROWN-SUSSKIND CONJECTURE – WHETHER A RESOURCE THEORY OF UNCOMPLEXITY CAN BE FORMULATED – BE SHOWN?

It seems so: Governed by the complexity entropy

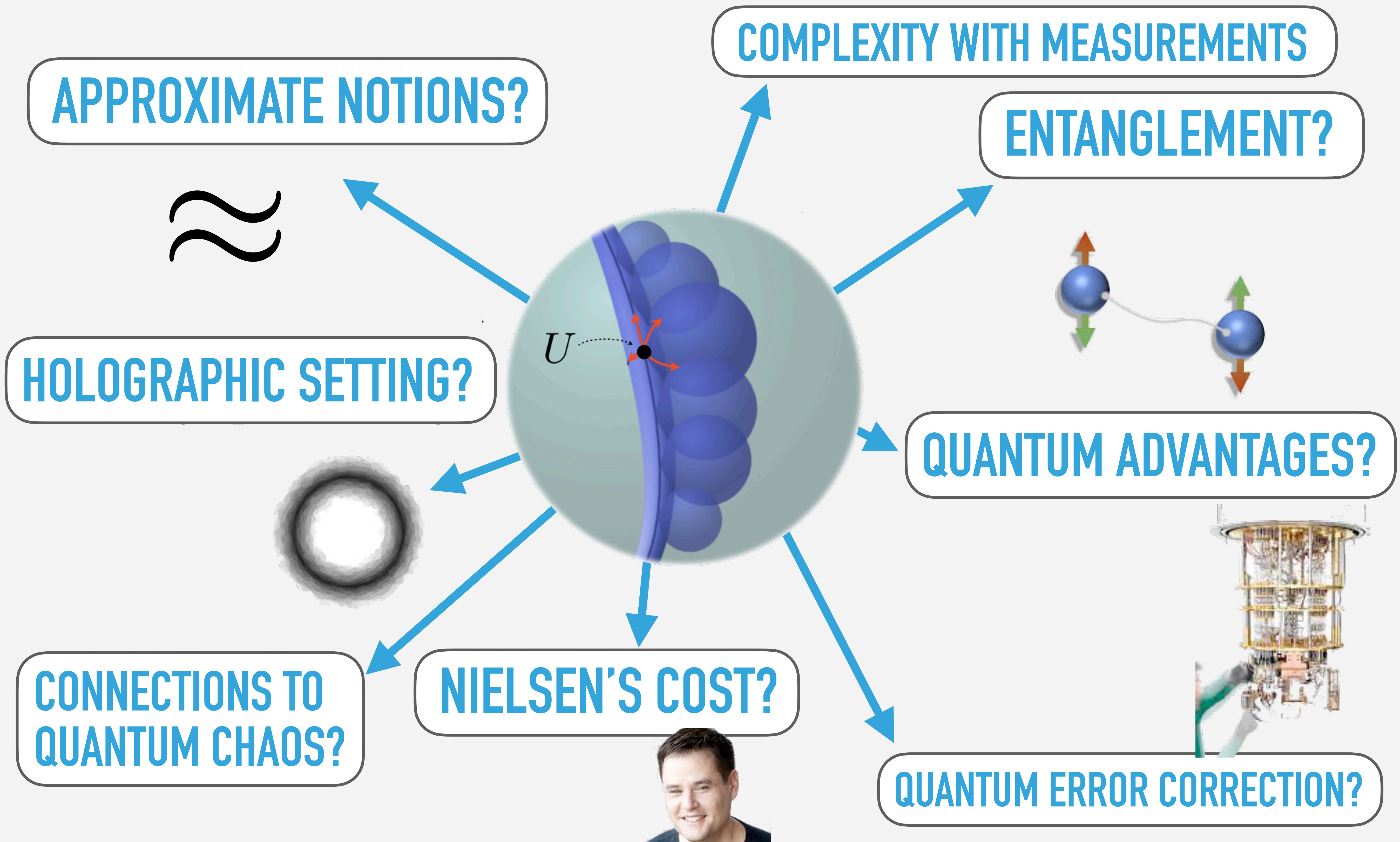
Yunger Halpern, Kothakonda, Haferkamp, Munson, Eisert, Faist, Phys Rev A 106, 062417 (2022) (2021)
Munson, Kothakonda, Haferkamp, Yunger-Halpern, Eisert, Faist, in preparation (2023)


Was ist ein schwarzes Loch?

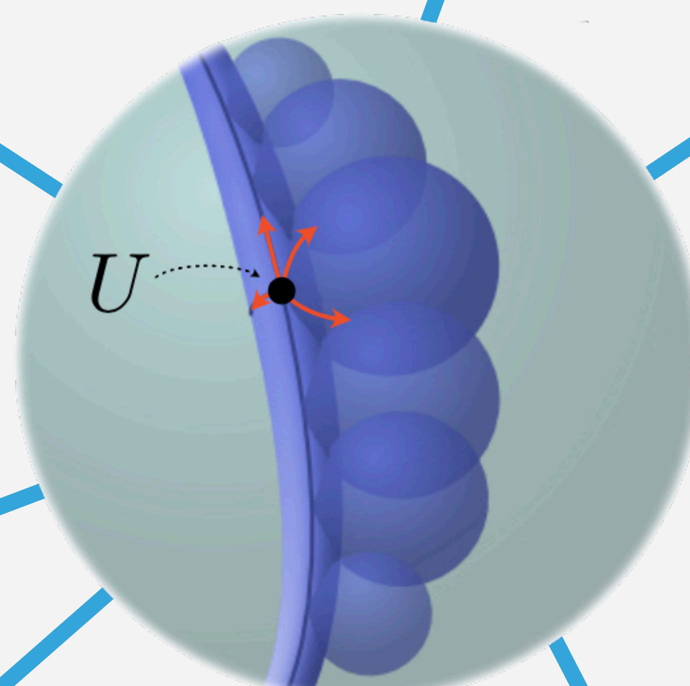
OUTLOOK

OPEN QUESTIONS

- Settled the **linear growth conjecture** due to Brown and Susskind



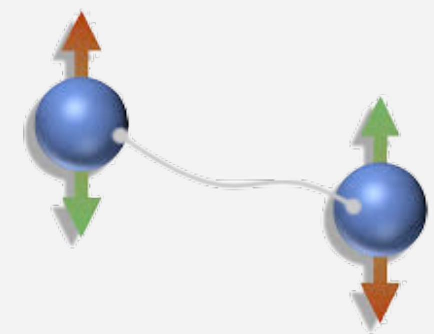
THANKS FOR YOUR ATTENTION!



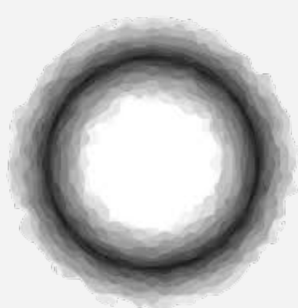
- APPROXIMATE NOTIONS?

$\approx$

- COMPLEXITY WITH MEASUREMENTS
- ENTANGLEMENT?



- HOLOGRAPHIC SETTING?



- QUANTUM ADVANTAGES?

- CONNECTIONS TO QUANTUM CHAOS?
- NIELSEN'S COST?

- QUANTUM ERROR CORRECTION?