

Feb. 13<sup>th</sup>, 2024

# Extreme Universe

## The 22nd COLLOQUIUM

### February 13<sup>th</sup> (Tue.) ONLINE

TALK 10:00 - 11:00 (JST)  
February 13<sup>th</sup> (Tue.) 1:00 - 2:00 (UTC)

ONLINE COFFEE TIME  
11:00 - 12:00 (JST)

Registration required (click [HERE](#))

Extreme Universe, JAPAN 



Speaker  
Prof. Masayuki Hashisaka  
The University of Tokyo

**Title**  
**Electron dynamics at boundaries  
of quantum many-body systems**

**Abstract**  
Scattering processes of particles and waves at a boundary provide a common diagnostic method for investigating unknown properties of bulk matter. This fact has a much more profound meaning in quantum many-body physics, where electron correlation is often linked to the emergence of exotic particles. This talk discusses electron dynamics at the boundaries of various quantum many-body systems, mainly for the quantum Hall systems. Coupling and fractionalization of elementary excitations show up in scattering processes at the boundaries, highlighting the quantum and correlated nature of electrons in condensed matter.



MEXT -KAKENHI- Grant-in-Aid for Transformative Research Areas (A)  
The Natural Laws of Extreme Universe -A New Paradigm for Spacetime and Matter from Quantum Information-

2024

## Electron dynamics at boundaries of quantum many-body systems

**Masayuki Hashisaka**

ISSP, The University of Tokyo

## Research Interest

- Experiments
- Condensed matter physics
  - Mesoscopic physics

### Keywords:

Quantum Hall, Topological materials,  
Transport measurements, Electronics,  
etc.



東京大学 物性研究所

THE INSTITUTE FOR SOLID STATE PHYSICS  
THE UNIVERSITY OF TOKYO

2023-  
2017-2023  
2009-2017

Associate Prof. @ISSP  
NTT Basic Research Labs.  
Assistant Prof. @Tokyo Tech.

Mar. 2009 Ph.D from Kyoto University



Master, Ph.D. @京大化研



Prof. Ono



Prof. Kobayashi  
(Supervisor)



Assistant prof. @東工大



Prof. Fujisawa



Reseacher @NTT BRL



Dr. Muraki



Dr. Kumada

# Boundary of a material

## ◆ Measurement for bulk properties

Reaction from a boundary

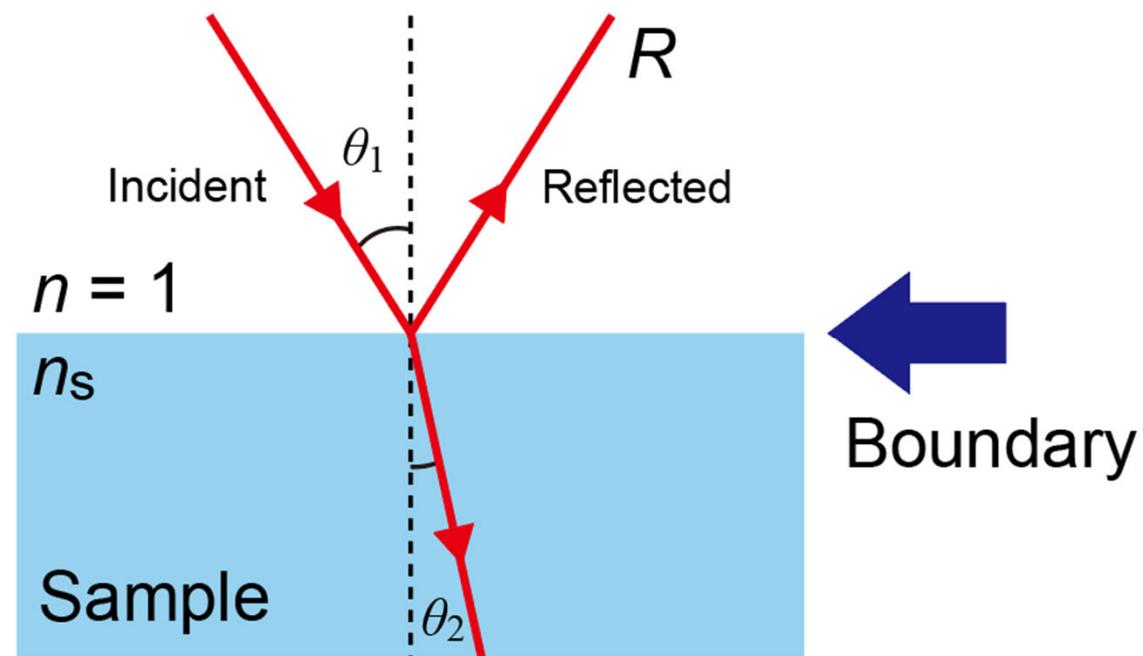
- Mismatch between systems forming the boundary

A boundary between a sample and a well-known system

- Bulk property of the sample

An example:

**Fresnel law**    
$$R = \left| \frac{1 - n_s}{1 + n_s} \right|^2$$



# Measurement for a quantum many-body system

An example:

## ➤ Andreev reflection at a Normal metal/Superconductor junction

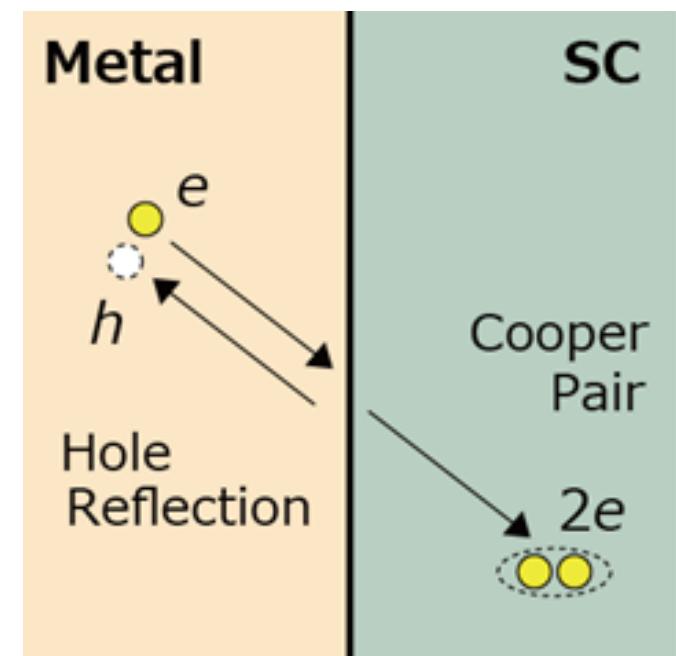
### Mismatch between carrier charges

- ✓ Normal metal: single particles (well-known)
- ✓ Superconductor: Cooper pair

### Charge conservation

Injection	Transmission	Reflection
$e$ (electron) = 2 $e$ (Cooper pair) - $e$ (hole)		

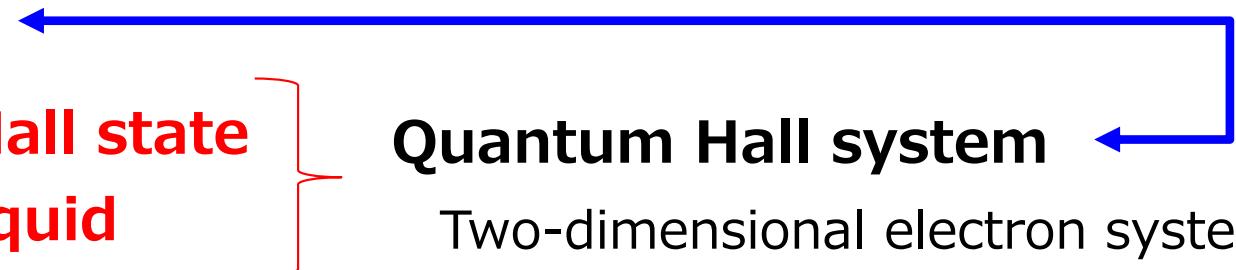
↑  
Action      Bulk property      Reaction



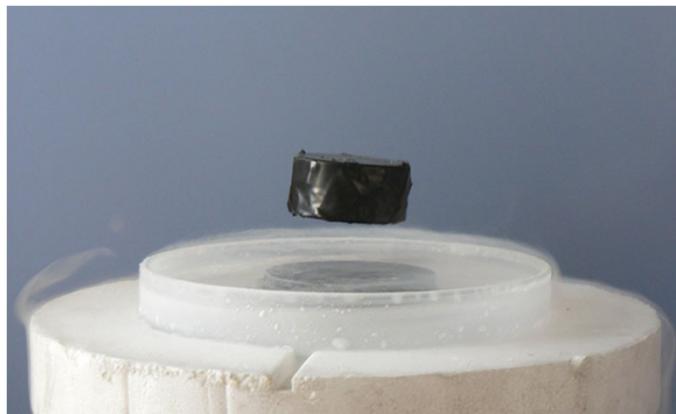
# Quantum many-body systems

- **Superconductor**
- **Fractional quantum Hall state**
- **Tomonga-Luttinger liquid**
- **Kondo effect**
- . . .

Some analogies appear in electron dynamics at boundaries

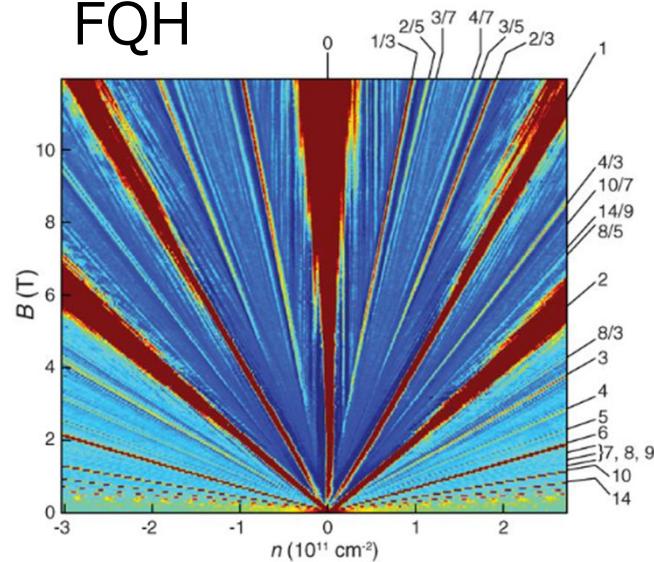


Superconductor



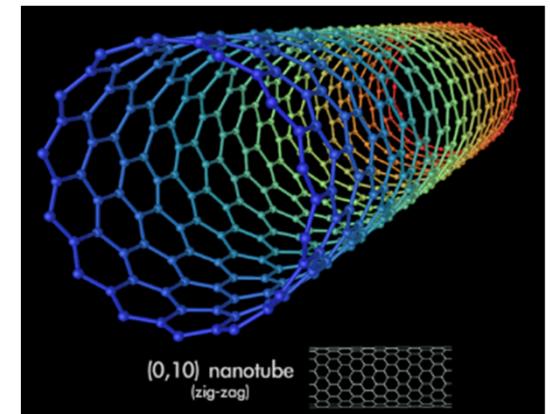
Wikipedia

FQH



Feldman et al., Science 2012.

1D electron system



Wikipedia

# Contents

Background

## ➤ **Quantum many-body effects in quantum Hall systems**

- ✓ Fractional quantum Hall (FQH) effect
- ✓ Tomonaga-Luttinger liquid nature of edge channels

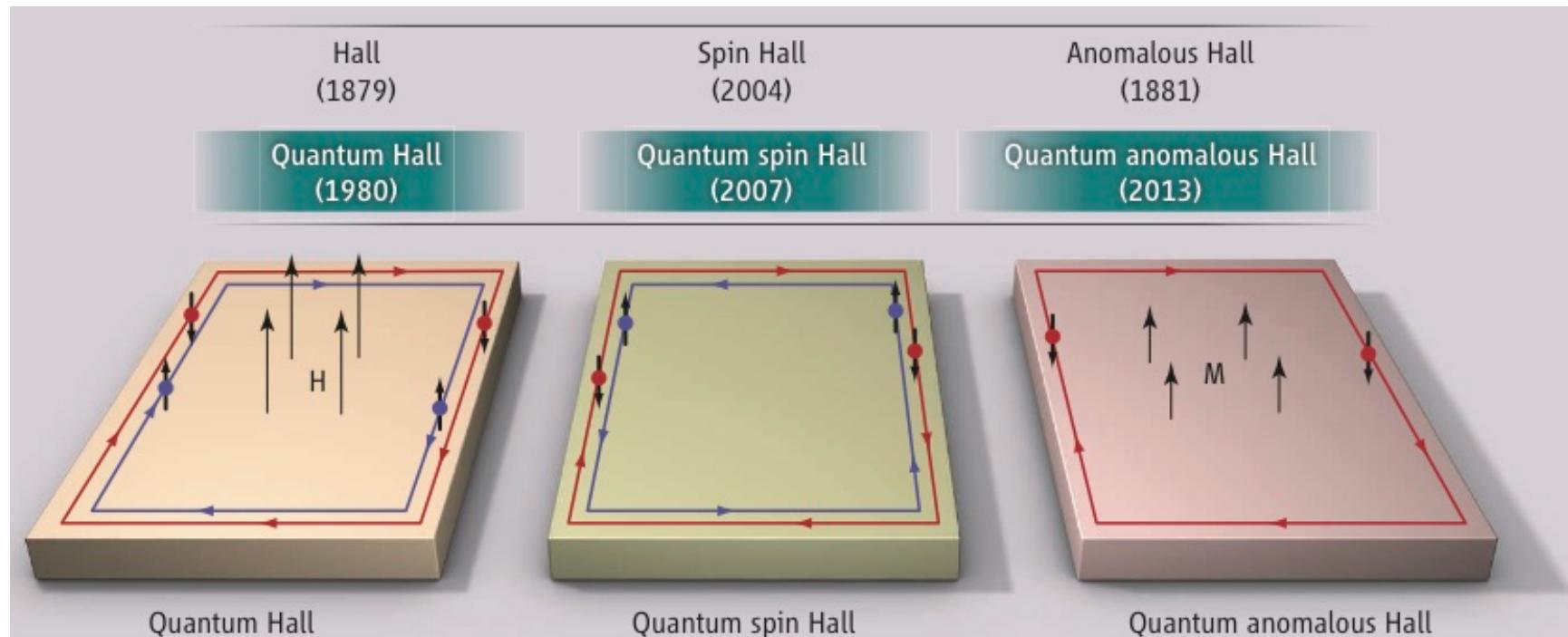
Main

## ➤ **Electron dynamics at boundaries of quantum many-body systems**

- ✓ Fractional-Integer quantum Hall junction  
*Hashisaka et al.*, Nat. Commun. **12**, 2794 (2021).
- ✓ A boundary of a Tomonaga-Luttinger liquid  
*Hashisaka et al.*, Nat. Phys. **13**, 559 (2017).

# Two-dimensional topological systems

S. Oh, Science 2013.



## ➤ Integer quantum Hall (IQH) effect

Klaus von Klitzing 1980. ➔ Nobel prize 1985.



## ➤ Fractional quantum Hall (FQH) effect

Tsui, Stomer, Gossard 1982. ➔ Nobel prize 1998.  
Laughlin 1983.



# Quantum Hall effect

## ➤ Integer quantum Hall (IQH) effect

von Klitzing 1980.

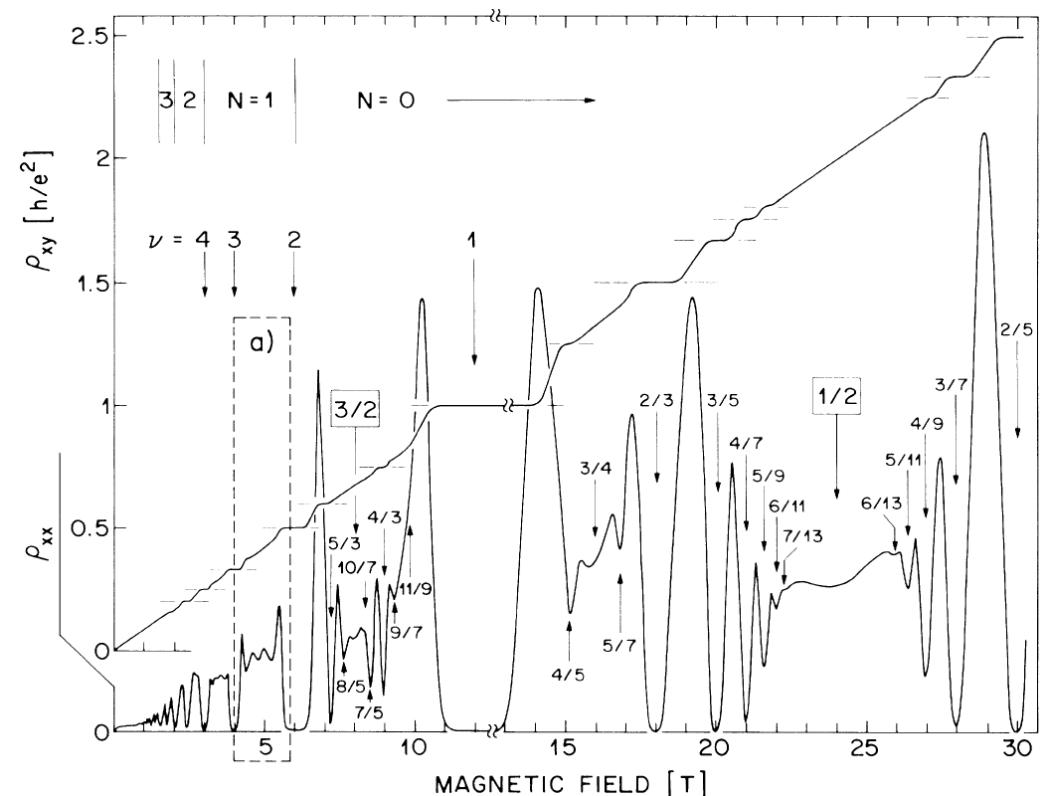
- ✓ Landau-level Filling factor :  
 $\nu = (\text{integer})$ 
  - Charge carrier:  $e^* = e$
  - Quantum statistics:  $\theta = \pi$  (Fermion)

## Quantum many-body state

## ➤ Fractional QH (FQH) effect

Tsui, Stomer, Gossard 1982.

- ✓ Laughlin state:  $\nu = 1/3$ , etc.
  - Charge carrier:  $e^* = e/3$
  - Quantum statistics:  $\theta = \pi/3$  (Abelian)
- ✓ Moore-Read Pfaffian state:  $\nu = 5/2$ 
  - Charge carrier:  $e^* = e/4$
  - Quantum statistics: Non-abelian (Ising)

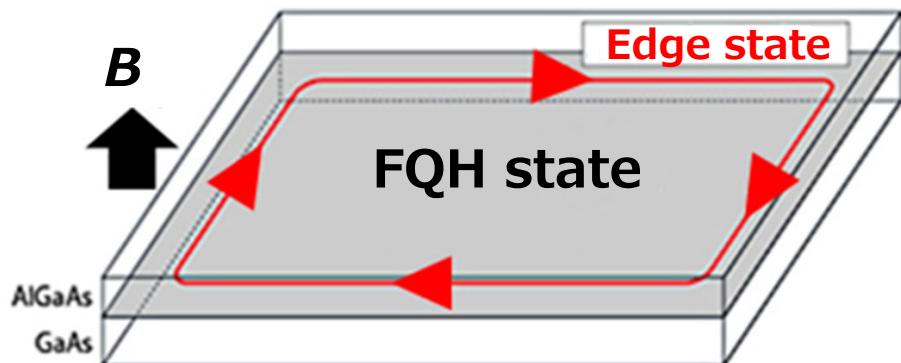


R. Willett et al.,  
Phys. Rev. Lett. **59**, 1776 (1987).

# Fractional quasiparticles

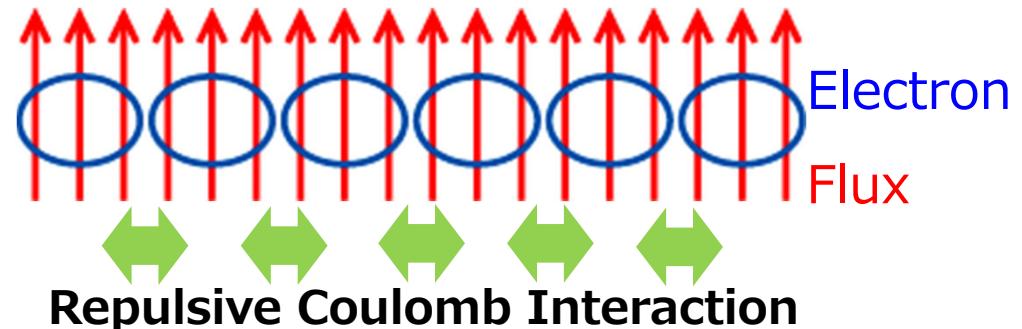
## FQH state

Bulk: incompressible (insulating)  
Edge: compressible (metallic)



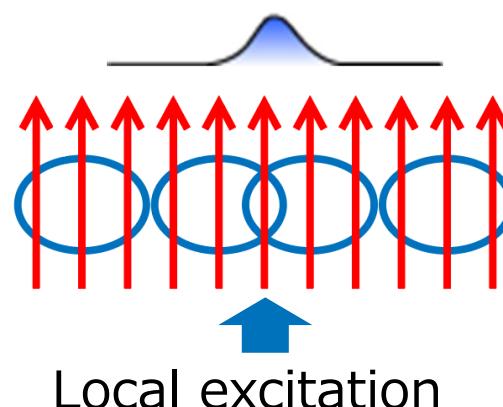
- **Laughlin state:**  $\nu = 1/3$ , etc.
  - ✓ Charge carrier:  $e^* = e/3$
  - ✓ Quantum statistics:  $\theta = \pi/3$

## Ground state ( $\nu = 1/3$ )



Uniform electron density:  
Three flux quanta per an electron.

## Elementary excitation (quasiparticle)



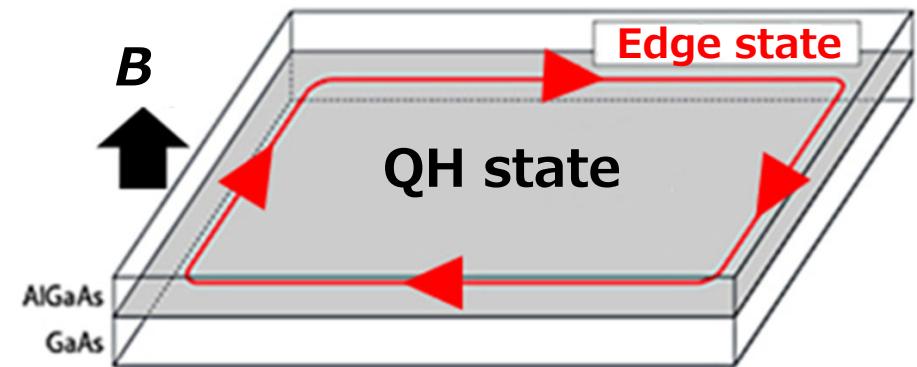
Minimal electron-density modulation

**Fractional charge**  
 $e^* = e/3$

# Edge channels

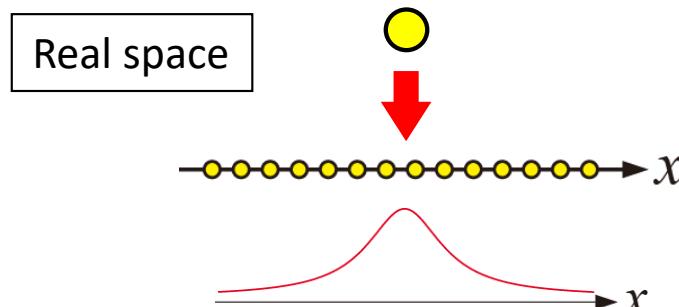
**Integer quantum Hall edge:**  
Chiral 1D channels

**Fractional quantum Hall edge:**  
Chiral or helical 1D channels



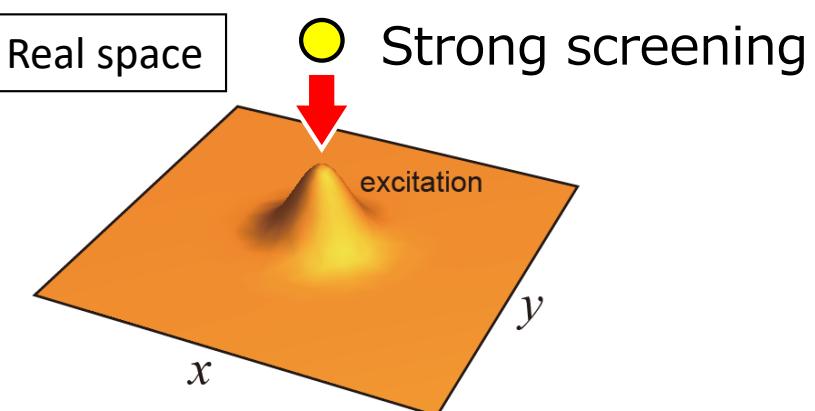
## ➤ Tomonaga-Luttinger liquid

Interacting 1D electron system



Correlation function:  
Long range (**power law**)

## c.f. Fermi liquid



Correlation function:  
**Local (exp. decay)**

# Luttinger-liquid nature of edge channels

## Quantum many-body state

### ➤ Fractional QH edge channel

Chiral Luttinger liquid

(Short-range interaction)

✓ Power-law behaviors.



### ➤ Integer QH edge channel

Non-interacting 1D system

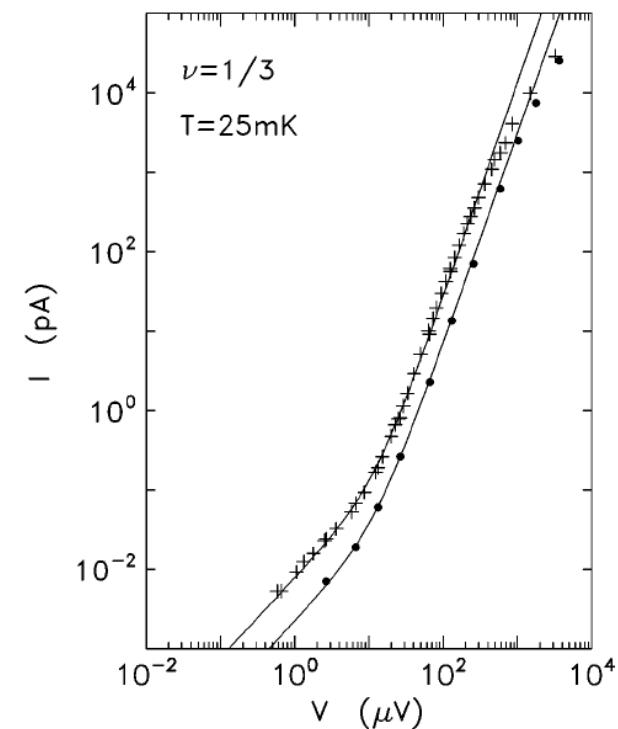
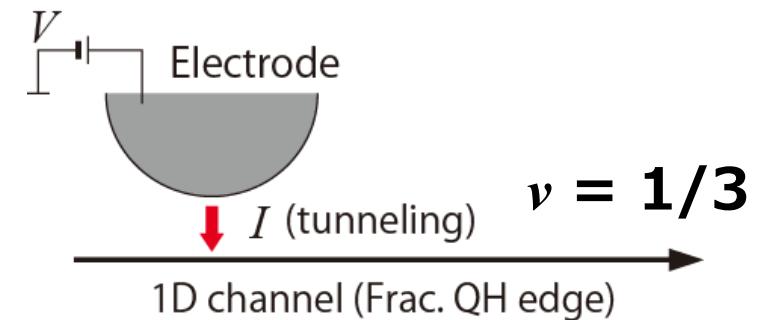
(Short-range interaction)

Artificial Luttinger liquid

(Long-range interaction)

- ✓ Charge fractionalization
- ✓ Spin-charge separation

(Discussions later)



Power-law behavior  
Phys. Rev. Lett. **77**, 2538 (1996).

# Contents

Background

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- ✓ Tomonaga-Luttinger liquid nature of edge channels

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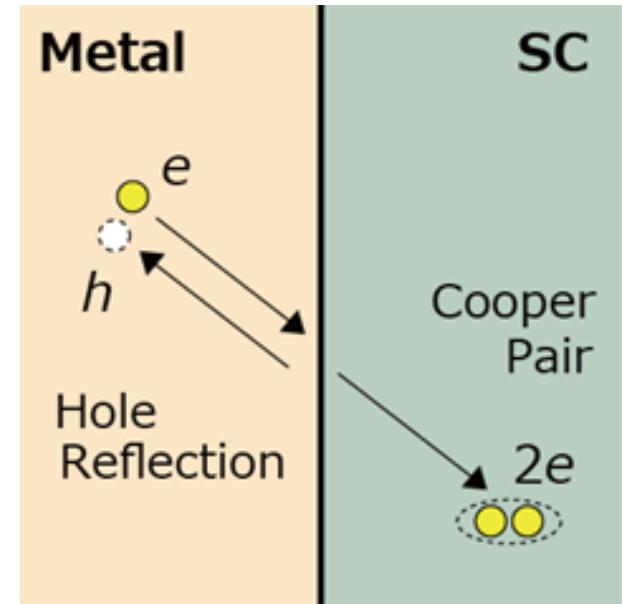
## ➤ **Electron dynamics at boundaries of quantum many-body systems**

- ✓ Fractional-Integer quantum Hall junction  
*Hashisaka et al.*, Nat. Commun. **12**, 2794 (2021).
- ✓ A boundary of a Tomonaga-Luttinger liquid  
*Hashisaka et al.*, Nat. Phys. **13**, 559 (2017).

# Andreev-reflection-like scatterings

## Electron dynamics at boundaries of quantum many-body systems

Electron scattering reflects the mismatch between systems forming the boundary.



### 1. Fractional-Integer quantum Hall junction

(Q Many-body)      (Normal)

Hashisaka *et al.*, Nat. Commun. **12**, 2794 (2021).

### 2. A boundary of a Tomonaga-Luttinger liquid

Hashisaka *et al.*, Nat. Phys. **13**, 559 (2017).

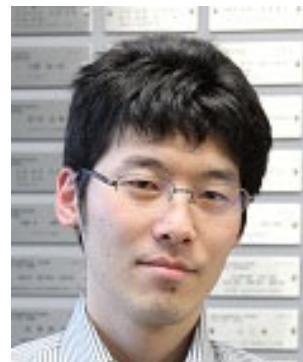
# Fractional-Integer QH junction

## Andreev reflection of fractional quantum Hall quasiparticles

M. Hashisaka, T. Jonckheere, T. Akiho, S. Sasaki, J. Rech, T. Martin, K. Muraki  
Nature Communications, **12**, 2794 (2021).

### NTT members

Experiments



### CPT, CNRS

Calculations

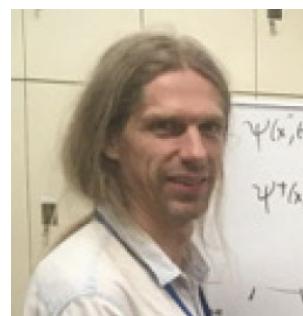
T. Akiho

S. Sasaki

K. Muraki

### Acknowledgements

N. Kumada, T. Ito,  
N. Shibata, T. Fujisawa,  
H. Murofushi, & M. Imai



T. Jonckheere

J. Rech

T. Martin

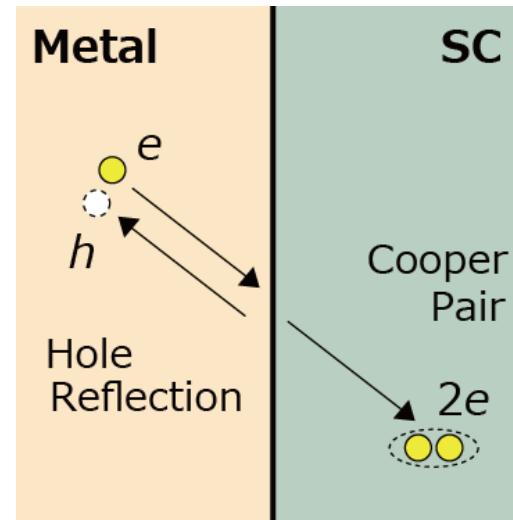
# Fractional-Integer QH junction

My first thought:

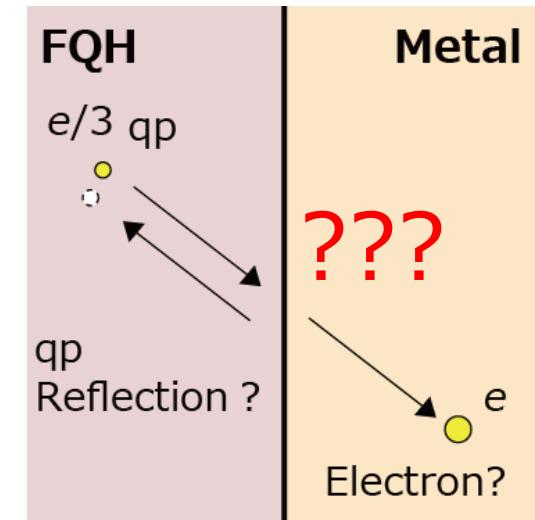
**Charge conservation** at a FQH/norma metal interface.

Fractionally-charged carrier dynamics

## Andreev reflection



## Fractional AR?



## Conventional Andreev reflection at a Metal/Superconductor interface

### Injection

$$e \text{ (electron)} = 2e \text{ (Cooper pair)} - e \text{ (hole)}$$

### Transmission

### Reflection

## Similar scattering process at an FQH/Metal interface (?)

### Injection

### Transmission

### Reflection

$$e/3 \text{ (quasiparticle)} = e \text{ (electron)} - 2e/3 \text{ (quasihole)}$$

# Theoretical proposals

Conductance through the 1/3-1 junction:  
**tunneling between Luttinger liquids.**

→ Analytical result of the single-impurity case:

$$G = 1.5 \times e^2/3h \text{ (Strong coupling)}$$

Safi & Schulz, PRB **52**, R17040 (1995).

Chamon & Fradkin, PRB **56**, 20120(1997).

Chklovskii & Halperin, PRB **57**, 3781 (1998).

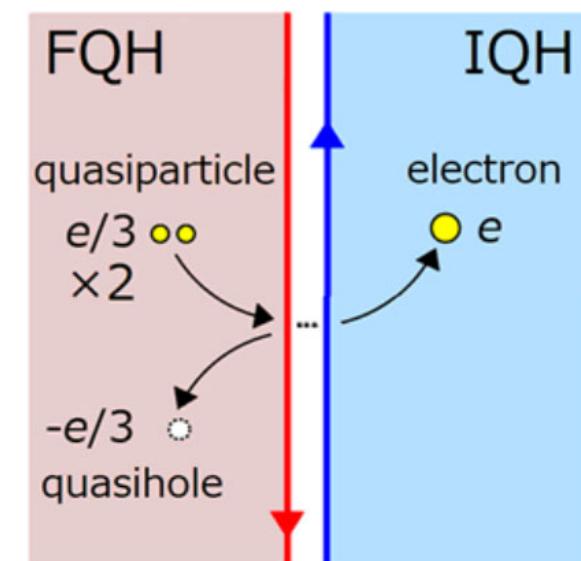
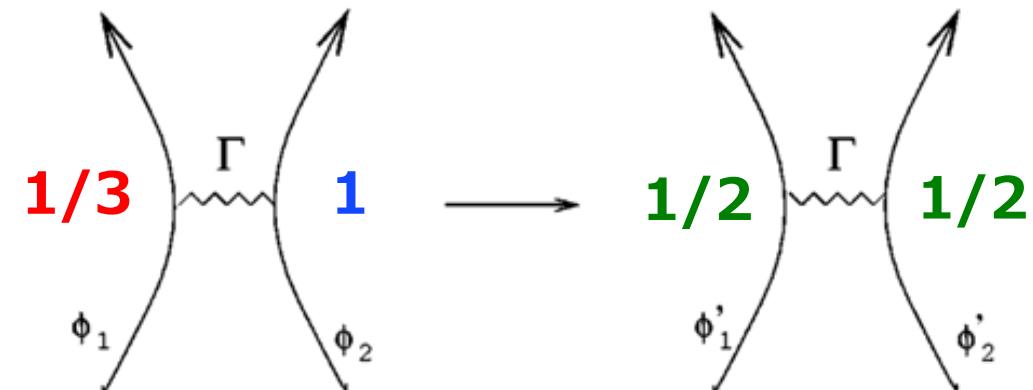
Sandler, Chamon & Fradkin, PRB **59**, 12521 (1999).

Conductance exceeding  $e^2/3h$ :

→ Interpretation:

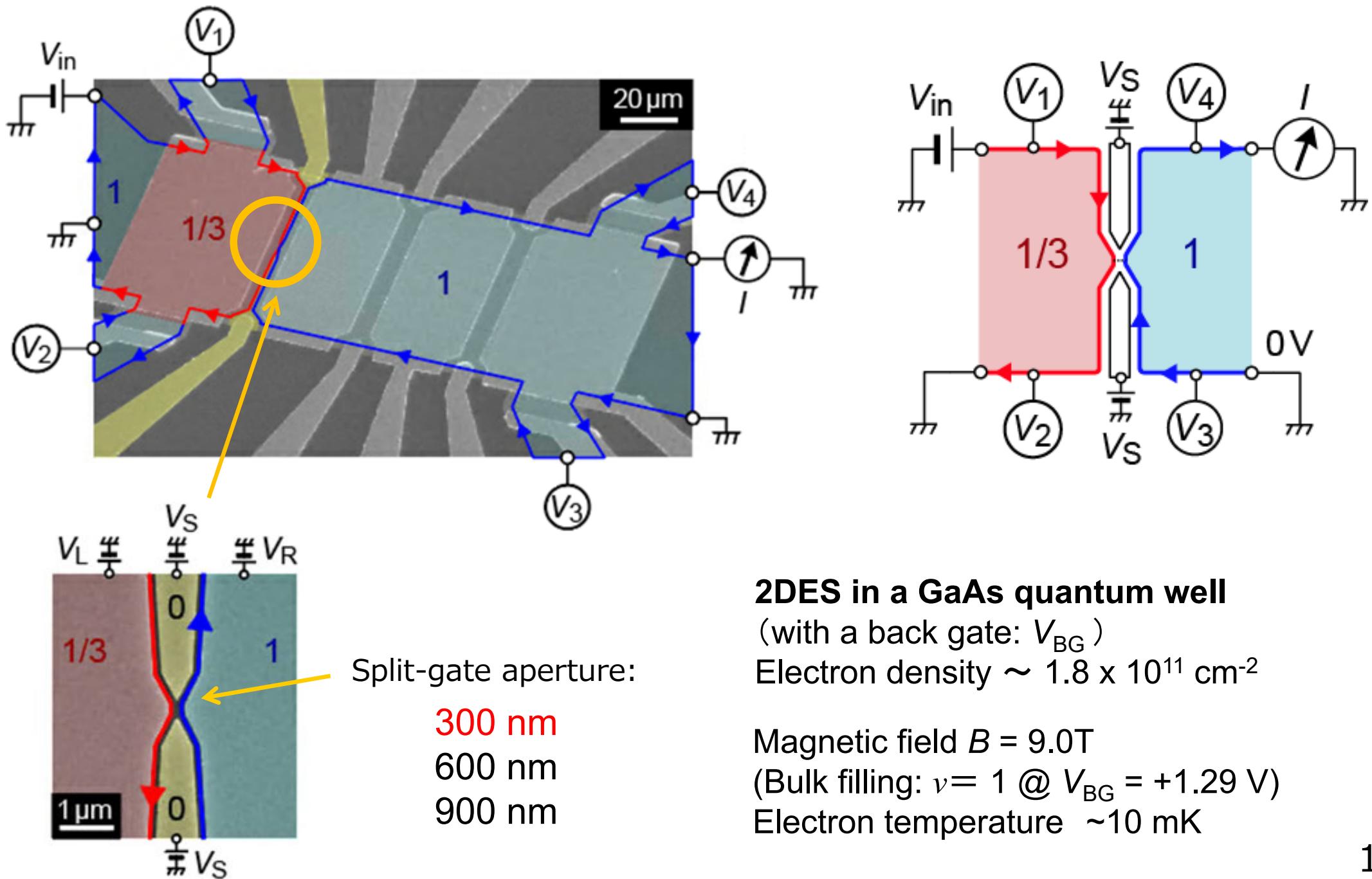
**Andreev reflection of fractional quasiparticles.**

Sandler, Chamon, Fradkin PRB **57**, 12324 (1998).



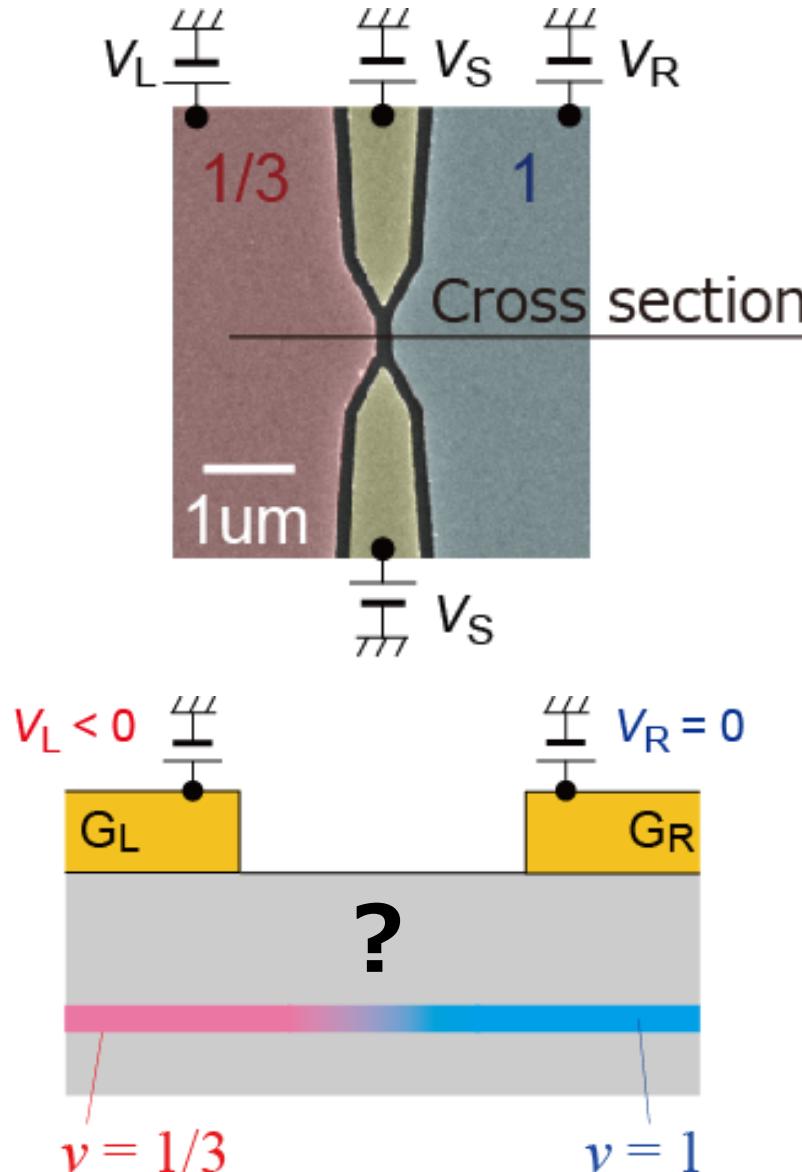
No experimental demonstration since the theoretical prediction in the 1990s.

# Sample

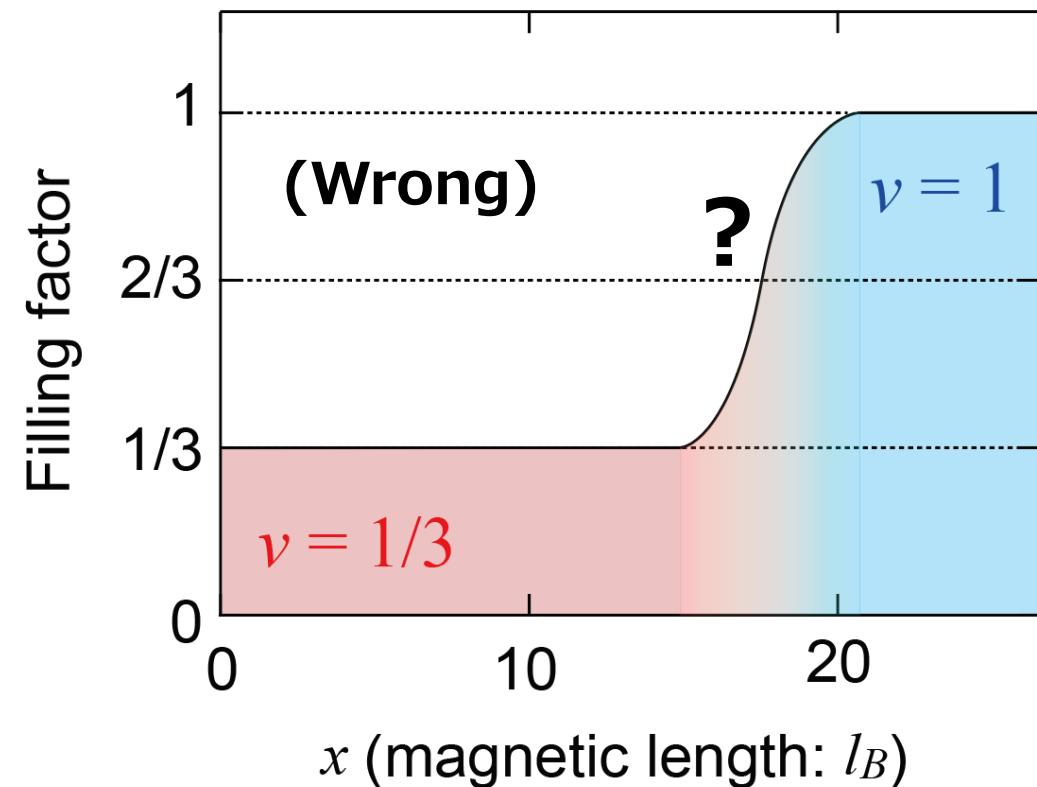


# Boundary of the FQH and IQH systems

Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).



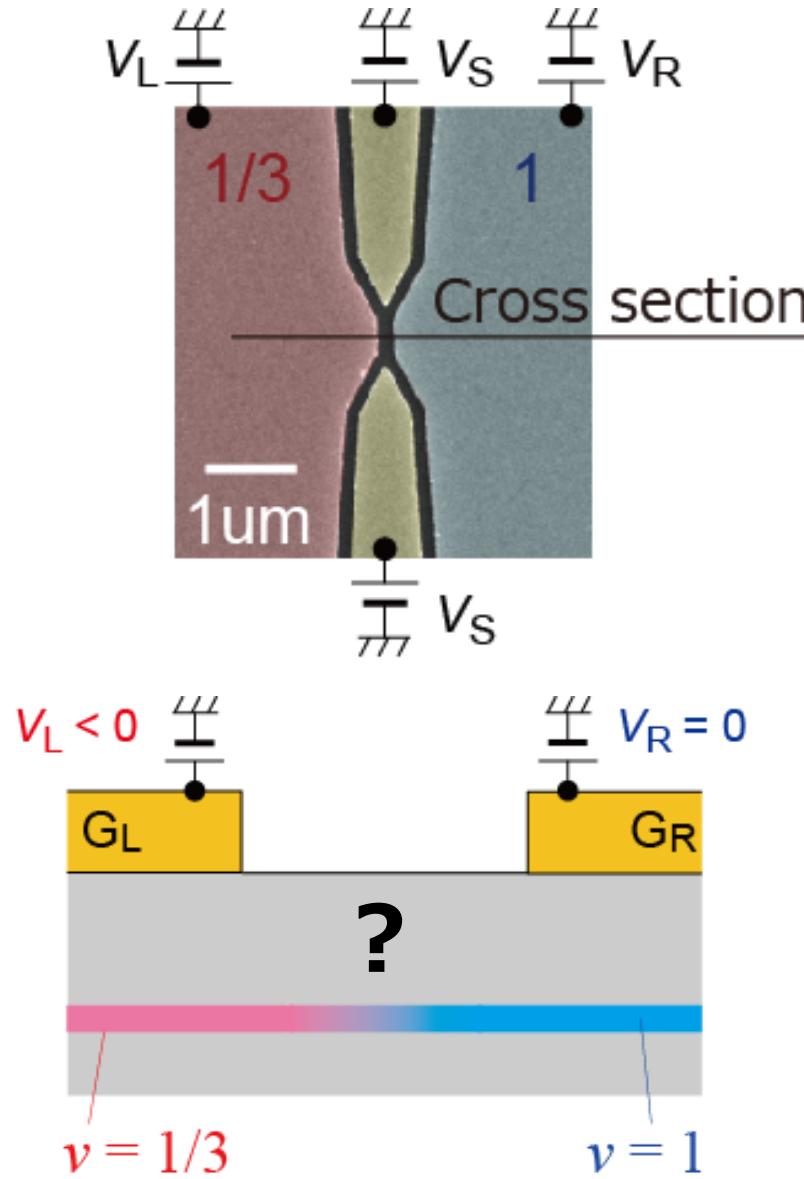
## Electron-density profile



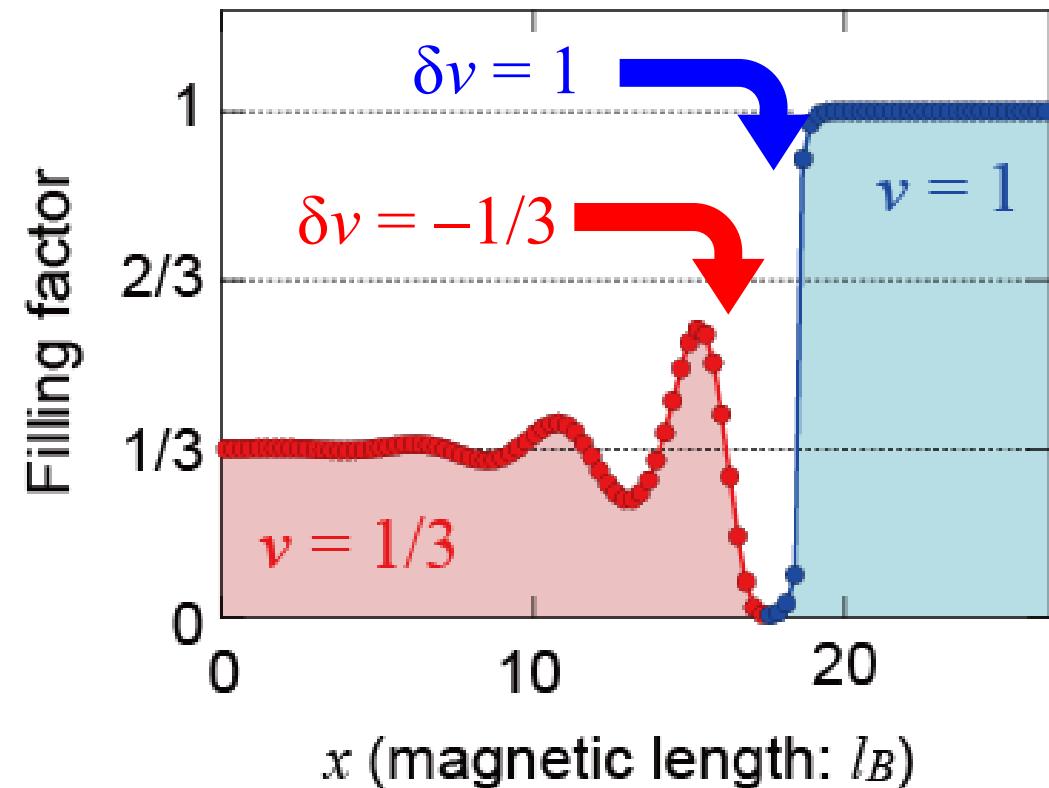
We need to consider  
electron correlation in the  $\nu = 1/3$  region.

# Boundary of the FQH and IQH systems

Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).



## Electron-density profile



## Density-Matrix Renormalization Group (DMRG) calculation

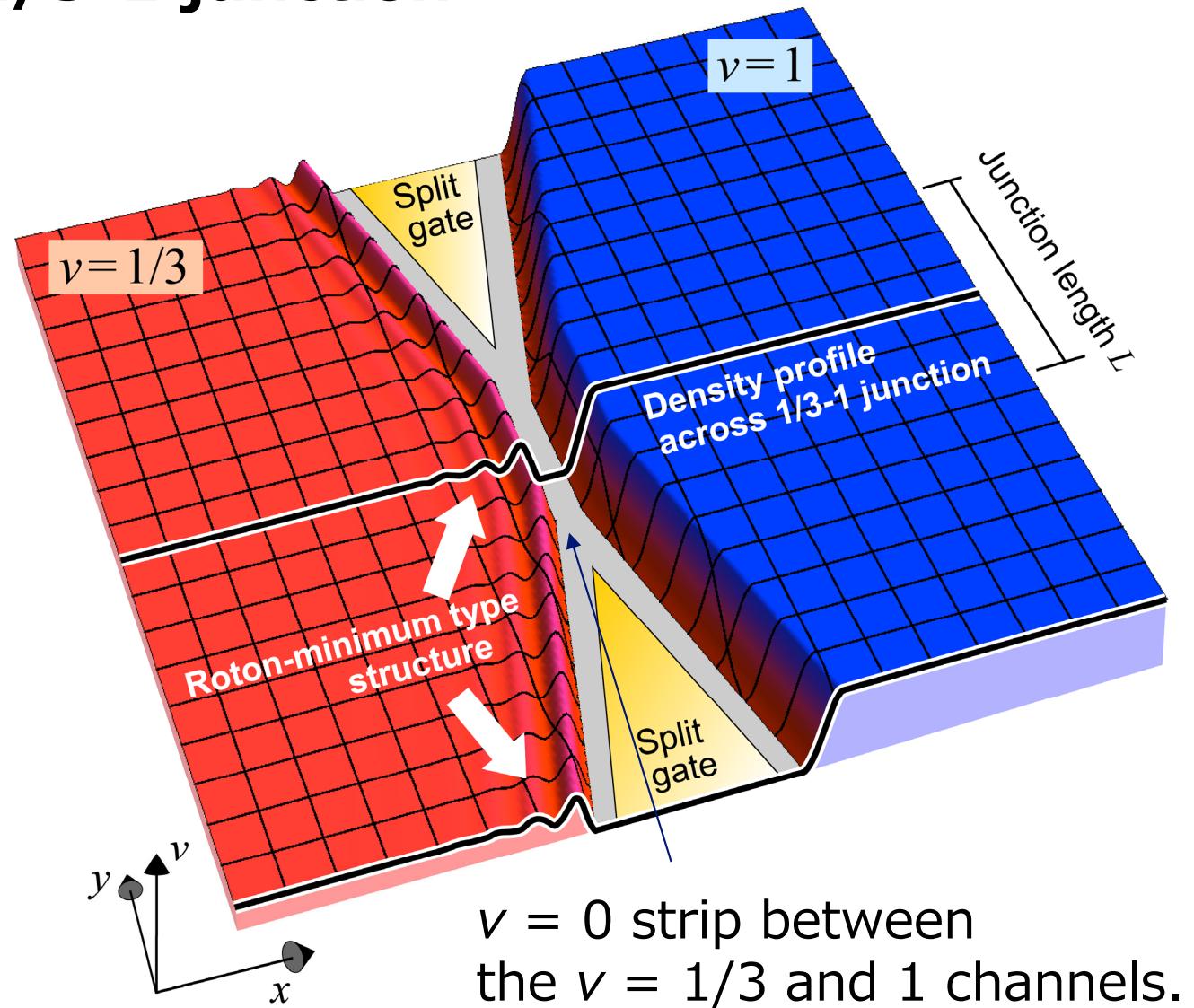
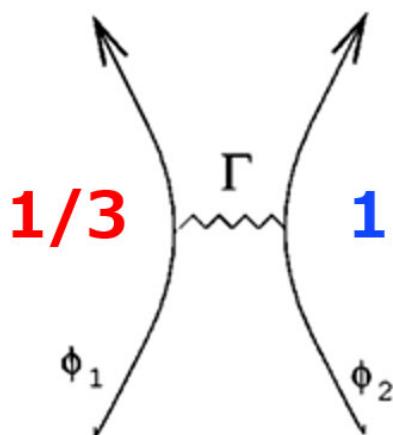
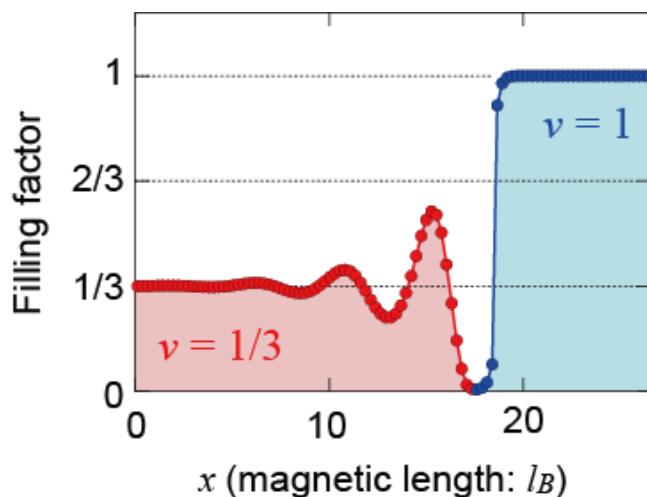
Ito & Shibata (Tohoku Univ.)

c.f. Ito, T. & Shibata, PRB **103**, 115107 (2021).

# Tunneling between edge channels

Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).

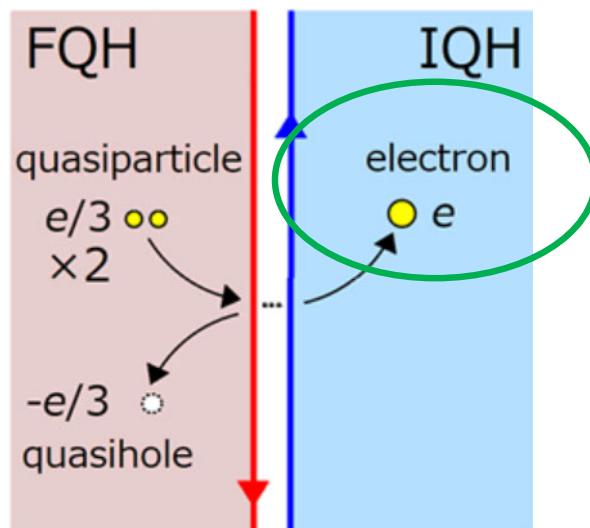
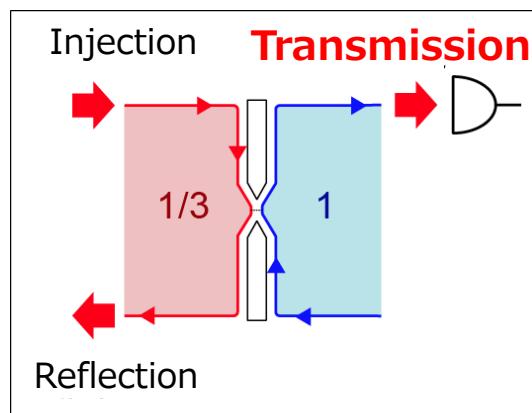
## Schematic of the $1/3\text{-}1$ junction



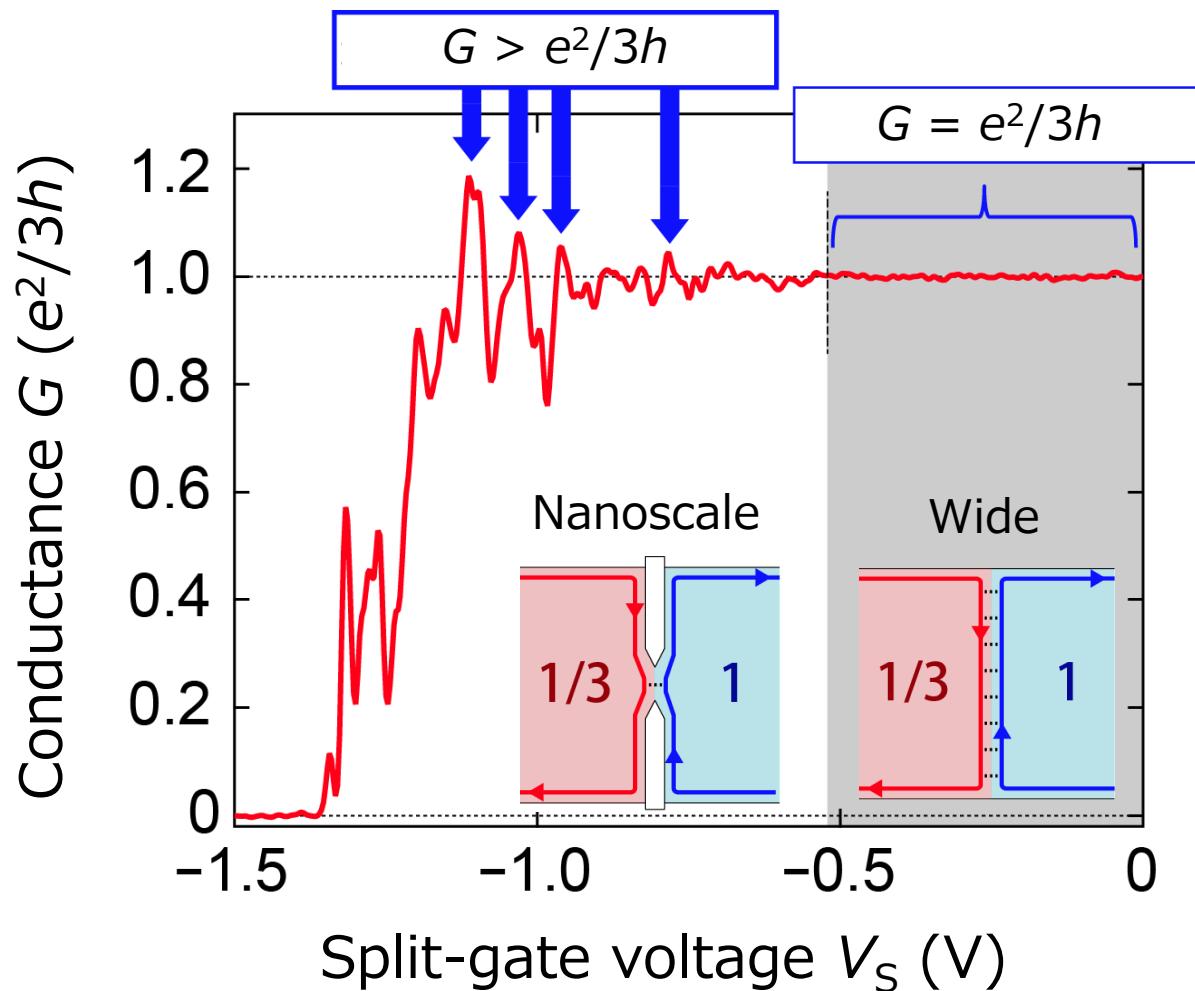
# Experimental result 1: Conductance oscillations with $G > e^2/3h$

**Maximum conductance:**  $\sim 1.2 \times e^2/3h$  ( $1.5e^2/3h$ : strong coupling limit)

Oscillation amplitude increases with decreasing the junction width.



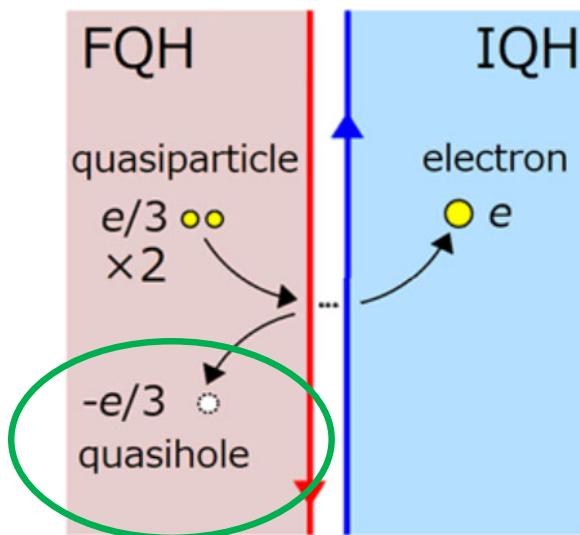
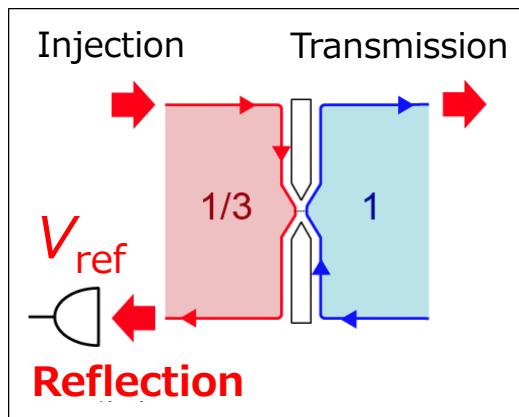
**Evidence of the Andreev process**



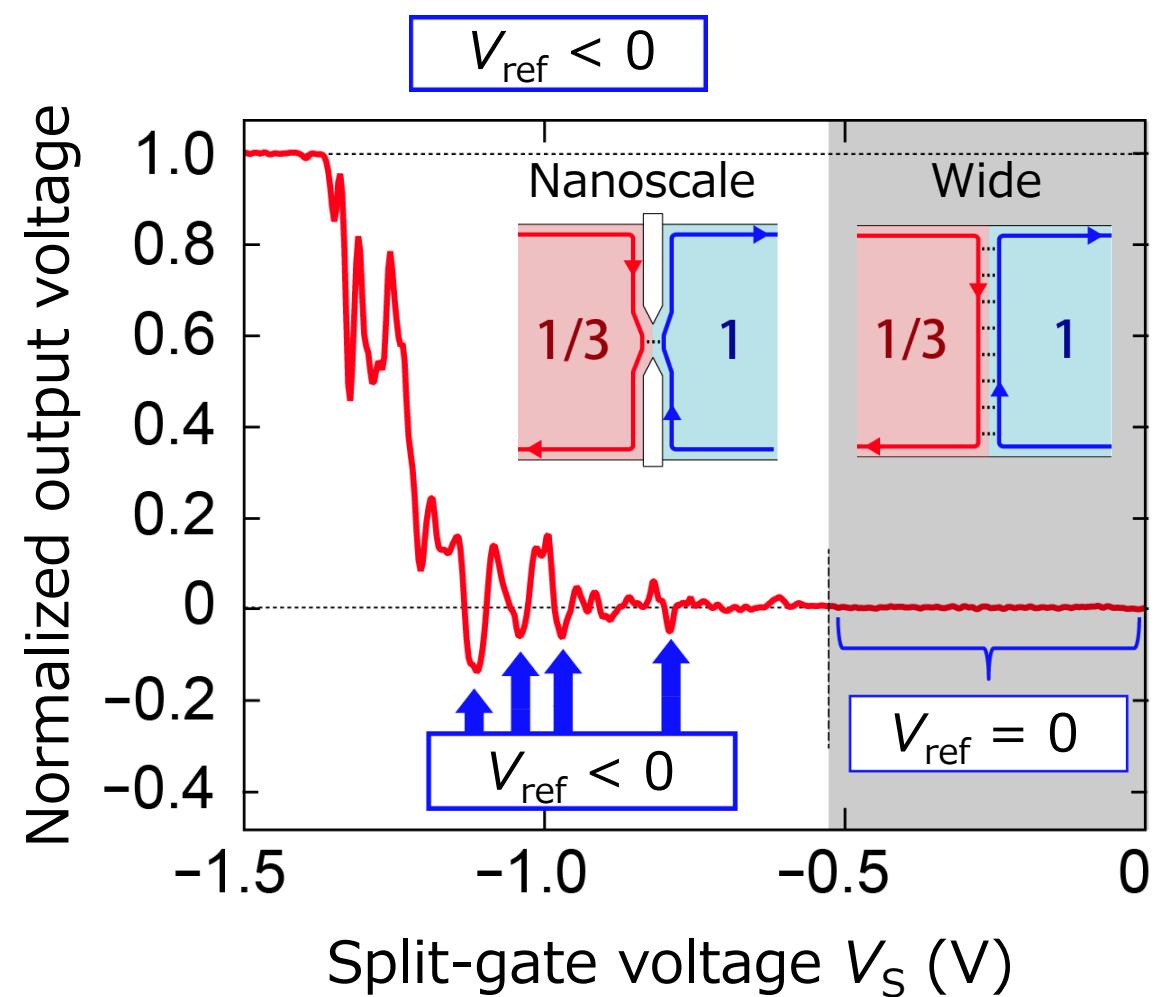
## Experimental result 2: Negative reflection signal

**Negative output voltage of the reflected channel.**

Similar oscillations are observed as a function of  $V_S$ .



**Evidence of the Andreev process**



# Conductance oscillations

Why does the conductance oscillate?

Multiple impurities

$G$  oscillates between 0 and  $1.5 \times e^2/3h$ .

Theory:

Chamon, Fradkin PRB **56**, 20120(1997).

Ponomarenko et al., PRB **67**, 035314 (2003).

Zülicke and Shimshoni, PRB **69**, 085307 (2004).

Our Experiment:

(GaAs 2DES) Hashisaka *et al.*, Nat. Commun. 2021.

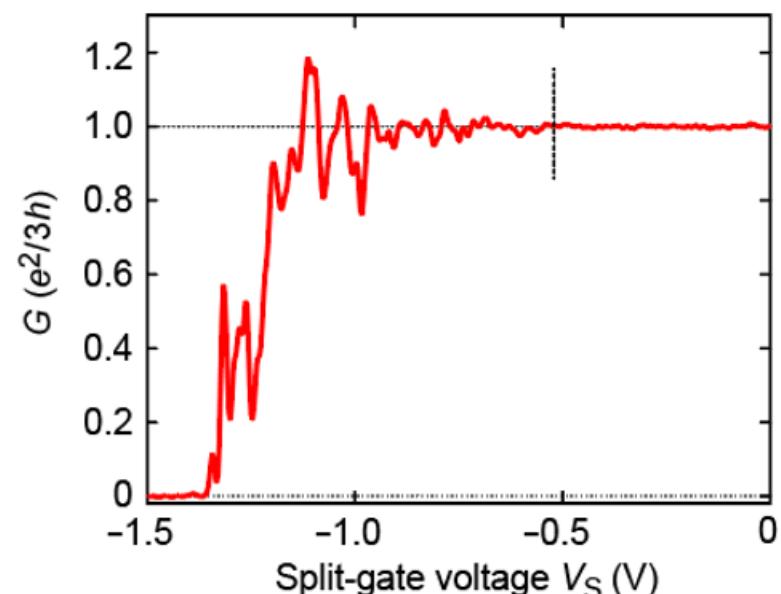
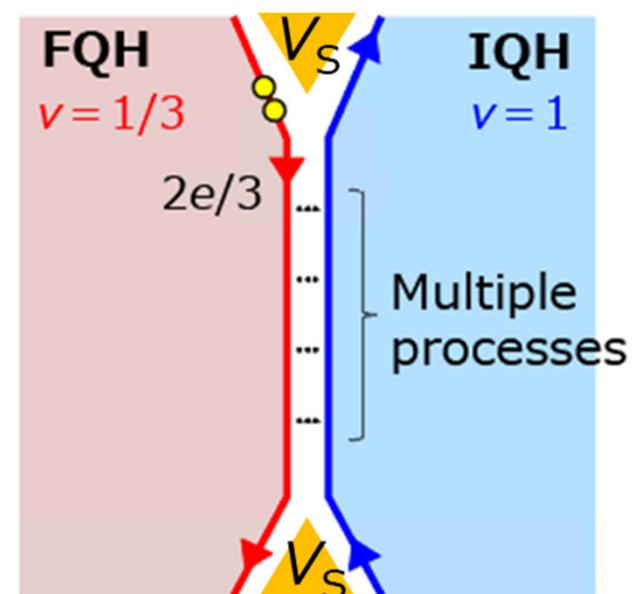
Single impurity

$G = 1.5 \times e^2/3h$  (Strong coupling)

Recent Experiment:

(Graphene) Cohen *et al.*, Science 2023.

Multiple impurities

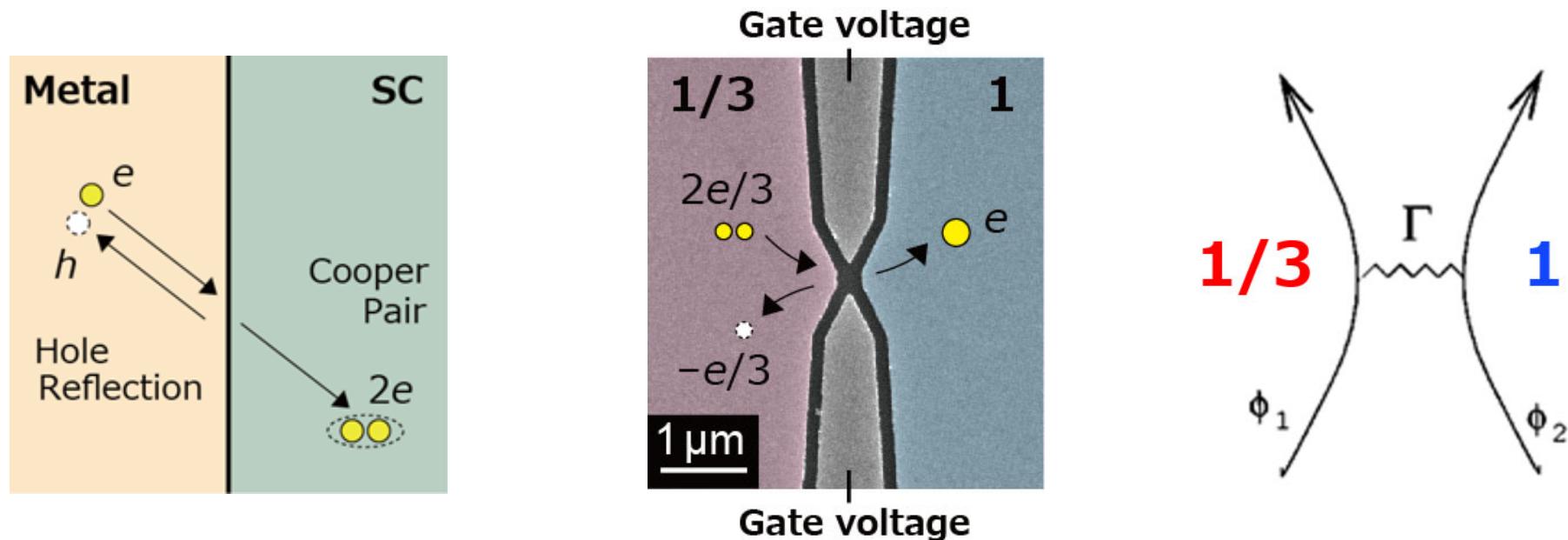


# Summary of the first topic

## Andreev-reflection-like scatterings at an FQH-IQH interface

Nature Communications, **12**, 2794 (2021).

Theory: Safi & Schulz, PRB **52**, R17040 (1995).  
Chamon & Fradkin, PRB **56**, 20120(1997).  
Sandler, Chamon, Fradkin PRB **57**, 12324 (1998).  
Chklovskii & Halperin, PRB **57**, 3781 (1998).  
Sandler, Chamon & Fradkin, PRB **59**, 12521 (1999).



c.f. Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).  
Cohen et al., Science **382**, 542 (2023).

# Contents

Background

## ➤ **Quantum many-body effects in quantum Hall systems**

Fractional quantum Hall (FQH) effect

Edge channel (Tomonaga-Luttinger liquid)

Main

## ➤ **Electron dynamics at boundaries of quantum many-body systems**

✓ Fractional-Integer quantum Hall junction

Hashisaka *et al.*, Nat. Commun. **12**, 2794 (2021).

✓ A boundary of a Tomonaga-Luttinger liquid

Hashisaka *et al.*, Nat. Phys. **13**, 559 (2017).

# Boundary of a Tomonaga-Luttinger liquid

## Waveform measurement of charge- and spin-density wavepackets in a chiral Tomonaga-Luttinger liquid

M. Hashisaka, N. Hiyama, T. Akiho, K. Muraki, and T. Fujisawa

Nature Physics **13**, 559 (2017).

**Tokyo Tech.**

Measurements



**N. Hiyama**



**T. Fujisawa**

**NTT members**

Crystal growth



**T. Akiho**



**K. Muraki**

Acknowledgements

H. Kamata, N. Kumada,  
Y. Tokura

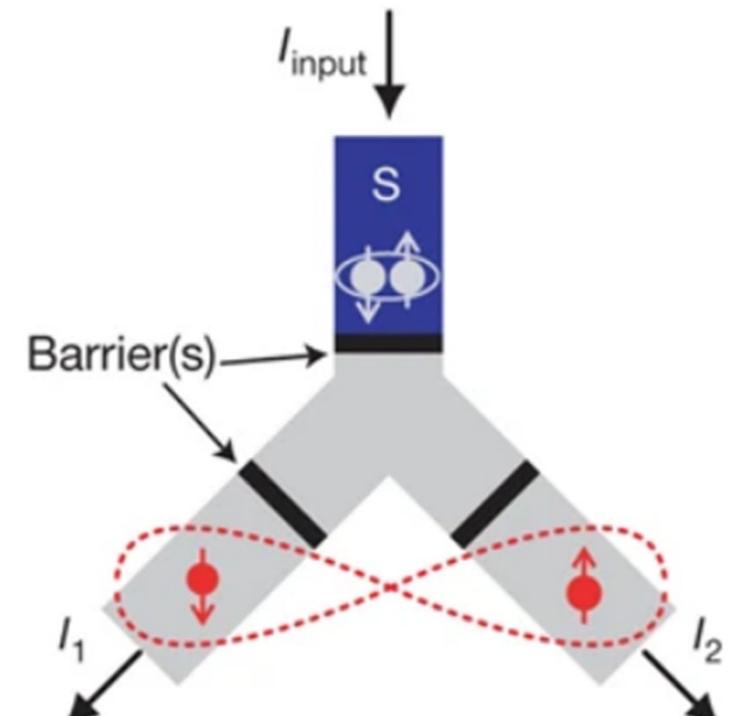
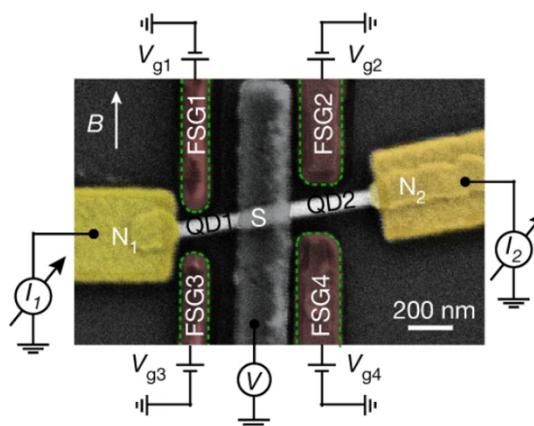
# Cooper-pair splitting

## Andreev reflection at a Y-shaped Metal-Superconductor junction

A promising device for generating pairwise entangled electrons in solids.

Experiments:

- Das *et al.*, Nat. Commun. 2012.
- Deacon *et al.*, Nat. Commun. 2015.
- Rani *et al.*, Nat. Commun. 2021.
- Bordoloi *et al.*, Nature 2022.
- etc.



Hofstetter *et al.*, Nature 2009.

## Mode transformation at a boundary of a Tomonaga-Luttinger liquid



Electron fractionalization (Spin-Charge separation)

Hashisaka *et al.*, Nat. Phys. 13, 559 (2017).

# Artificial Luttinger liquid

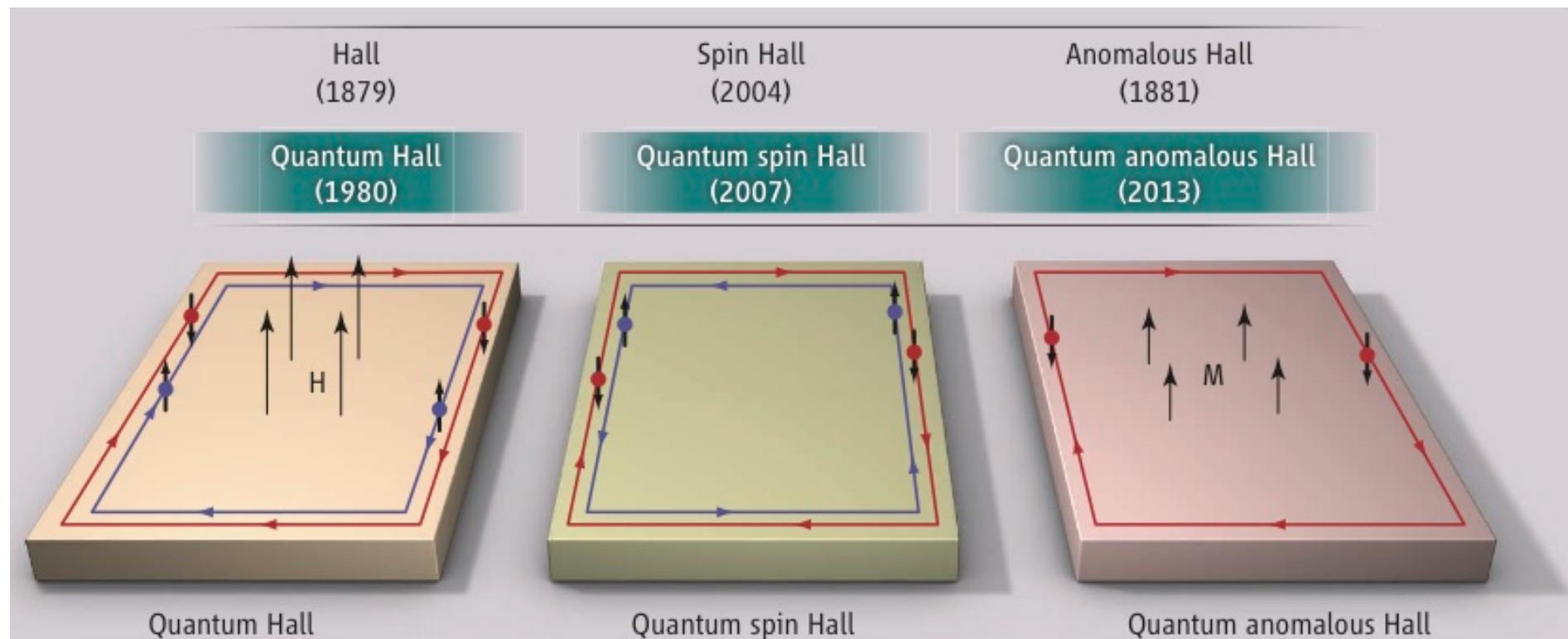
## ➤ Integer QH edge channel

(Short-range interaction) Non-interacting 1D system

(Long-range interaction) **Artificial Luttinger liquid**

- ✓ Charge fractionalization
- ✓ Spin-charge separation

} RF transport



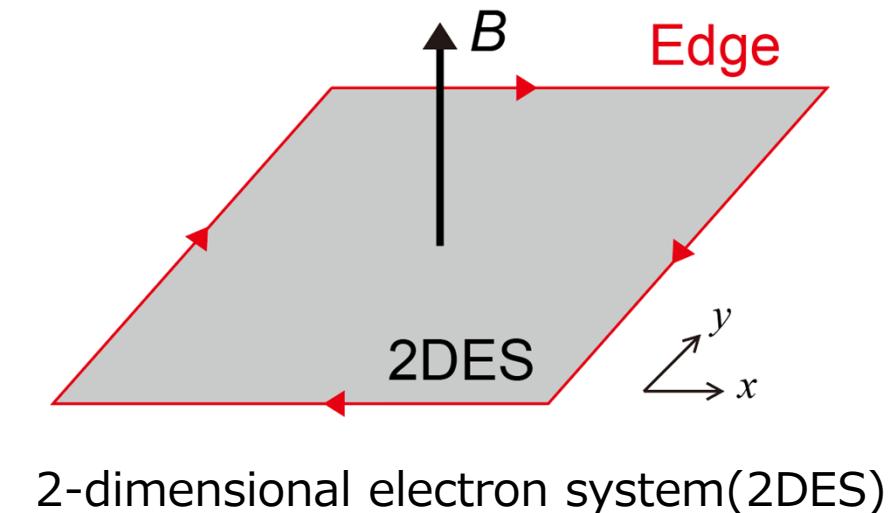
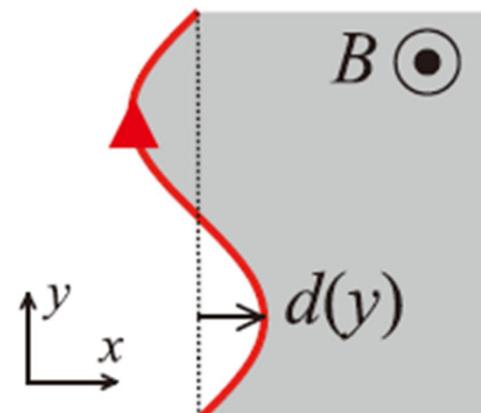
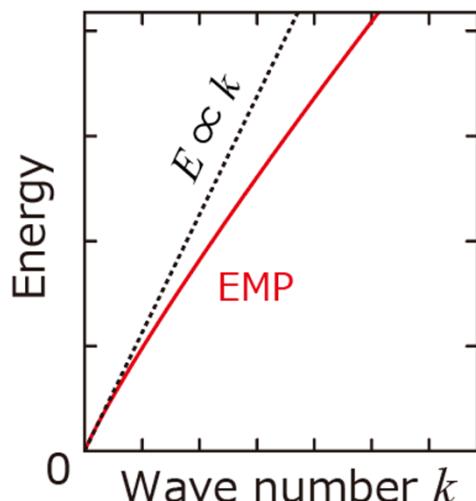
# Low-energy charge dynamics

## Quantum Hall system

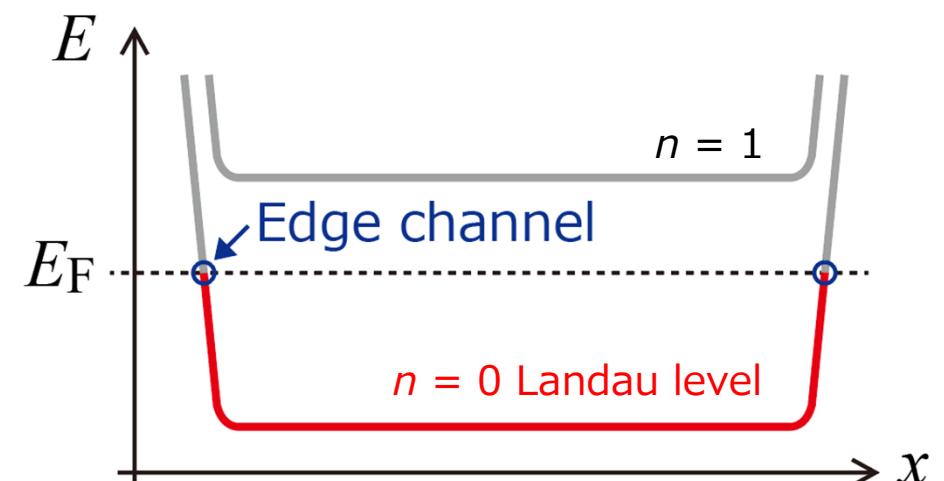
- ✓ Bulk: (Energy gap)  $> \sim 100$  GHz
- ✓ Edge: gapless

### Edge magneto-plasmon (EMP)

- Change in the boundary shape
- Almost linear dispersion relation



2-dimensional electron system(2DES)



# Inter-channel interaction

## ➤ Circuit model of edge magnetoplasmons

- 1D continuity equation  $\rightarrow \frac{\partial I}{\partial x} = -\frac{\partial \rho}{\partial t}$
- Electron correlation  $\rightarrow \rho = c_{ch}V$
- Equation of motion  $\rightarrow I(x, t) = \sigma_{xy}V(x, t)$

Hashisaka *et al.*, PRB 2012; PRB 2013.

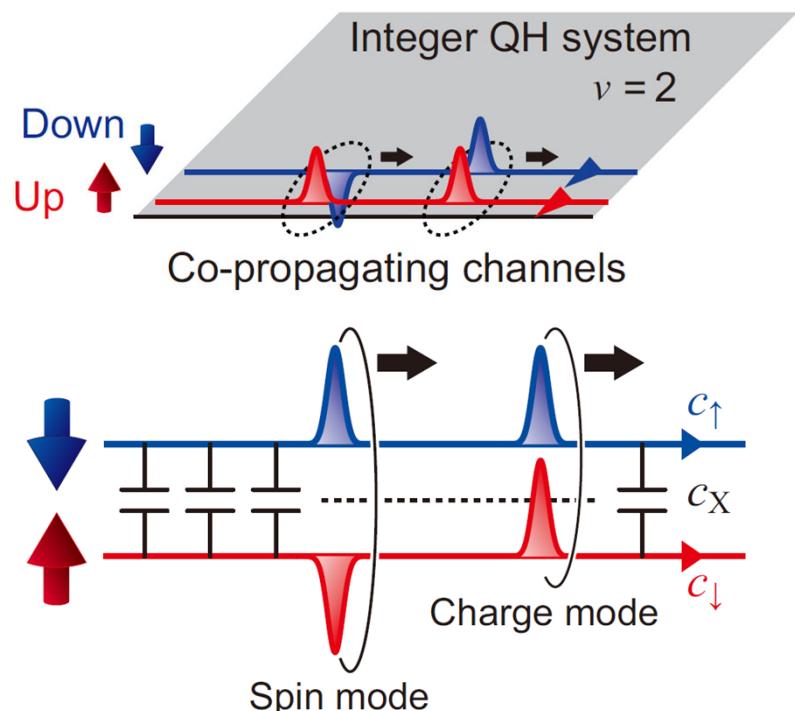
## Inter-channel Coulomb interaction

- ✓ 1D wave equation:

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho_{\uparrow} \\ \rho_{\downarrow} \end{pmatrix} = - \begin{pmatrix} v_{\uparrow} & U_X \\ U_X & v_{\downarrow} \end{pmatrix} \frac{\partial}{\partial y} \begin{pmatrix} \rho_{\uparrow} \\ \rho_{\downarrow} \end{pmatrix}$$

$$v_{\uparrow} = \frac{\sigma_{xy}(c_{\downarrow} + c_X)}{c_{\uparrow}c_{\downarrow} + c_{\uparrow}c_X + c_{\downarrow}c_X}, \quad v_{\downarrow} = \dots, \quad U_X = \dots$$

Hashisaka & Fujisawa, Rev. Phys. 2018.



# Artificial Luttinger liquid

Interacting integer edge channels show Luttinger-liquid-like behaviors

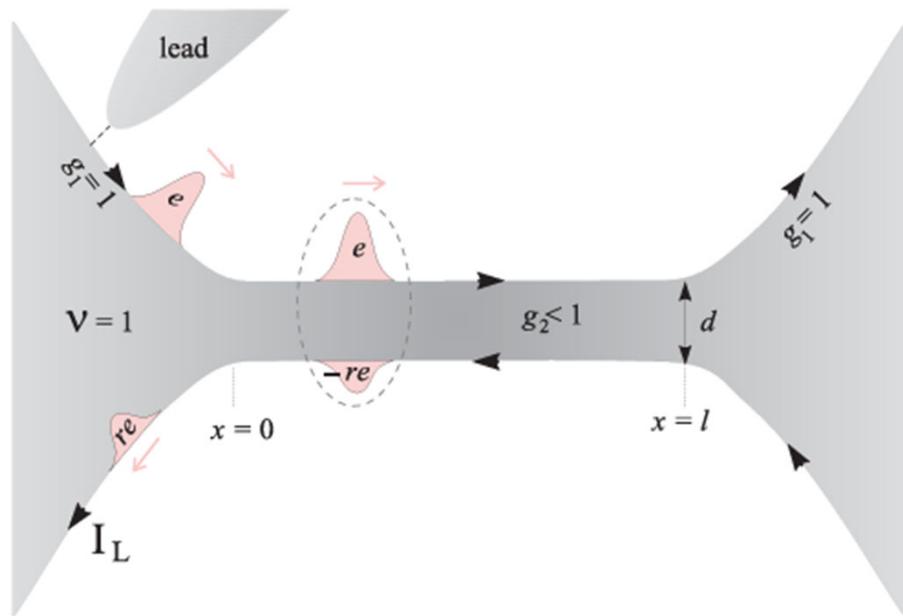
Theory: Berg, Oreg, Kim, von Oppen, Phys. Rev. Lett. **102**, 236409 (2009).

Charge conservation:  $e = er + e(1 - r)$

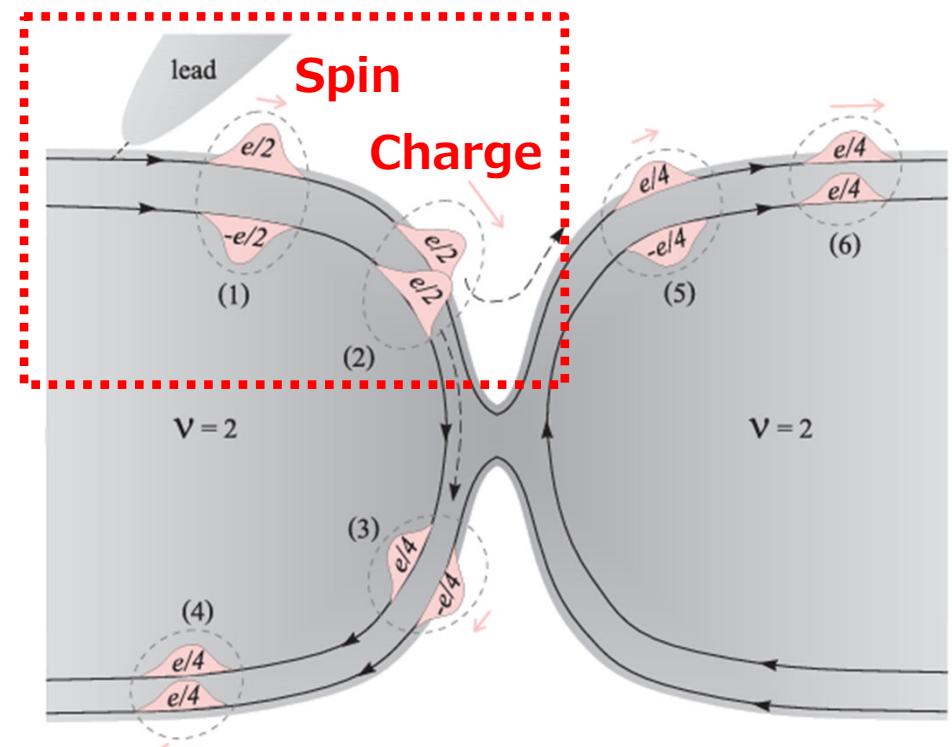
electron      Mode A + Mode B

---

## Charge fractionalization



## Spin-Charge separation

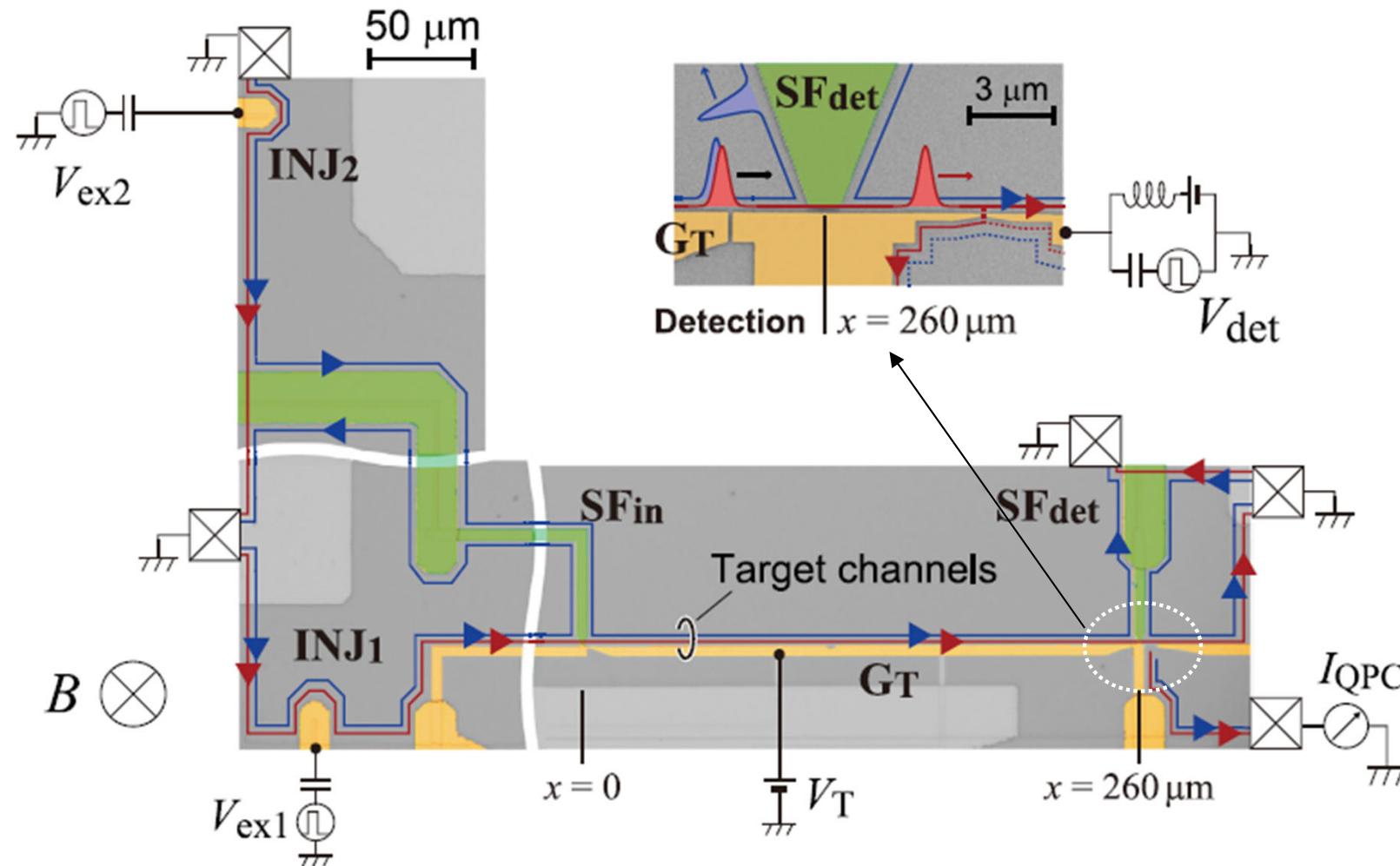


Kamata, Hashisaka, et al., Nat. Nanotechnol. 2014.  
Hashisaka et al., PRB 2012.

# Waveforms of charge & spin excitations

Direct time-domain measurement of the spin-charge separation in a Tomonaga-Luttinger liquid.

Hashisaka et al., Nat. Phys. **13**, 559 (2017).

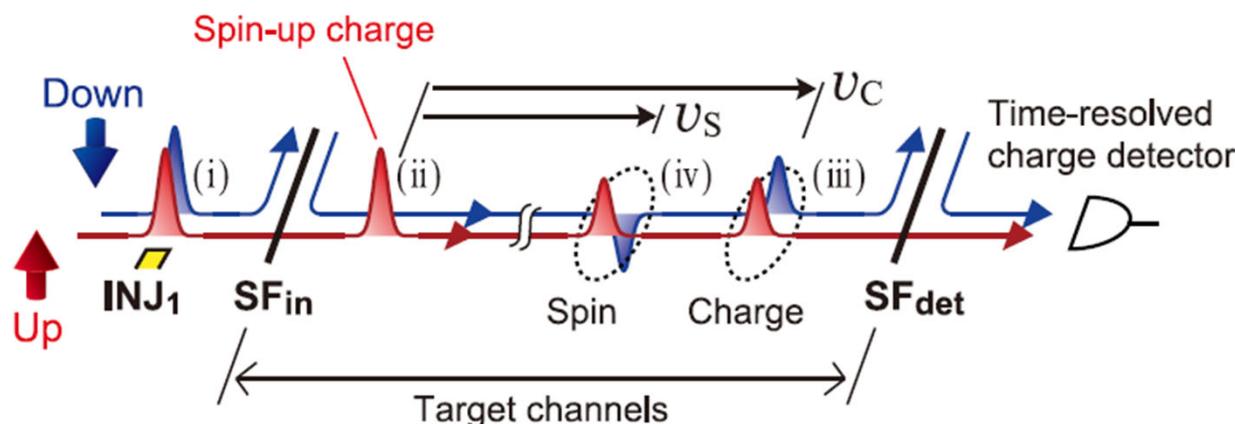


Optical micrograph of the sample ( $v = 2$  IQH system in a GaAs quantum well).

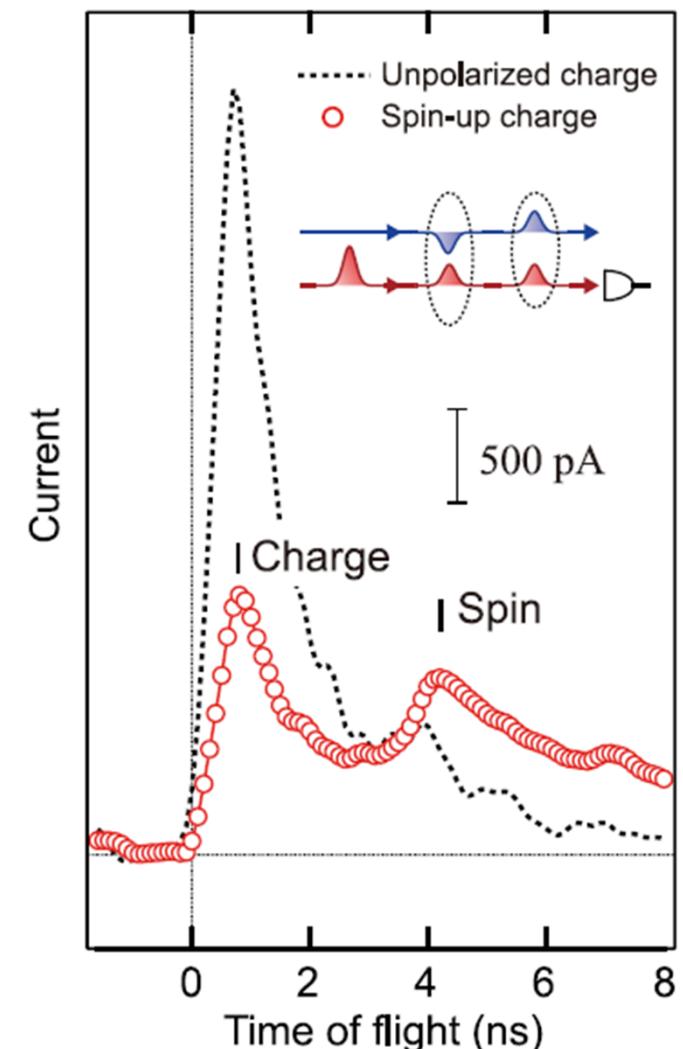
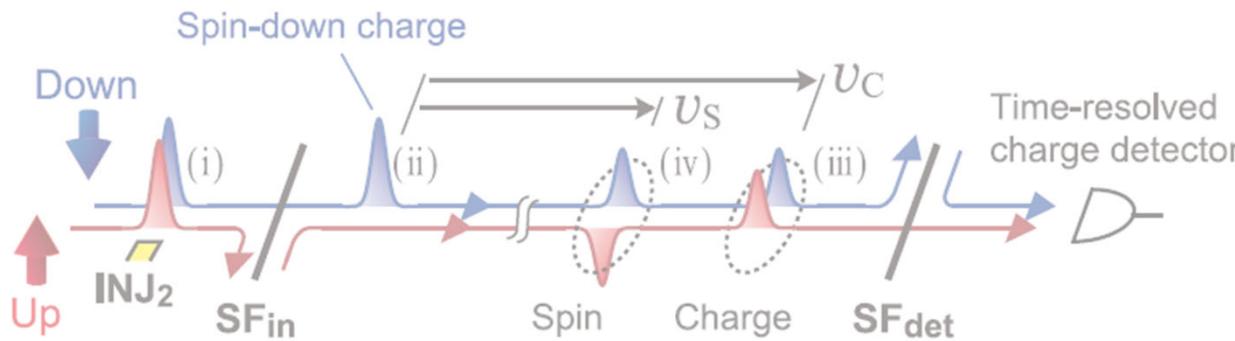
# Spin-charge separation

Hashisaka et al., Nat. Phys. **13**, 559 (2017).

## ➤ Spin-up charge injection



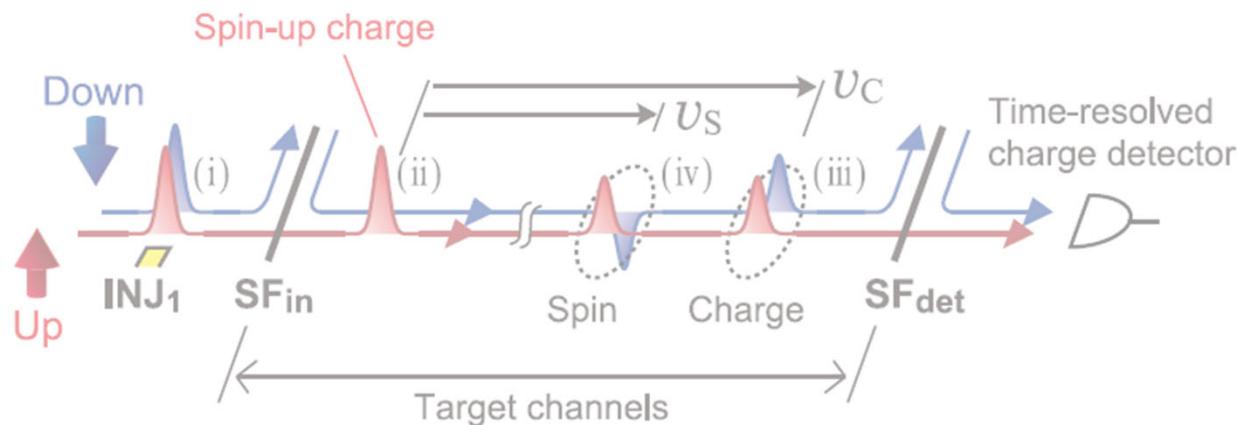
## ➤ Spin-down charge injection



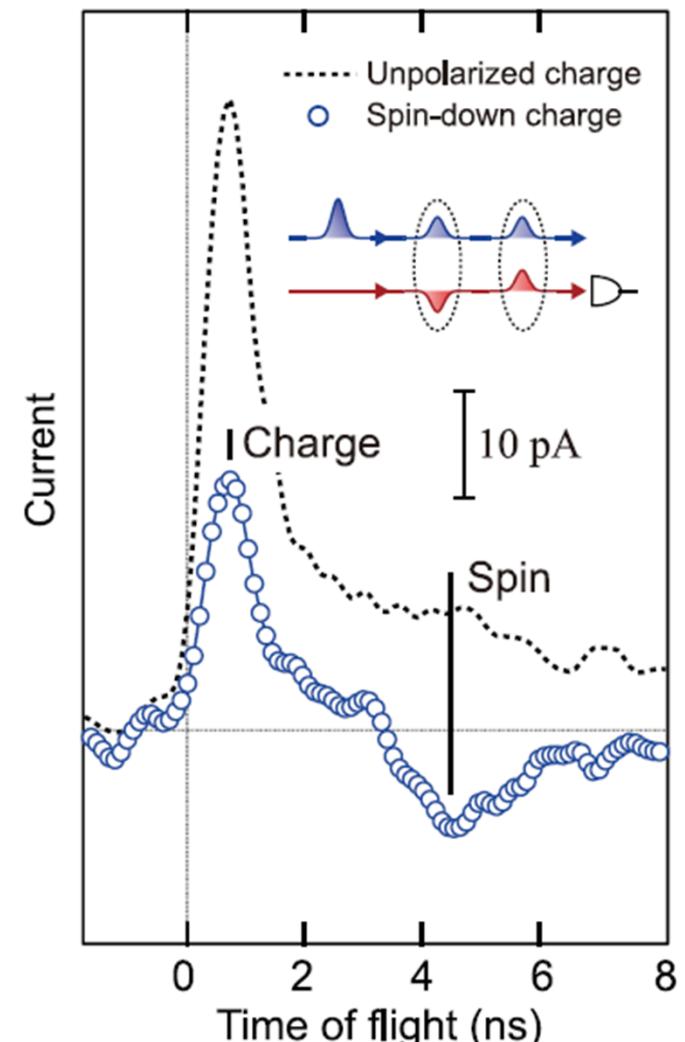
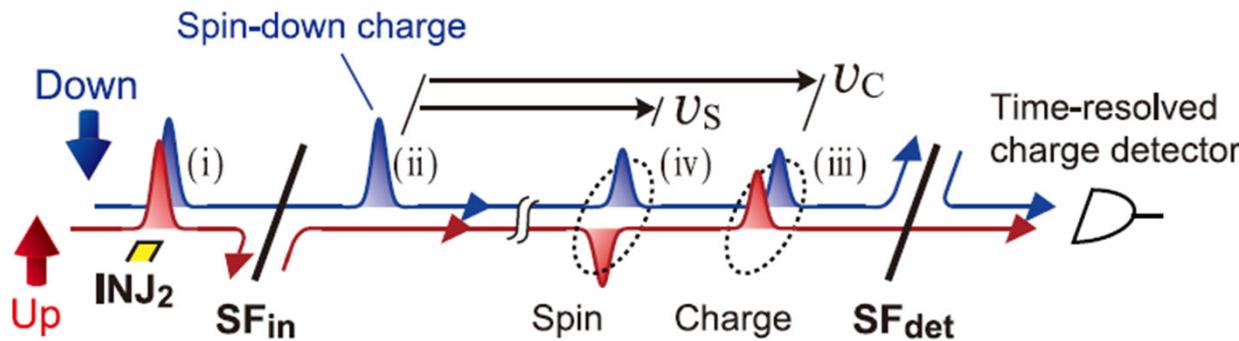
# Spin-charge separation

Hashisaka et al., Nat. Phys. **13**, 559 (2017).

## ➤ Spin-up charge injection



## ➤ Spin-down charge injection



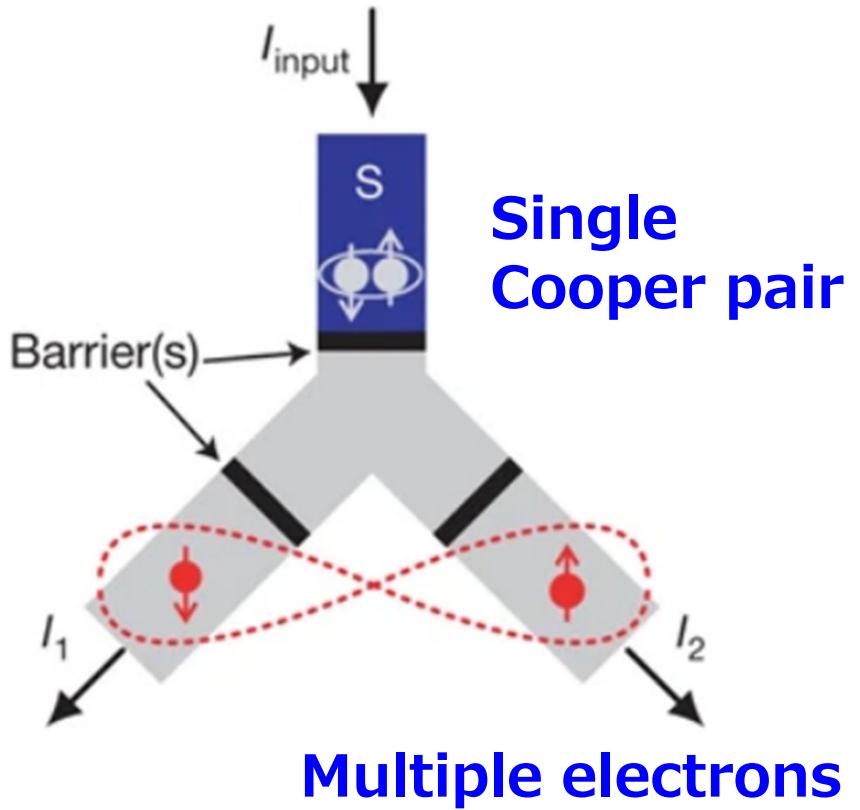
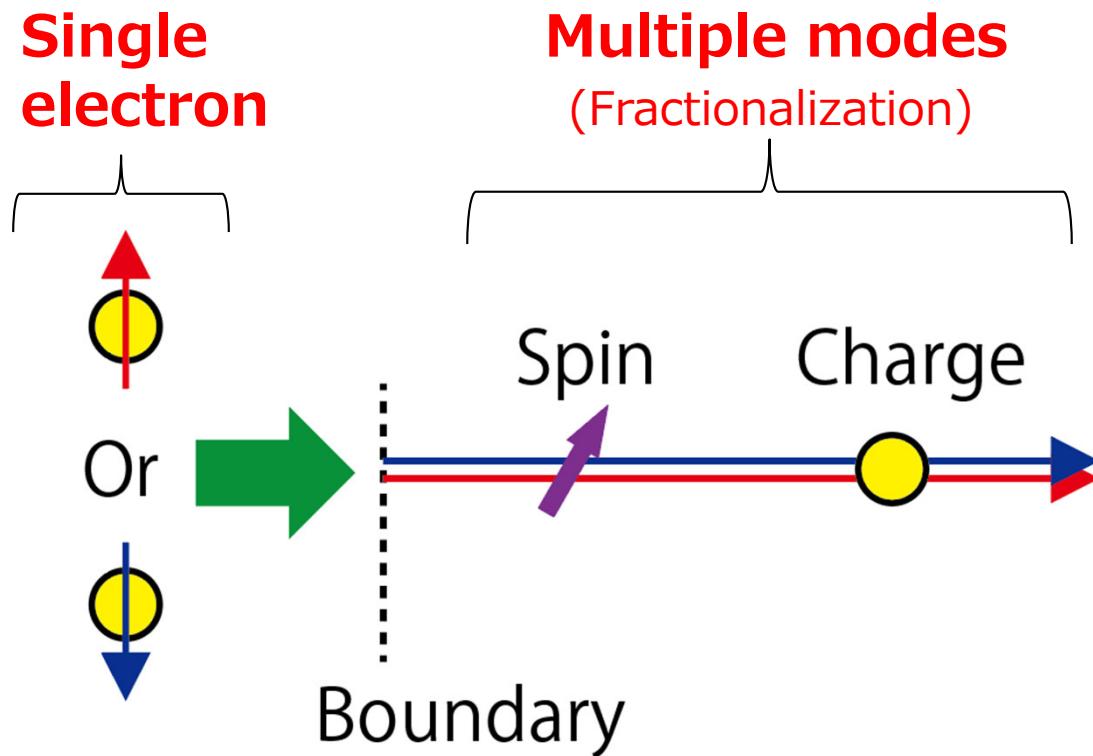
# Summary of the second topic

## Waveform measurement of charge- and spin-density wavepackets in a chiral Tomonaga-Luttinger liquid

Nature Physics **13**, 559 (2017).

Theory: Berg, Oreg, Kim, von Oppen, Phys. Rev. Lett. **102**, 236409 (2009).

## Mode transformations at boundaries



Hashisaka et al., Nat. Phys. **13**, 559 (2017).

Hofstetter et al., Nature 2009.

# Summary

## Background

### ➤ **Quantum many-body effects in quantum Hall systems**

Fractional quantum Hall (FQH) effect

Edge channel (Tomonaga-Luttinger liquid)

## Main

### ➤ **Electron dynamics at boundaries of quantum many-body systems**

- ✓ Fractional-Integer quantum Hall junction

Hashisaka *et al.*, Nat. Commun. **12**, 2794 (2021).

- ✓ A boundary of a Tomonaga-Luttinger liquid

Hashisaka *et al.*, Nat. Phys. **13**, 559 (2017).

## Appendix

### ➤ **Quantum many-body phase of interacting 1D channels**

Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).

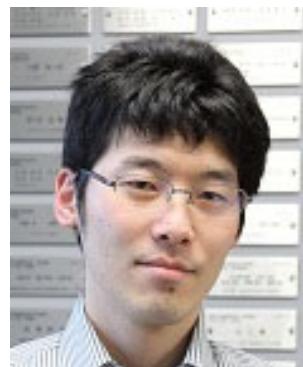
# Edge-mode transformation

**Coherent-Incoherent Crossover of Charge and Neutral Mode Transport  
as Evidence for the Disorder-Dominated Fractional Edge Phase**

Hashisaka et al., Phys. Rev. X **13**, 031024 (2023).

**NTT members**

Experiments



**T. Akiho**



**S. Sasaki**



**N. Kumada**



**K. Muraki**

**Tohoku Univ.**  
Calculations



**T. Ito**



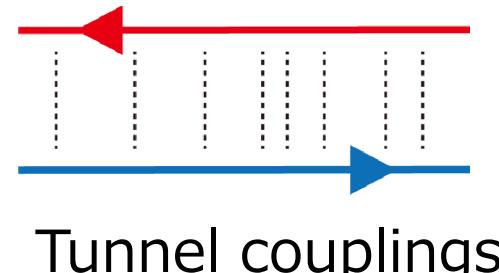
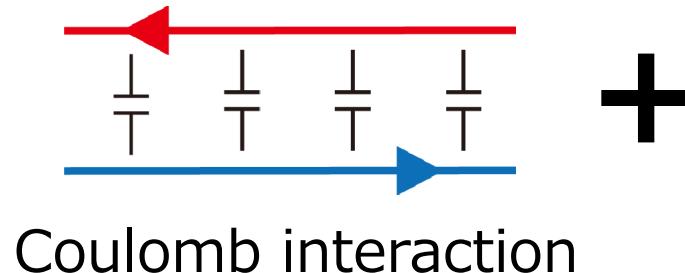
**N. Shibata**

Acknowledgements

T. Jonckheere, J. Rech,  
T. Martin, T. Fujisawa,  
H. Murofushi, & M. Imai

# Appendix: Coulomb interaction & Tunnel couplings

## ➤ Helical edge channels

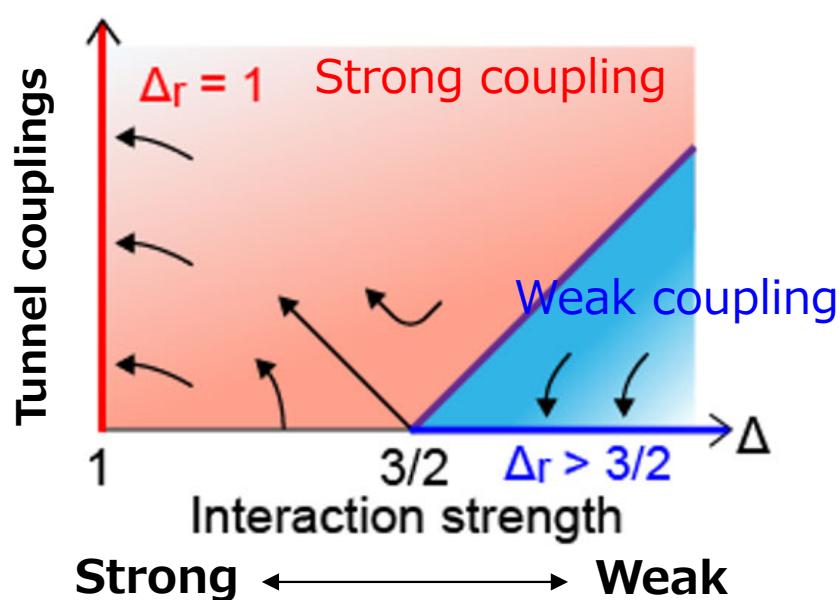


Example:

- ✓ 2D topological insulator (HgTe, etc.)

Quantized  
electrical conductance (?)

## ➤ Equilibration in the coupled helical channels.



Phase diagram of the  
coupled helical channels  
(from renormalization group theory)

The 1D phase can be identified by the  
**Electrical and thermal transport meas.**

MacDonald, PRL 1990.  
Kane, Fisher, Polchinski, PRL 1994.  
Kane, Fisher, PRB 1995.  
Protopopov, Gefen, Mirlin, Ann. Phys. 2017.

# Appendix: Coulomb interaction

## Mechanism: How do the charge and charge-neutral modes emerge?

Wave equation:  
(Coulomb interaction only)

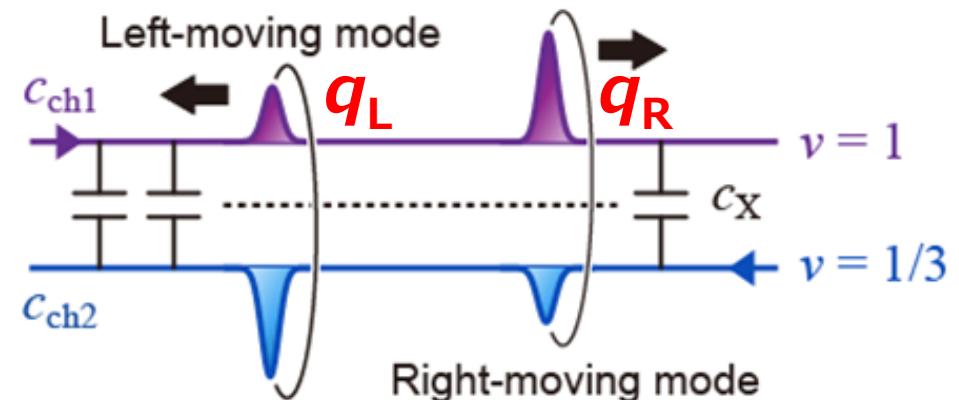
$$\frac{\partial}{\partial t} \begin{pmatrix} \rho_1 \\ \rho_{1/3} \end{pmatrix} = \begin{pmatrix} \nu_1 & U_X \\ -U_X & -\nu_{1/3} \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} \rho_1 \\ \rho_{1/3} \end{pmatrix}$$

Hashisaka & Fujisawa, Rev. Phys. 3, 32 (2018).

### Coulomb interaction only

Right- and left-moving modes carrying finite charges.

↓ Renormalization toward the strong coupling limit



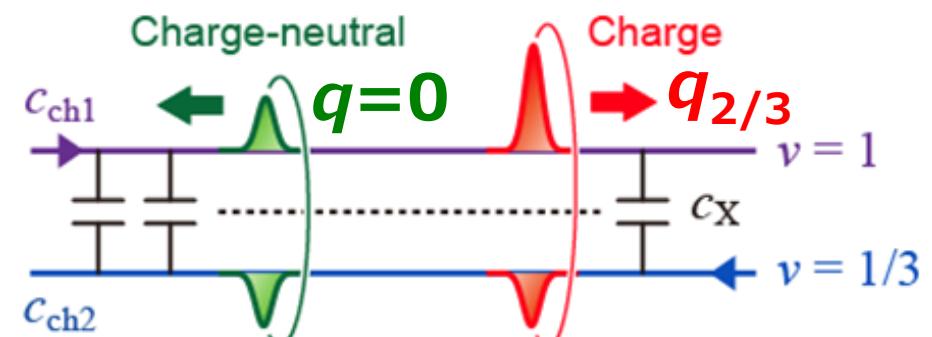
### Coulomb + Tunnel couplings

**Downstream charge and upstream charge-neutral modes**

Kane, Fisher, Polchinski 1994.

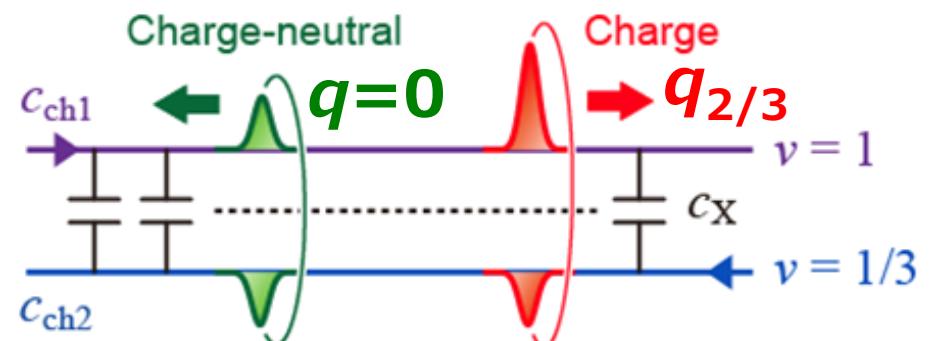
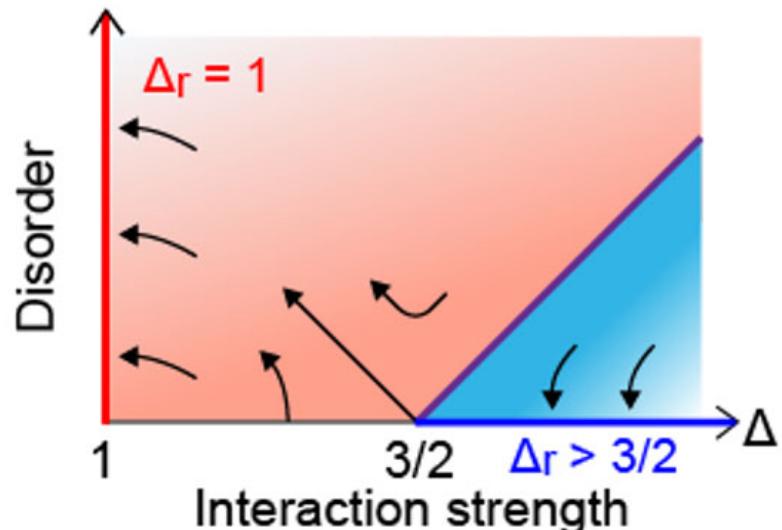
Kane, Fisher 1995.

Protopopov, Gefen, Mirlin 2017.



# Appendix: Renormalization

## Disorder-dominated FQH edge phase



Charge and neutral modes.

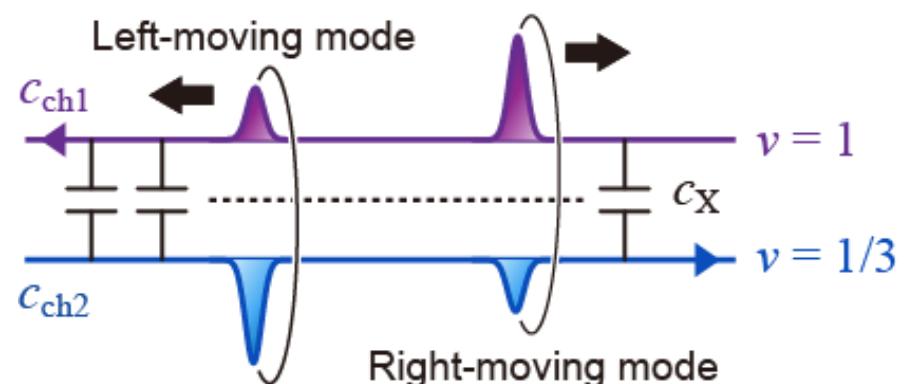
## Weak coupling phase ( $\Delta > 3/2$ )

Right- and left-moving modes both carrying charges.

Kane, Fisher, Polchinski 1994.

Kane, Fisher 1995.

Protopopov, Gefen, Mirlin 2017.



# Appendix: Upstream heat transport

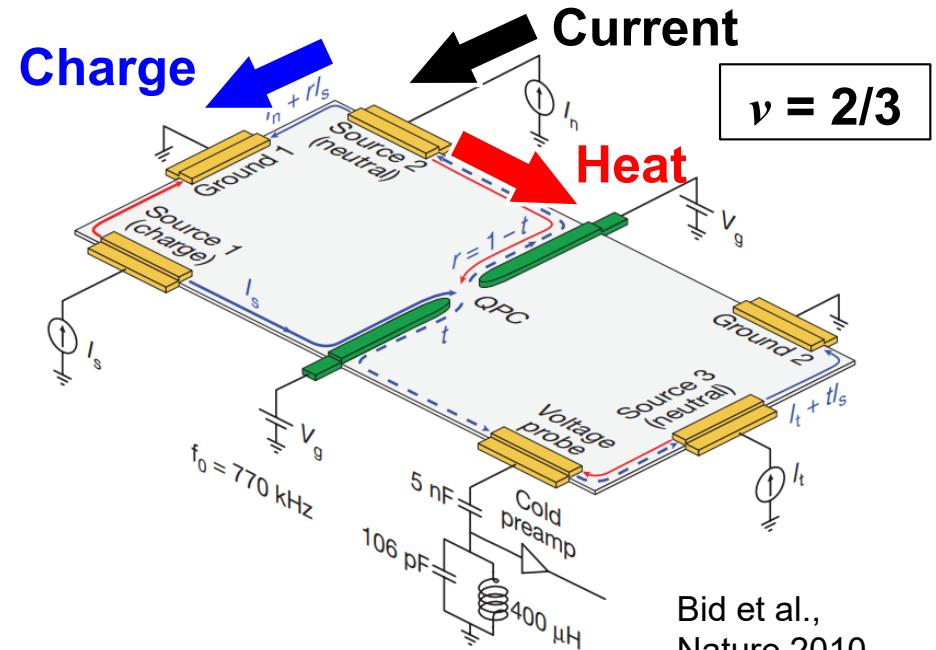
## Experimental evidence (?) of the strong coupling phase

- ✓ Electrical conductance:  $2e^2/3h$
- ✓ Upstream heat transport  
(Charge-neutral mode)

**"Standard model" for fractional QH edge transport**

- Downstream charge mode
- Upstream neutral mode

Banerjee et al., Nature **545**, 75 (2017).  
Banerjee et al., Nature **559**, 205 (2018).  
Nakamura et al., Nat. Phys. **15**, 563 (2019).  
Dutta et al., Science **375**, 193 (2022).  
Dutta et al., Science **377**, 1198 (2022). etc.



Bid et al., Nature **466**, 585 (2010).  
Venkatachalam et al., Nat. Phys. **8**, 676 (2012).  
Altimiras et al., PRL **109**, 026803 (2012).  
Inoue et al., Nat. Commun. **5**, 4067 (2014).  
Sabo et al., Nat. Phys. **13**, 491 (2017).  
Kumar et al., Nat. Commun. **13**, 213 (2022).  
Melcer et al., Nat. Commun. **13**, 376 (2022).  
Breton et al., PRL **129**, 116803 (2022).  
Srivastav et al., Nat. Commun. **13**, 5185 (2022).  
Nakamura et al., PRL **130**, 076205 (2023). etc.

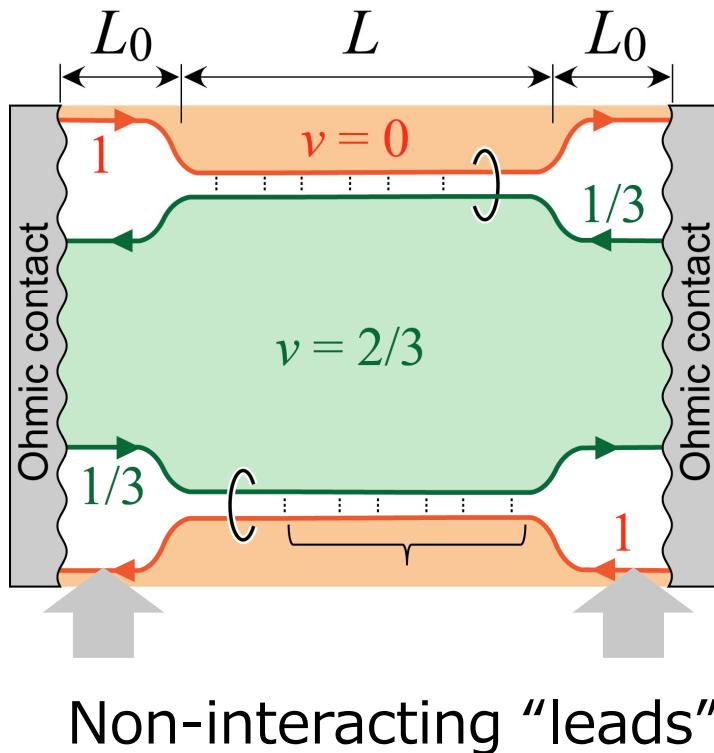
# Appendix: Problems in experiments and theories

## ◆ Non-interacting “leads” near ohmic contacts

Protopopov, Gefen, Mirlin, Ann. Phys. **385**, 287 (2017).

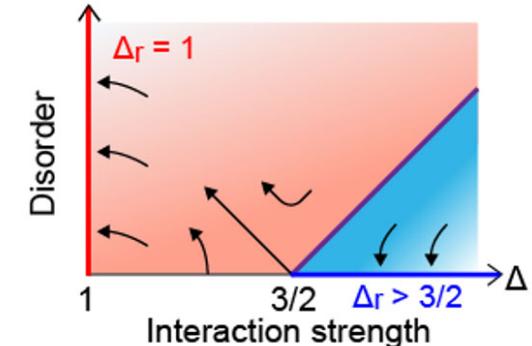
“Strong” and “Weak” coupling phases have very similar electrical & thermal transport properties except for **the mesoscopic regime**.

$$l \ll \min(L_0, L_T) \ll L \ll L_{\text{in}}$$



Interaction	Strong $\Delta_r \approx 1$	Weak $\Delta_r > 3/2$
<b>Eigenmode</b>	DS charge and US neutral modes	DS and US charged modes
<b>Coherent</b>	$G_E^{(2)} = 4/3$ $G_T^{(2)} = 2$	$G_E^{(2)} = 4/3$ $G_T^{(2)} = 2$
<b>Mesoscopic</b>	$1/3 \leq G_E^{(2)} \leq 4/3$ (fluctuating) $G_T^{(2)} = 1$	$2/3 \leq G_E^{(2)} \leq 4/3$ (monotonic) $1.88 \leq G_T^{(2)} \leq 2$
<b>Incoherent</b>	$G_E^{(2)} = 2/3$ $G_T^{(2)} \propto 1/L$	$G_E^{(2)} = 2/3$ $G_T^{(2)} \propto 1/L$

↑  
L  
↓ long



# Appendix: Coherence in a disordered system

Protopopov, Gefen, Mirlin, Ann. Phys. **385**, 287 (2017).

## Previous experiments:

Long counter-propagating channels with **inelastic scatterings**.

Bid *et al.*, Nature 2010, etc.

This study: crossover between long and short channel regimes.

Short counter-propagating channels with **only elastic scatterings**.

### Mesoscopic regime

$$L < L_{\text{in}}$$

$L$ : 1/3-1 channel length

$L_{\text{in}}$ : Inelastic scattering length

Cocurrence of

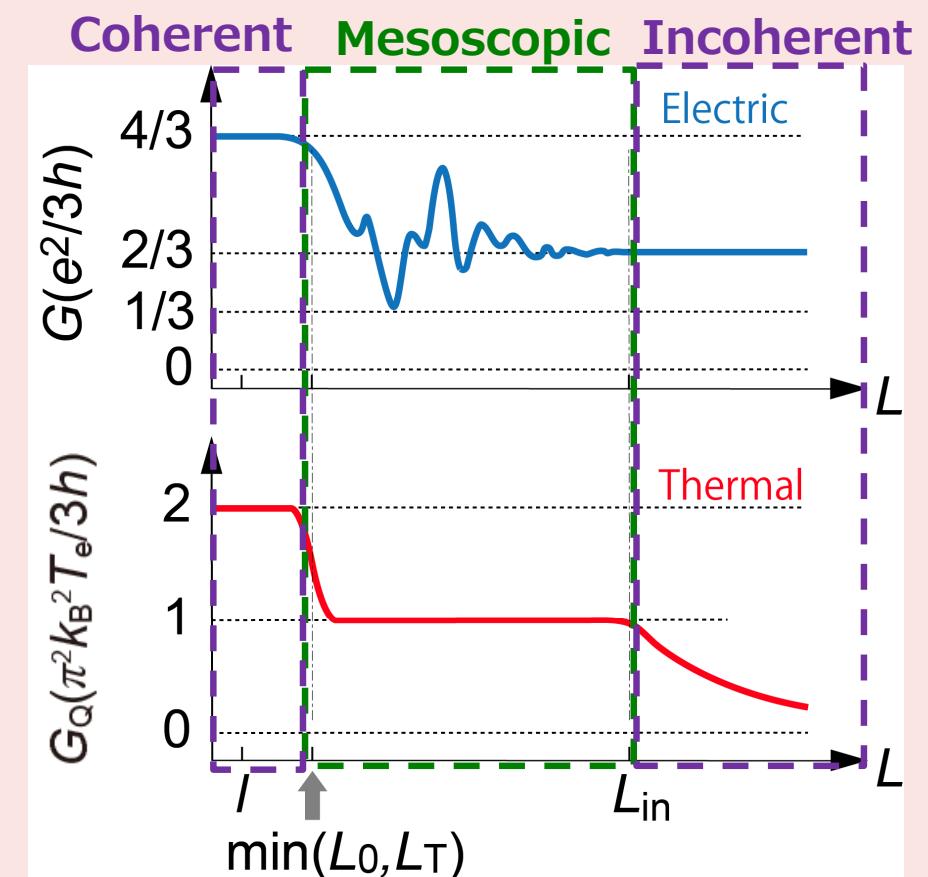
- ✓ Electrical conductance fluctuations
- ✓ Quantized thermal conductance

c.f.

Cohen *et al.*, Nat. Commun. 2019.

Kumar *et al.*, Nat. Commun. 2022.

Melcer *et al.*, Nat. Commun. 2022.



# Appendix: Half-quantized thermal conductance

## Strong coupling phase

- ✓ Electrical conductance:  $2e^2/3h$
- ✓ Upstream heat transport

Bid et al., Nature 2010.  
Inoue et al., Nat. Commun. 2014.  
...



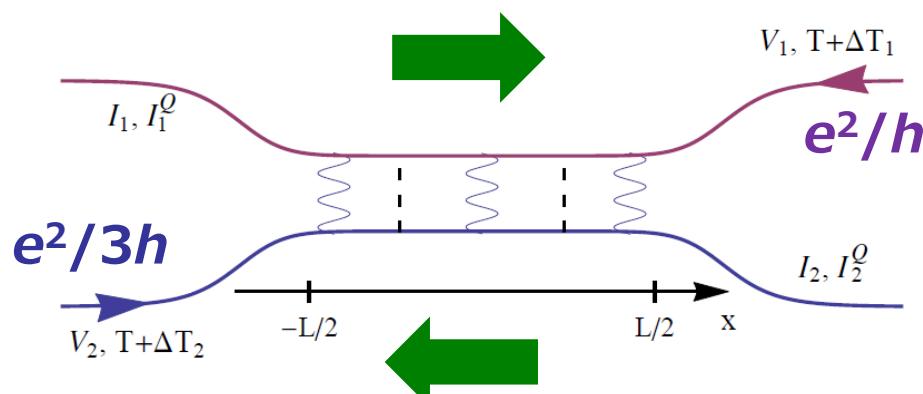
## Experimental hallmark

$$G_Q = 0.5 \frac{\pi^2 k_B^2 T}{3h}$$

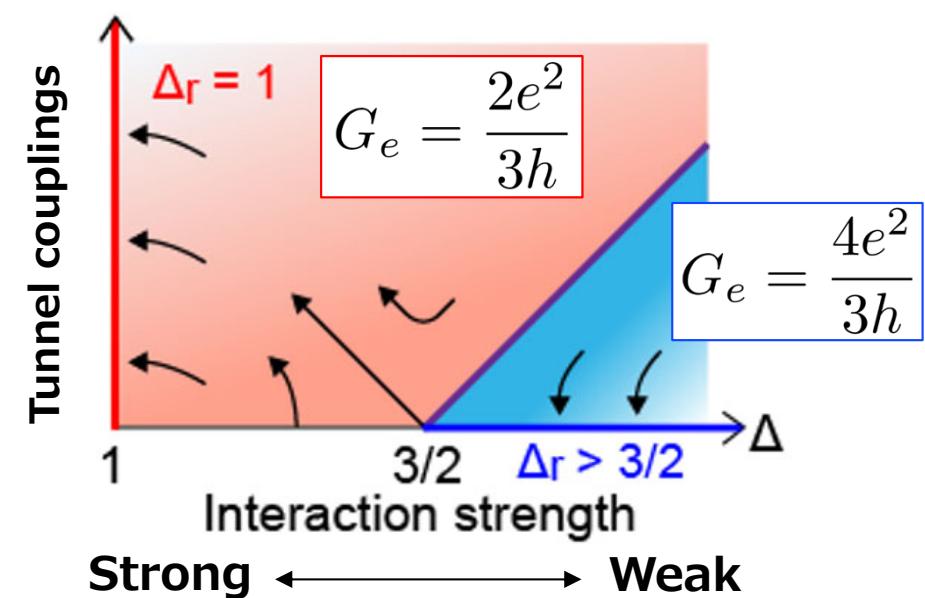
Half-quantized thermal conductance  
in the dissipationless transport regime.

Protopopov, Gefen, Mirlin 2017.

## Upstream heat transport

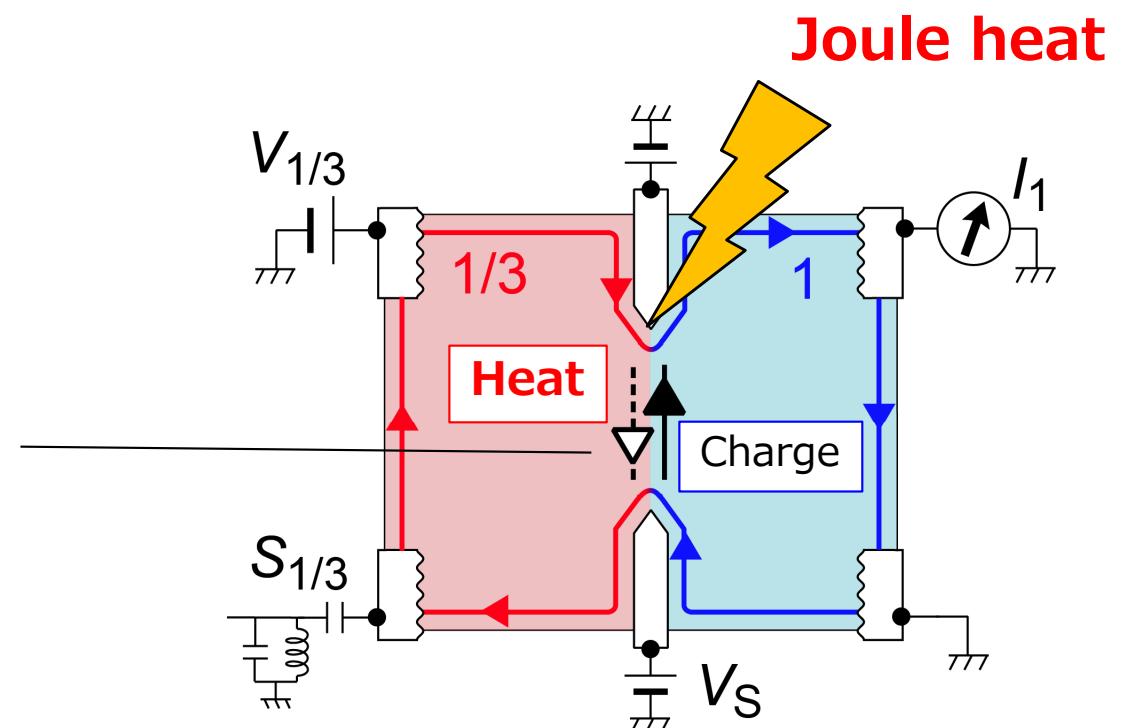
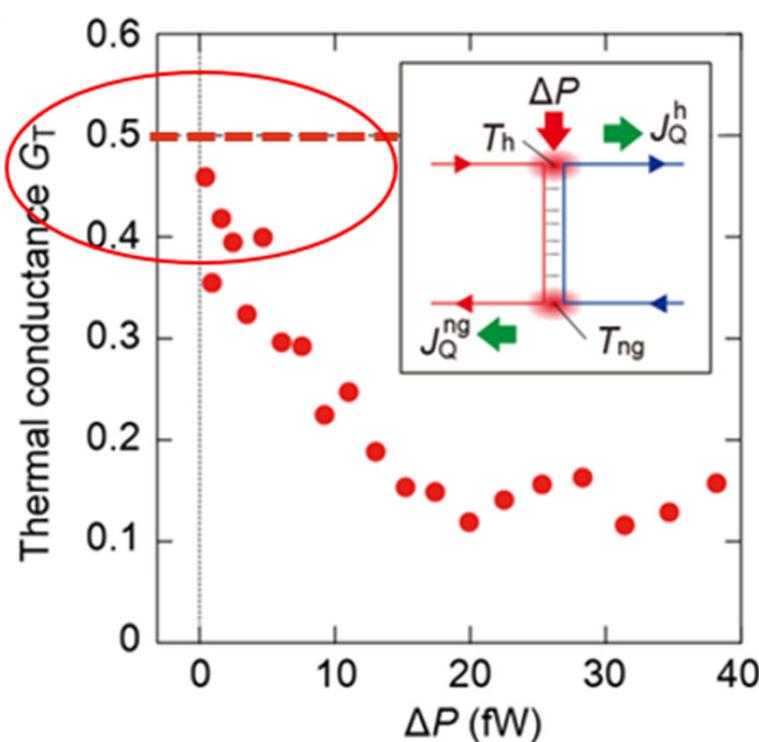
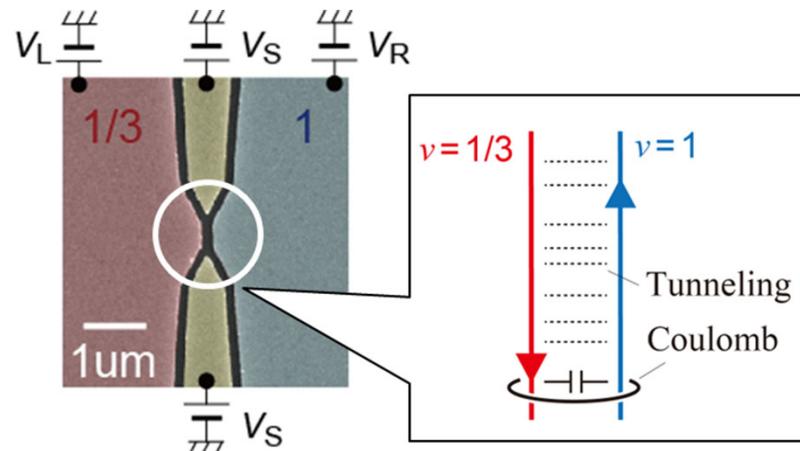


Electrical transport  $2e^2/3h$   
( $1 - 1/3$ )



# Appendix: Thermal conductance measurement

## 2DES in a GaAs quantum well



## Thermal Noise

(Thermal conductance from top to bottom)

Half-quantized thermal conductance  $G_Q/2$ :  
Evidence of the strong coupling phase

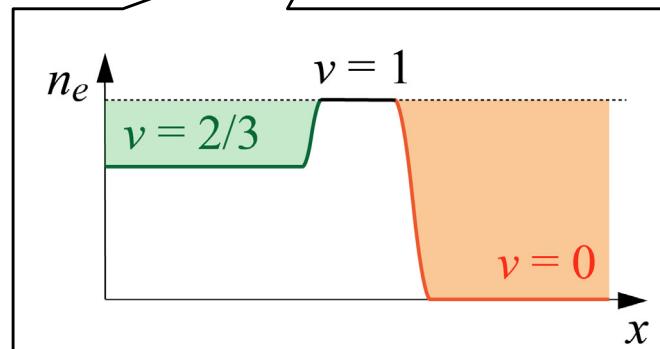
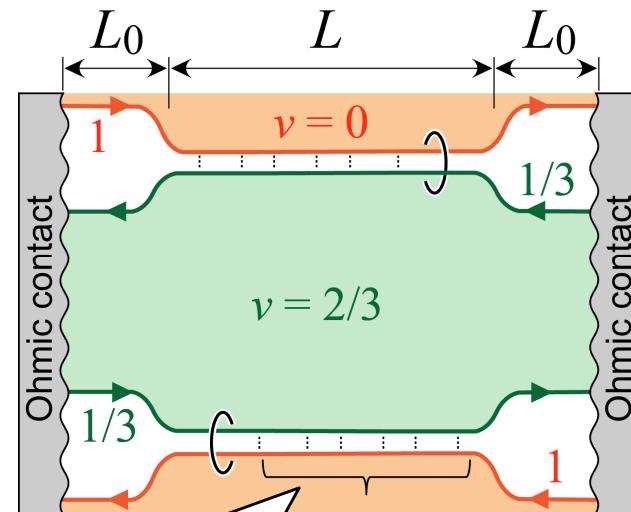
# Appendix: Particle-hole symmetry

## Coulomb interaction + Tunneling

M. Hashisaka et al., PRX **13**, 031024 (2023).

### Generalized quantum Hall (QH) edge system

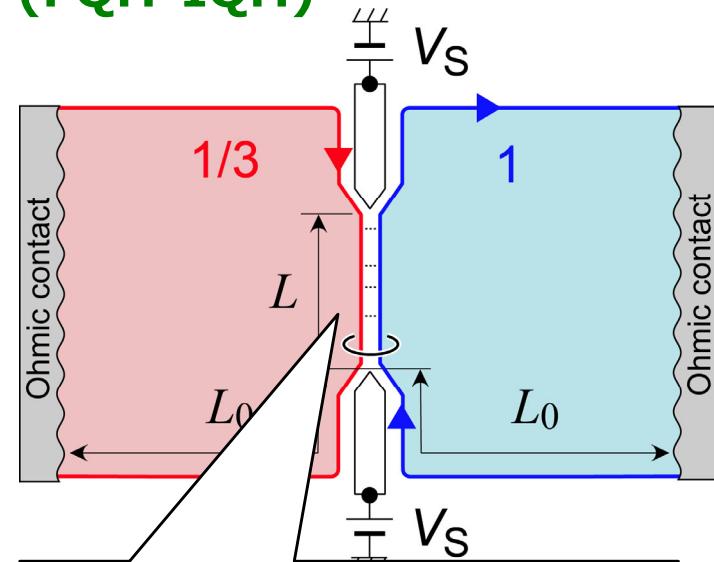
#### Conventional $v = 2/3$ device



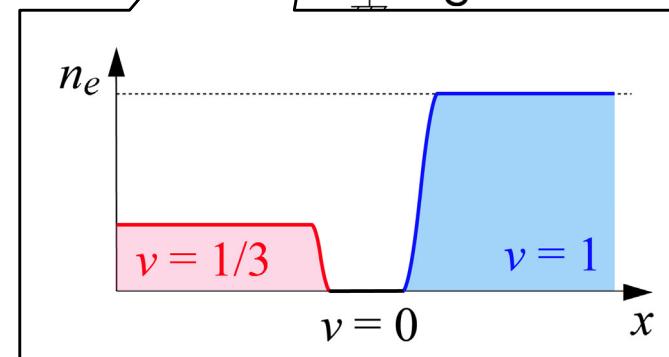
$$v(x) \leftrightarrow 1 - v(x)$$

Particle-hole  
symmetry

#### Fractional-Integer QH junction (FQH-IQH)

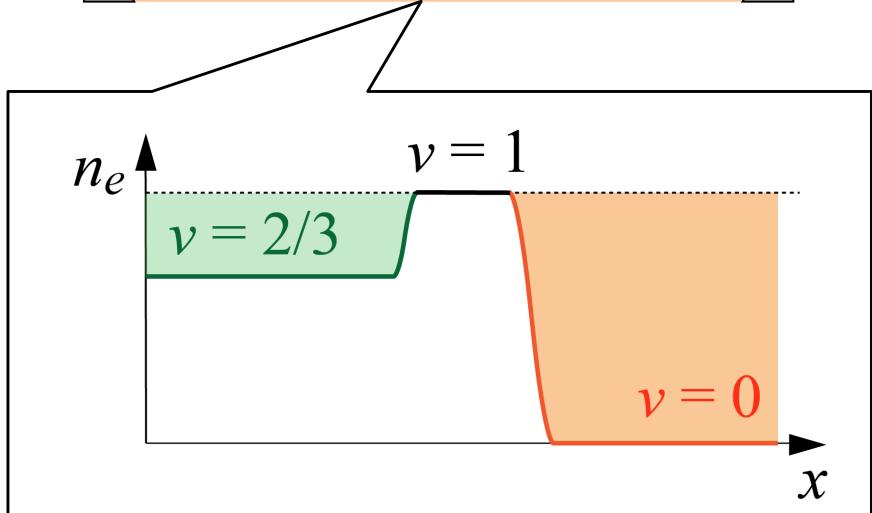
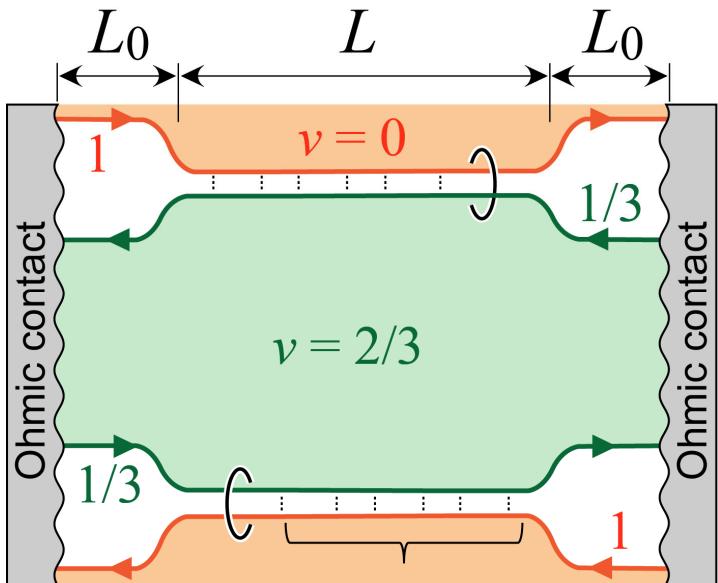


Spontaneously  
appeared  
 $1/3$ - $1$  channels



# Appendix: Bulk-edge correspondence

## ◆ Counter-propagating $\nu = 1/3$ and $\nu = 1$ edge channels



Bulk

Spin-polarized  $\nu = 2/3$  state:

$\nu_h = 1/3$  fractional quantum Hall state

( $\nu = 2/3 = 1 - 1/3$ )

Bulk-Edge correspondence

Edge

Counter-propagating  $1/3$  and  $1$  channels

(MacDonald's model: 1-1/3 channels)

MacDonald, PRL **64**, 220 (1990).

Johnson, MacDonald, PRL **67**, 2060 (1991).

Meir, PRL **72**, 2624 (1994).

Chamon, Wen, PRB **49**, 8227 (1994).

...

# Summary of the Appendix part

## Experiment:

### Coherent-Incoherent Crossover of Charge and Neutral Mode Transport as Evidence for the Disorder-Dominated Fractional Edge Phase

M. Hashisaka, T. Ito, T. Akiho, S. Sasaki, K. Kumada, N. Shibata, K. Muraki  
Physical Review X **13**, 031024 (2023).

## Theory:

### Transport in a disordered $v = 2/3$ fractional quantum Hall junction

I. V. Protopopov, Y. Gefen, A. D. Mirlin  
Annals of Physics **385**, 287 (2017).

## Key background theory:

Neutral modes in fractional QH systems.

Kane, Fisher, Polchinski, PRL **72**, 4129 (1994).  
Kane, Fisher, PRB **51**, 13449 (1995).

