Quantum transport with ultracold atomic gases

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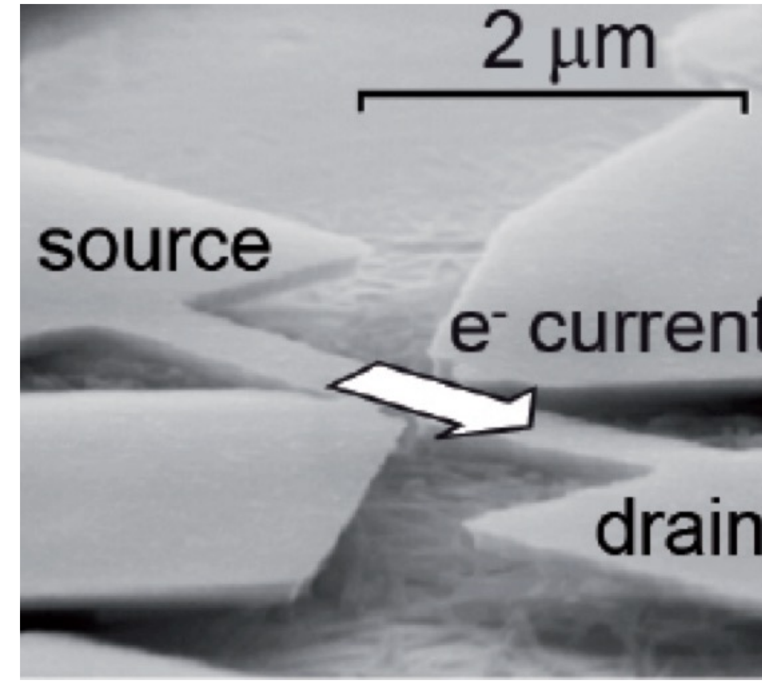
科研費 KAKENHI

卓越研究員事業
Leading Initiative for Excellent Young Researchers (LEADER)

Matsuo Foundation
公益財団法人松尾学術振興財団

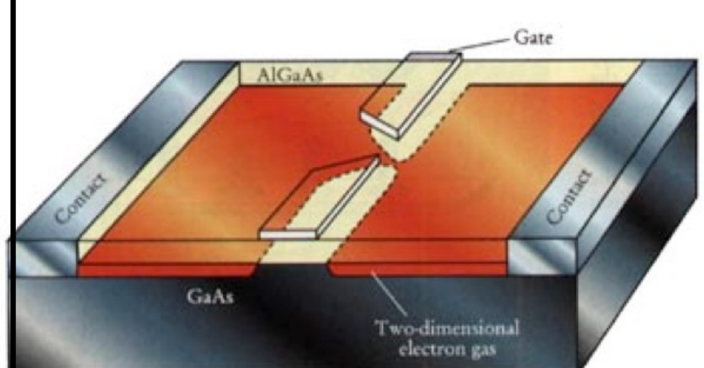
Mesoscopic systems in condensed matter

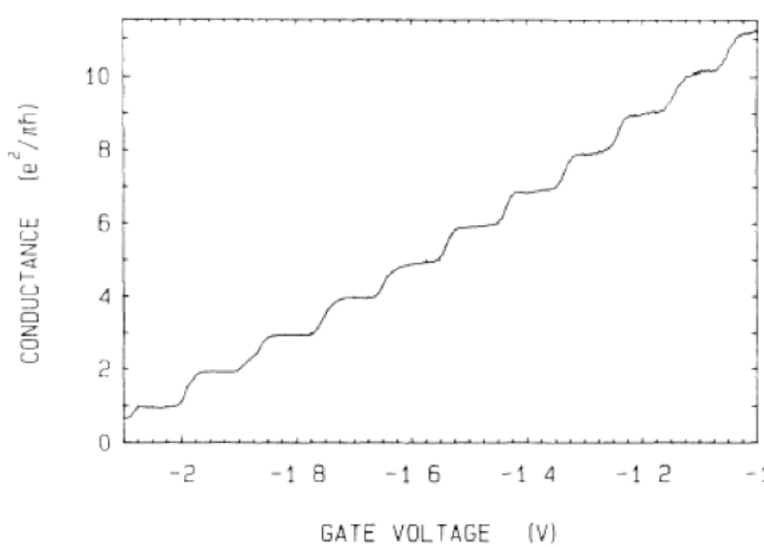
- Small sample (conduction channel) attached to macroscopic reservoirs
- Electric current induced by external bias voltage



C. Rossler et al., APL **93**, 071107 (2008)

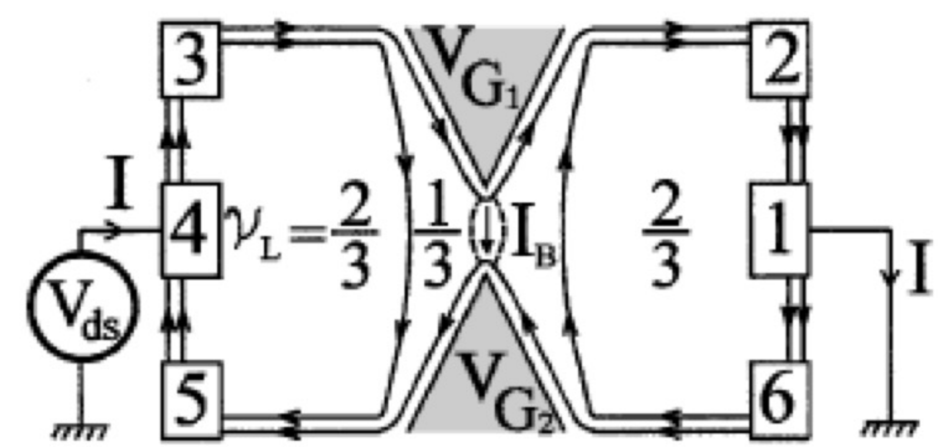
Conductance quantization in quantum point contact





B. J. van Wees et al., PRL **60**, 848 (1988).

Fractional charge measurement in FQHE

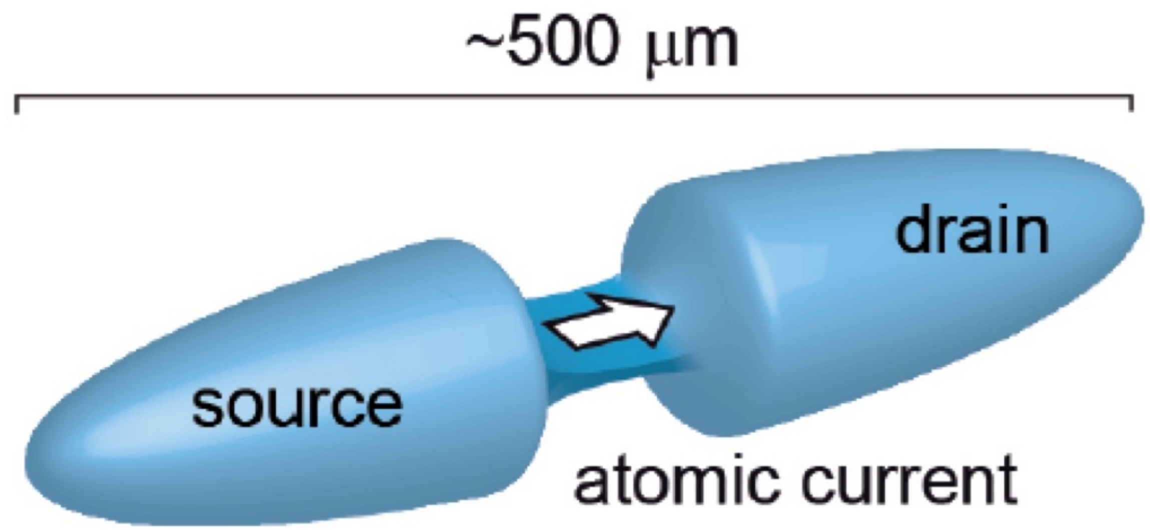


R. de-Picciotto et al., Nature **389**, 162 (1997); L. Saminadyar et al., PRL **79**, 2526 (1997).

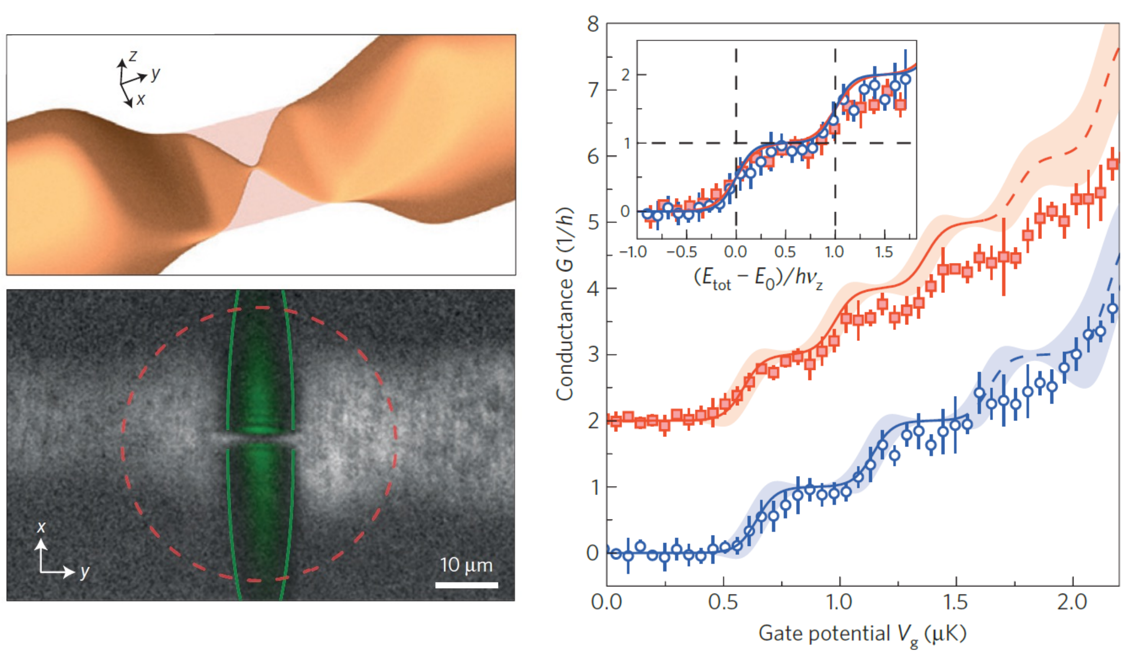
Mesoscopic system w/ cold atoms

Two-terminal setup realized by Esslinger's group at ETH

- Atomic current (charge neutral)
- Current induced by biases on thermodynamic quantities (chemical potential, temperature)

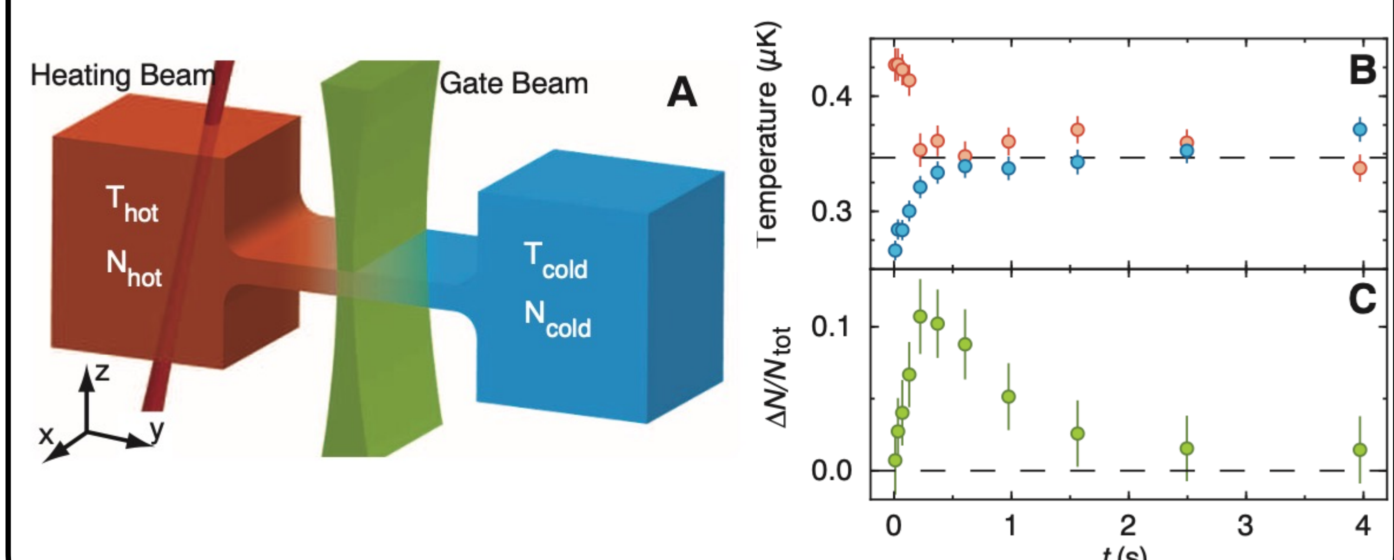


Conductance quantization in quantum point contact



S. Kriner et al., Nature **517,** 6467 (2015).

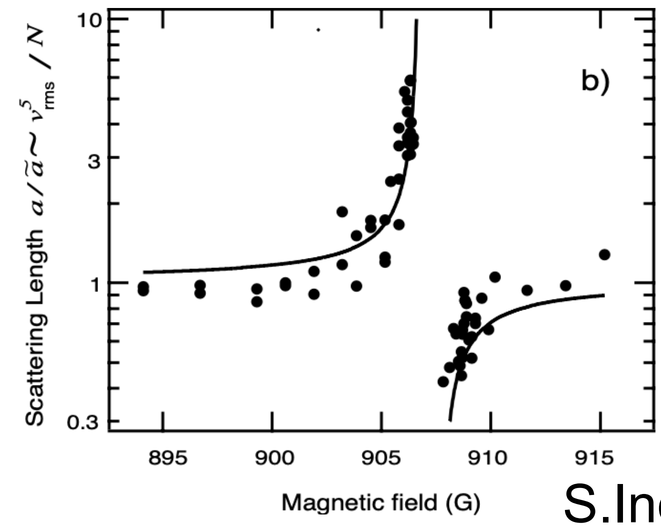
Heat transport



J.-P. Brantut et al., Science **342**, 713 (2013).

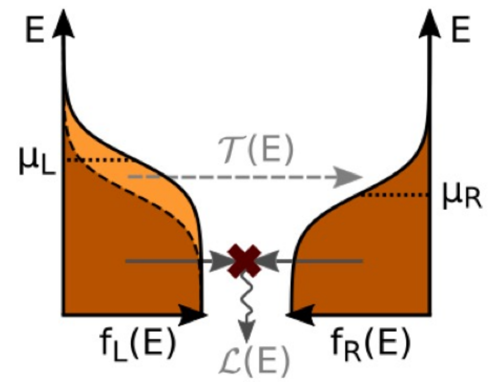
Why cold atoms?

- Control of interaction



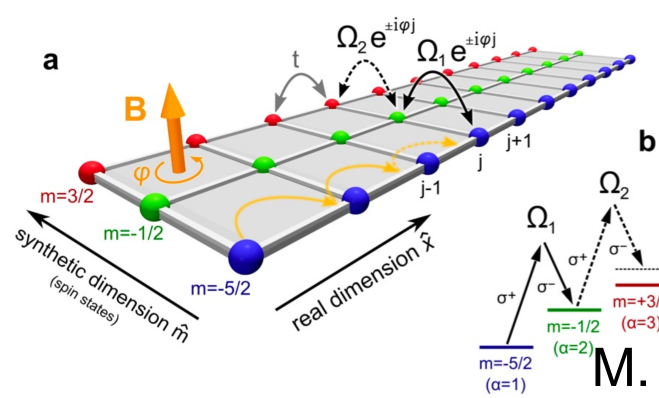
S.Inouye et al., Nature **392**, 151 (1998).

- control of dissipation



L. Corman et al., PRA **103**, 059902 (2021).

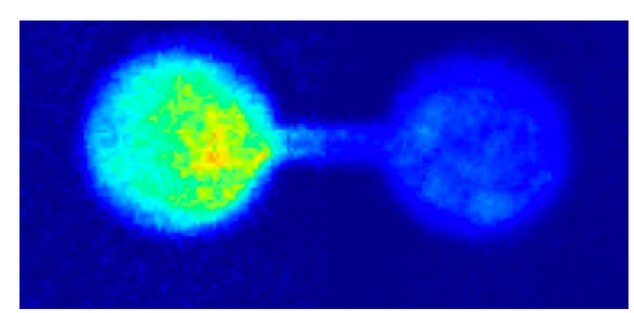
- Synthetic dimensions



M. Mancini et al., Science **349**, 1510 (2015).

- Control of quantum statistics

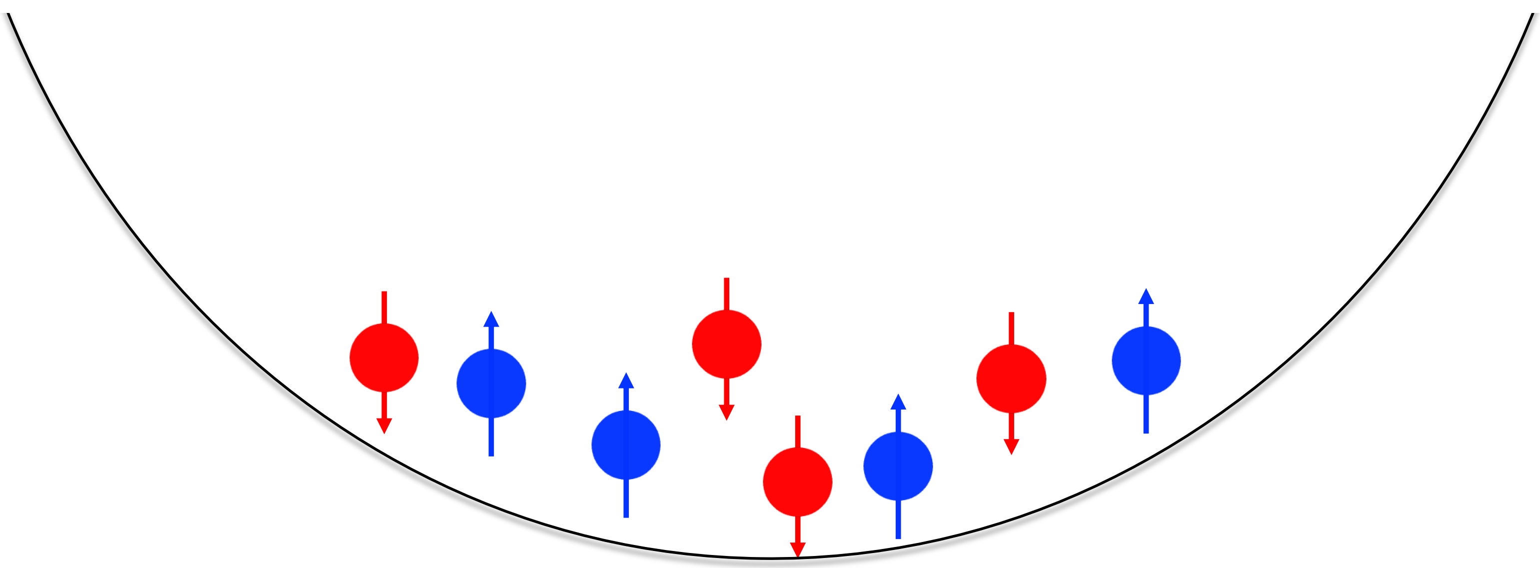
  Bosons as well as fermions



S. Eckel et al., PRA **93**, 063619 (2016).

Transport in strongly-interacting Fermi gases

$$\mathcal{H} = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$



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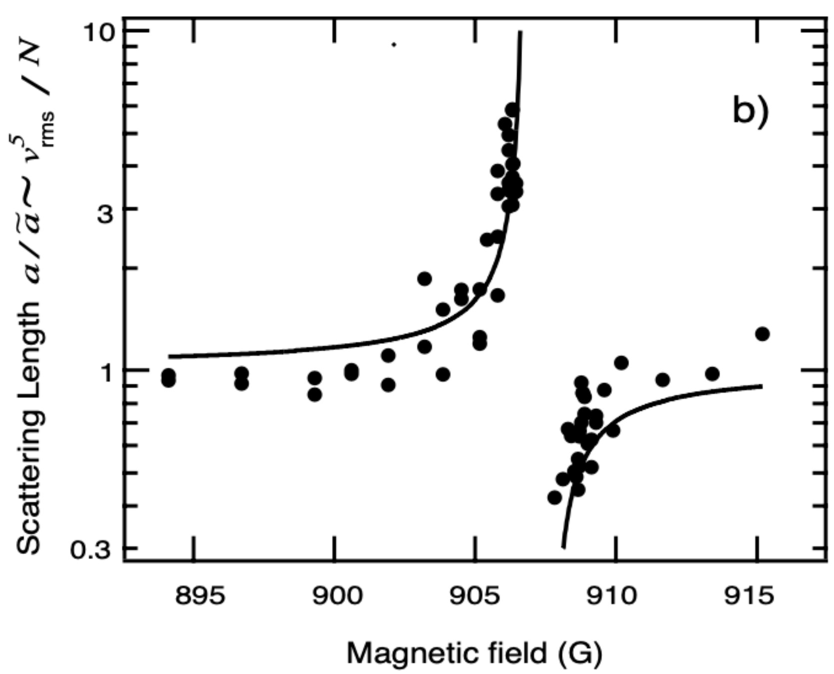
g (attractive interaction) can be tuned with Feshbach resonance!



S.Inouye et al., Nature **392**, 151 (1998).

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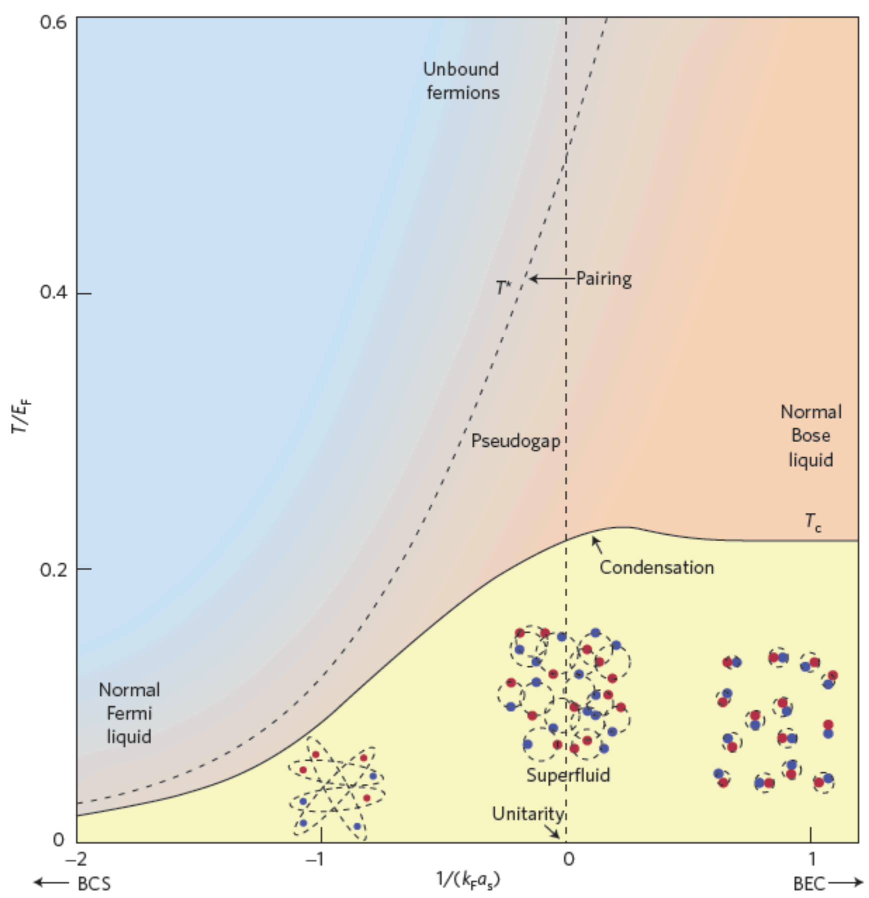
Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

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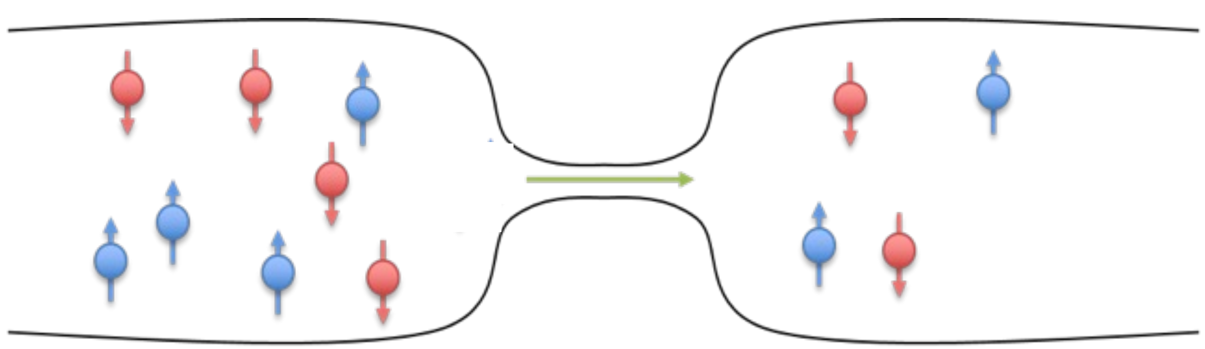

Transport in strongly-interacting Fermi gases

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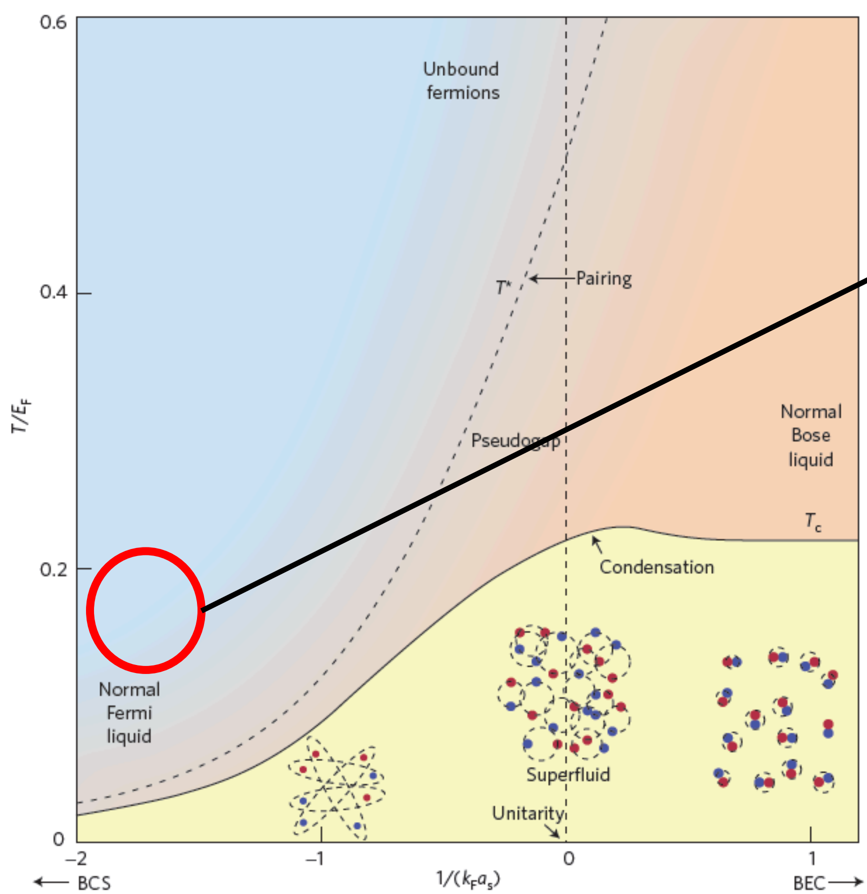
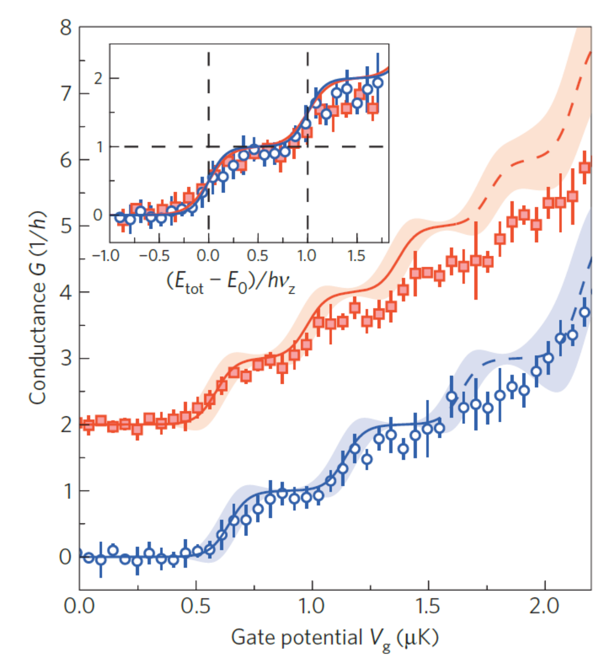
Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

Transport of noninteracting fermions

Conductance quantization



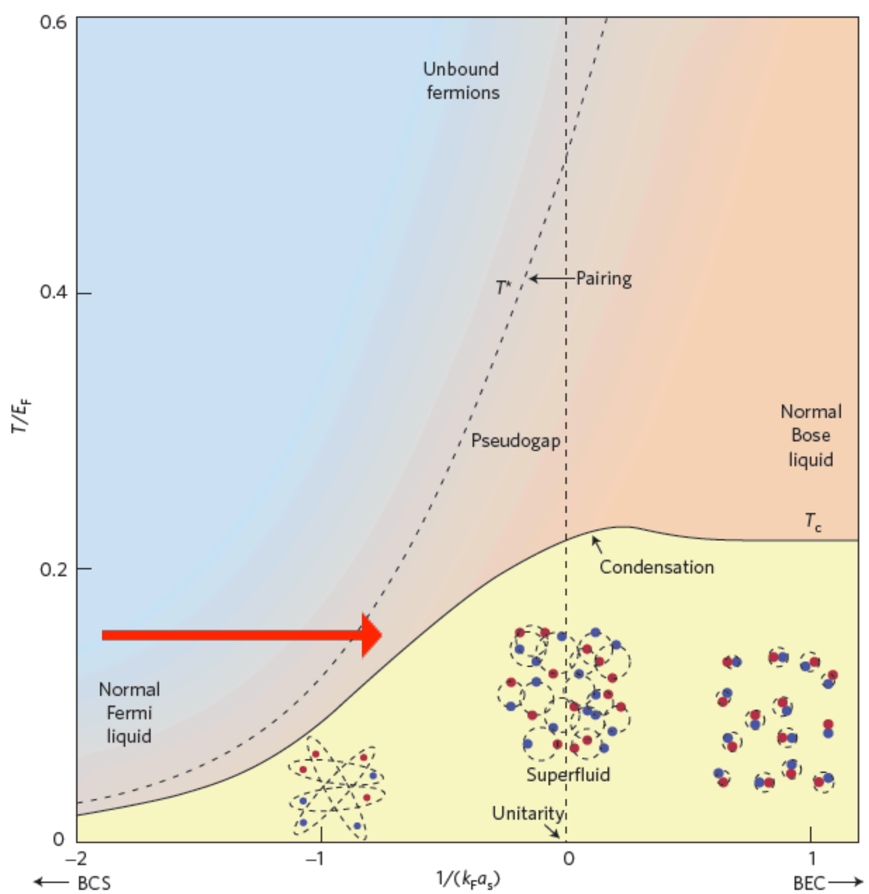
S. Kriner et al., Nature **517,** 6467 (2015).

Transport in strongly-interacting Fermi gases

$$\mathcal{H} = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$

Interaction inside the wire

Phase diagram



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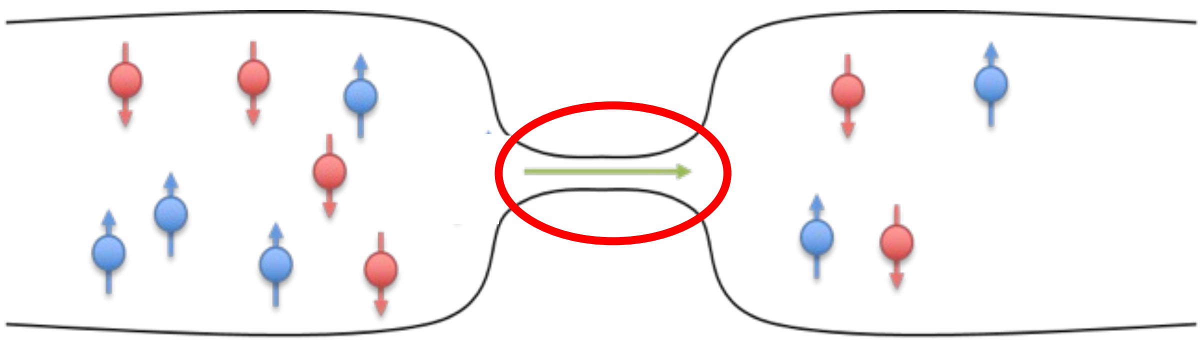


Old theory predicted $G = \frac{K}{h}$

K : Luttinger parameter

Experiment concluded $G = \frac{1}{h}$

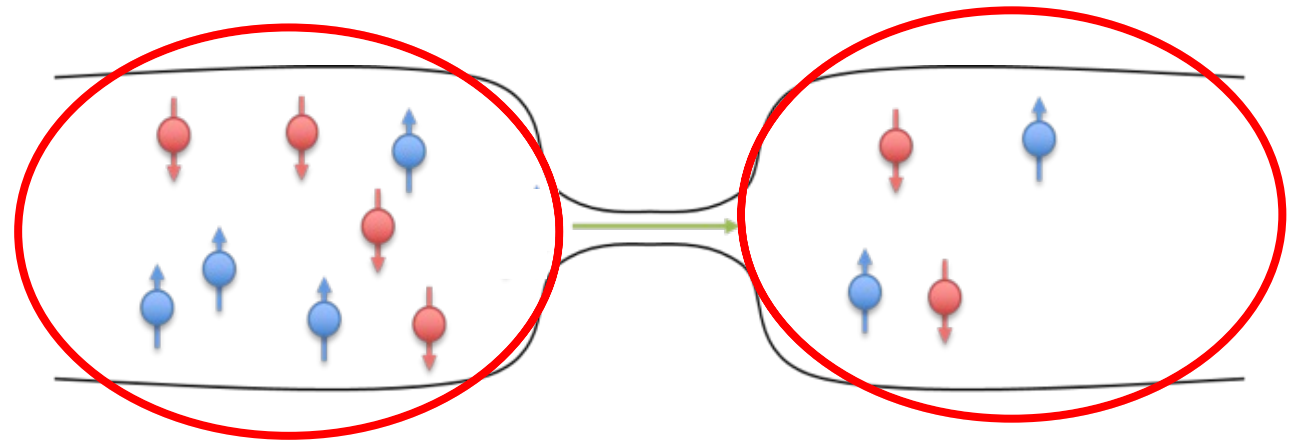
S. Tarucha et al., Solid State Commun. **94,** 413 (1995).

Later on, this discrepancy was resolved with inhomogeneous Tomonaga-Luttinger theory.

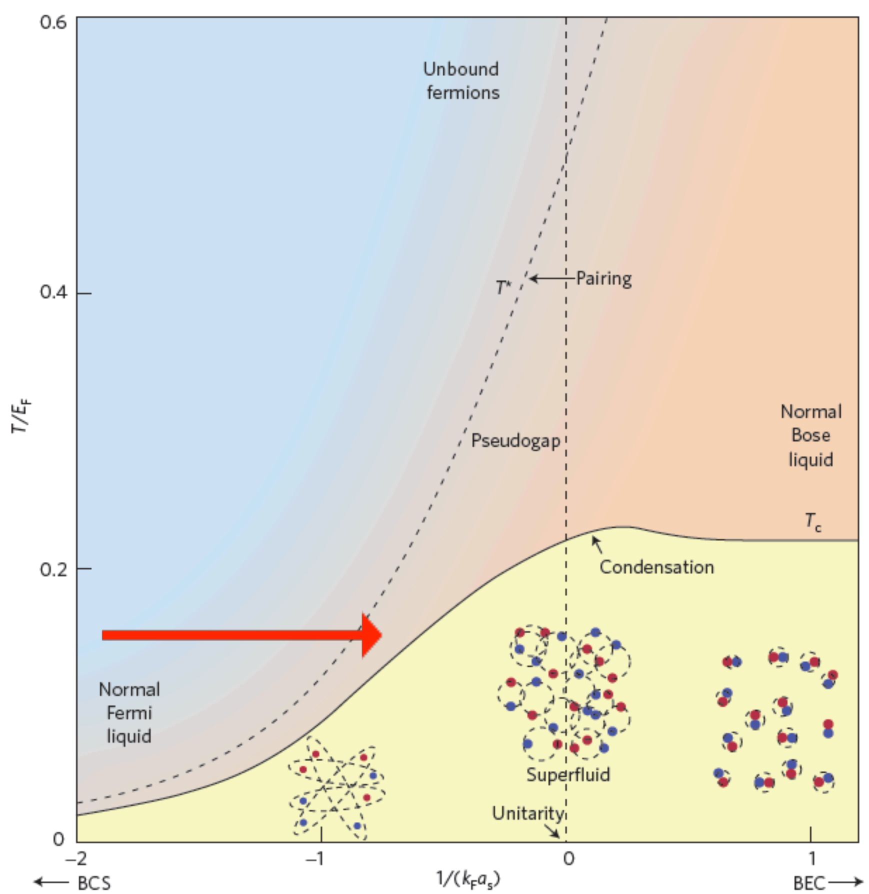
Transport in strongly-interacting Fermi gases

$$\mathcal{H} = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$

Interaction inside reservoirs

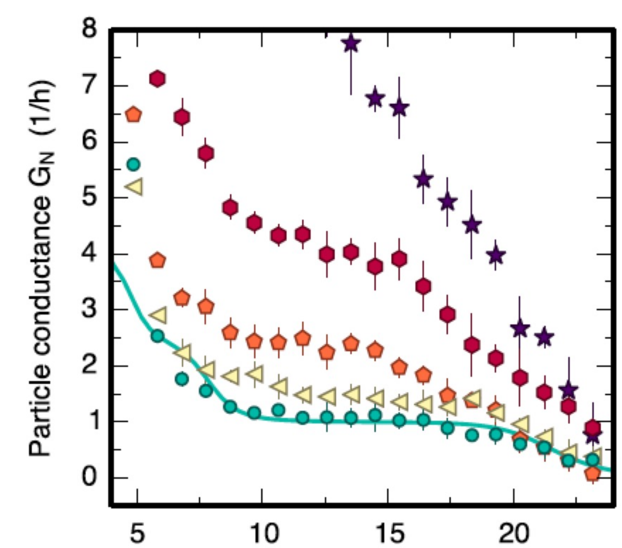


Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

Breakdown of conductance quantization



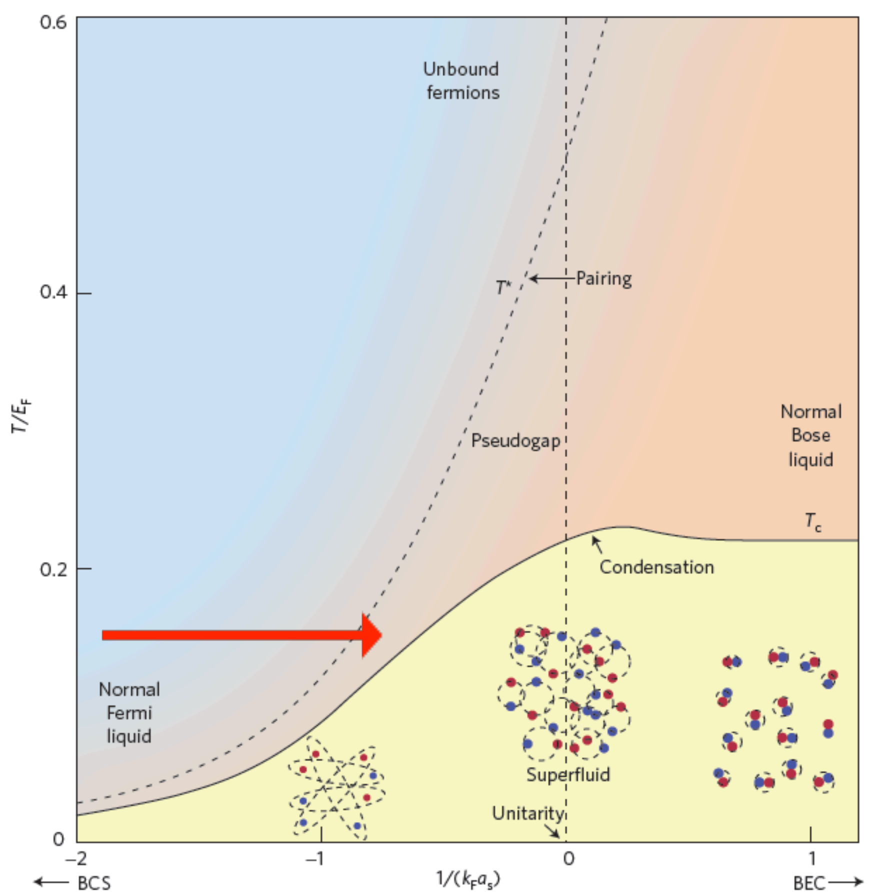
S. Krinner et al., PNAS **13**, 8144 (2016).

New phenomenon found in cold atoms!

Transport in strongly-interacting Fermi gases

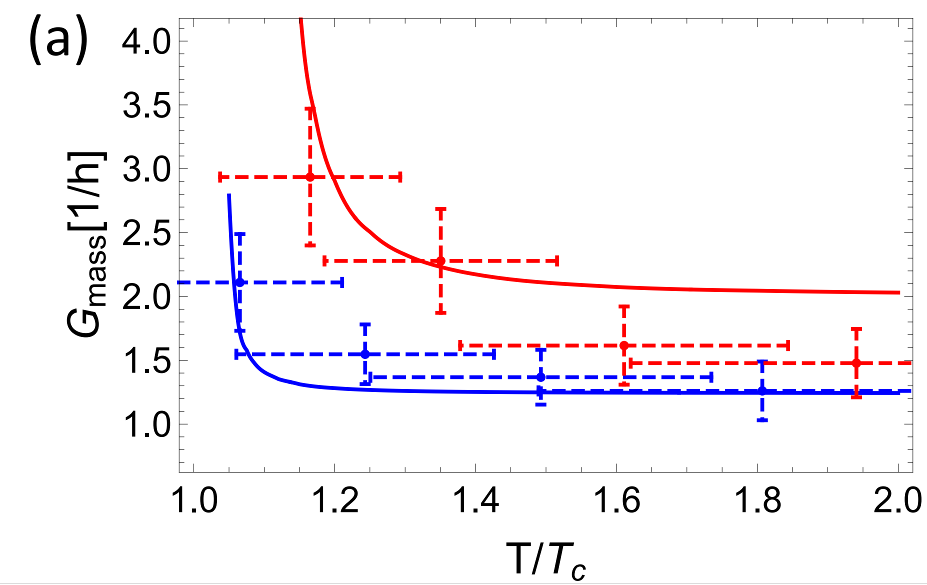
$$\mathcal{H} = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$

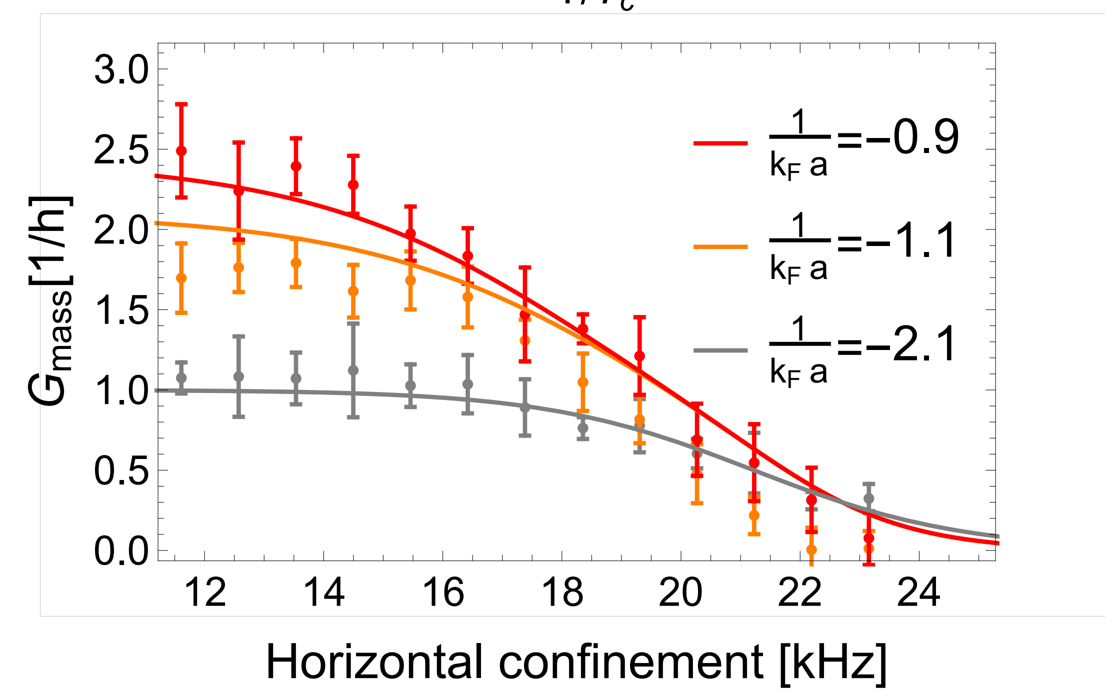
Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

Theory with preformed pairs

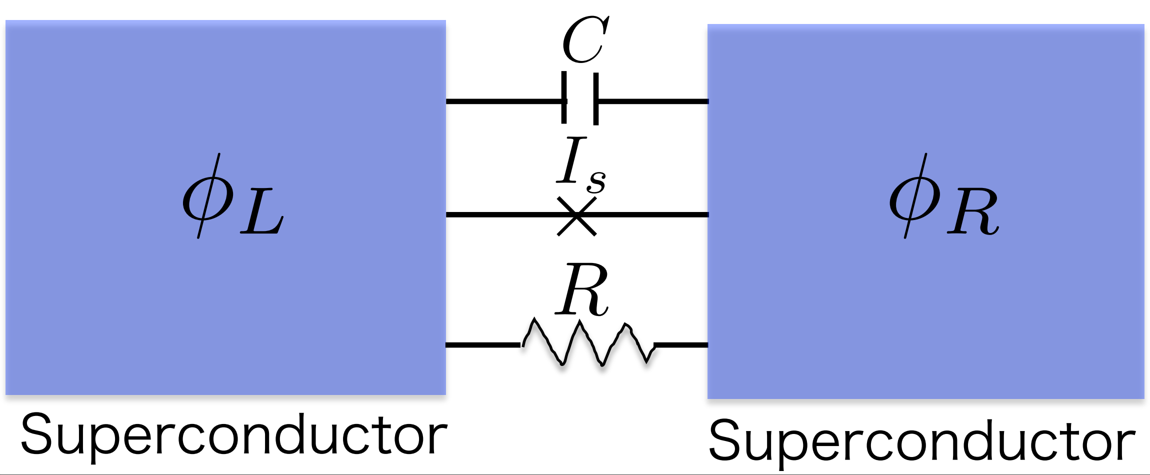




SU and M. Ueda, PRL **118**, 105303 (2017).

Dissipation in quantum transport

Condensed matter example: dissipation in Josephson junctions



RCSJ model

$$I = C\frac{dV}{dt} - I_s \sin(\Delta\phi) + \frac{V}{R}$$

$$V = -\frac{d\Delta\phi}{dt}$$

Coupling to environment: Caldeira & Leggett

PRL **46**, 211 (1981)

$$\mathcal{L} = \frac{C}{2}\left(\frac{d\Delta\phi}{dt}\right)^2 + I_0\cos(\Delta\phi) + \sum_i \frac{\lambda_i^2}{m_i\omega_i^2}\Delta\phi + \frac{1}{2}\sum_i m_i\left[\left(\frac{dx_i}{dt}\right)^2 - \omega_i^2 x_i^2\right] - \sum_i \frac{\lambda_i^2}{2m_i\omega_i^2}(\Delta\phi)^2$$

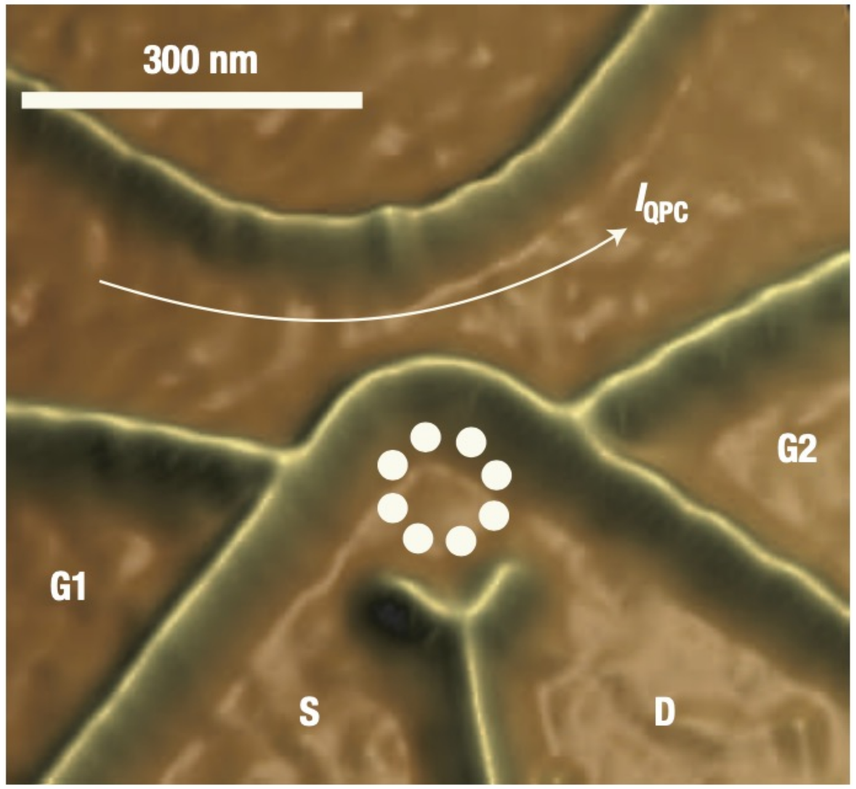
$$\pi\sum_i \frac{\lambda_i^2}{2m_i\omega_i}\delta(\omega-\omega_i) = \frac{\omega}{R}$$

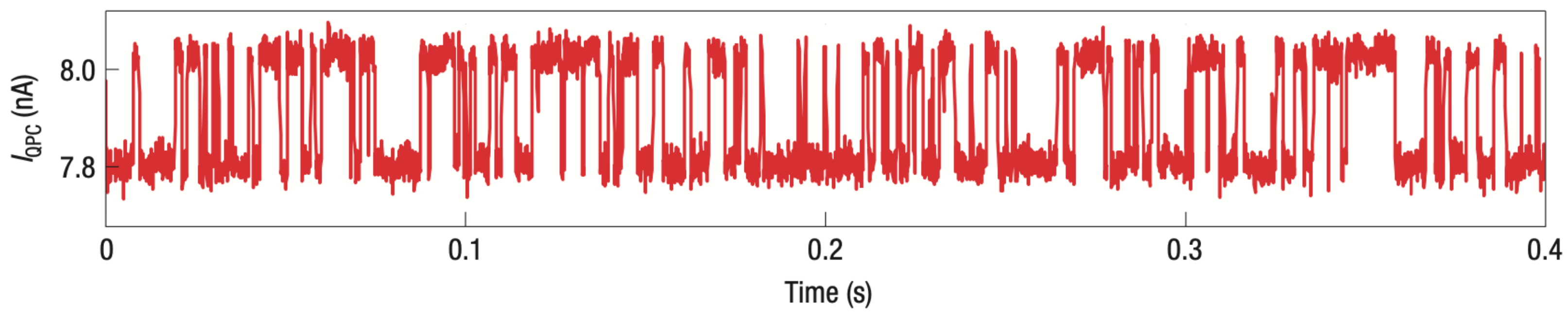
⇒ Reproduce the RCSJ model

Dissipation effect in quantum transport

Condensed matter example2: continuous measurement of mesoscopic currents

E.V. Sukhorukov et al., Nat. Phys. **3**, 243, (2007)

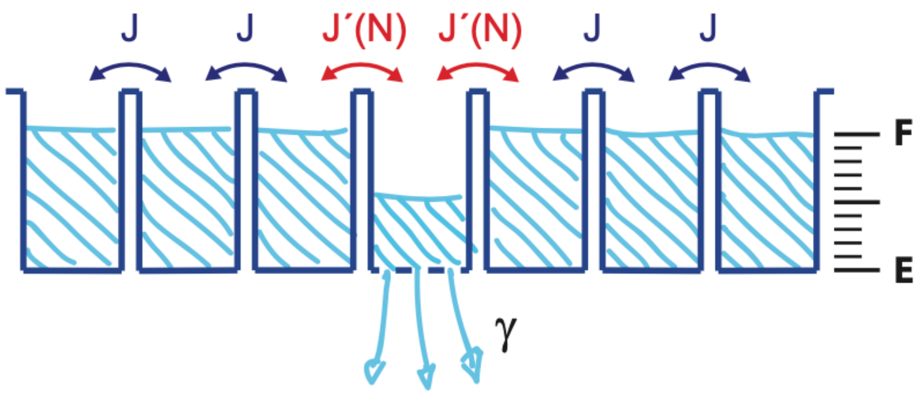




The quantum dot distribution affects the current in the quantum point contact .

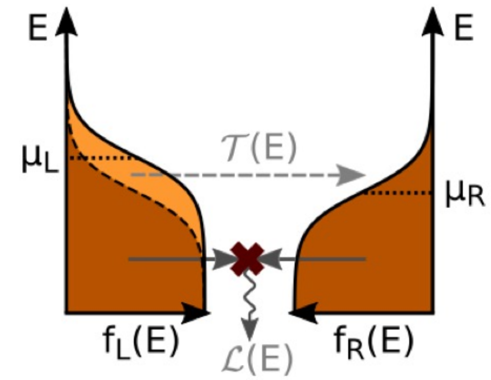
Dissipation in cold atoms

Jesephson junction array of Bose-Einstein condensates



R. Labouvie et al., PRL**116**, 235302 (2016).

Point contact transport in noninteracting Fermi gases



L. Corman et al., PRA **103**, 059902 (2021).

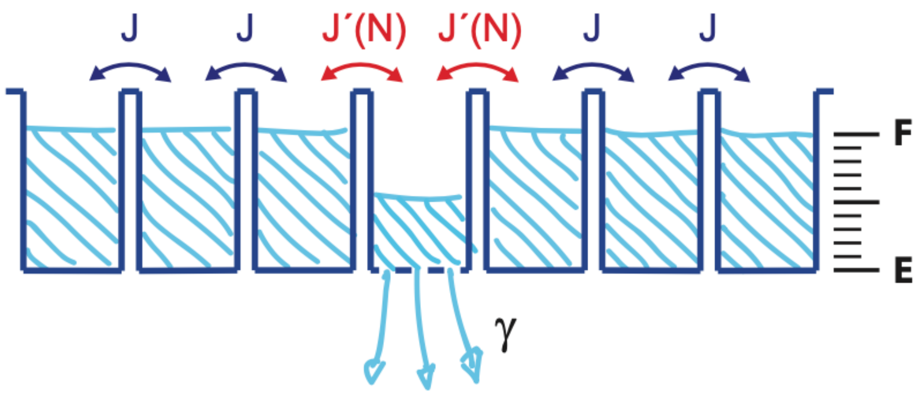
Local single particle loss as dissipation

Quantum master equation:

$$\partial_t \rho = i[\rho, H] + \gamma \left[\psi \rho \psi^\dagger - \frac{\{\psi^\dagger \psi, \rho\}}{2} \right]$$

Dissipation in cold atoms

Jesephson junction array of Bose-Einstein condensates



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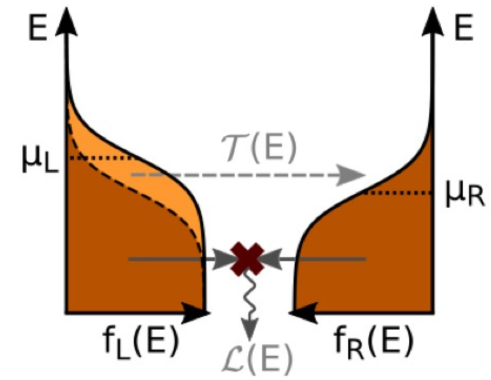
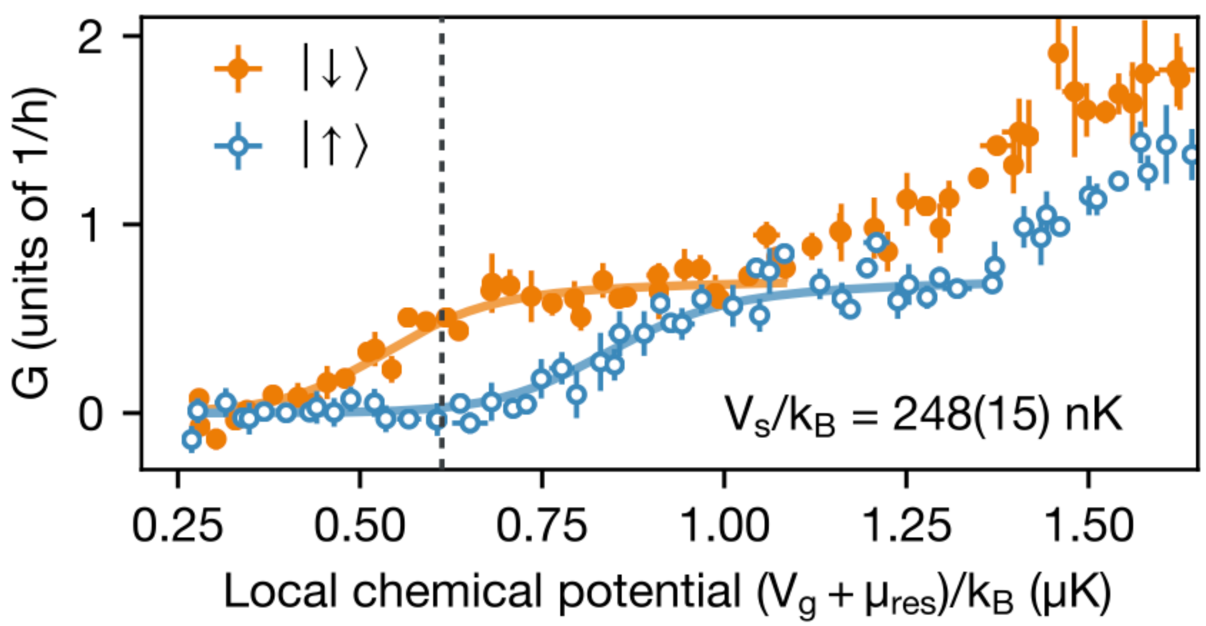


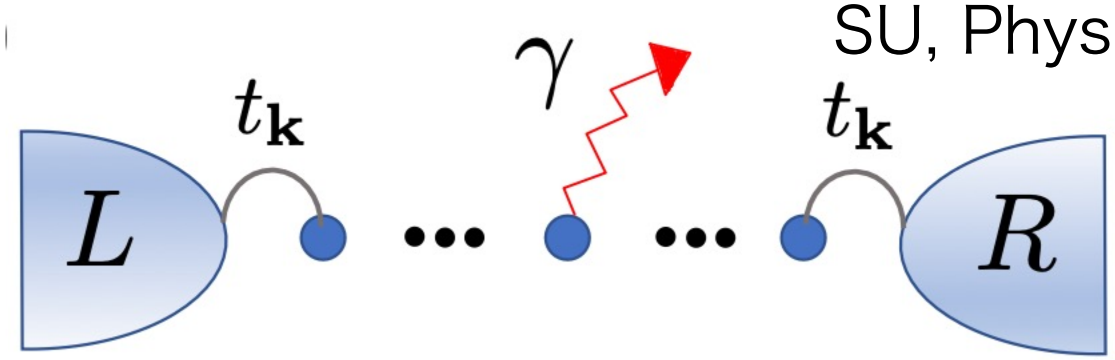
L. Corman et al., PRA **103**, 059902 (2021).



Experimental results were interpreted with a phenomenological analysis

Theoretical model 1

SU, Phys. Rev. A **106**, 053320 (2022).



$$H = H_L + H_R + H_{1D} + H_T$$

$$\partial_t \rho = i[\rho, H] + \gamma \left[d_0 \rho d_0^\dagger - \frac{\{d_0^\dagger d_0, \rho\}}{2} \right]$$

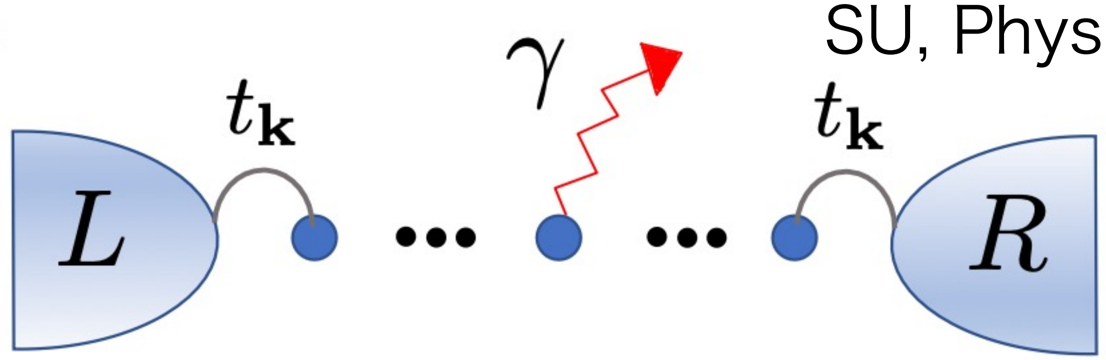
Physical quantities of interest

$$I = \frac{\langle (\partial_t N_R - \partial_t N_L) \rangle}{2}$$

$$-\dot{N} = -\langle (\partial_t N_R + \partial_t N_L) \rangle$$

Theoretical model 1

SU, Phys. Rev. A **106**, 053320 (2022).



$$H = H_L + H_R + H_{1D} + H_T$$

$$\partial_t \rho = i[\rho, H] + \gamma\left[d_0 \rho d_0^\dagger - \frac{\{d_0^\dagger d_0, \rho\}}{2}\right]$$

L.M. Sieberer et al., Rep. Prog. Phys. **79**, 096001 (2016).

Result based on Keldysh+Lindblad formalism

$$I = \int \frac{d\omega}{2\pi}\left[\mathcal{T}(\omega) + \frac{\mathcal{L}(\omega)}{2}\right][n_L(\omega) - n_R(\omega)]$$

$$-\dot{N} = -\int \frac{d\omega}{2\pi}\mathcal{L}(\omega)[n_L(\omega) + n_R(\omega)]$$

$\mathcal{T}(\omega)$: transmittance

$\mathcal{L}(\omega)$: loss probability

SU, Phys. Rev. A **106**, 053320 (2022).

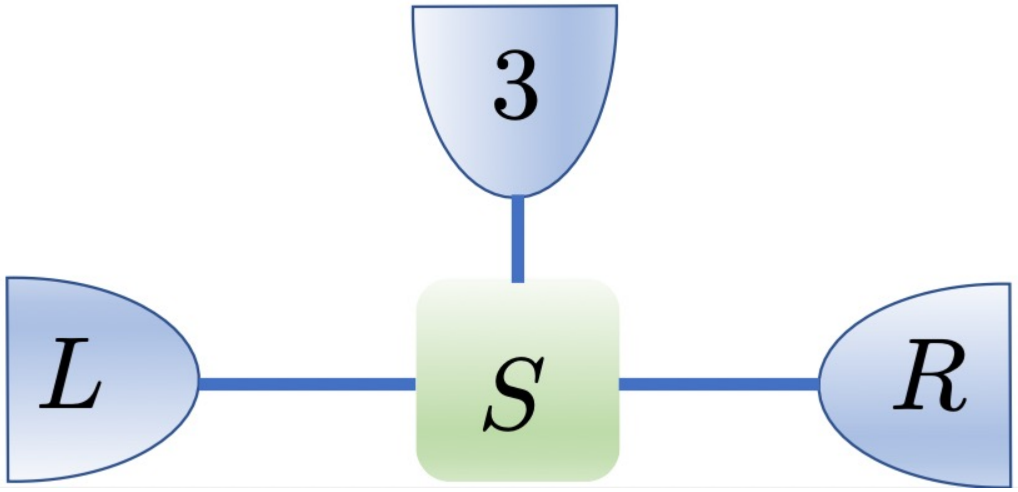
Three-terminal Landauer-Büttiker analysis

$$I_j(t) = \int \frac{d\omega}{2\pi} \int d\omega' e^{i(\omega-\omega')t} [a_j^\dagger(\omega) a_j(\omega') - b_j^\dagger(\omega) b_j(\omega')]$$

$$\begin{pmatrix} b_L \\ b_R \\ b_3 \end{pmatrix} = \begin{pmatrix} r & t & t_L \\ t' & r' & t_R \\ t'_L & t'_R & r_3 \end{pmatrix} \begin{pmatrix} a_L \\ a_R \\ a_3 \end{pmatrix}$$

$$\mathcal{T} = |t|^2 = |t'|^2$$

$$\mathcal{L} = |t_L|^2 = |t_R|^2$$



Result based on the scattering formalism

$$I = \int \frac{d\omega}{2\pi} \left[\mathcal{T}(\omega) + \frac{\mathcal{L}(\omega)}{2} \right] [n_L(\omega) - n_R(\omega)]$$

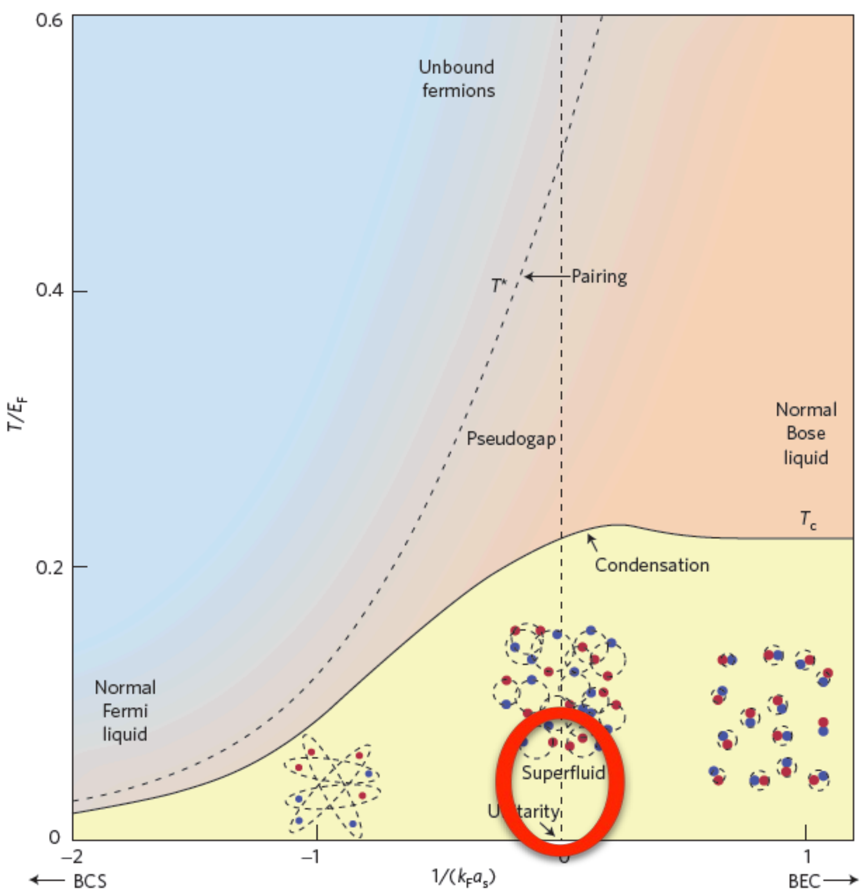
$$-\dot{N} = -\int \frac{d\omega}{2\pi} \mathcal{L}(\omega) [n_L(\omega) + n_R(\omega)]$$

provided that the injection from the third reservoir is absent $(\mathcal{L} n_3 = 0)$.

Strongly-interacting superfluid + dissipation

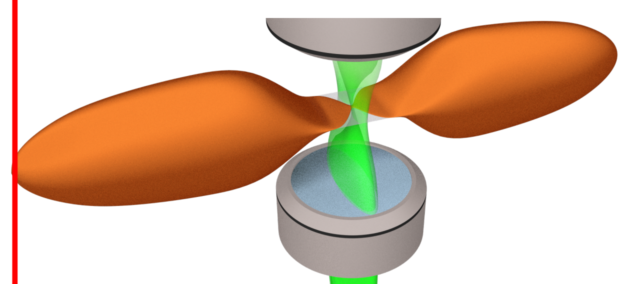
$$\mathcal{H} = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$

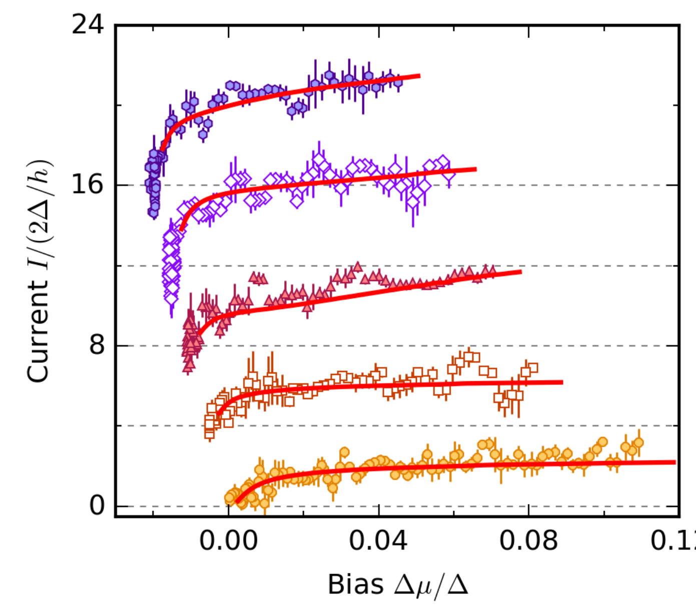
Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

Transport without dissipation





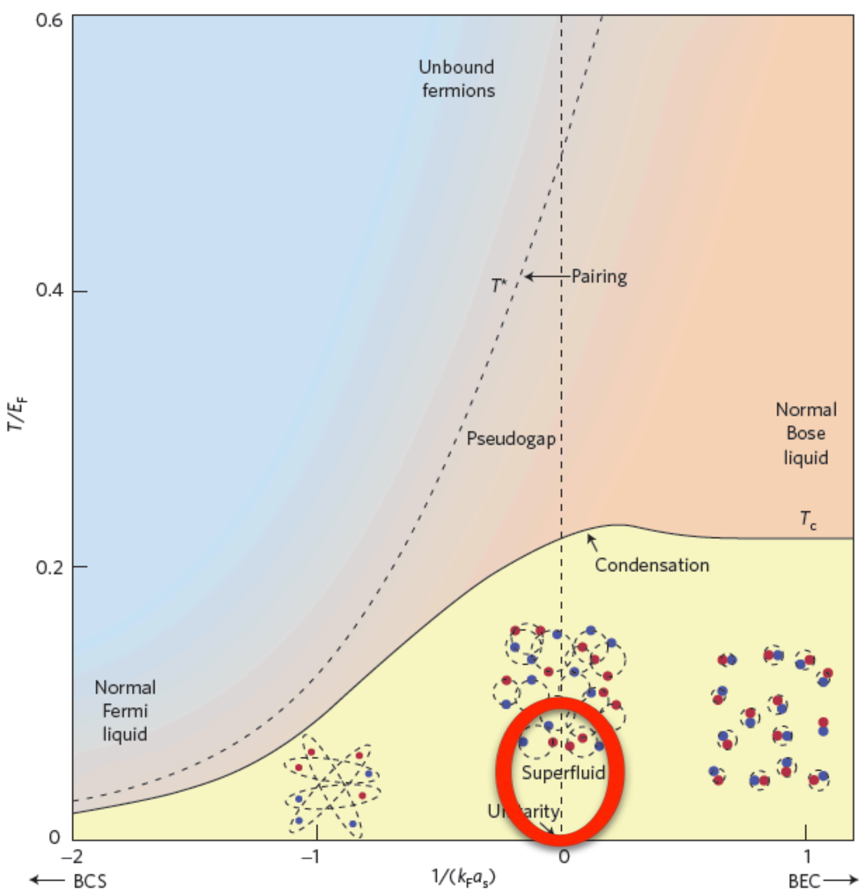
D. Husmann et al., Science **350**, 1498 (2015).

Nonlinear current-bias characteristics stemming from multiple Andreev reflections

Strongly-interacting superfluid + dissipation

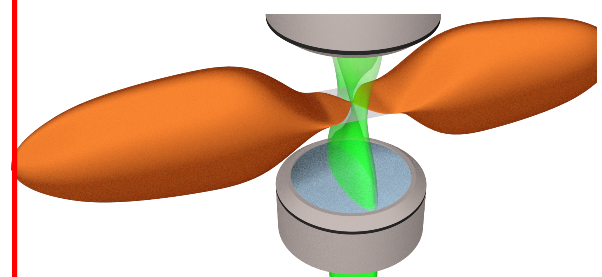
$$\mathcal{H} = \int d^3 r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big\{ -\frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$

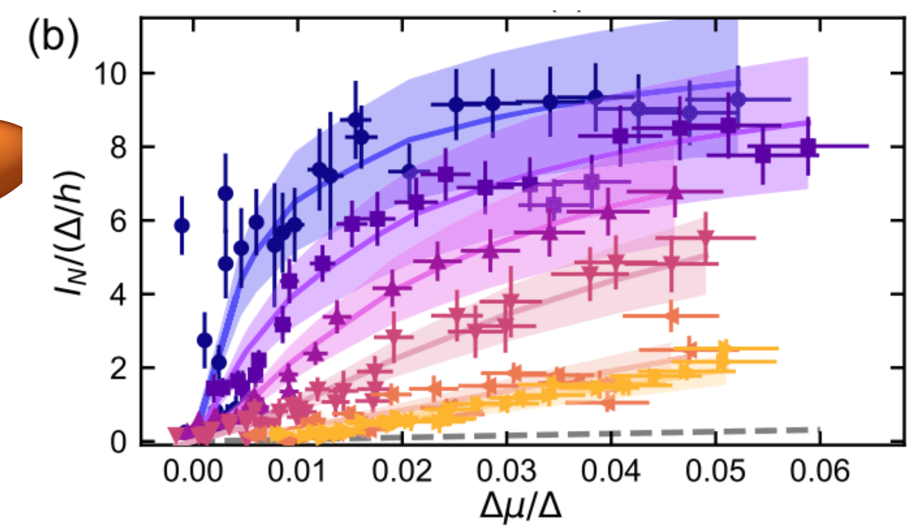
Phase diagram



M. Randeria, E. Taylor, Annual Review of Condensed matter physics **5,** 209 (2014).

Transport with dissipation



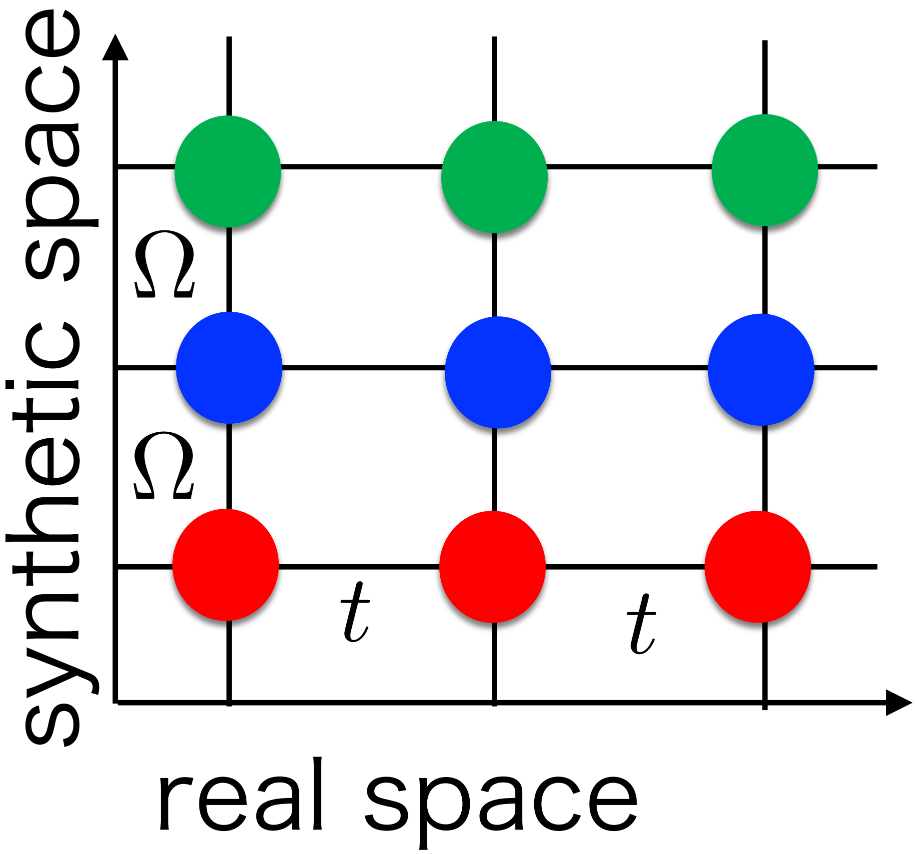


M.-Z. Huang et al., arXiv:2210.03371 (Phys. Rev. Lett., In press).

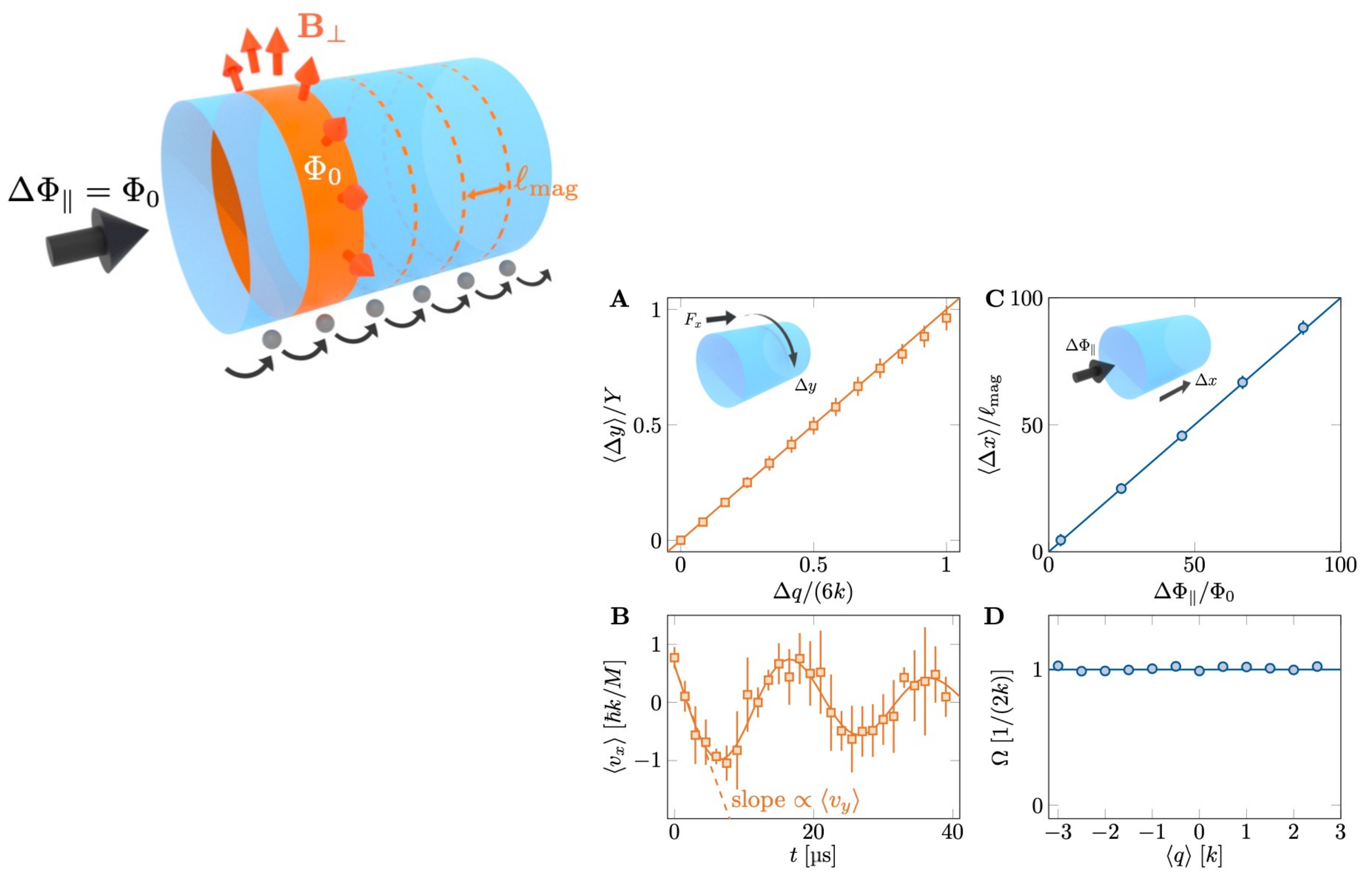
Nonlinearity is weakened by atom loss.

Lindblad+Keldysh analysis gives consistent results.

Synthetic dimensions



Laughlin's charge pump



A. Fabre et al., PRL **128**, 173202 (2022).

t : hopping amplitude along real space

Ω: hopping amplitude along synthetic space

Mesoscopic transport via magnetic impurity

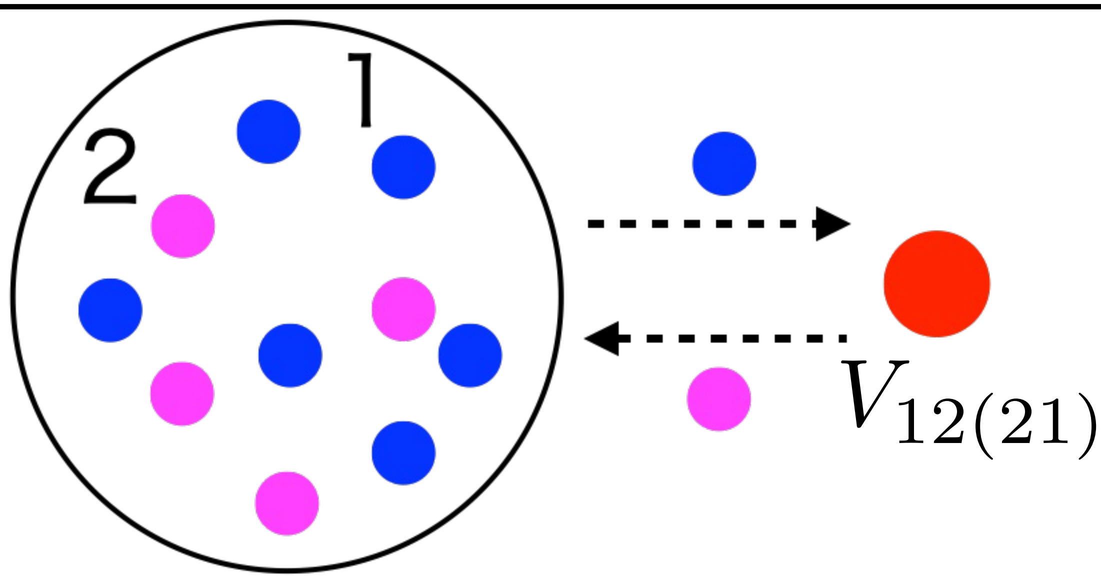
S. Nakada et al., PRA 102, 031302(R) (2020).

$$\mathcal{H} = \int d^3 r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi^\dagger_\sigma \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\sigma - g\psi^\dagger_\uparrow \psi^\dagger_\downarrow \psi_\downarrow \psi_\uparrow \Big] + \sum_\sigma V_\sigma \psi^\dagger_\sigma \psi_\sigma(\mathbf{0})$$

spin rotation $|\sigma\rangle \to |\alpha\rangle = \sum_\sigma |\sigma\rangle U^\dagger_{\sigma\alpha}$

$$\mathcal{H} = \int d^3 r \Big[\sum_{\alpha=1,2} \psi^\dagger_\alpha \Big\{ -\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) \Big\} \psi_\alpha - g\psi^\dagger_1 \psi^\dagger_2 \psi_2 \psi_1 \Big] + \sum_{\alpha,\beta=1,2} \psi^\dagger V_{\alpha\beta} \psi_\beta(\mathbf{0})$$

$V_{12(21)} \neq 0 \quad \text{if } V_\uparrow \neq V_\downarrow$



Landauer-Büttiker formula

$$I = \int \frac{d\epsilon}{h} \mathcal{T}(\epsilon)[f_1(\epsilon) - f_2(\epsilon)]$$

JAEA

Realization with Yb atoms

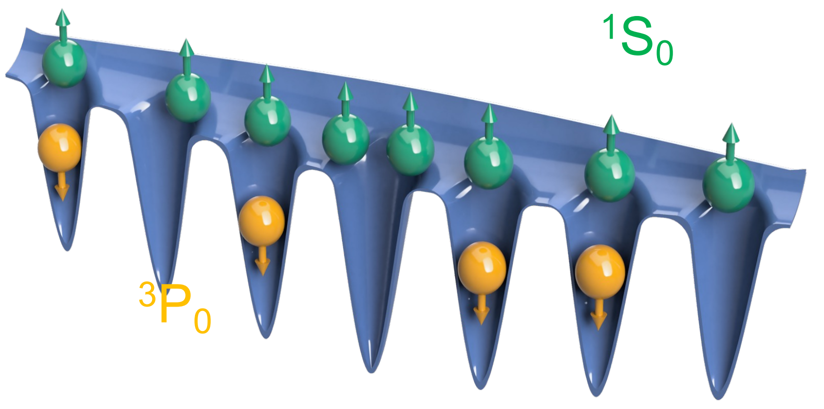
ASR

K. Ono et al., Nat. Commun. **12,** 6724 (2021).

- Two-orbital lattice system with ^{173}Yb

$^1S_0$ atoms: itinerant fermions

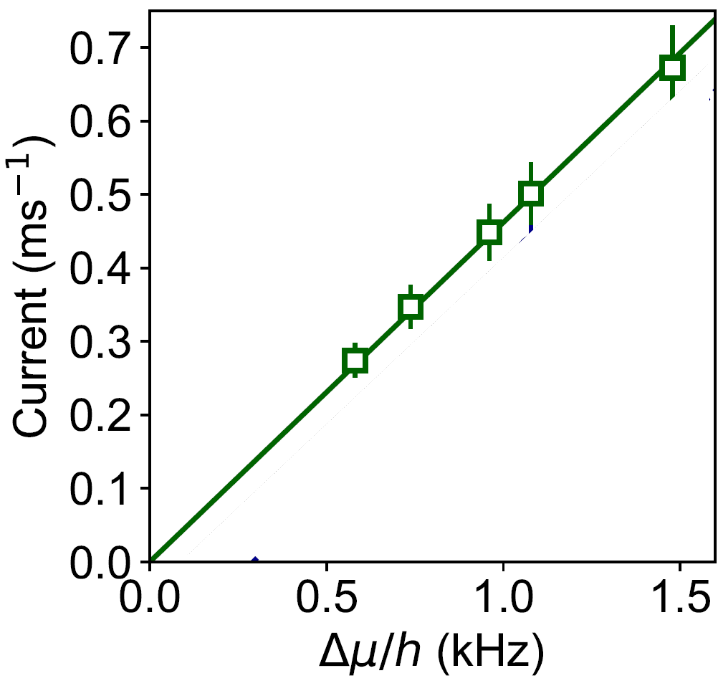
$^3P_0$ atom: localized impurity



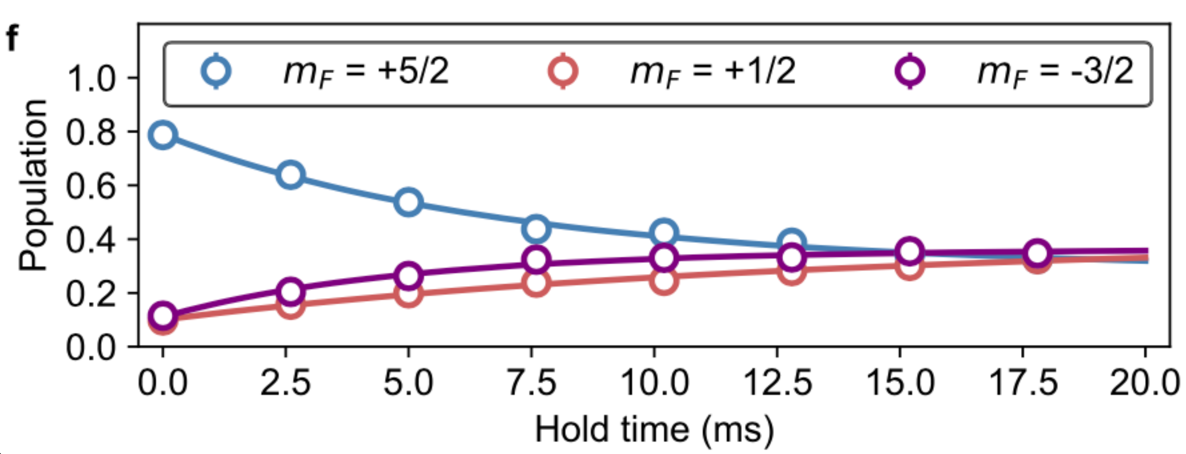
Spin-dependent potential can be tuned with the orbital Feshbach resonance

G. Pagano et al., PRL **115**, 265301 (2015); M. Hofer et al., PRL **115**, 265302 (2015).

Ohmic current



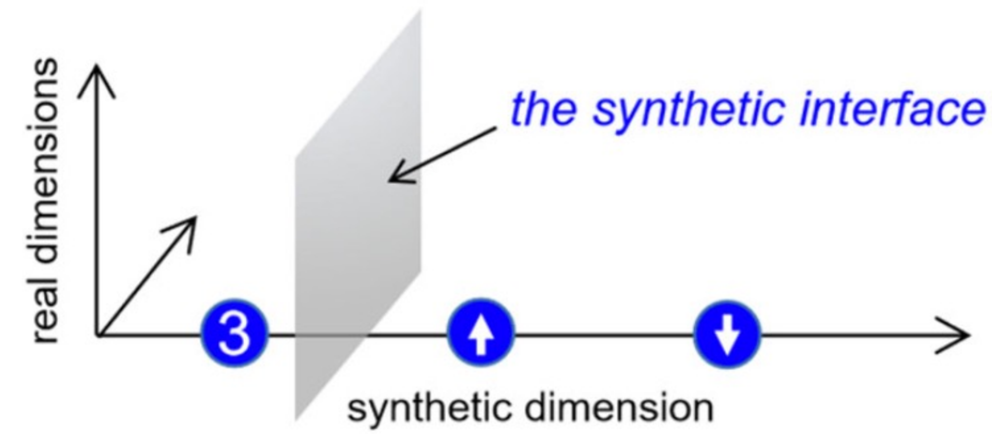
Three-terminal transport



Dominant Andreev current in an internal space junction

T. Zhang et al., Commun. Phys. **6**, 86 (2023).

$$H = H_{\mathrm{SF}} + H_3 + H_T$$

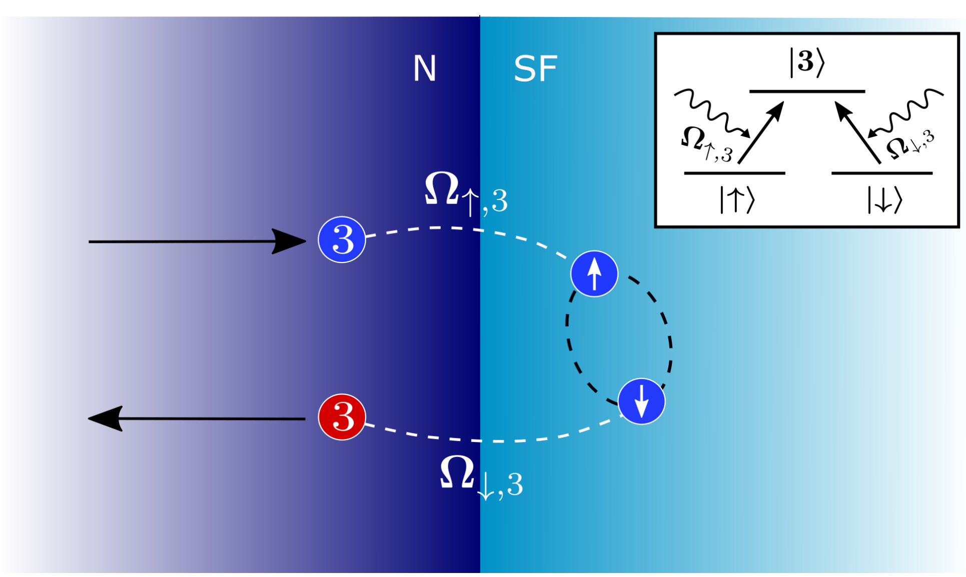


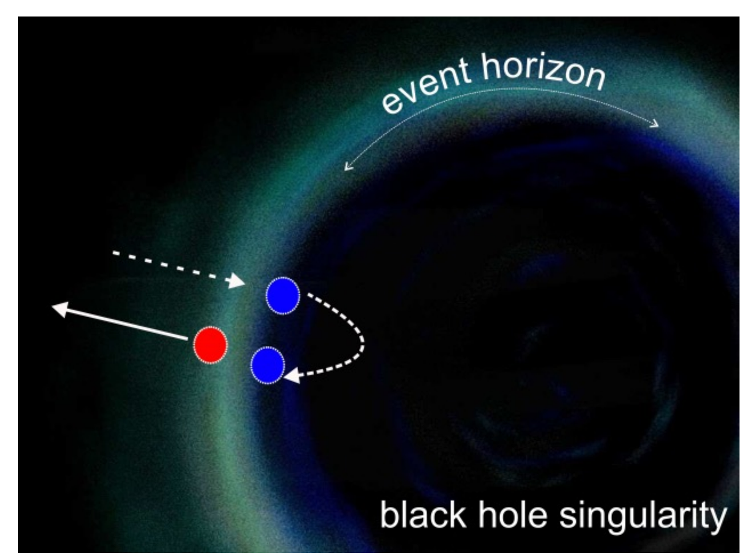
$$H_T = \sum_{\mathbf{k},\sigma} \left(e^{-i\omega_{L,\sigma} t} \Omega_{\sigma,3} d^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},3} + h.c. \right)$$

$$I = \dot{N}_3 = -i[N_3, H_T]$$

Tunneling w/ momentum conservation

➡ Analogy between Andreev reflection and physics of black holes





PRD **96**, 124011 (2017);PRD **98**, 124043 (2018);
PRD **102**, 064028 (2020).

Dominant Andreev current in an internal space junction

T. Zhang et al., Commun. Phys. **6**, 86 (2023).

$$H = H_{\mathrm{SF}} + H_3 + H_T$$

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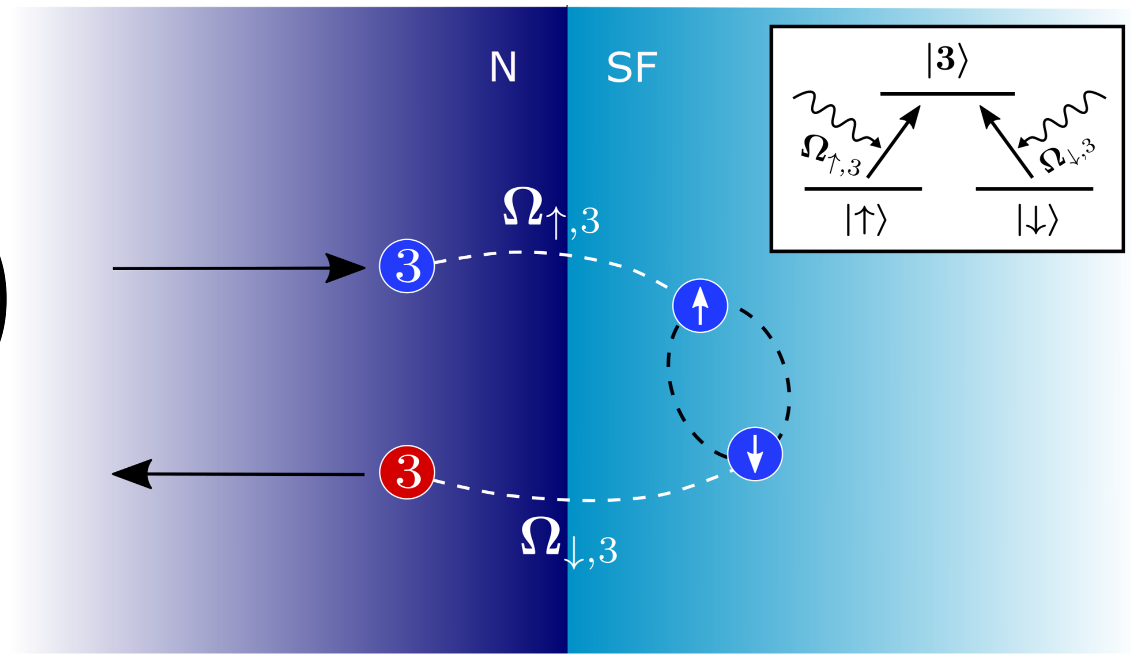
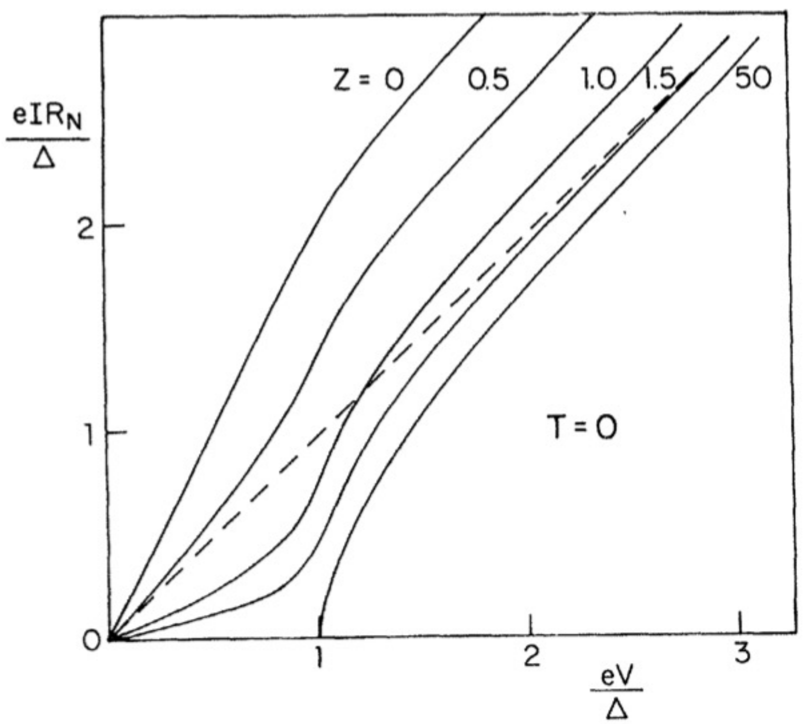
$$I = \dot{N}_3 = -i[N_3, H_T]$$



e.g., analysis up to $\Omega_{\sigma,3}$

Rf spectroscopy

$\approx$ tunneling junction of superfluid & normal states

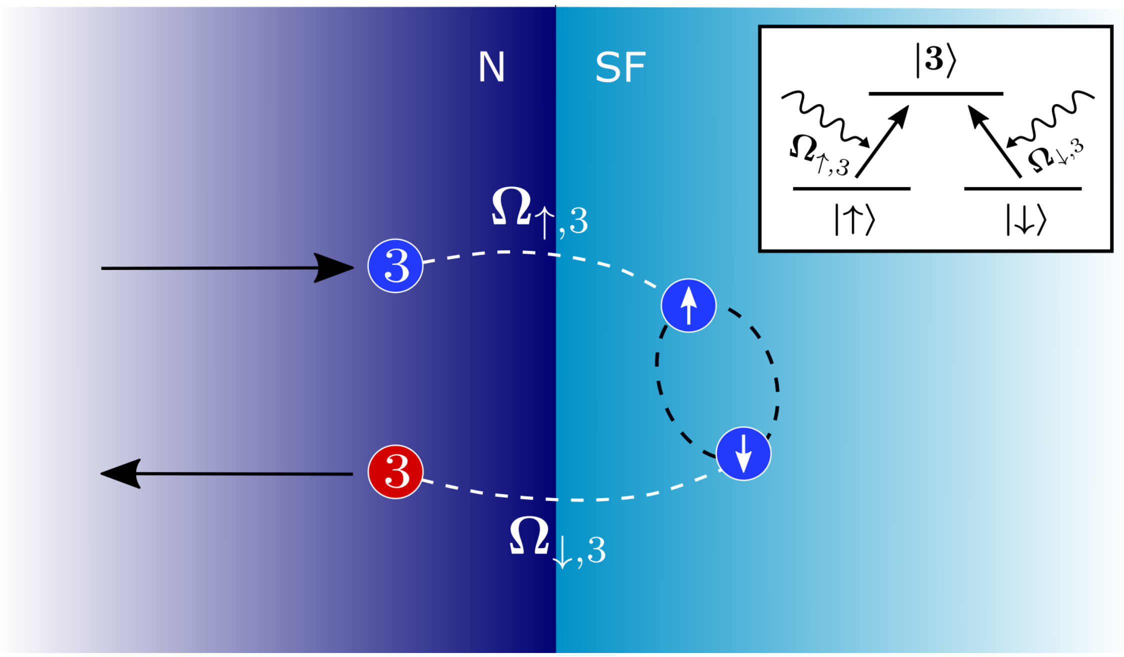


Blonder,Tinkham, Klapwijk, PRB **25**, 4515 (1982).

Dominant Andreev current in an internal space junction

T. Zhang et al., Commun. Phys. **6**, 86 (2023).

$$H = H_{\mathrm{SF}} + H_3 + H_T$$

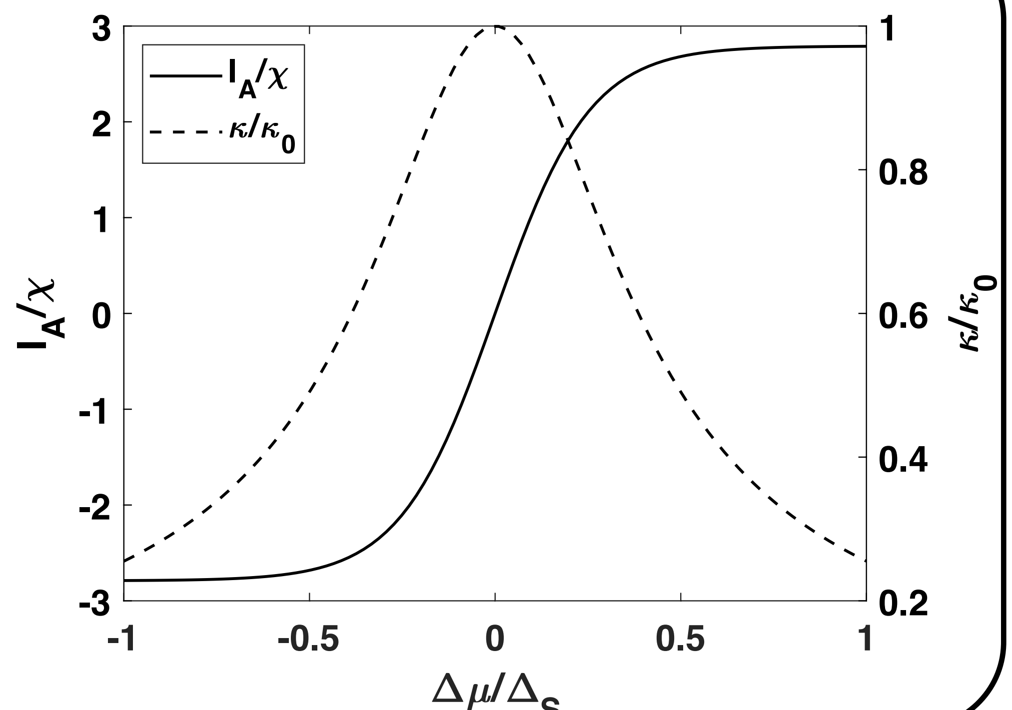


$$H_T = \sum_{\mathbf{k},\sigma} \left(e^{-i\omega_{L,\sigma} t} \Omega_{\sigma,3} d^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},3} + h.c. \right)$$

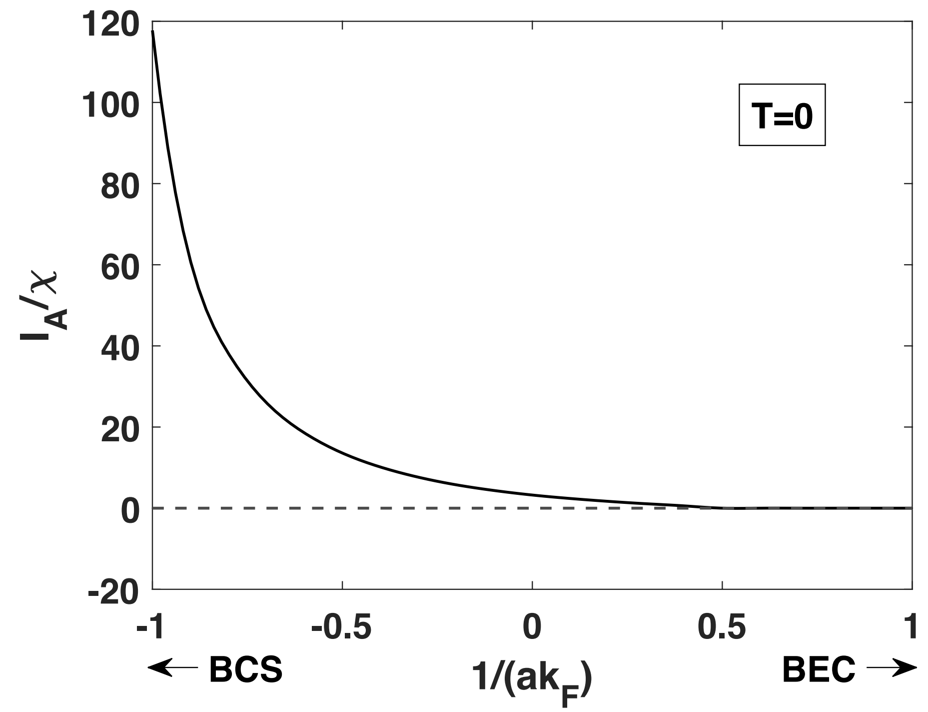
$$I = \dot{N}_3 = -i[N_3, H_T]$$

Analysis up to $\Omega^3_{\sigma,3}$

Current-bias curve

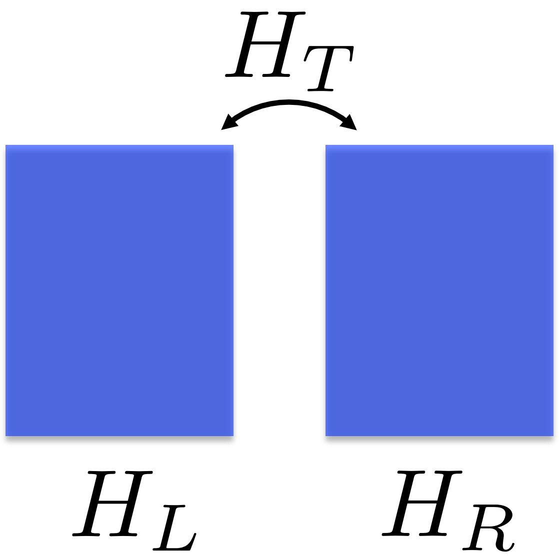


Interaction dependence



Mesoscopic transport of Bose gases

- Tunneling Hamiltonian formalism



$$H = H_L + H_R + H_T$$

$$I = -\dot{N}_L = i[N_L, H_T]$$

$$H_T = \sum_{\mathbf{k},\mathbf{p}} \left(e^{-i\Delta\mu\tau} t_{\mathbf{k},\mathbf{p}} b^{\dagger}_{\mathbf{k},L} b_{\mathbf{p},R} + h.c. \right)$$

absence of the momentum conservation

There must be the conversion process
between condensation and normal elements.

Linear response theory: F. Meier & W. Zwerger PRA **64** 033610 (2001).
Beyond linear response effect: SU & J.P. Brantut, PRR **2**, 023284 (2020);
SU, PRR **2**, 023340 (2020).

Experiment: G. Del Pace et al., PRL **126**, 055301 (2021).

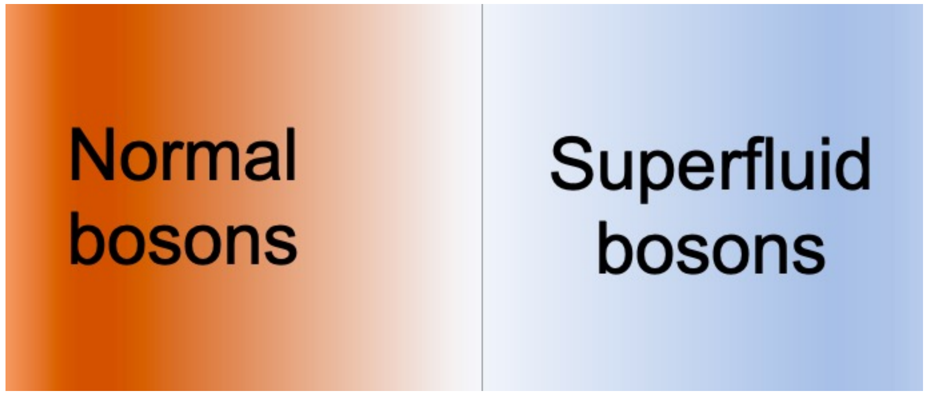
Superfluid-normal state junction

PHYSICAL REVIEW A **106**, L011303 (2022)

Letter | Editors' Suggestion

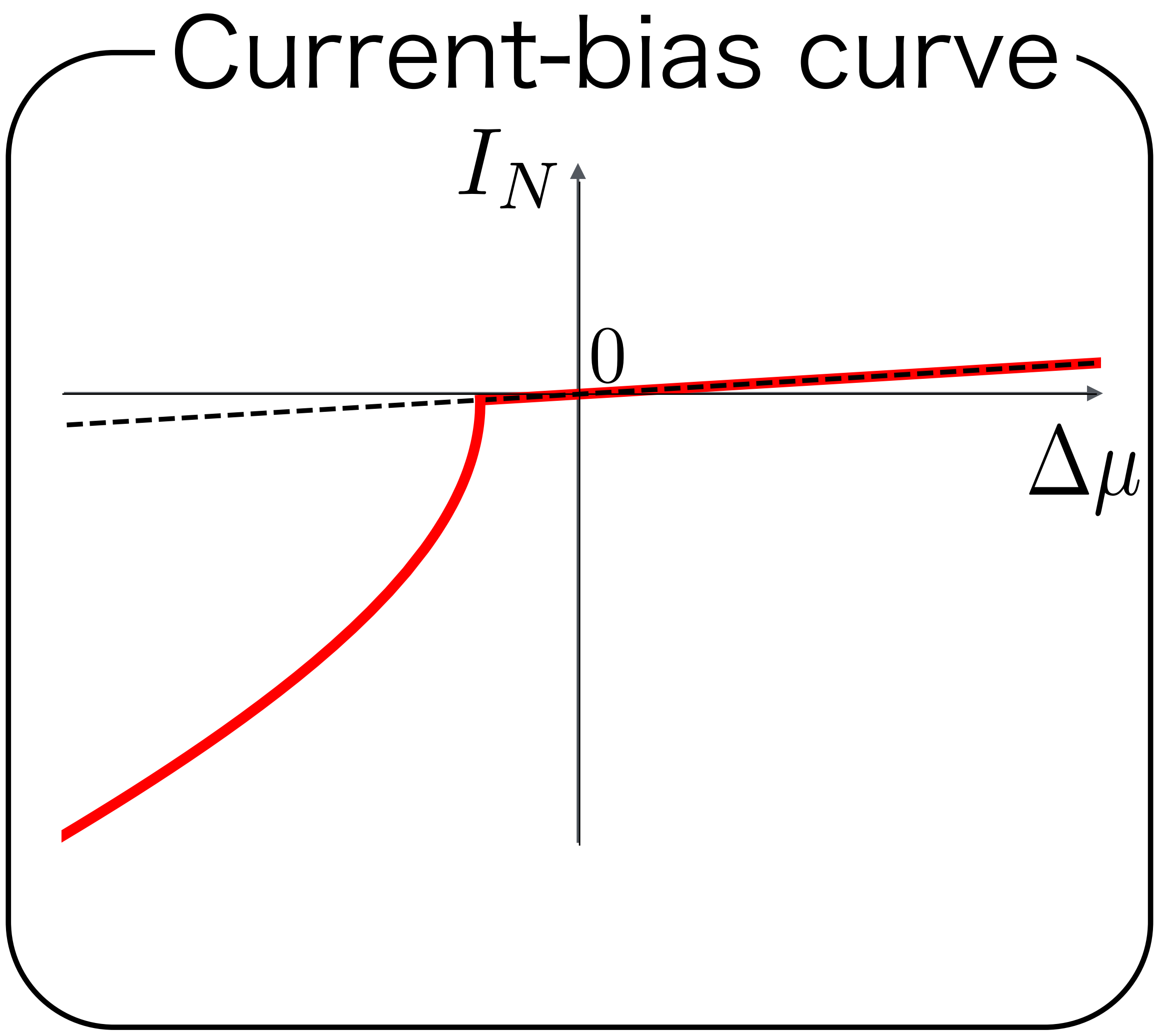
Asymmetry and nonlinearity of current-bias characteristics in superfluid–normal-state junctions of weakly interacting Bose gases

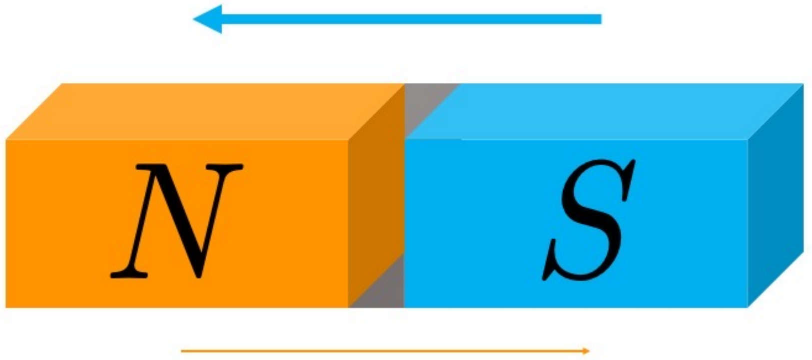
Shun Uchino



Normal bosons: Hartree-Fock theory

Superfluid bosons: Bogoliubov theory





- Asymmetry arises from the conversion process between condensation and normal elements, and the bosonic Andreev reflection.

Rectification effect
(e.g., diode)

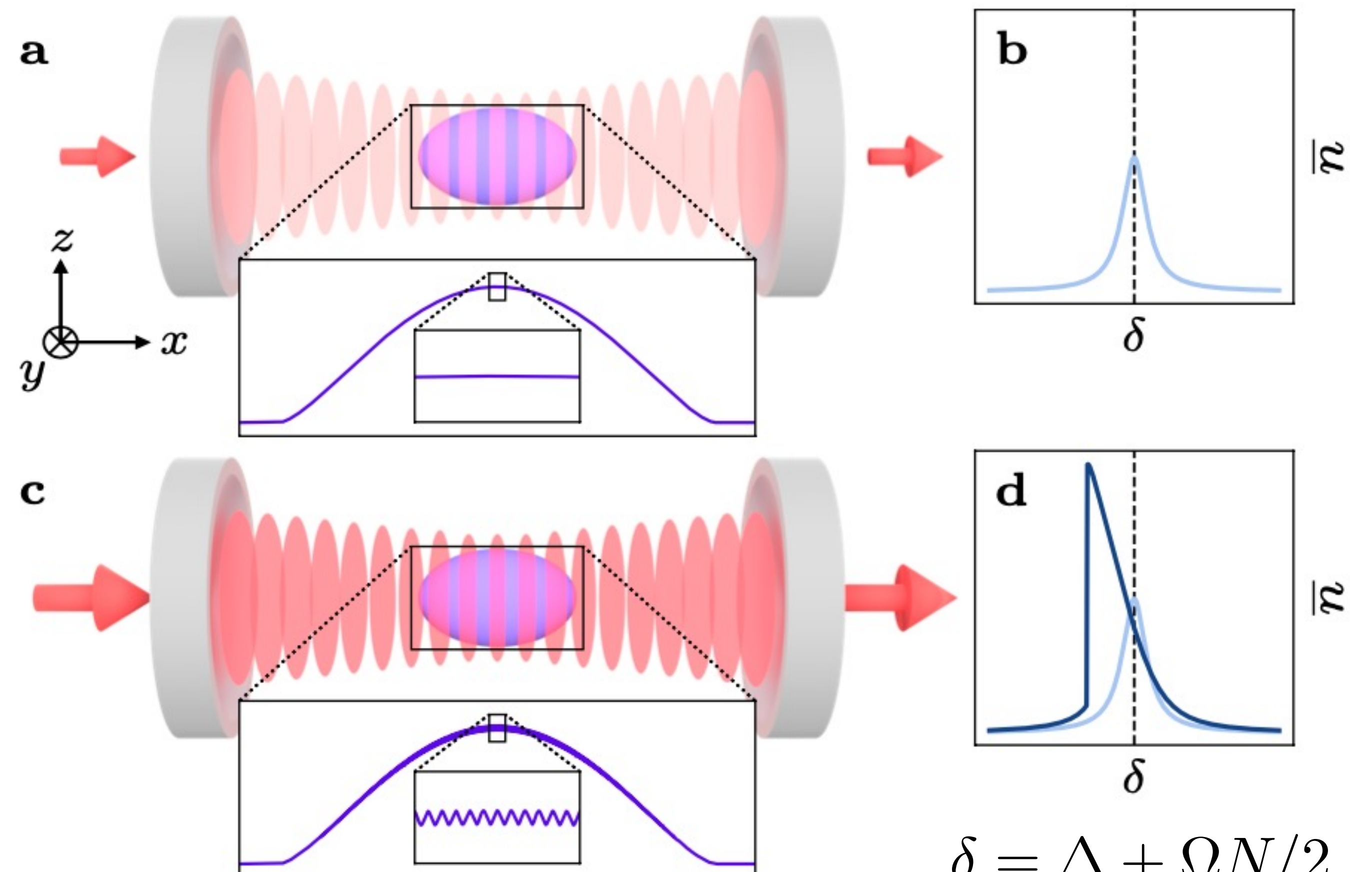
Strongly interacting Fermi gas in a cavity

V. Helson et al., Phys. Rev. Res. **4**, 133199 (2022)

$$H = H_{\mathrm{atom}} + H_c + H_{\mathrm{int}}$$

$$H_c = \Delta a^\dagger a$$

$$H_{\mathrm{int}} = \Omega a^\dagger a \int d^3 r n(\mathbf{r}) \cos^2 \mathbf{k}_c \cdot \mathbf{r}$$

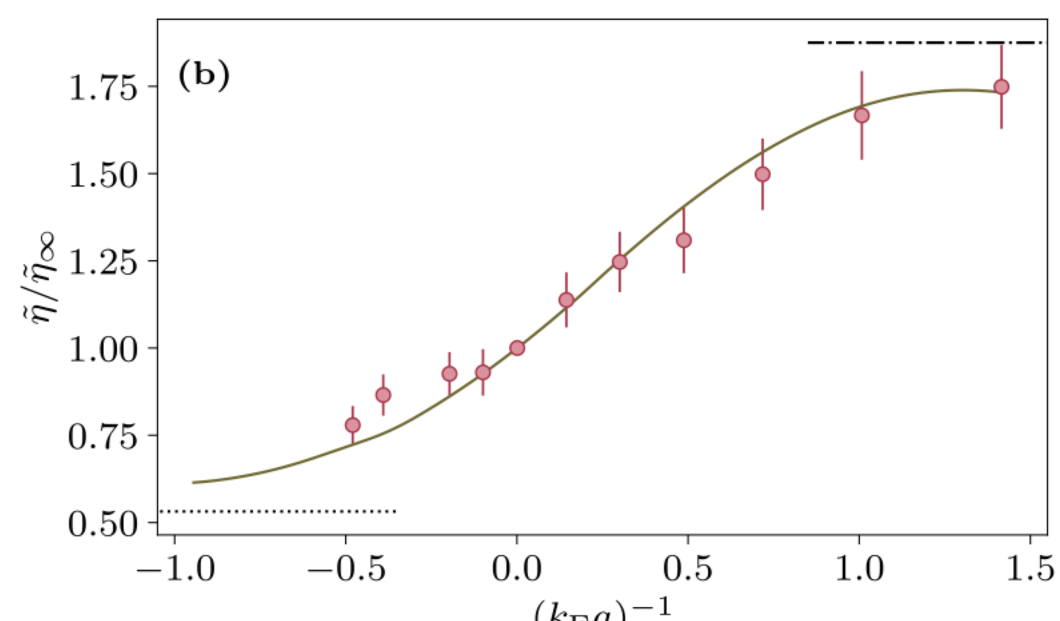


$$\delta = \Delta + \Omega N/2$$

- Photon measurement reflects density-density correlation of atoms

$$\chi^R(\mathbf{q}, \omega = 0) = -\int d\omega \left[\frac{S(\mathbf{q}, \omega) + S(-\mathbf{q}, \omega)}{\omega} \right]$$

compressibility sum rule



- Agreement with a theory with the operator product expansion

Summary

- Two-terminal transport of Fermi gases

  Nonlinear current-bias characteristics D. Husmann et al., Science **350**, 1498 (2015).

  Breakdown of conductance quantization

  SU and M. Ueda, PRL **118**, 105303 (2017).

  Particle loss effect in mesoscopic transport

  SU, PRA106, 053320 (2022).

  M.-Z. Huang et al., arXiv:2210.03371 (PRL, In press)

- Transport with synthetic junctions

  Realization of multi terminal mesoscopic transport

  S. Nakada et al., PRA 102, 031302(R) (2020).

  K. Ono et al., Nat. Commun. **12,** 6724 (2021).

  Andreev reflection transport with rf lasers

  T. Zhang et al., Commun. Phys. **6**, 86 (2023).

- Transport of bosons

  Asymmetry and nonlinearity in SN junction

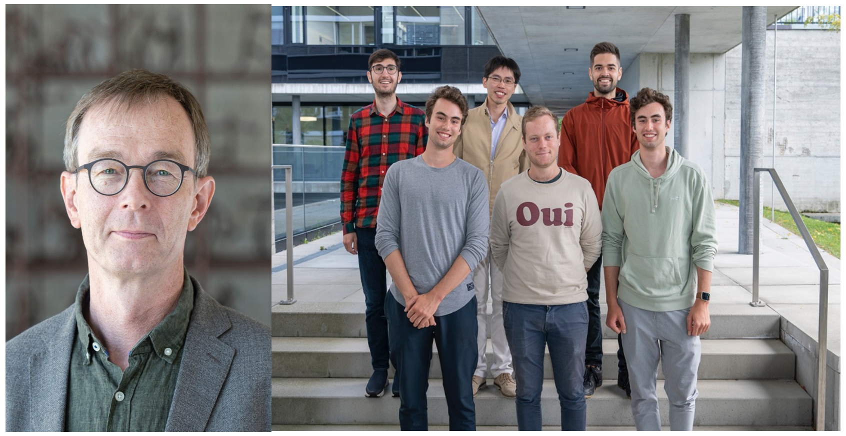
  SU, PRA **106**, L011303 (2022).

  Compressibility sum rule via optical cavity

  V. Helson et al., PRR **4**, 133199 (2022).

Collaborators

- Transport of Fermi gases



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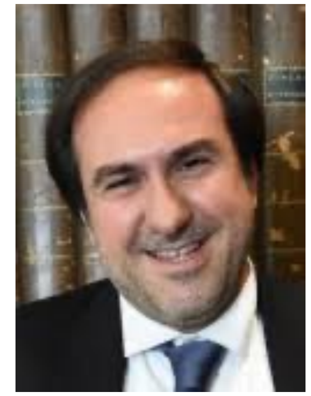

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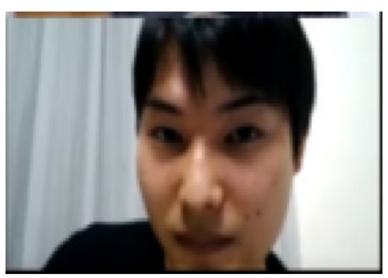
- Transport with synthetic dimensions



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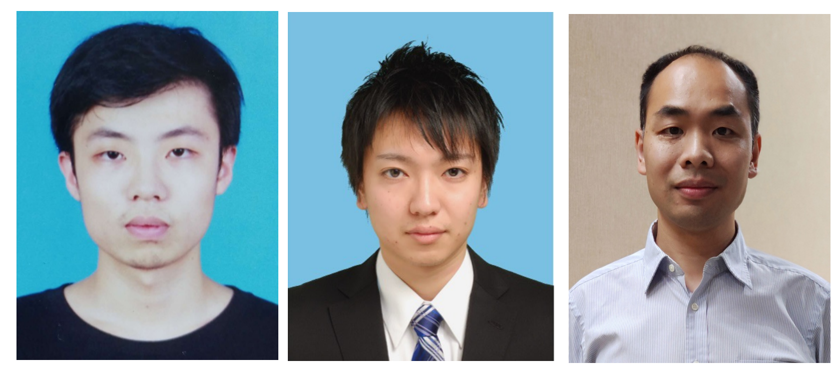

T. Zhang, H. Tajima, H. Liang
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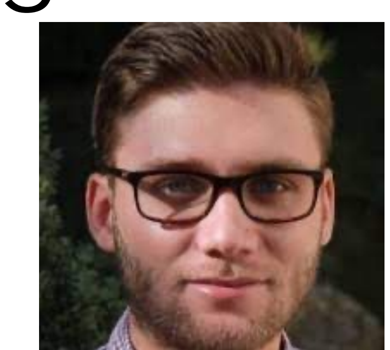
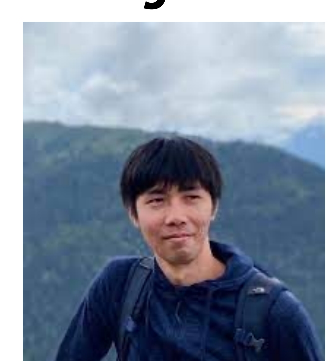
Y. Sekino
RIKEN

- Strongly-interacting Fermi gas inside a cavity

Brantut group @ EPFL
J.P. Brnatut, V. Helson,
T. Zwettler



K. Roux
UGA



H. Konishi
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