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Joint ExU and Nagoya Physics Colloquium

Negative Energy

February 5, 2026

Stefan Hollands

Artwork: Michelangelo

Absolute energy

Common statement

"The actual value of energy is not physically significant; only *differences* are"

- ✓ Electrostatics, thermodynamics, ...
- ✗ General relativity

Special relativity

In special relativity, E is a component of the energy-momentum 4-vector

$$P^\mu = (E, \mathbf{P})$$

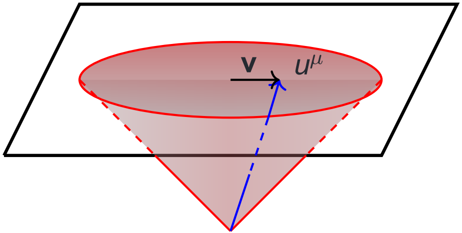
or more precisely:

Energy with respect to observer

$$E = -P_\mu u^\mu, \quad u^\mu = (1, \mathbf{v}) = \text{4-velocity of observer, } v < 1$$

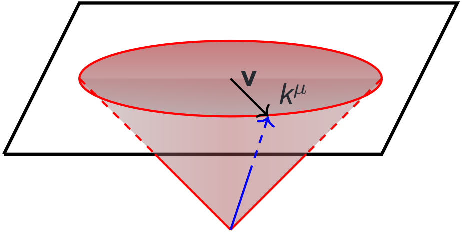
Timelike causal 4-vectors

timelike 4-velocity u^μ



Lightlike causal 4-vectors

lightlike (null) 4-velocity k^μ



General relativity

In general relativity, energy density of matter is a component of the stress-energy-momentum tensor:

Energy density with respect to observer

$$\rho = T_{\mu\nu} u^\mu u^\nu, \quad u^\mu = (1, \mathbf{v}) = \text{4-velocity of observer}$$

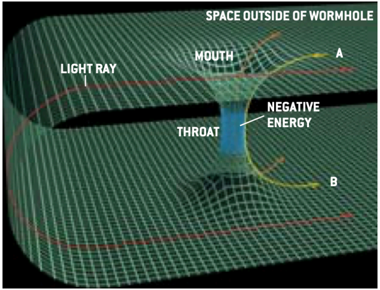
Einstein's equation:

$$\underbrace{G_{\mu\nu}}_{\textit{Einstein curvature tensor}} = 8\pi \underbrace{T_{\mu\nu}}_{\textit{energy density, not difference}}$$

NEGATIVE ENERGY AND TOPOLOGY

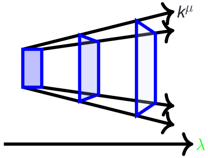
Negative energy density enables wormholes [Artwork: Scientific American Jan 2000]

Wormholes



Geodesics defocus when exiting the wormhole [Artwork: Scientific American Jan 2000]

(De-)focussing of lightbundles in GR



Raychaudhuri's equation

$$\frac{d}{d\lambda}(\text{expansion}) = -(\text{squares}) - 8\pi T_{\mu\nu}k^\mu k^\nu$$

$\Longrightarrow$ if we want to turn lightbundle from focussing $\to$ defocussing, we require sufficient amount of negative energy ($T_{\mu\nu}k^\mu k^\nu < 0$)!

Topological censorship

Rough statement

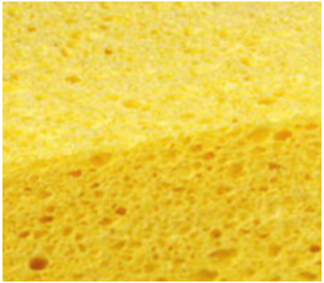
If $T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null k^μ, then any spacetime region containing non-trivial topology collapses too fast in order for us to be able to observe it, i.e. any probe gets ”pinched off”.

Technical statement

If $T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null k^μ, then any causal curve γ starting and ending in the asymptotic region (“null infinities”) of a globally hyperbolic spacetime can be continuously deformed so that it lies entirely within the asymptotic region.

Poincaré “conjecture” $\implies$ topology of time-slice $= \mathbb{R}^3$ minus balls (BHs).

Quantum foam

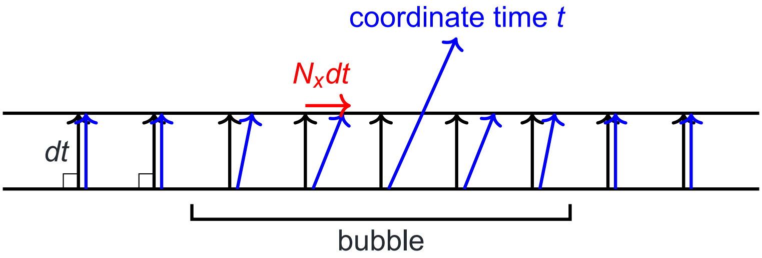


as non-trivial topology possible in principle on *microscopic* scales $\lesssim l_{\text{Planck}}$.

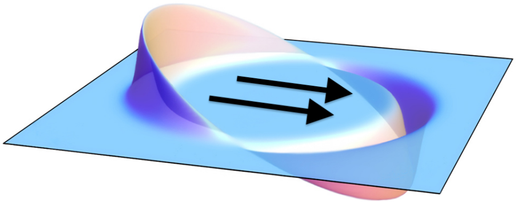
NEGATIVE ENERGY AND WARP DRIVE

Negative energy density enables spacetimes enabling *globally faster than light* travel. *Locally* speed of light *remains* 1. Constructible e.g., by suitable shift N_x creating a sort of *bubble*

$$g_{\alpha\beta}dx^\alpha dx^\beta = -dt^2 + (dx + N_x dt)^2 + dy^2 + dz^2$$

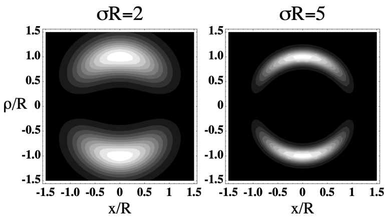


Opposing regions of expanding and contracting spacetime:



Negative energy surrounding bubble

$$N_x(t,\mathbf{r}) = \underbrace{\dot{x}_{\text{bubble}}(t)}_{\textit{bubble velocity}} \quad \underbrace{\chi_{\sigma,R}}_{\textit{bubble profile (smoothed top hat function)}} \left(\left|\mathbf{r} - \underbrace{\mathbf{r}_{\text{bubble}}(t)}_{\textit{bubble trajectory in } x-\textit{direction}}\right| \right)$$



Class.Quant.Grav. 14 (1997) 1743-1751

$R =$ radius of bubble, $\sigma =$ thickness of bubble walls, $\rho \perp$ to $x-$ direction

Gao-Wald no-go-theorem

Gao-Wald no-go-theorem (rough statement)

If energy density is non-negative, then a ”time advance” is not possible

- **Locally**, speed **not superluminal**
- **Impossible** to **globally** take a shorter ”detour”

Perturbative version of Gao-Wald no-go-theorem

Perturbed Minkowski metric

$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\textit{Minkowski}} + \underbrace{h_{\mu\nu}}_{\textit{ripple}}$$

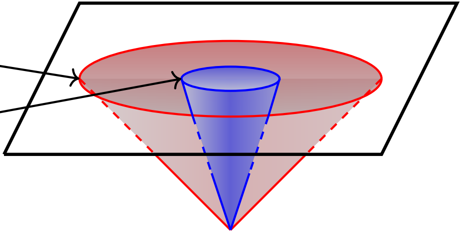
Linearized Einstein equation in Lorenz gauge

$$\Box h_{\mu\nu} = 8\pi T_{\mu\nu}$$

with $T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null $k^\mu \implies h_{\mu\nu}k^\mu k^\nu \geq 0$ (no incoming radiation)

Lightcone of $\eta_{\mu\nu}$

Lightcone of $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



"ENERGY CONDITIONS" AND THEIR VIOLATIONS

Energy "conditions"

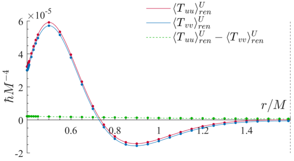
NEC

The ”null energy condition” states $T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null k^μ

- ✓ Holds for *classical* matter (e.g., EM fields, perfect fluids, Vlasov-model, …)
- ✗ Can be violated for *quantum* matter, e.g. Hawking process of BH evaporation!

Hawking flux violates NEC!

In the so-called *Unruh quantum state* $|U\rangle$, the expectation value of the both ingoing $\langle \hat{T}_{vv} \rangle_U$ and outgoing $\langle \hat{T}_{uu} \rangle_U$ null components of the expected stress energy tensor of a massless scalar quantum field can become negative at the *event horizon* of a Kerr black hole of mass M:



[Zilberman et al. PRD 111 (2025)]

Total amount of negative quantum energy outside a solar mass black hole:

$$E_{\text{Hawking}} \approx -10^{-32}\,\text{Joule}$$

Energy ``conditions"

ANEC

The "averaged null energy condition" states $\int_\gamma T_{\mu\nu} k^\mu k^\nu \, d\lambda \geq 0$ for all inextendible null-curves γ (tangent $=$ velocity $= \dot{\gamma}^\mu = k^\mu$)

- ✓ Holds for expectation value of *quantum* stress energy tensor for null-rays in Minkowski, still sufficient for topological censorship
- ✗ Severely violated for expectation value of *quantum* stress energy tensor inside a charged/rotating BH

Negative energy density through interference

Free KG quantum field

$$\hat{\phi}(t,\mathbf{x}) = \sum_{\mathbf{k}} \underbrace{\hat{a}_{\mathbf{k}}}_{\text{annihilates mode }\mathbf{k}} e^{-i\omega(\mathbf{k})t + i\mathbf{k}\mathbf{x}} + \text{c.c.}$$

Coherent superposition of vacuum and 2-particle state:

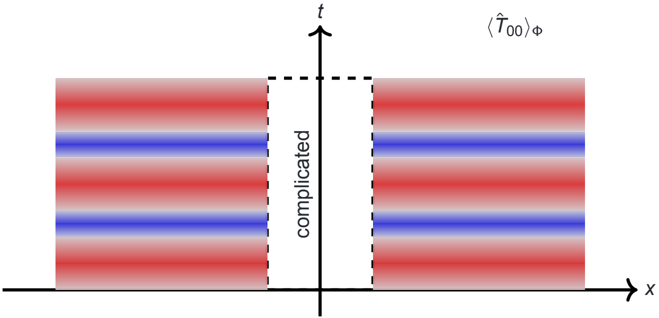
$$|\Phi\rangle = \alpha|0\rangle + \beta|2\rangle$$

Momentum eigenbasis decomposition:

$$|2\rangle = \sum_{\mathbf{k},\mathbf{p}} \underbrace{c_{\mathbf{k},\mathbf{p}}}_{\textit{Fourier coefficients}} \underbrace{\hat{a}^\dagger_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{p}}|0\rangle}_{\textit{momentum eigenstate}}$$

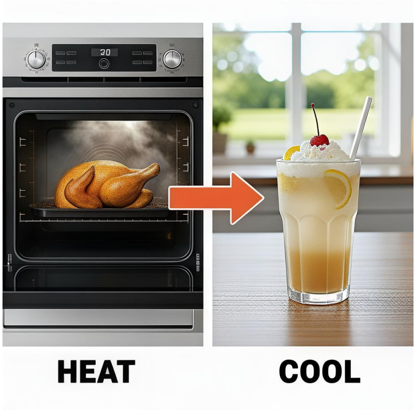
Expected energy density

- Tune $c_{\mathbf{k},\mathbf{p}}$ to be peaked for $\mathbf{k} \approx -\mathbf{p}$
- Populate modes for $|k| \lesssim \Lambda$.



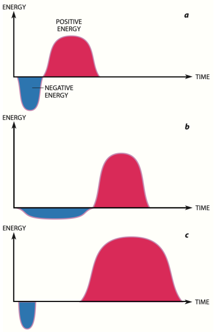
QUANTUM ENERGY INEQUALITIES

Negative energy vs second law



”Artwork”: Promeai

Quantum inequalities I



"Quantum energy inequalities" limit the shapes of pulses of negative energy [source: Ford & Roman, Scientific American 2000]

Quantum inequalities II

Concretely, a quantum energy inequality may look e.g., like this (here a $1+1$-dimensional ”conformal” field theory):

$$\int d\lambda\, f^2 \langle \hat{T}_{\mu\nu}(\gamma)\rangle \dot{\gamma}^\mu \dot{\gamma}^\nu \geq -\frac{c}{6}\frac{\hbar}{2\pi}\int d\lambda\,(f')^2$$

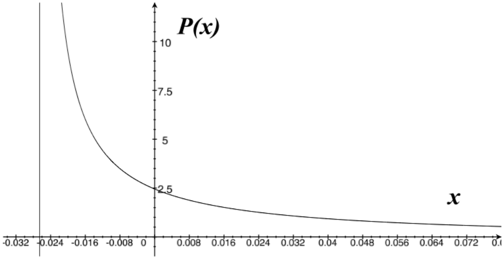
for:

- **all** null worldlines γ with tangent $k^\mu = \dot{\gamma}^\mu$
- **all** quantum states
- **all** sampling functions $f(\lambda)$ (e.g. Gaussian)

Probability distributions for energy in vacuum

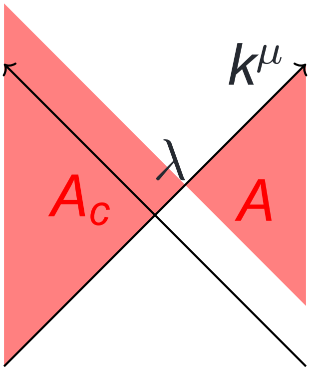
$$f(\lambda) \propto e^{-\lambda^2/2\tau^2} = \text{Gaussian sampling function with width } \tau$$

Since $|0\rangle$ is not an eigenstate of $\hat{\rho} = \int d\lambda\, f^2 \hat{T}_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu$, its measurements x fluctuate!



Probability for x. Negative energies much more likely ($\approx 90\%$)! [Fewster & Ford, PRD 81 (2010)]

Entanglement entropy of a "causal wedge"

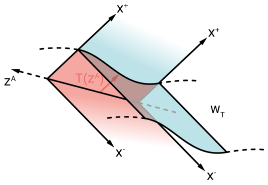


$$S_{EE}(\lambda) = S_{vN}(\hat{\rho}_A), \quad \hat{\rho}_A = \mathrm{Tr}_{A_c} |\psi\rangle\langle\psi|$$

The quantum NEC

QNEC in $1+1$D

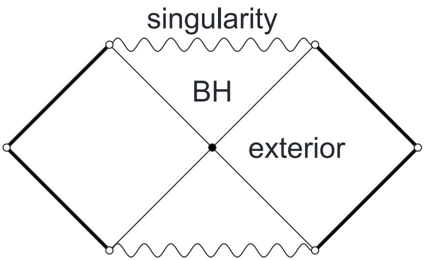
$$\langle T_{\mu\nu}\rangle k^\mu k^\nu \geq \frac{\hbar}{2\pi} S''_{\text{EE}}$$



Second shape derivative of S_{EE} in $D \geq 3$ dimensions

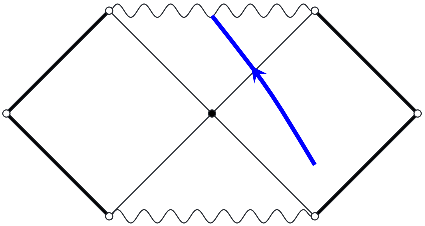
NEGATIVE ENERGY INSIDE BLACK HOLES

Schwarzschild black hole



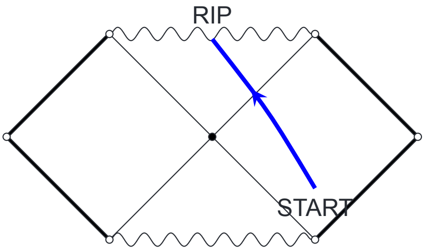
Singularity is spacelike

Schwarzschild black hole



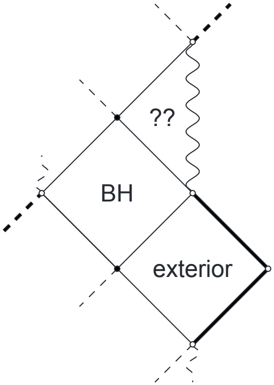
Observers diving into BH end their existence

Schwarzschild black hole

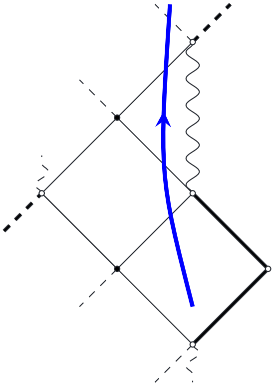


... in finite proper time

Charged black hole

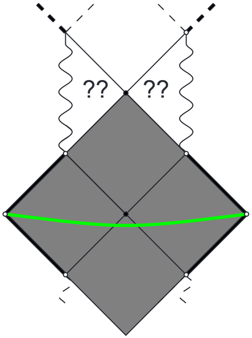


Charged black hole



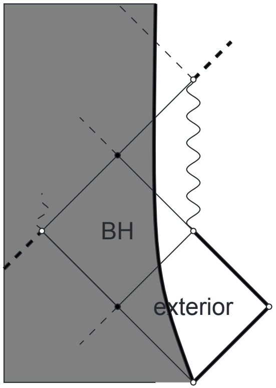
Observers diving into BH may stay clear of singularity

Determinism violation

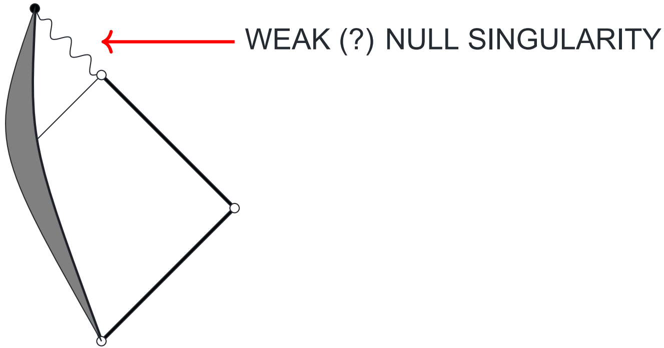


Domain of dependence of Cauchy slice

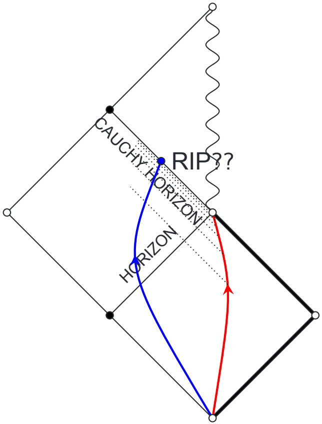
Collapsing shell [Boulware]



What really happens: Null singularity



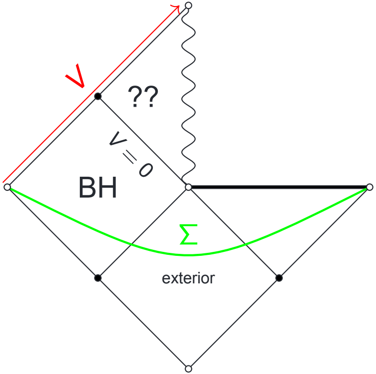
(Strong) "cosmic censorship" in the RNdS spacetime



- Cosmological redshift $\rightarrow$ competition with blueshift
- This competition is mathematically expressed by

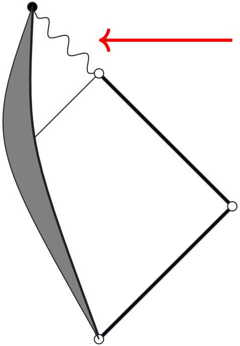
$$\beta = \frac{\alpha}{\kappa_-} = \frac{\text{spectral gap of QNMs}}{\text{temperature of CH}}$$

sCC in the RNdS spacetime



- sCC violated if $\beta > \frac{1}{2}$ because $T_{VV} \propto V^{-2+2\beta}$ for generic smooth initial data
- $\beta > 1/2$ for large Q
- sCC *fails* for near extremal RNdS BHs!
- sCC *holds* for asympt. flat RN BHs because $T_{VV} \propto V^{-2}(\log V)^{-p}$, where $p =$ Price law exponent!

Asymptotically flat case: *weak* null singularity



$T_{VV} \to \infty$ but horizon area $\sim \int \int T_{VV} < \infty$

QUANTUM FIELDS

Quantum energy blowup

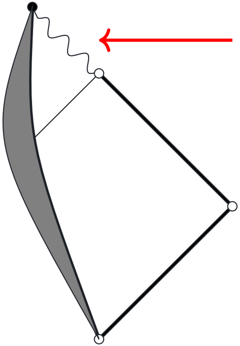
Behavior near $\mathcal{CH}$

$$\langle T_{VV} \rangle \sim \hbar \tilde{C} V^{-2} + t_{VV} \,.$$

Here:

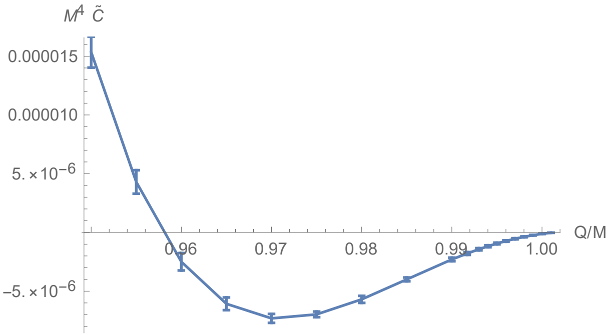
- $\tilde{C}$ universal
- $t_{VV} \sim V^{-2+2\beta}$ near $\mathcal{CH}$.
- Quantum singularity **stronger** than classical!

Quantum case: *strong* singularity



$T_{VV} \to \infty$ and horizon area $\sim \int \int \langle T_{VV} \rangle = \infty$

Sign of the energy flux



[Hollands et al. PRD 102 (2020)]

Energy flux $\langle T_{VV}\rangle \sim \hbar\tilde{C}V^{-2}$ at $\mathcal{CH}$ as a function of Q/M. **Both signs appear**!

Sign of energy density

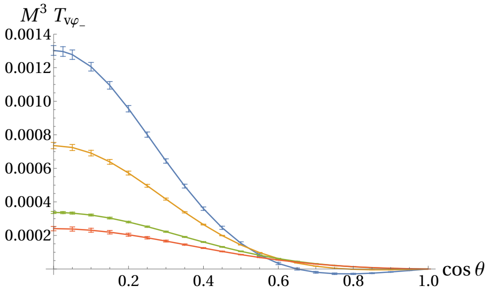
If we could impose semi-classical Einstein equation

$$G_{\mu\nu} = 8\pi \left(T_{\mu\nu}^{\text{class}} + \langle \hat{T}_{\mu\nu} \rangle \right)$$

- $\tilde{C} > 0$: Observers crossing CH get crushed to death
- $\tilde{C} < 0$: Observers crossing CH get stretched to death

This should be analyzed more fully on evaporating BH spacetime

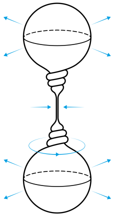
Angular momentum flux in Kerr-deSitter



[Klein, Soltani, Casals, SH, PRL 132 (2024)]

$\Longrightarrow$ Unusual sign of angular moment flux $\langle T_{V\varphi_-}\rangle = \hbar\tilde{C}V^{-1} + t_{V\varphi_-}$ near poles at CH

Shape of CH in Kerr-deSitter



[Artwork: SH, Tom Endler]

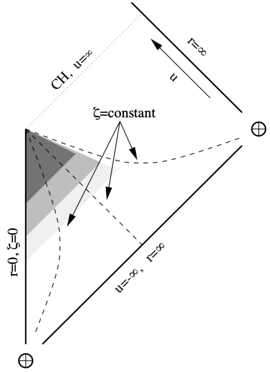
$\implies$ infinite **twisting**/streching/squeezing of observers possible at CH

Self-similar collapse

Self-similar spacetime ($u =$ time):

$$g_{\mu\nu}(u+\lambda, x^i) = e^{-2\lambda} g_{\mu\nu}(u, x^i)$$

- relevant for critical collapse (Choptuik scaling)
- discrete ($\lambda \in \mathbb{Z}$) or continuous versions ($\lambda \in \mathbb{R}$)
- E.g. $ds^2 = e^{-2u}[-G(\zeta)du^2 - 2H(\zeta)dud\zeta + \zeta^2 d\omega^2]$



[Brady & Ottewill PRD 58 (1998)]

Quantum stress tensor

Self-similar states: $\langle \phi(u+\lambda)\phi(u'+\lambda)\rangle = e^{2\Delta\lambda}\langle\phi(u)\phi(u')\rangle$

Late time SET in self-similar state

$$\langle T_{\alpha\beta}\rangle \sim \frac{c}{4\pi} u e^{2u} (\text{curvature tensor})_{\alpha\beta} \quad u \to \infty$$

see Zahn CQG 43 (2026), Brady & Ottewil PRD 58 (1998)

Self-similar collapse

Examples:

- Minkowski patch (!)
- Choptuik spacetime Choptuik PRL 70 (1993), ..., Reiterer & Trubowitz CMP 368 (2013)
- Hayward spacetime Hayward CQG 17 (2000)
- Critical Roberts spacetime Roberts GRG 21 (1989)

$$ds^2 = e^{2(\zeta - u)}[2(1 - e^{-2\zeta})du^2 - 4du\,d\zeta + d\omega^2]$$

Late time SET:

$$\langle T_{\zeta\zeta} \rangle \sim u e^{-2(\zeta - u)}(5 - 17e^{-2\zeta})$$

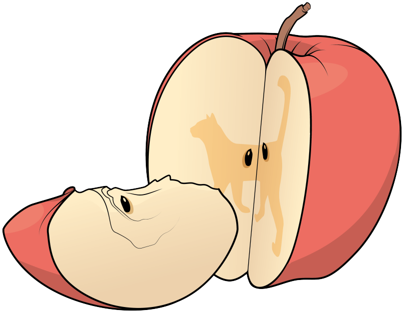
Can have *both signs*, quantum corrections to critical exponents Brady & Ottewill PRD 58 (1998)

Conclusions

- In GR, the *absolute* energy (-momentum) density matters
- Bubbly spacetime *topology* and *warp drive* require negative energy
- Quantum effects can *enable* negative energy
- But severely constrained by inequalities
- Negative energy fluxes can be *macroscopically large inside a charged or rotating black hole*

Some references

- Topological censorship: Friedman et al. PRL 76 (1993)
- Gao-Wald theorem: Visser et al., NPB 88 (2000), Gao & Wald CQG 17 (2000)
- Hawking fluxes: Hawking, CMP 43 (1975), ..., Zilberman et al. PRD 111 (2025)
- Quantum inequalities: Ford, PRSL A364 (1978), Ford & Roman PRD 53 (1996), 55 (1997), Fewster CQG 17 (2000), Ford et al. PRD 66 (2002), Hollands & Fewster RMP 17 (2005), ...
- QNEC: Bousso et al. PRD 93, 024017 & 064044 (2016), 102 (2020), Wall PRL 118 (2017), Ceyhan & Faulkner CMP 277 (2020), Hollands & Longo, CMP (to appear) arXiv:2503.04651,
- (Quantum) fluxes inside BHs: Penrose (1974), Dafermos, Ann. Math. 158 (2003), Dias et al. JHEP 10 (2018), Zilberman et al. PRL 124 (2019), 129 (2022), Hollands et al. CQG 37 (2020), Hollands et al. PRD 102 (2020), PRL 127 (2021), 132 (2024), ...
- Critical collapse: Eardley CMP 37 (1974), Ori & Piran PRL 59 (1987), Choptuik PRL 70 (1993), Gundlach, Hildich, Arrachea (2025), Reiterer & Trubowitz CMP 368 (2013), Brady & Ottewill PRD 58 (1993), Koike et al. PRL 74 (1995), ...
- **Not covered:** Negative Casimir energies/repulsive forces Carpasso et al. Nature 457 (2009), ..., negative energy modes in BECs Teocharis et al. PRA 76 (2007), many more!



Artwork: SH & Tom Endler

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QUESTIONS WELCOME!

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