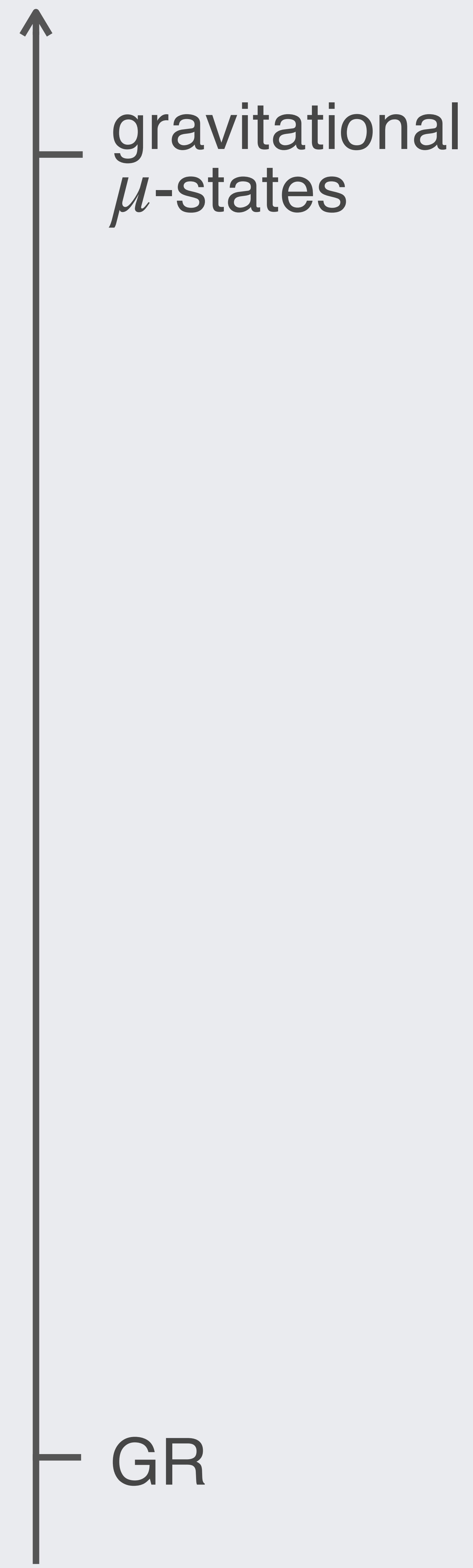
holography

$\boxed{\text{chaos}}$

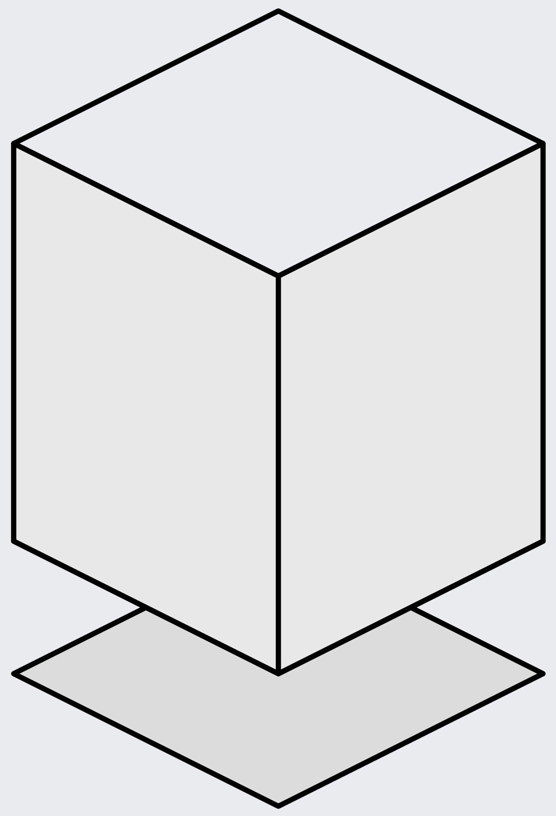
$\hookrightarrow$

quantum
gravity

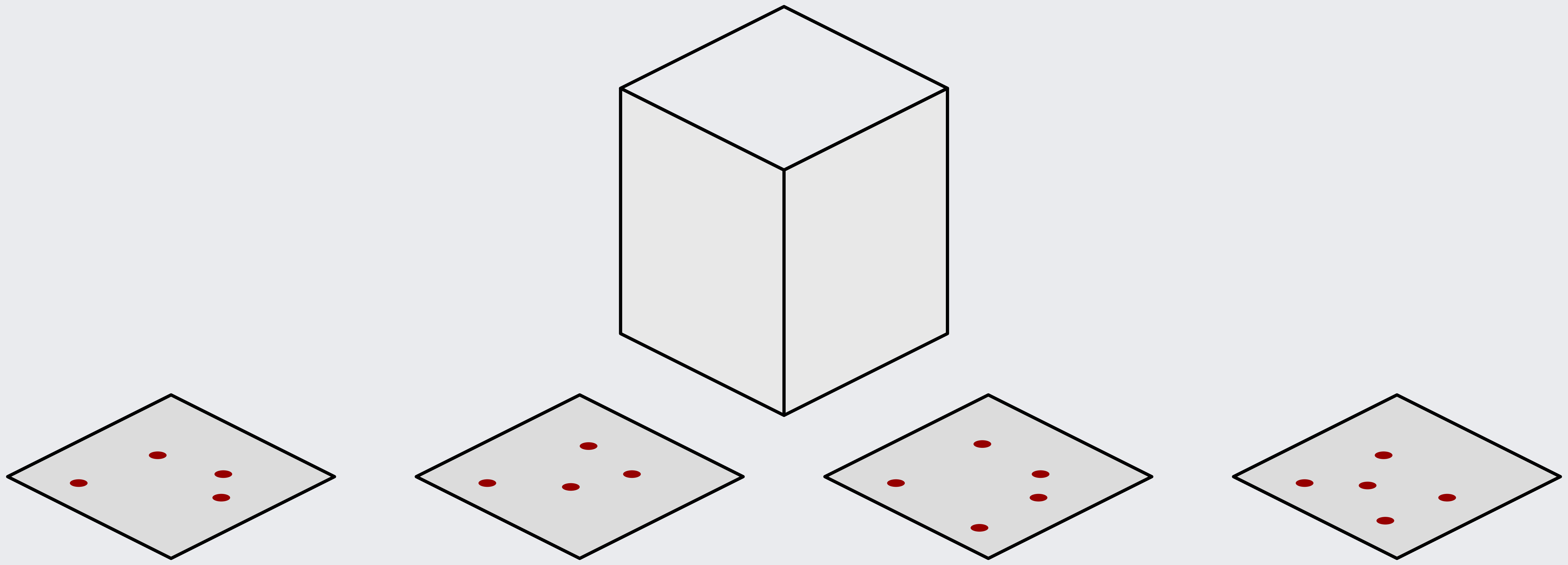


Holographic principle

Orthodox reading: duality between a quantum theory and a gravitational theory in one dimension higher (cf. Maldacena, 97).



Low dimensional holography: duality between an ensemble of quantum theories a gravitational theory



This talk:

- ☐ Review of late time quantum chaos in 2d holography
- ☐ The gravitational ensemble
- ☐ Random matrix theory through a topological lens
- ☐ Microstate spectrum of 2d gravity

Late time chaos in 2d gravity

Kyoto, Sept. 25

Alexander Altland (Cologne),
Julian Sonner (Geneva),
Boris Post, Jeremy van der Hayden, Erik Verlinde (Amsterdam)
Jac Verbaarschot (Stony Brook)

quantum chaos review

semiclassical chaos in gravity

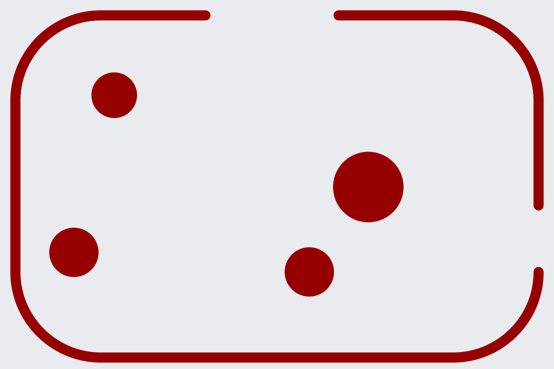
quantum chaos in gravity

arXiv:2008.02271, SciPost. 22
arXiv:2204.07583, SciPost 23
arXiv:2403.13516

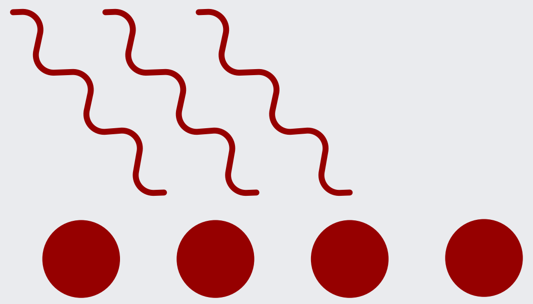
chaos (review)

The ergodic phase of quantum chaos

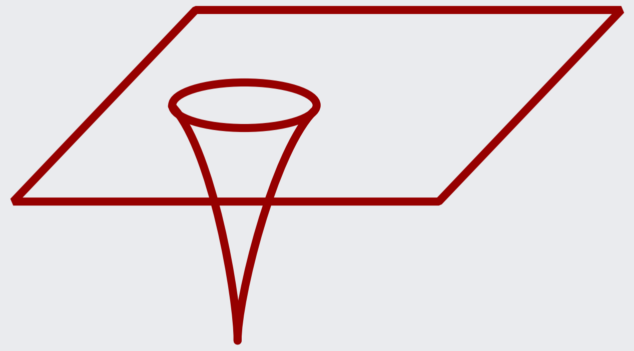
Realized in



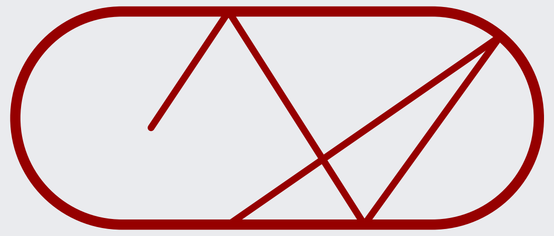
cond-mat



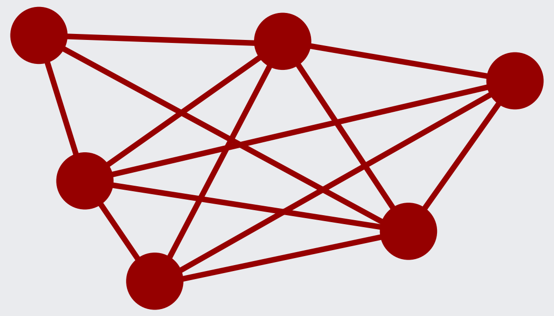
AMO



gravity



few body quantum chaos

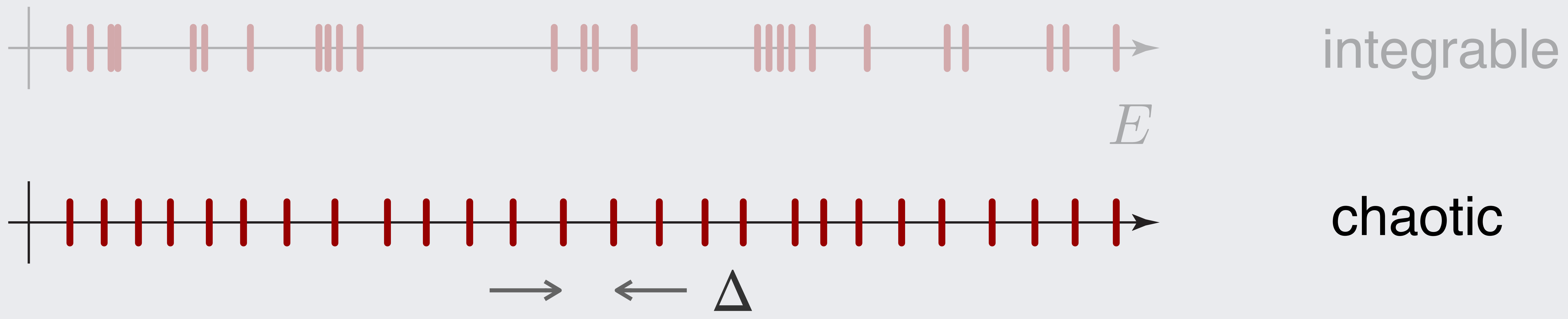


strongly correlated systems

The ergodic phase of quantum chaos

- states uniformly Gaussian distributed in Hilbert space (max. entropy, cf. ETH)

- spectrum shows high degree of order



$$\boxed{s = \frac{\pi\omega}{\Delta}}$$

quantitatively described by

$$R_2(\omega) \equiv \frac{1}{\Delta^2}\langle \rho(E+\omega)\rho(E)\rangle_c \longrightarrow R_2(s) \longrightarrow \begin{cases} \text{10 different antilinear symmetry classes:} \\ \text{A, AI, AII, AIII, BDI, C, CI, CII, D, DIII} \end{cases}$$

aa, Zirnbauer, 97

Spectral correlation function

$$\frac{c_1}{s^2} + \frac{c_2}{s^3} + \ldots$$

deep quantum

(non-analytic in s^{-1})

semiclassical

(expansion in s^{-1})

Spectral form factor (favored by holography community)

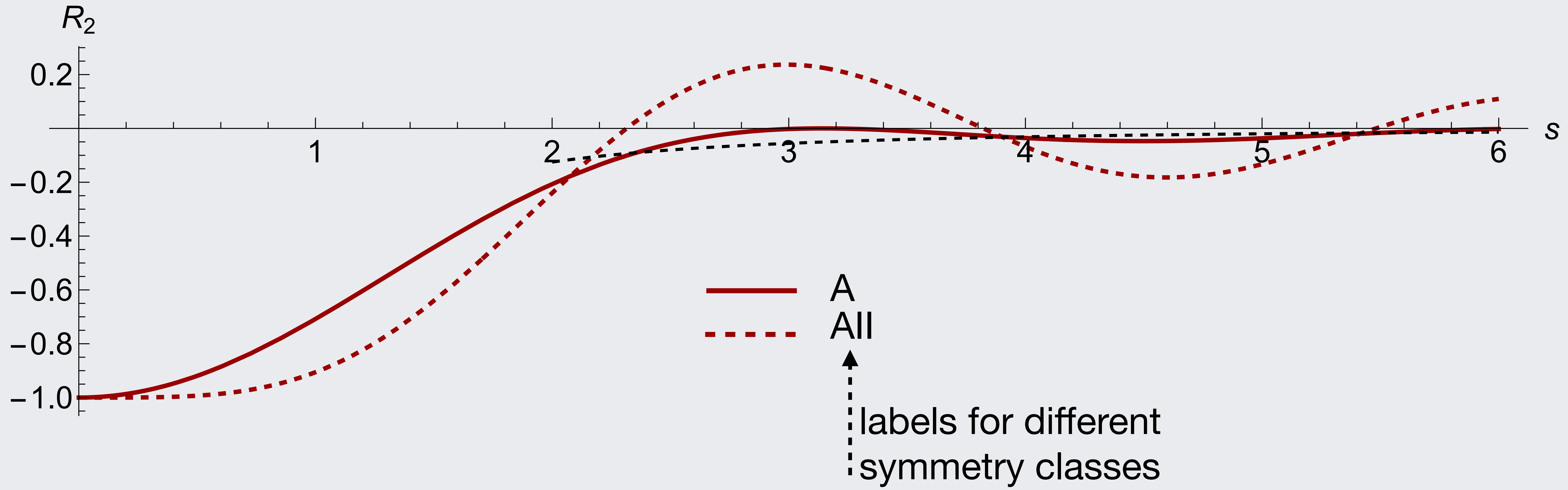
semiclassical

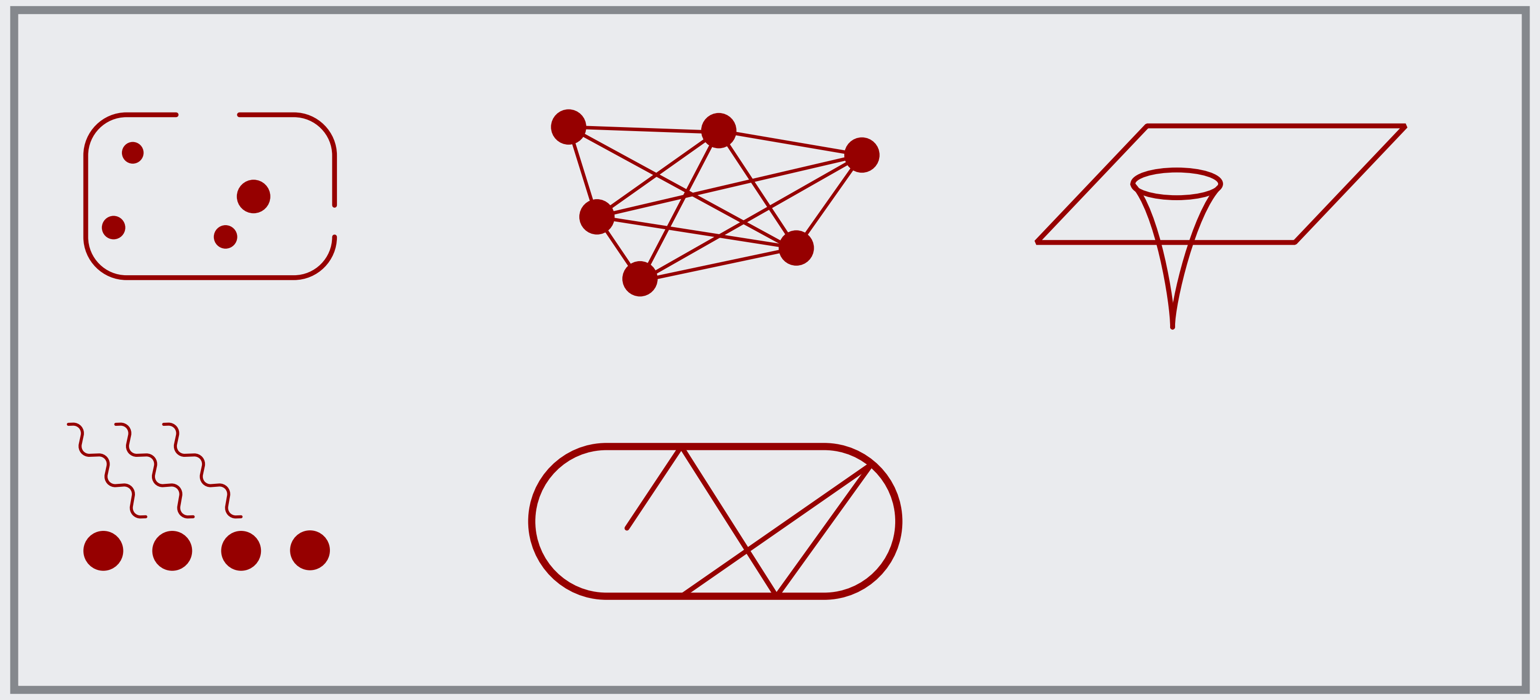
(expansion in τ)

deep quantum

(non-analytic in τ)

Spectral universality





Shown by a large class of chaotic systems.

$$R_2(s) = -\frac{\sin^2(s)}{s^2}$$ A (GUE)

$$R_2(s) = \frac{\sin^2(2s)}{(2s)^2} - \frac{d}{d(2s)}\frac{\sin(2s)}{2s}\int_0^1 \frac{\sin(2st)}{t}$$ AII (GSE)

Q: How can this level of universality be understood?

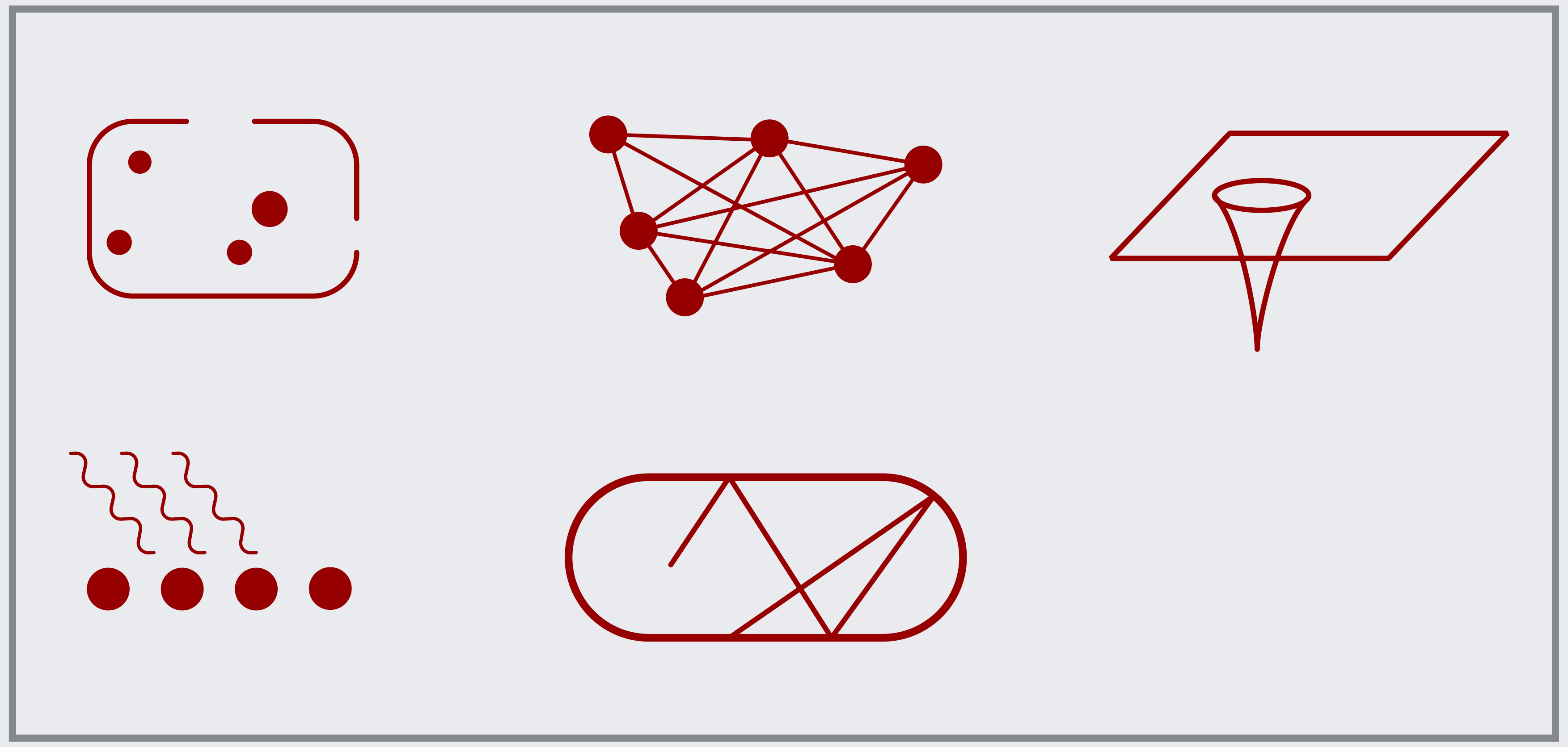
Understanding universal spectral correlations

Constructive approach (aka Bohigas–Gianonni–Schmit (GGS) conjecture)

Wigner 1955

RMT $\boxed{H = \{H_{\mu\nu}\}, \qquad \mathrm{var}(H_{\mu\nu}) = \mathrm{const}\,.}$

$\downarrow$



chaotic systems in cond-mat, AMO, …, gravity.

$\xrightarrow[\text{long times}]{\text{low energies}}$ universal correlations

cf. “Ising model of magnetism”

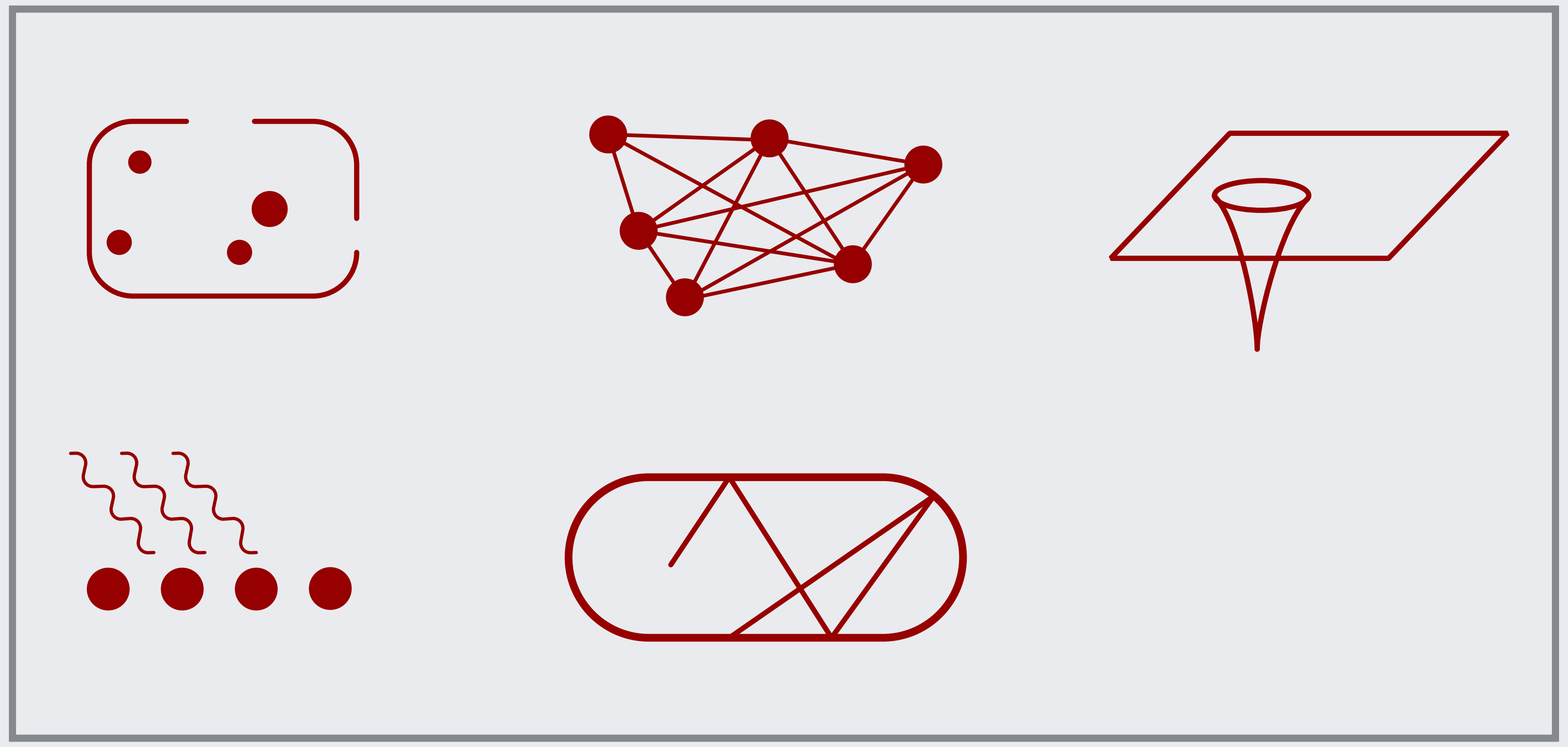
Understanding universal spectral correlations

Conceptual approach

susy nonlinear
σ-model

$$Z = \int_{X(n|n)} dQ\, e^{is\,\mathrm{tr}(Q\tau_3)}$$

Wegner 1980
Efetov 1981



chaotic systems in cond-mat, AMO, …, gravity.

low energies

long times

effective theory

cf. "ϕ^4 theory of magnetism"

Understanding universality (from matrix theory)

$$H = \{H_{\mu\nu}\} \longrightarrow \left\langle \mathrm{tr}\frac{1}{E + i0 - H} \;\; \mathrm{tr}\frac{1}{E' - i0 - H} \right\rangle, \qquad E - E' \sim s\Delta$$

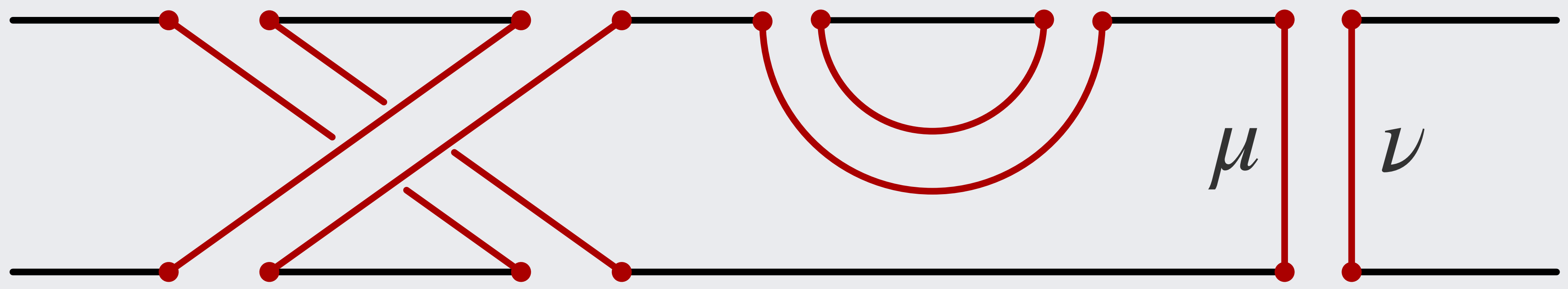
hep-th $\rightarrow$ topology

cond-mat $\rightarrow$ correlations

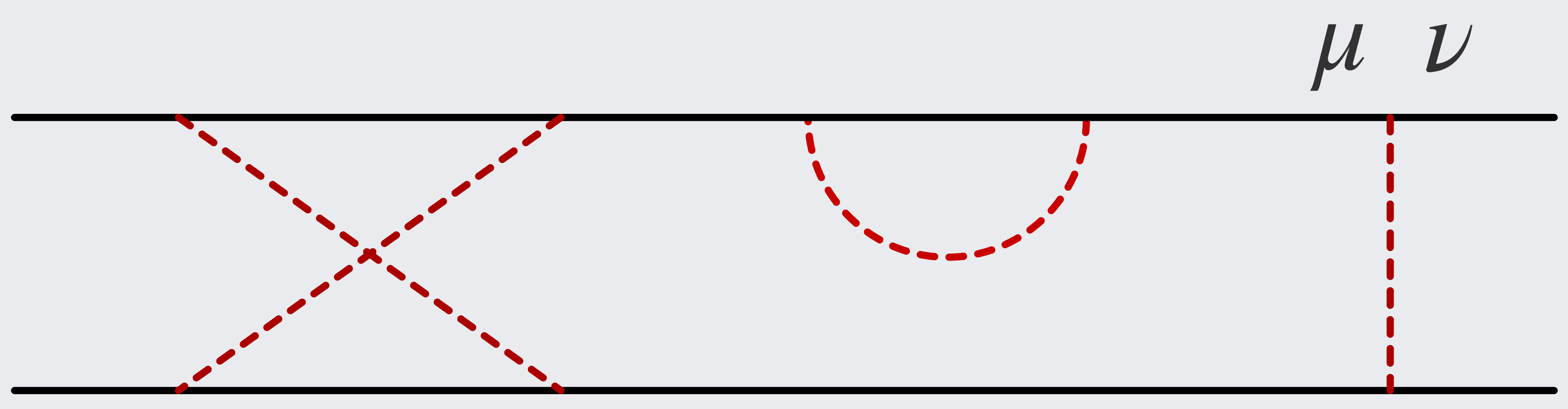
$$\downarrow$$

$\boxed{\text{expansion in } H}$

$$\downarrow$$



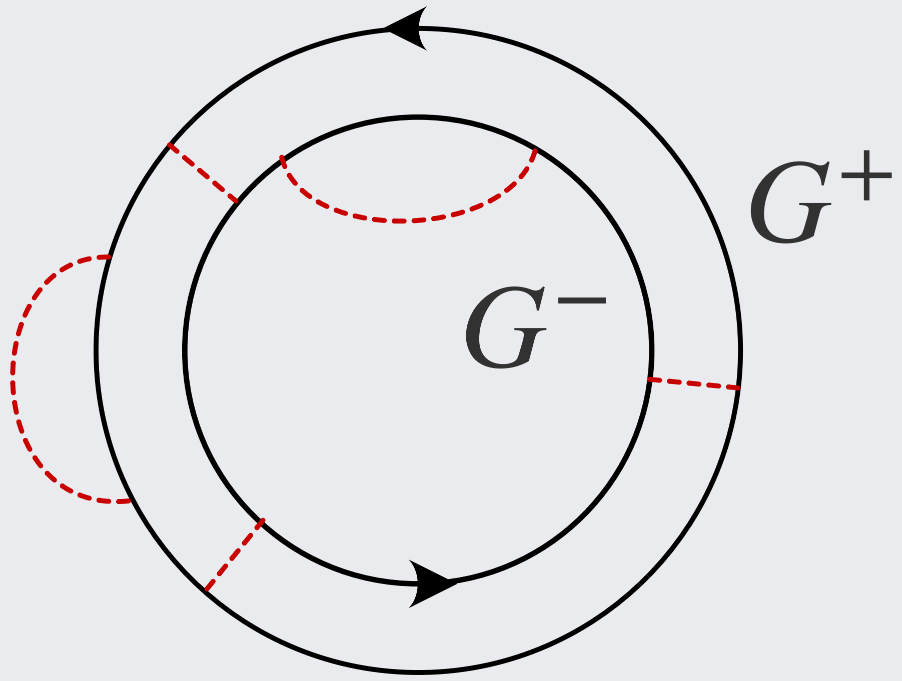
hep-th: “double line notation”



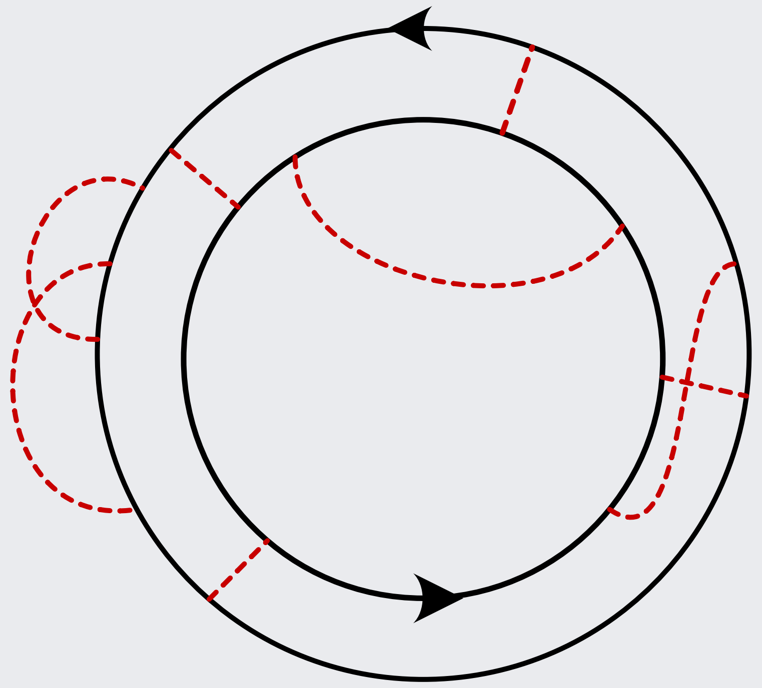
cond-mat: “impurity diagram notation”

Perturbative expansion – topological perspective

$g = 0$



$g = 1$



$\vdots$

topological expansion for 2-point function:

$$\langle G^+ G^- \rangle = \sum_g D^{-2g} R_{g,2}$$

topological recursion (symbolically):

Eynard-Orantin, 07

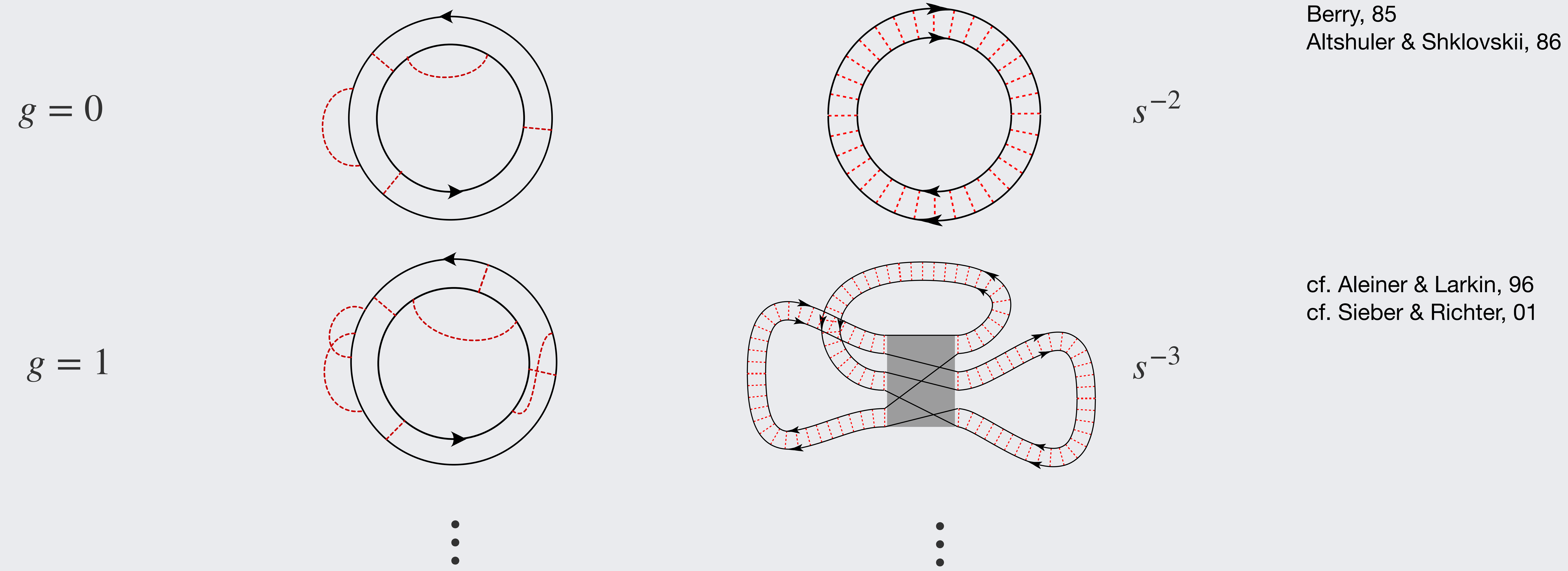
$$R_{g,n} = \mathscr{F}(R)$$

$$R = \{R_{h,n} \,|\, n = 1,2,3; h \leq g\}$$

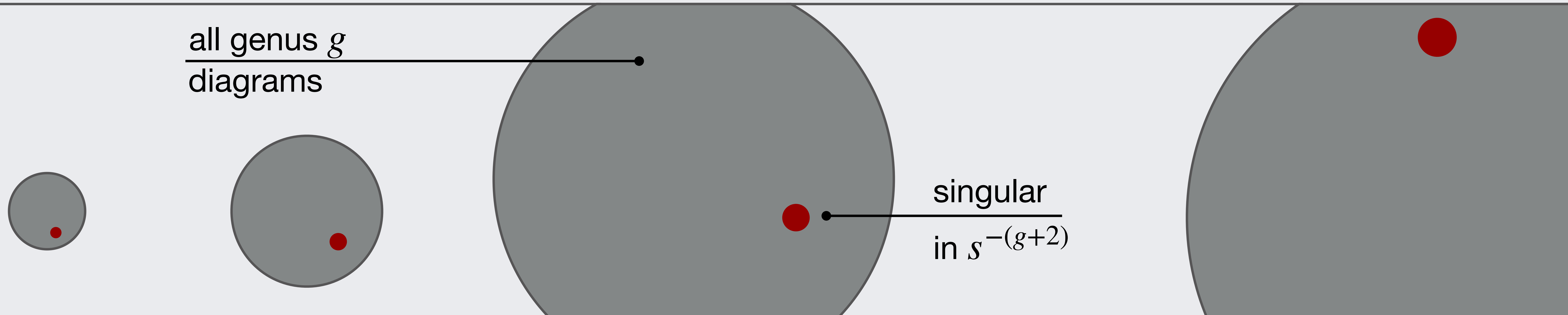
Disclaimer: for ensembles with T-invariance (AI, AII, CI, ⋯) need non-orientable surfaces

Brezin & Neuberger, 90
Stanford & Witten, 19

Perturbative expansion — correlation perspective

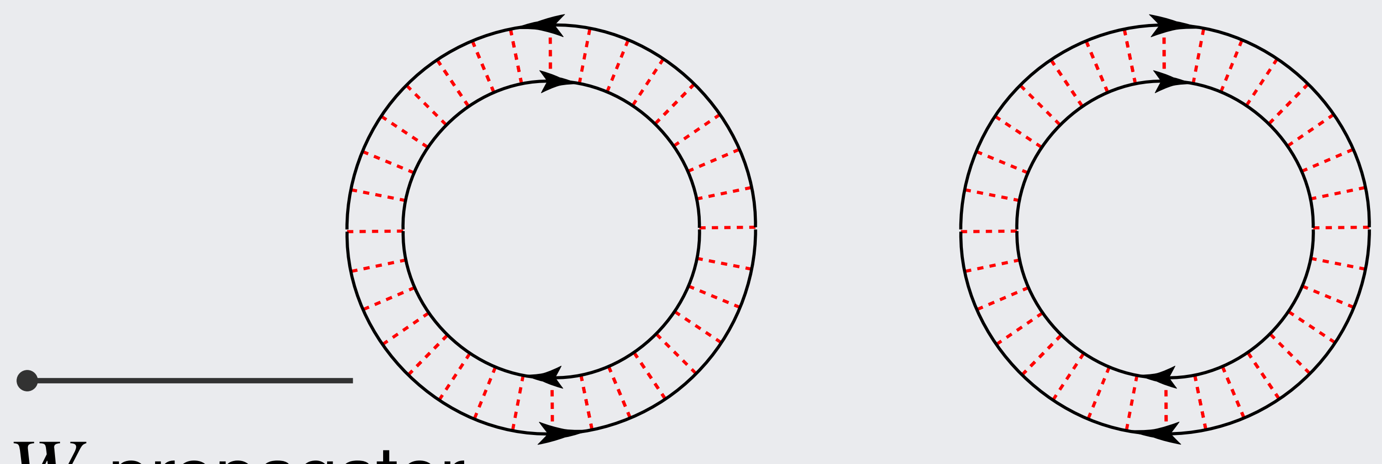


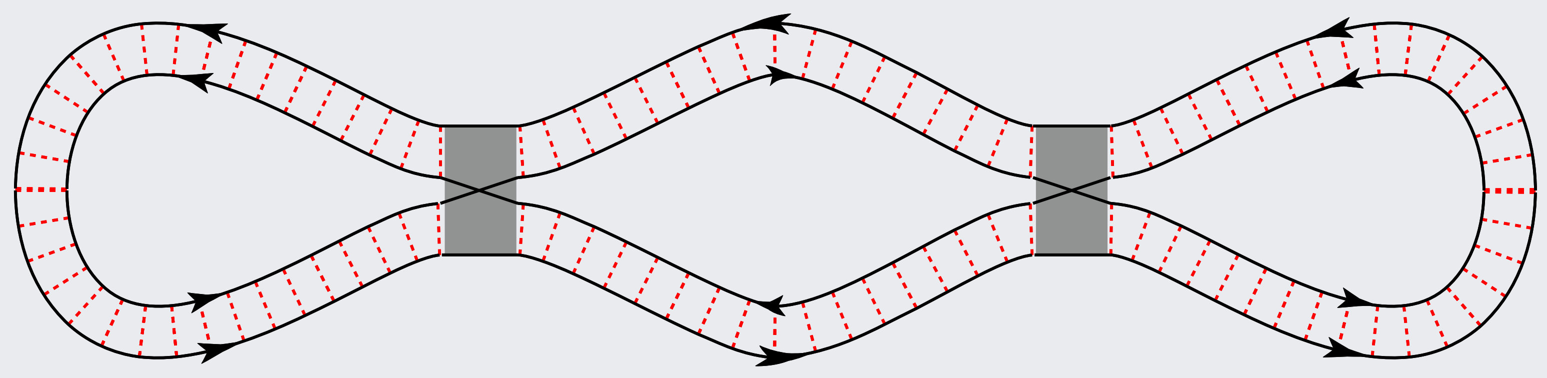
$s^{-(g+2)}$ diagrams:
small subset of $R_{g,2}$

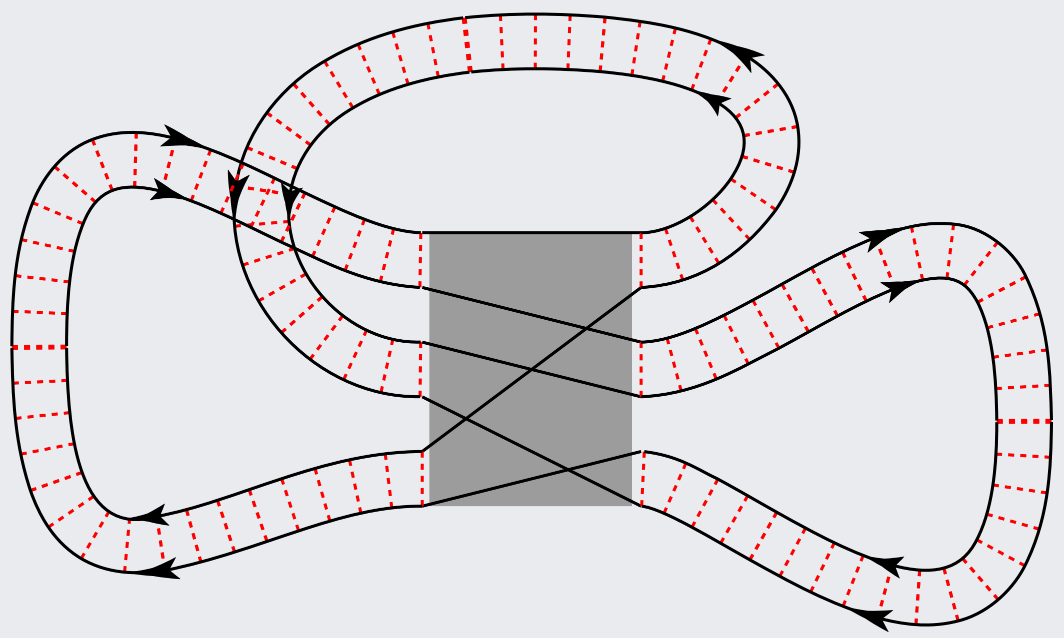


Perturbative expansion — (mean) field theory

$$Q = T\tau_3 T^{-1}, \quad T = \exp(W)$$







$$\boxed{Z = \int_{X(n|n)} dQ\, e^{is\,\mathrm{tr}(Q\tau_3)}}$$

Wegner 1980
Efetov 1981

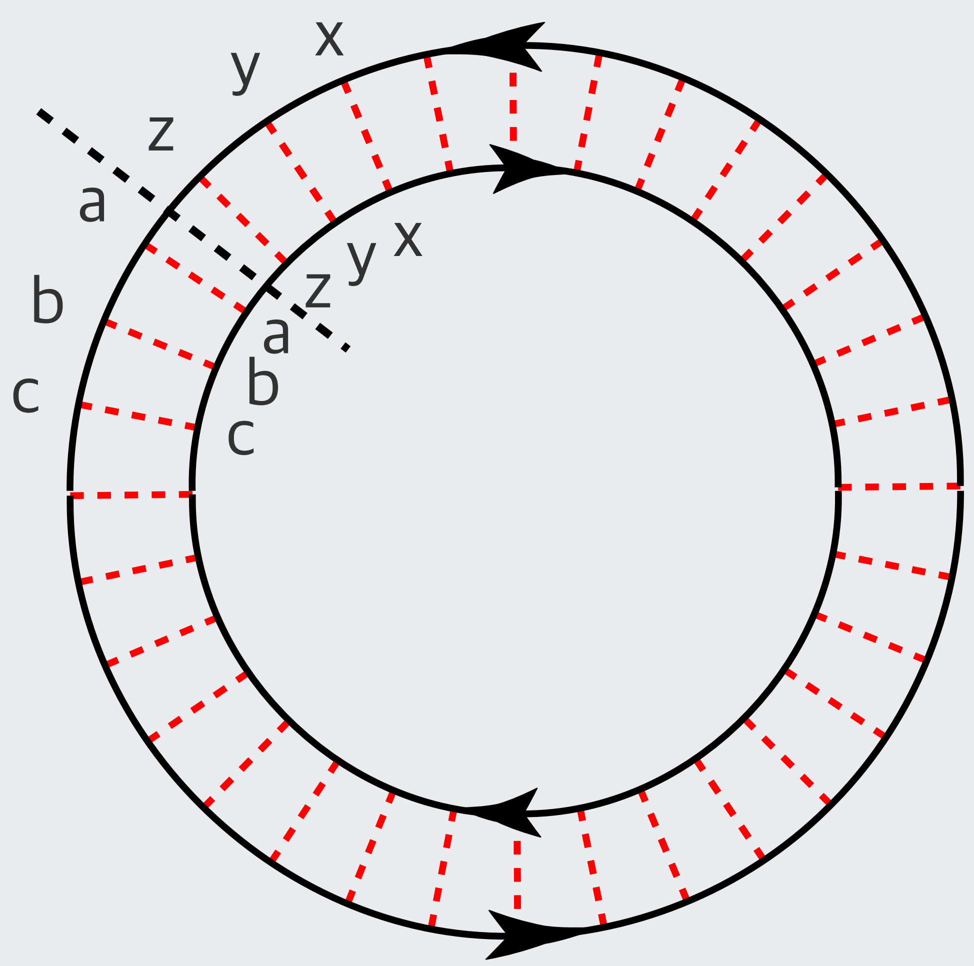
effective theory of
ergodic quantum chaos

- Q: low dimensional (flavor) matrices
- diagrams: loop expansion
- full integration: correlations beyond perturbation theory

Note (for info people)

Diagrams probing chaotic correlations are information encoders (→symbolic dynamics)

cf. Hirata & Amigó 23



a b c … x y z
a b c … x y z

semiclassical chaos in gravity

Holography background

The holographic principle: d-dimensional gravitational systems cast $(d-1)$-dimensional holographic shadows.*

Black holes are chaotic systems.

't Hooft 93, Susskind 95

> 2015 search for a simple dimensional holographic correspondence between 1-dimensional quantum chaotic boundary theory and 2-dimensional gravity theory.

Kitaev 15, …

---

* classic example: gravity in $\mathrm{AdS}_5 \times S^5$ $(d = 5) \to \mathcal{N} = 4$ super Yang-Mills $(d = 4)$

Maldacena 97

Sachdev-Ye-Kitaev Model (15)

A model of *N* randomly interacting *Majorana* fermions

$$\hat{H} = \sum_{ijkl}^{N} \underset{\text{random}}{J_{ijkl}}\, \hat{\chi}_i \hat{\chi}_j \hat{\chi}_k \hat{\chi}_l, \qquad \{\hat{\chi}_i, \hat{\chi}_j\} = 2\delta_{ij}$$

SYK model

Kitaev 15

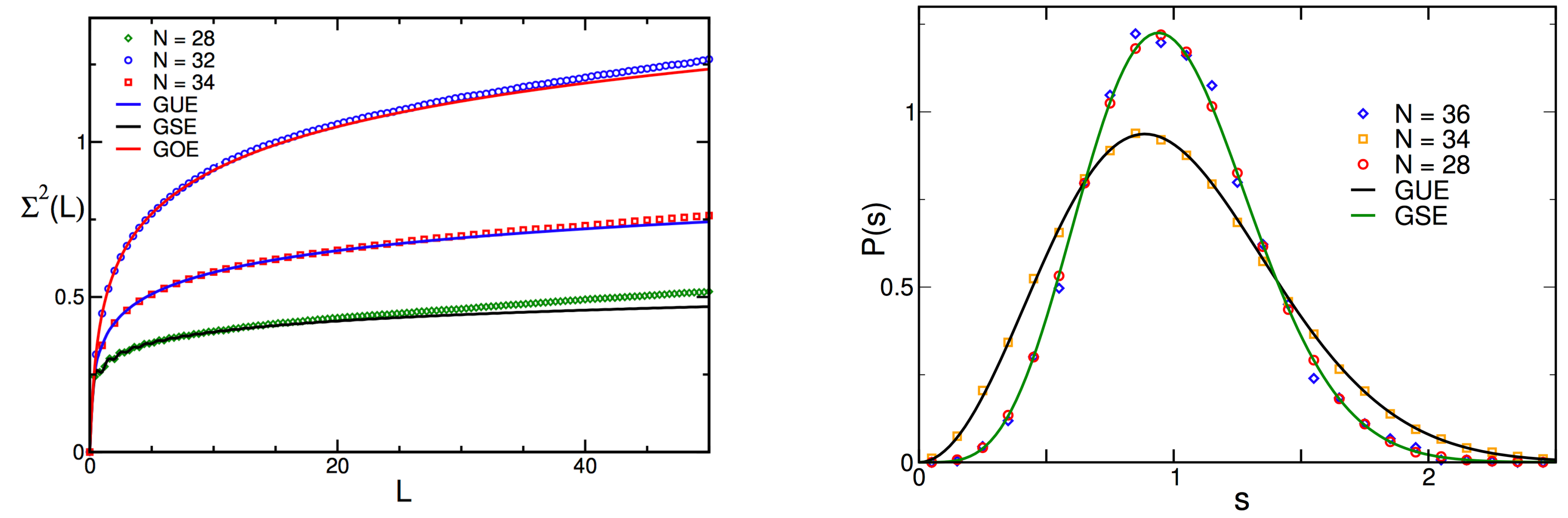
cf. Sachdev, Ye 90

cf. Bohigas, French,
Weidenmüller, …
early 70s

- ☑ one-dimension (quantum mechanics)
- ☑ hard quantum chaos
- ☑ (weakly broken) conformal invariance

SYK – quantum chaos

ergodic spectral correlations witnessed by correlation functions



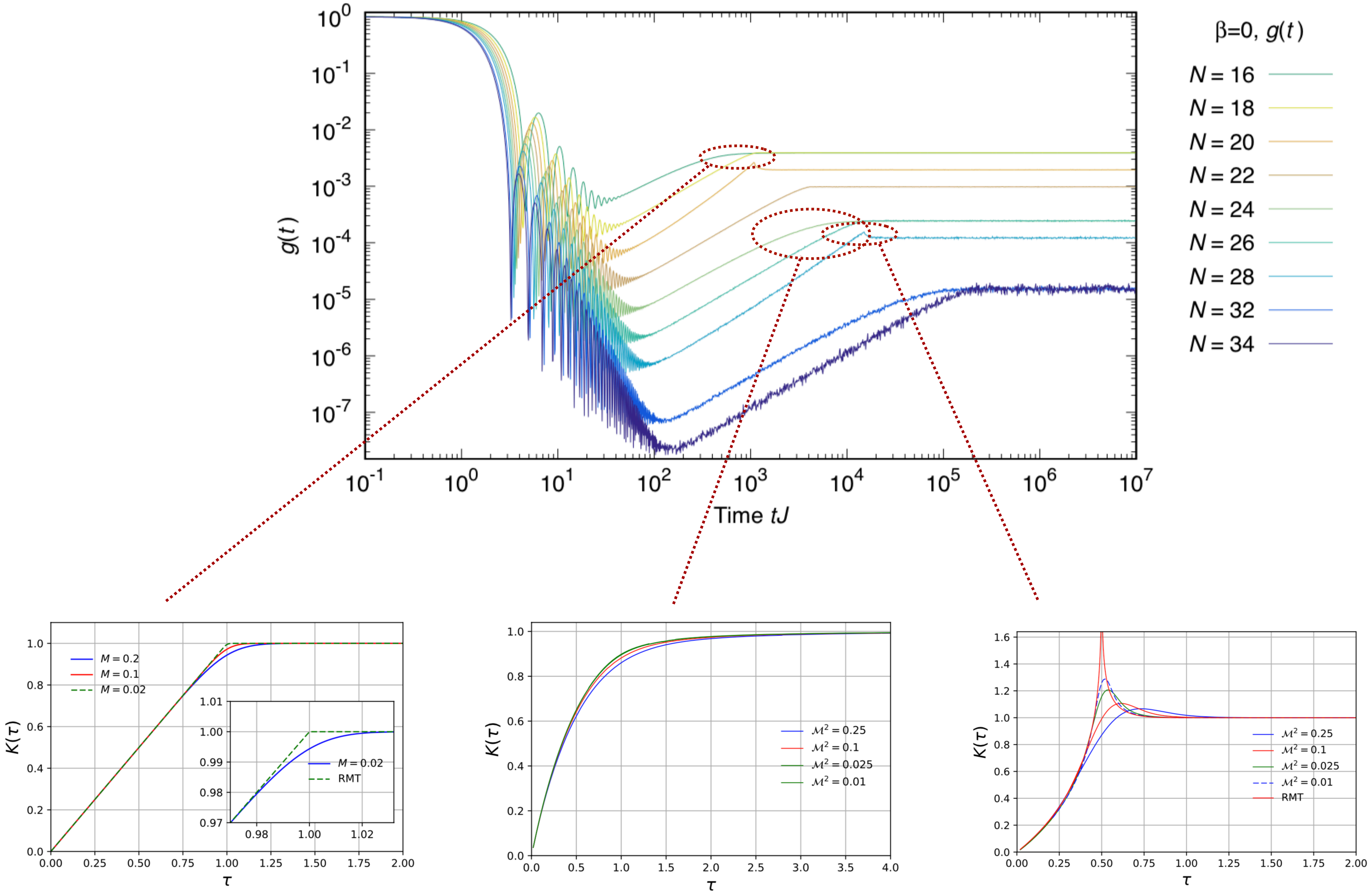
Verbaarschot, Garcia-Garcia, 16

effective field theory identified

Bagrets, aa, 18

SYK ensemble *not* in RMT universality class

aa, Micklitz, Verbaarschot, *et al.* 24



Cotler et al. 17

aa, Micklitz, Verbaarschot *et al.* 24

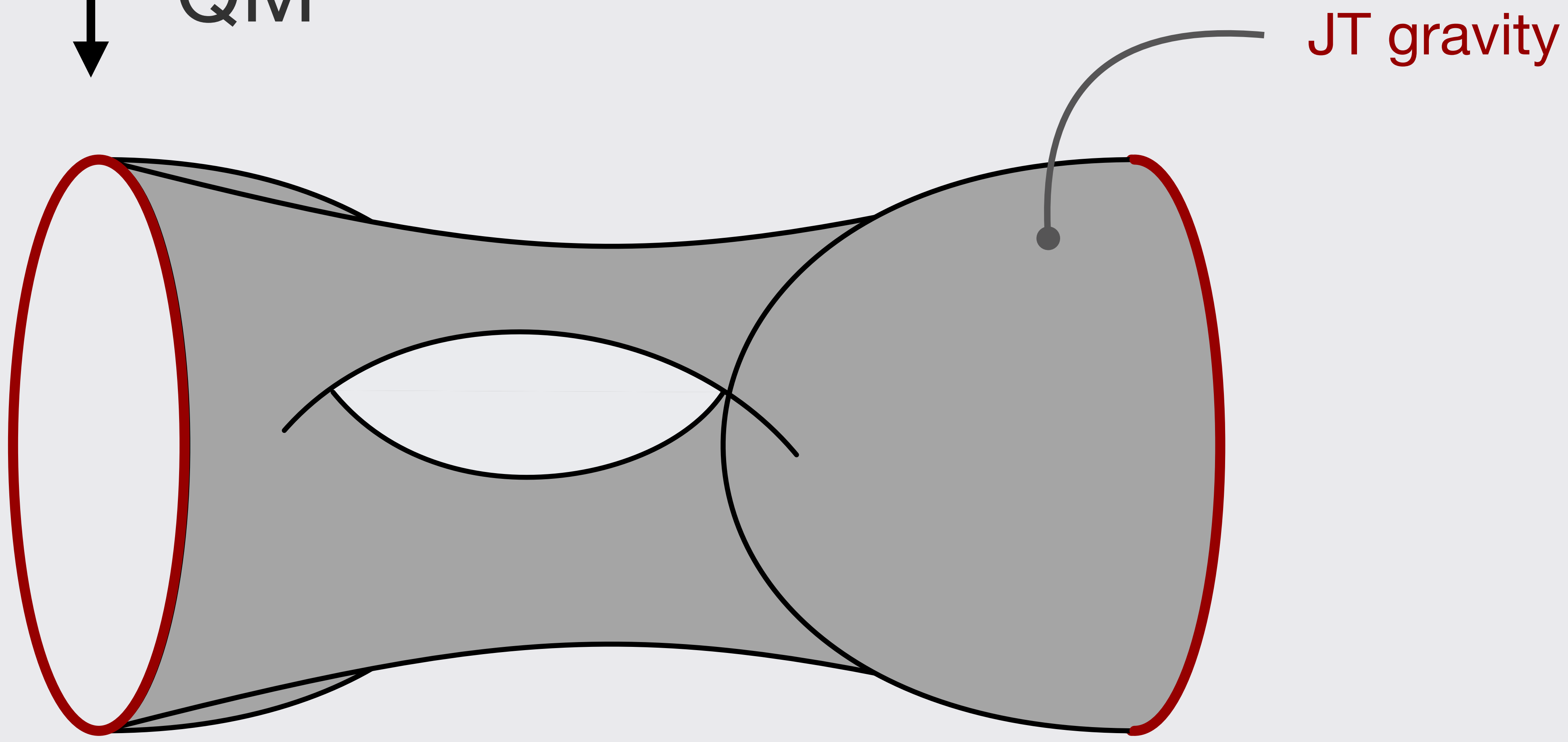
SYK holographic correspondence

$$\left\langle \quad G = \frac{1}{z-H} \qquad\qquad G = \frac{1}{z-H} \quad \right\rangle_{\text{disorder}}$$

$\updownarrow$ imag. time FT

$$\left\langle \quad Z = \operatorname{tr} e^{-\beta H} \qquad\qquad Z = \operatorname{tr} e^{-\beta H} \quad \right\rangle$$

$\downarrow$ imag. time QM



holographic principle

“euclidean wormhole”

JT gravity

Jackiw, 83
Teitelboim, 85

2d Einstein–Hilbert action coupled to dilaton field

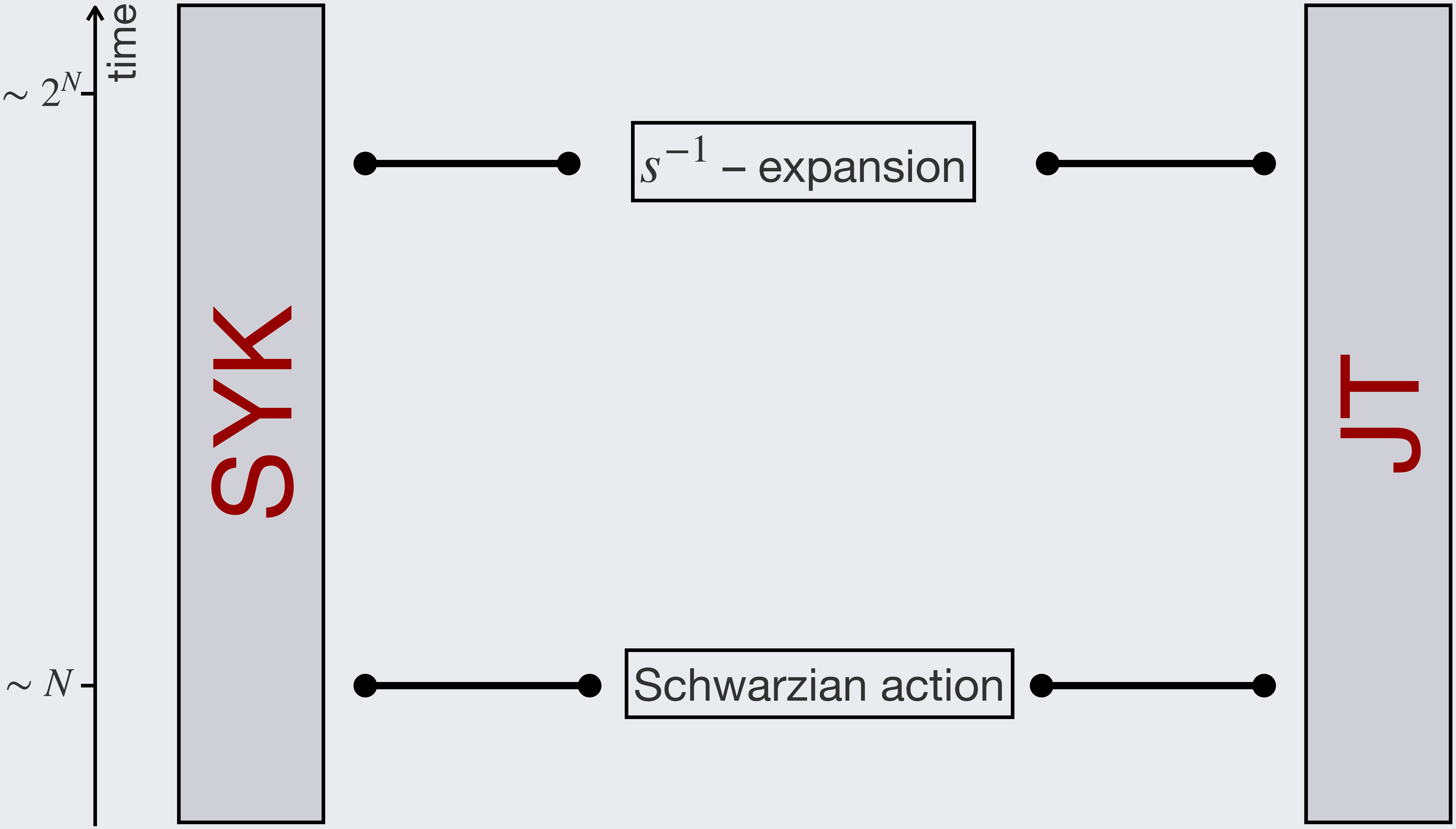
dilaton field

negative cosmological constant

$$S = \frac{1}{16\pi G}\int \sqrt{g}\,\phi(R + \Lambda) + \ldots$$

curvature

Jackiw Teitelboim gravity



Eynard, Orantin, 08
Mirzhakani, 06
Shenker *et al. 19*
Stanford & Witten 19

Kitaev, Suh,
Stanford,
Maldacena, aa,
Bagrets, …
2015+

The gravitational path integral

JT partition sum

topological action

Schwarzian action

$$Z = \sum_g e^{-S_0 g} \int (\text{moduli space}) \int (\text{boundary wiggles})\, e^{-\int_{\text{boundary}} \mathcal{K}\phi}$$

genus expansion

these
parameters

“an integral over geometries …”

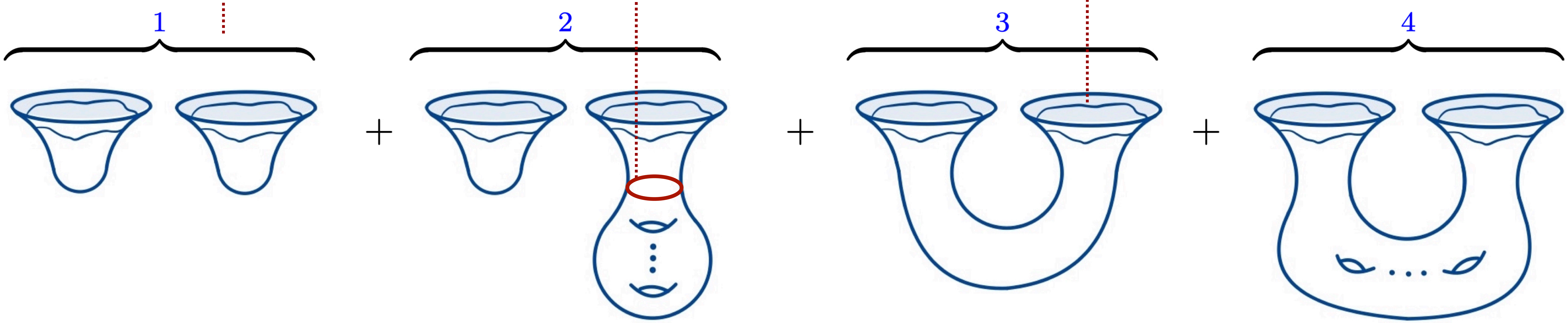


figure: Post *et al.* 22

Gravity topological recursion

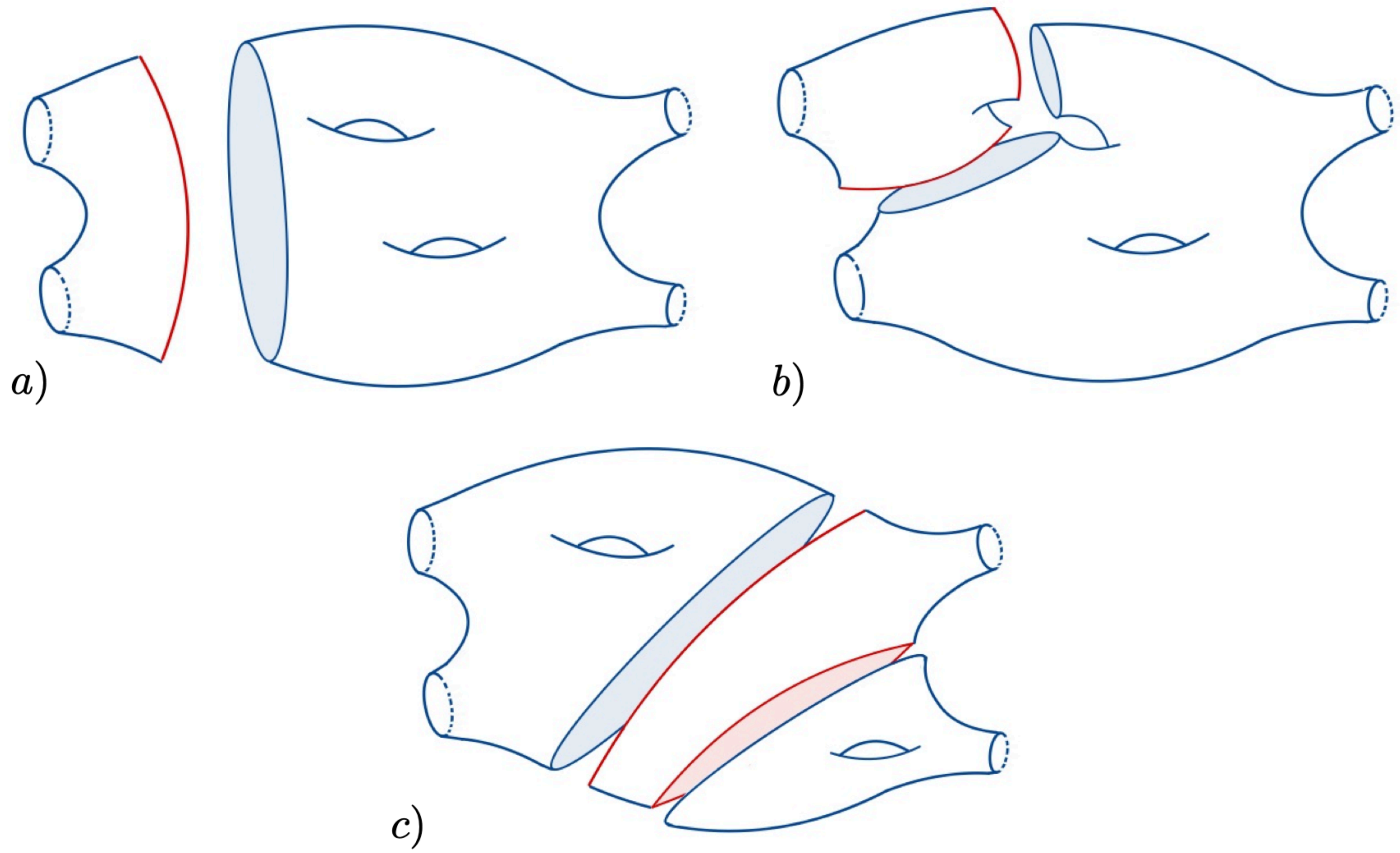
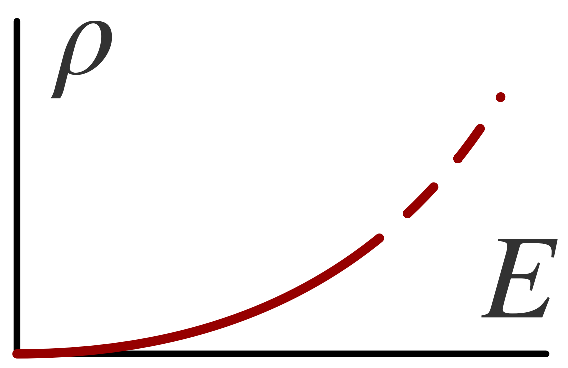


figure: Post *et al.* 22

I: Formulate recursive relation for topological expansion of JT

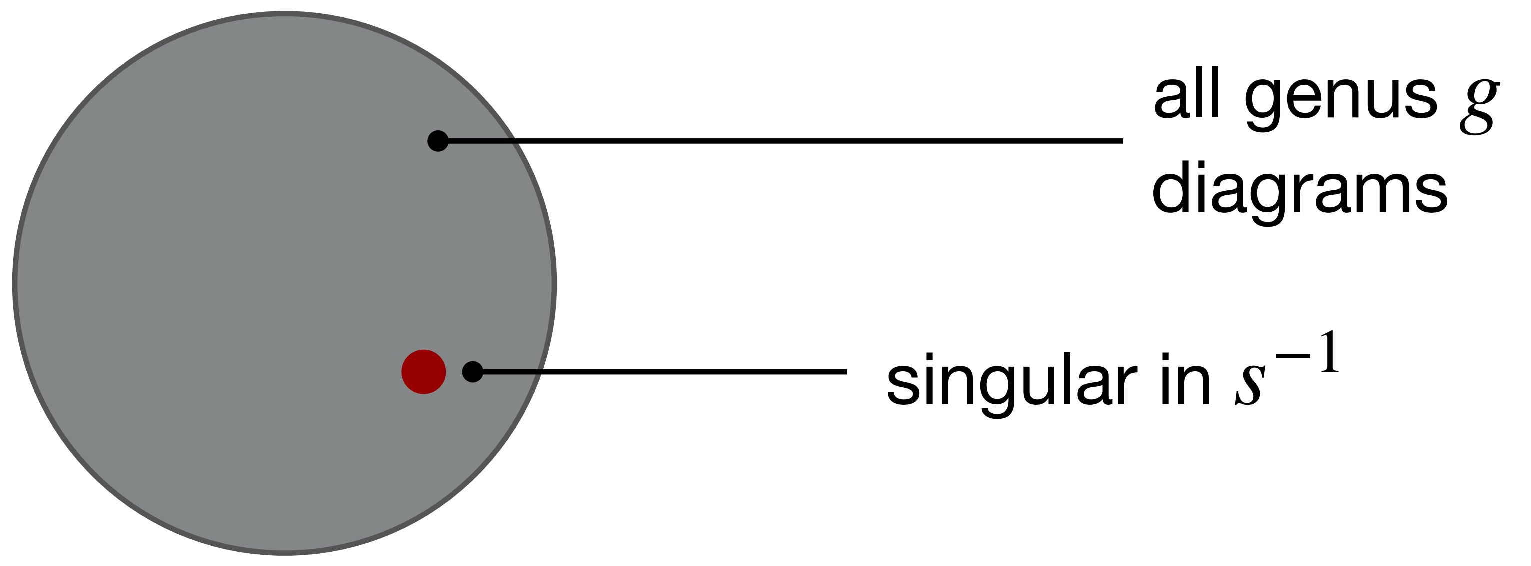
Mirzhakani, 06

II: compare to topological recursion of matrix model with spectral density matching that of SYK.



→ perfect match

Shenker *et al.* 19



Expansion of JT partition sum captures perturbative expansion of spectral correlations

Gravity topological recursion

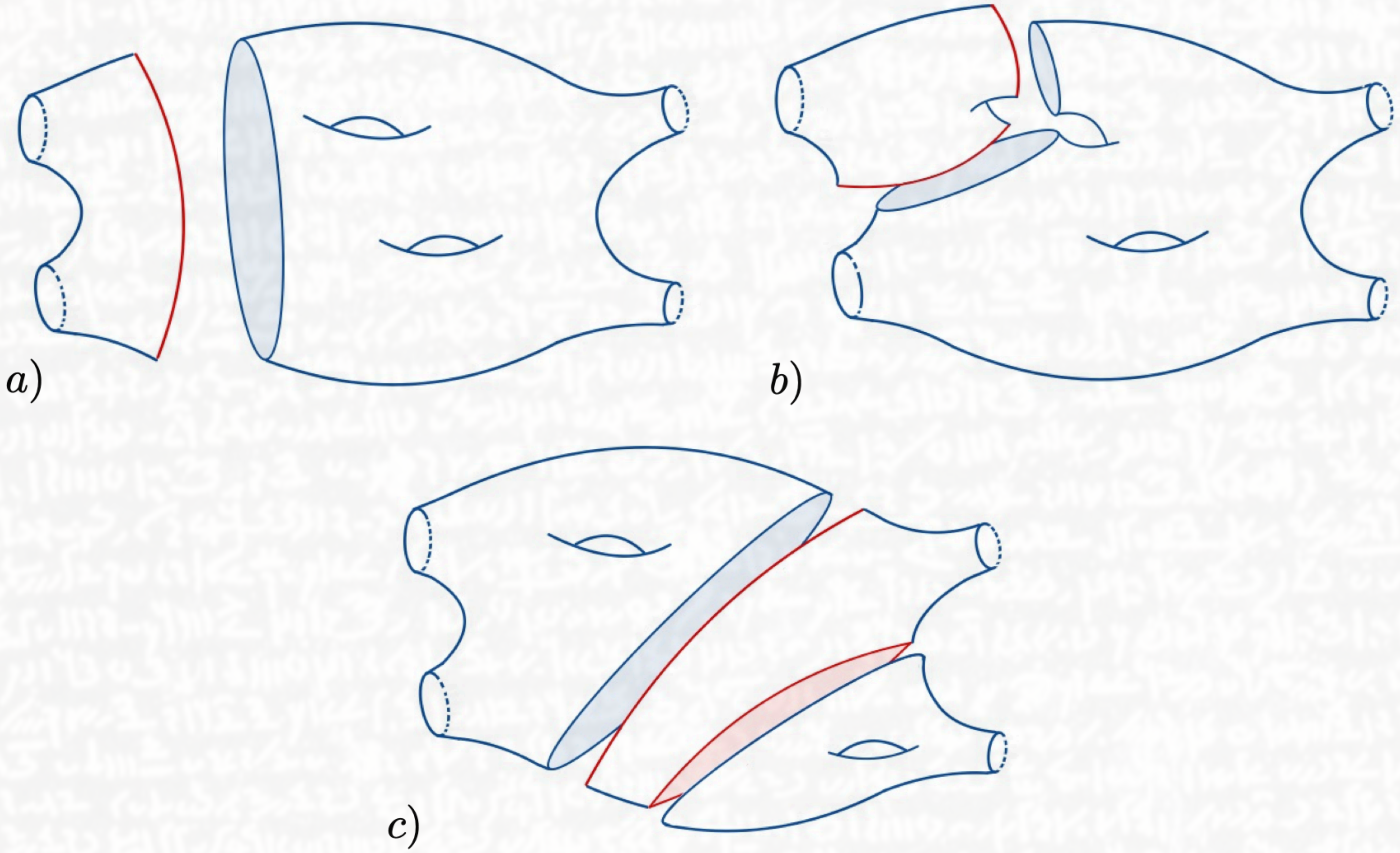
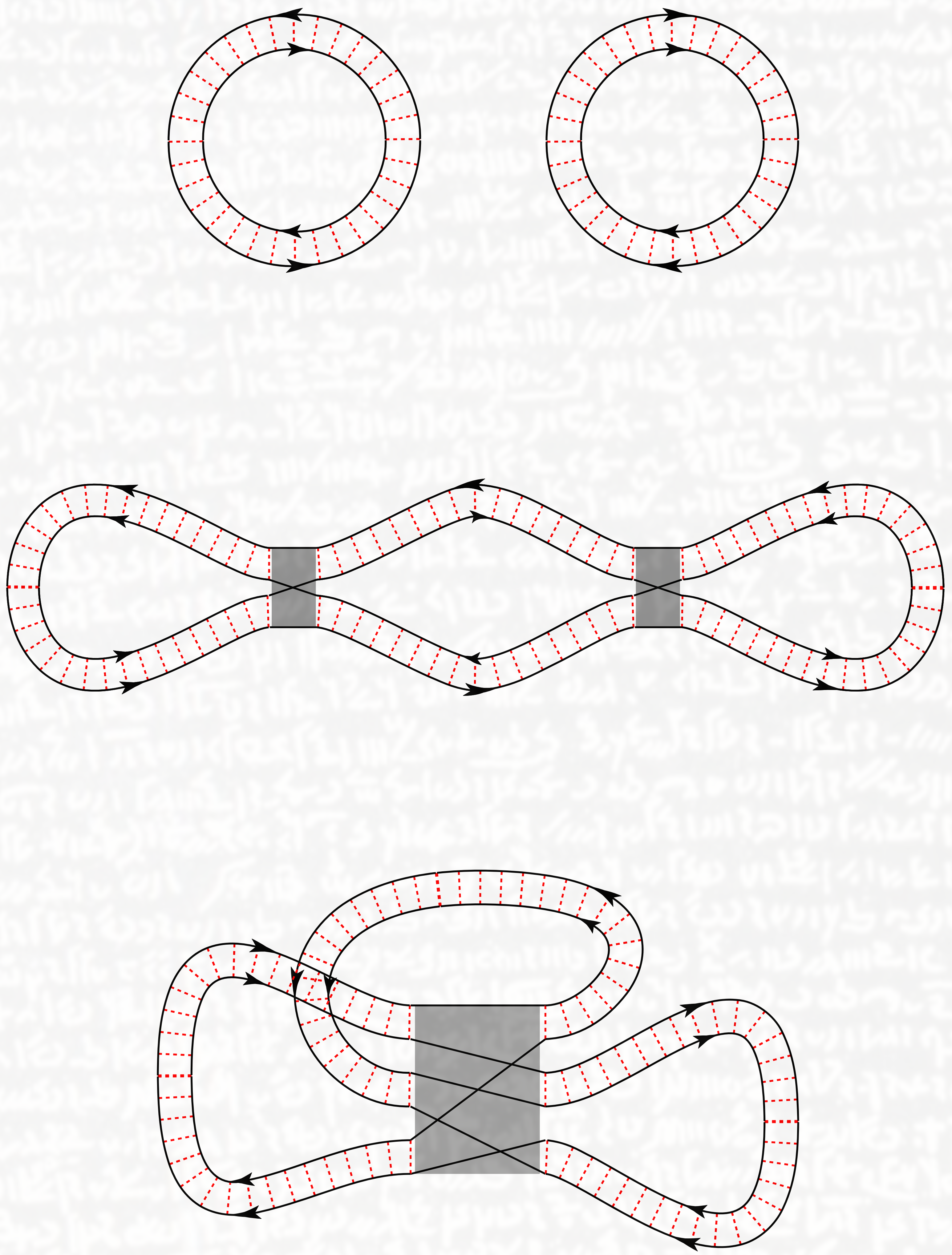
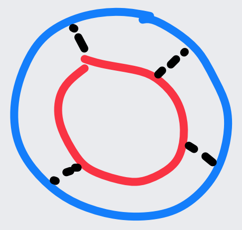


figure: Post *et al.* 22

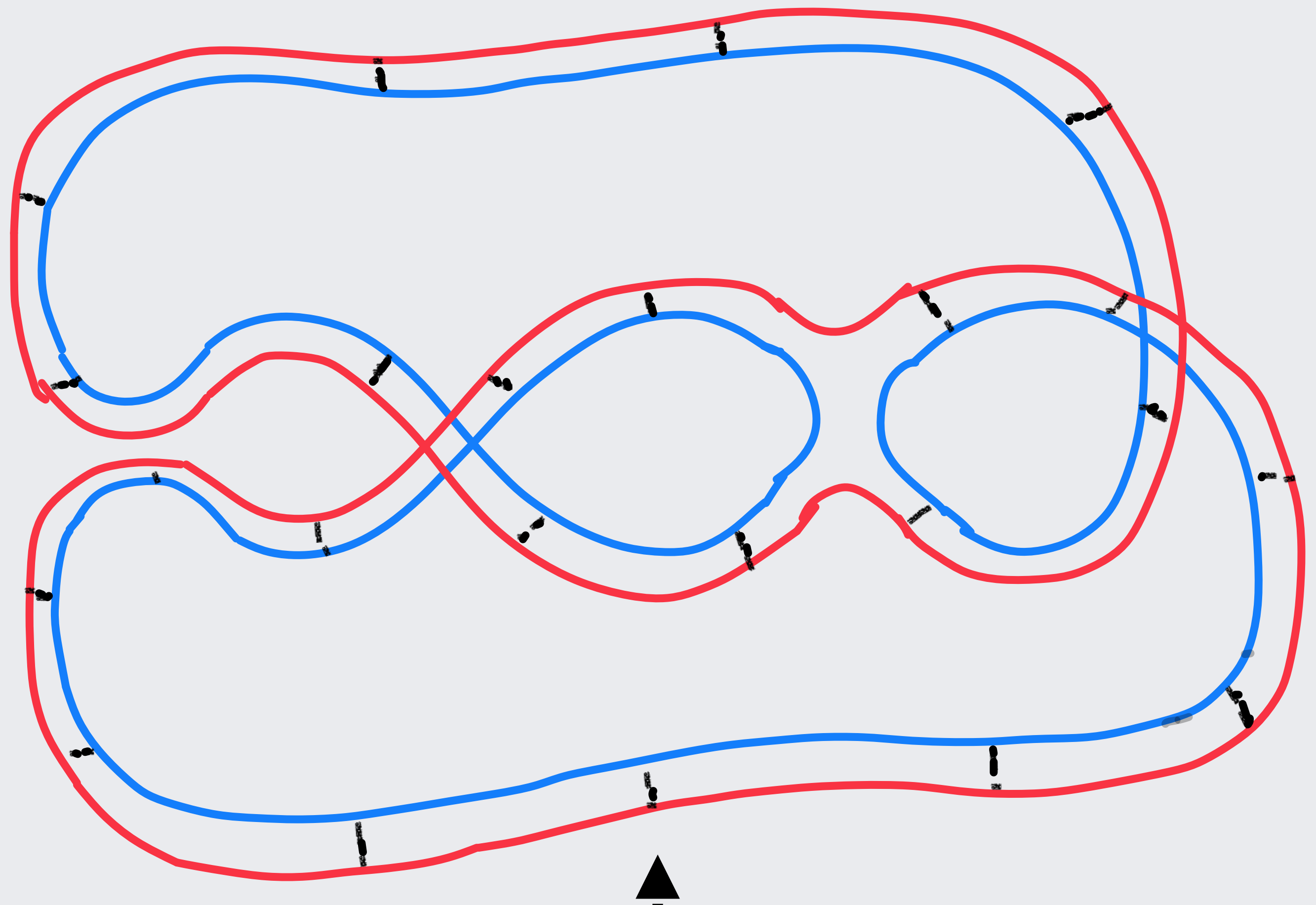


Intuitive interpretation of diagram - surface analogy

diagram representation



genus $g = 0$ — “planar”

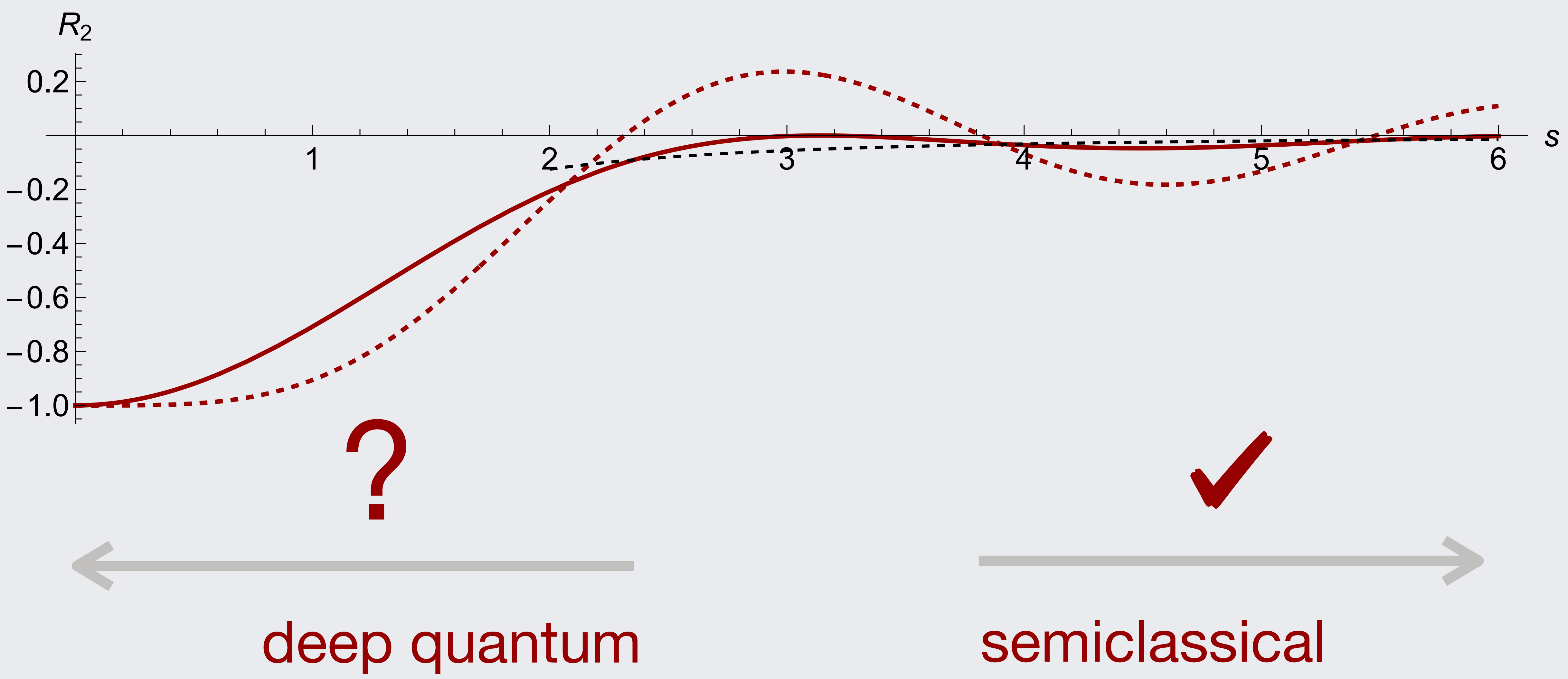


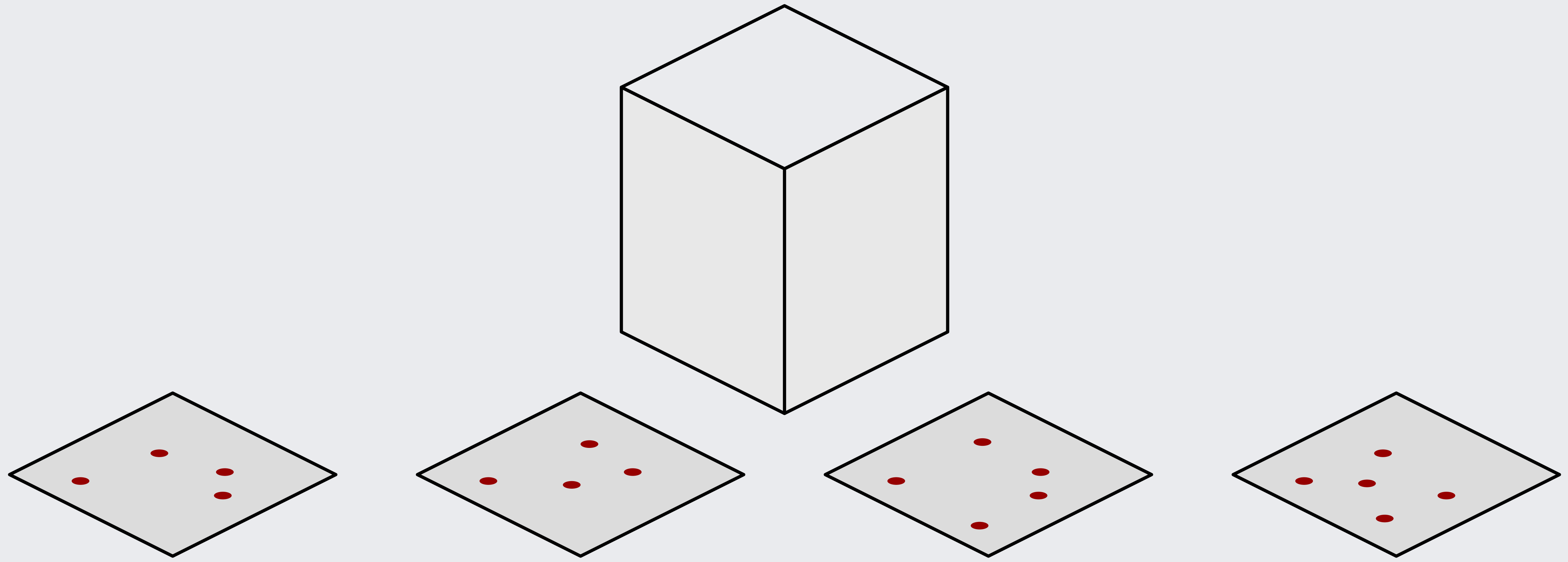
$g = 1$

information

information $\rightarrow$ ‘strings’

$\rightarrow$ ‘geometry’





Q1: Where is the non-perturbative quantum chaos?*

Q2: Interpretation of gravitational ensemble?

A: Think of gravity as semiclassical limit of a string theory.

* Extreme infrared ($\mathcal{O}(1)$ level spacing above spectral edged) amenable to non-perturbative resummation (Saad, Stanford, *et al.*)

chaos in low-dimensional string theory

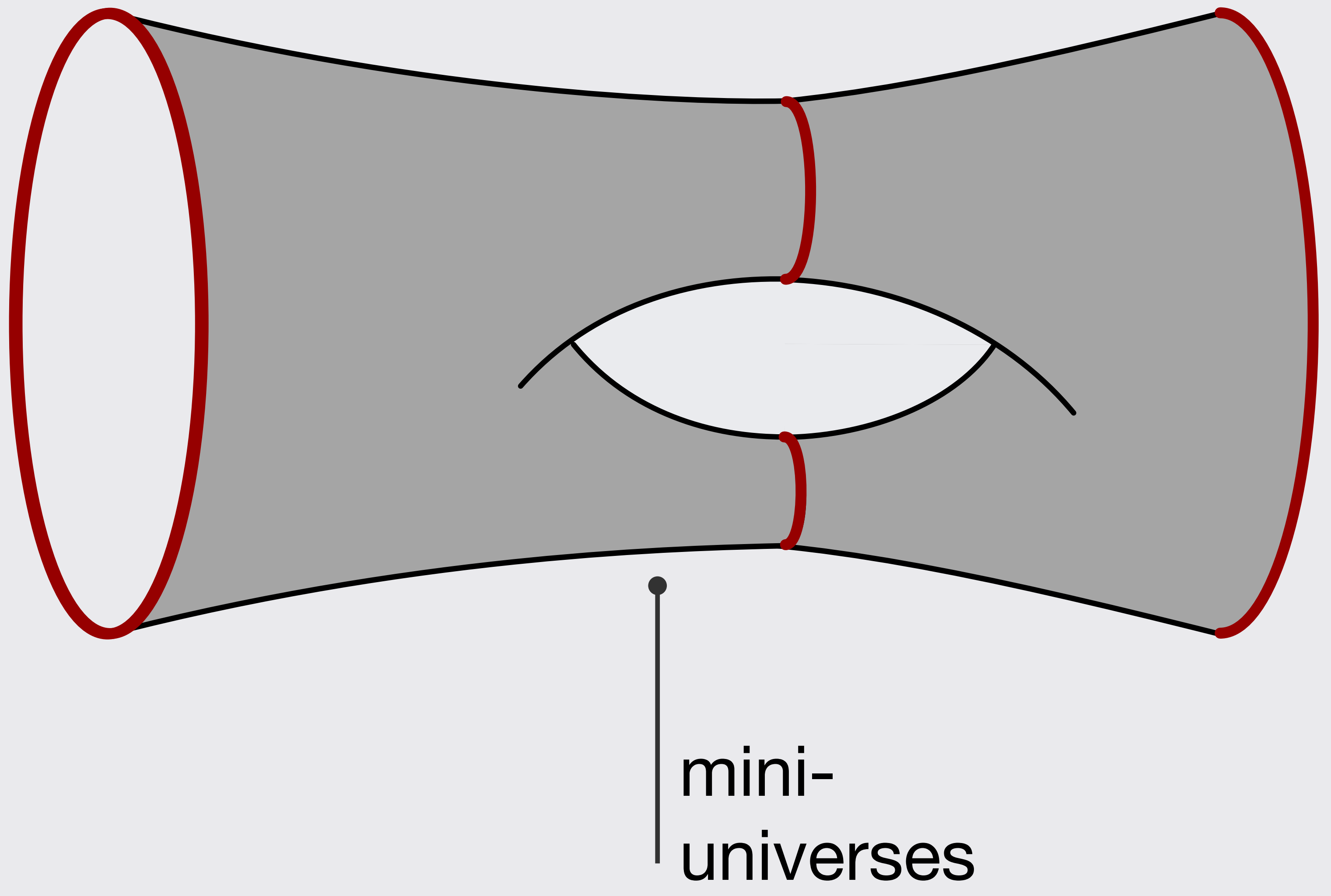
Motivation I: closed string perspective

– Think of JT partition sum as asymptotic expansion of some quantum theory

– Need quantum field theory of scattering of universes or closed strings

– KS field theory: A $2d$ CFT of chiral boson with with cubic nonlinearity

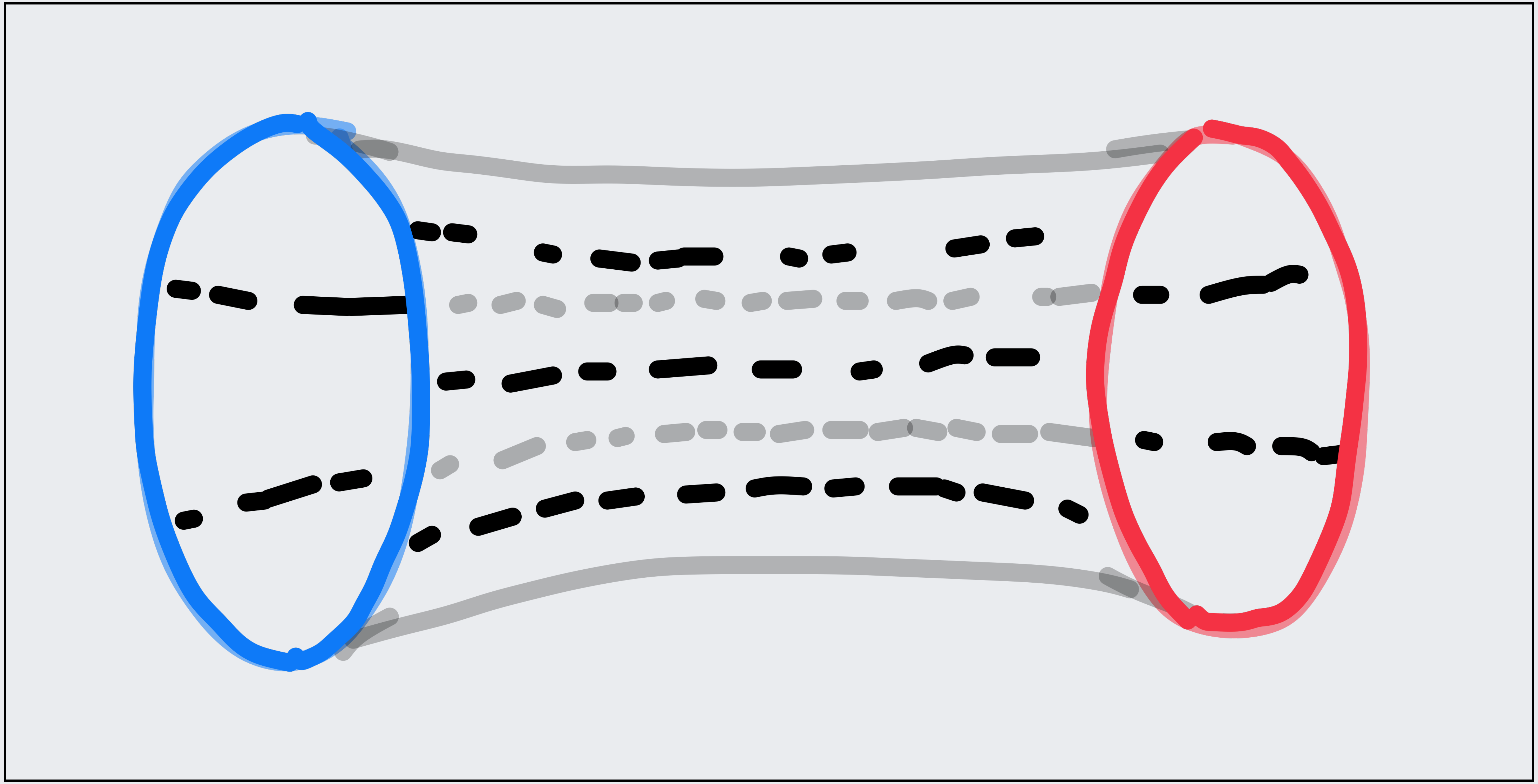
– perturbation theory in nonlinearity $\rightarrow$ JT partition sum.



Post *et al.* 22

Kodaira, Spencer 58
Bershadsky *et al.* 93

Motivation II: open string perspective

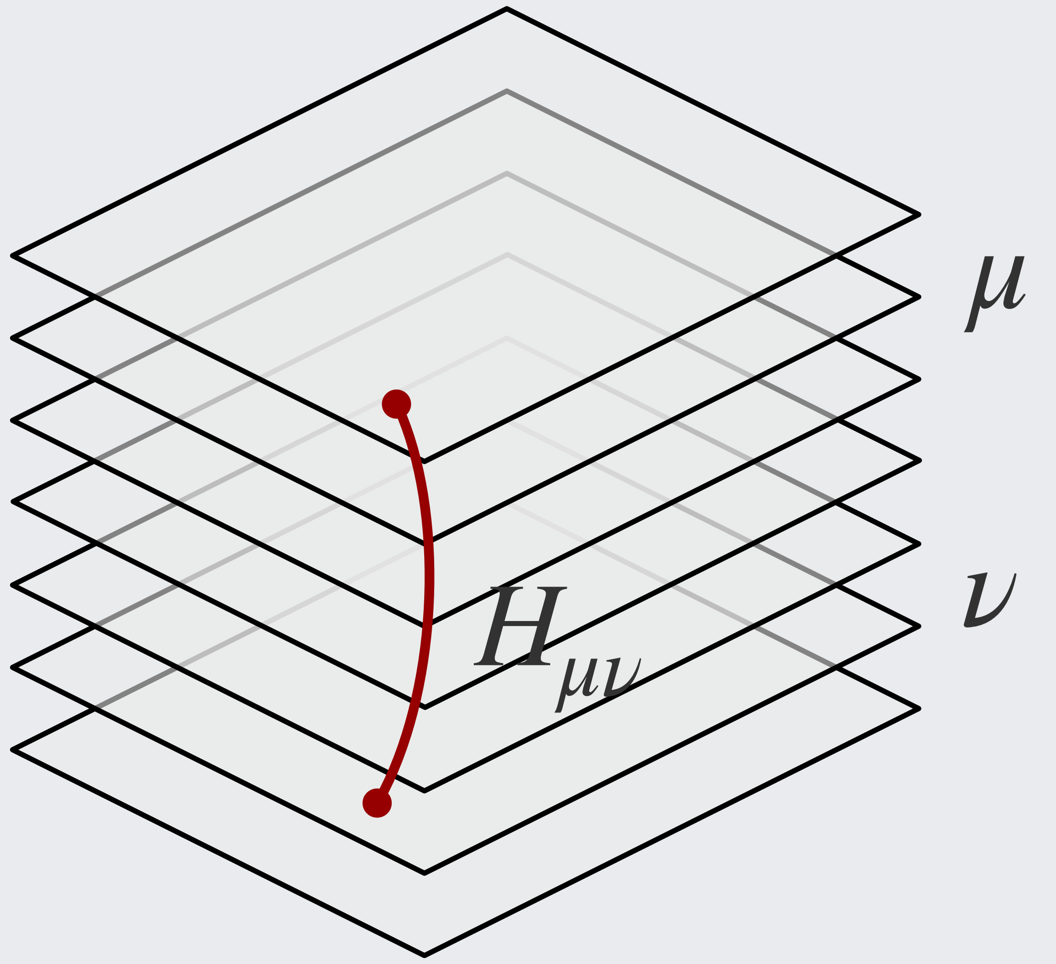


JT theory from dimensional reduction

Calabi-Yau
(6-dim.)

dimensional reduction
(closed string perspective)

dimensional reduction
(open string perspective)



EFT of ergodic
quantum chaos

Kodaira-Spencer field theory
(2d CFT)

perturbation theory
(in cubic nonlinearity)

JT

chaos from KS

Introducing boundary sources (flavor probe branes) and taking low energy limit can show:

aa, Sonner, *et al.* 22

$$\left\langle :e^{\phi(z_1)}e^{-\phi(z_2)}e^{\phi(z_3)}e^{-\phi(z_4)}:\right\rangle_{\mathrm{KS}}$$

$$= \int dY\, e^{\frac{i}{\lambda}X\cdot Y}\,\Delta(Y)\left\langle :e^{+\phi(y_1)}\,e^{-\phi(y_2)}\,e^{+\phi(y_3)}\,e^{-\phi(y_4)}:\right\rangle_{\mathrm{KS}}$$

$$= \int dY\, e^{\frac{i}{\lambda}X\cdot Y}\,\Delta(Y)\,e^{-\Gamma(Y)} = \Delta(X)\int_{\mathrm{GL}(2|2)} dA\, e^{\lambda^{-1}(i\,\mathrm{str}(AX)-\Gamma(A))}$$

$$\longrightarrow \int_{\mathrm{A}_{2|2}} dQ\, e^{\frac{i}{\lambda}\,\mathrm{str}(QX)}$$

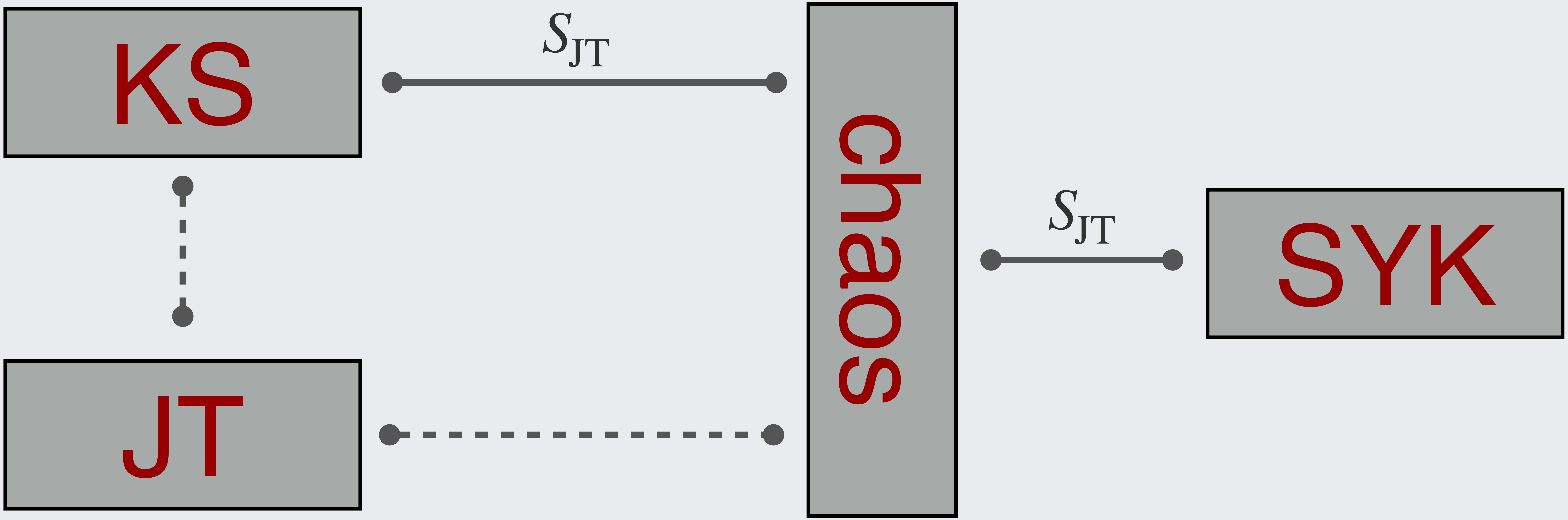
Summary

2d gravity encodes chaotic correlations in smooth geometric data.

Ensemble average implied by 2d holographic principle implied by dimensional reduction.

Topolological string theory $\to$ chaotic with universal spectral correlation.

conclusions



perturbation theory/info on μ-structure lost

non-perturbative