Tue 9/17/2024
2024 JPS meeting@Hokkaido University
"Dynamics in quantum many-body systems ~ ExU symposium"

強相関物性の量子シミュレーションとスパイラル基底量子トモグラフィ
(Simulating strongly correlated quantum systems via spiral-basis quantum tomography)

Daisuke Yamamoto
CHS, Nihon Univ., Japan

「極限宇宙」
B02:人工量子物質による
量子ブラックホールの解明

さきがけ PRESTO

Today’s talk

☐ Recent trends in “quantum simulation” of many-body systems

☐ Ultracold atom systems for strongly correlated electron physics

☐ Basics of quantum state tomography (QST)

- Reconstruction of density matrix

☐ **Spiral quantum state tomography (spiral-QST)**

- <u>Efficient</u> reconstruction of density matrix

- Demonstration:

  How to measure entanglement entropy via spiral-QST

Quantum computer/quantum simulator

<u>Nature isn't classical</u>, dammit, and **if you want to make a simulation of nature, <u>you'd better make it quantum mechanical</u>**, and by golly it's a wonderful problem, because it doesn't look so easy.

- Richard P. Feynman -

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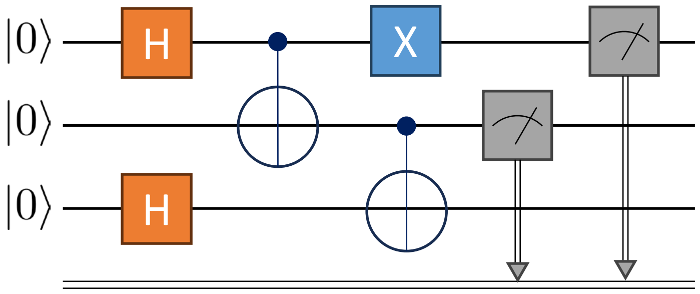
Digital quantum computer

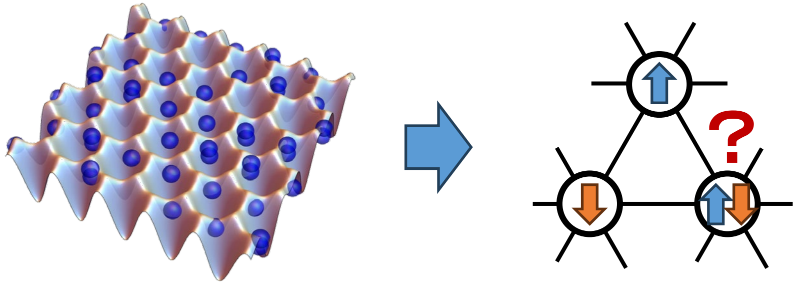
Use quantum gates to perform **general-purpose** quantum computations and require error correction.



Superconducting qubits, trapped ions, photonic qubits, semiconductor quantum dots, neutral atom qubits, …

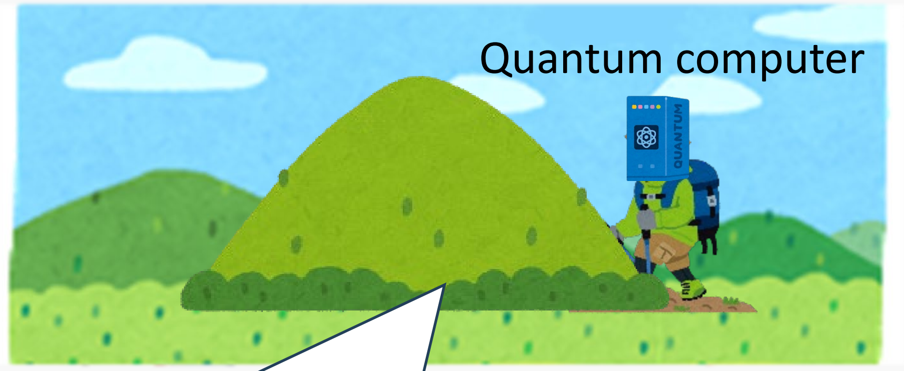
Analog quantum simulator

Mimics the behavior of **a specific quantum system** by using another controllable quantum system.



Cold atoms in optical lattices, trapped ions, Rydberg atom arrays, superconducting circuits, photonic simulators, …

Quantum *simulation* on digital quantum computers



Challenges:

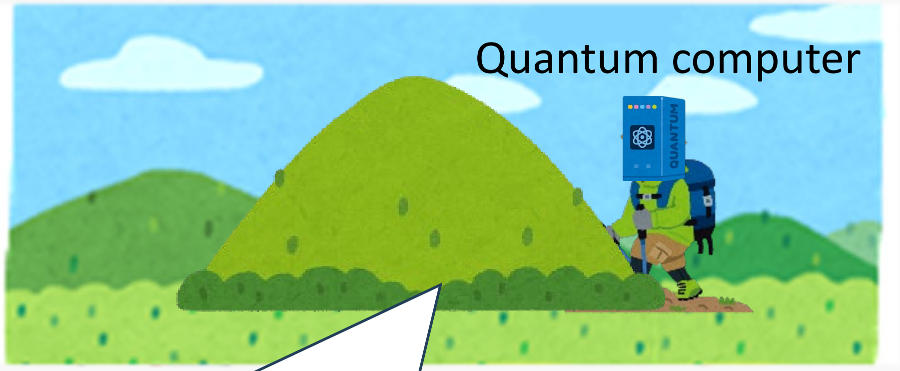
- ☐ Quantum Error Correction
- ☐ Fault-Tolerance (耐障害性)
- ☐ Quantum Decoherence and Noise
- ☐ Scalability
- ☐ High-Fidelity Gates and Operations
- ☐ Algorithm and Software etc.

NISQ era ---------------> **FTQC era**

(Noisy Intermediate-Scale Quantum) (Fault-Tolerant Quantum Computing)

Mid-term targets for applications are highly desired!

Quantum *simulation* on digital quantum computers



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npj | Quantum Information 5, 106 (2019)

ARTICLE OPEN

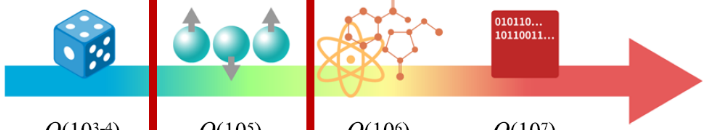
Simulating quantum many-body dynamics on a current digital quantum computer

Adam Smith[1,2*], M. S. Kim[1], Frank Pollmann[2] and Johannes Knolle[1]

npj | quantum information 10, 45 (2024) Article

Hunting for quantum-classical crossover in condensed matter problems

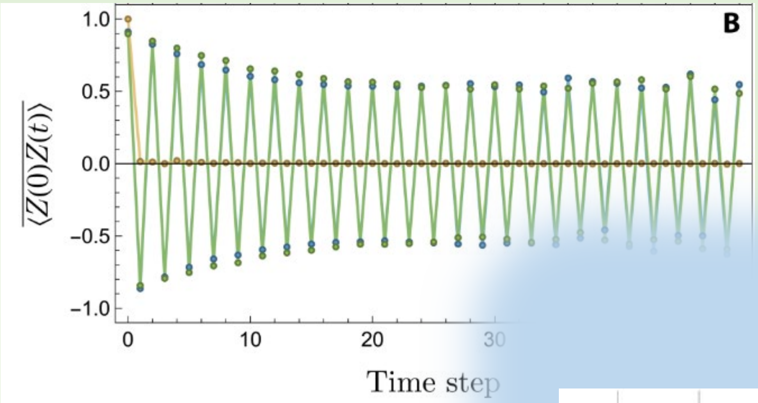
Nobuyuki Yoshioka[1,2,3], Tsuyoshi Okubo[3,4], Yasunari Suzuki[3,5], Yuki Koizumi[5] & Wataru Mizukami[3,6,7]



Problems	Random Circuit	Cond-mat Physics	Quantum Chemistry	Factoring
#Qubits	$O(10^{3\text{-}4})$	$O(10^5)$	$O(10^6)$	$O(10^7)$
Runtime	Hours	Hours	Days	Days

Quantum *simulation* on digital q

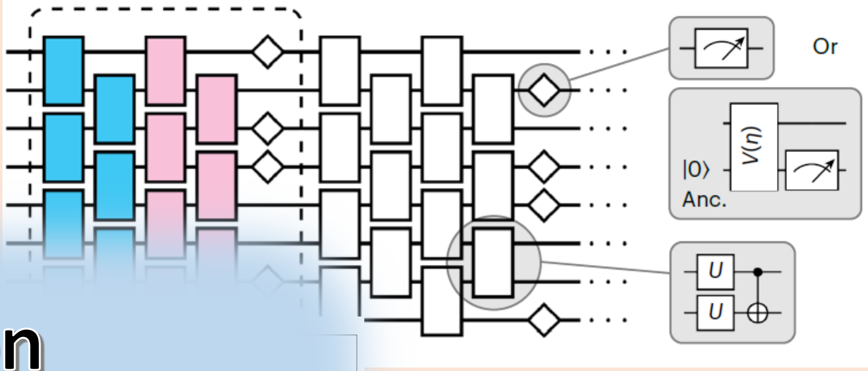
Discrete time crystal



SC qubits（IBM）

Sci. Adv. **8**, 7652 (2022)

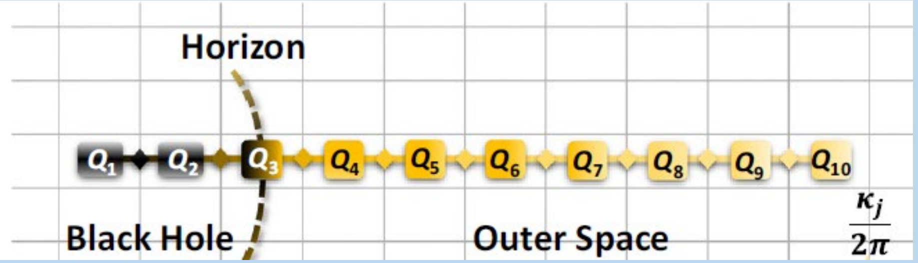
Measurement-induced entanglement transitions

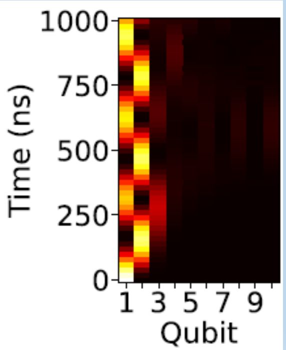


SC qubits（IBM）

Nat. Phys. **19**, 1314 (2023)

Event horizon





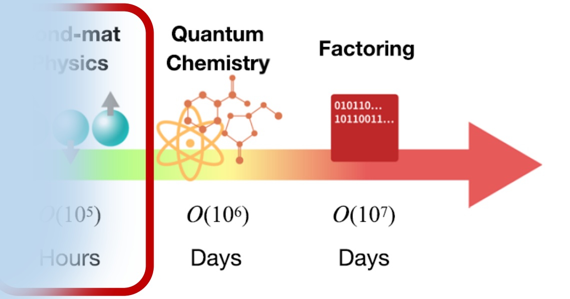
SC qubits

Nat. Commun. **14**, 3263 (2023)

☐ High-Fidelity Gates and O

☐ Algorithm and Software

Okubo, Yasunari Suzuki, Yuki Koizumi &

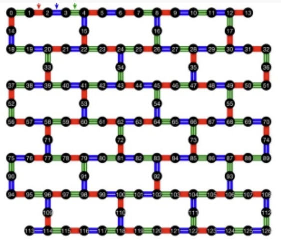


NISQ era ---------------> F

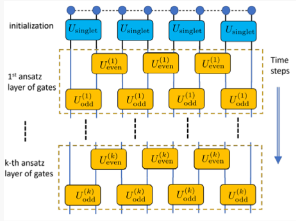
(Noisy Intermediate-Scale Quantum) (Fault-Tolera

Mid-term targets for applications are

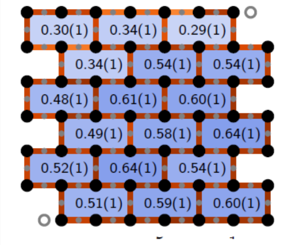
Quantum *simulation* on digital quantum computers



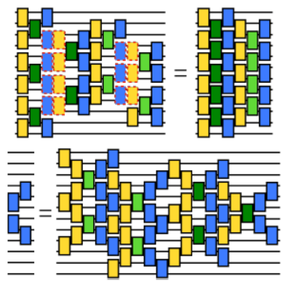
Evidence for the utility of quantum computing before fault tolerance
Nature **618**, 500 (2023)



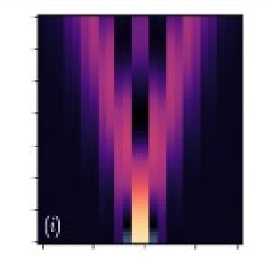
Simulating large-size quantum spin chains on cloud-based superconducting quantum computers
Phys. Rev. Research **5**, 013183 (2023)



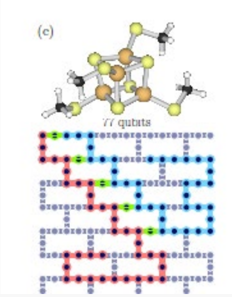
Realizing the Nishimori transition across the error threshold for constant-depth quantum circuits
arXiv:2309.02863



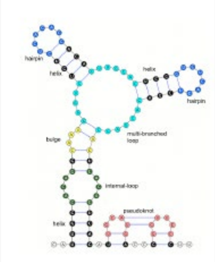
Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits
PRX Quantum **5**, 020315 (2024)



Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits
Phys. Rev. D **109**, 114510 (2024)



Chemistry Beyond Exact Solutions on a Quantum-Centric Supercomputer
arXiv:2405.05068

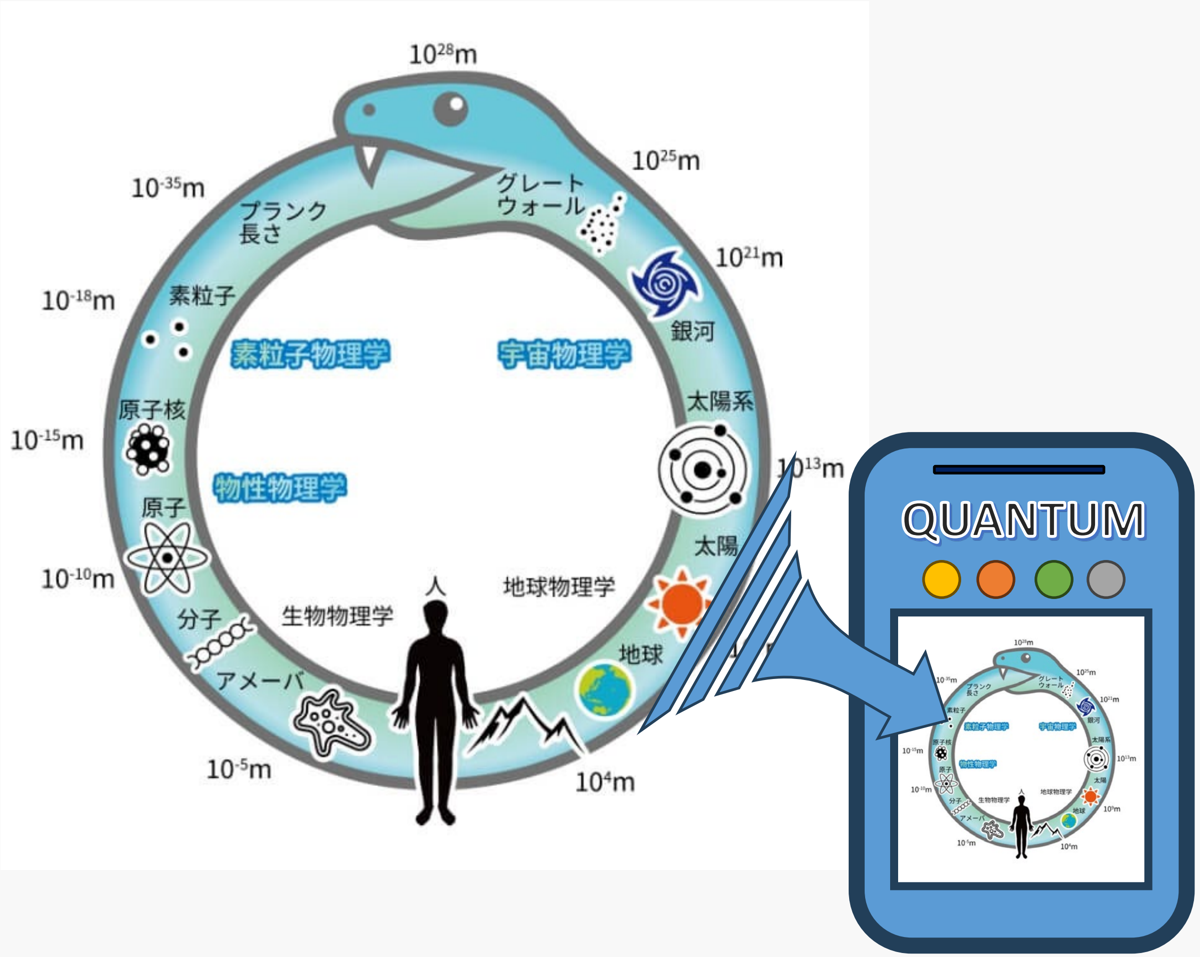


mRNA secondary structure prediction using utility-scale quantum computers
arXiv:2405.20328

and many others...

Why is “quantum simulation” important?

Physics across **different length/time/energy scales** **can be interconnected** through the language of **“quantum information.”**



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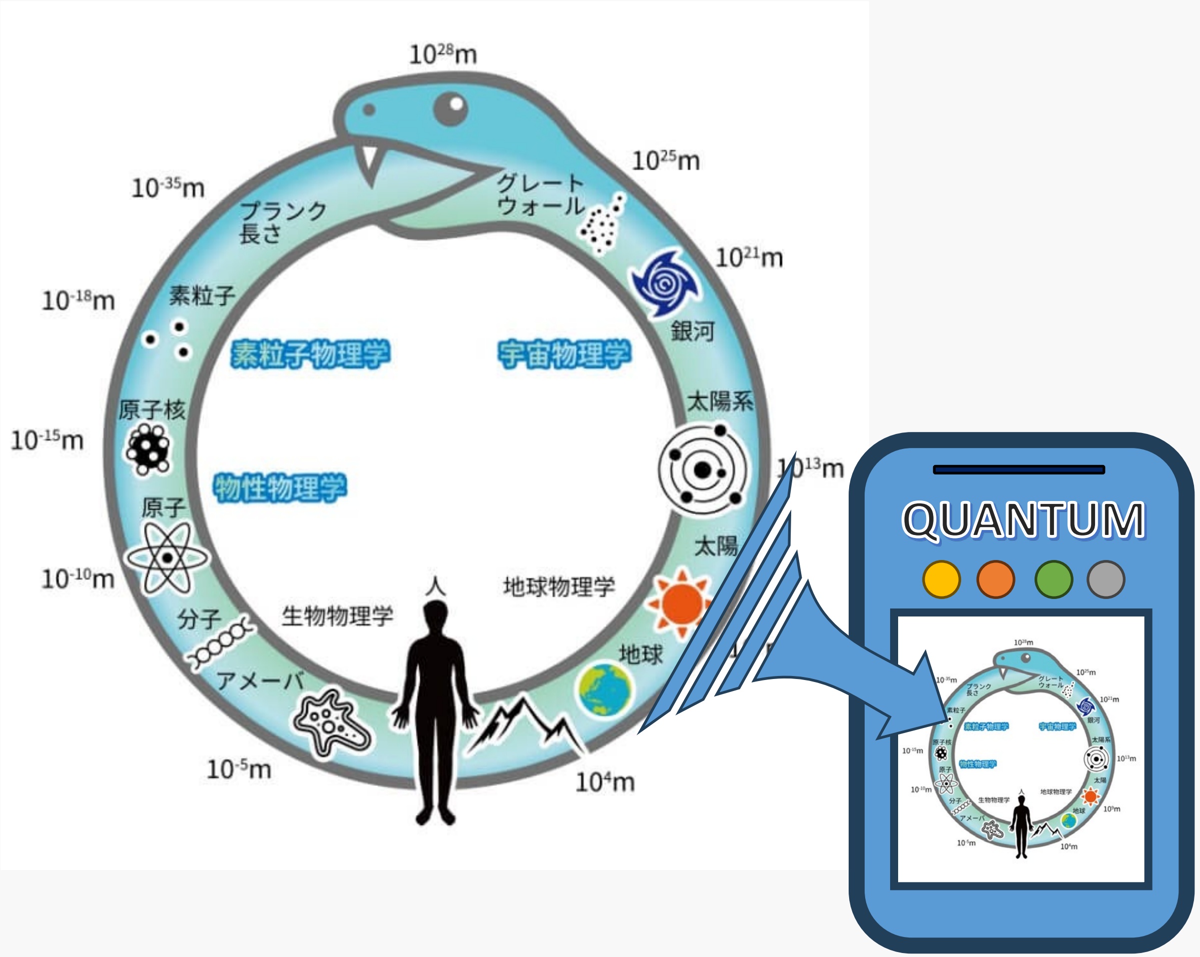


e.g.) Ryu-Takayanagi Formula PRL **96**,181602 (2006)

$$S_{\mathrm{A}} = \frac{\mathrm{Area}(\gamma_{\mathrm{A}})}{4G_{\mathrm{N}}}$$

Entanglement entropy in quantum materials

Gravitational theory for the universe

Grant-in-Aid for Transformative Research Areas (A)
Extreme Universe

quantum information
(i) The limit of space
(ii) The limit of time
(iii) The limit of matter

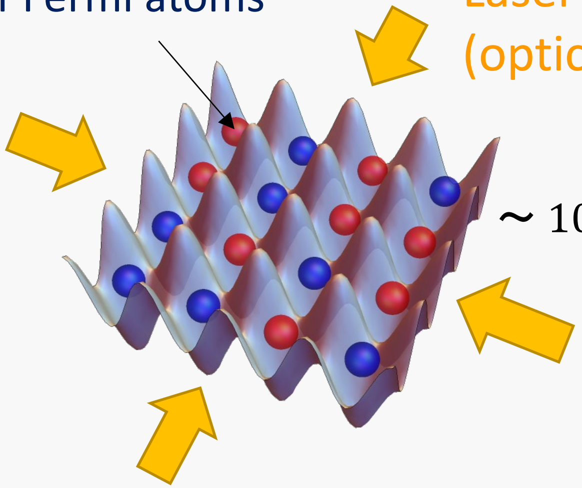
Cold-atom analog quantum simulator for SCES

***SCES — Strongly Correlated Electron System**

Cold-atom "Hubbard model" simulation

Two different hyperfine states of Fermi atoms

Laser beams (optical lattice)



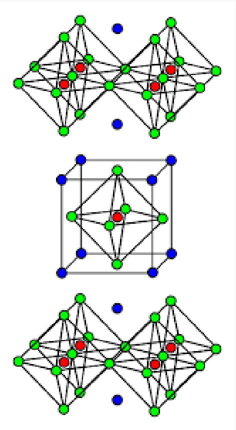
$\sim 10^2$-10^3 nm

Ideal Hubbard model system

$$\hat{\mathcal{H}} = -t \sum_{\langle i,j \rangle} \sum_{\sigma} \left(\hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \mathrm{H.c.} \right) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

mimicking the SCES physics!

High-T_c superconductivity
Quantum magnetism
Spin liquid
Heavy fermion *etc.*



LBCO

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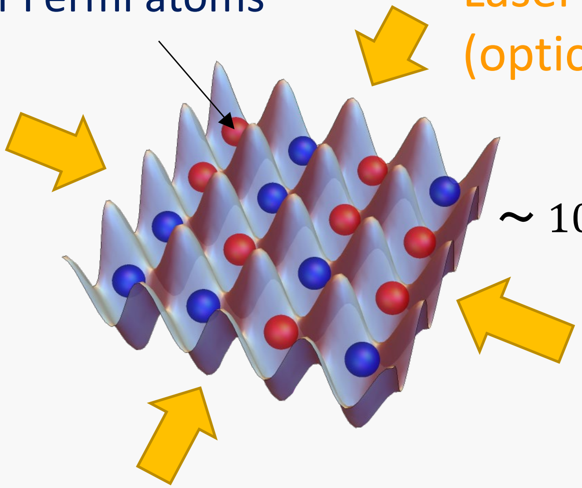

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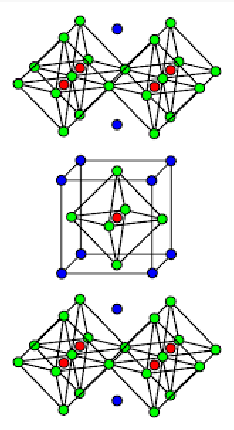
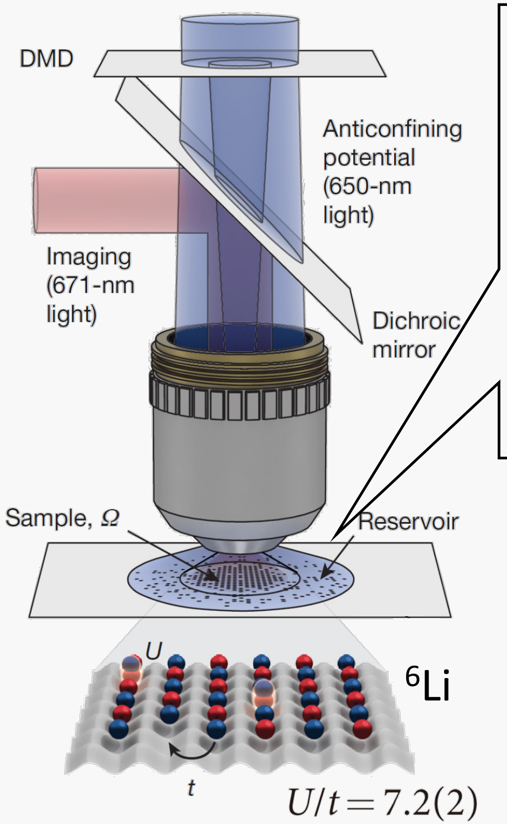


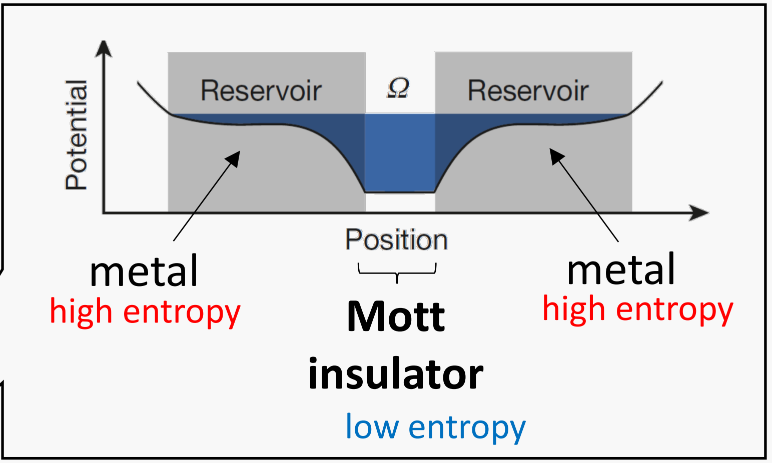
LBCO

Nature **545**, 462 (2017)

A cold-atom Fermi–Hubbard antiferromagnet

Anton Mazurenko[1], Christie S. Chiu[1], Geoffrey Ji[1], Maxwell F. Parsons[1], Márton Kanász-Nagy[1], Richard Schmidt[1], Fabian Grusdt[1], Eugene Demler[1], Daniel Greif[1] & Markus Greiner[1]





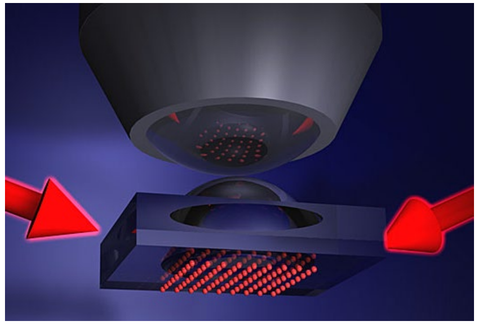
Spin exchange: $J = \dfrac{2t^2}{U} \sim 12\,n\mathrm{K}$

Temperature: $T \sim 10\,n\mathrm{K}\,(\sim J/k_{\mathrm{B}})$

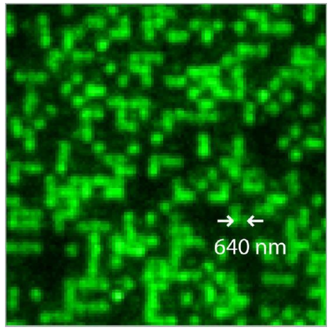
Ad

How to achieve lower temperatures?
⇒ DY, K. Morita, PRL **132**, 213401 (2024).

Quantum-gas microscope (QGM) imaging

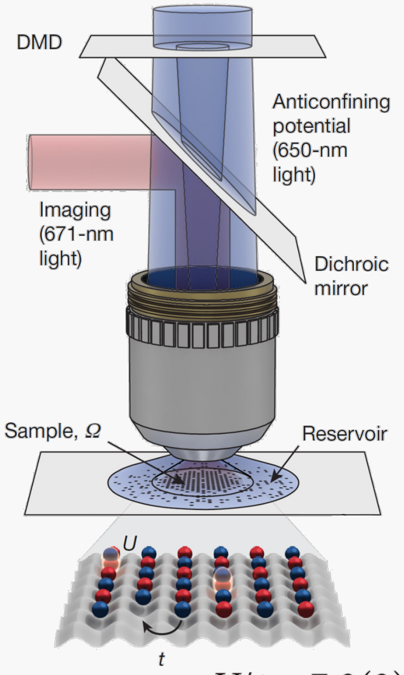


Harvard University website



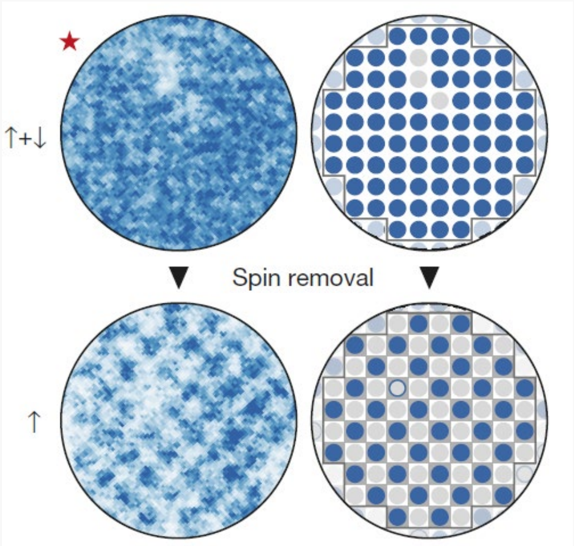
Single-site resolved imaging for **particle occupancy**

W. S. Bakr *et al.*, Nature **462** (2009).



$U/t = 7.2(2)$

A. Mazurenko *et al.*, Nature **545** (2017).



Removing one spin component enables us to perform "**all-*Z* measurement**."

$$M = Z \otimes Z \otimes \cdots \otimes Z$$

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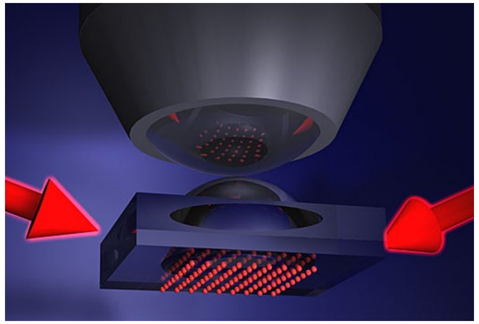
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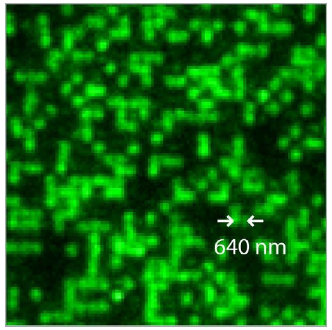

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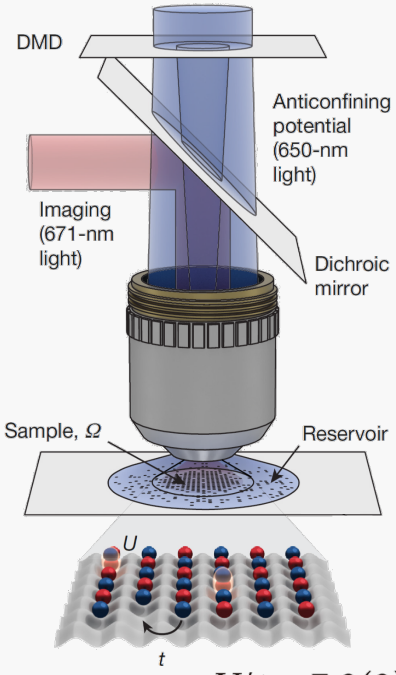

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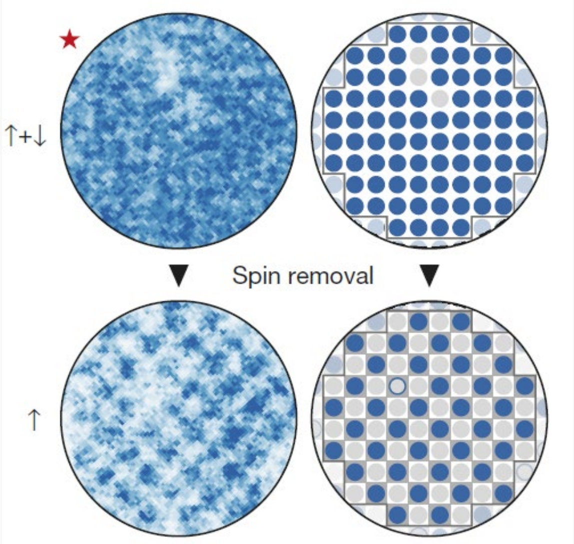

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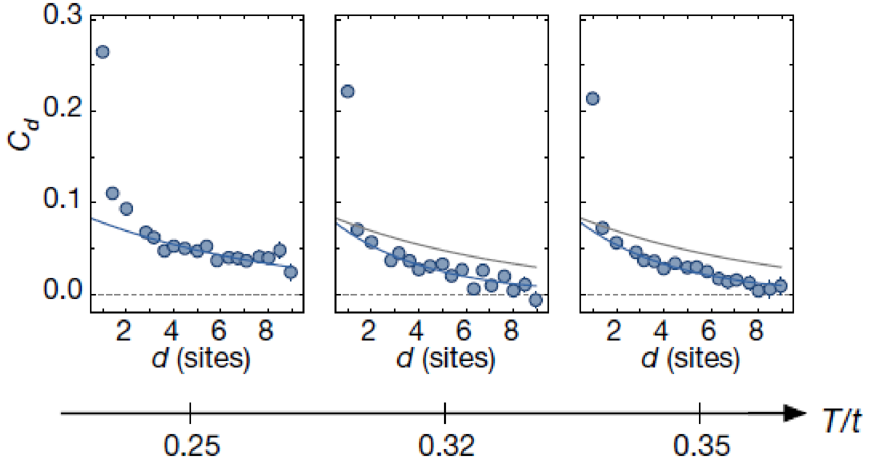
***n*-point correlation functions** can be calculated by averaging the QGM shots over many samples.

$$\langle \hat{S}_i^z \hat{S}_j^z \cdots \hat{S}_k^z \rangle \approx \frac{1}{N_{\text{sample}}} \sum_{l}^{N_{\text{sample}}} \text{QGM outcomes}$$

e.g.) two-point AF correlations

$$C_d = \frac{1}{N^2} \sum_{i,j} (-1)^{i+j} \left(\langle \hat{S}_i^z \hat{S}_j^z \rangle - \langle \hat{S}_i^z \rangle \langle \hat{S}_j^z \rangle \right)$$



Correlation length ξ ~ 10 sites at the lowest T.

Entanglement entropy

In order to advance an artificial quantum system from being ***"just another platform for cond-mat experiments"*** to a ***"quantum information processor,"*** it is essential to achieve the capability to measure **entanglement**.

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von Neumann "entanglement" entropy (EE)

Quantum states are described by *density matrix ρ.*

pure state

$$\rho = |\Psi\rangle\langle\Psi| \Rightarrow S(\rho) \equiv -\mathrm{Tr}\left[\rho \log \rho\right] = 0$$

state vector

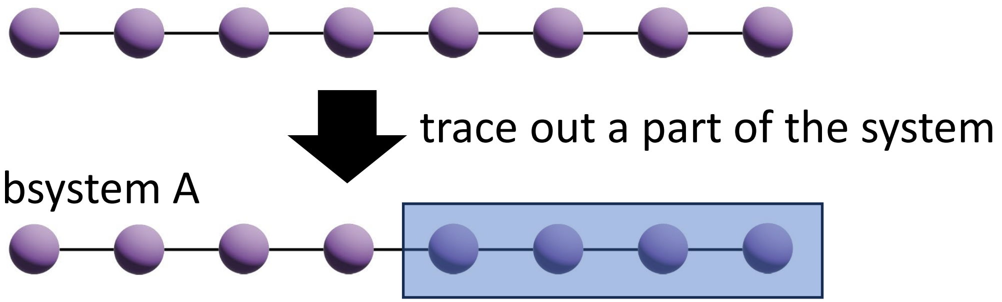
von Neumann entropy

mixed state

$$\rho = \sum_n p_n |\Psi_n\rangle\langle\Psi_n| \Rightarrow S(\rho) > 0$$

mixing fraction

When the entire state is pure, $\rho = |\Psi\rangle\langle\Psi|$



$\rho_{\mathrm{A}} = \mathrm{Tr}_{\bar{\mathrm{A}}}[\rho]$ (reduced density matrix)

If $S(\rho_{\mathrm{A}}) = 0$ (still pure) ⇒ A and $\bar{\mathrm{A}}$ are unentangled

If $S(\rho_{\mathrm{A}}) > 0$ (become mixed) ⇒ entangled

Quantum state tomography (QST) of density matrix

How do you **measure** *quantum state* or density matrix ρ?

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⇒ Doable by collecting enough information of **expectation values**. (but how many?)

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For N spins (S = 1/2), dimension is d=2^N. ⇒ ρ is $2^N \times 2^N$ Hermitian matrix with trace 1. ⇒ **4^N-1** parameters!

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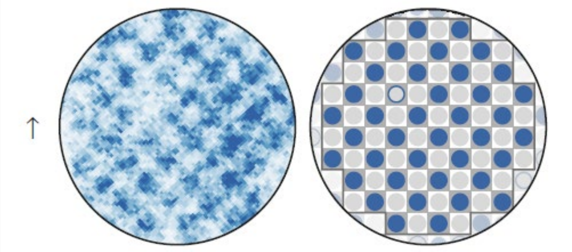
QGM



$$M = Z \otimes Z \otimes \cdots \otimes Z$$

$$\langle \hat{S}_i^z \hat{S}_j^z \cdots \hat{S}_k^z \rangle$$ **2^N** values

2 options (included or not)
for each site

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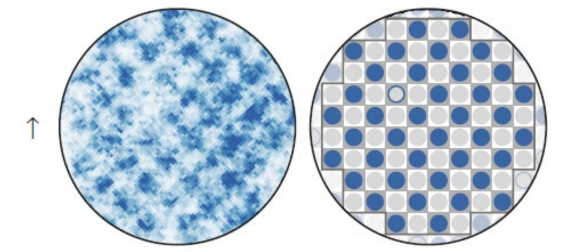

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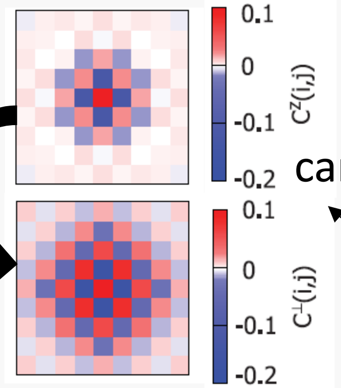
QGM after global spin rotations

π/2 Rabi pulse



canted AF

Brown *et al.*, Science **357** (2017).

$$M = X \otimes X \otimes \cdots \otimes X$$
$$M = Y \otimes Y \otimes \cdots \otimes Y$$

2^N values $\times 2$

Quantum state tomography (QST) of density matrix

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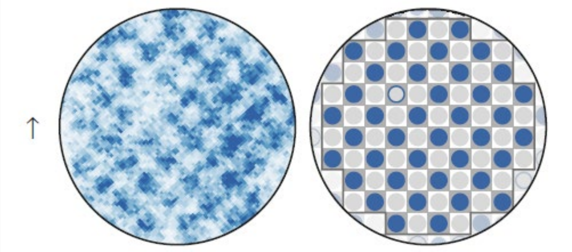
For N spins (S = 1/2), dimension is $d=2^N$. $\Rightarrow$ ρ is $2^N \times 2^N$ Hermitian matrix with trace 1. $\Rightarrow$ **4^N-1** parameters!

QGM



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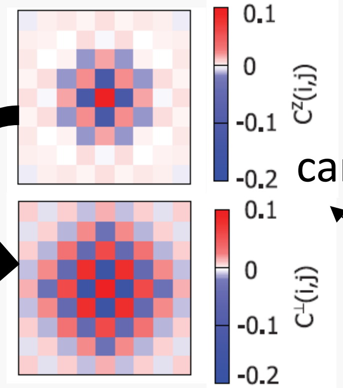

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Brown *et al.*, Science **357** (2017).

$$M = X \otimes X \otimes \cdots \otimes X$$
$$M = Y \otimes Y \otimes \cdots \otimes Y$$

2^N values $\times$ 2

For full tomography of ρ, only the ***global*** rotations are not enough.

$$M = \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix} \otimes \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix}$$

All **4^N** combinations (**3^N** setups) are needed!

Challenges:

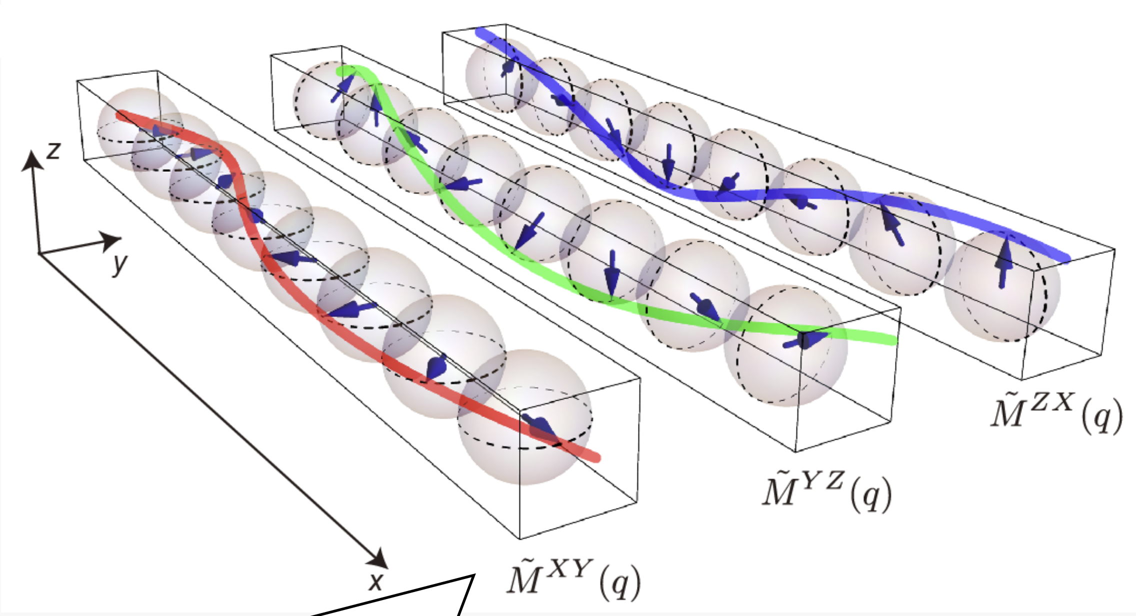
- ☐ **Exponential growth as system size *N***
- ☐ **Individual single-site addressing**

Spiral quantum state tomography (spiral-QST)

Collaborators: Giacomo Marmorini (Nihon Univ., ExU postdoc),
Hideki Ozawa, Takeshi Fukuhara (RIKEN)

Spiral measurements (e.g. for 1D)

The measurement axis **twisted in spiral** with pitch angle q



Experimental advantages:

☑ Preparable only with ***global*** manipulations

- Temporary magnetic field gradient

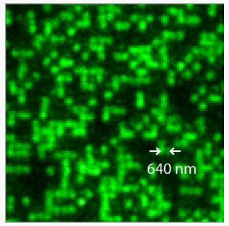
- Global π/2 pulses

☑ Different settings obtained only by changing the application time of magnetic field gradient

Spiral operators in 3 planes:

$$\tilde{M}^{XY}(q) = Z^{(1)\prime} \otimes Z^{(2)\prime} \otimes Z^{(3)\prime} \otimes \cdots \otimes Z^{(N)\prime},$$
$$\tilde{M}^{YZ}(q) = Z^{(1)\prime\prime} \otimes Z^{(2)\prime\prime} \otimes Z^{(3)\prime\prime} \otimes \cdots \otimes Z^{(N)\prime\prime},$$
$$\tilde{M}^{ZX}(q) = Z^{(1)\prime\prime\prime} \otimes Z^{(2)\prime\prime\prime} \otimes Z^{(3)\prime\prime\prime} \otimes \cdots \otimes Z^{(N)\prime\prime\prime}$$

$$\left(\begin{aligned} Z^{(i)\prime} &= \cos(qi+\theta)X + \sin(qi+\theta)Y, \\ Z^{(i)\prime\prime} &= \cos(qi+\theta)Y + \sin(qi+\theta)Z, \\ Z^{(i)\prime\prime\prime} &= \cos(qi+\theta)Z + \sin(qi+\theta)X \end{aligned}\right)$$

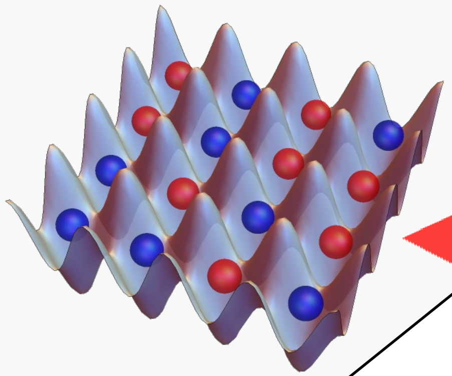


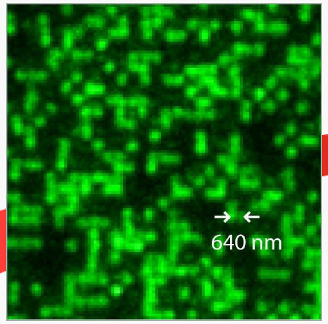
2^N expectation values × **3** planes × $\boldsymbol{n_q}$

# of different pitch angles (~ N)

* Still not enough for full tomography (4^N-1)

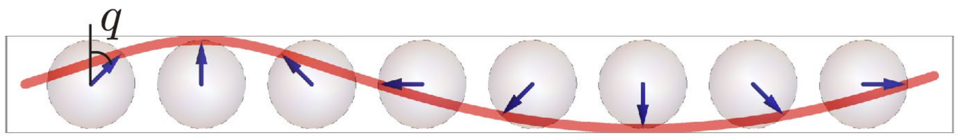
Spiral quantum state tomography (spiral-QST)





Taking many QGM shots
for each setting

Rotation into spiral-spin basis



global π/2 Rabi pulse
+ magnetic-field gradient

Collecting expectation values of different spiral operators

$\langle \tilde{M}^{XY}(q) \rangle,\ \langle \tilde{M}^{YZ}(q) \rangle,\ \langle \tilde{M}^{ZX}(q) \rangle$ with different *q*

Compressed sensing
for low-rank matrix
Gross *et al*., PRL **105** (2010)

Reconstruct density matrix ρ

Entanglement entropies

$$S(\rho_{\mathrm{A}}) \equiv -\mathrm{Tr}\left[\rho_{\mathrm{A}} \log \rho_{\mathrm{A}}\right] \quad \text{(von Neumann)}$$

$$S_n(\rho_{\mathrm{A}}) \equiv \frac{1}{1-n} \log \mathrm{Tr}\left[\rho_{\mathrm{A}}^n\right] \quad \text{(n-th Rényi)}$$

Compressed sensing for a low-rank matrix

Recover a low-rank matrix ***M*** by approximate ***X***:

Minimize $\mathrm{rank}(\boldsymbol{X})$ subject to $X_{ij} = M_{ij}$ $\left[(i,j) \in \Omega\right]$

sampled set

convex relaxation

Minimize $\tau \|\boldsymbol{X}\|_* + \frac{1}{2}\|\boldsymbol{X}\|_F^2$ $\left(\tau\text{ : a large value}\right)$

subject to $P_\Omega(\boldsymbol{X}) = P_\Omega(\boldsymbol{M})$ $\left[P_\Omega(\cdot)\text{ : orthogonal projector}\right]$

Singular value thresholding (SVT) algorithm:

Repeat $\begin{cases} \boldsymbol{X}^k = \mathcal{D}_\tau(\boldsymbol{Y}^{k-1}), \\ \boldsymbol{Y}^k = \boldsymbol{Y}^{k-1} + \delta_k \mathcal{P}_\Omega(\boldsymbol{M} - \boldsymbol{X}^k) \end{cases}$ until convergence

$\left(D_\tau(\cdot)\text{ : singular value thresholding}\right)$

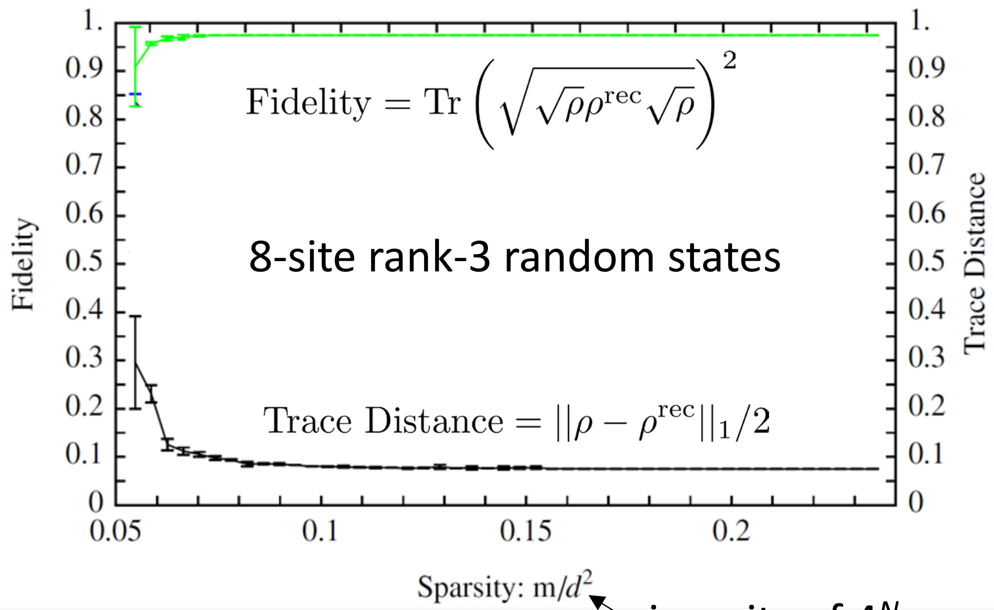
See Cai *et al.*, arxiv:0810.3286 for technical details

QST via compressed sensing

Gross *et al.*, PRL **105** (2010)

with *m* random Pauli measurements from 4^N-1 options:

$$M = \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix} \otimes \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} X \\ Y \\ Z \\ I \end{pmatrix}$$



in units of 4^N

Does it work also for spiral measurements?

Numerical experiments I: Random states (rank=1)

Target *ρ*: Random pure states (*N*=8 sites)

Noise:

- depolarizing noise λ = 0.05 for state preparation

$$\rho \to (1-\lambda)\rho + \frac{\lambda}{d} I$$

- Gaussian noise $\sigma = 0.1/2^N$ for measurement

$$\langle \tilde{M}(q) \rangle \to \mathcal{N}(\mu = \langle \tilde{M}(q) \rangle, \sigma)$$

Spiral measurements:

Random *m* measurements from

$$\left\{ \begin{array}{l} \tilde{M}^{XY}(q) \\ \tilde{M}^{YZ}(q) \\ \tilde{M}^{ZX}(q) \end{array} \right.$$

Pitch angles:

$$q = \frac{\pi}{N} l \quad (l = -N+1, -N+2, \ldots, N)$$

Phase shift: $\theta = -q(L+1)/2$

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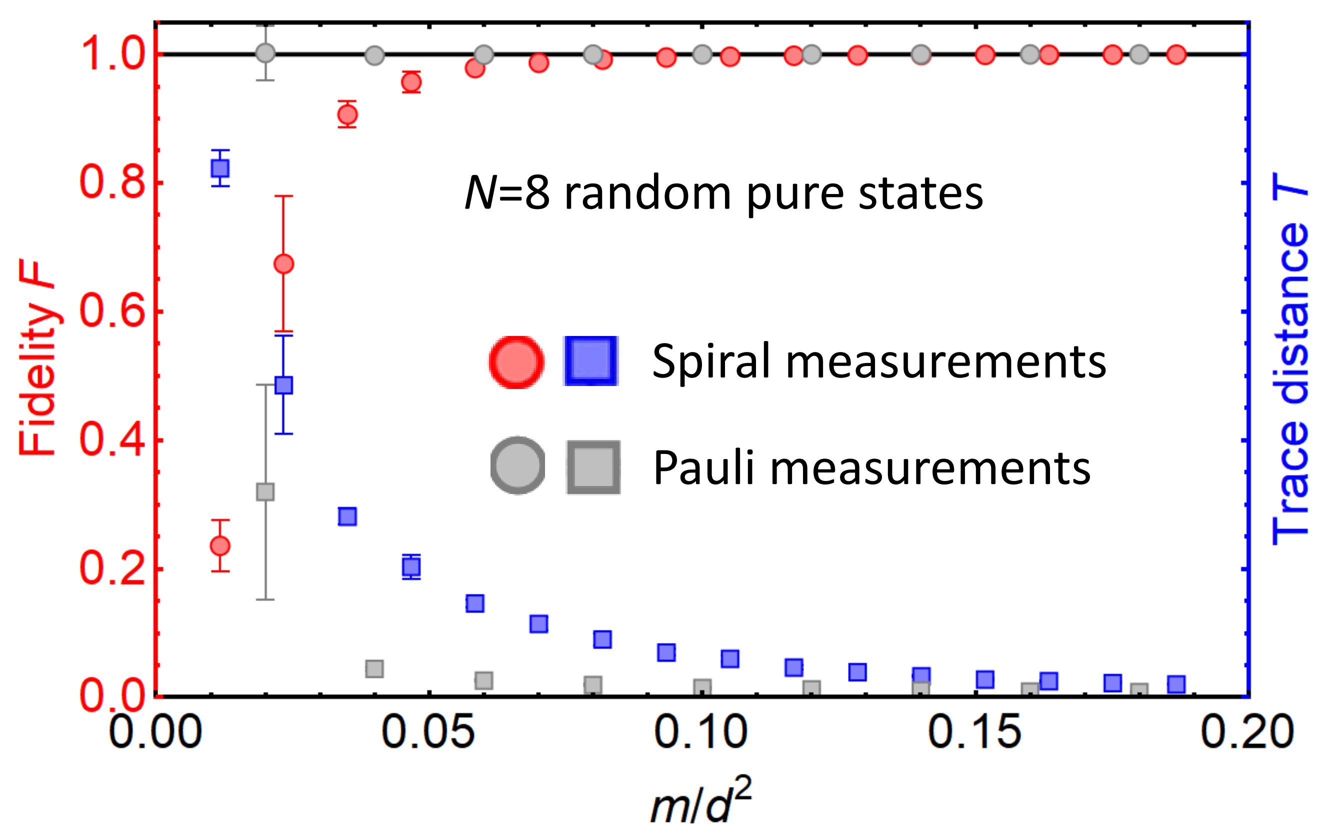
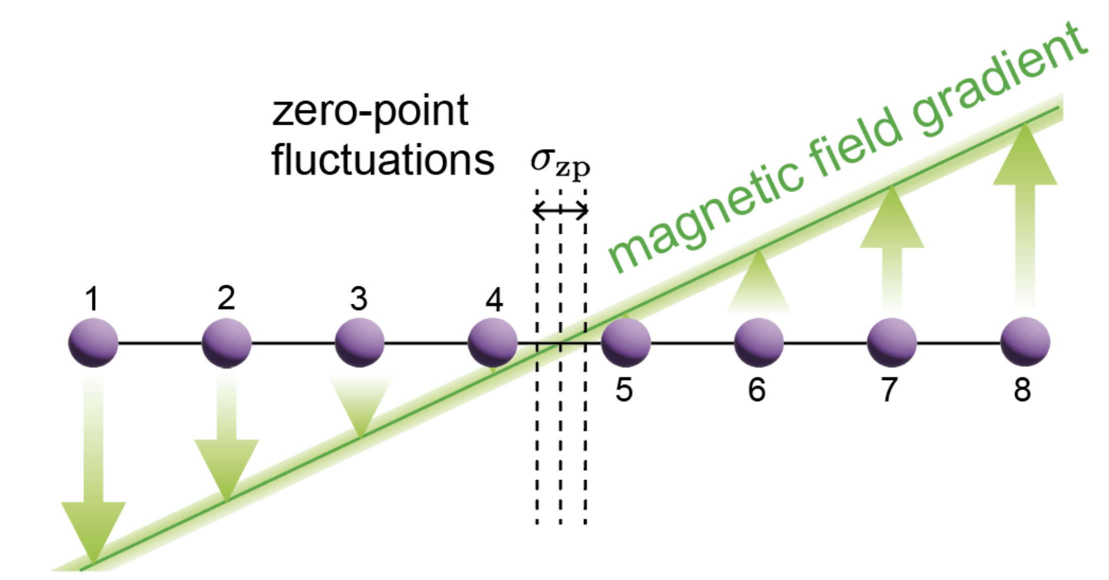


☑ Accuracy of the matrix recovery by spiral QST is quite good with only less than **10% of 4^N** expectation values!

($\sim$ **26** experimental setups)

Realistic experimental source of noise



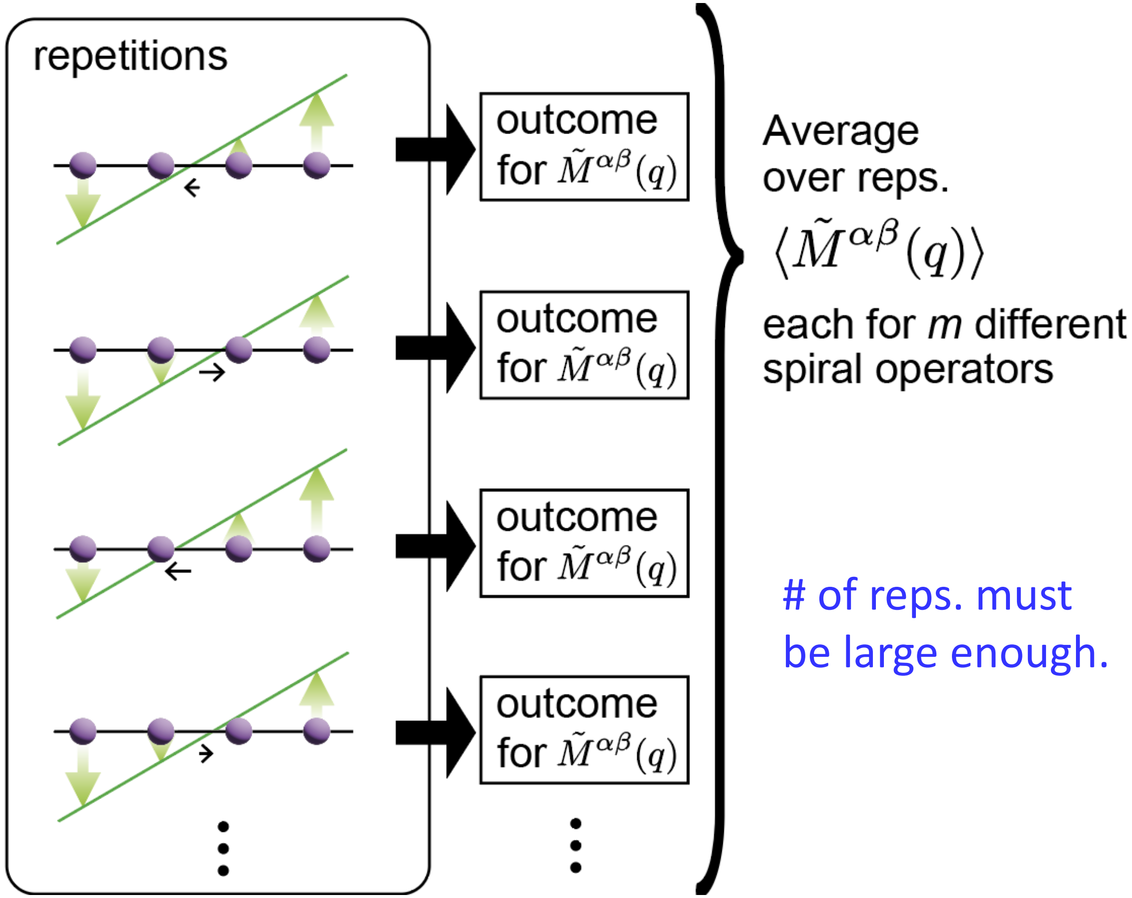
Main source of measurement noise:
zero-point fluctuations of the magnetic field gradient

$$qi \to q(i - \delta_{\text{zp}})$$

$$\left(\delta_{\text{zp}} = \mathcal{N}(\mu = 0, \sigma_{\text{zp}})\right)$$

*We will not consider any other sources hereafter.

Each QGM shot is affected.



# of reps. must be large enough.

Numerical experiments II: Heisenberg ground state

Target ρ:

Ground state of the *N*=8 Heisenberg model

$$\hat{\mathcal{H}} = \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} \quad \text{(AFM)}$$

Noise: Magnetic field zero-point fluctuations

$\sigma_{\mathrm{zp}} = 0.01, 0.05, 0.1$ in units of lattice spacing

Scheduled spiral measurements:

$$q = \frac{\pi}{N} l \quad (l = 0, 1, 2, \dots) \quad \text{for} \begin{cases} \tilde{M}^{XY}(q) \\ \tilde{M}^{YZ}(q) \\ \tilde{M}^{ZX}(q) \end{cases}$$

Phase shift: $\theta = -q(L+1)/2$

# of reps. for expectation values:

100, 500, 1000

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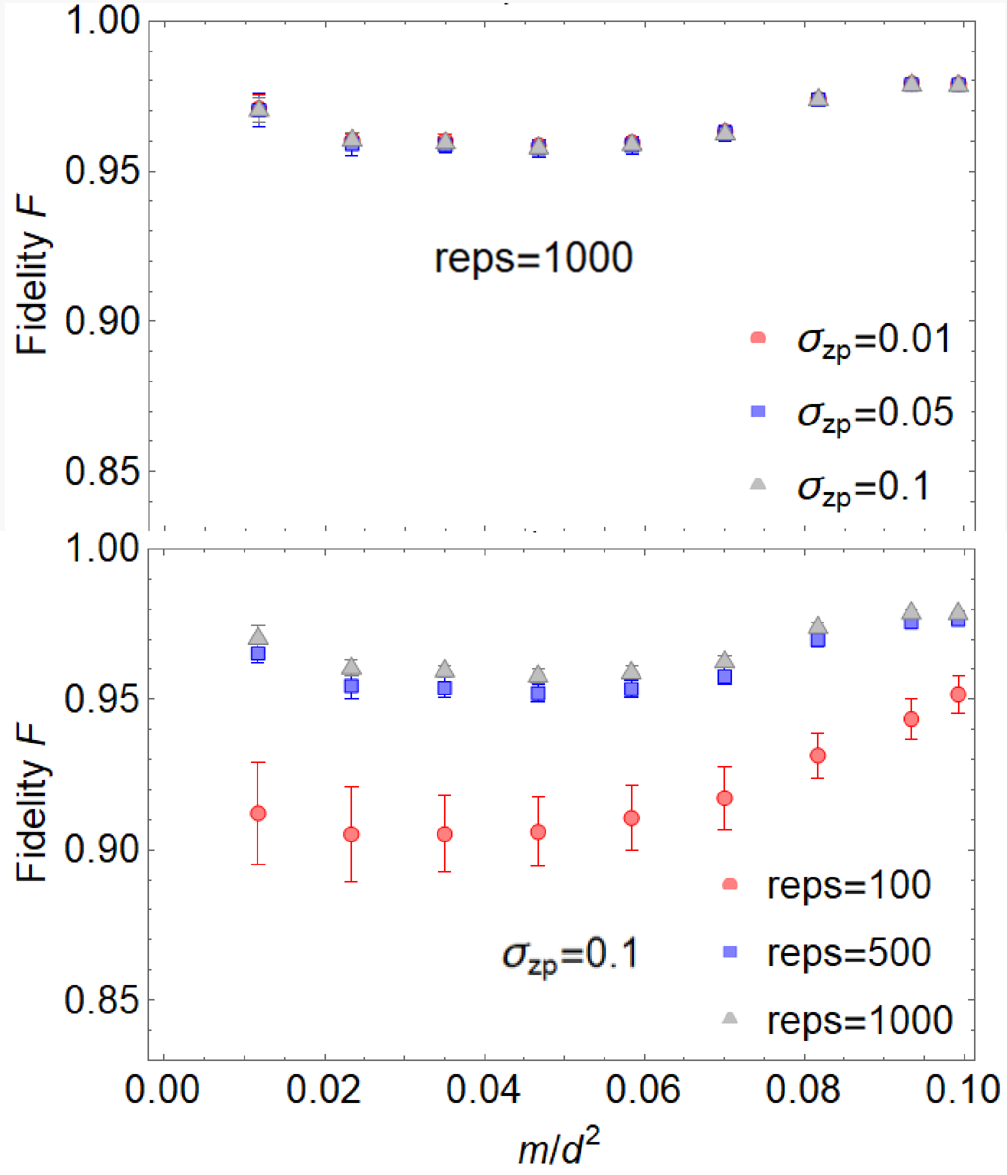

The dependence on σ_{zp} is quite weak!

σ_{zp} = 0.01 ⇔ experimental accuracy of 10 μGauss

$$H = \sum_i S_i \cdot S_{i+1}$$

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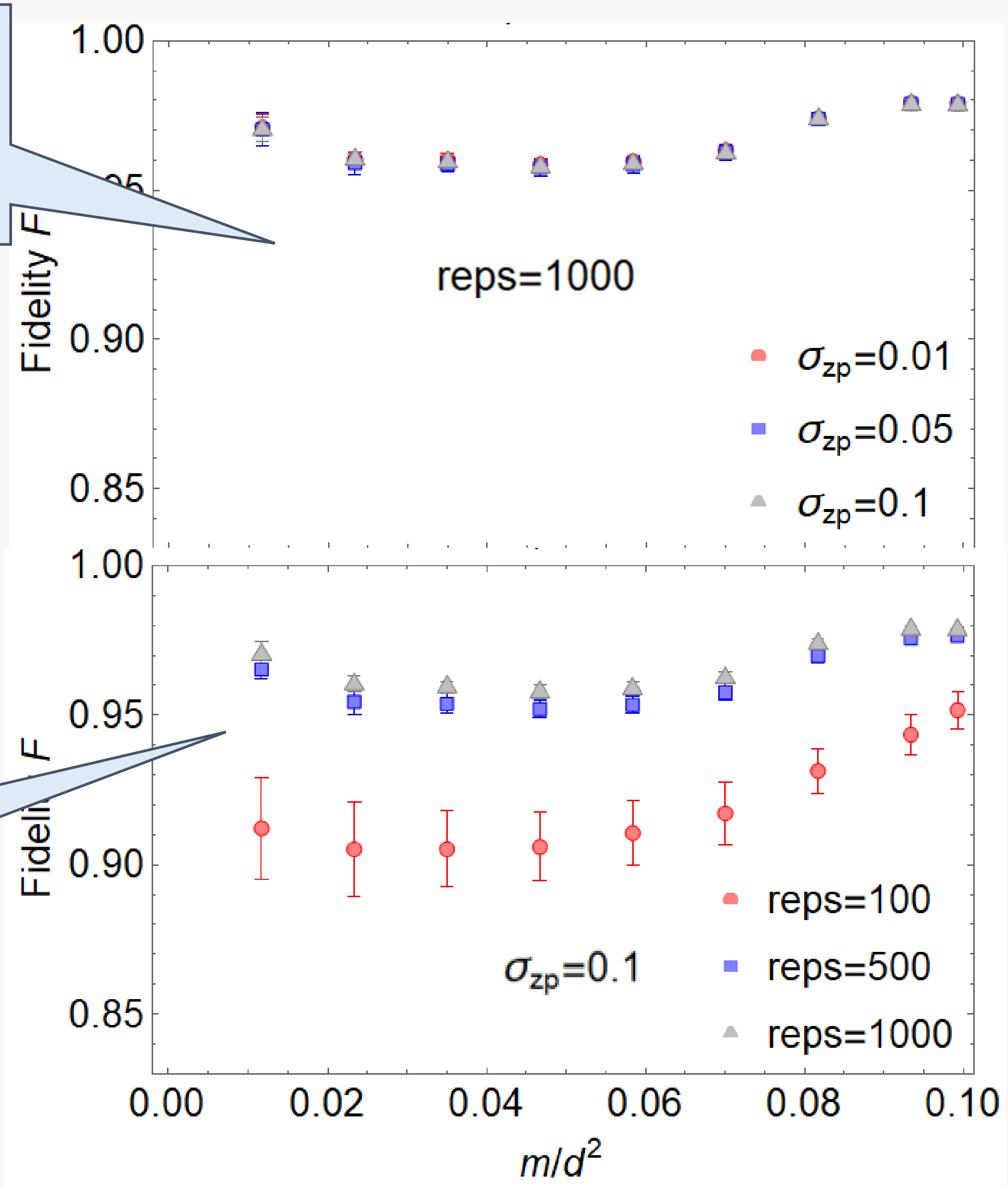
P

#

100, 500, 1000

# of repetitions ≃ 500 is sufficient to obtain accurate expectation values for each spiral measurement.



Numerical experiments II: Heisenberg ground state

The dependence on σ_{zp} is quite weak!

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$H = \sum_i S_i \cdot S_{i+1}$ (AFM)

Noise: Magnetic field ze

$\sigma_{zp} = 0.01, 0.05, 0.1$ in

The lowest m (= 3×2^N) already gives a quite good fidelity.

***Only global measurements in 3 planes.**

Scheduled spiral measurements:

$$q = \frac{\pi}{N} l \quad (l = 0, 1, 2, \dots) \text{ for } \begin{cases} \tilde{M}^{XY}(q) \\ \tilde{M}^{YZ}(q) \end{cases}$$

P

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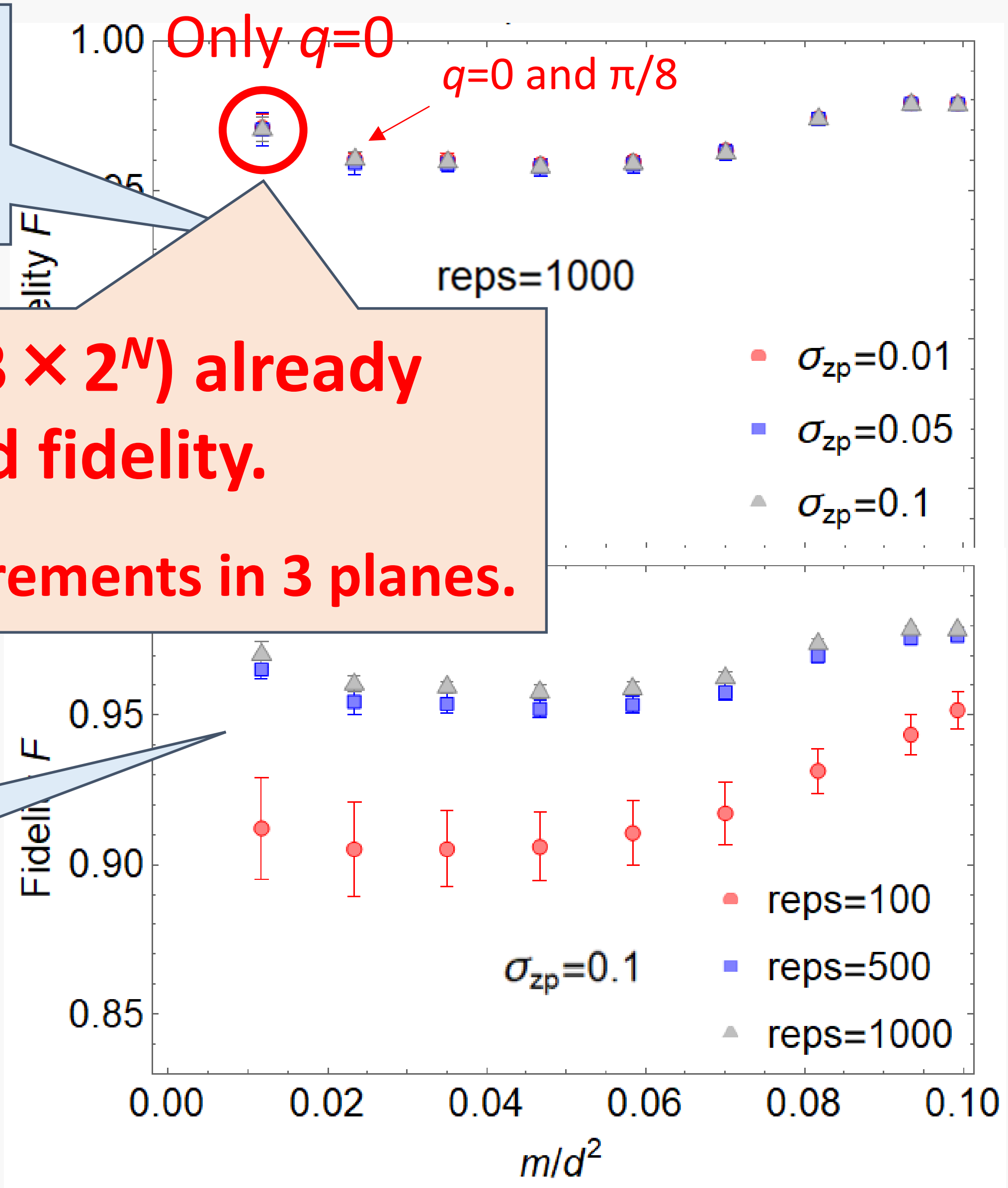

Numerical experiments III: Heisenberg + DM g.s.

Target ρ:

Ground state of the N=6 Heisenberg +DM

$$\hat{\mathcal{H}} = -\sum_i (\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \hat{S}_i^x \hat{S}_{i+1}^y - \hat{S}_i^y \hat{S}_{i+1}^x)$$

(FM) (Dzyaloshinskii-Moriya)

Noise: Magnetic field zero-point fluctuations

$\sigma_{\mathrm{zp}} = 0.1$ in units of lattice spacing

Scheduled spiral measurements:

$$q = \frac{\pi}{N} l \quad (l = 0, 1, 2, \dots) \quad \text{for} \begin{cases} \tilde{M}^{XY}(q) \\ \tilde{M}^{YZ}(q) \\ \tilde{M}^{ZX}(q) \end{cases}$$

Phase shift: $\theta = -q(L+1)/2$

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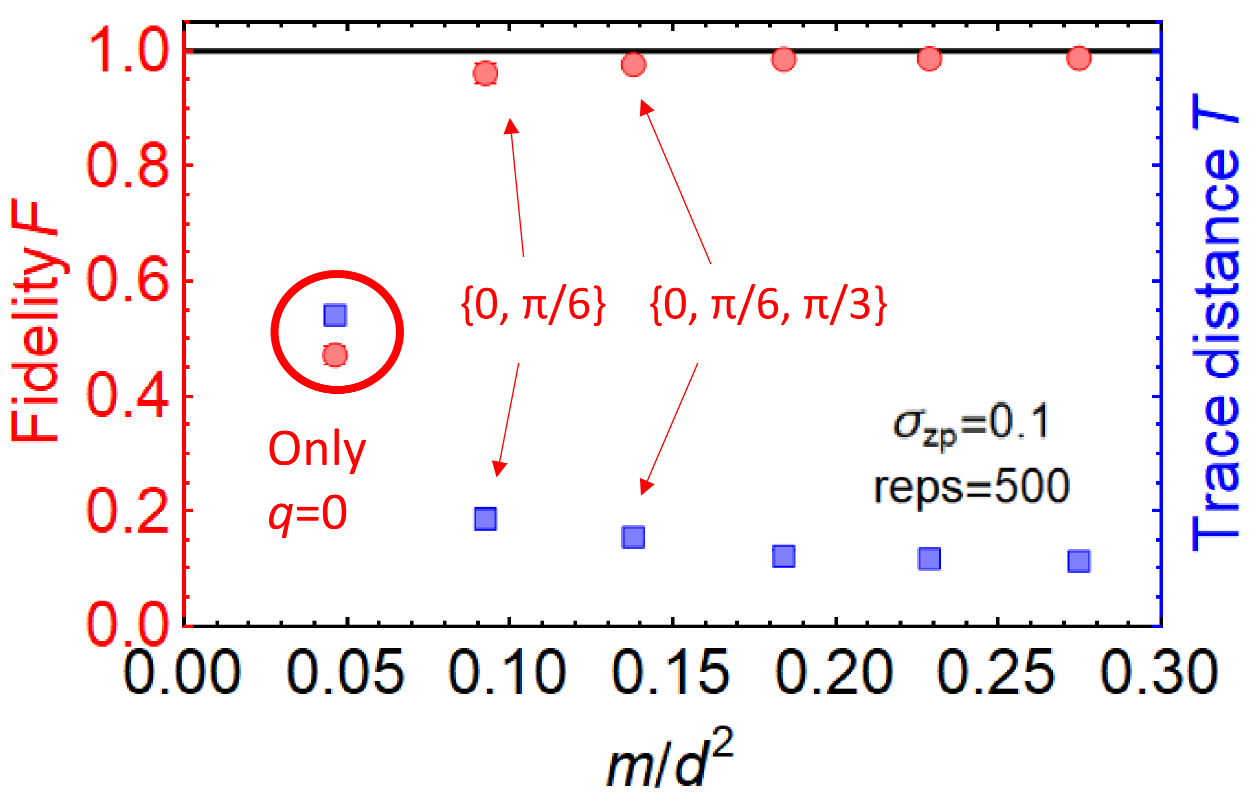
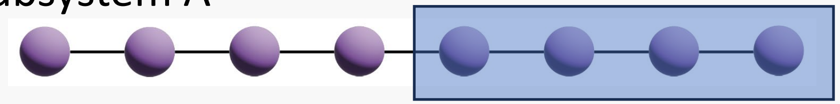


☑ In general, only q=0 measurements are not enough.

☑ Appropriate scheduling on the q's sequence may reduce the experimental cost for spiral-QST.

Numerical experiments IV: Reduced density matrix

subsystem A



$\rho_{\mathrm{A}} = \mathrm{Tr}_{\bar{\mathrm{A}}}[\rho]$ (reduced density matrix)

***ρ*:** Ground state of the N=14 Heisenberg model

$$\hat{\mathcal{H}} = \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1}$$

(AFM)

Target $\boldsymbol{\rho}_{\mathrm{A}}$:

$\rho_{\mathrm{A}}(N_{\mathrm{A}})$ for $N_{\mathrm{A}} = 2, 3, 4, 5, 6, 7$

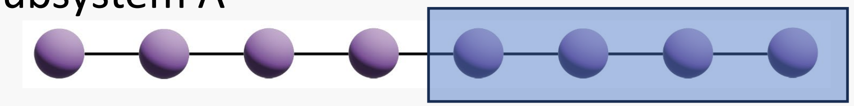
*The other conditions are the same as before.

☑ Entire density matrix ρ is **not** needed.

☑ **Spiral-QST directly reconstruct the reduced matrix $\boldsymbol{\rho}_{\mathrm{A}}$!**

Numerical experiments IV: Reduced density matrix

subsystem A



$\rho_{\mathrm{A}} = \mathrm{Tr}_{\bar{\mathrm{A}}}[\rho]$ (reduced density matrix)

ρ: Ground state of the
N=14 Heisenberg model

$$\hat{\mathcal{H}} = \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1}$$

(AFM)

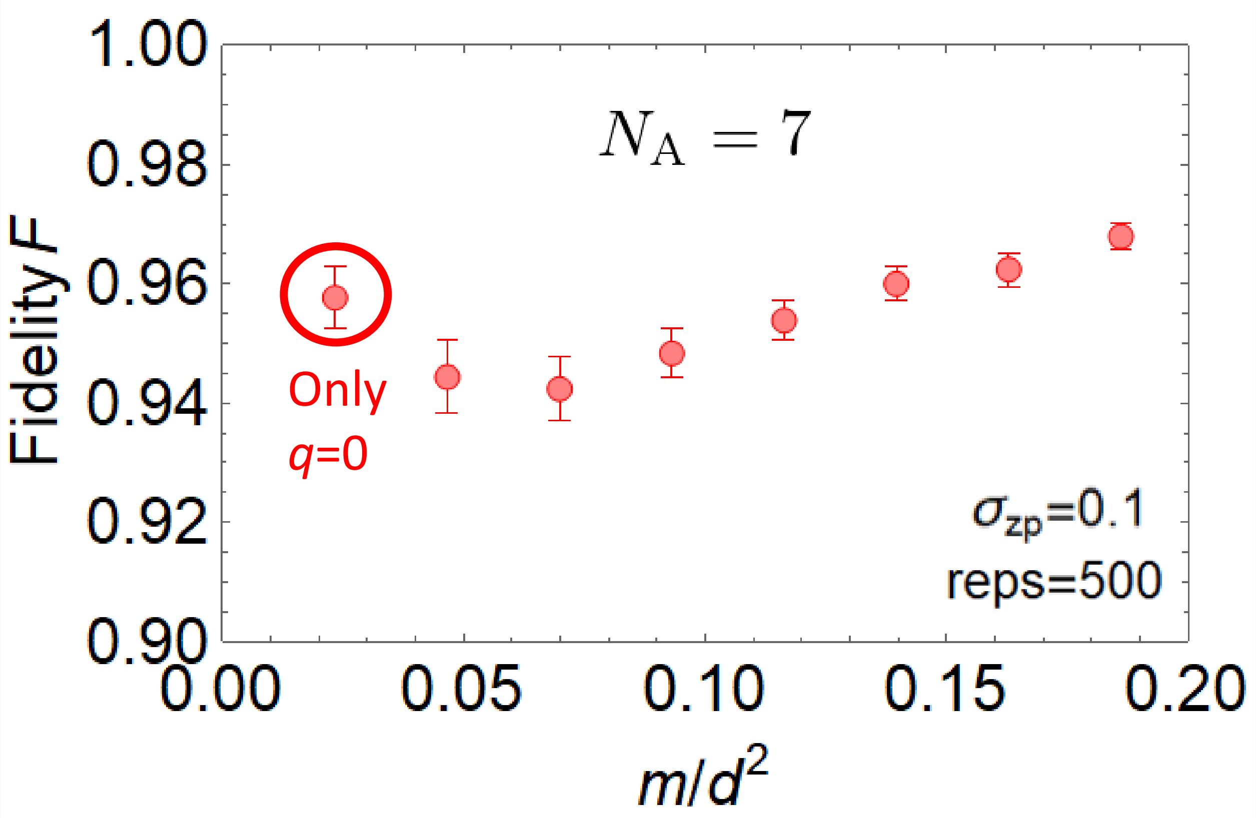
Target ρ_{A}:

$\rho_{\mathrm{A}}(N_{\mathrm{A}})$ for $N_{\mathrm{A}} = 2, 3, 4, 5, 6, 7$

*The other conditions are the same as before.

☑ Entire density matrix ρ is **not** needed.

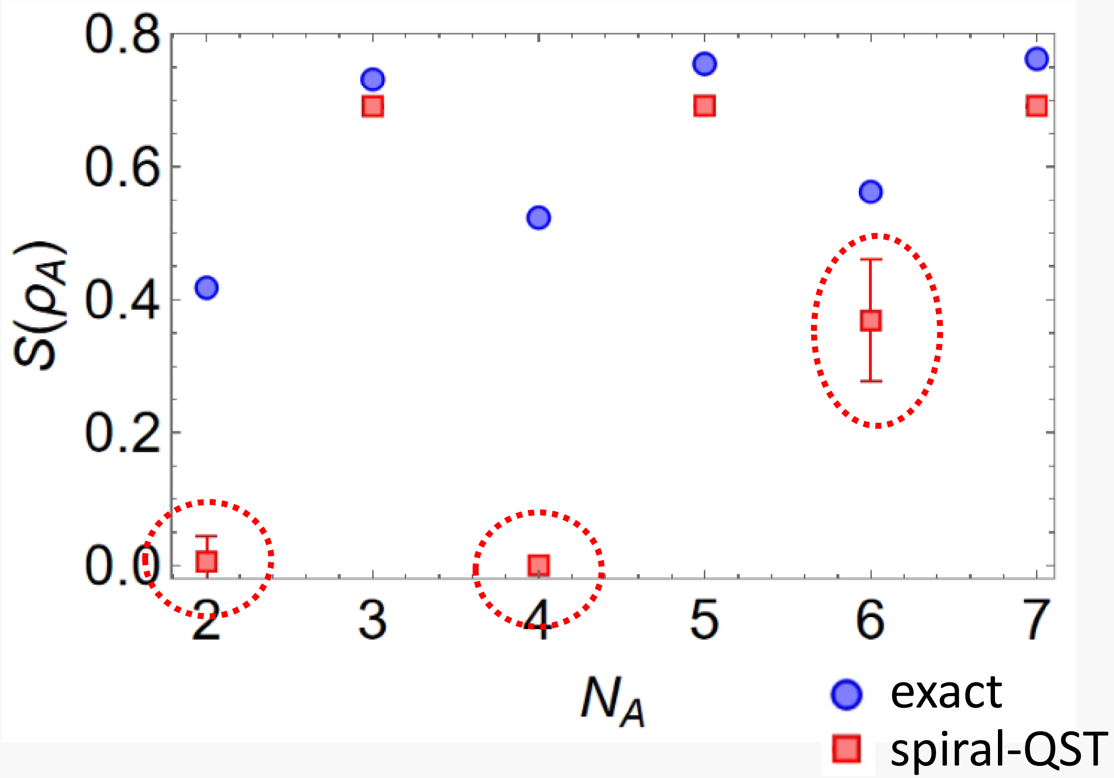
☑ **Spiral-QST directly reconstruct the reduced matrix ρ_{A}!**



☑ Reduced density matrix (mixed) can also be reconstructed with good accuracy.

Evaluation of entanglement entropy

$$S(\rho_{\mathrm{A}}) \equiv -\mathrm{Tr}\left[\rho_{\mathrm{A}} \log \rho_{\mathrm{A}}\right]$$ (von Neumann)



☑ **For low-entangled cases, the accuracy seems bad.**

☑ Compressed sensing tends to **lose the information of the smaller eigenvalues** of the reduced density matrix.

Evaluation of entanglement entropy

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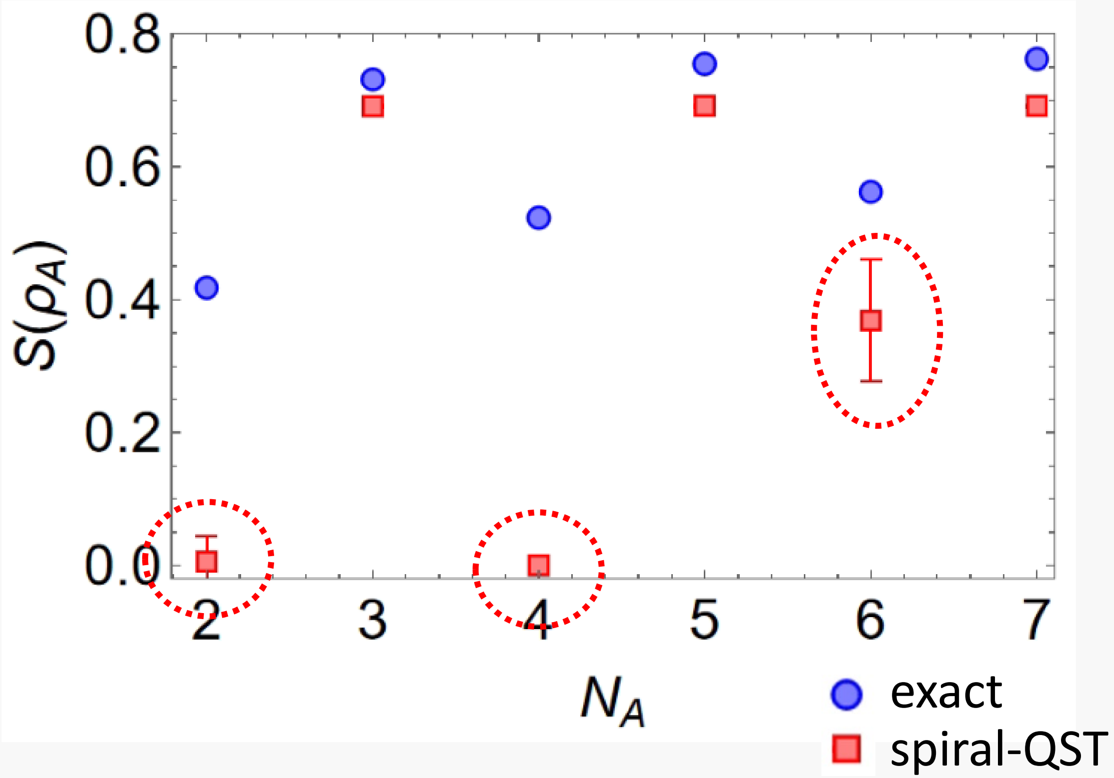
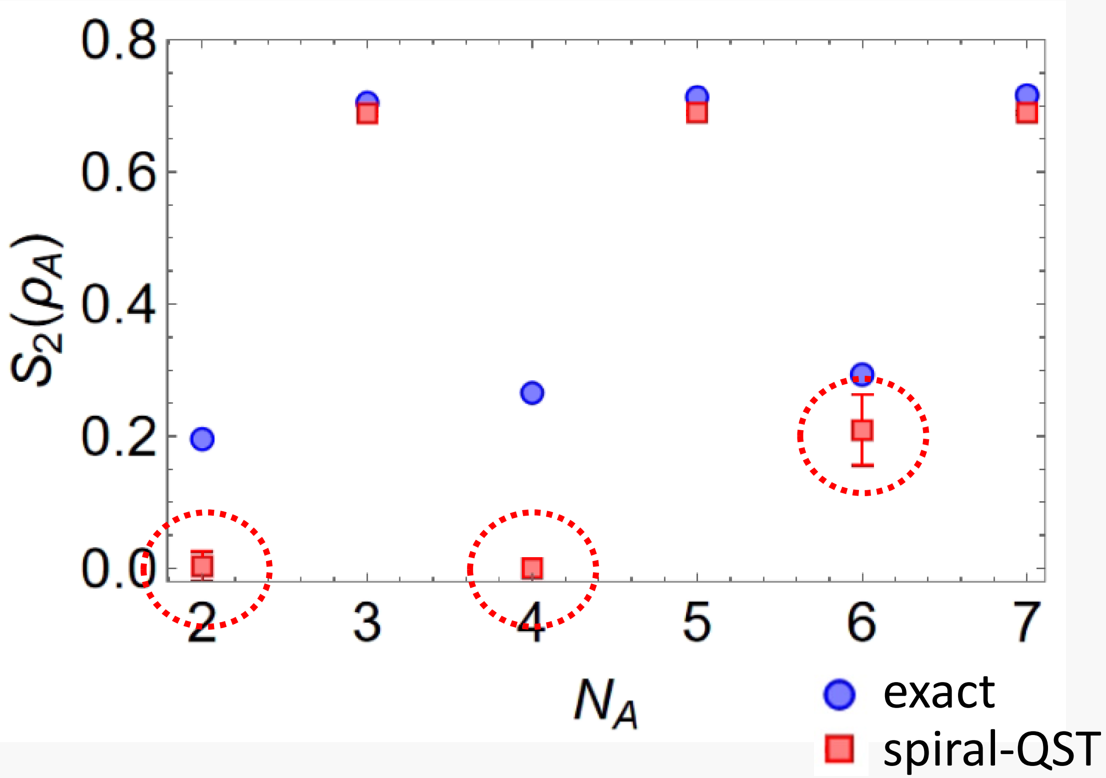
$$S_n(\rho_\mathrm{A}) \equiv \frac{1}{1-n}\log\mathrm{Tr}\left[\rho_\mathrm{A}^n\right]$$ (n-th Rényi)



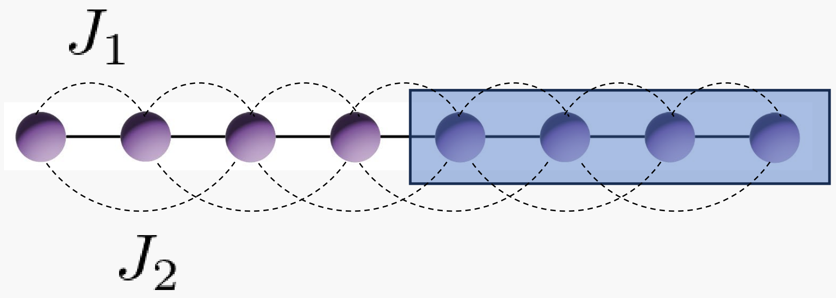


☑ **For low-entangled cases, the accuracy seems bad.**

☑ Compressed sensing tends to **lose the information of the smaller eigenvalues** of the reduced density matrix.

☑ For larger-order Rényi entropy, the difference from the exact value gets smaller.

Case of frustrated *NN*+*NNN* model g.s.

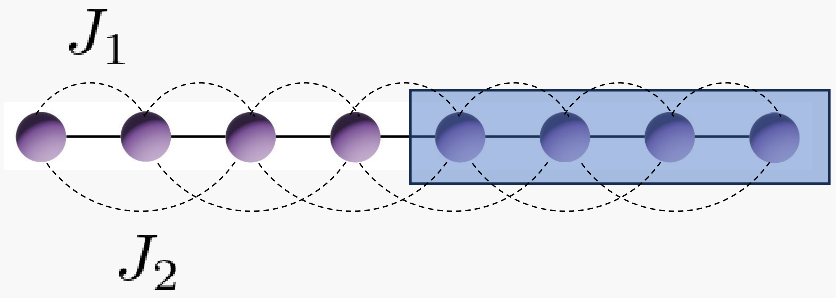


$$\hat{\mathcal{H}} = J_1 \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + J_2 \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+2} \qquad J_2 = J_1$$

(NN AFM) (NNN AFM)

N=14

Case of frustrated *NN*+*NNN* model g.s.



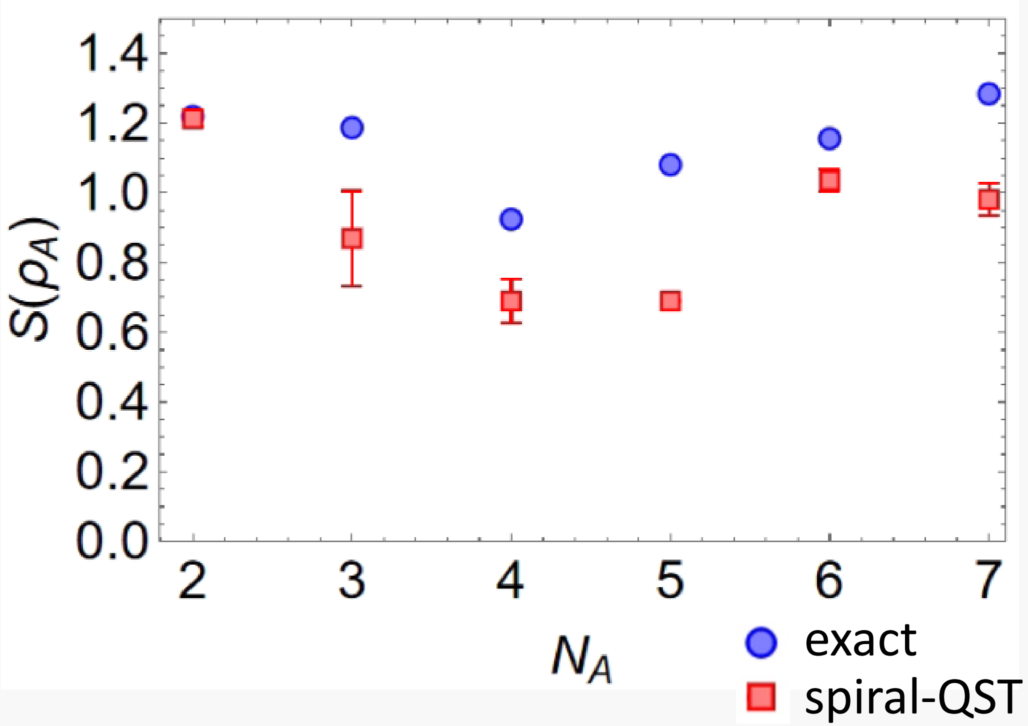
$$\hat{\mathcal{H}} = J_1 \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + J_2 \sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+2} \qquad J_2 = J_1$$

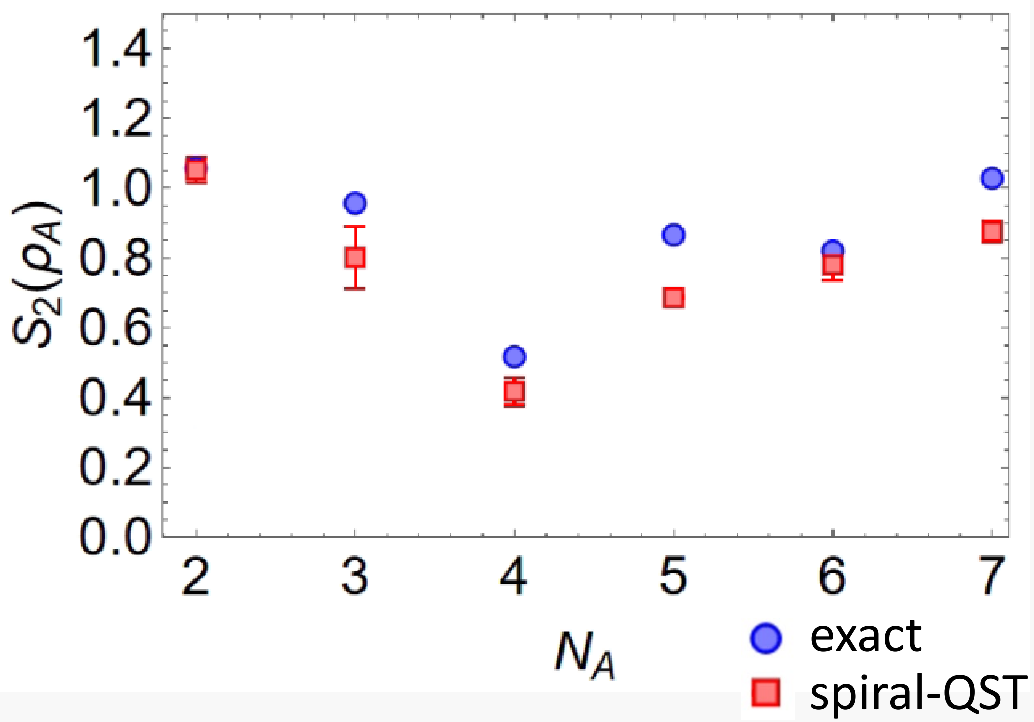
(NN AFM) (NNN AFM)

N=14

$$S(\rho_{\mathrm{A}}) \equiv -\mathrm{Tr}\left[\rho_{\mathrm{A}} \log \rho_{\mathrm{A}}\right]$$ (von Neumann)

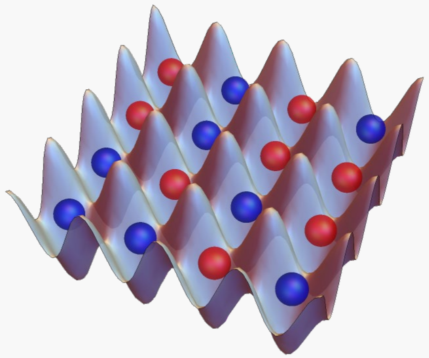
$$S_n(\rho_{\mathrm{A}}) \equiv \frac{1}{1-n} \log \mathrm{Tr}\left[\rho_{\mathrm{A}}^n\right]$$ (*n*-th Rényi)





Summary

Cold atoms in an optical lattice

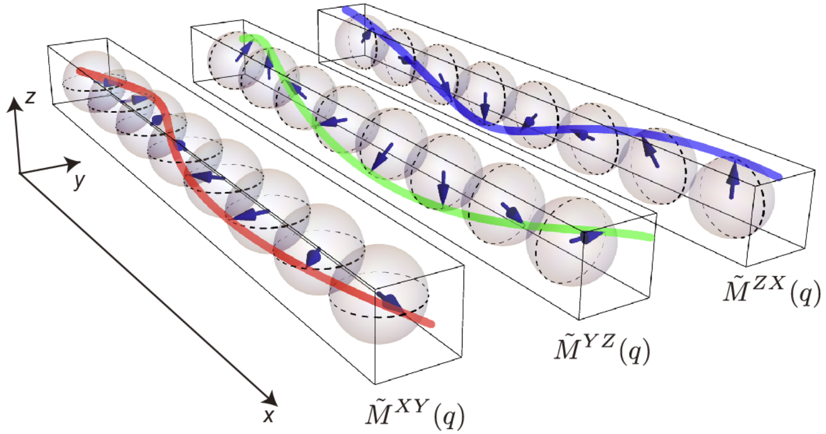


"Just another platform for experiments"
⇒ ***"Quantum information processor"***

*But the application is not limited to cold atoms.

- **Cooling** of system ⇒ DY, K. Morita, PRL **132**, 213401 (2024).
- **Detection** of entanglement

Spiral quantum state tomography!



☑ No single-site addressing is required!
☑ Strategy in the sequence of q's can reduce the costs.
☑ Reasonable expectation of entanglement measurement in upcoming experiments.

paper (*coming soon*)

Future works

☐ Experimental realizations

☐ Combine with other QST methods, like tensor network description of quantum state

☐ Applications to "extreme universe"