冷却原子におけるエフィモフ状態とその普遍性
+
関連する少数多体研究

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科研費
KAKENHI

Outline

- What is cold atoms?

- Quantum 3-body problem : Efimov state
  ⇒Universality

- From few-body to many-body: cluster (virial) expansion
  – 3rd order expansion with Efimov states
  – 4th order expansion for unitary Fermi gas

Cold atoms: “Coolest system” in the universe

Dilute atoms cooled down to extreme low temperature

1995 Nobel Prize in Physics

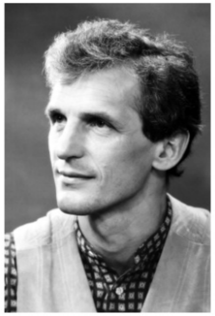
Laser Cooling of atomic gases

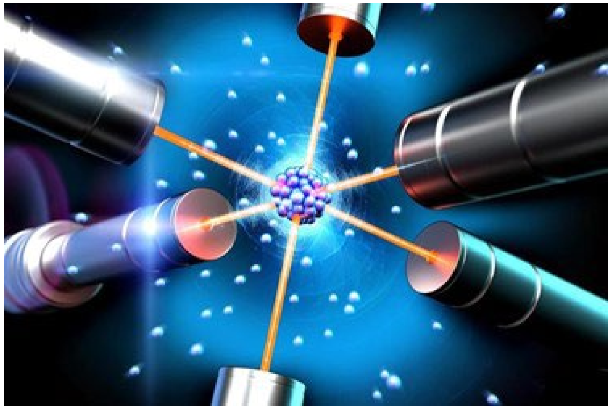


2001 Nobel Prize in Physics

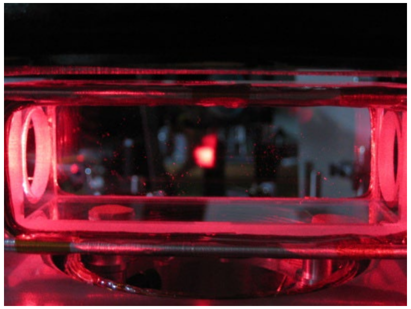
Superfluid of atomic gases

(Bose-Einstein condensation)

$T \sim 10^{-7}$-10^{-10} K



Cold atom: Highly controllable

- Statistics (boson/fermion) of particles
- Mass ratio between particles
- Dimension (1d,2d,3d,mixed-d)
- Optical lattice
- Interaction
- Molecule with strong dipole-dipole interacton
- Ramdomness
- Synthetic gauge field

+ very clean (coherent) experiment
high resolution observations

Quantum simulation

Cold atom: Highly controllable

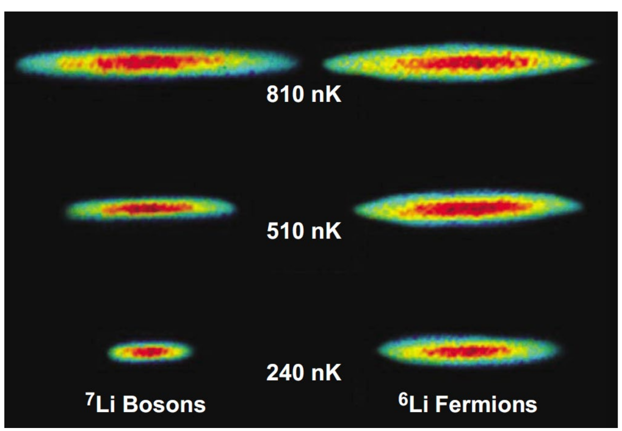
- Statistics (boson/fermion) of particles
- Mass ratio between particles

陽子+中性子+電子 = 偶数 ⇒ Bosonoic particle

陽子+中性子+電子 = 奇数 ⇒ Fermionic particle

Bosons	Fermions
H, ^{4}He (excited state), ^{7}Li, ^{23}Na, ^{39}K, ^{41}K, ^{85}Rb, ^{87}Rb, ^{133}Cs, ^{52}Cr, ^{40}Ca, ^{84}Sr, ^{86}Sr, ^{88}Sr, ^{174}Yb	^{6}Li, ^{40}K, ^{171}Yb, ^{173}Yb, ^{53}Cr

Bose原子とFermi原子の比較



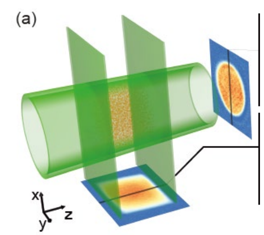
Truscott, et al., Science (2001)

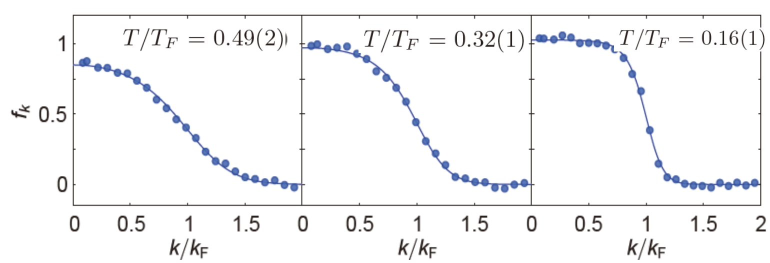
一様Fermi気体。Fermi分布

箱型(Box) ポテンシャル

曲線：理想Fermi分布

$$f_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}$$





Mukherjee, et al, PRL. (2017): ^{6}Li

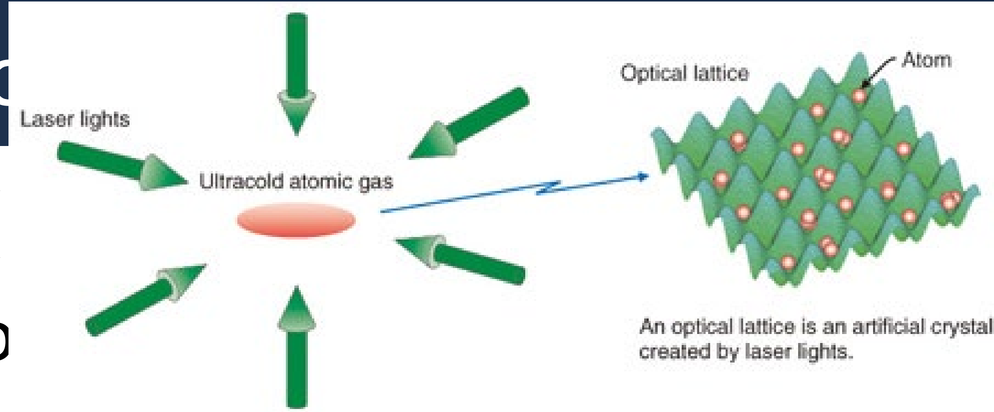
Mass-imbalanced mixture

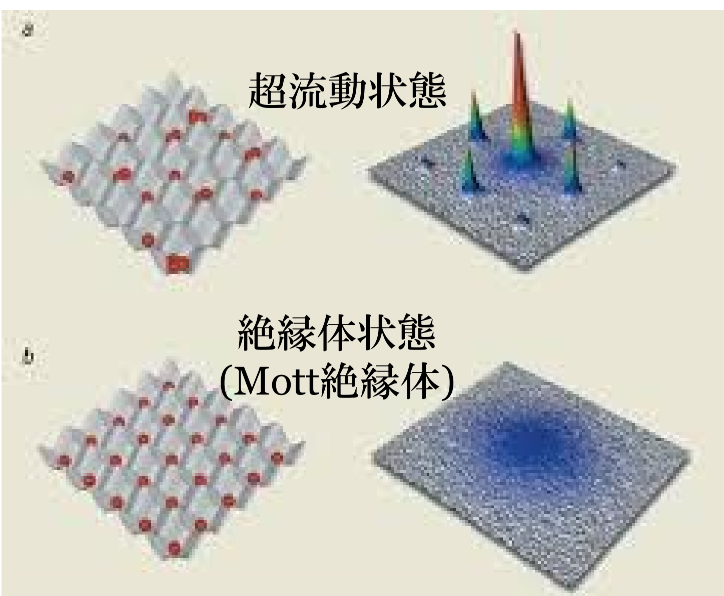
Li-K	6.7
Li-Cr	8.8
Li-Rb	15
Li-Cs	22
Li-Er	28
Li-Yb	29
Dy-Er	1.0
Na-K	1.7
K-Rb	2.2
K-Dy	4.0

…..

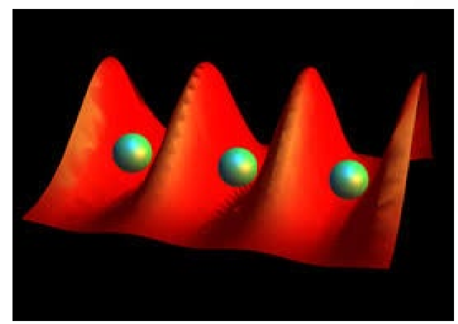
Co ble

- Statistics (
- Mass ratio
- Dimension (1d,2d,3d,mixed-d)
- Optical lattice

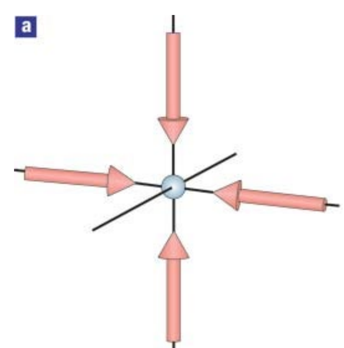




2D



1D



Controllable interaction: Feshbach resonance

- Low temperature T ～100nK
- Dilute $n^{-1/3}$ ～1μm

$$r_{\rm eff} \sim r_{\rm vdw} \sim 100 \text{ nm}$$

$$r_{\rm eff}, r_{\rm vdw} \ll \lambda_{dB}, n^{-1/3}$$

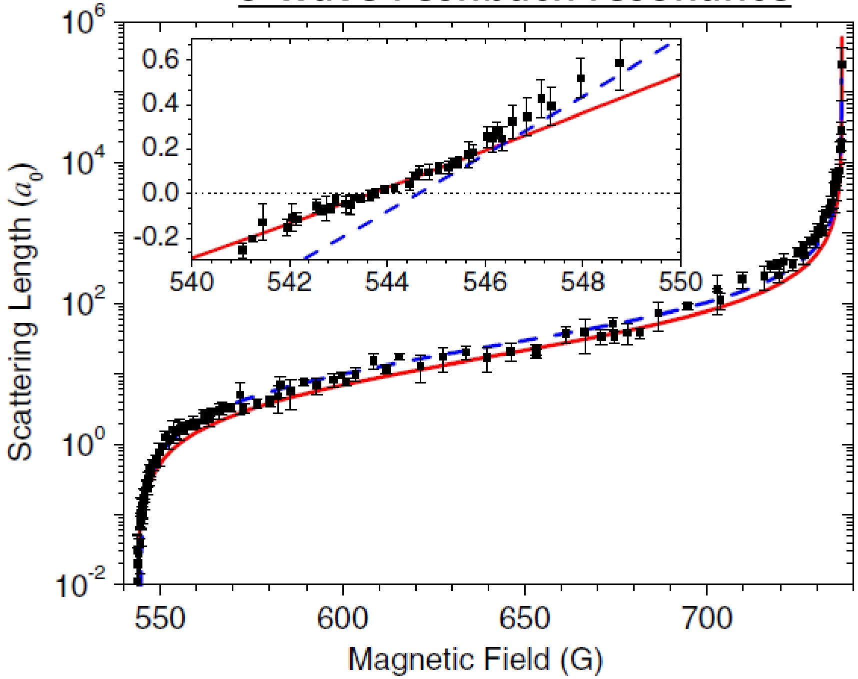
⇒(Leading order) Interaction described by s-wave scattering length

$$H_{tot} = H_0 + H_{int} \qquad H_{int} = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \hat{\psi}^\dagger_\uparrow(\boldsymbol{r}_1)\hat{\psi}^\dagger_\downarrow(\boldsymbol{r}_2) V(\boldsymbol{r}_1 - \boldsymbol{r}_2)\hat{\psi}_\downarrow(\boldsymbol{r}_2)\hat{\psi}_\uparrow(\boldsymbol{r}_1)$$

$$V(\boldsymbol{r}) = \frac{4\pi\hbar^2 a}{m}\delta^{(3)}(\boldsymbol{r})\frac{\partial}{\partial r}r$$

Contact interaction Hamiltonian for cold atoms

S-wave Feshbach resonance



S. E. Pollack *et al.*, PRL. **102** 090402, (2009): ^{7}Li

Universal 2-body physics for large scattering length

- Scattering amplitude at low energy: (short-ranged, central potential)

$$f_\ell(k) = \frac{k^{2\ell}}{k^{2\ell+1}\cot\delta_\ell(k) - ik^{2\ell+1}}.$$

$\delta_\ell(k)$: l-th wave phase shift

$$k^{2\ell+1}\cot\delta_\ell(k) = \text{Const.} + O(k^2).$$

: Effective range expansion

⇒ Higher partial wave scatterings are negligible.

$$f(k) = -\frac{1}{1/a + ik}.$$

a : s-wave scattering length

- Bound state is determined by a.

$$E = -\frac{\hbar^2}{2\mu a^2} \qquad \psi(r) = \frac{e^{-r/a}}{\sqrt{2\pi a}\, r}$$

⇒ Valid for various kinds of short-range potentials (Universality)

Universal 2-body problem

- Condition: Low energy. Large s-wave scattering length

$$f(k) = -\frac{1}{1/a + ik}.$$

Binding energy of deutron 2.22MeV (proton + neutron)

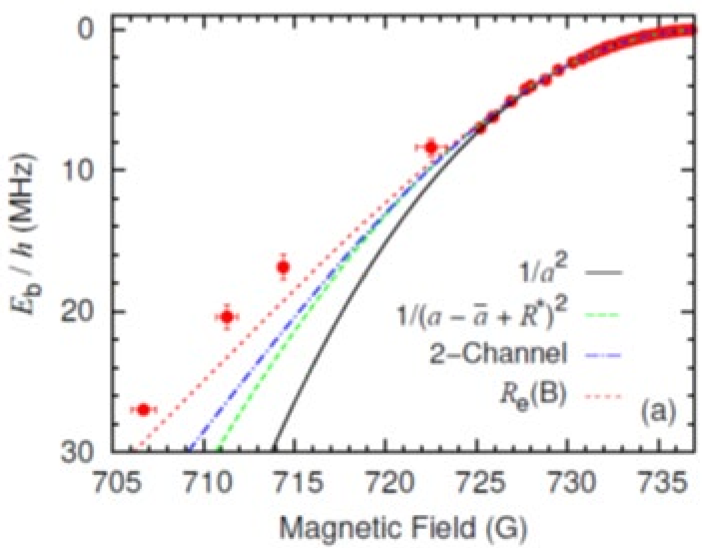
$$E = -\frac{\hbar^2}{ma^2}$$ → 1.41MeV

+ effective range correction

→ 2.22MeV

$$a = 5.4112(15)\ \mathrm{fm}$$
$$r_{\mathrm{eff}} = 1.7436(19)\ \mathrm{fm}$$

Binding energy of a Feshbach molecule in cold atoms



10 neV = 10^{-8} eV

^{7}Li Rice Univ.: Dyke, et al., PRA (2013)

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  ⇒Universality

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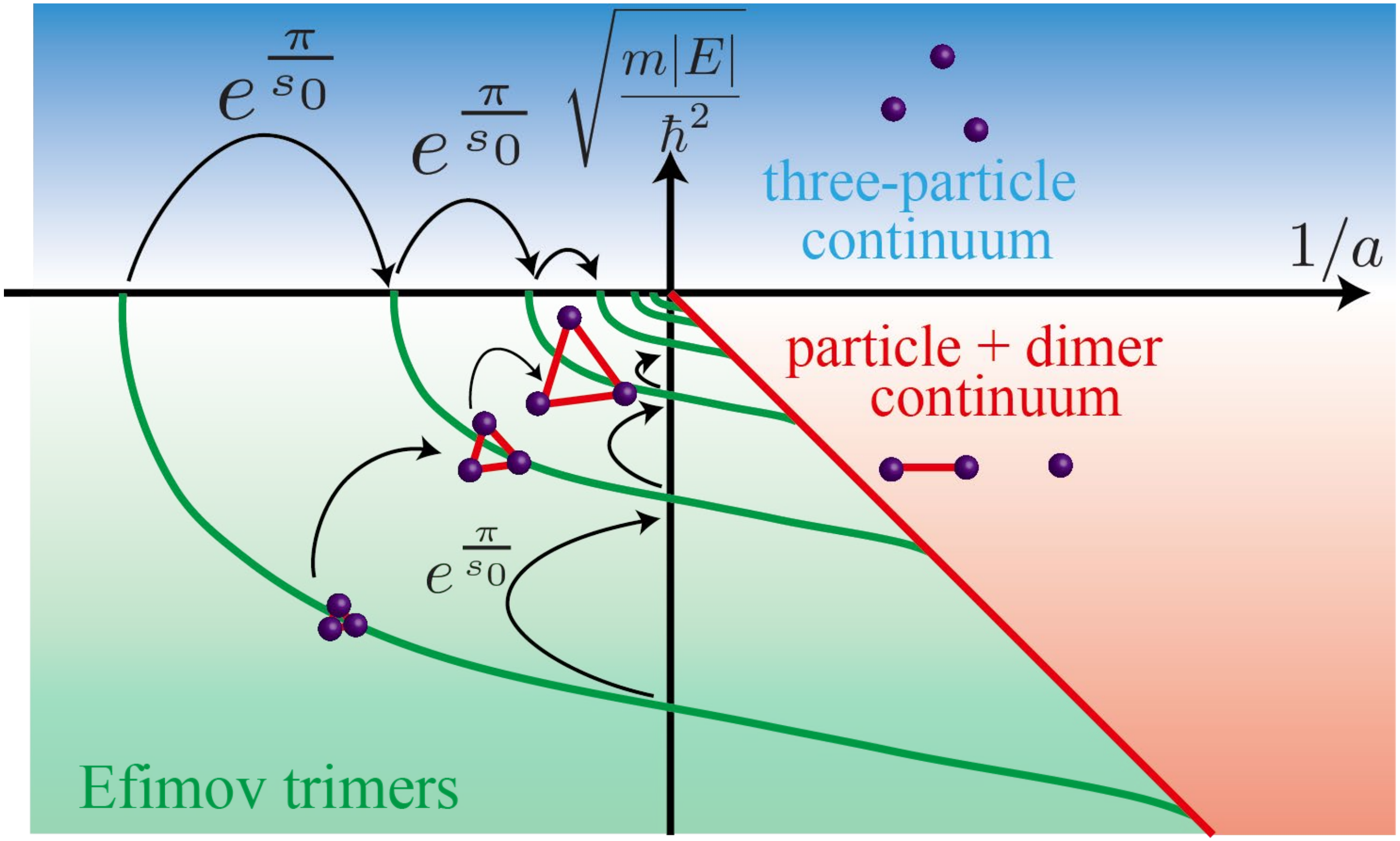
  - 3^{rd} order expansion with Efimov states
  - 4^{th} order expansion for unitary Fermi gas

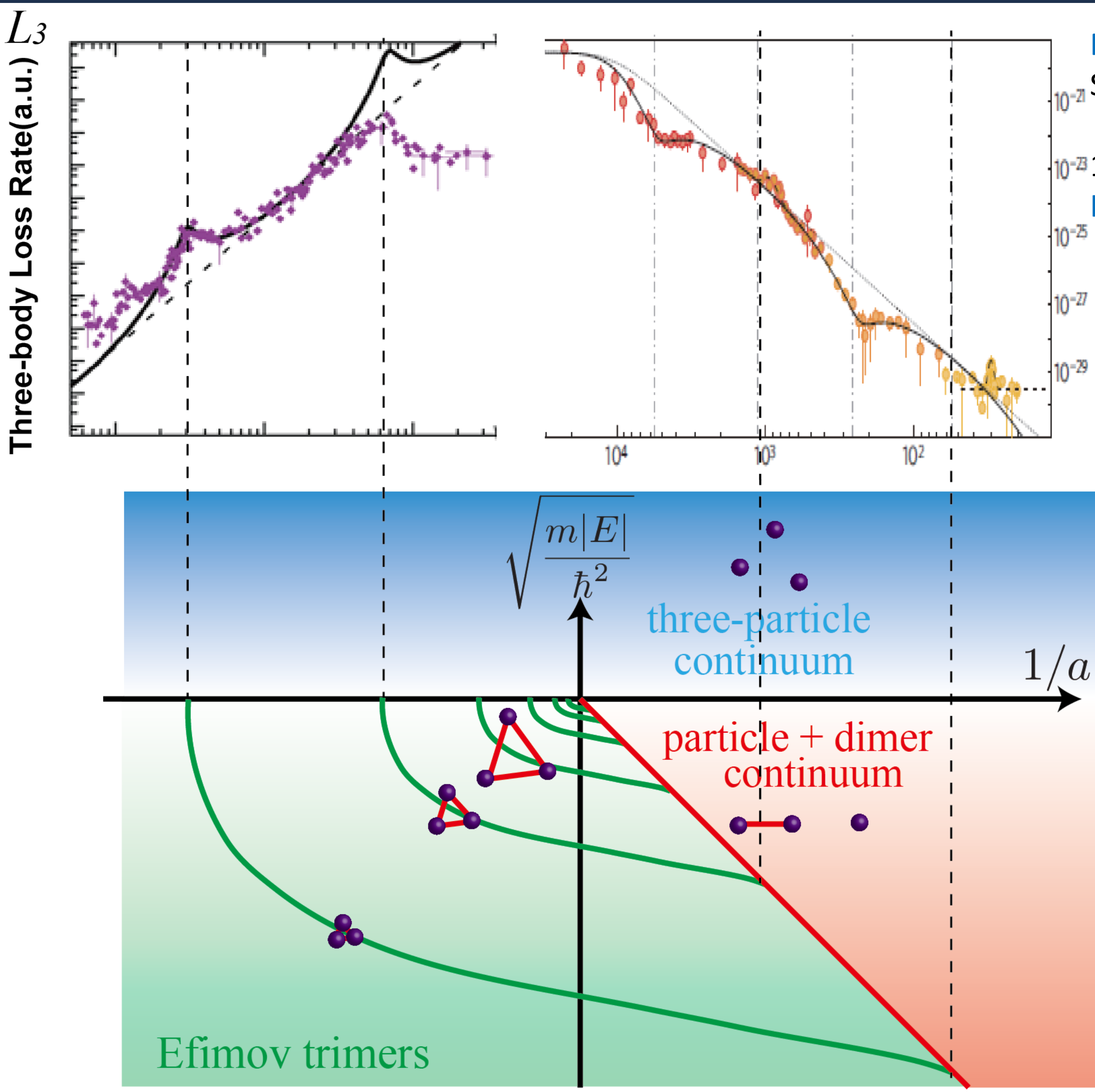
3-body energy spectra of 3 bosons at a≈±∞

- Discrete scale invariance
  - Binding energy $E_{n+1} = e^{-2\pi/s_0} E_n$, $\quad e^{\pi/s_0} = 22.7\ldots\ (s_0 = 1.0024\ldots)$
  - Wave function $\Psi_{n+1}(r_i) = \Psi_n(r_i e^{-\pi/s_0})$,

"Efimov physics: a review", Naidon, Endo, Rep. Prog. Phys. (2017)
「講義ノート：量子少数系におけるユニバーサリティとEfimov状態の物理」遠藤晋平, 原子核研究 (2019)



Observation of the Efimov states



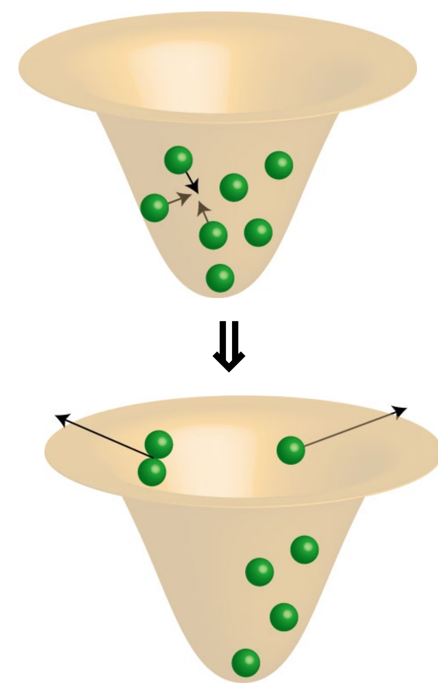
F. Ferlaino, R. Grimm Physics (2010)

Solid curve: zero-range model (2 fit parameters)

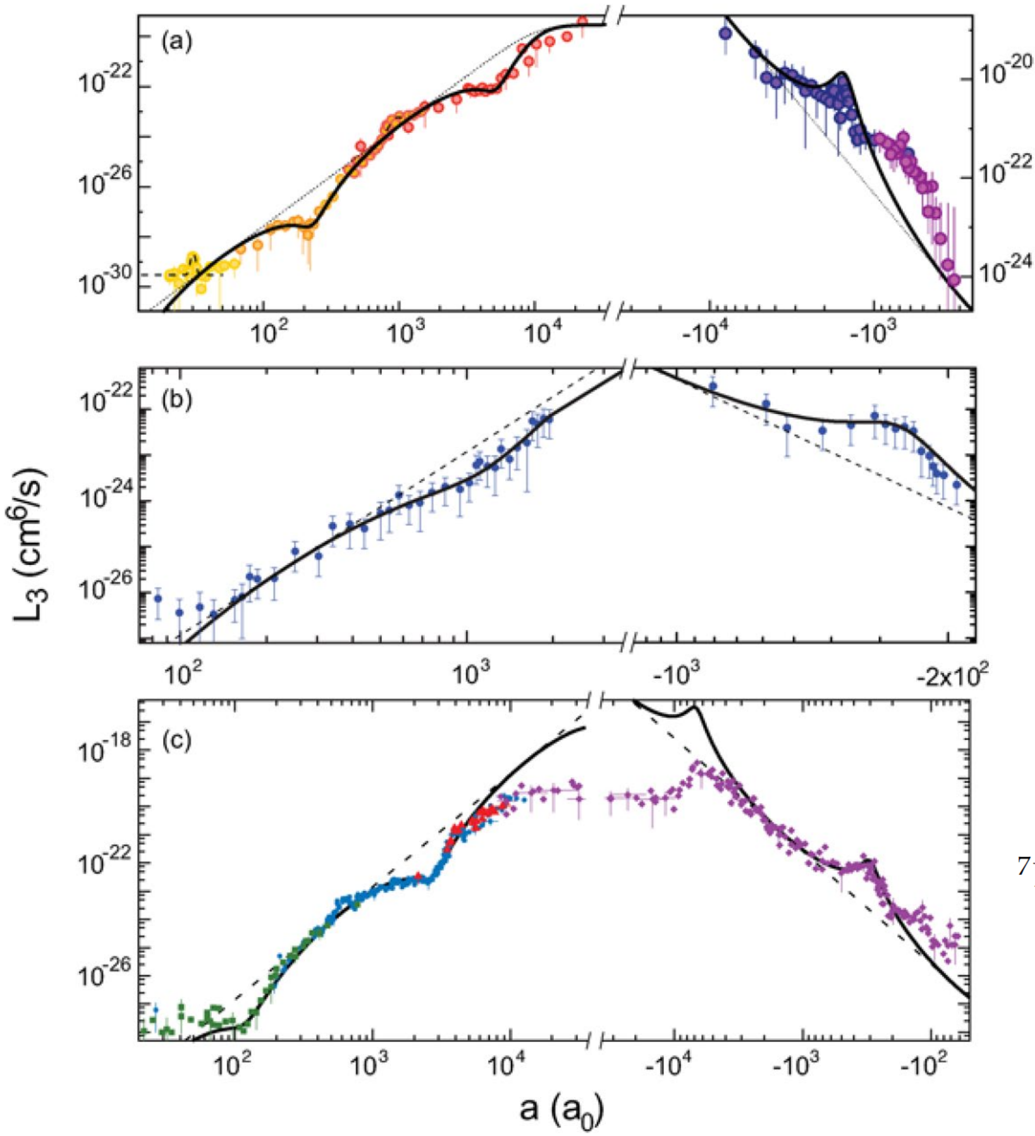
1st observation in cold atoms

Kraemer et al, Nature (2006)

$$\frac{dn}{dt} = -L_3 n^3.$$



Efimov states with various atoms, spins



^{39}K *Zaccanti, et al. Nature Physics 5, 586 (2009)*

Roy, et al. PRL 111, 053202 (2013)

^{7}Li *Gross,et al. PRL. 103, 163202 (2009).*

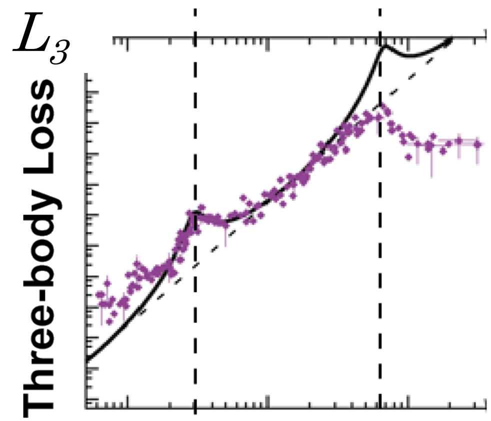
^{7}Li *Pollack, Dries, Hulet, Science 326, 1683 (2009).*

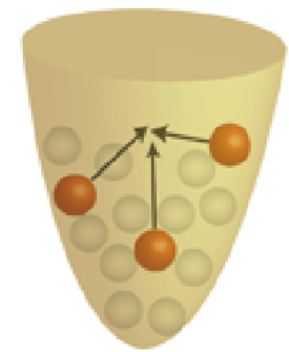
Dyke, Pollak, Hulet PRA (2013)

3-body parameter of the Efimov states

- Scattering length： low-energy phase shift of 2-body scattering.
- 3-body parameter： low-energy phase shift of 3-body scattering .
  ⇒ These 2 parameters characterizes the Efimov states.

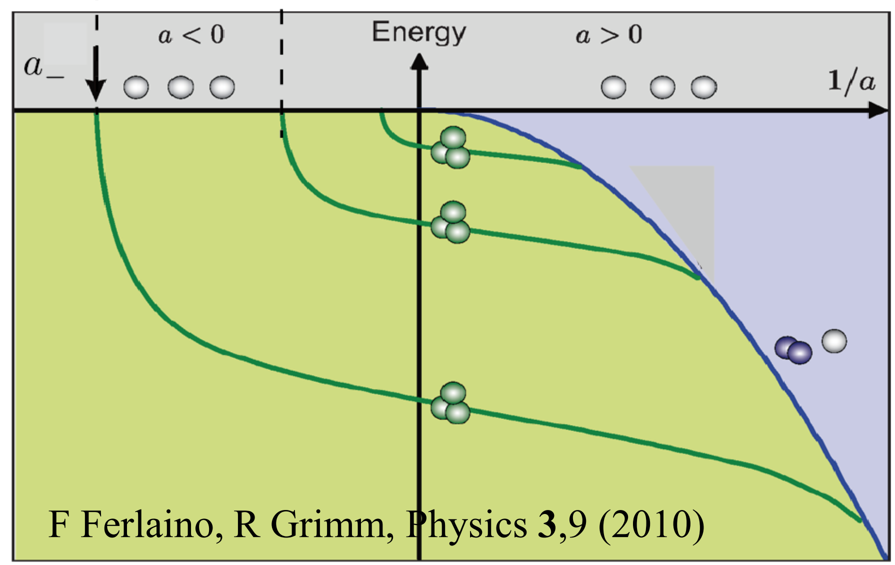
- 3-body parameter can be measured experimentally from the peak of the 3-body loss rate.





- Scattering length depends sensitively on atomic species and internal states (non-universal).

➡ 3-body parameter should also be non-universal.



F Ferlaino, R Grimm, Physics **3**,9 (2010)

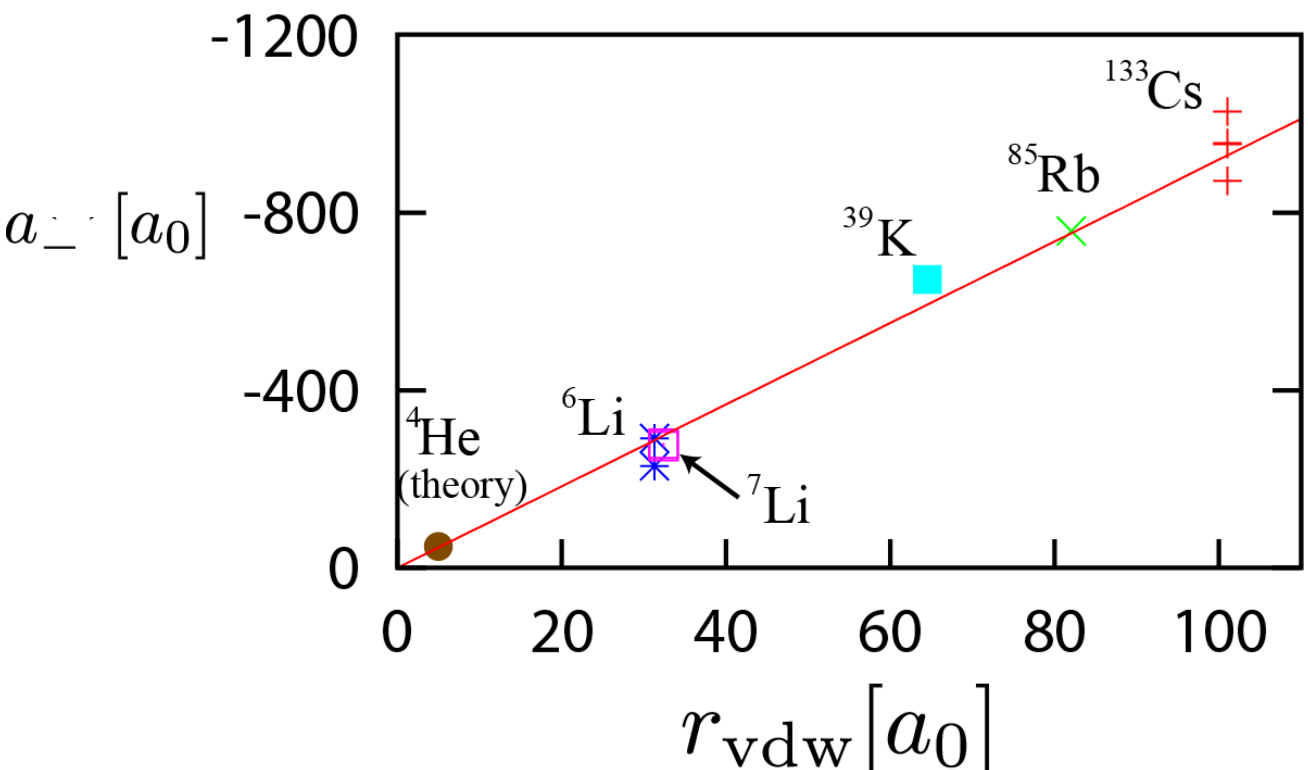
Universal 3-body parameter

- **3-body parameter measured for various atomic species (～2012)**

⇒Universallv characterized by the van der Waals length!

$$a_- = -(9.4 \pm 0.2) r_{\mathrm{vdw}} \quad r_{\mathrm{vdw}} = \frac{1}{2}\left(\frac{mC_6}{\hbar^2}\right)^{\frac{1}{4}}$$ van der Waals length

M. Berninger, *et al.* PRL. **107**, 120401 (2012)



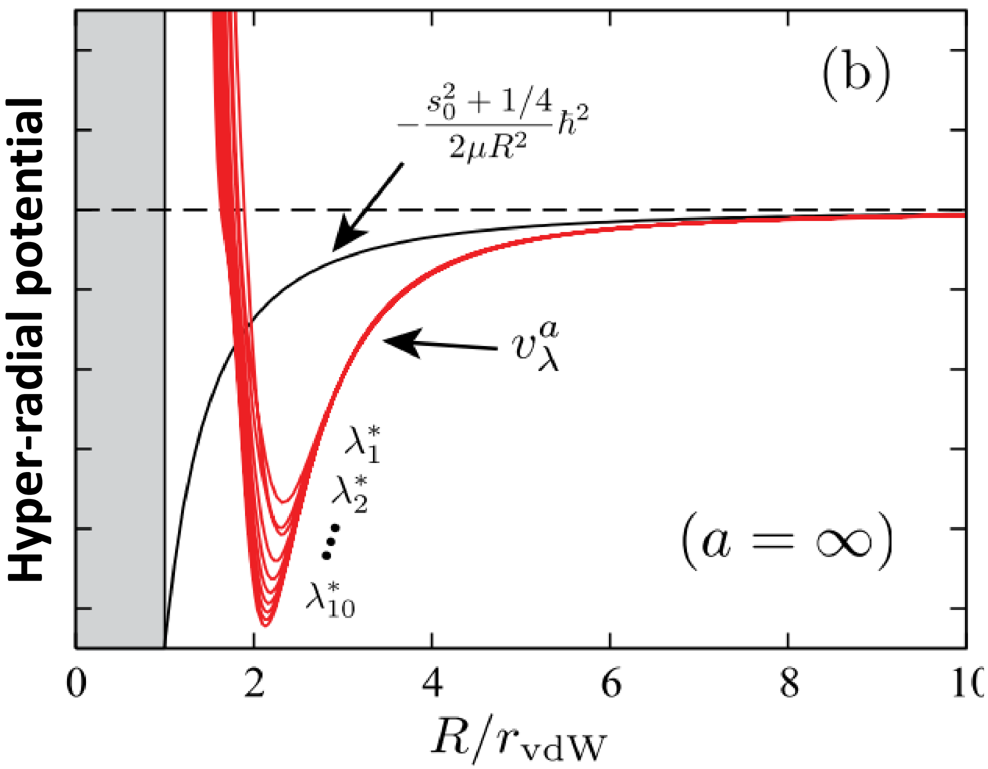
a_0 Bohr radius

Previous study and unanswered questions

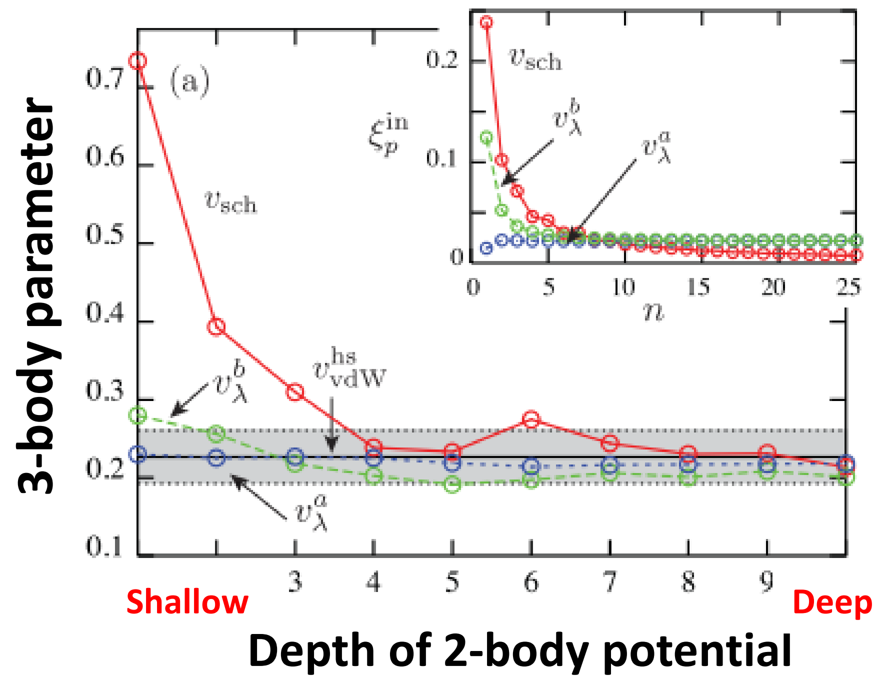
- **2-body potential is either strongly attractive or has a hard-core**

  ⇒ Strong Hyper-radial repulsion appears, rendering the 3-body parameter universal.

J. Wang *et al.*, PRL. **108**, 263001 (2012)

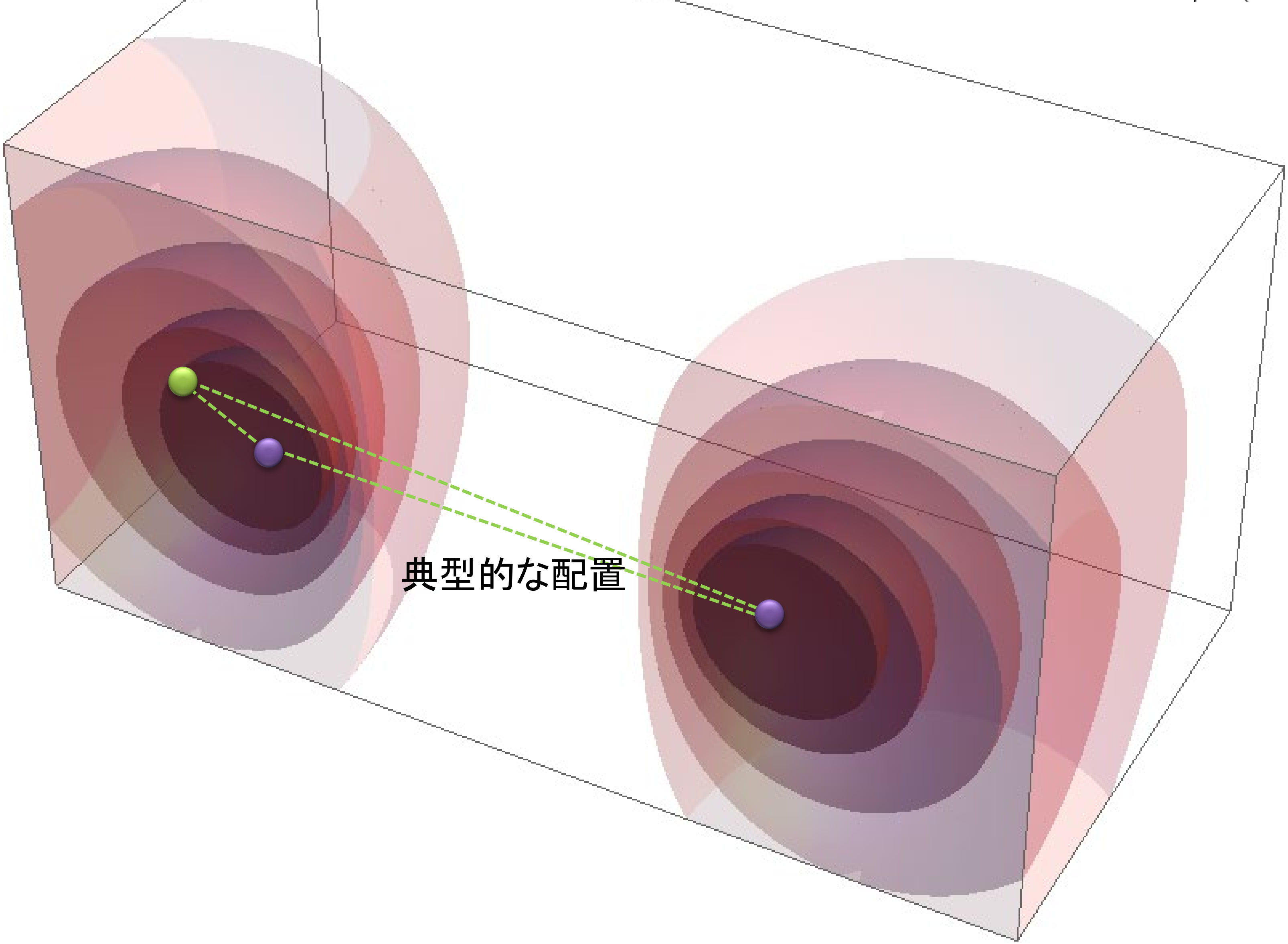


$$R = 1.31\sqrt{r_{12}^2 + r_{13}^2 + r_{31}^2}$$



Zero-range Efimov theory

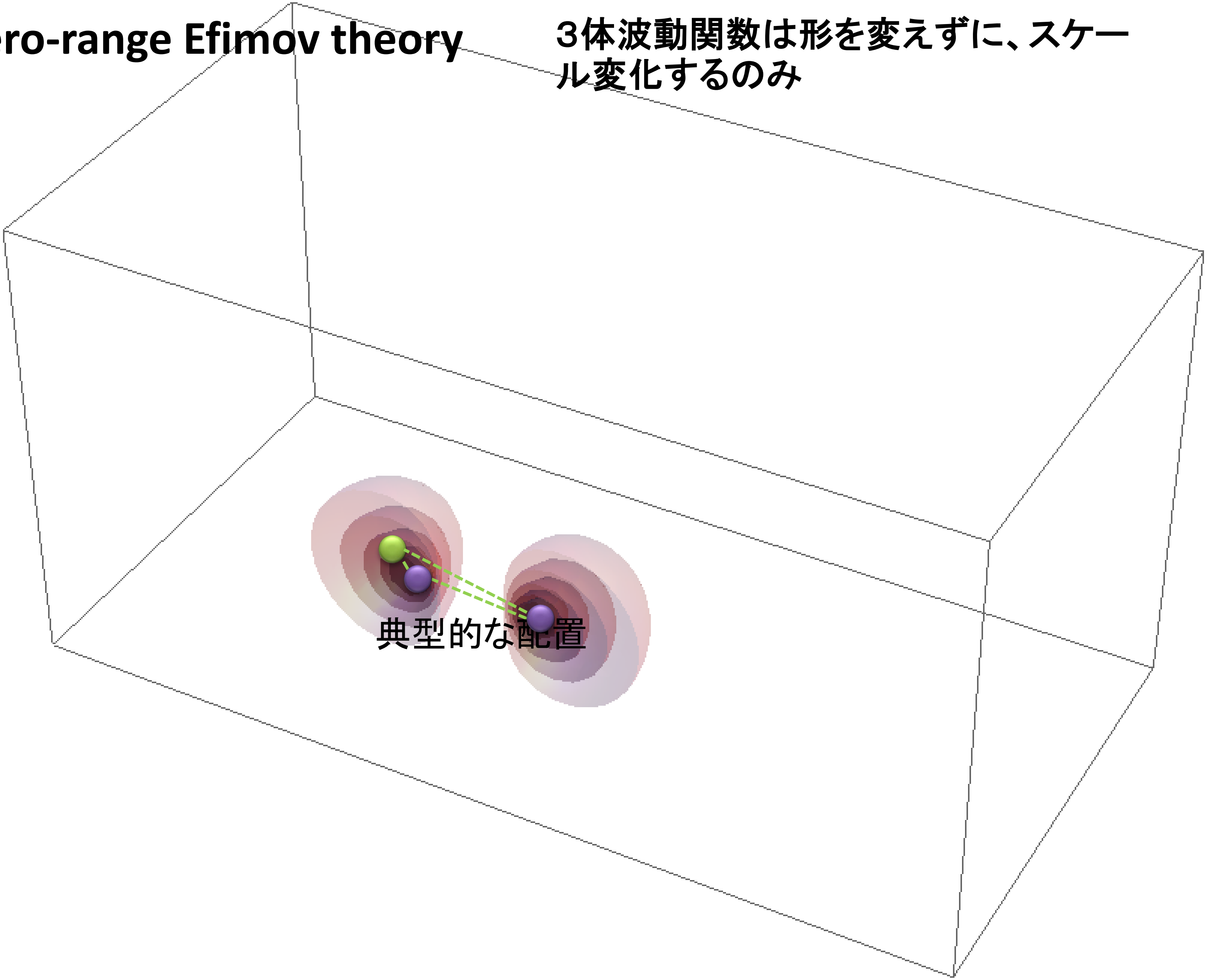
2粒子の位置を固定した際の、3つめの粒子の存在確率分布 $|\tilde{\Phi}(R,\Omega)|^2$



Naidon, Endo, Ueda, PRA & PRL (2014)

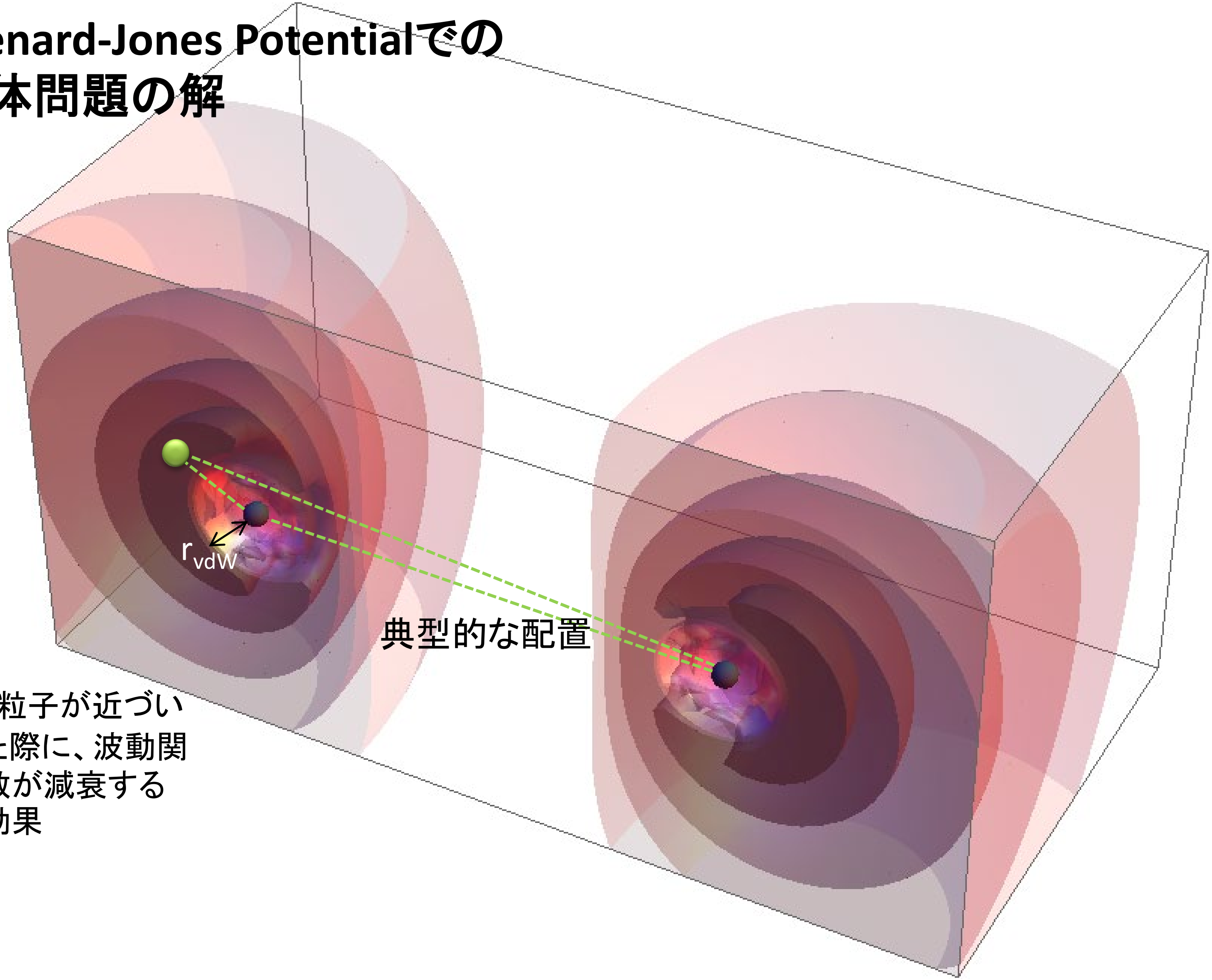
Zero-range Efimov theory

3体波動関数は形を変えずに、スケール変化するのみ



Naidon, Endo, Ueda, PRA & PRL (2014)

Lenard-Jones Potentialでの3体問題の解

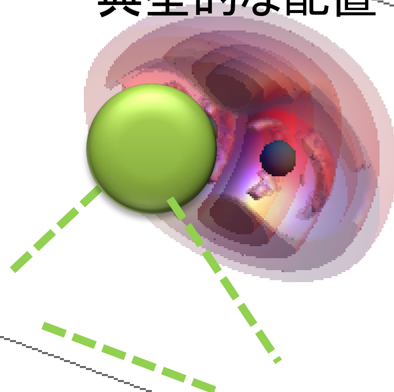


2粒子が近づいた際に、波動関数が減衰する効果

Naidon, Endo, Ueda, PRA & PRL (2014)

Lenard-Jones Potentialでの 3体問題の解

典型的な配置



3粒子がvan der Waals長程度まで近づくと、波動関数が急激に変形

急激な変形
⇒ 斥力ポテンシャルの起源

$$\left[-\frac{\hbar^2}{m}\frac{\partial^2}{\partial R^2}+\frac{\hbar^2}{mR^2}\left(\lambda_n(R)+4\right)+Q_{nn}(R)-\frac{\hbar^2}{4mR^2}\right]f_n(R)=Ef_n(R)$$

$$Q_{nn}(R)=\frac{\hbar^2}{m}\int d\Omega\left|\frac{\partial\Phi_n(R,\Omega)}{\partial R}\right|^2$$

Naidon, Endo, Ueda, PRA & PRL (2014)

Universal Efimov 3-body parameter in nuclei?

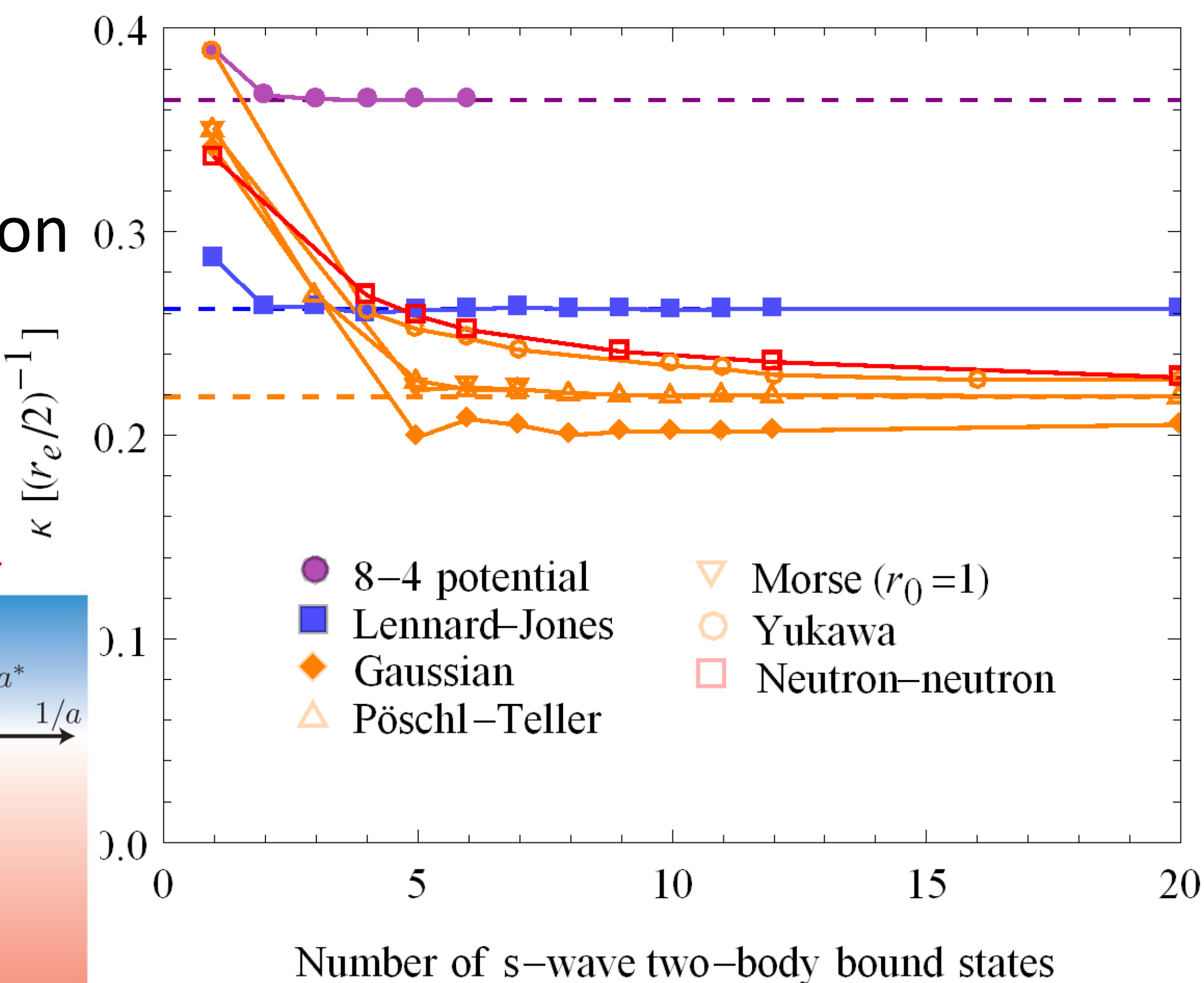
- 3-body parameter mostly characterized by effective range

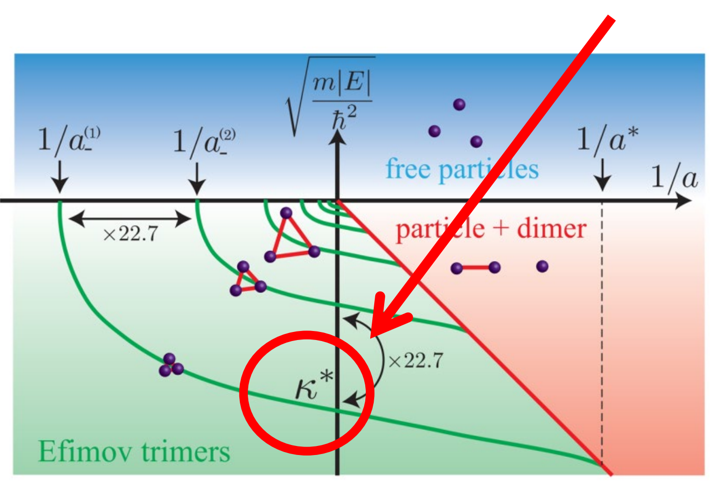
$$\kappa^* = (0.2 - 0.4)\left(\frac{r_{\rm eff}}{2}\right)^{-1}$$

Naidon, Endo, Ueda, PRL (2014)

- Small difference

➡ Difference in 2-body correlation

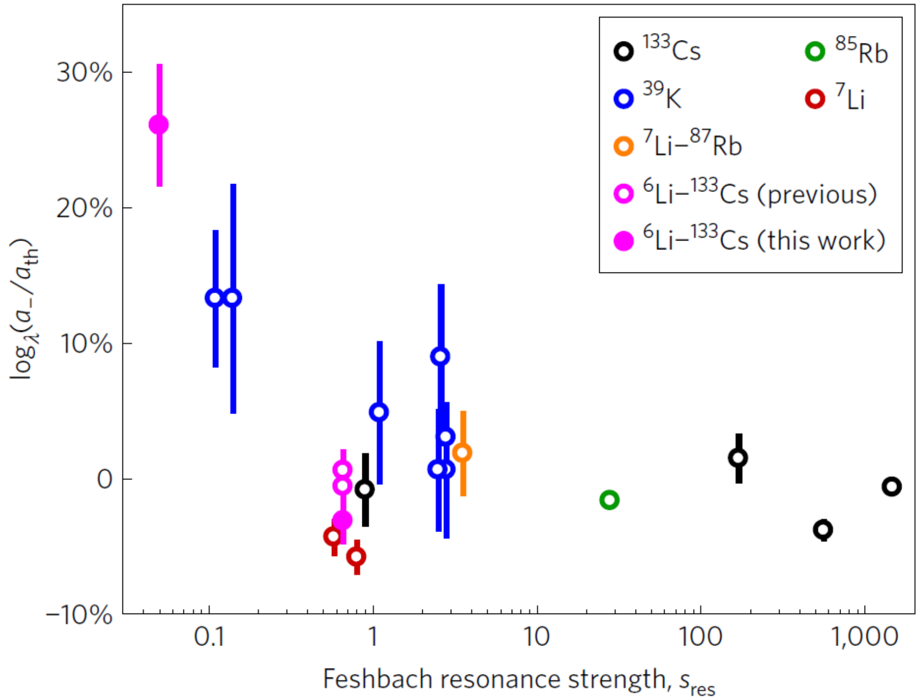




Efimov states with narrow Feshbach resonance

- Broad Feshbach resonance $r_{\text{eff}} \sim r_{\text{vdw}} > 0$
- Narrow Feshbach resonance $r_{\text{eff}} < 0$

Johansen, DeSalvo, Patel, Chin, Nature (2017)



$r_{\text{eff}} < 0$ $r_{\text{eff}} = 0$ $r_{\text{eff}} > 0$

Ultra-narrow resonance limit (exact result)

$$|a_-| \sim |r_{\text{eff}}| \to \infty$$

3 identical boson: Petrov PRL. (2004)
2-component mass-imbalanced boson/fermions: Endo, Castin, EPJD (2016)

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  ⇒Universality

- From few-body to many-body: virial (cluster) expasnion
  - 3rd order expansion with Efimov states
  - 4th order expansion for unitary Fermi gas (no Efimov effect)

Cluster (Virial) Expansion

$$\frac{\Omega}{V} = -\frac{k_B T}{\lambda^3}\left[b_1 e^{\beta\mu} + b_2 e^{2\beta\mu} + b_3 e^{3\beta\mu} + \ldots\right]$$

λ : thermal de Brogie length
μ : chemical potential

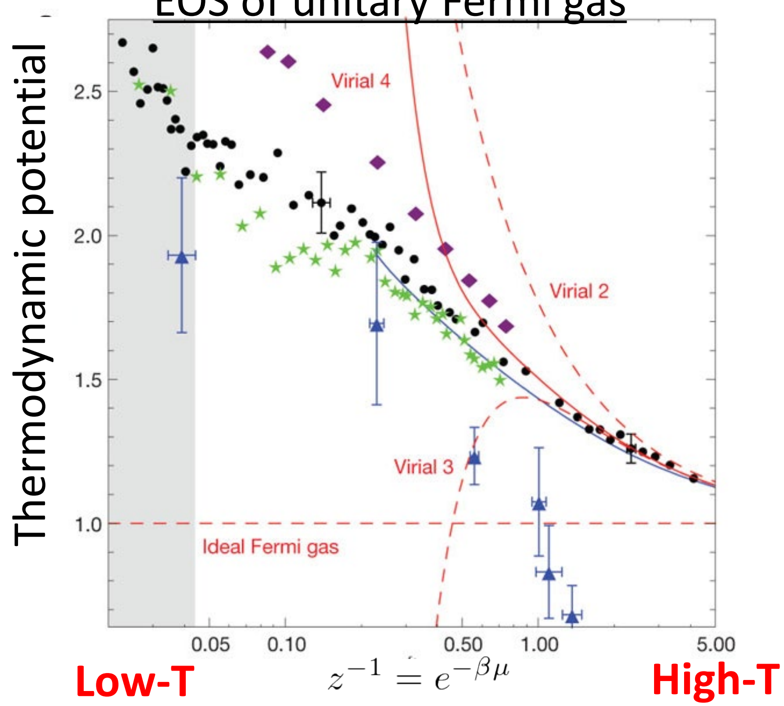
- Works well at high T

$$e^{\beta\mu} \approx n\lambda^3 \ll 1$$

- b_n can be calculated from *n*-body solution

➡ Few-body approach to many-body physics

EOS of unitary Fermi gas



S. Nascimbène, et al. Nature (2010)

Cluster (Virial) expansion and few-body problem

Cluster (Virial) Expansion

Liu, Phys. Rep. (2013)

$$\frac{\Omega}{V} = -\frac{k_B T}{\lambda^3}\left[b_1 e^{\beta\mu} + b_2 e^{2\beta\mu} + b_3 e^{3\beta\mu} + \ldots.\right]$$

Canonical & Grand Canonical Ensenble

$$\Omega = -k_B T \log\left(1 + Z_1 e^{\beta\mu} + Z_2 e^{2\beta\mu} + \ldots\right)$$

$$Z_N = \mathrm{Tr}[e^{-\beta H_N}] = \sum_i e^{-\beta E_N^{(i)}}$$:canonical partition function of N-body Hamiltonian H_N

1-body problem	Z_1	⇒	b_1
2-body problem	Z_1, Z_2	⇒	b_2
		⋮	
N-body problem	$Z_1, Z_2, ..., Z_N$	⇒	b_N

Few-body problem gives many-body EOS at high T

Harmonic regulator technique to deal with all the continuum *Liu,Hu, Drummond, PRL (2009)*

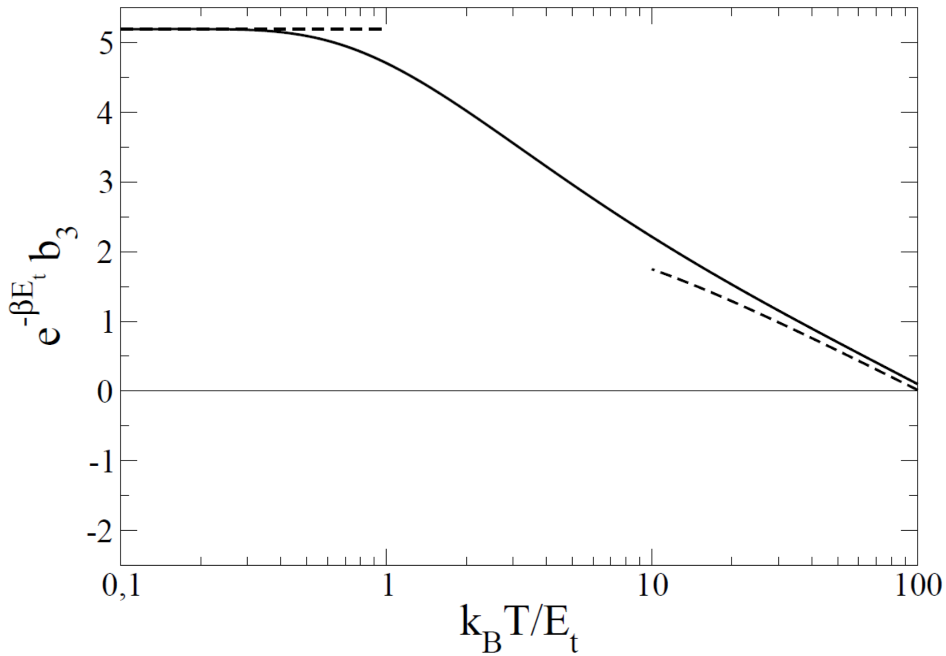
Contour integral conjecture to take the canonical sum
Exact solutions of fermionic 3-body &4-body problems *SE, Castin, PRA (2015) & J. Phys. A (2016)*

3rd order virial expansion with Efimov effect at a=±∞

- Efimov effect. 3-body parameter R_* is necessary

$$b_3\left(\frac{k_BT}{E_t}\right) \qquad E_t \sim \frac{2}{mR_*^2}\exp\left[\frac{2}{|s_0|}\mathrm{Im}\ \ln\Gamma(1+s_0)\right]$$

b3 of 1-component Bose gas @ a=∞
(broad resonance)



Castin, Werner, Can. J. Phys. (2013)

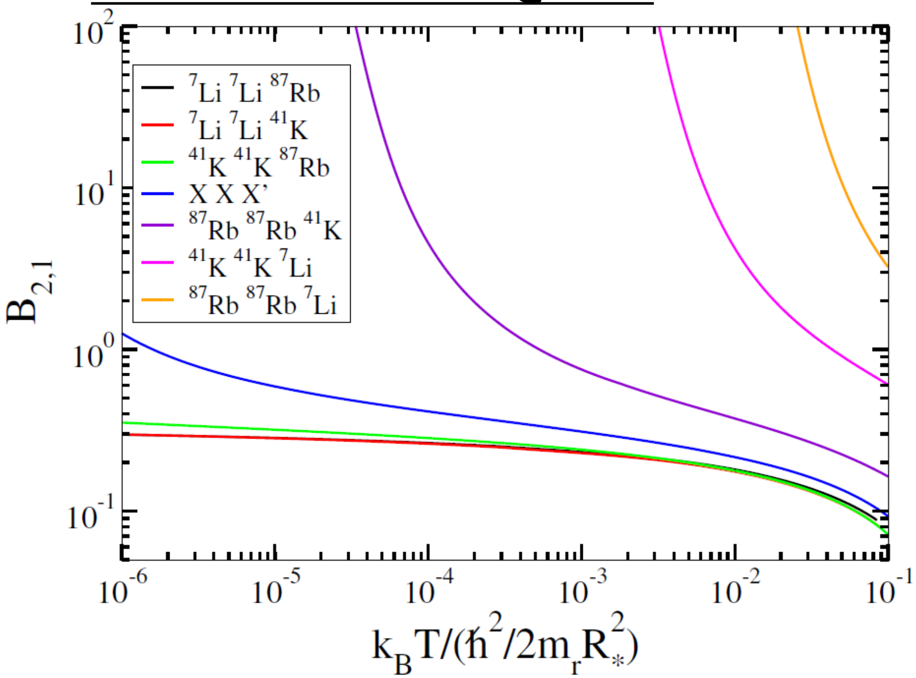
b_3 with Efimov effect for mass-imbalanced mixtures

- Efimov effect. 3-body parameter R* is necessary

$$b_3\left(\frac{k_BT}{E_t},\frac{m_1}{m_2}\right) \qquad\qquad b_3 = b_{2,1}$$

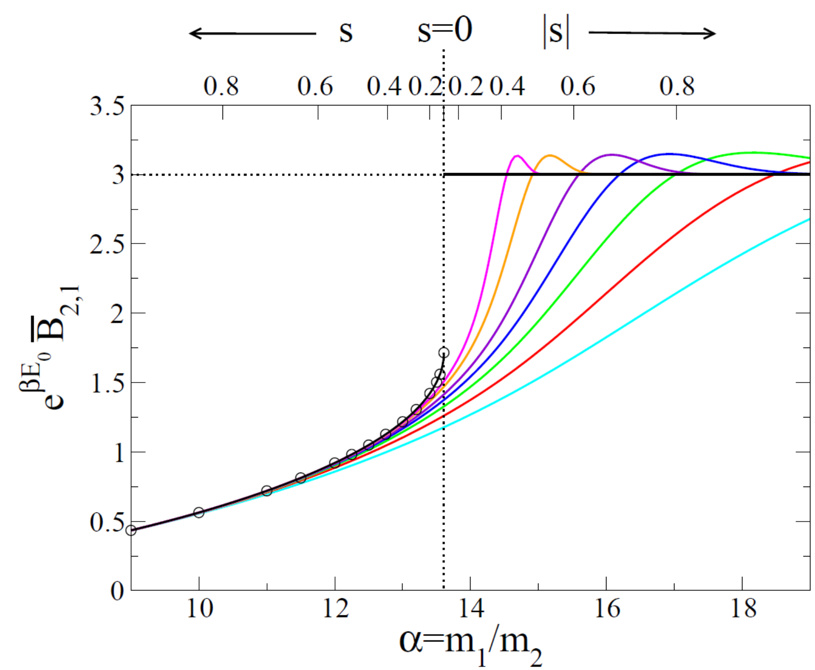
- Efimov effect appears for m_1/m_2 >13.6 fo identical fermions

B_3=$B_{2,1}$ of mass-imbalanced system of Boson+Boson+Other @ a=∞



Endo, Castin, EPJD (2016)

B_3=$B_{2,1}$ of mass-imbalanced system of Fermion+Fermion+Other @ a=∞



Gao, Endo, Castin, EPL (2015)

b_4 in unitary Fermi gas: Equal mass

- Upto 3rd order: excellent agreement

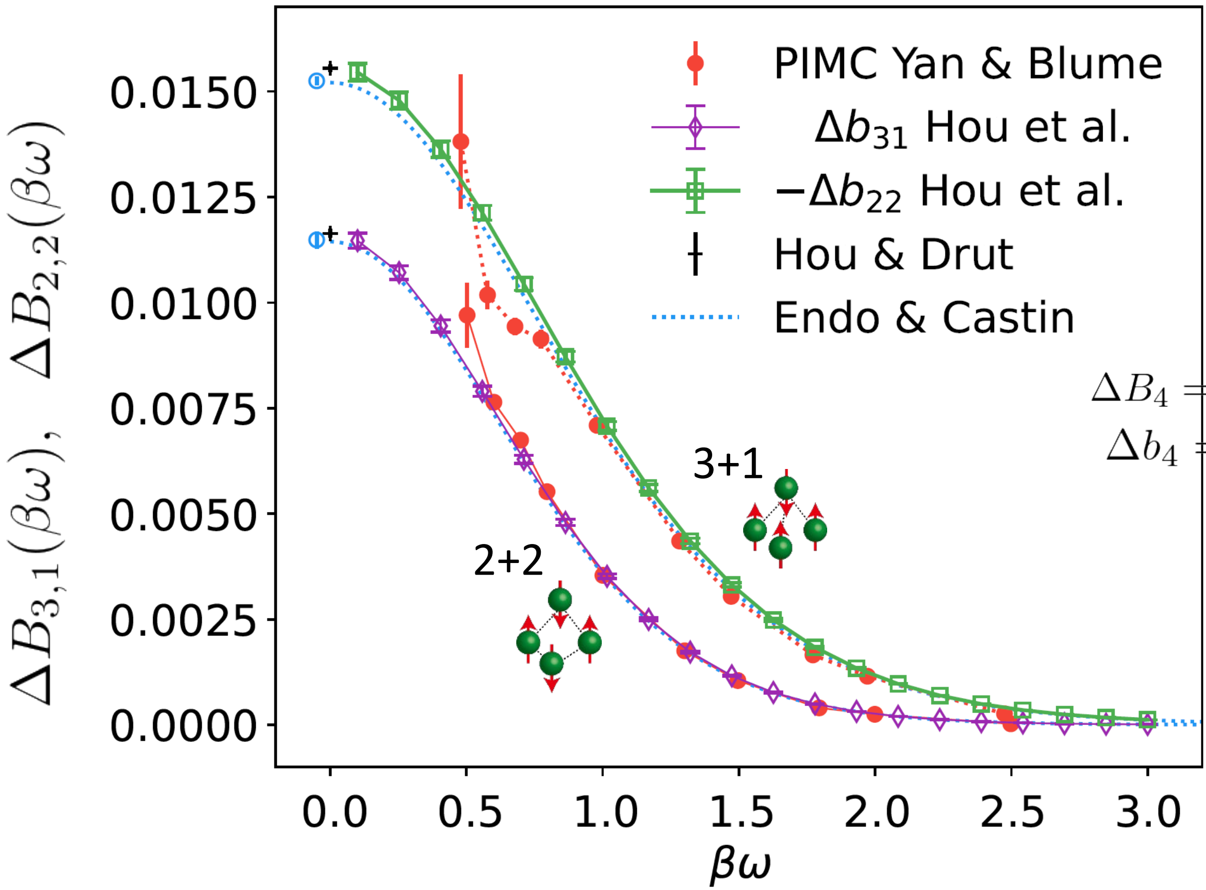
$b_3^{\rm exp} = -0.29(2)$ *MIT, ENS experiments* $b_3^{\rm theory} = -0.2909..$ *Liu,Hu, Drummond,PRL (2009)*

- 4th order: has been challenging

$b_4^{\rm MIT} = 0.065(10)$ $b_4^{\rm ENS} = 0.065(15)$ *MIT, ENS experiments*

Theoretical Method	Theory result b_4	Referenece
Sum 4-body energies in a harmonic trap numerically & ω→0 extrapolation	-0.047(4)	*Rakshit, Daily Blume, PRA (2012)*
Contour integral conjecture + 3- & 4-body exact solutions	0.031(1)	***SE**, Y. Castin, J Phys. A (2016)*
Feynman diagram	≈0.03	*Ngampruetikorn, Parish,Levinsen PRA (2015)*
Path Integral Monte Carlo	0.047(18)	*Yan, Blume, PRL (2016)*
Numerical Suzuki-Trotter expansion	0.031(2)	*Hou, Drut, PRL (2020)*

4th virial expansion of a harmonically trapped unitary Fermi system

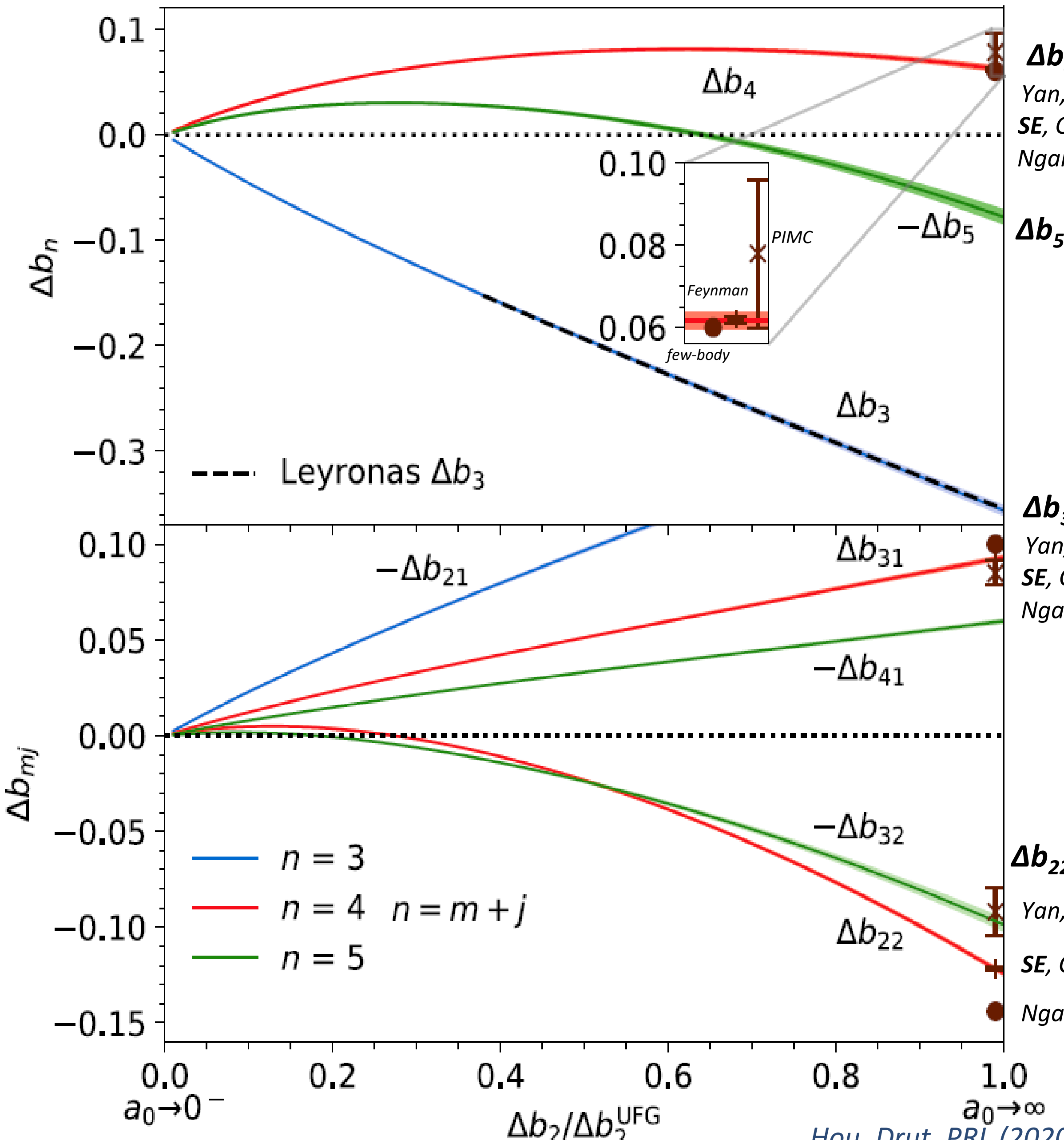


$$\Delta B_4 = \frac{1}{2}\left[\Delta B_{3,1} + \Delta B_{1,3} + \Delta B_{2,2}\right]$$
$$\Delta b_4 = 8\Delta B_4(\omega = 0)$$

Czejdo, Drut, Hou, Morrell, Condens. Matter 2022,

- Virial coefficients in a harmonic trap calculated with our conjecture(blue dotted) agree excellently with other theories:
  - Path-Integral Monte Carlo *[Yan Blume, PRL. (2016)]*
  - Suzuki-Trotter expansion *[Hou, Drut, PRL. (2020)]*

Recent theory work with semiclassical expansion



$\Delta b = b - b^{\mathrm{non-int}}$

Δb_4 = 0.062(2)

Yan, Blume (PIMC): **0.078(18)**

SE*, Castin (3 & 4-body exact sol. + contour):* **0.0620(8)**

Ngampruetikorn, Parish,Levinsen (Feynman diagram): **0.06**

Δb_5 = 0.078(6) >0

Δb_{31} = 0.0931(8)

Yan, Blume (PIMC): **0.0848(64)**

SE*, Castin (3 & 4-body exact sol. + contour):* **0.09188(16)**

Ngampruetikorn, Parish,Levinsen (Feynman diagram): **0.100**

Δb_{22} = -0.1244(7)

Yan, Blume (PIMC): **-0.0920(128)**

SE*, Castin (3 & 4-body exact sol. + contour):* **-0.1220(8)**

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Hou, Drut, PRL (2020)

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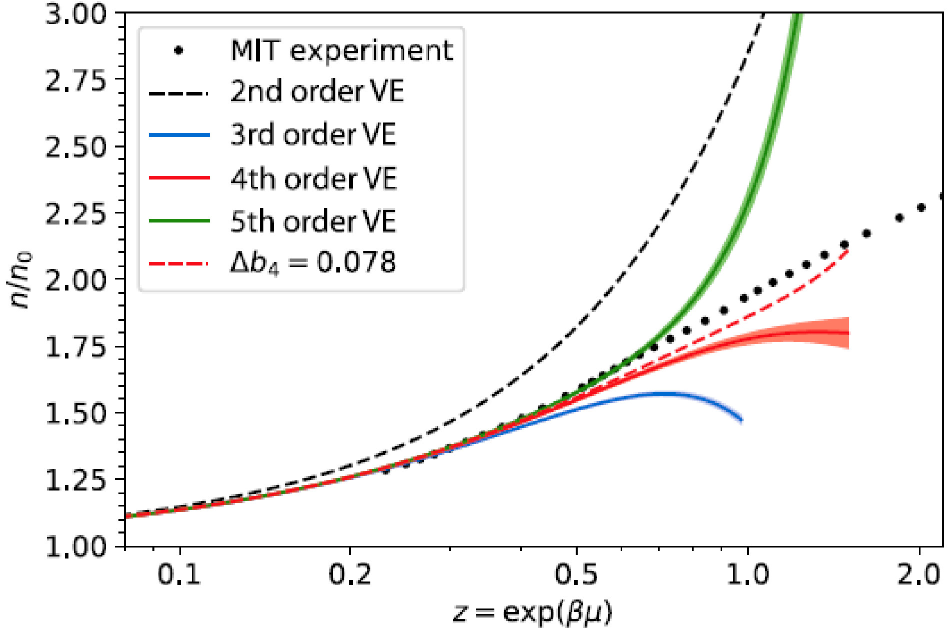

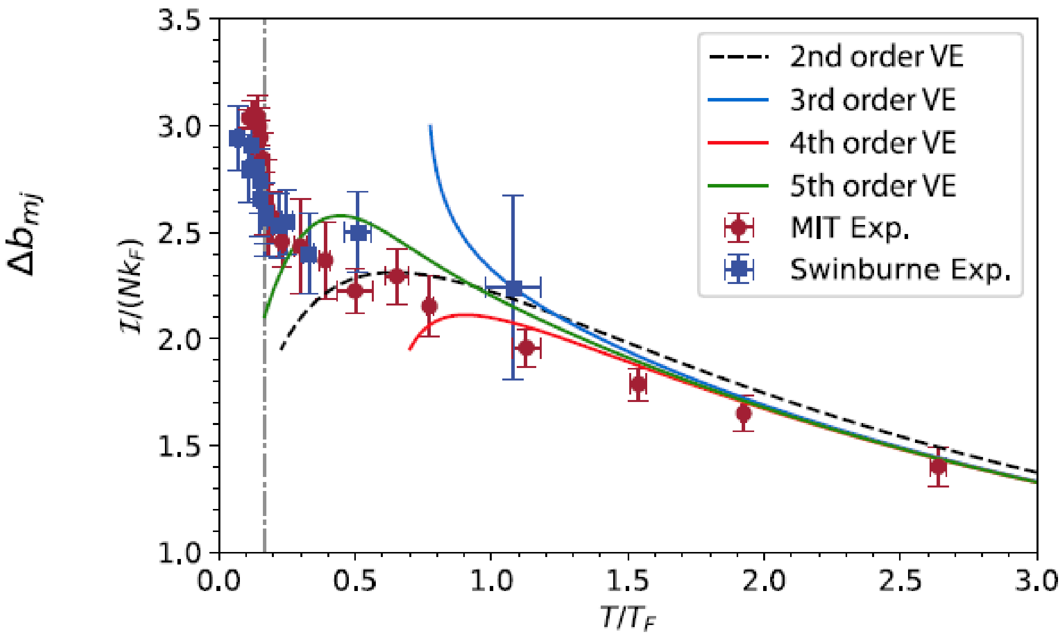

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1.0

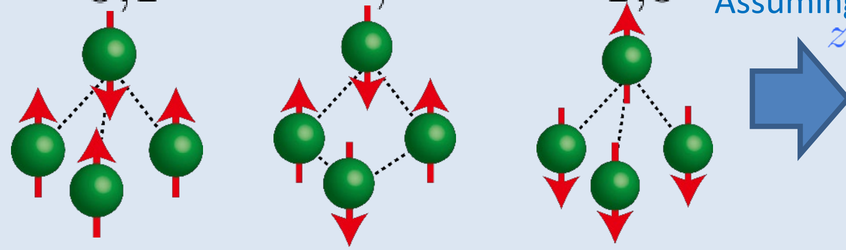
$a_0 \to \infty$

RL (2020)

4th order $b_{3,1}$, $b_{2,2}$, b_4 in mass-imbalanced unitary Fermi gas

$$\frac{P\lambda^3}{k_B T} = \sum_{(n_\uparrow,n_\downarrow)\in\mathbb{N}^{2*}} b_{n_\uparrow,n_\downarrow} z_\uparrow^{n_\uparrow} z_\downarrow^{n_\downarrow}$$

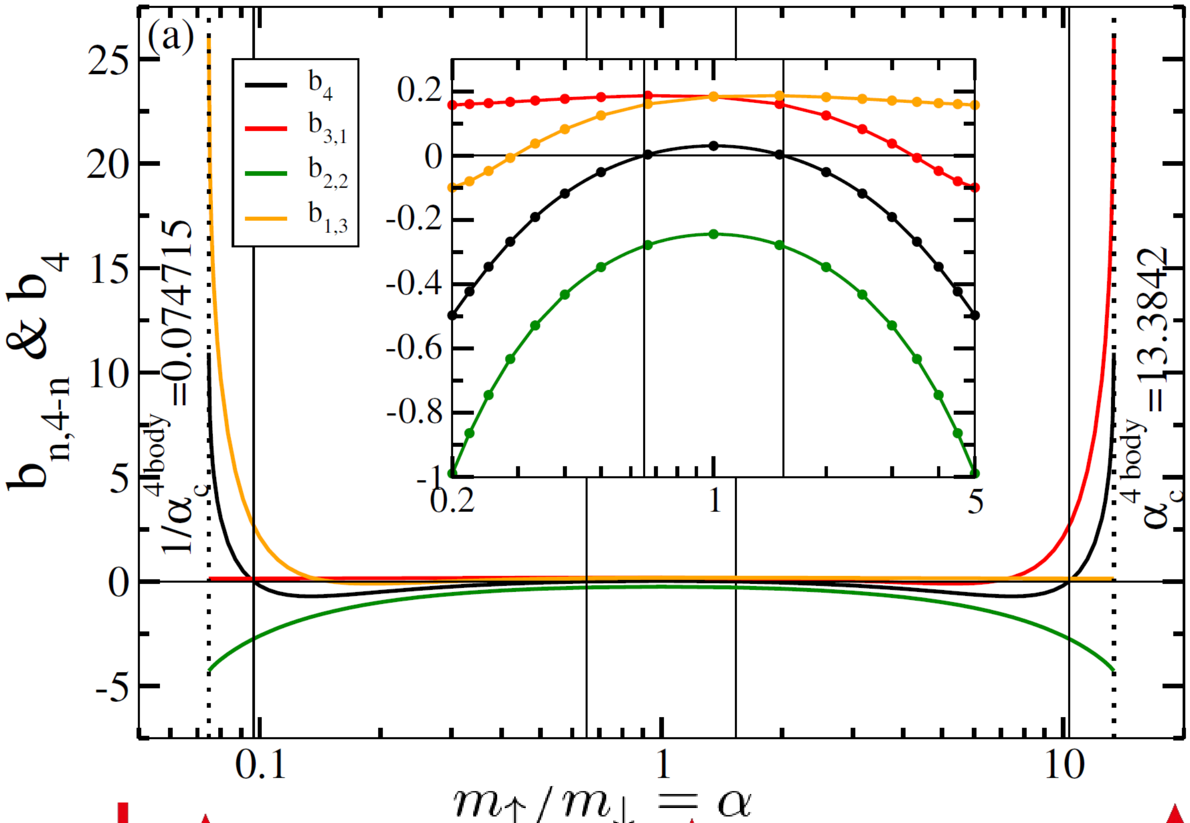
$b_{3,1}$ $b_{2,2}$ $b_{1,3}$

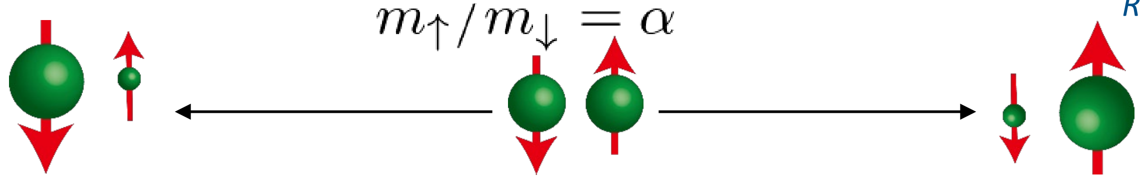


Assuming equal fugacity $z_\uparrow = z_\downarrow$

b_4

$$\Delta b_4 = \Delta b_{3,1} + \frac{1}{2}\Delta b_{2,2}$$



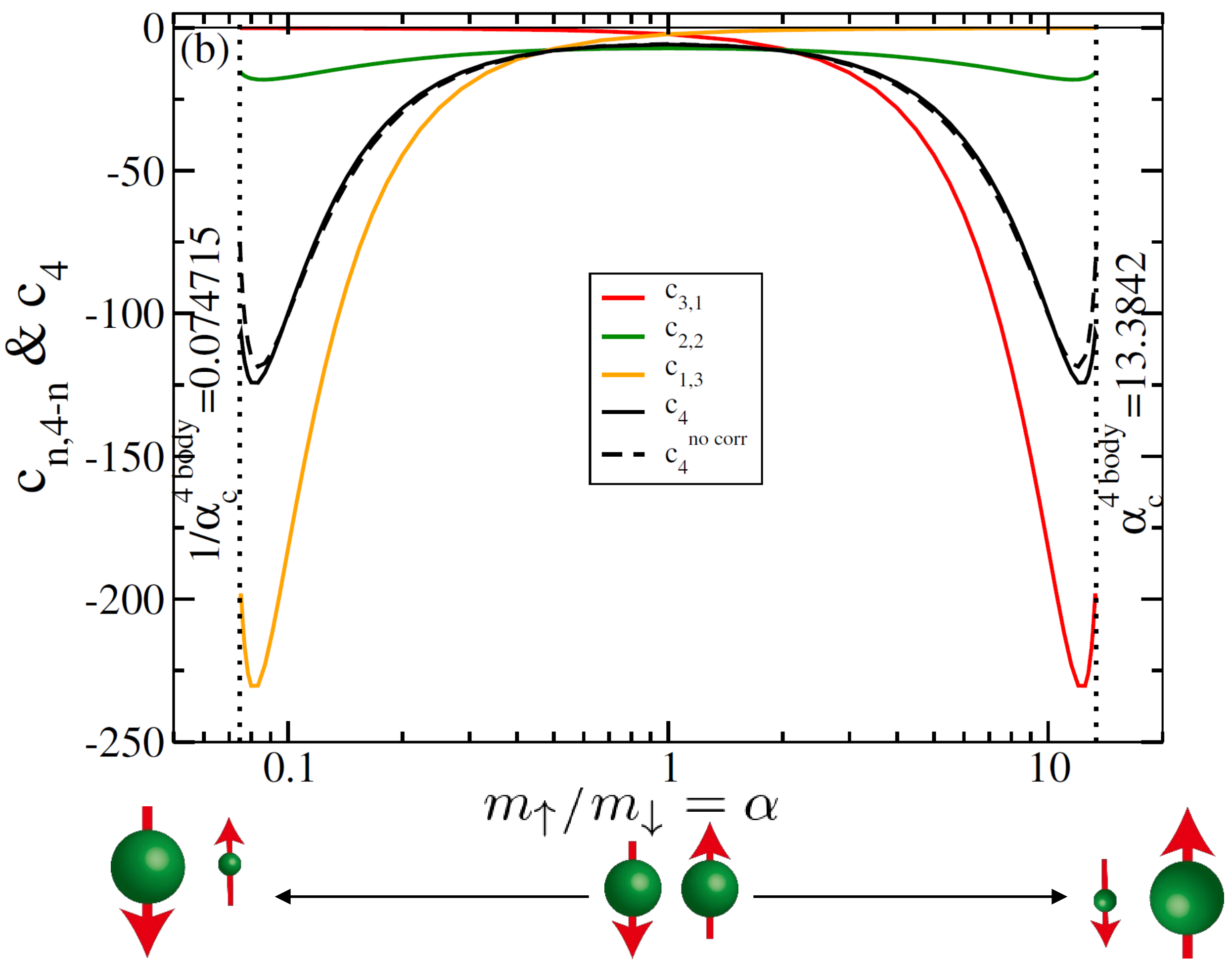


S. Endo, *Y. Castin,*
arXiv:2203.00916 (2022)
To be publisehd in Comp. Rend. Phys.

Cluster expansion and Virial expansion

$$\frac{P\lambda^3}{k_B T} = \sum_{(n_\uparrow,n_\downarrow)\in\mathbb{N}^{2*}} b_{n_\uparrow,n_\downarrow} z_\uparrow^{n_\uparrow} z_\downarrow^{n_\downarrow} = \sum_{(n_\uparrow,n_\downarrow)\in\mathbb{N}^{2*}} c_{n_\uparrow,n_\downarrow} (\rho_\uparrow \lambda_\uparrow^3)^{n_\uparrow} (\rho_\downarrow \lambda_\downarrow^3)^{n_\downarrow}$$

fugacity — *phase-space density*



Exact mapping btw. dissipative UFG & dissipationless UFG

Quantum dissipative dynamics (Caldirola-Kanai model)

Classical

*Tokieda, **Endo** , Front. Phys. 9, 730761 (2021)*

$$\ddot{\mathbf{x}}_i(t) + \gamma \dot{\mathbf{x}}_i(t) + \frac{1}{m_i}\frac{\partial U}{\partial \mathbf{x}_i}(\mathbf{x}_1(t), \ldots, \mathbf{x}_N(t), t) = 0$$

Quantum

$$i\hbar \frac{\partial}{\partial t}\phi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t) = \left[-e^{-\gamma t} \sum_{i=1}^{N} \frac{\hbar^2 \nabla_{\mathbf{x}_i}^2}{2m_i} + e^{\gamma t} U(\mathbf{x}_1, \ldots, \mathbf{x}_N) \right] \phi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t).$$

Quantum dissipationless dynamics

Classical

$$\ddot{\mathbf{x}}_i(t) + \frac{1}{m} e^{\gamma t/2} \nabla_i U(\mathbf{x}_1 e^{-\gamma t/2}, \ldots, \mathbf{x}_N e^{-\gamma t/2}, t) - \frac{\gamma^2}{4}\mathbf{x}_i(t) = 0$$

Quantum

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t) = \left[-\sum_{i=1}^{N} \frac{\hbar^2 \nabla_{\mathbf{x}_i}^2}{2m_i} + e^{\gamma t} U(\mathbf{x}_1 e^{-\gamma t/2}, \ldots, \mathbf{x}_N e^{-\gamma t/2}, t) - \sum_{i=1}^{N} \frac{m_i \gamma^2}{8} \mathbf{x}_i^2 \right] \psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t)$$

Relation between the wavefunctions

$$\phi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t) = \exp\left(-ie^{\gamma t} \sum_{i=1}^{N} \frac{m_i \gamma}{4\hbar} \mathbf{x}_i^2 + \frac{dN\gamma t}{4} \right) \psi(\mathbf{x}_1 e^{\gamma t/2}, \ldots, \mathbf{x}_N e^{\gamma t/2}, t)$$

Unitary Fermi gas: Scale-invariant interaction ⇒ Equivalent

i.e. Dissipative UFG = Dissipationless UFG with negative harmonic pot.

Conclusion

- Cold atoms: highly controllable. Quantum simulation
- Universality of few-body systems @ $E \to 0$, $a \to \pm\infty$

- Efimov states
  - Universal 3-body parameter

    a_- (i.e. R_*) universally determined by effective range
- Few-body approach to many-body: Virial expansion
  - Accurate calculation using 3-body & 4-body technique
  - Virial coefficients in the presence of Efimov effect depends on 3-body parameter R_*
- Dissipative UFG = Dissipationless UFG + negative harmonic potential