Spacetime Singularities and Black Holes

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2020 Nobel Prize in Physics

Roger Penrose

Reinhard Genzel

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was for black holes

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was for black holes and singularities

Michell (1784) pointed out that since the gravitational escape velocity is obtained from

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

setting v = c yields

$$\boxed{r = \frac{2GM}{c^2}}$$

A mass M compressed to this radius would appear black.

Exactly this same expression holds in GR!

Surface of the black hole is the event horizon. The solution in general relativity describing a nonrotating black hole was found by Schwarzschild in 1916.

$r_0=2GM/c^2$ is called the Schwarzschild radius.

Nothing happens when you cross the event horizon.

Hawking (1974) showed that when quantum effects are included, black holes radiate an essentially thermal spectrum, so they have a temperature T (and entropy) and follow the laws of ordinary thermodynamics.

If nothing falls in, black holes will evaporate.

Outline

I. Singularities inside black holes

II. Singularities on black holes

III. Singularities outside black holes

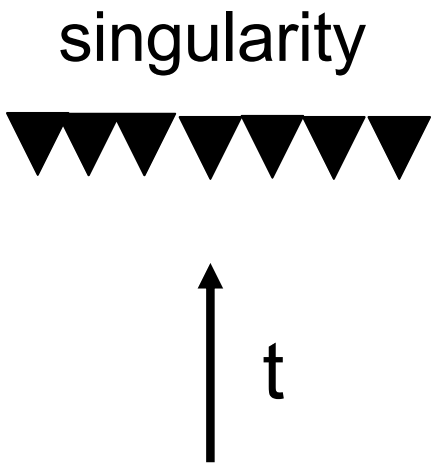
Singularities inside black holes

Spacetime singularity: a region of spacetime where general relativity breaks down due to infinite curvature i.e. infinite gravitational fields.

Penrose showed that generic gravitational collapse produces a spacetime singularity (1965).

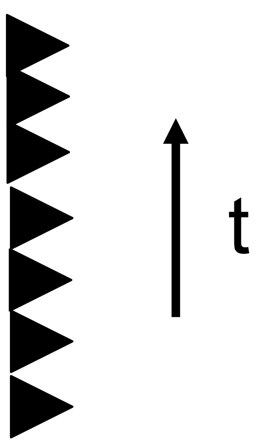
This usually happens deep inside the black hole.

For a Schwarzschild black hole ($J = 0$, $Q = 0$) the singularity is not at the center. It is in your future.



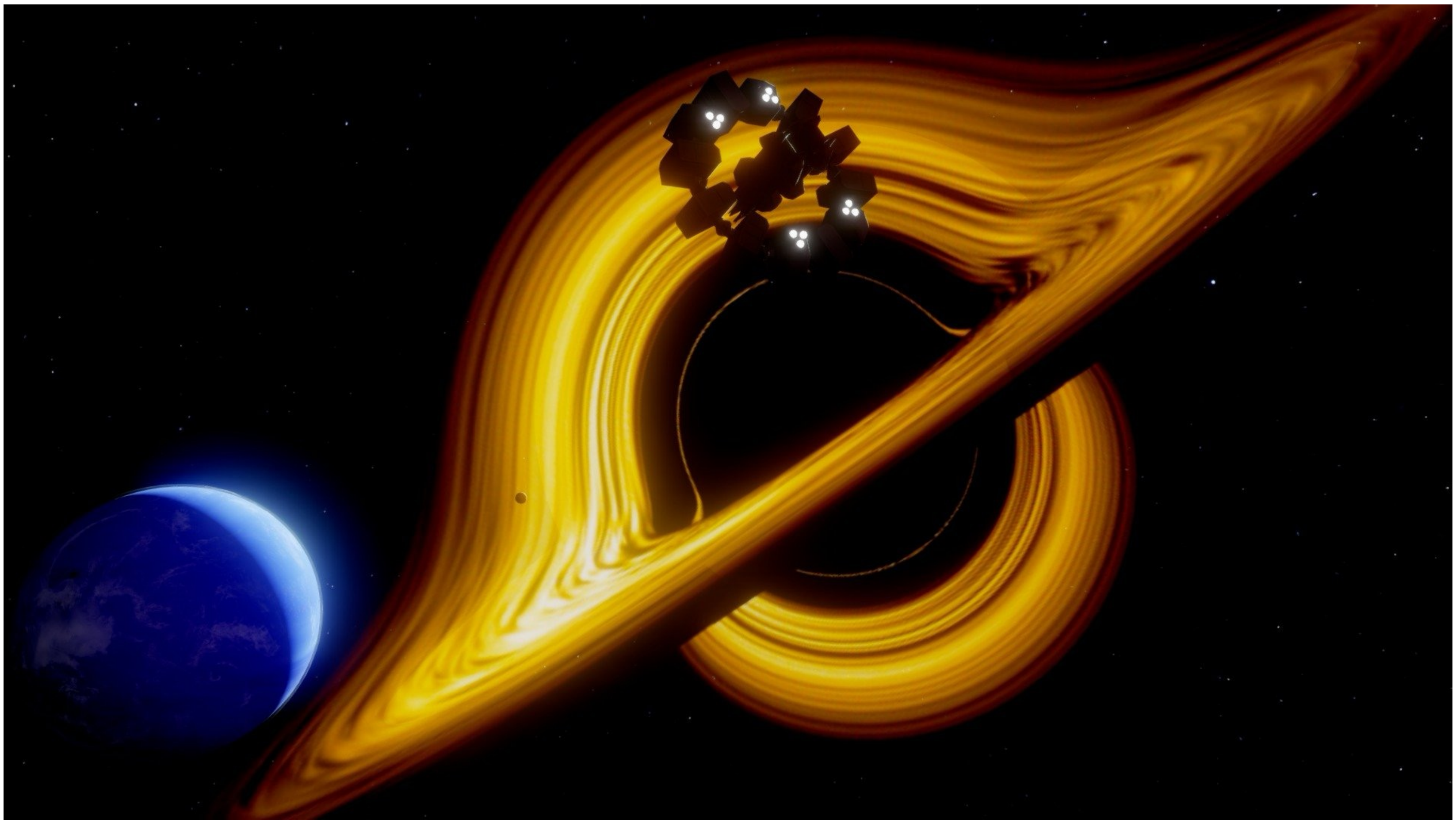
It is like an anisotropic big crunch in cosmology.

But real black holes rotate, and the story inside is now different. The stationary solution was found by Kerr (1963) and looks like this deep inside, so you can travel through.



You can also travel into your past near the singularity!

So there was some truth in the movie Interstellar



Unfortunately, this region of the rotating black hole interior is unstable.

(Simpson and Penrose 1973,…, Dafermos 2005)

Small perturbations will form a singularity that blocks off this entire region.

However there appears to be an exception.

Charged black holes have a similar singularity inside (but no time travel).

The universe is accelerating and the simplest explanation is to add a cosmological constant $\Lambda > 0$.

It was shown in 2017 that when $\Lambda > 0$, the interior of a black hole with large charge would be stable.

(Hintz and Vasy, Cardoso et al)

However, quantum effects appear to restore the instability. The quantum stress energy tensor $\langle T_{ab} \rangle$ diverges on a surface inside the black hole preventing any travel through.

(Hollands, Wald, and Zahn 2020)

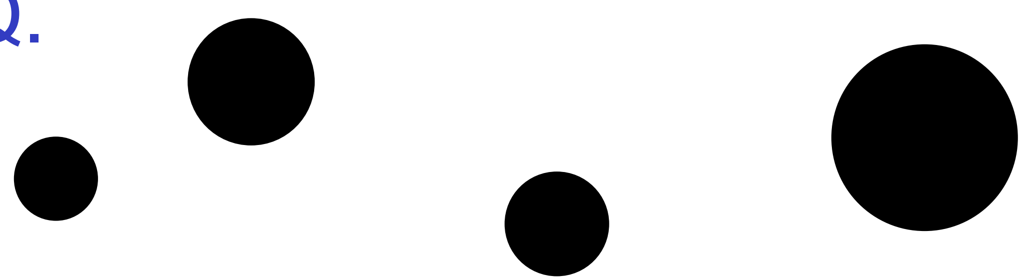
Singularities on black holes

There is a maximal Q or J that a black hole of given M can carry.

Black holes that saturate this bound are called extremal.

Extremal black holes have T = 0.

No force between extremal BHs with J = 0 and same sign of Q.



Some black hole solutions

D = 4 Einstein-Maxwell theory:

$Q \neq 0$: Reissner (1916) Nordstrom (1918)

$J \neq 0$: Kerr (1963)

$Q \neq 0$ and $J \neq 0$: Newman et. al. (1965)

D > 4 Einstein-Maxwell theory:

$Q \neq 0$: Tangherlini (1963)

$J_i \neq 0$: Myers & Perry (1986)

These all have smooth extremal limits.

Strongly suggests that all extremal black holes have smooth horizons.

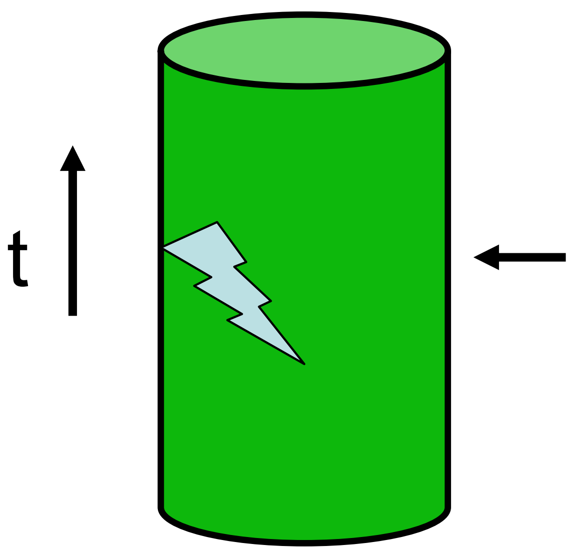
This is completely wrong!
Generic extremal black holes have singular horizons.

Near extremal black holes generically have anomalously large (but finite) curvature.

Anti-de Sitter space

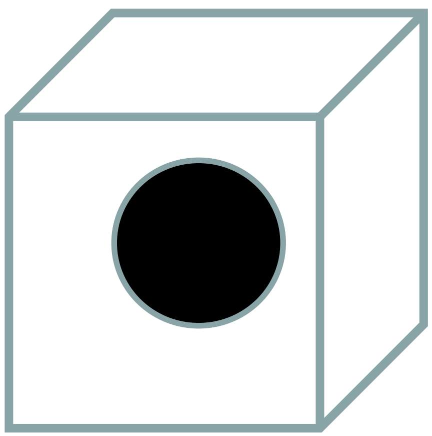
It is hard to confine gravity inside a box.

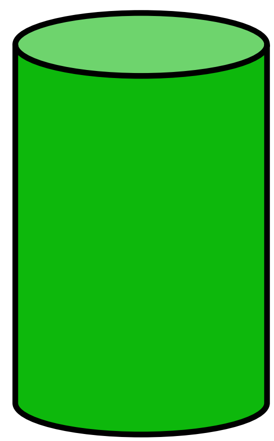
The best "box" is anti-de Sitter space (AdS). Add cosmological constant $\Lambda < 0$.



Boundary at infinity.
Light can reach it in finite time.
Need to impose boundary conditions.

Suppose you put a black hole inside a steel cage, or inside AdS with inhomogeneous boundary conditions at infinity. What happens to the horizon?

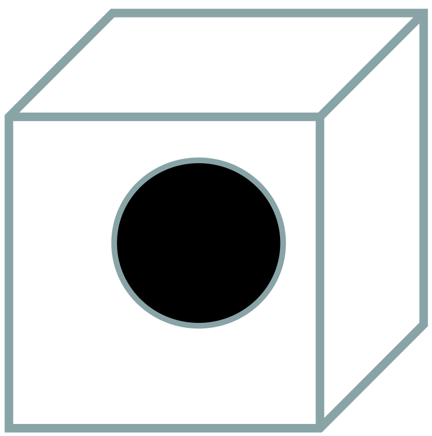


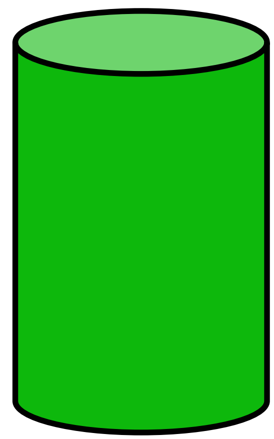


A_t not constant or S^2 not round on boundary

For nonextremal black holes, the horizon becomes distorted but remains smooth.

Suppose you put an extremal black hole inside a steel cage, or inside AdS with inhomogeneous boundary conditions at infinity. The horizon is now infinitely far away.





A_t not constant or S^2 not round on boundary

GR with $\Lambda = 0$:

Horizon is unaffected

GR with $\Lambda < 0$:

Horizon becomes singular

Generic, nonspherical extremal black holes in AdS are singular.

(Kolanowski, Santos, and GH, 2022)

They have a metric that is continuous but not differentiable at the horizon. The horizon is singular, and tidal forces diverge for ingoing observers.

Quantum and stringy effects produce higher derivative corrections to general relativity.

They are usually negligible when describing large macroscopic objects.

So we often treat general relativity as the leading term in a low energy effective action and expect it to give a good description of large objects.

This is not true for extremal black holes!

(Kolanowski, Remmen, Santos, and GH, 2023 and 2024)

Extreme rotating and charged black holes are very sensitive to higher derivative corrections to GR. Even small higher derivative terms cause large extremal black holes to have singular horizons.

Effective field theory breaks down.

In 5 spacetime dimensions, black holes can have two angular momenta J_i.

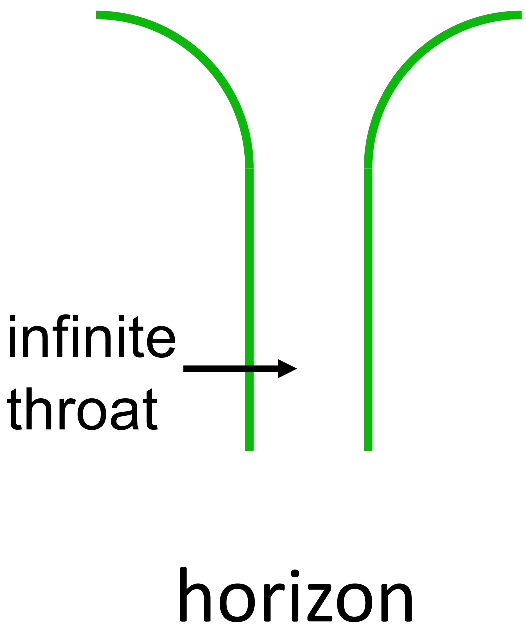
The general solution for rotating, charged black holes in 5D Einstein-Maxwell theory has never been found.

Why?

Perhaps because: It has a singular extremal limit. Only smooth for $Q = 0$ and both $J_i \neq 0$, or both $J_i = 0$. (Santos and GH, 2024)

Basic reason for this

Spatial geometry of a extremal black hole:



There is a limiting geometry near the horizon with enhanced symmetry.

An extremal black hole can be reached in finite proper time for infalling observers.

Ingoing light rays have a natural (affine) parameter which remains finite at the surface of an extremal black hole.

Extremal black holes are not “at infinity”.

Near the horizon of an extremal black hole, the metric takes the form

$$g \approx g_{NH} + \rho^{\gamma} h$$

smooth

Smooth near horizon geometry

Affine distance to horizon

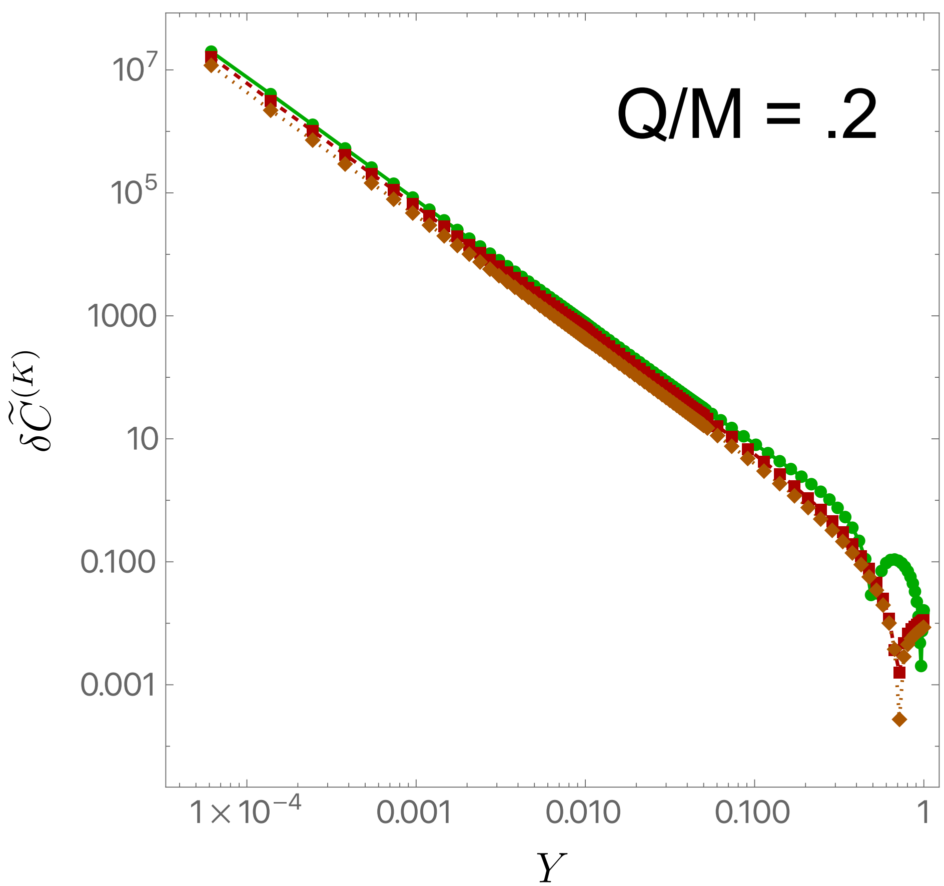
In the known exact solutions, $\gamma = 1$ and the horizon is smooth.

Generically, γ is not an integer, and the horizon becomes singular.

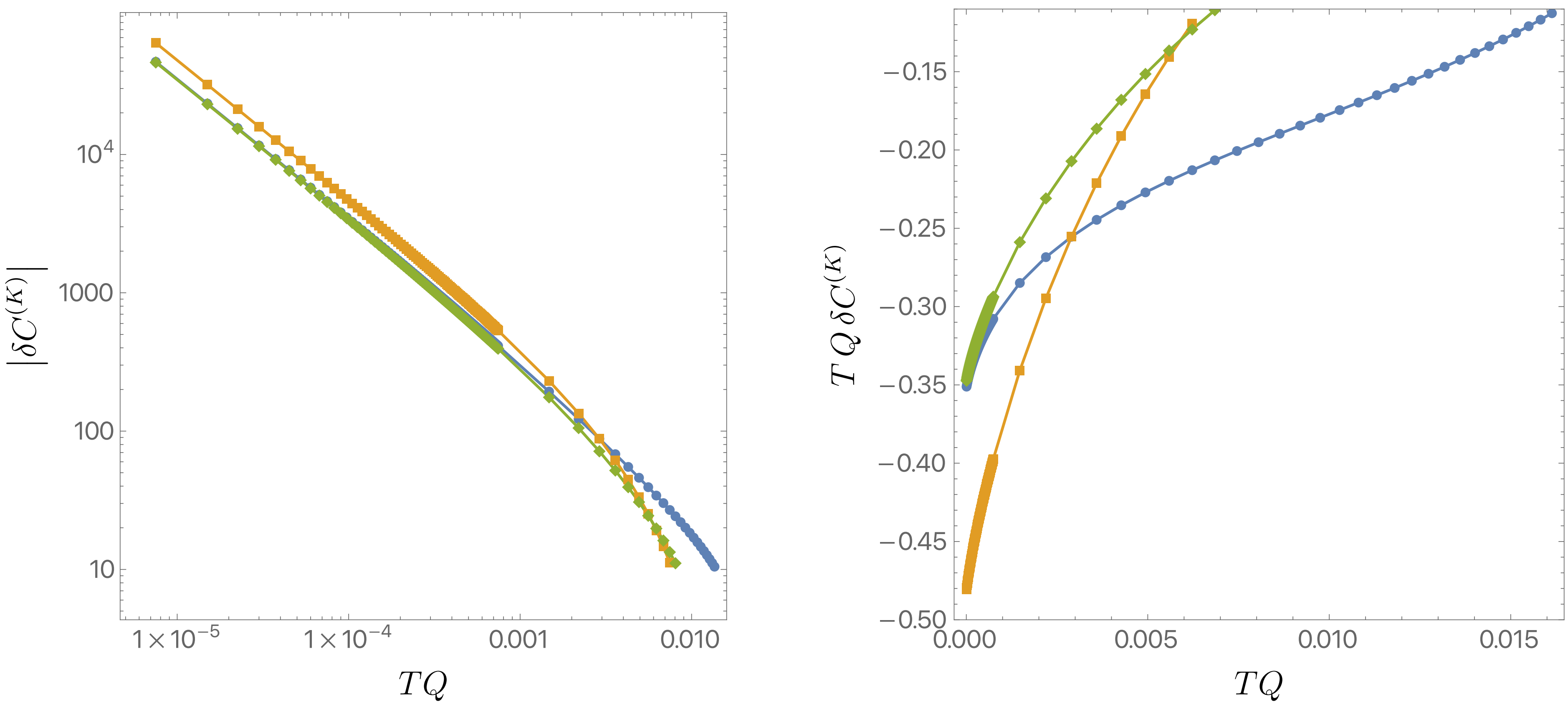
Example of effect of higher derivative corrections on extreme black hole

Log-log plot of difference between EFT corrected curvature and extremal Kerr-Newman black hole with same Q, J.

Y = 0 is horizon
Y = 1 is infinity



Approaching the extremal limit
(with Q = J/M)



Singularities outside black holes

Cosmic Censorship Conjecture

Penrose (1969): Smooth initial data in general relativity cannot evolve to form singularities visible from infinity, i.e., naked singularities.

50 years later, this conjecture is still open.

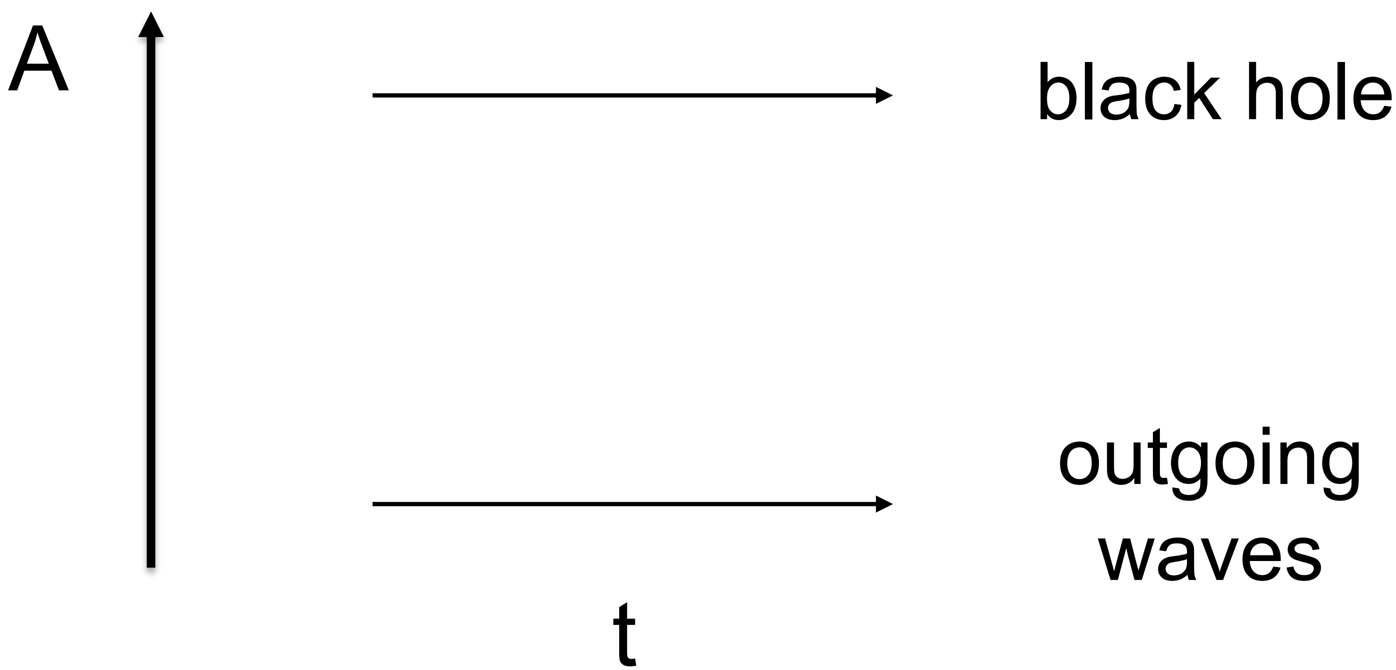
Approaches to Cosmic Censorship

- Prove mathematical theorems
- Evolve initial data numerically
- Look for counterexamples

Naked singularities can form!

(Christodoulou, Choptuik, 1990’s)

Start with a spherical wave with a profile: A p(r).
Evolve in general relativity with various A.



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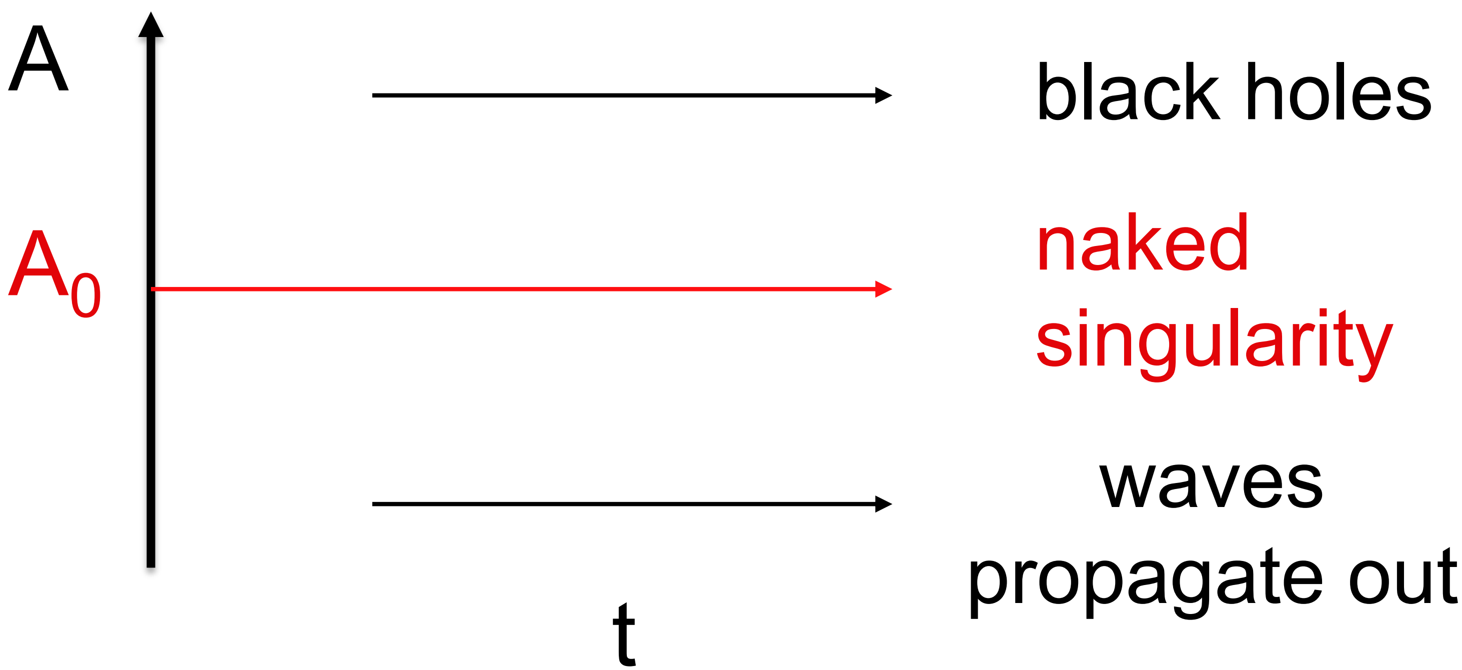


Cosmic Censorship Conjecture (revised)

Generic smooth initial data in general relativity cannot evolve to form naked singularities.

Many of us hope this is false, so we might directly observe effects of quantum gravity (expected at 10^{-33}cm, or 10^{19}GeV).

Generalizing Cosmic Censorship

Since this problem is so hard, people have studied a similar question in different settings:

1) What if space has more than three dimensions?
2) What if we add a cosmological constant Λ?

Black holes are less constrained in higher dimensions

- They are no longer characterized by just M, Q, J
- They can have nonspherical topology

- They can be unstable

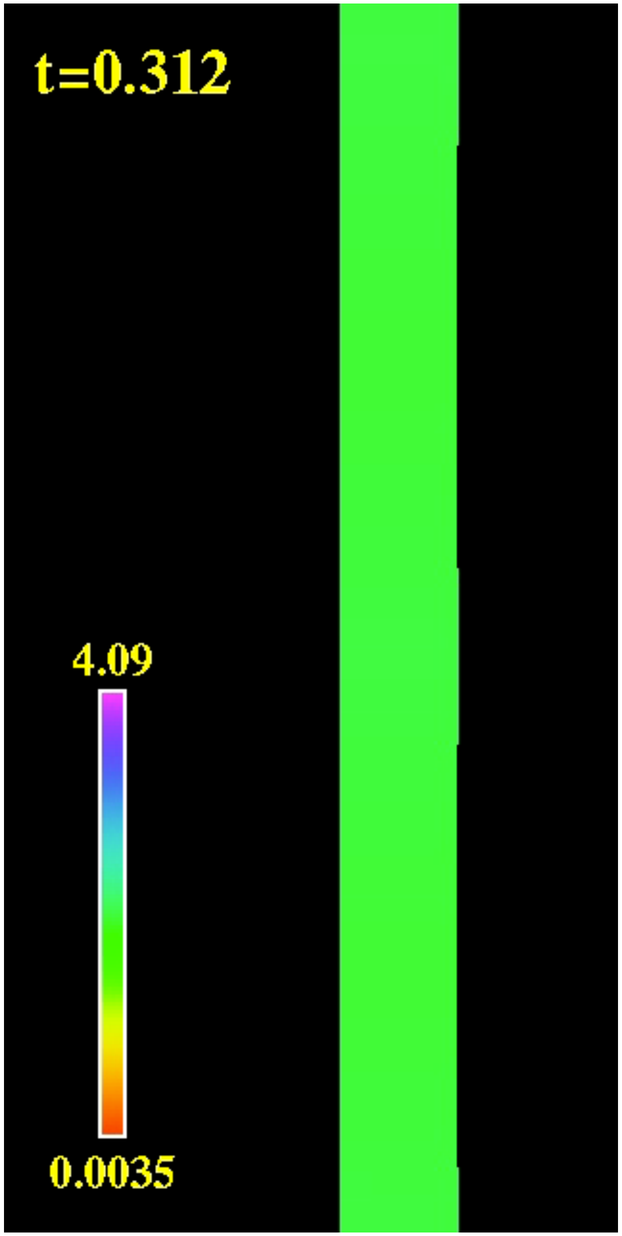
CC fails in higher dimensions

Small perturbations of an unstable black hole cause the horizon to pinch off. When it does, you generically form a naked singularity.

Example: In one extra dimension, there are "black strings" with horizon $S^2 \times R$ that are unstable (Gregory and Laflamme, 1993).

Evolution of perturbed black string

(Lehner and Pretorius, 2010)



The black string wants to break up into spherical (higher dimensional) black holes.

Add a cosmological constant

If $\Lambda < 0$, evolution requires boundary conditions at infinity.

For reasonable boundary conditions, one can violate (the spirit of) cosmic censorship generically (in just three space dimensions).

A counterexample

(Santos, Way, GH, 2016)

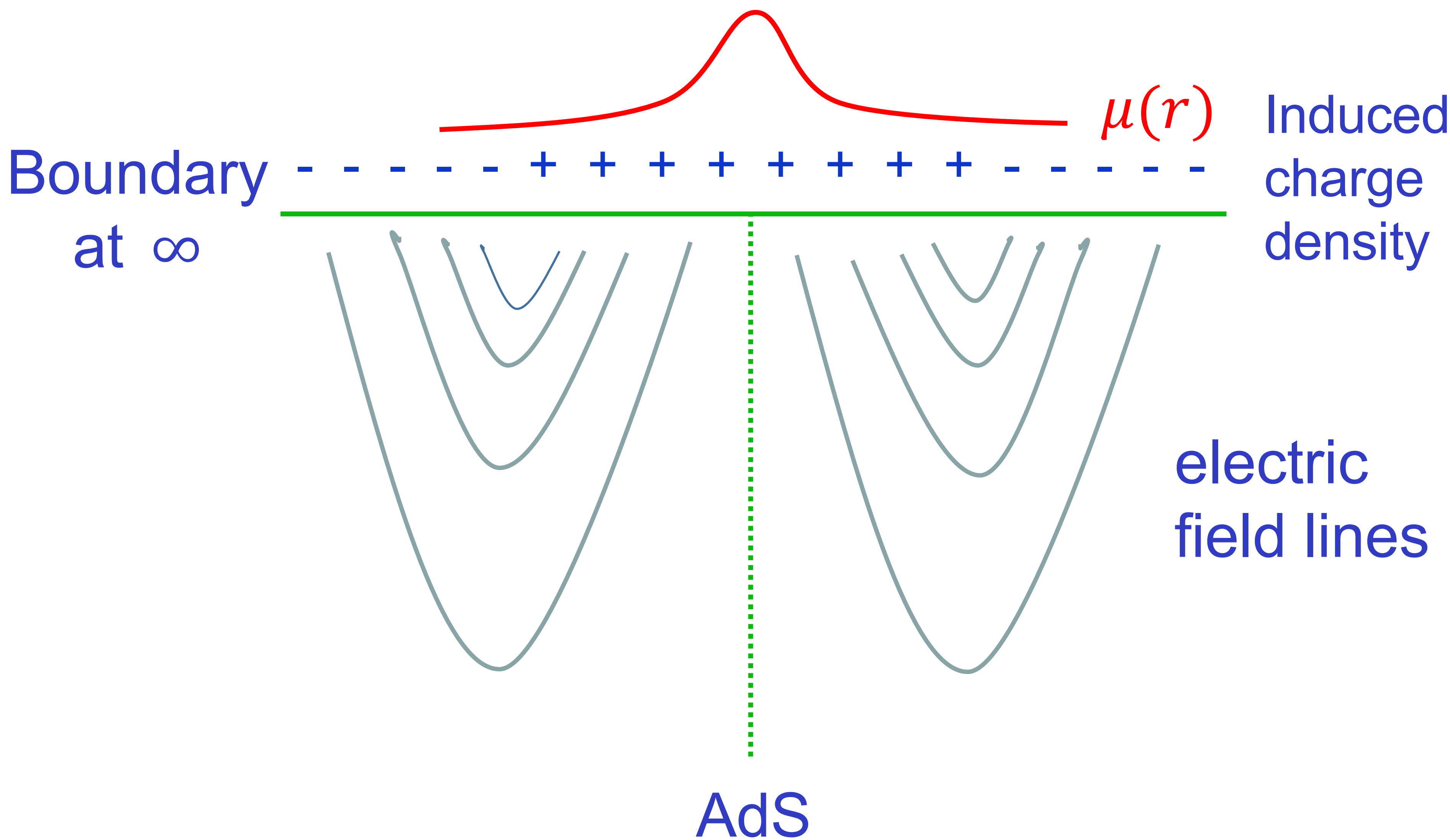
Consider gravity coupled to a Maxwell field with $\Lambda < 0$. Fix the metric and vector potential on the AdS boundary to be

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2, \quad A = \mu(r) dt$$

where $\mu(r) = a\, f(r)$ and $f \to 0$ as $r \to \infty$.

Solutions describe static, self-gravitating electric fields.

Smooth solutions exist only for $a < a_{max}$.


$\mu(r)$
Induced charge density
Boundary at ∞
- - - - - + + + + + + + + + - - - - -
electric field lines
AdS

Now make the amplitude time dependent: $a = a(t)$. Start with $a = 0$ and slowly increase to $a(\infty) > a_{max}$.

Bulk can't settle down to a smooth solution. Expect the curvature to grow without bound violating cosmic censorship.

This has been confirmed by a full time dependent numerical relativity calculation.

(Crisford and Santos, 2017)

Comments

1. The blow-up is not just on the axis, but over a large region.
2. The singularity does not form in finite time, but this clearly violates the spirit of cosmic censorship.
3. This violation is stronger than seen in higher dimensions.

Summary

- In principle one can travel through black holes to other regions of the universe, but in practice this is probably forbidden.
- Generic extremal black holes have singular event horizons.
- Naked singularities are common in D > 4 or $\Lambda < 0$, and might even occur in D = 4 with $\Lambda = 0$.