Entanglement, von Neumann algebras, the emergence of spacetime

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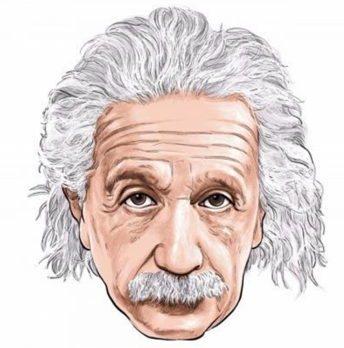
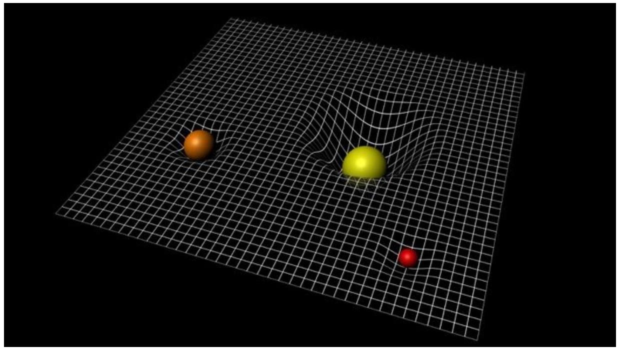
Spacetime



Special relativity : spacetime

General relativity: Gravity = spacetime

Spacetime is dynamical object, can evolve on its own, but is also curved by matter in it.



Dynamical variables: $g_{\mu\nu}(x^\lambda)$ (metric)

describe distances, time flow, curvature, ……

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

G_N: Newton's constant

characterize matter's ability in curving spacetime

Quantum mechanics and gravity

The other pillar of 20th century physics is quantum mechanics.

It is widely believed that spacetime/gravity must also be quantum even though there has not been any direct evidence.

In Nature gravity is very weak

detection of quantum gravitational effects is extremely extremely difficult.

$$\text{Planck length} : \ell_p = \sqrt{\frac{\hbar G_N}{c^3}} = 1.6 \times 10^{-35}\,\text{meter}$$

We now believe that smallness of G_N is needed to have a large universe

Nature is very good at hiding her secret ……

Quantum spacetime?

A natural idea:

General relativity is a classical field theory (metric field)

quantize this classical field theory, just as in the case of electromagnetism

many many difficulties

Maybe this is a wrong idea

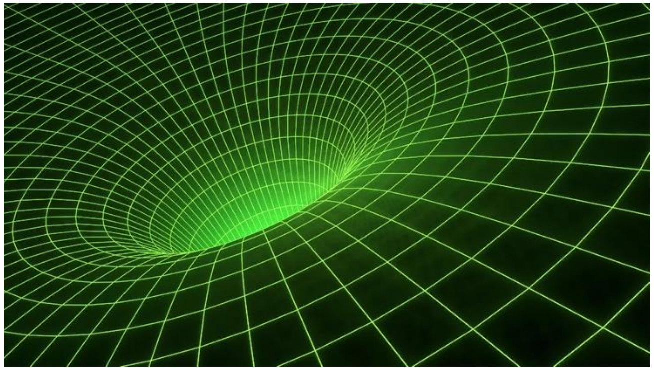
Maybe spacetime is not fundamental, but emergent in the limit

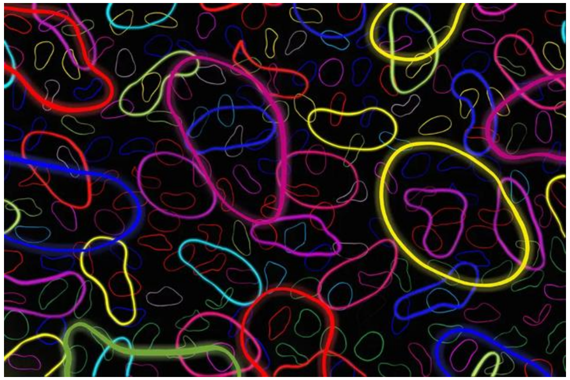
$$G_N \to 0$$

An analogue: fluid

Is spacetime emergent?

A strong hint that spacetime/gravity is emergent comes from string theory.





ℓ_s : string length

$L \gg \ell_s$

$L \sim \ell_s$

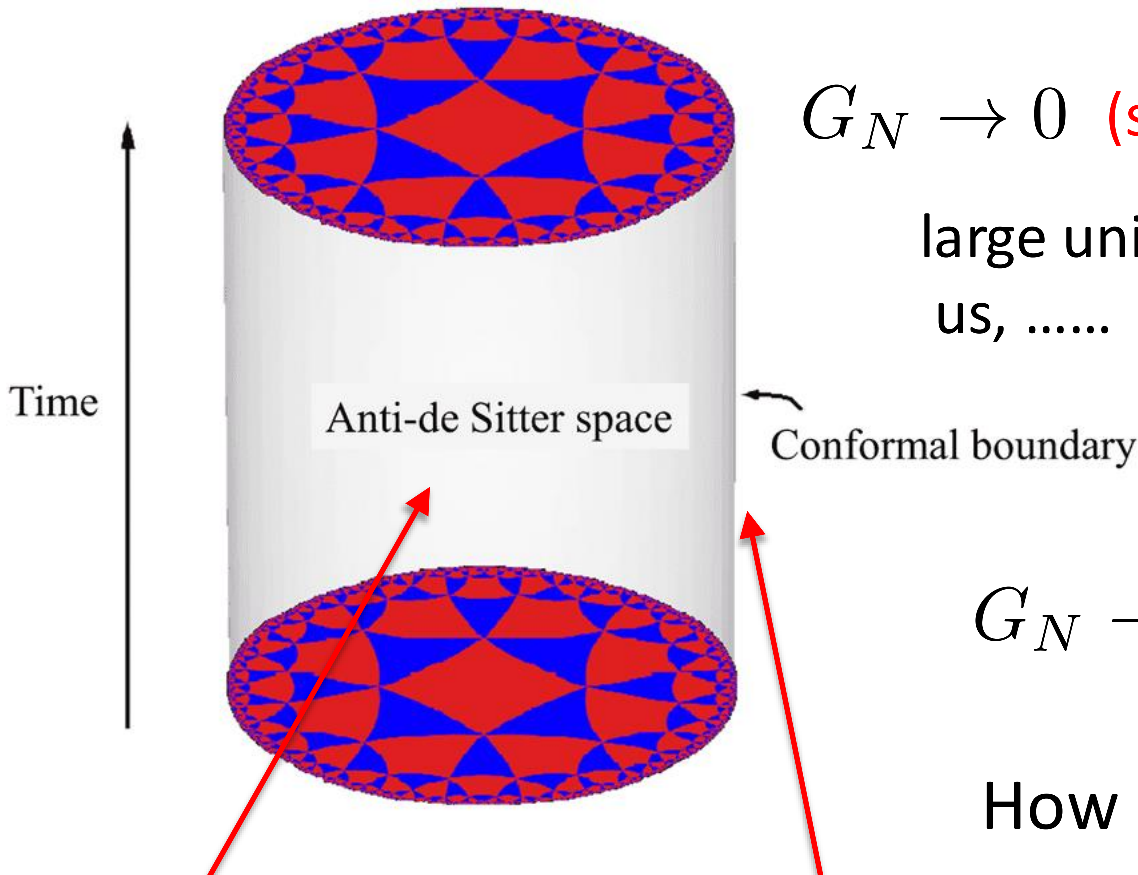
Spacetime is not fundamental, but rather an emergent concept at large-distances.

$\ell_p \ll \ell_s$

There are indications that stringy description may not be fundamental.

Holography: spacetime from matrices

Maldacena (1997)



$G_N \to 0$ (small cosmological constant)

large universe: galaxies, stars, us, ……

G_N ⟺ 1/N

$G_N \to 0$ ⟺ $N \to \infty$

quantum gravity = boundary theory (matrices)

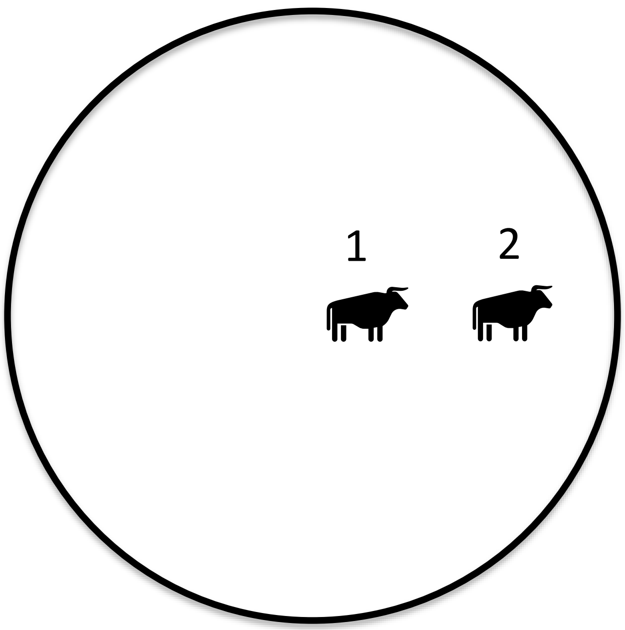
N: rank of matrices

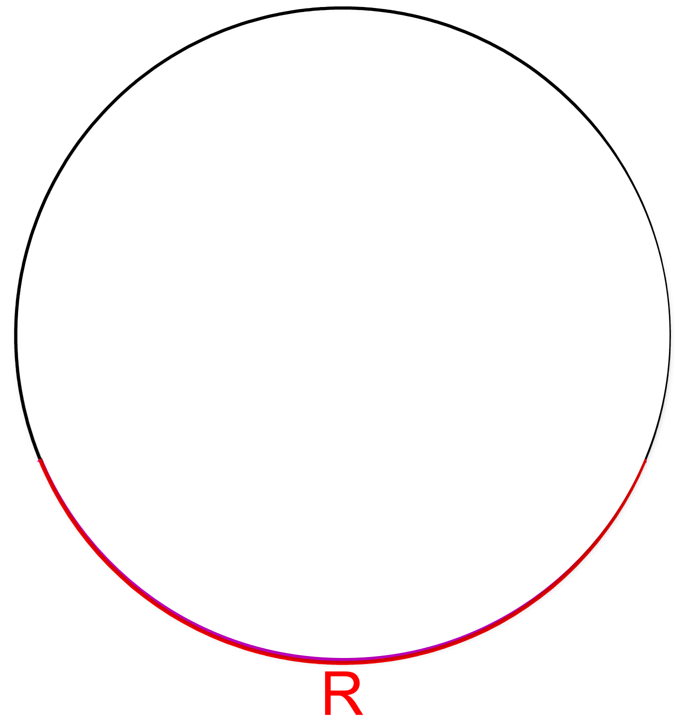
How does the bulk spacetime emerge from these matrix degrees of freedom in the $N \to \infty$ limit?

Hints

Where does the extra dimension come from:

Ryu, Takayanagi (2006)





scale ⟷ radial direction

????

Entanglement ⟷ Spacetime geometry

how spacetime emerges from entanglement?

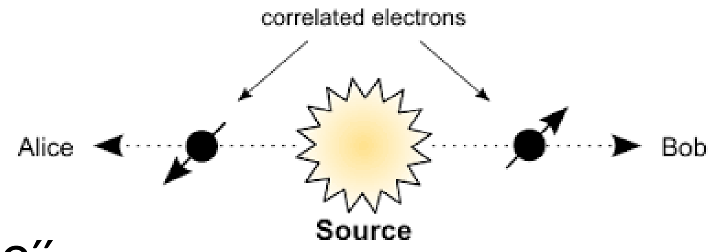
In this talk:

1. Von Neumann (vN) algebras provide a universal language in characterizing the entanglement structure of a quantum system.

2. Von Neumann algebra could provide a powerful language for understanding the emergence of spacetime and associated geometric notions.

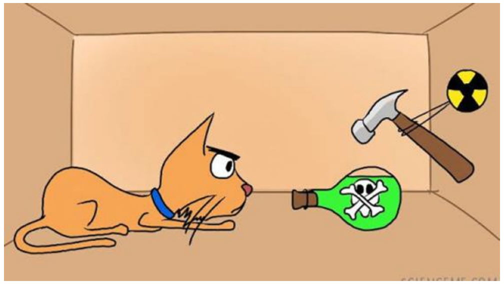
Entanglement and von Neumann algebras

Entanglement



Einstein, Podolsky and Rosen (1935):

"spooky action at a distance"



Schrodinger (1935): "entanglement"

Bell (1964): test quantum mechanics

1990's: resources for quantum computation, teleportation, communications

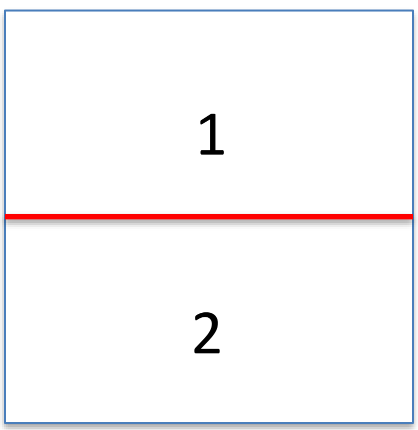
21st century: quantum many-body systems, quantum field theory, quantum gravity

Entanglement and subsystems

Heuristically, entanglement characterizes (quantum) correlations among different subsets of degrees of freedom.

(subsystem)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right)$$



Typical entangled states

$$\sum_{ij} a_{ij} |\psi_i\rangle_1 \otimes |\chi_j\rangle_2$$

Writing entangled states requires factorization of Hilbert space:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

While the concept of a subsystem is physically very intuitive

in textbook, defining it mathematically is highly unintuitive

It requires factorization of Hilbert space:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

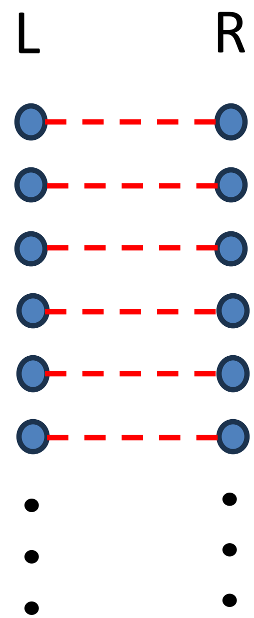
In fact, there are many physically interesting situations
where such a definition breaks down.

This happens when there is an infinite amount of entanglement.

Infinite entanglement

an infinite number degrees of freedom → an infinite amount of entanglement → breakdown of factorization of Hilbert space

Consider two infinite chains of spins which are pairwise entangled

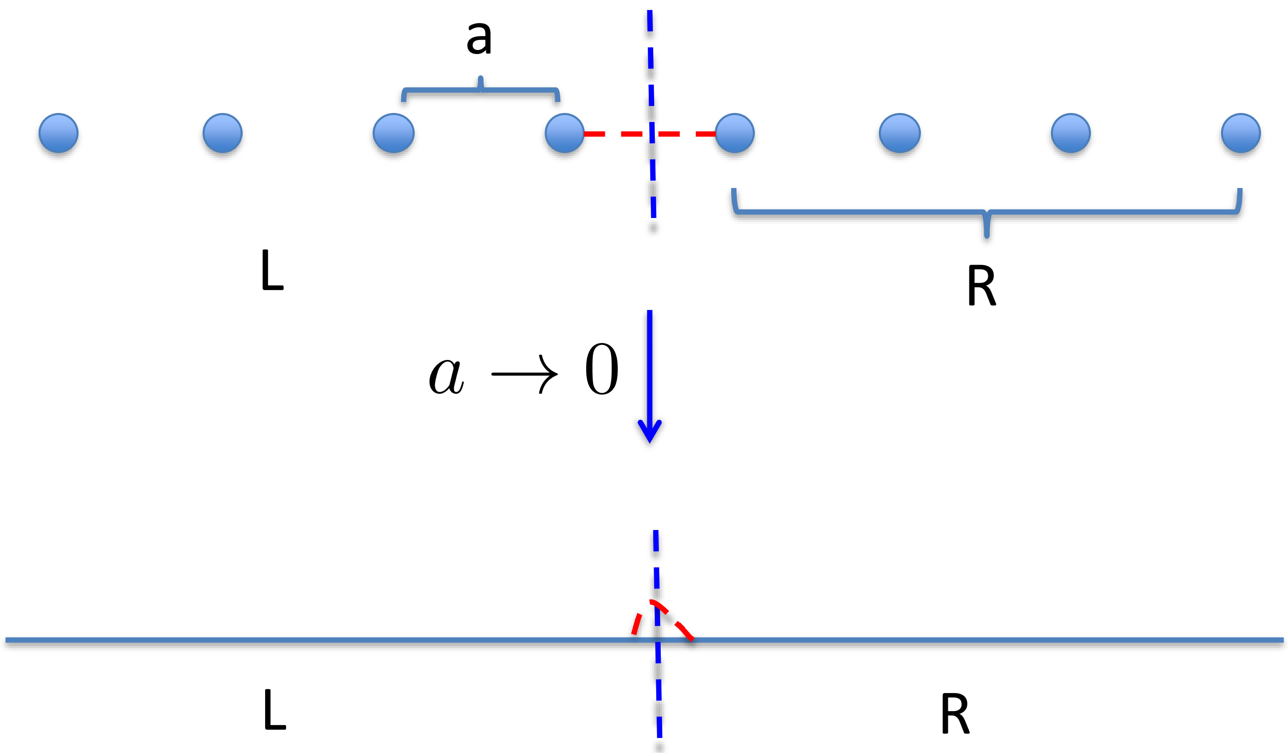


Physical Hilbert space: finite energy excitations on such a configuration

can only flip finite number of spins

$$\mathcal{H} \neq \mathcal{H}_R \otimes \mathcal{H}_L$$

Consider a 1+1-dimensional QFT on a lattice



Physical Hilbert space: finite energy excitations in the continuum limit

$$\mathcal{H} \neq \mathcal{H}_R \otimes \mathcal{H}_L$$

Factorization of Hilbert space does not exist $\mathcal{H} \neq \mathcal{H}_R \otimes \mathcal{H}_L$

???? $$\sum_{ij} a_{ij} |\psi_i\rangle_1 \otimes |\chi_j\rangle_2$$

How do we describe entanglement??

von Neumann algebras

von Neumann (1927-1929): mathematical foundation of quantum mechanics.

von Neumann algebras (1929): subalgebras of observables in a quantum system

Murray and von Neumann (1936-1940's): basic theory and classifications of von Neumann algebras

Type I, II, III

Since 1970's has become a vast mathematical subject ......

1960's: algebraic quantum field theory (Haag, Kastler, Araki, ...)

Only during the last few years, did people recognize the connection to entanglement

Infinite entanglement: von Neumann algebras to the rescue

A von Neumann algebra $\mathcal{A}$ in a quantum system is defined as:

a vector space of operators, closed under

Hermitian conjugation

operator products

limits in matrix elements

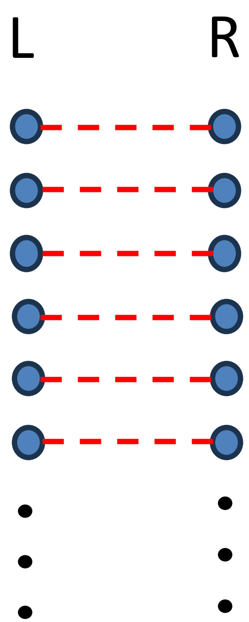
We can simply define a subsystem as a von Neumann algebra

$\mathcal{A}$ type I von Neumann algebra: $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \quad \mathcal{A} = \mathcal{B}(\mathcal{H}_1)$

This recovers the standard textbook definition of subsystem

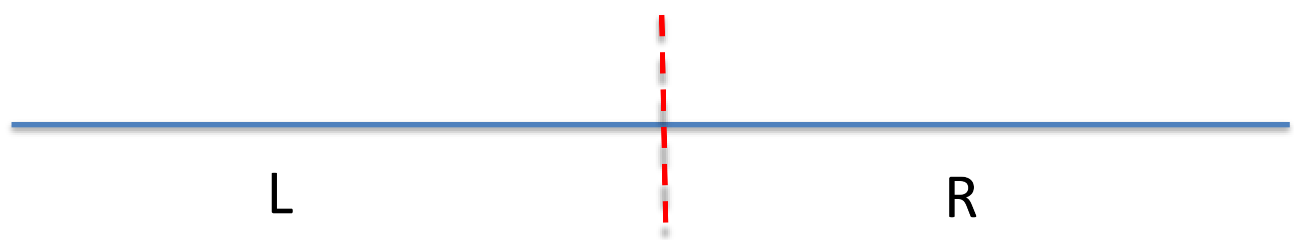
Type II and III: Situations where factorization of Hilbert does not exist

Examples



$\mathcal{A}_L, \mathcal{A}_R :$

type II
or III



$\mathcal{A}_L, \mathcal{A}_R :$ type III_1

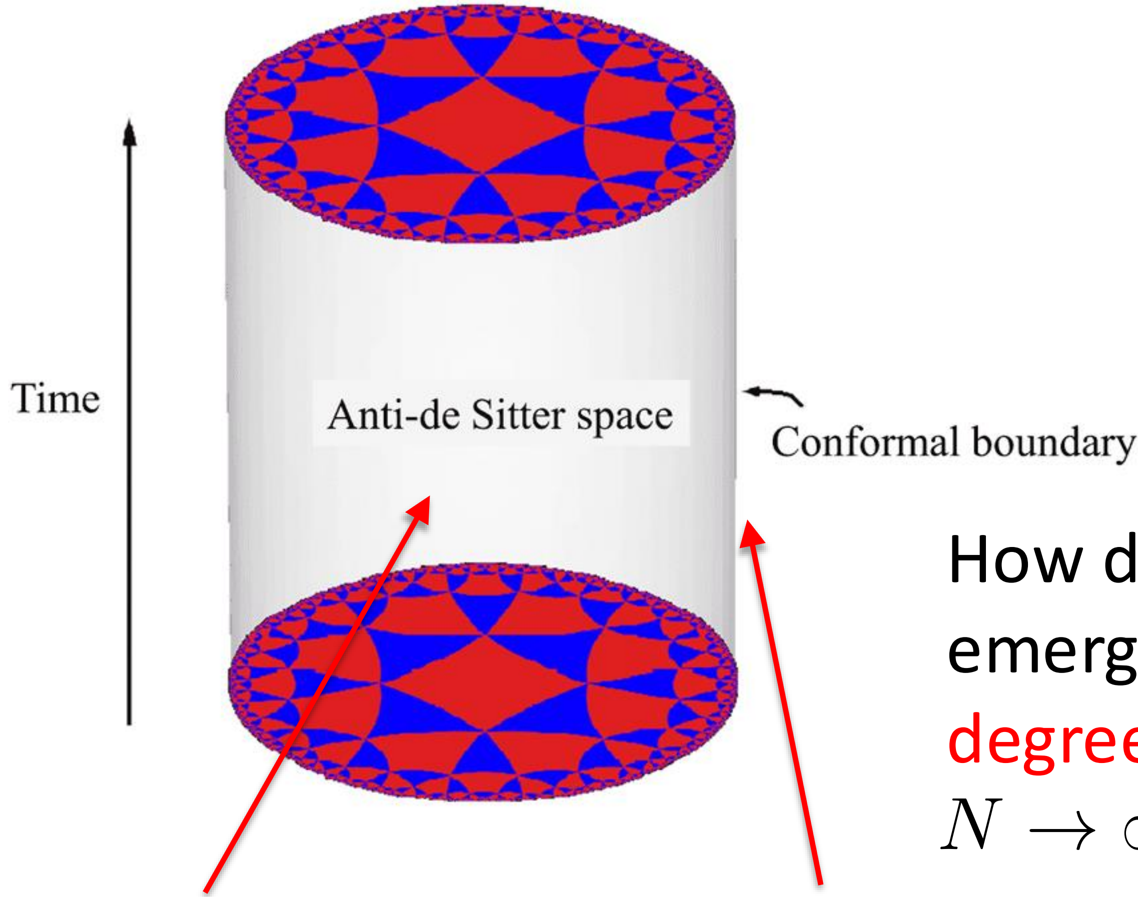
Algebra for a general open region in a quantum field theory: type III_1

continuum of spacetime and causal structure requires a very specific entanglement structure which is offered by type III_1

Classification of von Neumann algebras provides a classification of entanglement patterns of quantum systems !!

von Neumann algebras
and emergence of spacetime

Holography: spacetime from matrices



How does the bulk spacetime emerge from these matrix degrees of freedom in the $N \to \infty$ limit?

quantum gravity = boundary theory (matrices)

N: rank of matrices

Emergence of spacetime:

not only the spacetime itself (including all the geometric concepts such as causal structure), but also all the physics on that spacetime (QFT in curved spacetime).

Conventional thinking:

metric + Lagrangian of quantum matter on the spacetime

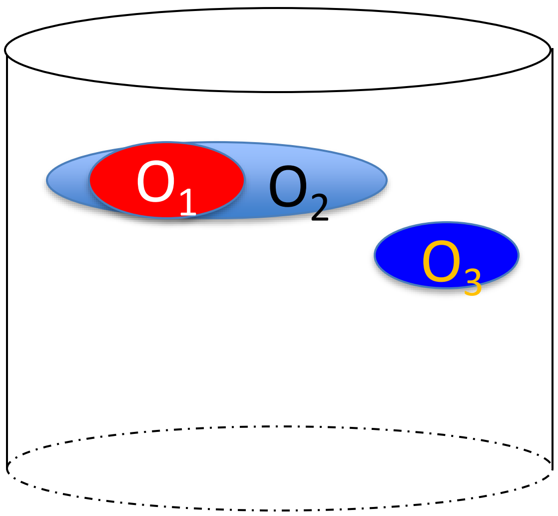
However, in this approach, local physics, causal structure, entanglement structure not manifest

These concepts have to do with subsystems

An alternative description

Metric + QFT in curved spacetime = Collection of physical operations in all open regions and their relations

(algebraic QFT)



$$O_1 \subset O_2$$

O_1 and O_3 are spacelike separated: physical operations in them commute (causal structure)

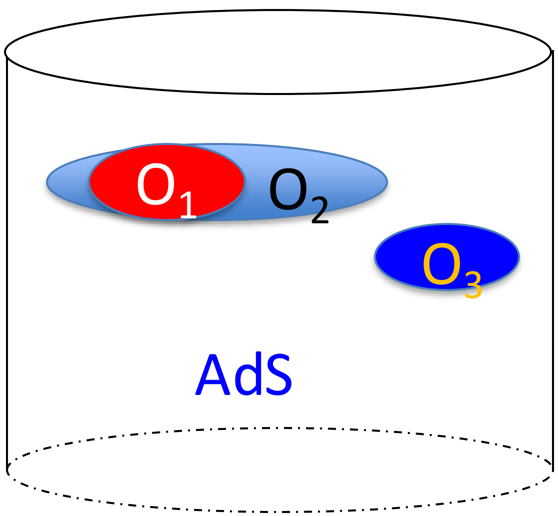
Emergence of spacetime ⟺ emergence of physical operations in all open regions and their relations

Subregion-Subalgebra duality

Leutheusser, HL (2022)

In the $N \to \infty$ limit

bulk subregion $\Longleftrightarrow$ **Emergent** type III_1 von Neumann subalgebra on the boundary



$O_1 \Longleftrightarrow \mathcal{A}_1$

$\mathcal{A}_1$: captures all bulk physical operations in O_1

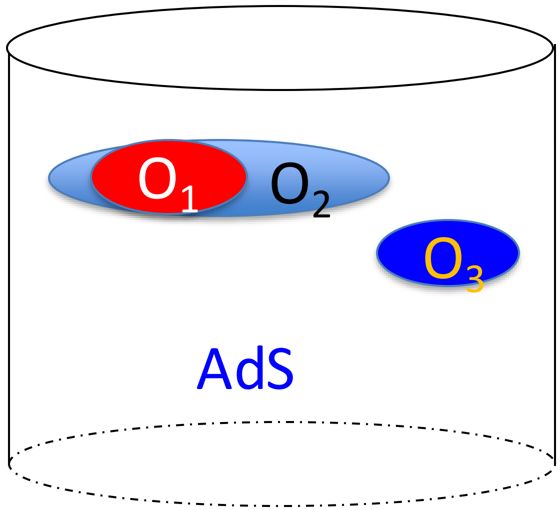
$\mathcal{A}_1$ emergent:

it arises in the $N \to \infty$ limit

due to an infinite amount of entanglement in the limit

type III_1 : needed to capture bulk local physics and causal structure

Subregion-Subalgebra duality

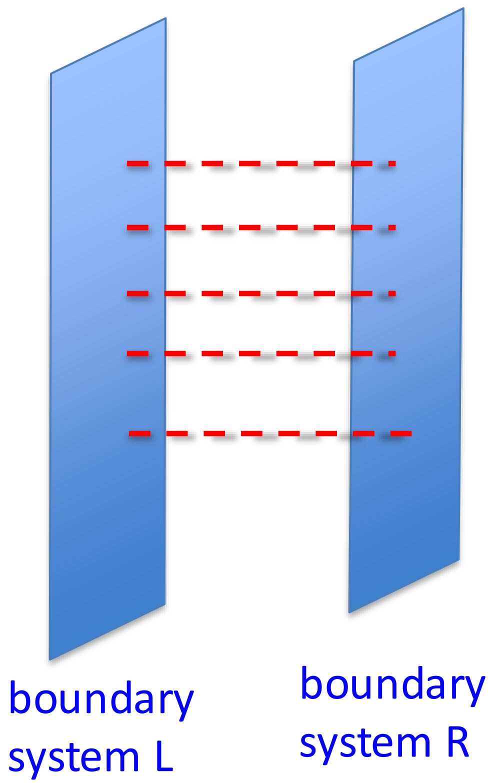


$O_1 \subset O_2$ ⟺ $\mathcal{A}_1 \subset \mathcal{A}_2$

O_1 and O_3 are spacelike separated (causal structure) ⟺ $[\mathcal{A}_1, \mathcal{A}_3] = 0$

bulk entanglement of O_1 ⟺ type III_1 structure of $\mathcal{A}_1$

Thermofield double (TFD) state



$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$$

In the TFD state, L and R systems are entangled in such a way that each subsystem appears thermal.

$\mathcal{B}_L$: operator algebra of the L system

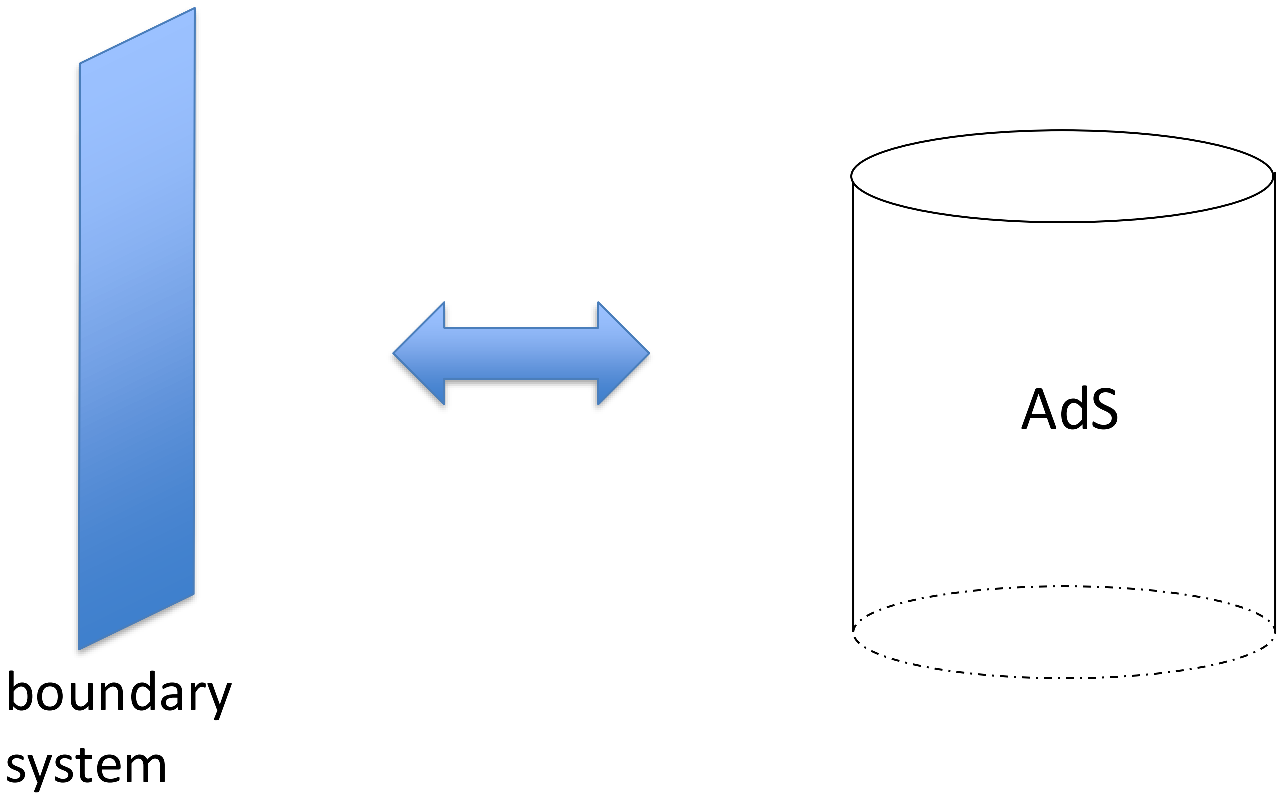
type I von Neumann algebra at finite N

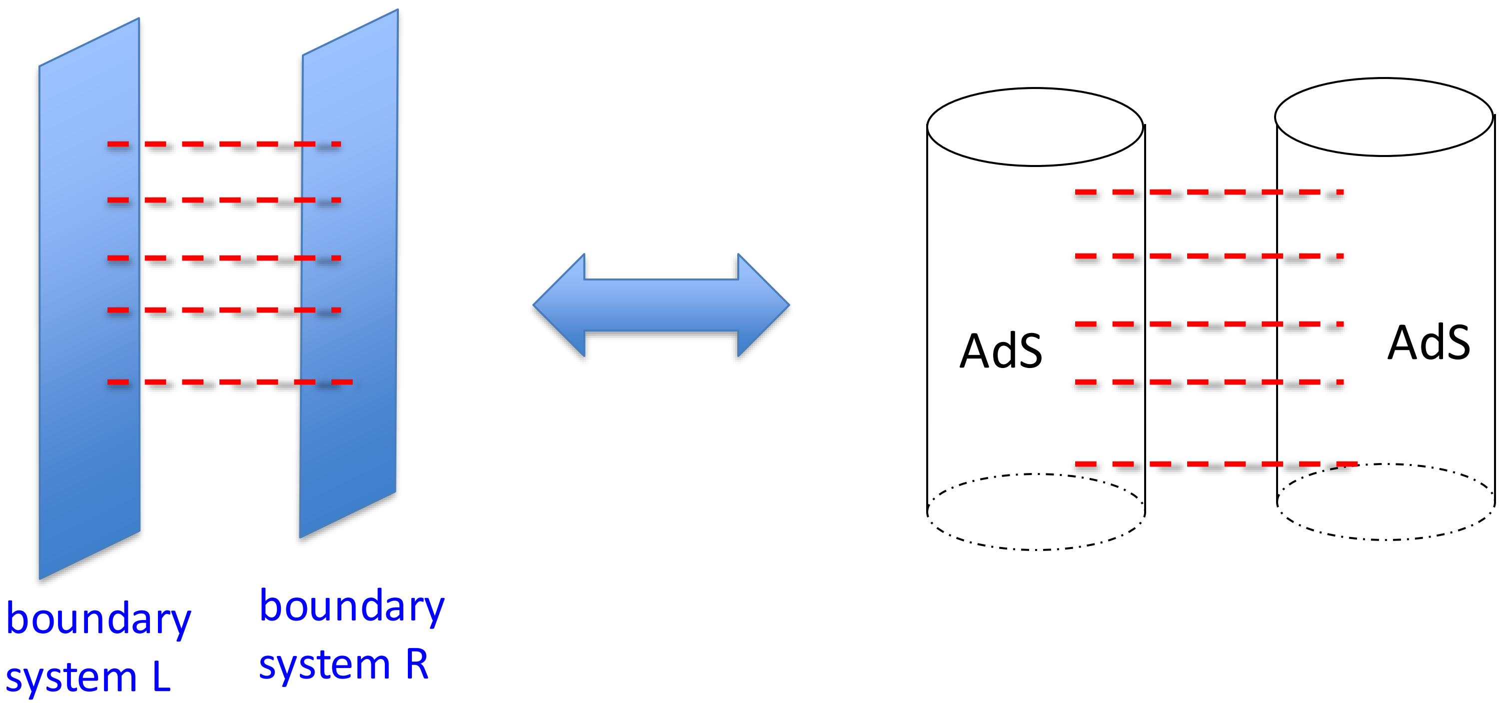
$$N \to \infty \qquad \lim_{N\to\infty} \mathcal{B}_L = \mathcal{X}_L$$

At low temperatures, entanglement between L and R remains finite as N goes infinity.

$\mathcal{X}_L$: type I

Hilbert space can still be factorized





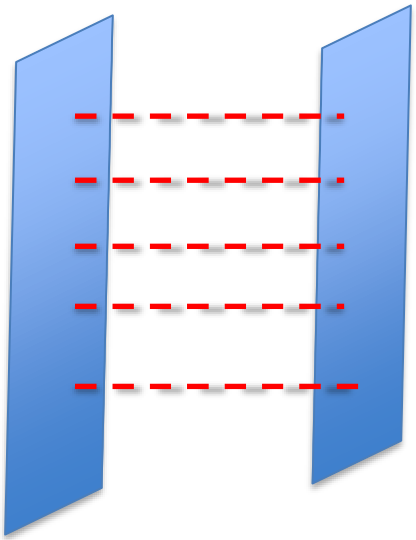
boundary
system L
boundary
system R
AdS
AdS

At sufficiently high temperature, entanglement between L and R systems becomes infinite

$$N \to \infty \quad \lim_{N\to\infty} \mathcal{B}_L = \mathcal{X}_L$$

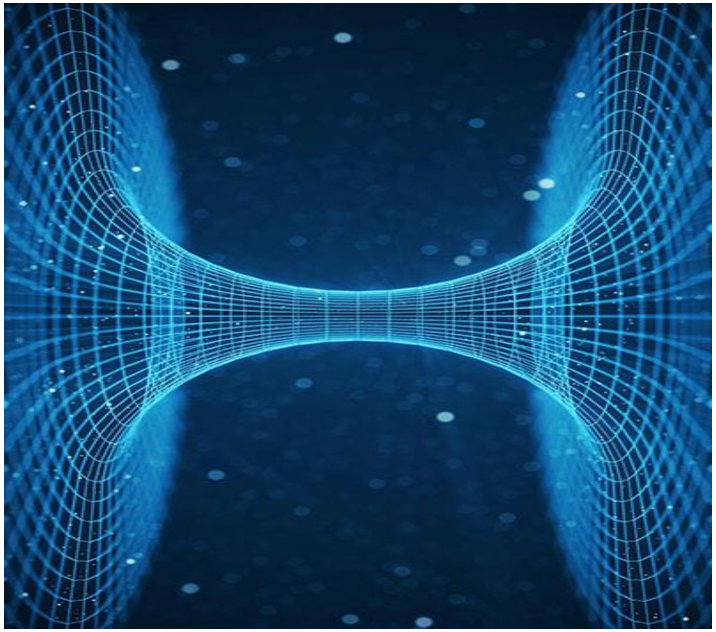
$\mathcal{X}_L$: type III_1

Hilbert space can NOT be factorized



boundary system L

boundary system R



two universes connected by a wormhole!

Subregion-subalgebra duality is reminiscent of the Gelfand duality in mathematics.

In mathematics, the Gelfand duality is the starting point of formulating the concept of noncommutative geometry (Connes).

Philosophically, subregion-subalgebra duality may be considered as a vast generalization of the Gelfand duality.

It gives us a more general/flexible language to talk about spacetime and physics in it,

which may be valid even in the stringy or quantum gravitational regime.

Summary

In the $N \to \infty$ limit

the boundary system becomes infinitely entangled

$\Rightarrow$ new emergent type III_1 algebras

$\Rightarrow$ bulk spacetime and all the physical operations in it

Using operator algebras to understand quantum gravity brings new powerful tools:

- New understanding of de Sitter entropy

Chandrasekaran, Longo, Penington, Witten

- New understanding of generalized entropies

Chandrasekaran, Penington, Witten

Jensen, Sorce, Speranza

Kudler-Flam, Leutheusser, Satishchandran

- Algebraic ER=EPR: emergence of spacetime connectivity

Engelhardt, HL

- Formulation of stringy black holes (stringy horizon)

Gesteau, HL

- Enhancement of quantum tasks

Leutheusser, HL

……

Mathematical structure of quantum gravity?

Hilbert space plays a fundamental role in quantum mechanics.

There are indications that Hilbert space is not fundamental in quantum gravity,

but a derived concept

Hopefully, we are not too far from it ……

Thank you!