量子多体系の非平衡ダイナミクス

―熱化の時間スケールをめぐって―

森貴司　慶應理工

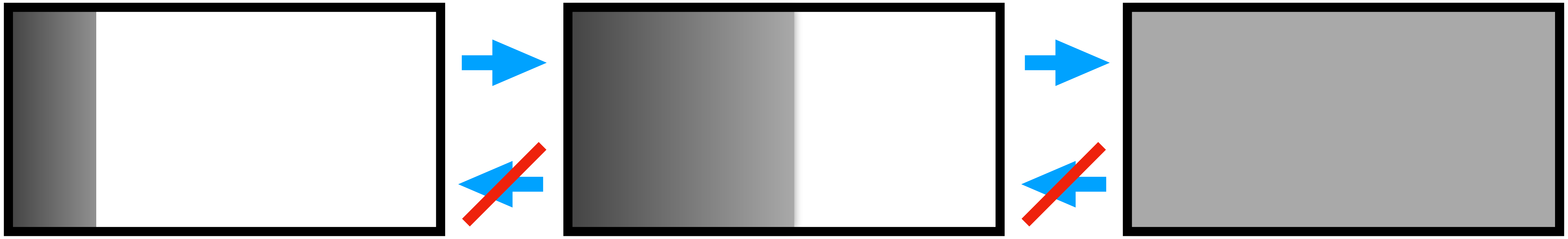
日本物理学会第79回年次大会

1. What is quantum thermalization?

microscopic reversibility and macroscopic irreversibility

microscopic mechanics are symmetric with respect to time

macroscopic processes have a preferred direction



thermalization: relaxation towards equilibrium in an isolated system

Boltzmann’s idea

ergodicity

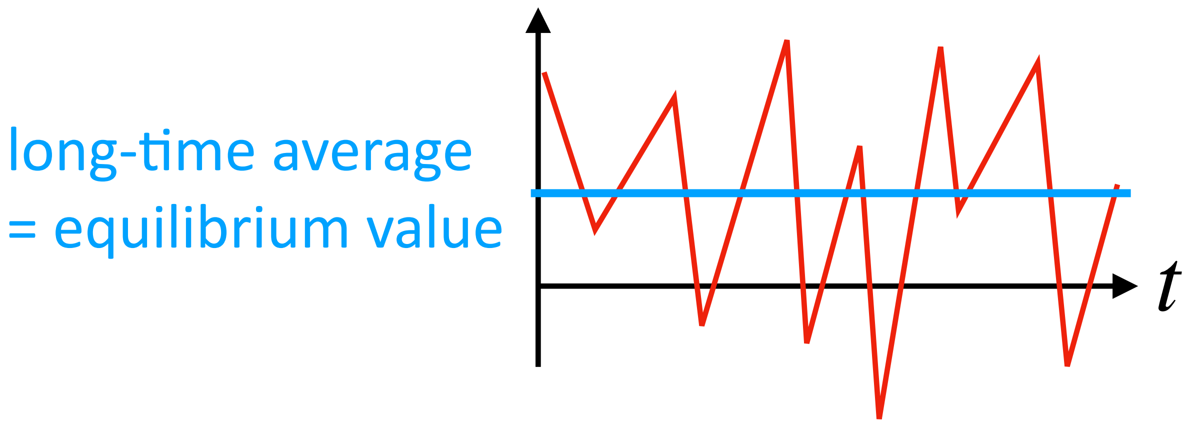
long-time average of a physical quantity coincides with its microcanonical average (equilibrium value)

typicality

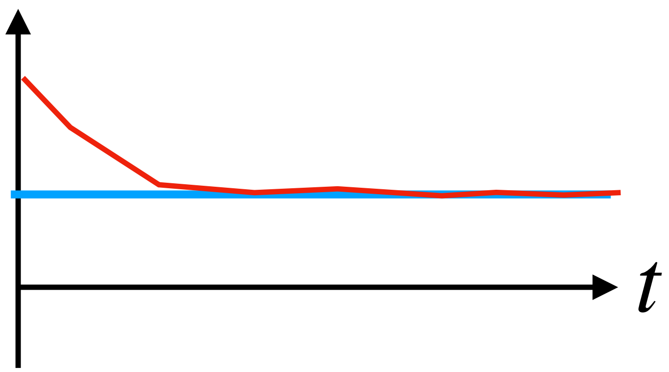
macroscopic quantities take on their equilibrium values for a vast majority of microscopic states

e.g. typicality of Maxwell distribution was proved by Jeans

ergodicity only



ergodicity and typicality



quantum systems

microscopic pure state: vector in the Hilbert space $|\psi(t)\rangle$

time evolution: Schrödinger equation $i\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

expectation value of a physical quantity $\langle\hat{O}\rangle_{\psi(t)} := \langle\psi(t)|\hat{O}|\psi(t)\rangle$

energy eigenstates and eigenvalues $\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle$

$$|\psi(t)\rangle = \sum_n C_n e^{-iE_n t}|\phi_n\rangle$$

quantum typicality

typicality: a vast majority of pure states corresponds to thermal equilibrium

S. Popescu, A. Short, A. Winter, Nat. Phys. (2006)

theorem: Consider an arbitrary bounded operator $\hat{O}$. For a large quantum system, most microscopic states in the energy shell share the same expectation value (that is nothing but the equilibrium value)

$$\langle \hat{O} \rangle_{\psi} \approx \langle \hat{O} \rangle_{\mathrm{eq}} \quad \text{for most } |\psi\rangle$$

$\hat{O}$ is not necessarily a macroscopic quantity

Typicality is almost trivial in quantum mechanics

ergodicity: Eigenstate Thermalization Hypothesis

ergodicity: long-time average of a physical quantity equals its equilibrium value

$$\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau \langle\hat{O}\rangle_{\psi(t)}dt \approx \langle\hat{O}\rangle_{\mathrm{eq}}$$ for any initial state

$$\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau \langle\hat{O}\rangle_{\psi(t)}dt = \sum_n |C_n|^2\langle\hat{O}\rangle_{\phi_n}$$ (assume that there is no energy degeneracy)

⇒ $\langle\hat{O}\rangle_{\phi_n} \approx \langle\hat{O}\rangle_{\mathrm{eq}}$ for *all* energy eigenstates

every energy eigenstate corresponds to thermal equilibrium

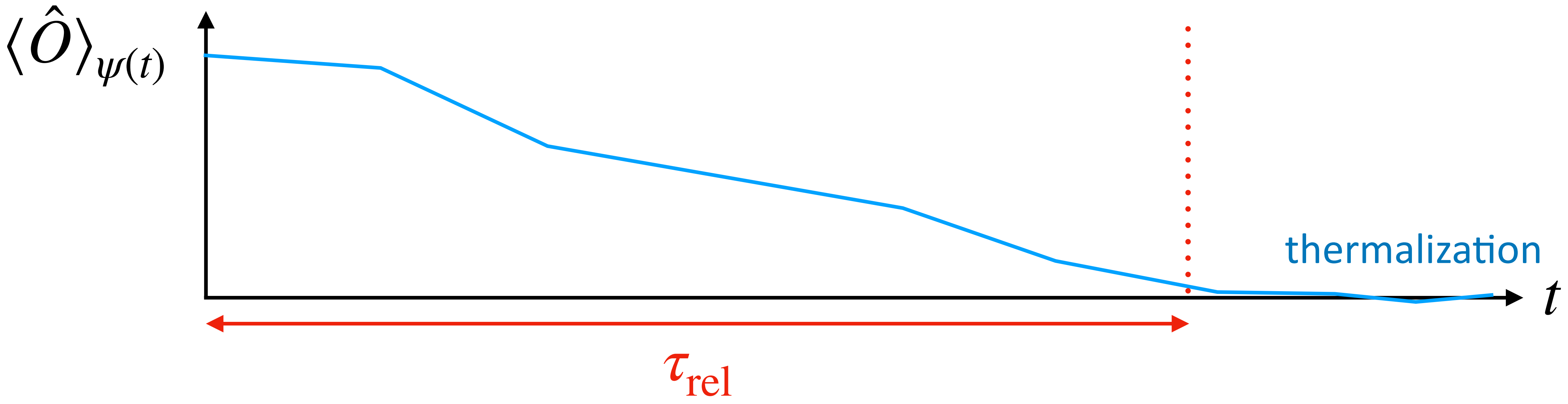
Eigenstate Thermalization Hypothesis (ETH)

J. von Neumann (1929); M. Srednicki (1994); M. Rigol, V. Dunjko, and M. Olshanii (2008)

2. timescale of thermalization

remaining issue: timescale of thermalization

ETH (and typicality) implies that the system thermalizes *after a sufficiently long time evolution*



ETH does not tell us about the *timescale of thermalization*

importance of the choice of observables

S. Goldstein, T. Hara, and H. Tasaki, Phys. Rev. Lett. (2013)

Any realistic quantum many-body system has an observable whose relaxation time behaves as $\tau_{\mathrm{rel}} = e^{O(N)}$ with N being the number of degrees of freedom (the number of particles, volume,…)

$$\hat{O} = \sum_n |\phi_{n+1}\rangle\langle\phi_n|$$

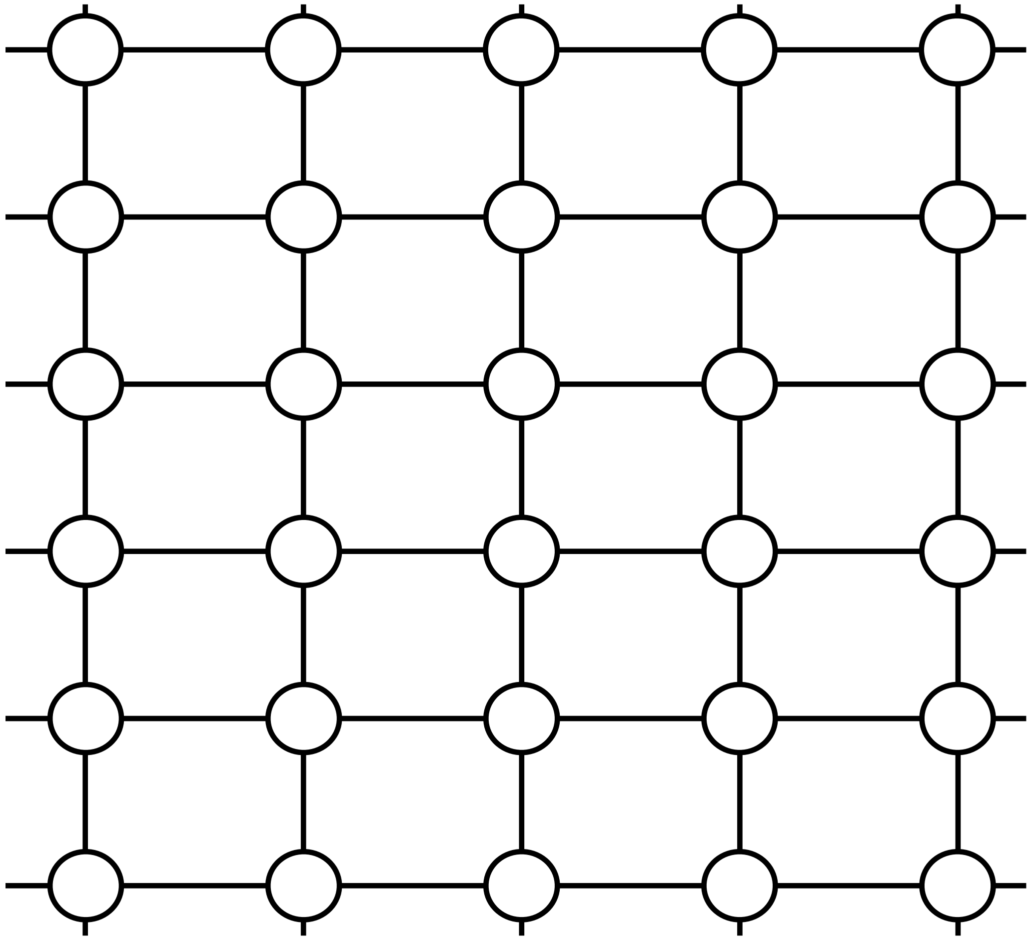
$$\langle\hat{O}\rangle_{\psi(t)} = \sum_n C^*_{n+1} C_n e^{i\underline{(E_{n+1}-E_n)}t}$$

energy level spacing $\sim e^{-O(N)}$

we should focus on a specific class of observables!

local or few-body observables

local operators: operators acting to spatially local region



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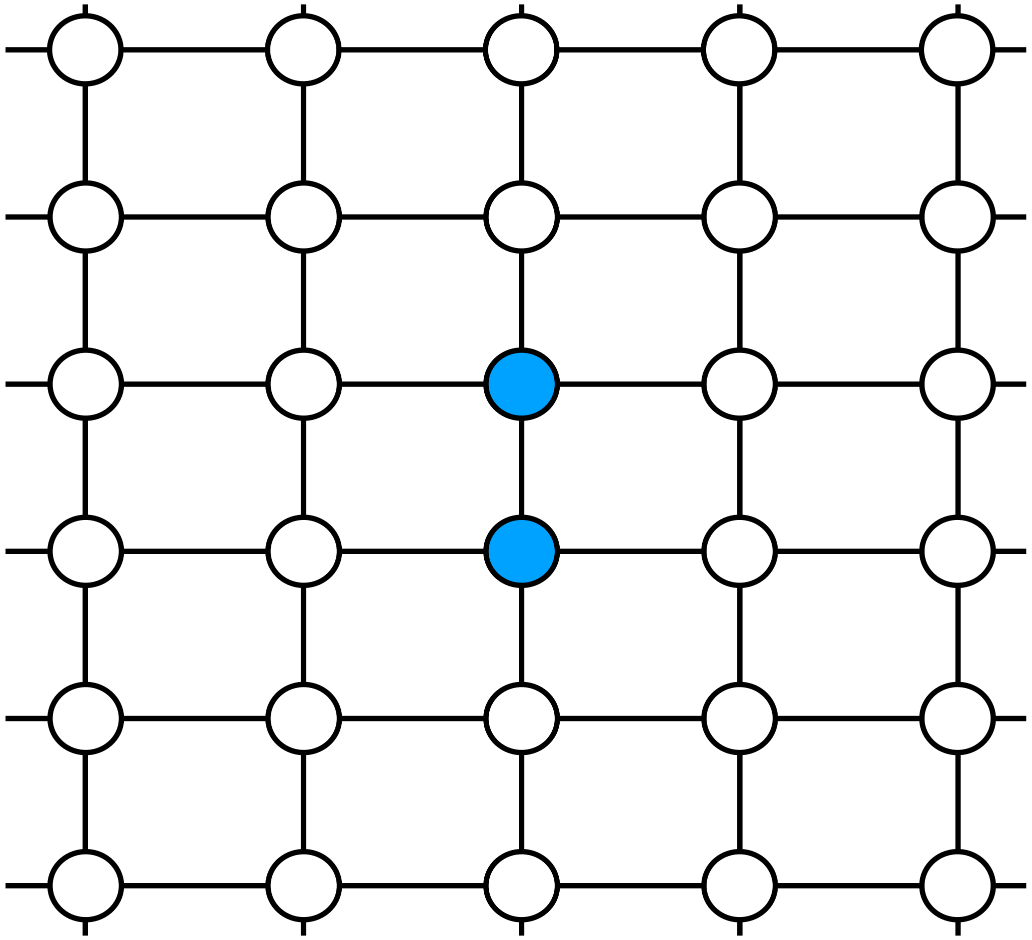

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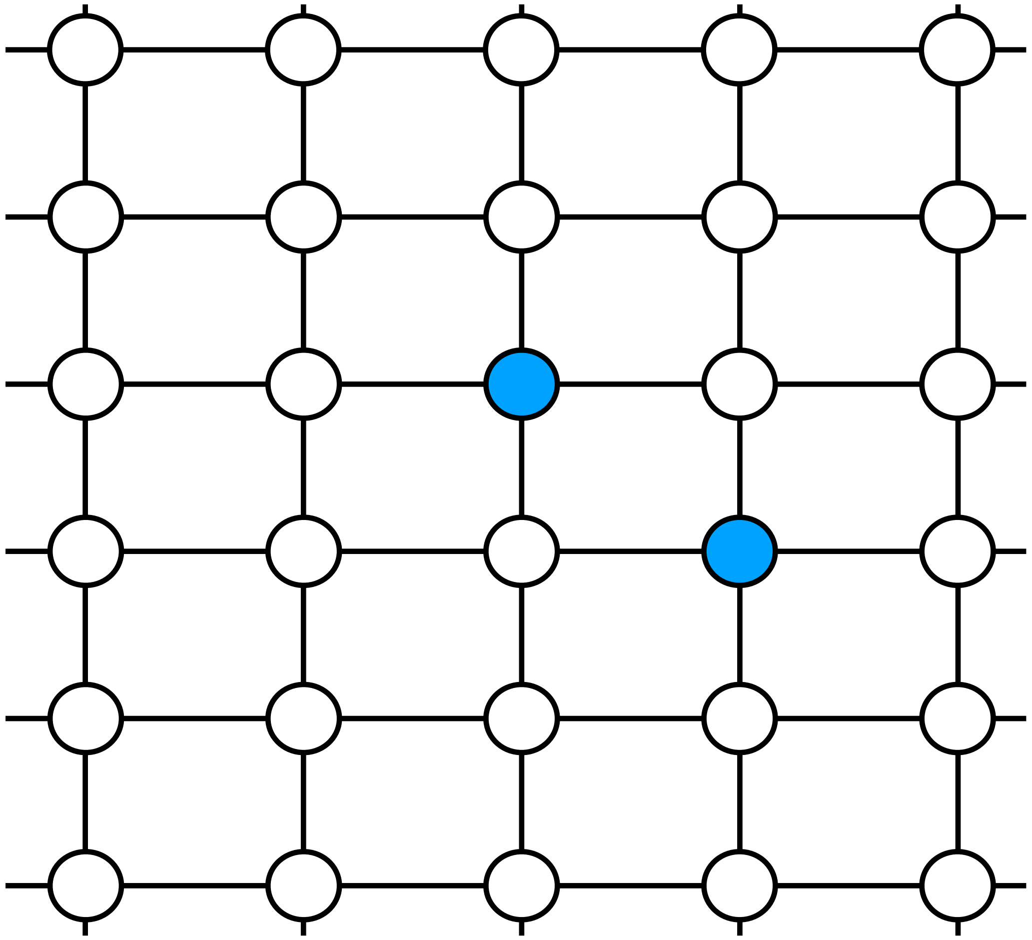

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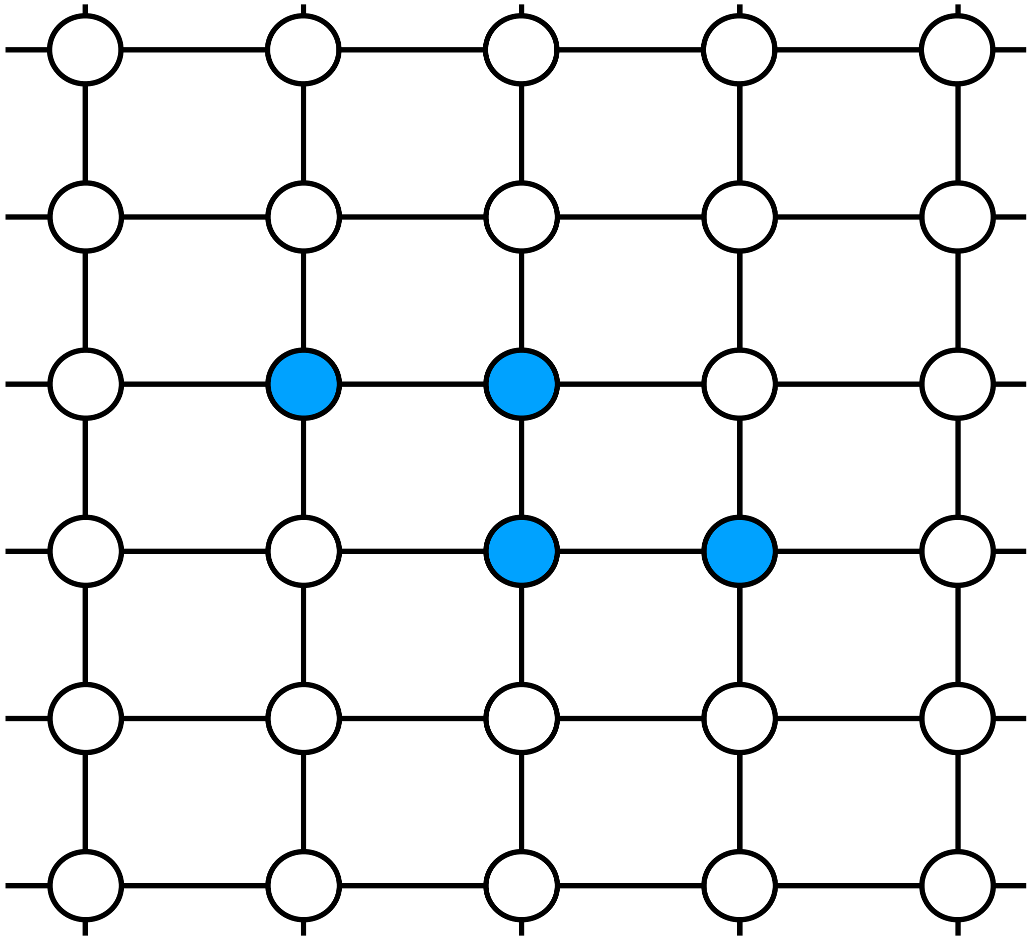
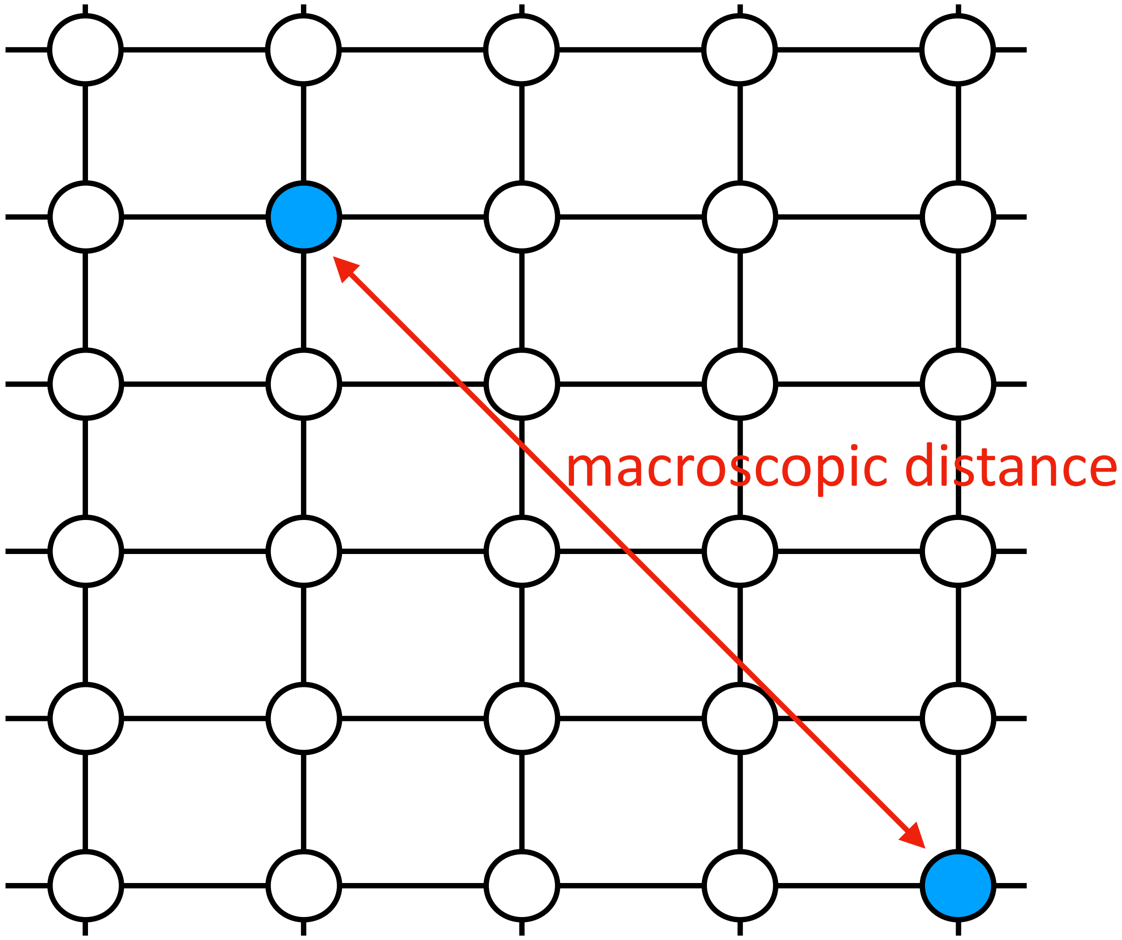


local or few-body observables

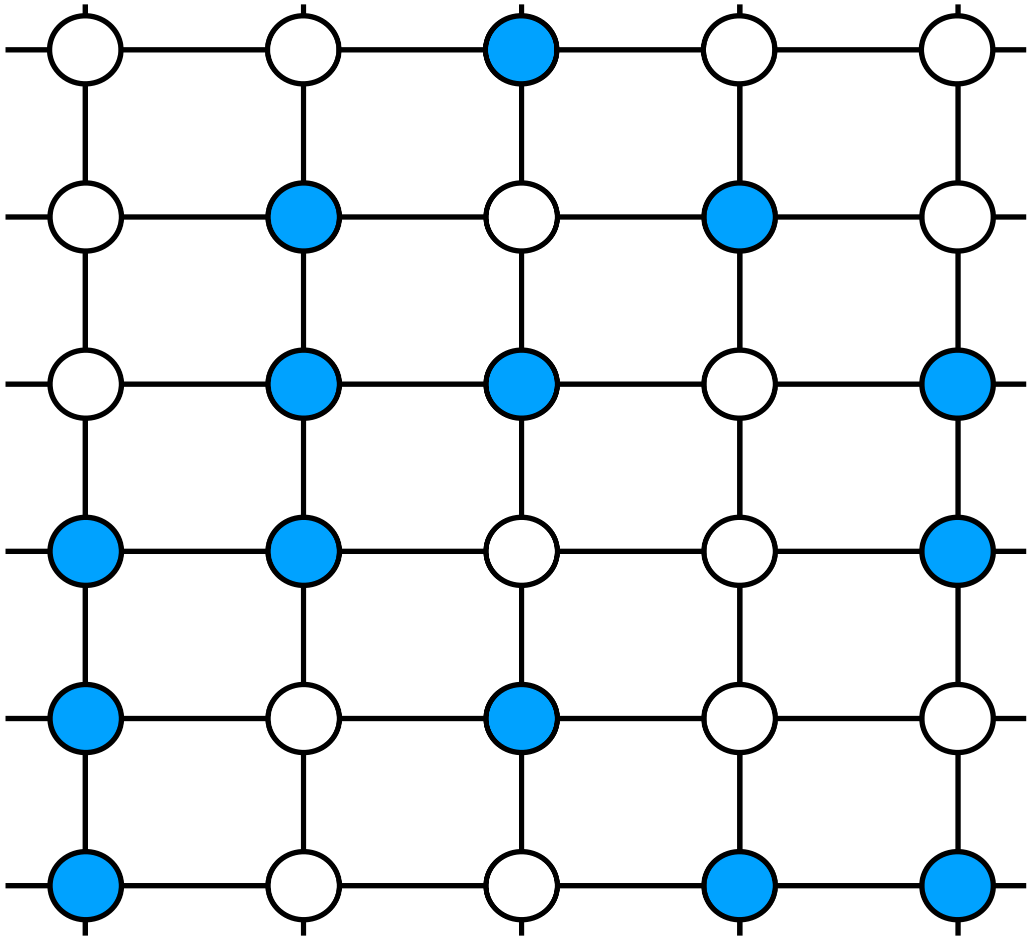
few-body operators: operators acting on a finite number of sites



(local operators) ⊂ (few-body operators)

local or few-body observables

many-body operators: operators acting on a macroscopically large number of sites



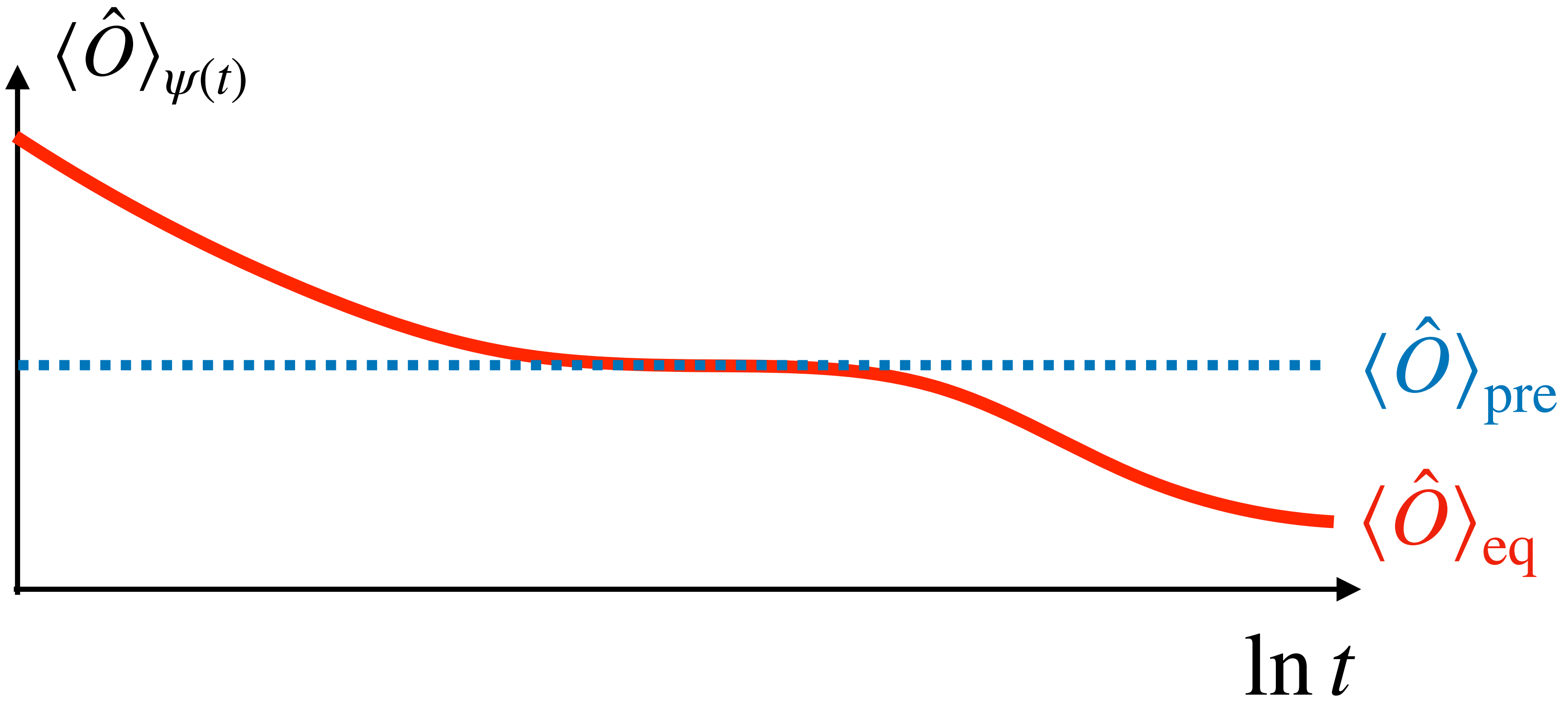
We do not consider this class of observables

In the following, we consider thermalization of local or few-body observables

3. slow relaxation

prethermalization

Let's focus on simple situations: **large separation of timescales**



two-step relaxation

initial relaxation (prethermalization): fast degrees of freedom

second relaxation (thermalization): slow degrees of freedom ⬅

review: TM, T. N. Ikeda, E. Kaminishi, and M. Ueda, J. Phys. B (2018)

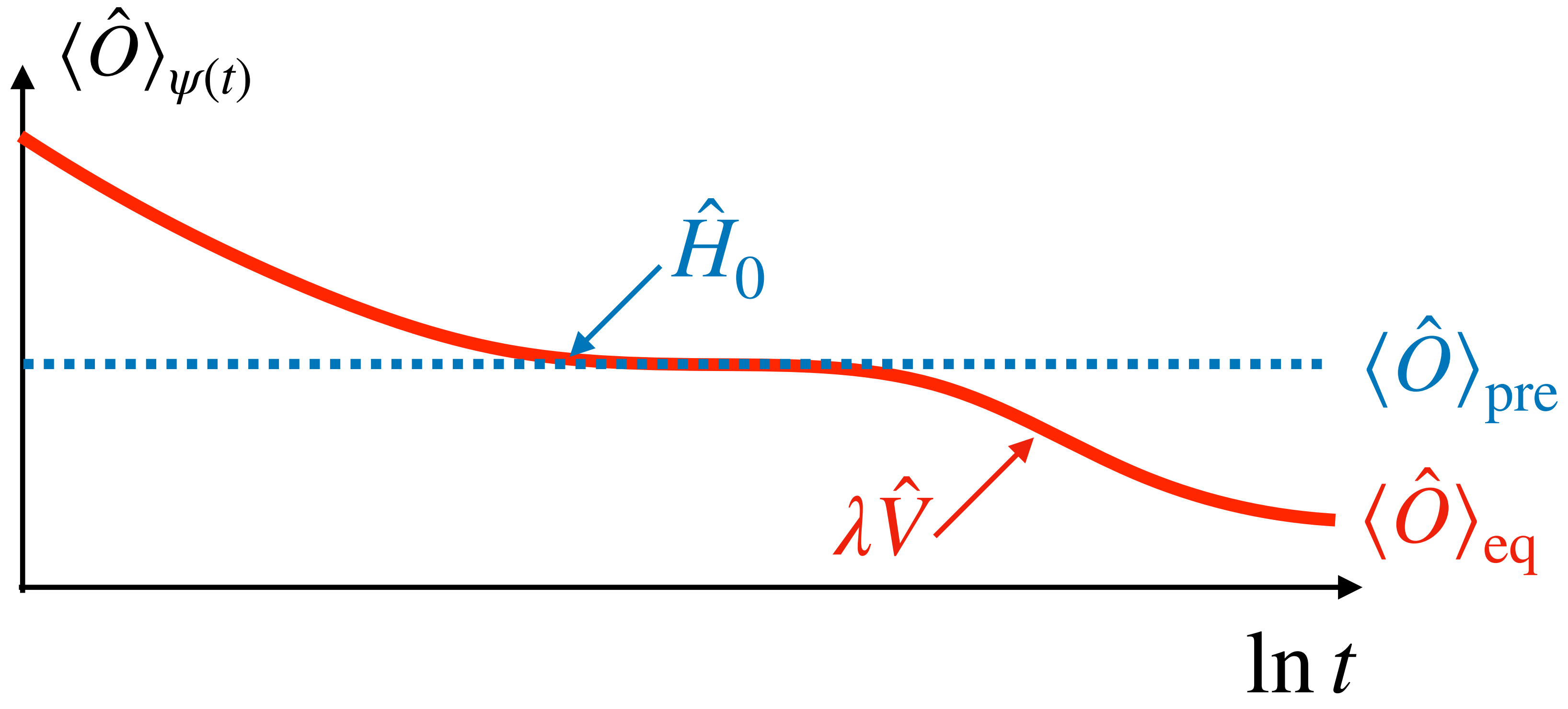
Fermi's golden rule

example: nearly integrable systems

$$\hat{H} = \hat{H}_0 + \lambda \hat{V}$$

integrable Hamiltonian

small perturbation



perturbative treatment: **Fermi's golden rule**

transition rate between two energy eigenstates of $\hat{H}_0 \propto \lambda^2$

relaxation time $\propto \lambda^{-2}$

anomalously slow thermalization

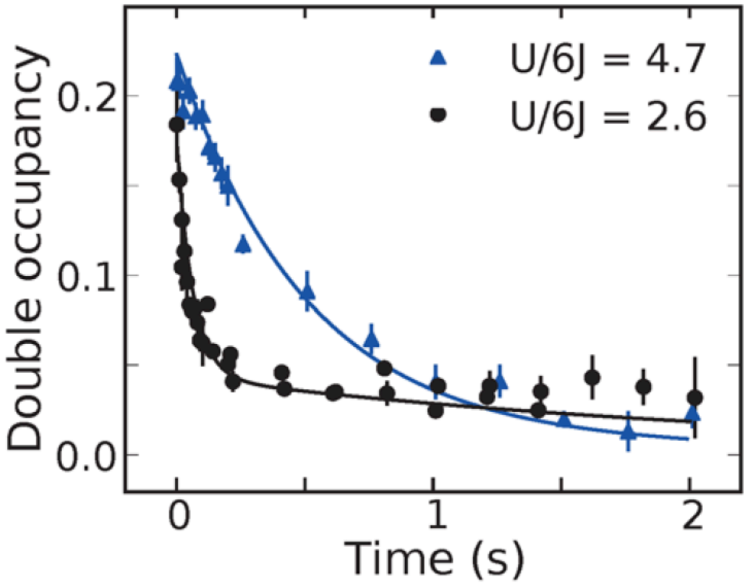
In some cases, the relaxation time can be much longer

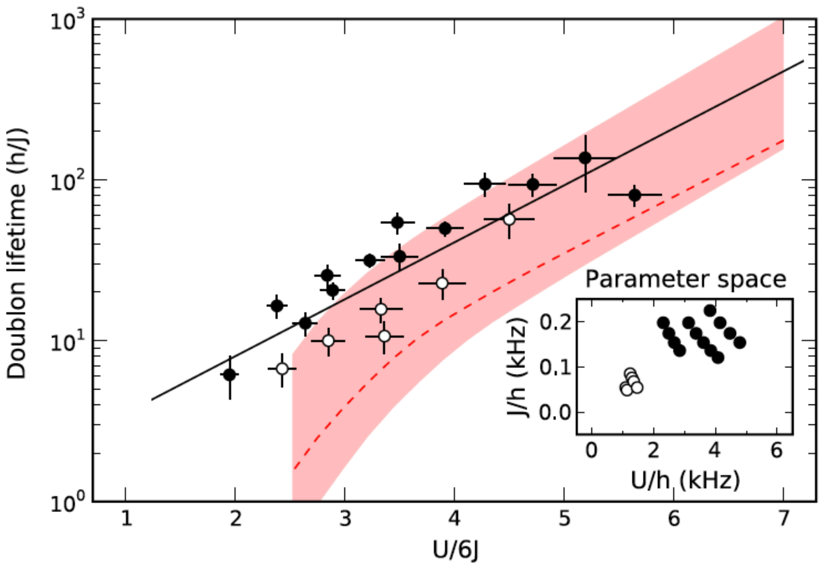
example: strong-coupling Hubbard model $U \gg J$ $\lambda \sim \dfrac{J}{U}$

$$\hat{H} = \underbrace{J\sum_{i=1}^{L}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.})}_{\text{small perturbation}} + U\sum_{i=1}^{L}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

exponentially slow relaxation $\tau \sim e^{O(1/\lambda)} = e^{O(U/J)}$

実験 N. Strohmaier et al. (2010)





rigorous theorem on Floquet systems

similar slow relaxations are found in periodically driven (Floquet) many-body systems *at high frequencies*

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t) \qquad \hat{V}(t) = \hat{V}(t+T) \qquad \omega = \frac{2\pi}{T}$$

$\omega \gg g$ (local energy scale of $\hat{H}_0$)

small parameter: $\lambda = g/\omega$

physically, the system absorbs energy from driving fields (heating) and eventually heats up to the infinite temperature

Theorem: *heating is exponentially slow with respect to* ω/g

$$\tau_{\mathrm{rel}} \sim e^{O(1/\lambda)} = e^{O(\omega/g)}$$

T. Kuwahara, TM, K. Saito (2016); TM, T. Kuwahara, and K. Saito (2016)
D. Abanin, W. De Roeck, W. W. Ho, F. Huveneers (2017)

strong-coupling Hubbard = Floquet

$$\hat{H} = \underbrace{J\sum_{i=1}^{L}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}_{i,\sigma}^{\dagger}\hat{c}_{i+1,\sigma} + \text{h.c.})}_{\text{small perturbation}} + \boxed{U\sum_{i=1}^{L}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}}$$

eigenvalues are **integer multiple** of U

unitary transformation

$$\mathscr{U}_t := e^{-itU\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}}$$

$$\Big\downarrow$$

$$\hat{H}'(t) = \mathscr{U}_t^{\dagger}\left[J\sum_{i=1}^{L}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}_{i,\sigma}^{\dagger}\hat{c}_{i+1,\sigma} + \text{h.c.})\right]\mathscr{U}_t$$

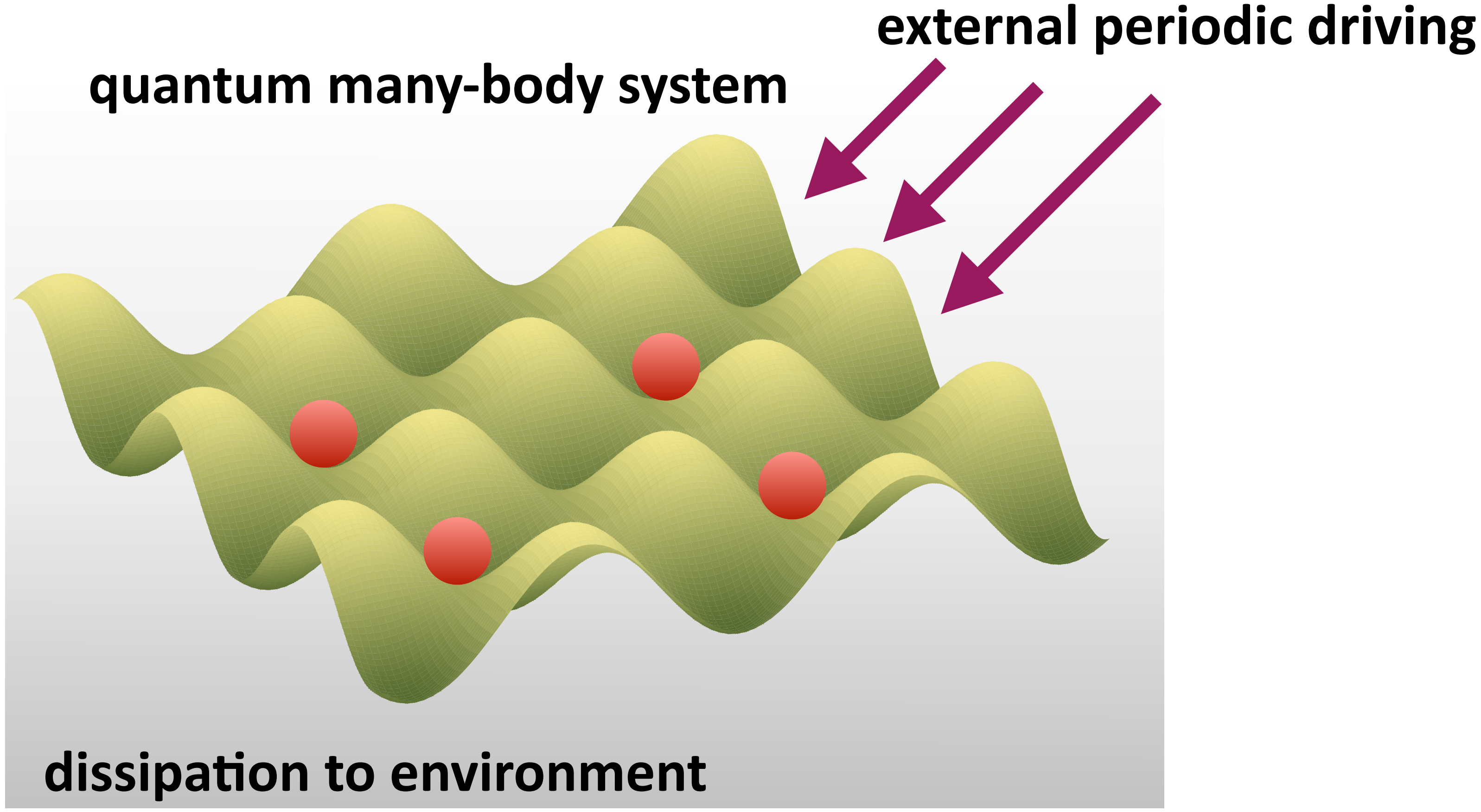
Note that $\mathscr{U}_t = \mathscr{U}_{t+T}$ with $T = 2\pi/U$

$$\hat{H}'(t) = \hat{H}'(t+T) \quad \omega = \frac{2\pi}{T} = U \gg J$$

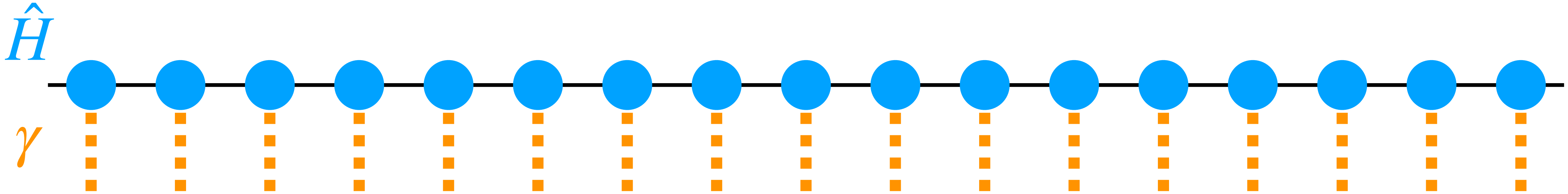
exponentially slow relaxation in the strong-coupling Hubbard model is explained by the theorem on Floquet systems

4. open-system analysis of thermalization timescale in isolated systems

open systems



Lindbladian (Liouvillian) and its eigenvalues



$$\frac{d}{dt}\rho(t) = -i[\hat{H}, \rho(t)] + \gamma \sum_{i=1}^{L} \left(\hat{L}_i \rho(t) \hat{L}_i^\dagger - \frac{1}{2} \left\{ \hat{L}_i^\dagger \hat{L}_i, \rho(t) \right\} \right)$$

$$=: \mathscr{L}\rho(t)$$ Lindbladian

eigenvalues of Lindbladian → Liouvillian gap g

$\|\rho(t) - \rho_{\mathrm{ss}}\|_{\mathrm{tr}} \sim e^{-gt}$ asymptotic decay rate in the long-time limit

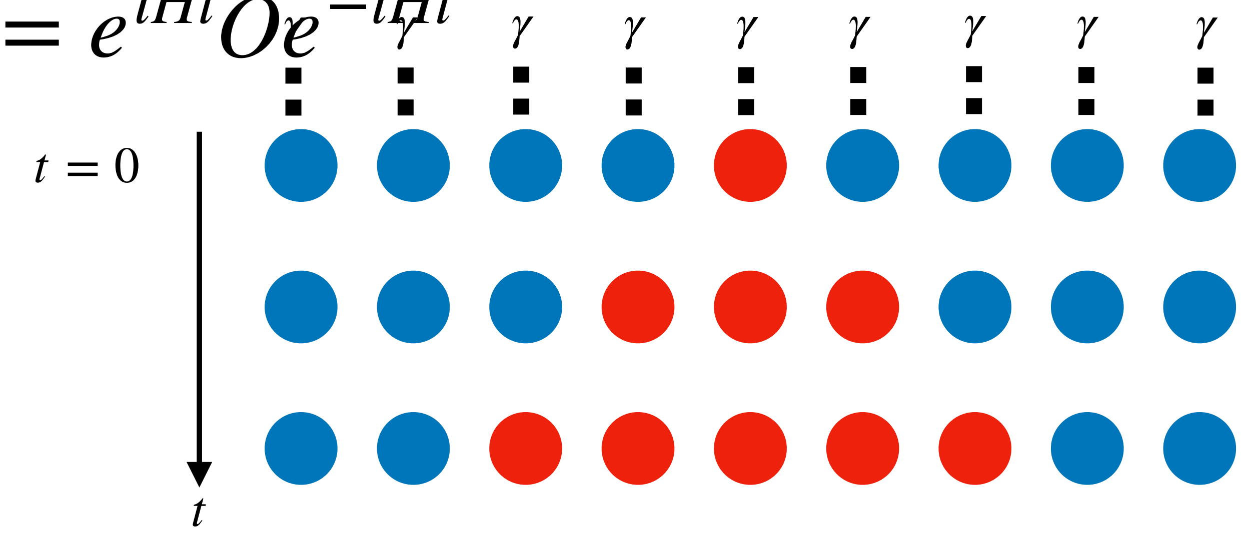
$$\boxed{\lim_{\gamma \to +0} g = 0}$$

operator growth and acceleration of dissipation

general property of *weak* bulk dissipation: **acceleration of dissipation**

mechanism: **operator growth + weak bulk dissipation**

$$\hat{O}(t) = e^{iHt}\hat{O}e^{-iHt}$$



dissipation

$\hat{O}_i \to e^{-\gamma \Delta t}$

$\hat{O}_{i-1}\hat{O}_i\hat{O}_{i+1} \to e^{-3\gamma \Delta t}$

$\hat{O}_{i-2}\hat{O}_{i-1}\hat{O}_i\hat{O}_{i+1}\hat{O}_{i+2} \to e^{-5\gamma \Delta t}$

T. Shirai and TM, Phys. Rev. Lett. (2024)

instantaneous decay rate $\propto \gamma \times$ (average operator size)

$$\sim \gamma v t$$

nontrivial Liouvillian gap in the weak dissipation limit

operator growth → amplification of dissipation

infinitesimally weak dissipation ($\gamma \to +0$) can provide a finite asymptotic decay rate (i.e. a finite Liouvillian gap)

$$\lim_{\gamma \to +0} \lim_{L \to \infty} g =: \bar{g} > 0$$

remark: if we take the limit of $\gamma \to +0$ before the thermodynamic limit, the Liouvillian gap always tends to zero

$$\lim_{L \to \infty} \lim_{\gamma \to +0} g = 0$$

finite Liouvillian gap in the weak dissipation limit

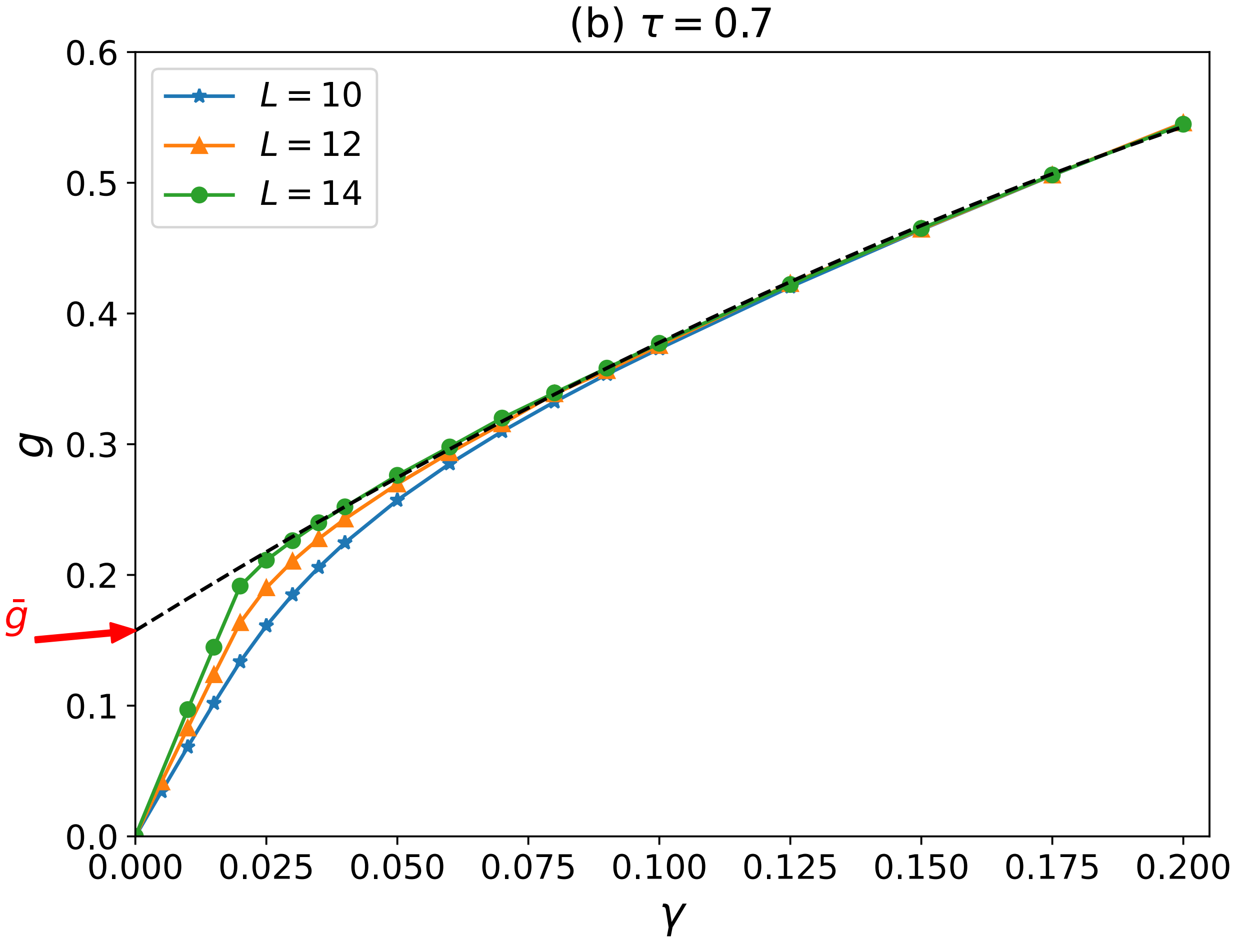
TM, Phys. Rev. B (2024)

kicked Ising chain $(J, h_z, h_x, \tau) = (1, 0.8090, 0.9045, 0.7)$

$$\hat{H}(t) = \sum_{i=1}^{L} \left(-J\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - h_z \hat{\sigma}_i^z \right) + \sum_{n=-\infty}^{\infty} \delta(t - nT) \sum_{i=1}^{L} (-h_x \hat{\sigma}_i^x)$$

+ bulk dephasing of strength γ $\quad \hat{L}_i = \hat{\sigma}_i^z$

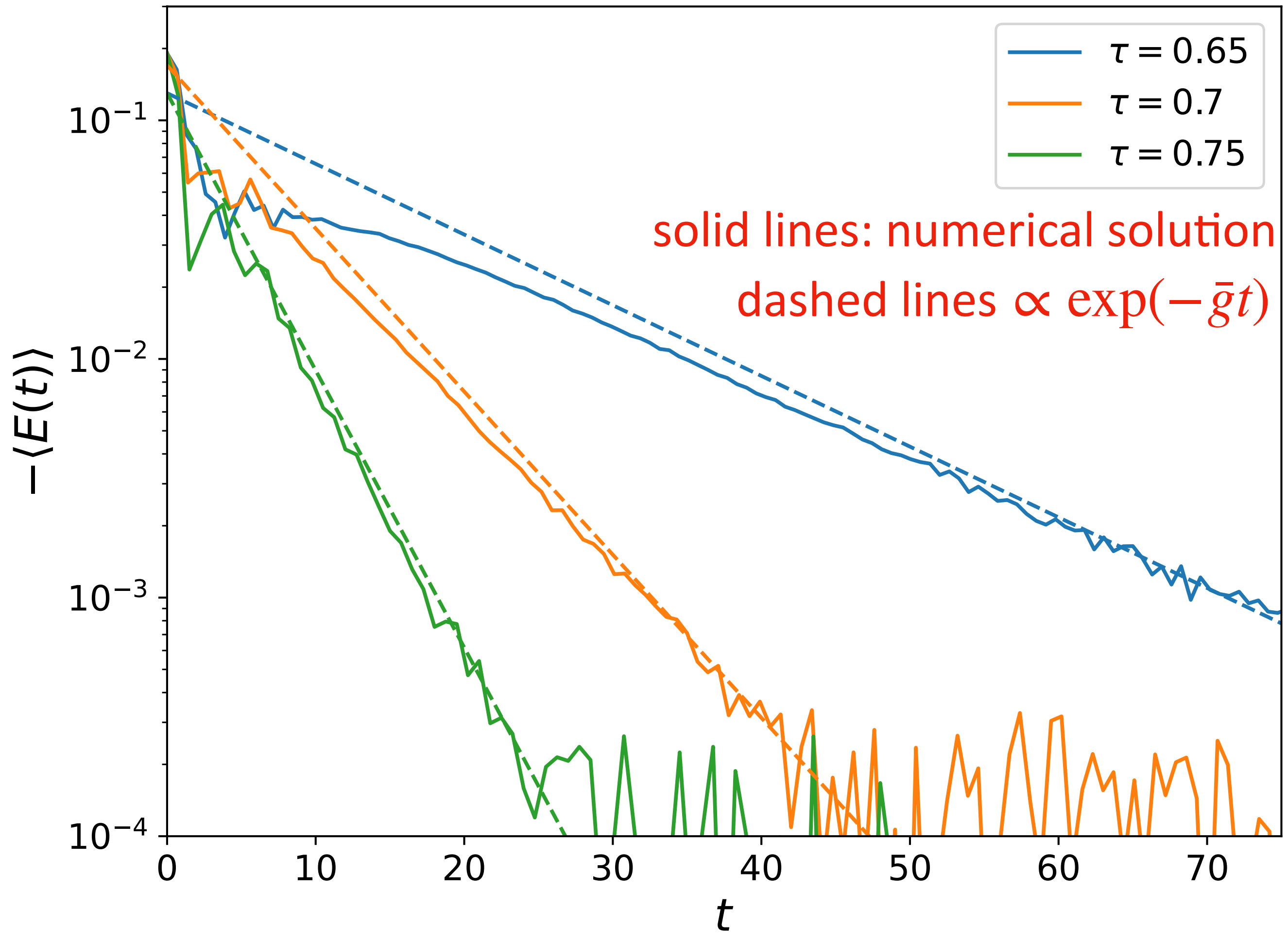
numerics



relation with the timescale of thermalization

TM, Phys. Rev. B (2024)

numerical result for the kicked Ising chain



solid lines: numerical solution of the Schrödinger eq. in the *isolated* system

dashed lines $\propto \exp(-\bar{g}t)$

The non-zero Liouvillian gap in the weak dissipation limit describes the timescale of thermalization!

note: this is an analogue of Ruelle-Pollicott resonance in classical chaos

M. Pollicott (1985); D. Ruelle (1986)

summary

- Typicality and ETH explain quantum thermalization
- Timescale of thermalization is non-trivial
(model dependent, observable dependent,…)
- Slow relaxation after prethermalization: perturbative approach (Fermi's golden rule) sometimes doesn't work
- Anomalously slow relaxation in Floquet systems and some static systems (e.g. Hubbard) is explained by our rigorous theorems
- New approach to thermalization: adding weak dissipation