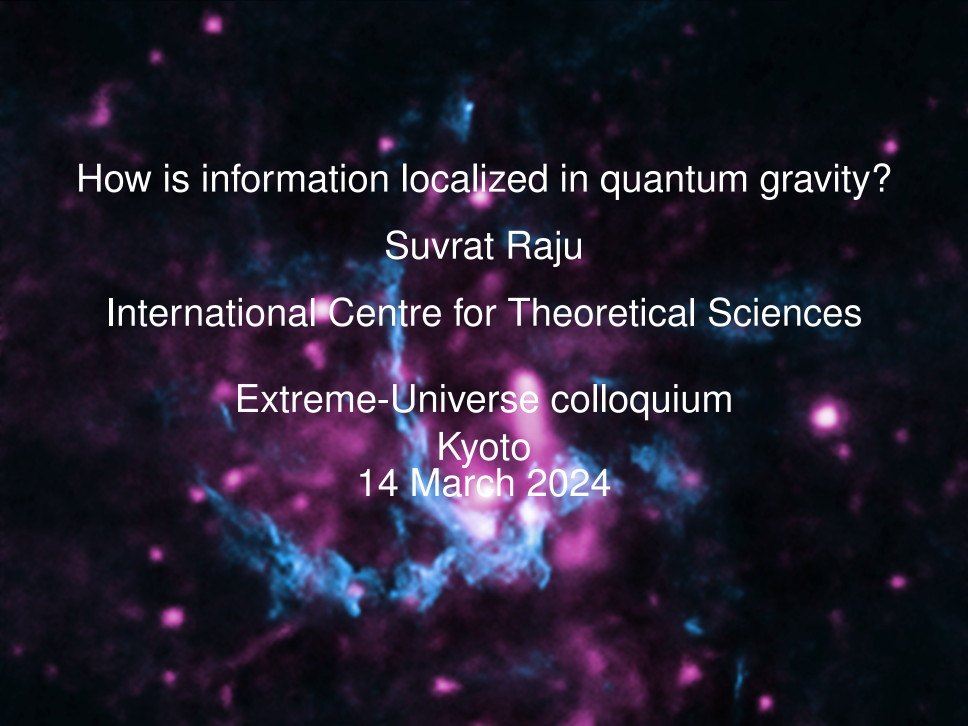

How is information localized in quantum gravity?

Suvrat Raju

International Centre for Theoretical Sciences

Extreme-Universe colloquium
Kyoto
14 March 2024

Collaborators and References

Joydeep Chakravarty

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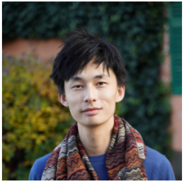
Tuneer Chakraborty



Victor Godet

- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

Collaborators and References

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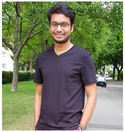

Pushkal Shrivastava

Chandramouli Chowdhury

Olga Papadoulaki

- `2002.02448`, Alok Laddha, Siddharth Prabhu, Pushkal Shrivastava, S.R.
- `2008.01740`, Chandramouli Chowdhury, Olga Papadoulaki, S.R.
- `2107.14802`, Chandramouli Chowdhury, Victor Godet, Olga Papadoulaki, S.R.

Collaborators and References

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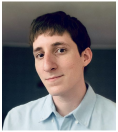

Marcos Riojas

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- `2107.03390`, Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi
- `2012.04671`, Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi

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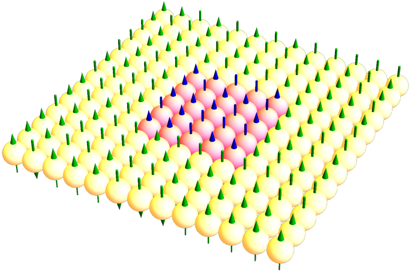
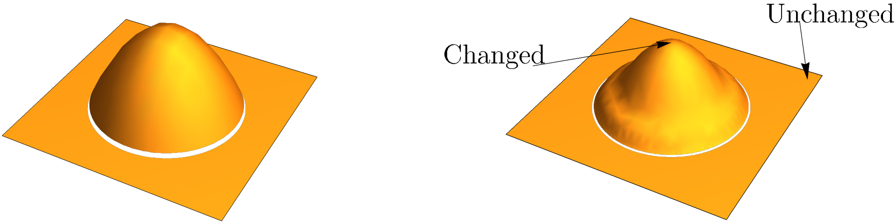
- 2012.05770, S.R.
- 2110.05470, S.R.

The split property in nongravitational theories



- In a nongravitational theory, the state inside and outside a bounded region can be specified independently. (Clear in a lattice regularization.)

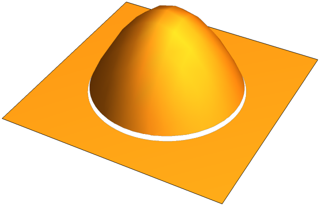
Ordinary localization of information in QFT

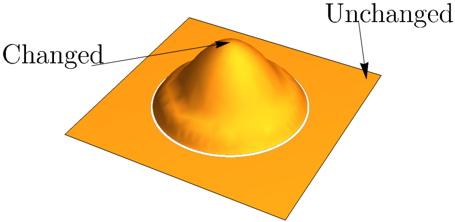


The split property is key to our usual idea that information is localized “inside” some region and is not available in its complement.

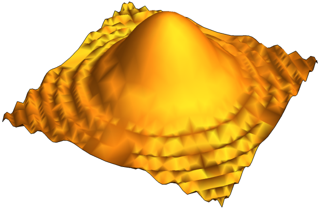
Complement-of-bounded-region has info about region

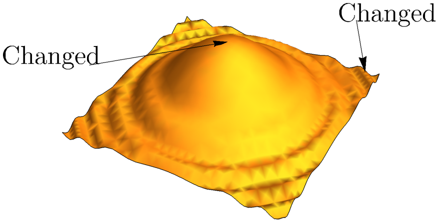
Nongravitational QFT:





Gravity:



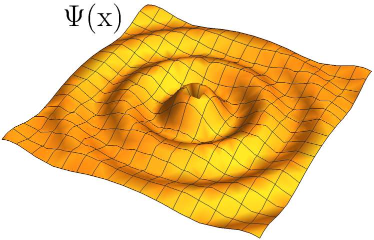


Overview

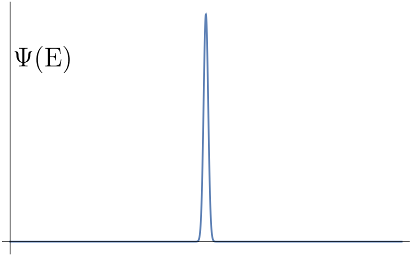
- This unusual localization of information can be inferred from a gravitational analysis.
- For simple states, this effect is visible perturbatively.
- This effect is important for the black-hole information problem.
- In some regimes, this effect becomes unimportant and gravity behaves like an ordinary QFT.
- Results pertain to information and not dynamical holographic dualities. So we term this “holography of information.”

From a physical point of view, why would one believe that gravity localizes information differently?

Uncertainty principle

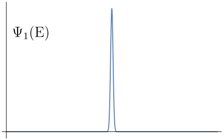


Position space



Energy spectrum

Localization via superposition



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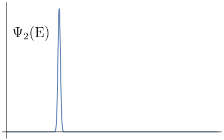

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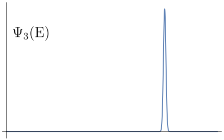

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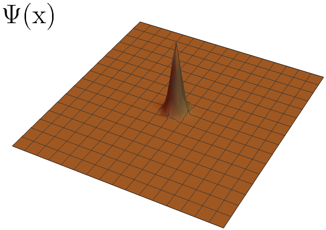
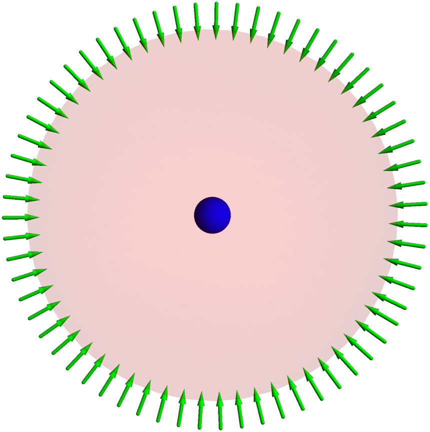
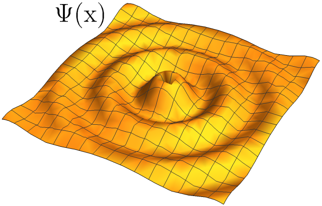


Gauss law



$$E = \frac{1}{16\pi G} \int \left(g_{jk,k} - g_{kk,j} \right) d^2\Omega_j$$

Gravitational tails



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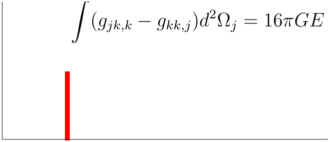
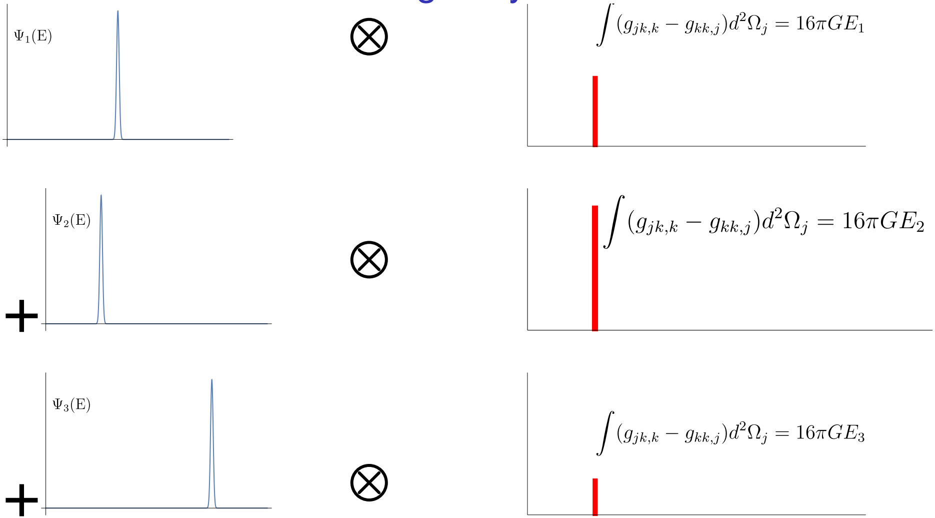


In the quantum theory, each excitation must be dressed with a gravitational tail.

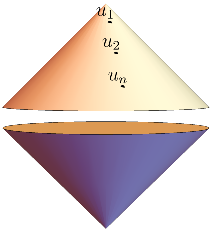
Failure of localization in gravity

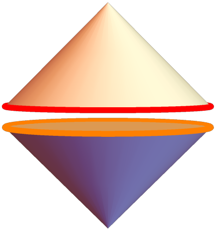


Tails prevent destructive interference outside a bounded region.
Correlators of the energy and other observables carry information about the state.

Precise results.

Holography of information in asymptotically flat space





In a nongravitational theory, asymptotic algebra at $\mathcal{I}^+$ comprises independent operators

$$\text{span}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}$$

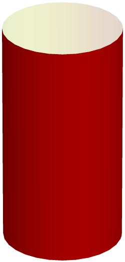
In gravity, any operator on $\mathcal{I}^+$ can be approximated arbitrarily well from

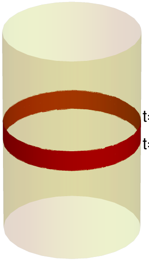
$$\text{span}\{\mathcal{Y}(u_1)\ldots\mathcal{Y}(u_n)\},$$

$$u_i \in [-\infty, -\frac{1}{\epsilon}]$$

Laddha, Prabhu, S.R., Shrivastava (2020); Marolf (2006–13)

Holography of information in asymptotically AdS





In a nongravitational theory, asymptotic algebra lives on the entire boundary

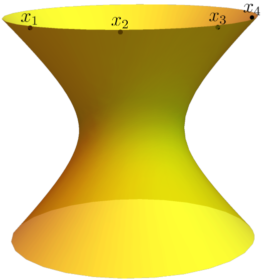
$$\text{span}\{\mathcal{Y}(t_1)\mathcal{Y}(t_2)\ldots\mathcal{Y}(t_n)\}$$

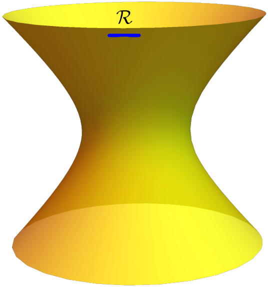
In gravity, any asymptotic operator can be approximated arbitrarily well from

$$\text{span}\{\mathcal{Y}(t_1)\ldots\mathcal{Y}(t_n)\},$$
$$t_i \in [0, \epsilon]$$

Laddha, Prabhu, S.R., Shrivastava (2020); R.de Mello Koch, Kemp (2022)

Holography of information in asymptotically dS





In a nongravitational theory, complete set of observables are

$$\langle \mathcal{Y}(x_1) \ldots \mathcal{Y}(x_n) \rangle$$

In gravity, complete set of observables are

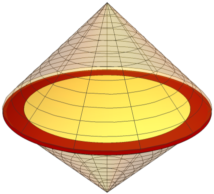
$$\langle \mathcal{Y}(x_1) \ldots \mathcal{Y}(x_n) \rangle, x_i \in \mathcal{R}$$

[Chakraborty, Chakravarty, Godet, Paul, S.R., 2023]

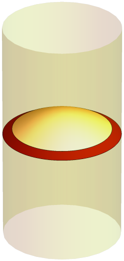
Physical interpretation: picture

Precise results are in terms of asymptotic algebras.

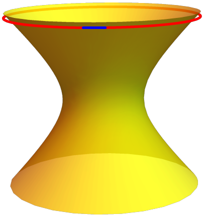
But physically, they imply that information about the whole state is available in a subregion.



Flat space



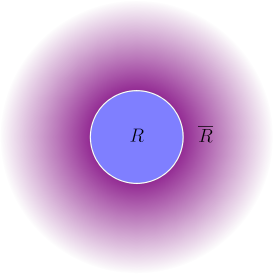
AdS



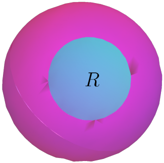
dS

Physical interpretation: picture

AdS and flat space



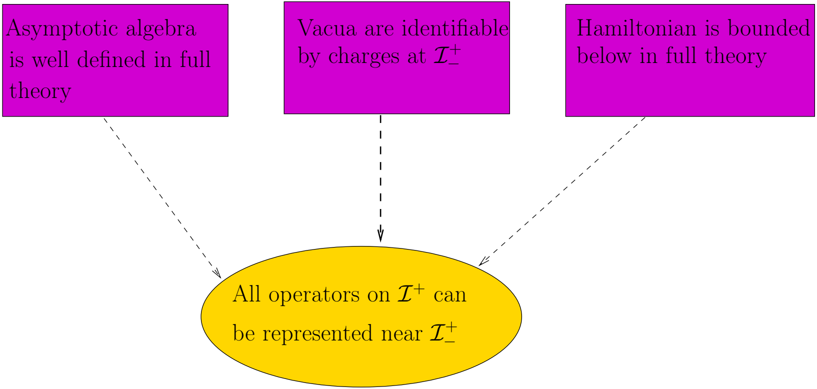
dS



The complement of a bounded region has all information about the state.

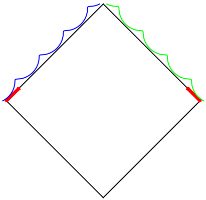
Assumptions: flat space

These results rely on weak assumptions about the full theory.



Elements of proof: algebra

Assumption: Asymptotic algebra makes sense in the UV theory.



- Study Hilbert space obtained by asymptotic quantization

$$\mathcal{H}^{(0)} = \text{span of}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}|0\rangle, \quad u_i \in (-\infty, \infty)$$

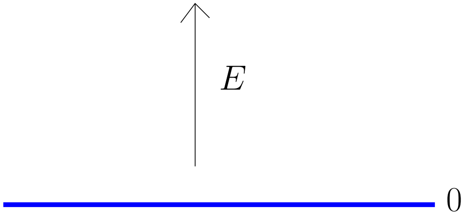
[Ashtekar, 1981]

- Define

$$\mathcal{A}_{-\infty,\epsilon} = \text{span of}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}, \qquad u_i \in (-\infty, -\frac{1}{\epsilon}]$$

Elements of proof: entanglement

Assumption: Hamiltonian is bounded below.



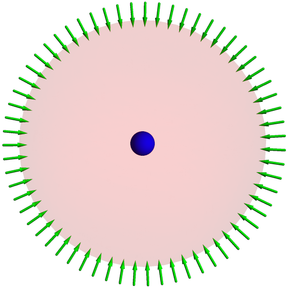
$\Longrightarrow$ any state $|n\rangle \in \mathcal{H}^{(0)}$ is approximated arbitrarily well by

$$|n\rangle \doteq X_n|0\rangle$$

where $X_n \in \mathcal{A}_{-\infty,\epsilon}$.

Elements of proof: Gauss law

Assumption: Vacuum can be identified by a boundary operator.



$$P_0 = |0\rangle\langle 0| \in \mathcal{A}_{-\infty,\epsilon}$$

A unique property of gravity is that the vacuum can be selected from the asymptotic region.

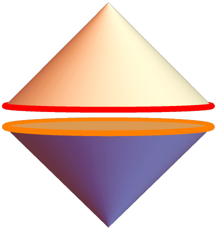
Proof of holography of information

$$A = \sum_{n,m} c(n,m)|n\rangle\langle m| \qquad \text{by step 1}$$

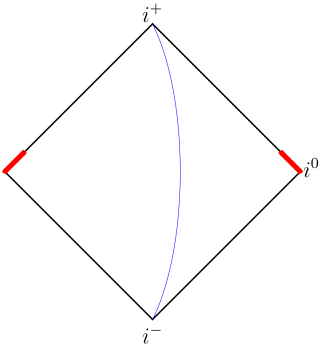
$$\doteq \sum_{n,m} c(n,m) X_n |0\rangle\langle 0| X_m^\dagger \qquad \text{by step 2}$$

$$= \sum_{n,m} c(n,m) X_n \mathcal{P}_0 X_m^\dagger \qquad \text{by step 3}$$

So any operator, $\mathcal{H}^{(0)} \to \mathcal{H}^{(0)}$, can be approximated arbitrarily well by an operator near $\mathcal{I}^+_-$.



Massive particles



Previous analysis does not account for massive particles that go from i^- to i^+.

No satisfactory asymptotic description of massive fields at $\mathcal{I}^\pm$.

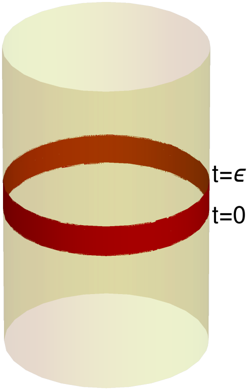
Some progress possible by studying massive particles at i^0.

[Laddha, Prabhu, S.R, Shrivastava, 2022]

[Anupam, Athira, Paul, S.R., (in progress)]

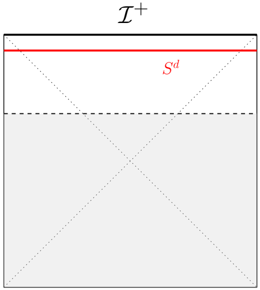
Holography of Information in AdS

Similar arguments imply that all operators on the boundary of AdS can be represented in a small time band $[0, \epsilon]$. (No subtlety with massive particles.)



This is consistent with AdS/CFT but does not assume it.

dS analysis



$$\mathcal{H}\Psi[g,\phi] = 0; \qquad \mathcal{H}_i\Psi[g,\phi] = 0.$$

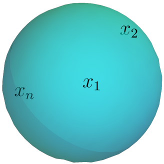
At late times, WDW equation forces

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

[Chakraborty, Chakravarty, Godet, Paul, S.R., 2023]

Related analysis: Freidel 2008

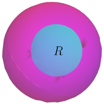
Cosmological correlators



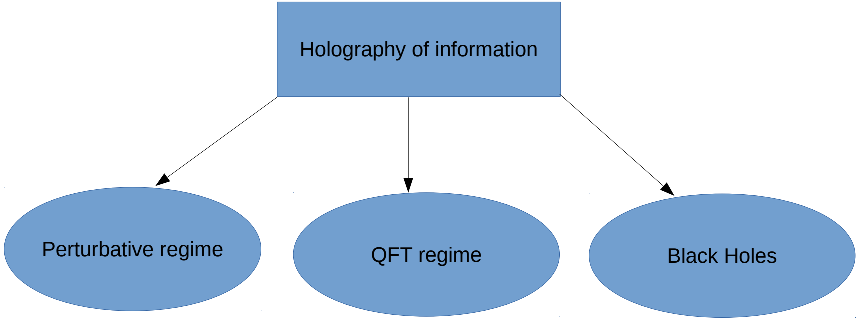
Cosmological correlators must be gauge fixed. Labelled by local points but secretly nonlocal.

$$\langle\!\langle \Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\!\rangle_{\text{CC}} = \int |\Psi|^2\chi(x_1)\dots\chi(x_n)\delta(\text{g.f})\Delta'_{FP}DgD\chi$$

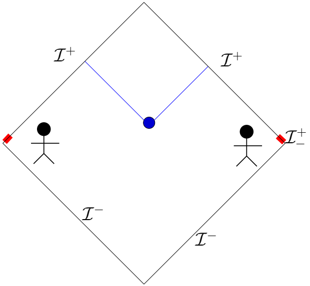
Constraints ⇒ form of the wavefunctional ⇒ symmetries of cosmological correlators.



Regimes


Holography of information
Perturbative regime
QFT regime
Black Holes

Perturbative evidence



- Excitation that hits null infinity between $u \in [0, 1]$.

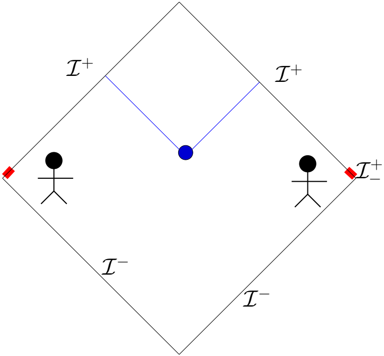
$$|f\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O(x,\Omega)} |0\rangle.$$

- **Challenge:** Using perturbative quantum correlators near $\mathcal{I}^+_-$ determine $f(x, \Omega)$.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Chowdhury, Papadoulaki, S.R., 2020]

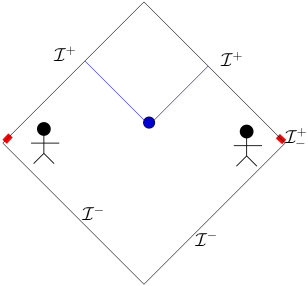
Perturbative evidence



$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = G\lambda \int_0^1 \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx + O\left(\lambda^2\right)$$

$$= -G\lambda \sum_{n=0}^{\infty} \frac{1}{u^{n+1}} \int_0^1 x^n f(x,\Omega')dx.$$

Significance of gravity

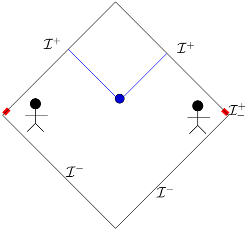


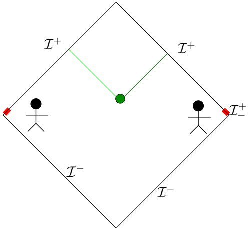
- This idea cannot work without gravity.
- Nongravitational gauge theories contain local gauge-invariant bulk operators.

$$|0\rangle \qquad \text{and} \qquad e^{i\mathrm{Tr}(F^2)(u=0)}|0\rangle$$

cannot be distinguished by any measurement near $\mathcal{I}^+_-$ without gravity.

Global symmetry





Say O_1 and O_2 are identical.

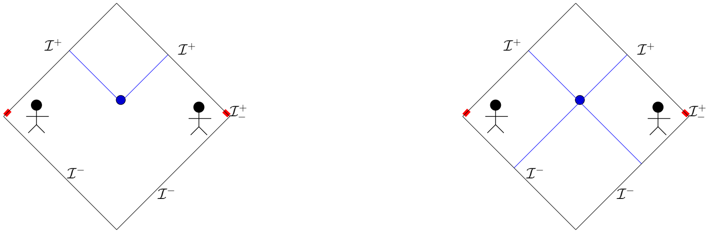
$$|f_1\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O_1(x,\Omega)}|0\rangle.$$

$$|f_2\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O_2(x,\Omega)}|0\rangle.$$

Then (up to $\mathrm{O}(\lambda)$)

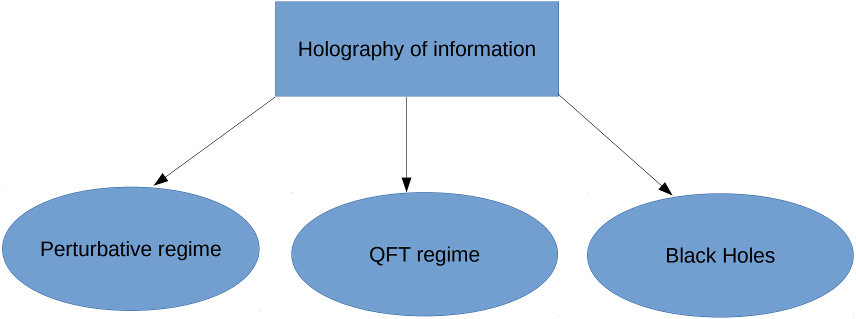
$\langle f_1 \| M(-\infty) O_1(u,\Omega') \| f_1\rangle \neq 0$	$\langle f_1 \| M(-\infty) O_2(u,\Omega') \| f_1\rangle = 0$
$\langle f_2 \| M(-\infty) O_1(u,\Omega') \| f_2\rangle = 0$	$\langle f_2 \| M(-\infty) O_2(u,\Omega') \| f_2\rangle \neq 0.$

Locality

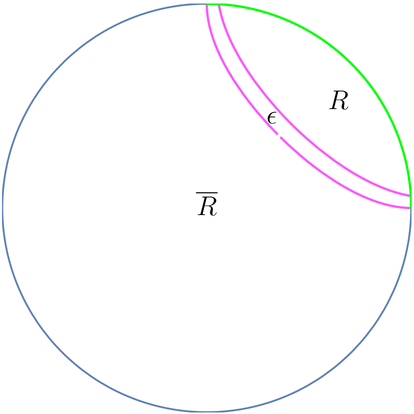


- Phenomenon visible in linearized theory; no complicated nonlocal dynamics.
- In gravity, after sending the excitation, it is impossible to erase information from infinity. (In nongravitational theories, this is possible).

Regimes

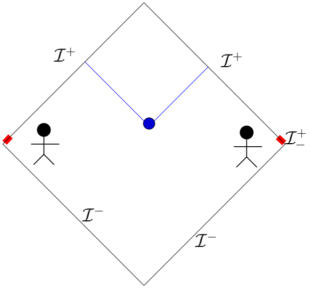

Holography of information
Perturbative regime
QFT regime
Black Holes

Two regions extending to infinity



When R and $\overline{R}$ both extend to infinity, the split property is obvious.

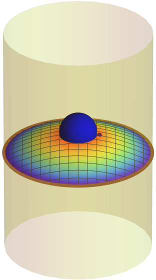
Classical limit



$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = \frac{\hbar G}{c^3}\lambda \int \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx + \mathrm{O}\left(\lambda^2\right).$$

Protocol fails when $G \to 0$ or $\hbar \to 0$.

Ordinary systems



Distinguishing similar states from infinity requires precision $O\left(e^{-S}\right)$ where S controls the density of states.

[Bahiru, Belin, Papadodimas, Sarosi, Vardian, 2022]

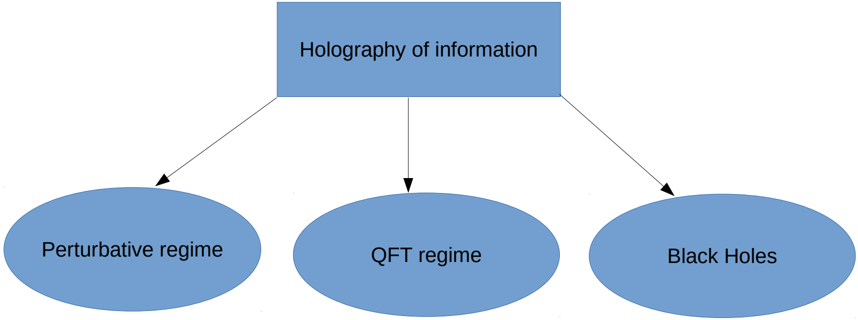
By coarse-graining observations, and introducing a classical "observer" we recover ordinary physics to good approximation.

[Jensen, Sorce, Speranza, 2023]

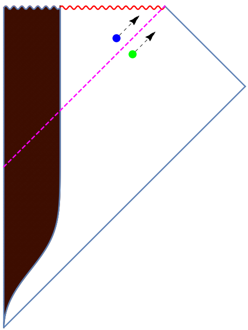
[Chandrasekaran, Longo, Penington, Witten, 2022]

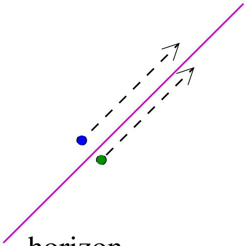
[Jensen, Speranza, S.R., in progress]

Regimes


Holography of information
Perturbative regime
QFT regime
Black Holes

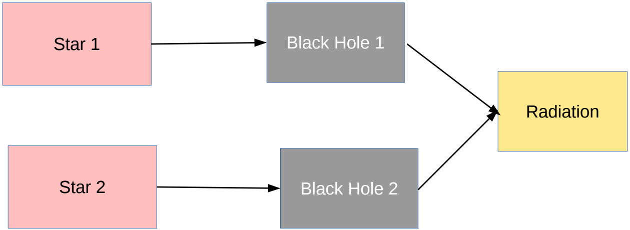
Pair creation





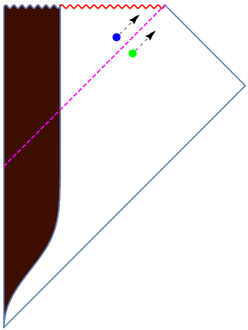
Hawking found that spontaneous pair creation leads to radiation that is seemingly uncorrelated with details of the initial state.

Information paradox



Hawking argued that since radiation is uncorrelated with the initial state, the time-evolution arrows cannot be reversed. This contradicts unitarity.

Significance of exponentially suppressed terms



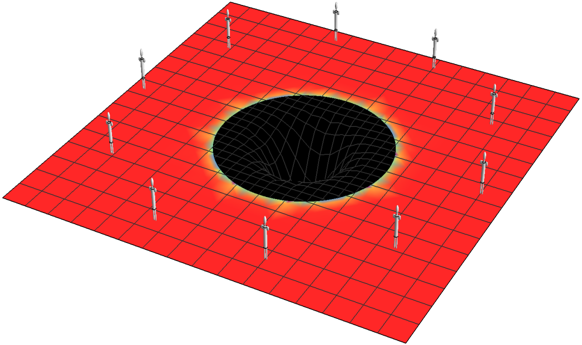
Typical pure states are exponentially close to mixed states

$$\langle\Psi|A|\Psi\rangle = \mathrm{Tr}(\rho_{\mathrm{micro}}A) + O\left(e^{-\frac{S}{2}}\right)$$

[Lloyd, 1988]

So to address the information paradox, we must allow discussion of exponentially precise observations at infinity.

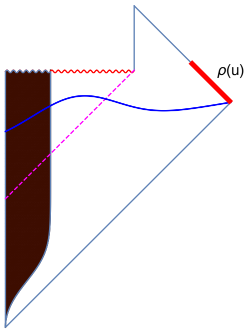
Black-hole information

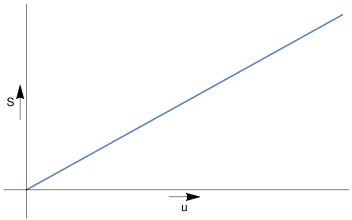


Holography of information implies that information about the black-hole microstate is always available outside with the right measurements.

[Related perspectives: Balasubramanian, Marolf, Rozali, 2006, Jacobson 2012, Jacobson, Nguyen 2019]

Entropy of Hawking radiation



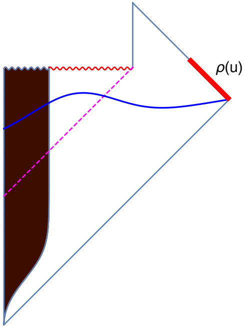


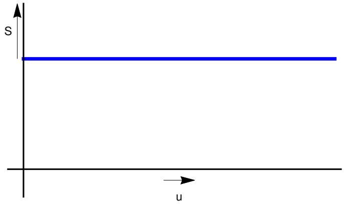
Hawking's arguments imply that $\frac{\partial S(u)}{\partial u} > 0$. This is a paradox since unitarity implies

$$S(\mathcal{I}^-) = S(\mathcal{I}^+)$$

and if $S(\mathcal{I}^-) = 0$ then $S(\mathcal{I}^+) = 0$.

von Neumann entropy on $\mathcal{I}^+$

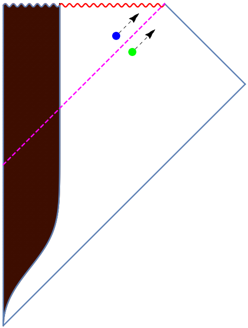




We can prove that the von Neumann entropy of a segment $(-\infty, u)$ of $\mathcal{I}^+$ is independent of u.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

Error in Hawking's argument?



Low point correlators

$$\langle a_\omega a_\omega^\dagger \rangle \approx \frac{1}{1 - e^{-\beta\omega}}$$

are insufficient to argue for information loss.

Where is the error in Hawking's argument?

One therefore has to introduce a hidden surface around each of these holes and apply the principle of ignorance to say that all field configurations on these hidden surfaces are equally probable provided they are compatible with the conservation of mass, angular momentum, etc. which can be measured by surface integrals at a distance from the hole.

Let H_1 be the Hilbert space of all possible data on the initial surface, H_2 be the Hilbert space of all possible data on the hidden surface, and H_3 be the Hilbert space of all possible data on the final surface. The basic assumption of quantum theory is that there is some tensor S_{ABC} whose three indices refer to H_3, H_2, and H_1, respectively, such that if

$$\xi_C \in H_1, \quad \zeta_B \in H_2, \quad \chi_A \in H_3,$$

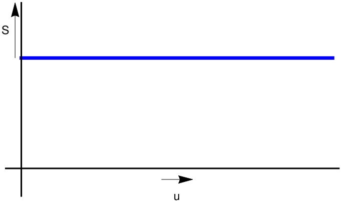
then

$$\sum\sum\sum S_{ABC}\chi_A\zeta_B\xi_C$$

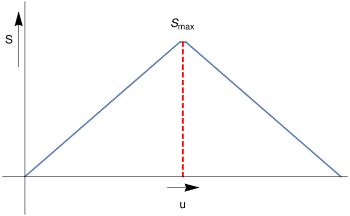
is the amplitude to have the initial state ξ_C, the final state χ_A, and the state ζ_B on the hidden surface. Given only the initial state ξ_C one cannot determine the final state but only the element $\sum S_{ABC}\xi_C$ of the tensor product $H_2 \otimes H_3$. Because one is ignorant of the state on the hidden surface one cannot find the amplitude for measurements on the final surface to give the answer χ_A but one can calculate the probability for this outcome to be $\sum\sum\sum\rho_{CD}\bar{\chi}_C\chi_D$, where

- Hawking assumed that the Hilbert space factorizes up to a few global constraints imposed by the no-hair theorem.
- Our previous discussion shows that this fails in quantum gravity.

Comparison with Page curve

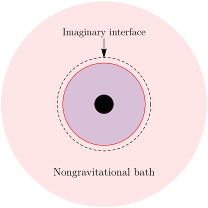


$\neq$



The idea that the von Neumann entropy of the region outside follows the Page curve also assumes factorization. (Other entropies do follow a Page curve (more below).)

Relationship to Page-curve computations



Recent computations of the Page curve do not contradict our results.

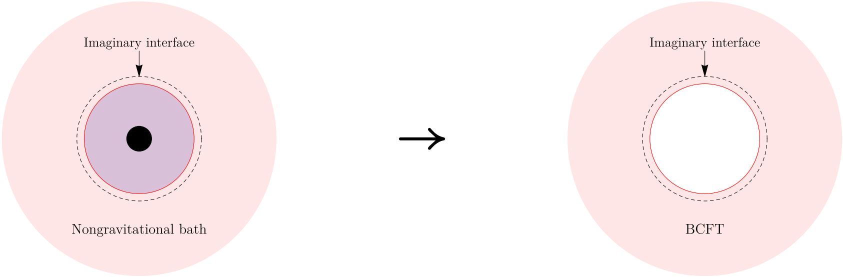
[Pennington, Almheiri, Engelhardt, Marolf, Mahajan,,]

[Maxfield, Maldacena, Hartman, Shaghoulian,]

[Tajdini, Stanford, Shenker, Yang..., 2019]

Gravity not dynamical throughout space. Models necessarily involve a nongravitational bath.

Nature of information transfer



Page curve computes information transfer between nongravitating subregions. (Phrase: “Page curve of Hawking radiation” is misleading.)

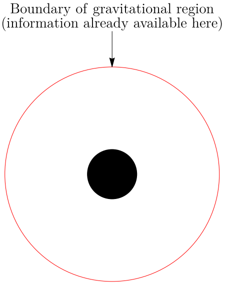
[Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi, 2020]

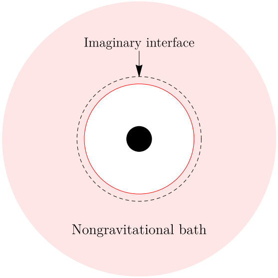
Nature of information transfer

Questions have been raised about how information “emerges” from the black holes.

[Martinec, 2022]

[Mathur, 2020–23]

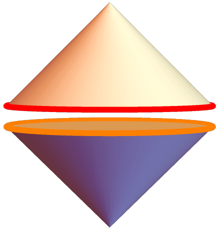




Holography of information $\Longrightarrow$ information is present near the boundary; merely transferred to the nongravitational region.

Summary

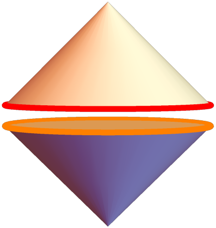
Standard theories of gravity store quantum information very differently from local quantum field theories.



- This is a nonperturbative result that relies on weak assumptions about the UV theory.
- But for simple states, it can be verified in perturbation theory.
- Sheds light on why gravitational theories are holographic.

Summary

Accounting for this unusual localization of information resolves several paradoxes about black holes.



- Forgetting about this property of gravity, and assuming factorization of the Hilbert space often leads to paradoxes!
- Recent computations of the Page curve involve a nongravitational bath where the Hilbert space does factorize.
- Holography of information helps to understand how information enters the bath.