冷却原子気体における少数量子系の普遍的性質
@ 京大基研、2022/08/23

冷却原子系における近藤効果

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MN and N. Kawakami, Phys. Rev. Lett. 115, 165303 (2015)
MN, N. Kawakami, and M. Ueda, Phys. Rev. Lett. 121, 203001 (2018)
M. Hasegawa, MN, and K. Saito, arXiv:2111.07771

Outline

1. Introduction
    - Universality of the Kondo effect
2. Kondo effect in ultracold atoms
3. New universality of the Kondo effect
    - Non-Hermitian Kondo effect
4. Summary

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Kondo effect

- Magnetic impurity coupled with conduction electrons in metal

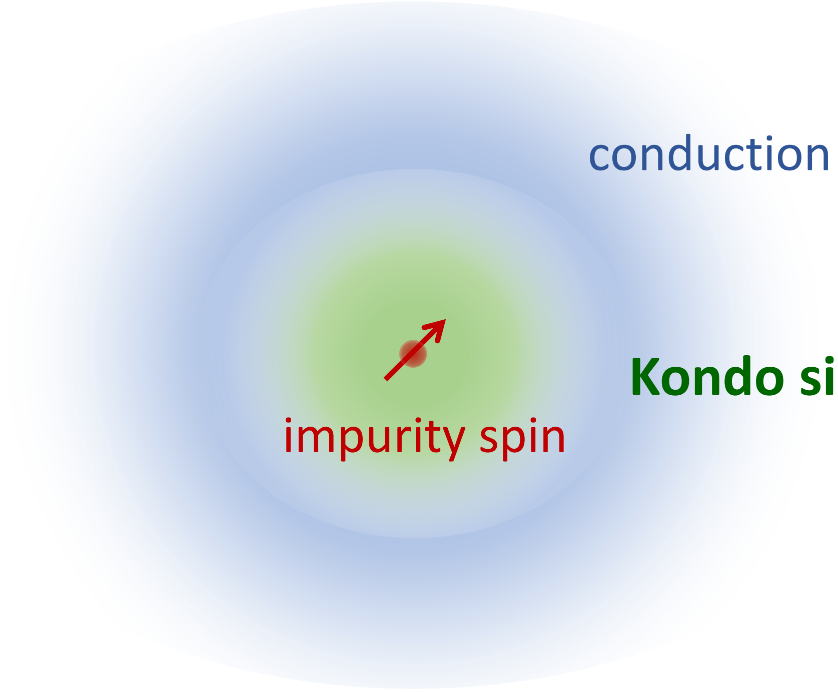
$$H = \sum_{\boldsymbol{k},\sigma} \varepsilon(\boldsymbol{k}) c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - J \boldsymbol{S}_{c0} \cdot \boldsymbol{S}_{\mathrm{imp}}$$

Kondo model

antiferromagnetic spin exchange

($J < 0$)

[J. Kondo, Prog. Theor. Phys. 32, 37 (1964)]



conduction electrons

Kondo singlet

impurity spin

Kondo temperature

$$T_K = D \exp\left[\frac{1}{\rho_0 J}\right]$$

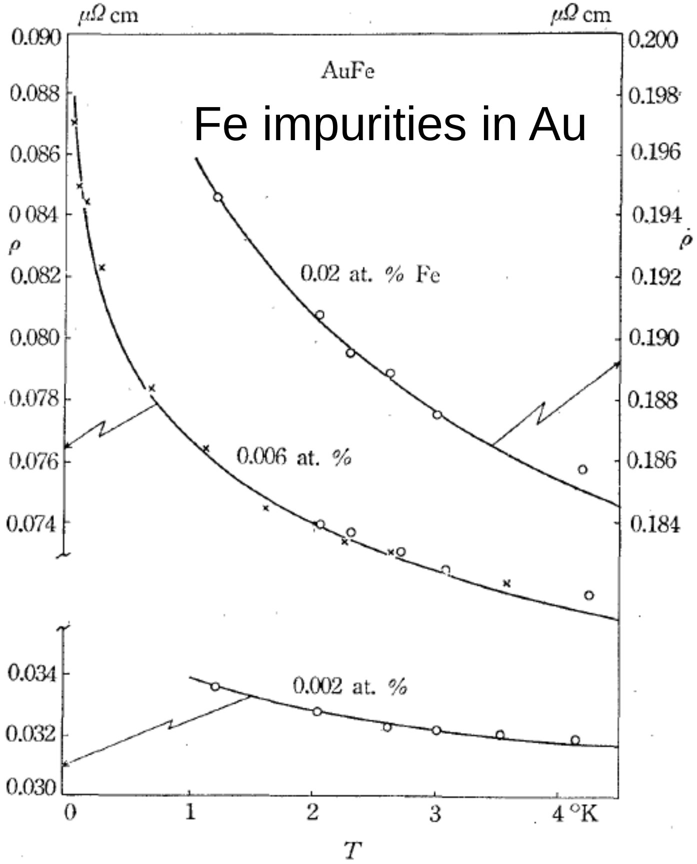
(D: bandwidth/2, ρ_0: DOS at Fermi energy)

Formation of many-body spin singlet

Kondo effect

- Formation of Kondo singlet → logarithmic **increase** of resistivity

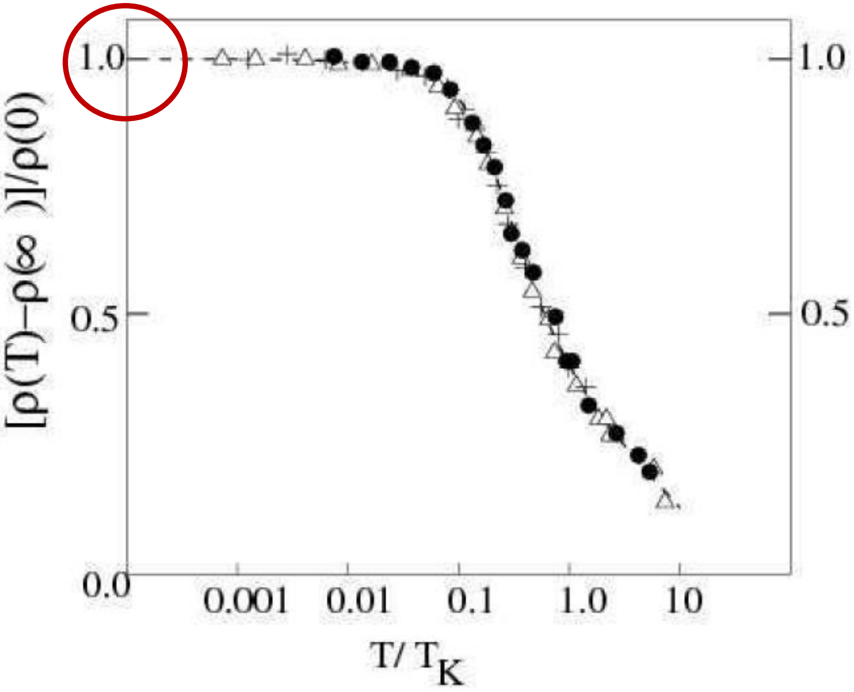
resistivity



Fe impurities in Au

temperature

“unitarity limit”



universal scaling
by Kondo temperature

[Figures from Kondo, PTP 32, 37 (1964) and Coleman, arXiv:0612006]

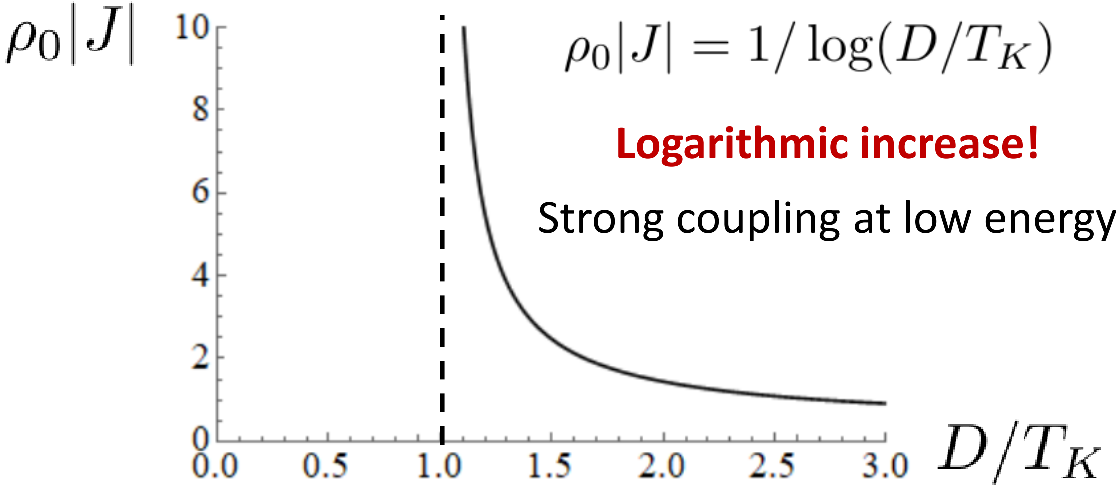
Kondo effect: renormalization group

Origin of universality: renormalization-group (RG) flow

$$\frac{dJ}{d\ln D} = \rho_0 J^2$$

RG equation

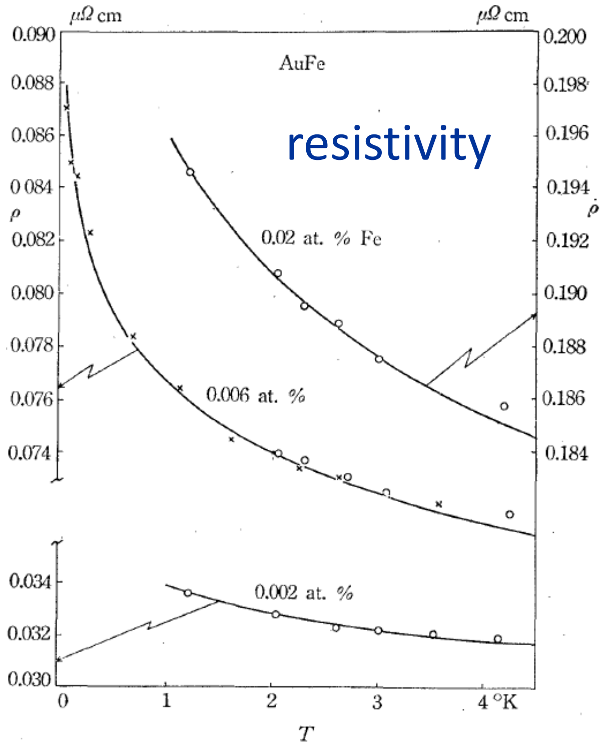
[Anderson, J. Phys. C 3, 2436 (1970)]



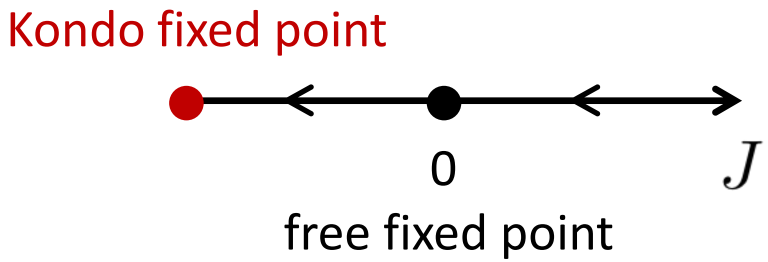
$$\rho_0|J| = 1/\log(D/T_K)$$

Logarithmic increase!

Strong coupling at low energy



resistivity



Kondo fixed point

0

J

free fixed point

Low-energy properties:

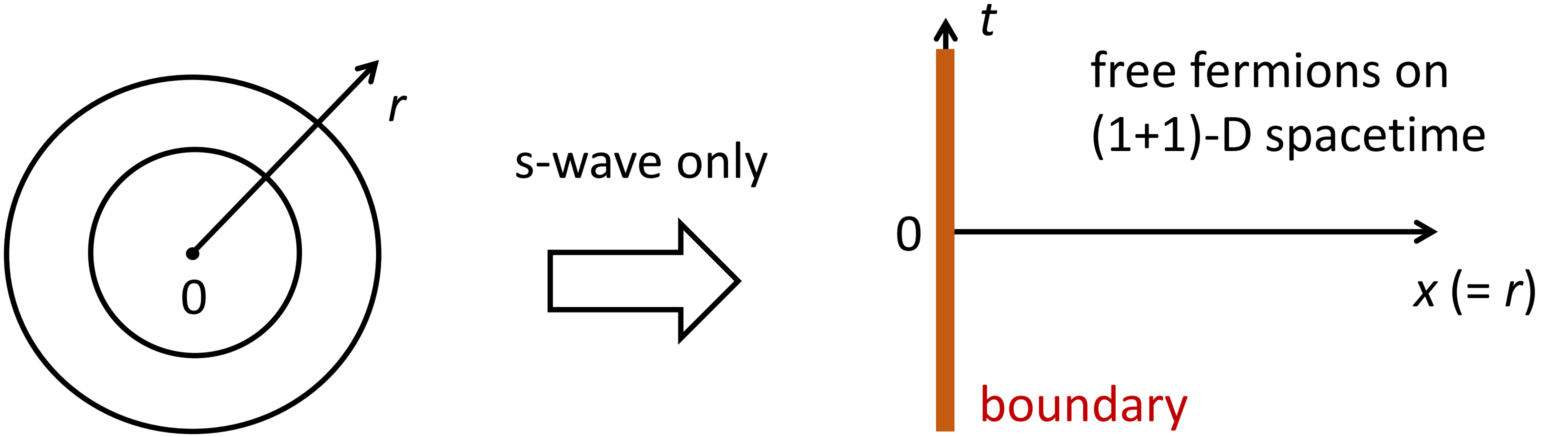
governed by Kondo fixed point

Kondo effect as a boundary critical phenomenon

- **Kondo effect = boundary quantum critical phenomenon**

[Affleck, Nucl. Phys. B 336, 517 (1990); Affleck & Ludwig, Nucl. Phys. B 352, 849 (1991)]

Impurity = "boundary" in radial direction



quantum critical (= gapless) system with boundary

→ boundary conformal field theory (boundary CFT)

→ conformally invariant boundary condition = universality class

Kondo effect = "Twisting" the boundary condition of quantum critical systems

Kondo effect and statistical physics

- Kondo effect and development of statistical physics

Kondo effect	Statistical physics
Kondo (1964)	
Anderson (1970): scaling theory (RG)	Wilson (1971): concept of RG
Andrei (1980), Wiegmann (1980), Kawakami-Okiji (1981): integrability	Bethe ansatz (1931) Yang-Baxter eq. (1968, 1971)
Affleck-Ludwig (1991): boundary CFT	BPZ (1984): CFT for critical phenomena
Affleck-Ludwig (1991): g-theorem	Zamolodchikov (1986): c-theorem

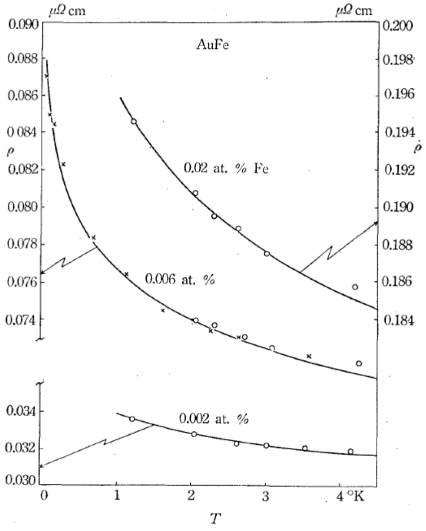
- Many extensions

SU(N) Kondo effect, multichannel Kondo effect, two-impurity Kondo, Kondo lattice, anisotropic Kondo, pseudogap Kondo, …

Milestone in solid-state & statistical physics

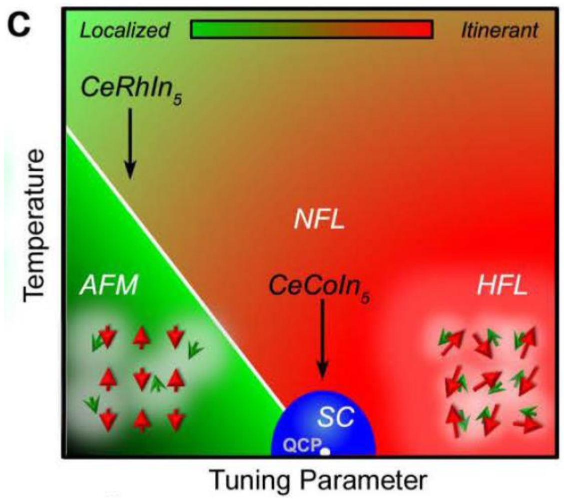
Experimental platforms for the Kondo effect

■ Magnetic impurity in metal



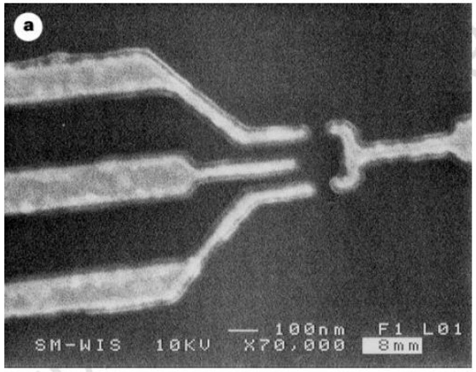
Kondo, PTP (1964)

■ Heavy-fermion materials



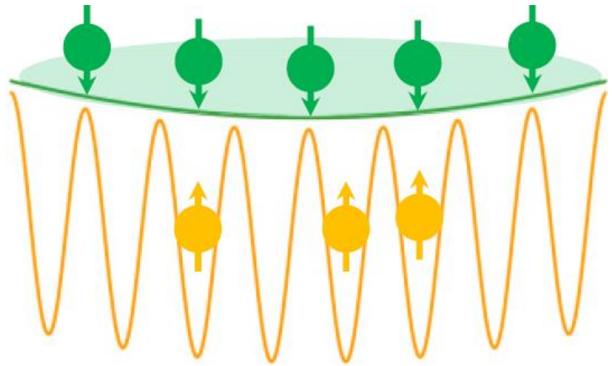
Aynajian *et al*, Nature (2012)

■ Quantum dot



Goldhaber-Gordon *et al*, Nature (1998)

■ Ultracold atoms: **new!**



Ono *et al.*, PRA (2021)

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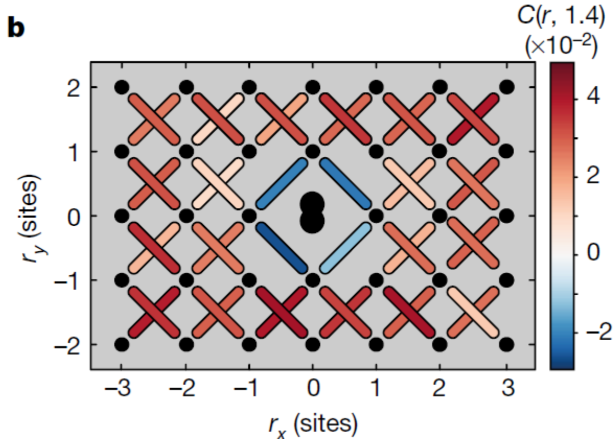
Kondo effect in ultracold atoms

Kondo effect in ultracold atoms?

- ◇ Test the universality of the Kondo effect
- ◇ Local correlation around an impurity
- ◇ Strongly correlated materials (Kondo lattice)
- ◇ Non-equilibrium dynamics
- ◇ New Kondo problem unique to ultracold atoms

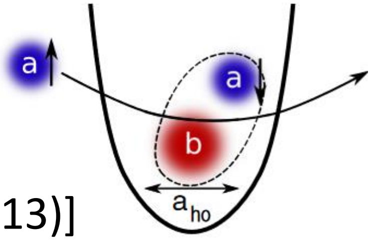


cf. quantum gas microscopy of magnetic polarons [Koespell *et al.*, Nature (2019)]

Proposals for alkali atoms

- ◇ Optical superlattice [Paredes *et al.*, PRA 71, 063608 (2005)]
- ◇ Mixture of two atomic species + Feshbach resonance

  [Bauer *et al.*, PRL 111, 215304 (2013); Nishida, PRL 111, 135301 (2013)]



Alkaline-earth(-like) atoms

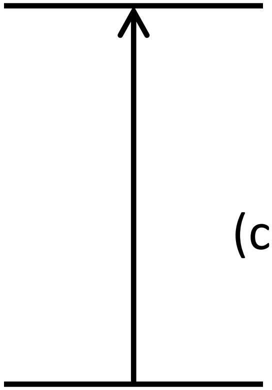
[Gorshkov *et al.*, Nat. Phys. 6, 289 (2010)]

[Foss-Feig *et al.*, PRA 81, 051603 (2010); PRA 82, 053624 (2010)]

Kondo effect in alkaline-earth-like atoms

- Most promising candidate: **alkaline-earth-like (^{171}Yb, ^{173}Yb, ^{87}Sr) atoms**

[Gorshkov *et al.*, Nat. Phys. 6, 289 (2010)]



$^3P_0$: **metastable** excited state (lifetime $\gtrsim$ 10s)

"clock transition"
(cf. optical lattice clock)

$^1S_0$: ground state

Kondo effect in alkaline-earth-like atoms

- Most promising candidate: **alkaline-earth-like (^{171}Yb, ^{173}Yb, ^{87}Sr) atoms**

  [Gorshkov *et al.*, Nat. Phys. 6, 289 (2010)]

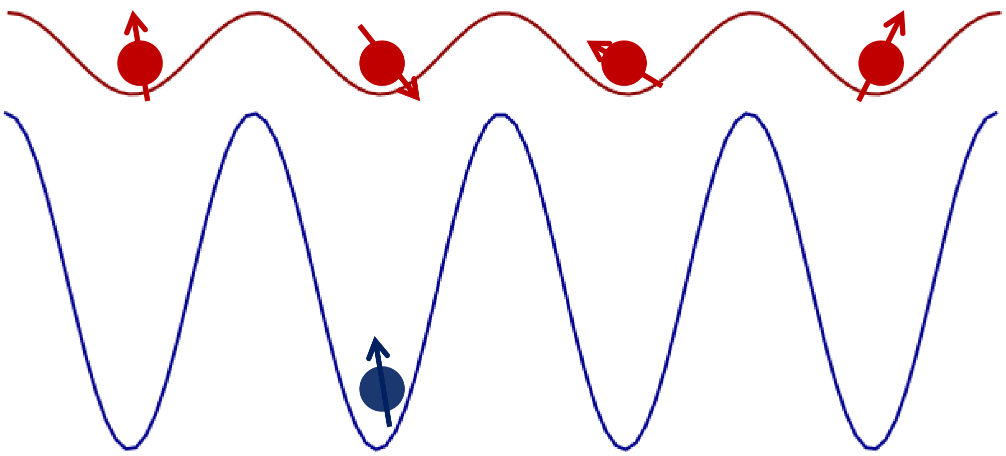
- Atomic ground state ($^1S_0$) → conduction electrons

  Metastable excited state ($^3P_0$) → localized impurity

- **Spin degrees of freedom: nuclear spin *I***

  ^{171}Yb: SU(2) (I = 1/2), ^{173}Yb: SU(6) (I =5/2), ^{87}Sr: SU(10) (I = 9/2)

- Difference of AC polarizability → **state-dependent optical lattice**



$^1S_0$ state: shallow lattice
→ **“conduction electrons”**

$^3P_0$ state: deep lattice
→ **“localized impurity”**

Experimental measurement of exchange interaction

- Experimental measurement of spin-exchange interaction btwn ${}^1S_0$ & ${}^3P_0$

☐ ^{173}Yb

[Scazza *et al.*, Nat. Phys. 10, 779 (2014)]

[Cappellini *et al.*, PRL 113, 120402 (2014)]

$$J \propto a_{eg}^{+} - a_{eg}^{-} = 1658 a_0 > 0$$

☐ ^{87}Sr

[Zhang *et al.*, Science 345, 1467 (2014)]

$$J \propto a_{eg}^{+} - a_{eg}^{-} = 101 a_0 > 0$$

Ferromagnetic spin-exchange interaction!

☐ ^{171}Yb

$$J \propto a_{eg}^{+} - a_{eg}^{-} = \begin{cases} -130 a_0 < 0 & \text{[Ono et al., PRA 99, 032707 (2019)]} \\ -149 a_0 < 0 & \text{[Bettermann et al., arXiv:2003.10599]} \\ -105 a_0 < 0 & \text{[Abeln et al., PRA 103, 033315 (2021)]} \end{cases}$$

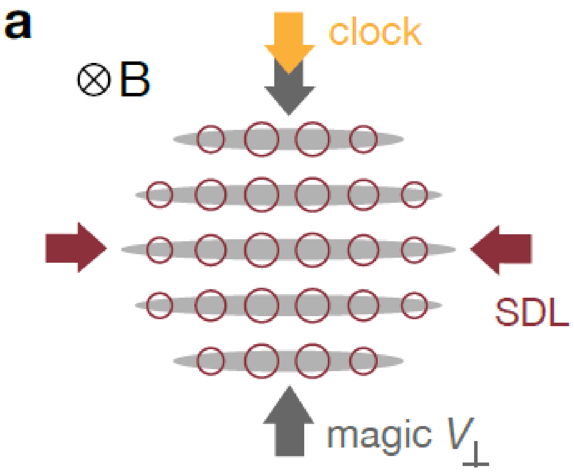
Antiferromagnetic spin-exchange interaction!

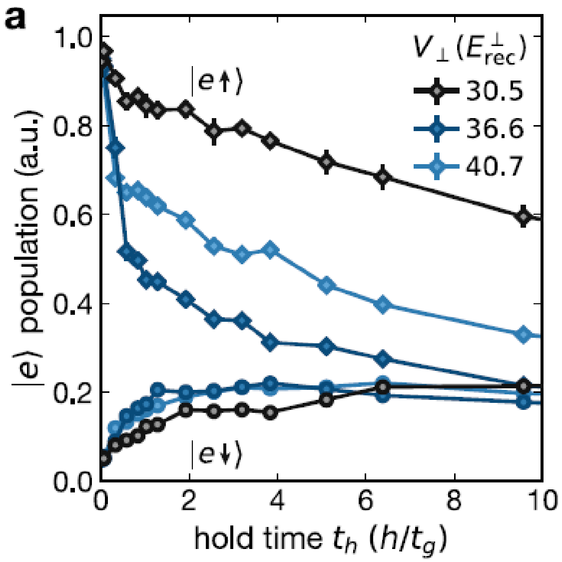
Experimental realization

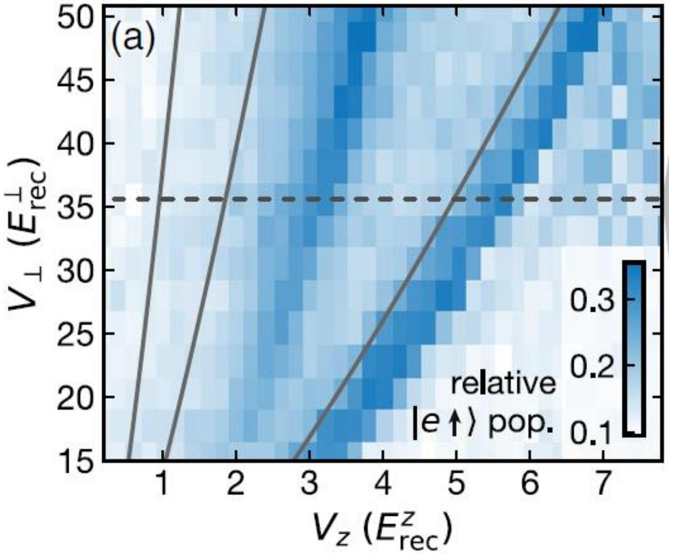
Experimental realization of the Kondo model with ultracold atoms

[Munich group: Riegger *et al.*, PRL 120, 143601 (2018)]

- ✓ **^{173}Yb, ferromagnetic** spin exchange, $T/T_F \simeq 0.2$
- ✓ State-dep. optical lattice → Kondo model on 1D lattice
- ✓ Kondo effect has not yet been observed
- ✓ Observation of spin-exchange dynamics
- ✓ Tunable spin-exchange interaction via confinement-induced resonances





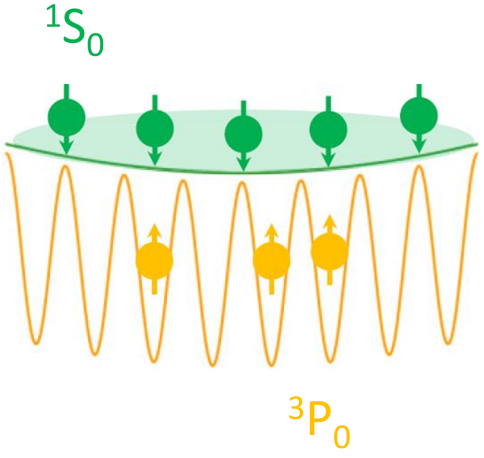


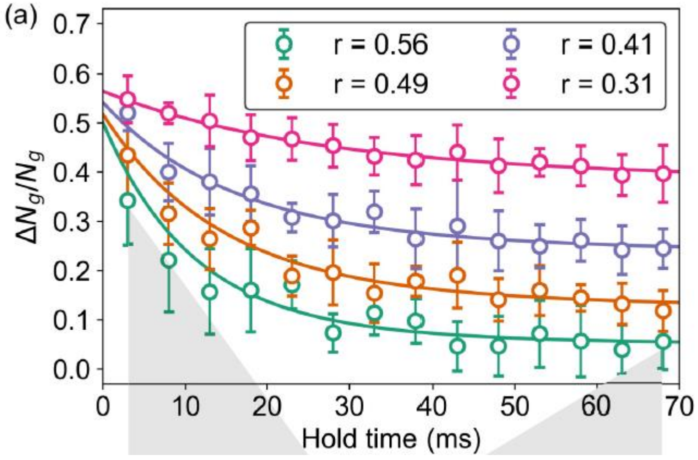
Experimental realization

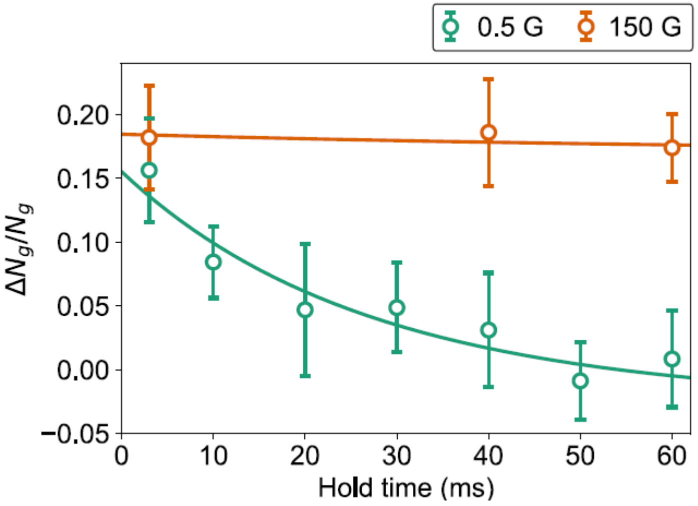
Experimental realization of the Kondo model with ultracold atoms

[Kyoto group: Ono *et al*., PRA 103, L041303 (2021)]

- ✓ **^{171}Yb**, **antiferromagnetic** spin exchange, $T/T_F \simeq 0.3$
- ✓ State-dep. optical lattice → $^1S_0$ without lattice + localized $^3P_0$ impurity
- ✓ Kondo effect has not yet been observed
- ✓ Observation of spin-exchange dynamics
- ✓ Suppression of spin-exchange dynamics under a magnetic field







Current status

Problems

- ◇ Temperature (vs. Kondo temperature)
- ◇ Control of the spin-exchange interaction

  (^{173}Yb, ^{87}Sr: ferromagnetic, ^{171}Yb: antiferromagnetic but relatively small)
- ◇ System size (a few tens of atoms per tube)

Toward Kondo physics in ultracold atoms

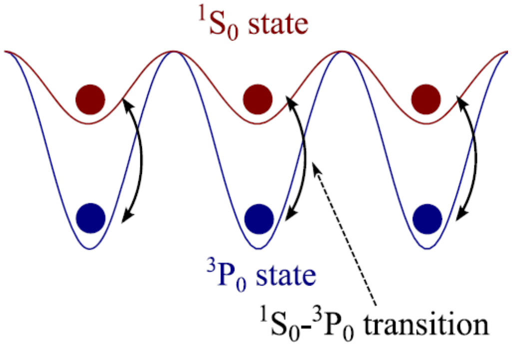
- ◇ Confinement-induced resonance

  [Zhang *et al.*, PRA 93, 043601 (2016); PRA 101, 013636 (2020)]
- ◇ $^1S_0$-$^3P_0$ transition: laser-induced Kondo effect

  [MN and N. Kawakami, PRL 115, 165303 (2015)]
- ◇ Floquet engineering

  [Kánasz-Nagy *et al.*, PRB 97, 155156 (2018)]
- ◇ $^3P_2$ state

  [Kuzumenko *et al.*, PRB 97, 075124 (2018); Yang *et al.*, PRR 4, 023173 (2022)]



Prospects

Prospects: novel Kondo physics with ultracold atoms?

- ✓ How to extract the universal feature of the Kondo effect in cold atoms?
- ✓ New Kondo problem unique to cold atoms?

  (cf. realization with a quantum dot → nonequilibrium Kondo problem)

◇ Real-time dynamics (cf. Lee-Low-Pines)

[Ashida *et al*., PRL 121, 026805 (2018)]

◇ Dynamics at finite temperature

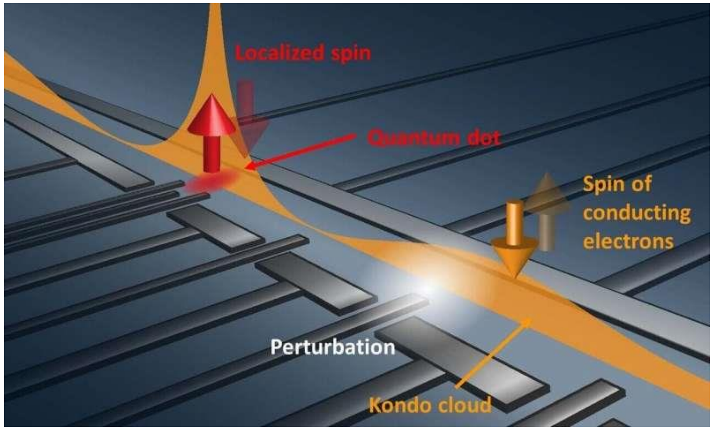
[Goto and Danshita, PRL 123, 143002 (2019)]

◇ Finite-size effect on T_K → Kondo cloud

[cf. Observation in a quantum dot: Nature 579, 210 (2020)]

◇ Dissipation

[MN, N. Kawakami, and M. Ueda, PRL 121, 203001 (2018)]



https://phys.org/news/2020-03-world-experimental-kondo-cloud.html

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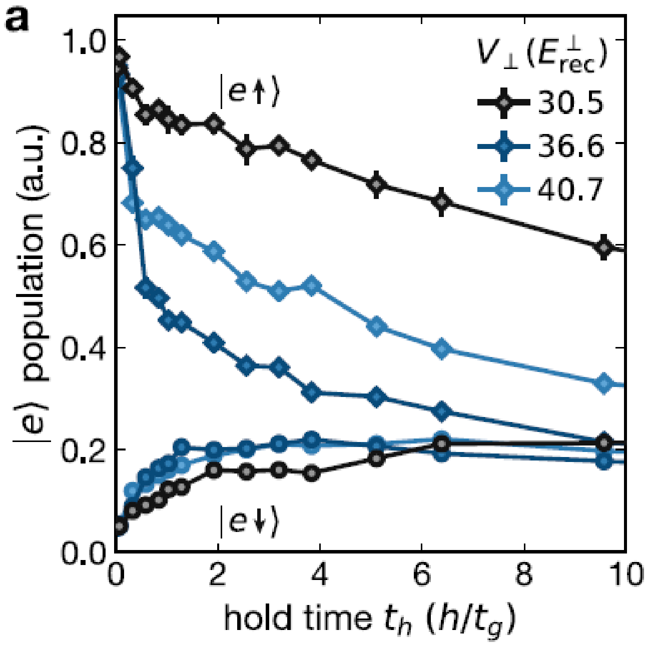
Inelastic collision

- Experimental realization of the Kondo model with ultracold atoms

[Riegger *et al.*, PRL 120, 143601 (2018)]



Atom number decreases!

Loss of atoms:

Inelastic collisions

between $^1S_0$ & $^3P_0$ states

Problem

Kondo effect vs. inelastic collisions

Non-Hermitian Kondo Hamiltonian

- Atom loss → described by a quantum master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho] + \sum_{\alpha=\pm,\uparrow\uparrow,\downarrow\downarrow} \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right)$$

$$= -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger) + \sum_{\alpha=\pm,\uparrow\uparrow,\downarrow\downarrow} L_\alpha \rho L_\alpha^\dagger$$

Loss event:
change the particle #

Non-Hermitian Hamiltonian : Dynamics between loss events

- Loss operators (*c*: majority fermion, *f*: impurity fermion)

$$L_\pm = \sqrt{2\gamma_{eg}^{\mp}}\frac{1}{\sqrt{2}}(f_\downarrow c_\uparrow(0) \pm f_\uparrow c_\downarrow(0)),\ L_{\uparrow\uparrow} = \sqrt{2\gamma_{eg}^{-}} f_\uparrow c_\uparrow(0),\ L_{\downarrow\downarrow} = \sqrt{2\gamma_{eg}^{-}} f_\downarrow c_\downarrow(0),$$

$$H_{\mathrm{eff}} = H - \frac{i}{2}\sum_\alpha L_\alpha^\dagger L_\alpha = \sum_{\boldsymbol{k},\sigma} \varepsilon(\boldsymbol{k}) c_{\boldsymbol{k}\sigma}^\dagger c_{\boldsymbol{k}\sigma} - J \boldsymbol{S}_{c0}\cdot \boldsymbol{S}_{\mathrm{imp}} \quad \boxed{(J = J_r + iJ_i)}$$

Imaginary int. due to inelastic collisions

Non-Hermitian Kondo model
w/ complex-valued interaction

[MN, N. Kawakami, and M. Ueda, PRL 121, 203001 (2018)]

Quantum trajectory

- Equivalent "quantum trajectory" description [Dalibard *et al.*, PRL 68, 580 (1992)]

$$\frac{d\rho(t)}{dt} = -i(H_{\rm eff}\rho - \rho H_{\rm eff}^{\dagger}) + \sum_{\alpha} L_{\alpha}\rho L_{\alpha}^{\dagger}$$

"Unraveling" ⇩ ⇧ Ensemble average

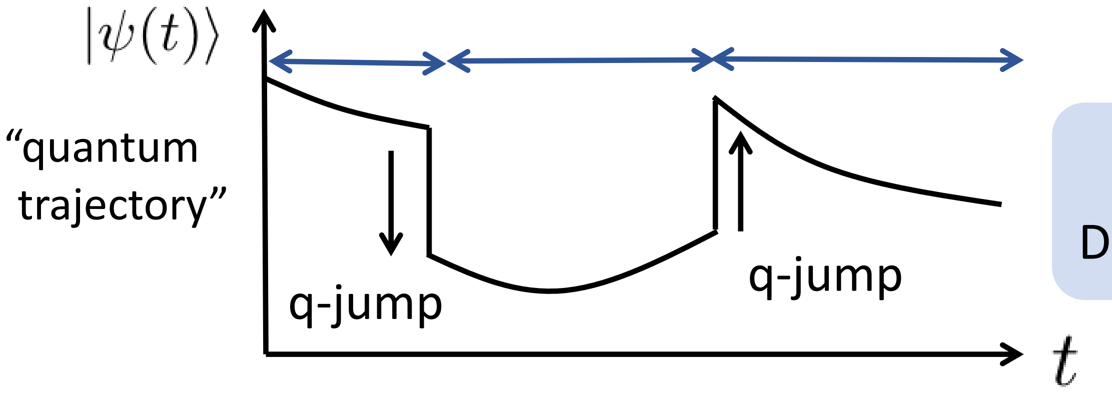
$$i\partial_t \left|\psi\right\rangle = H_{\rm eff}\left|\psi\right\rangle$$

✚

$$\left|\psi\right\rangle \to \frac{L_{\alpha}\left|\psi\right\rangle}{\sqrt{\left\langle\psi\right| L_{\alpha}^{\dagger} L_{\alpha}\left|\psi\right\rangle}}$$

Non-Hermitian Schrödinger evolution

Stochastic quantum jump
(loss event)

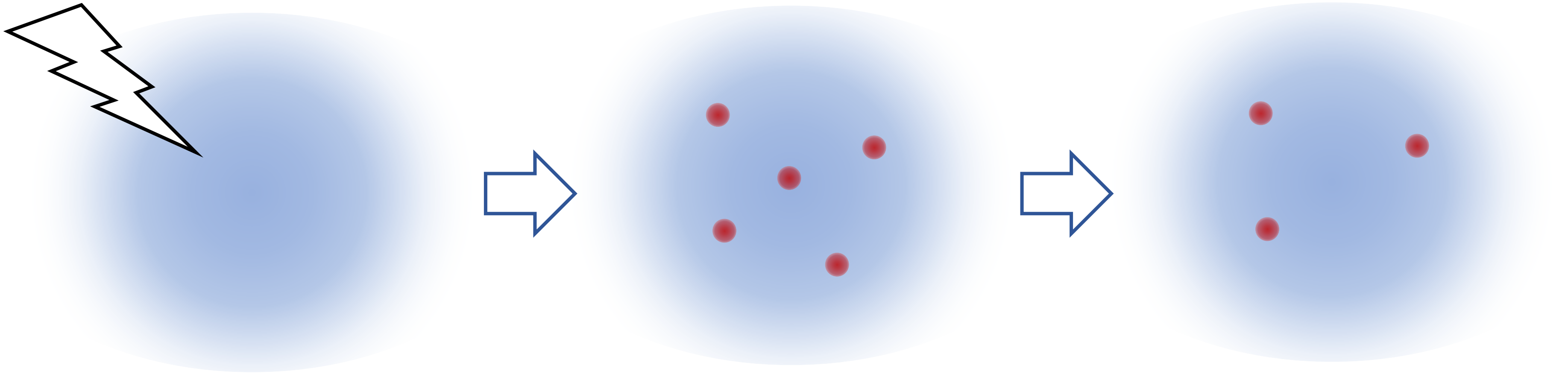


Non-Hermitian Hamiltonian :

Dynamics between quantum jumps

Setup

Excitations to $^3P_0$ state



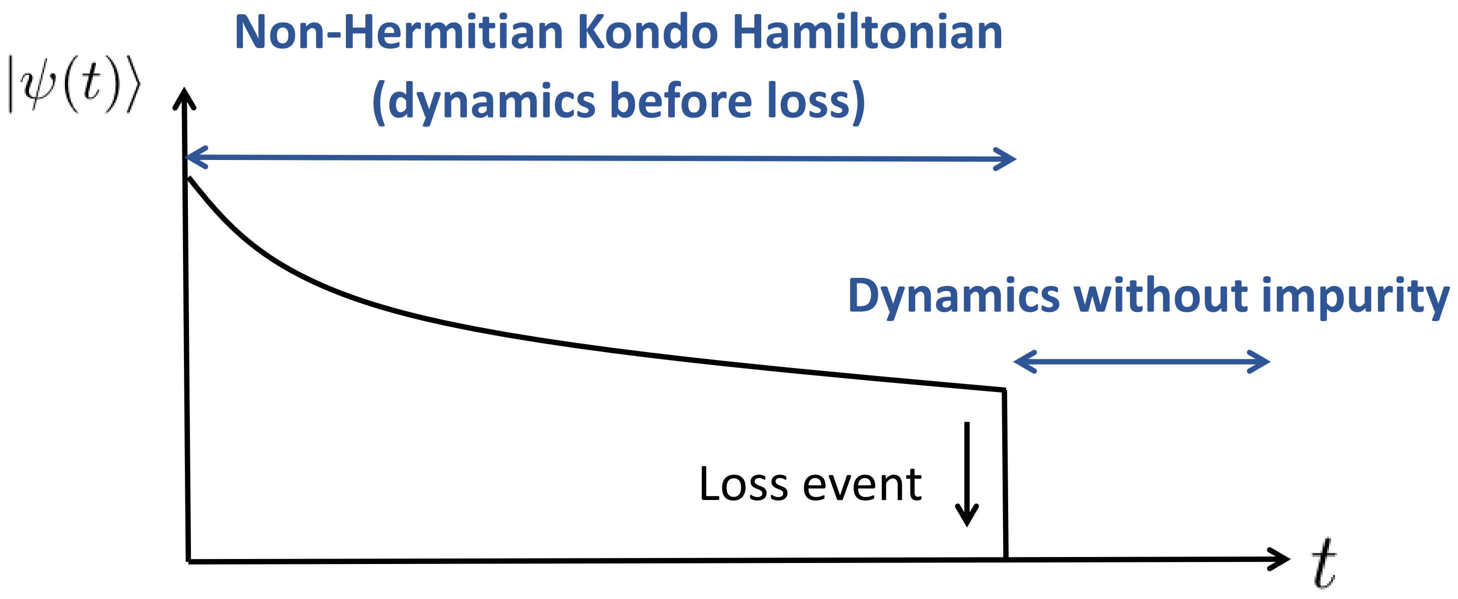
Equilibrium gas
All atoms are in $^1S_0$ state

$^3P_0$ state as impurities:
Kondo system

Some impurities are lost
due to inelastic collisions

Emergence of non-Hermitian dynamics

- Dynamics of a “surviving” impurity (quantum-trajectory method)



Non-Hermitian Kondo Hamiltonian (dynamics before loss)

Dynamics without impurity

Loss event

“Surviving impurity”

= Non-Hermitian quantum impurity

Renormalization group

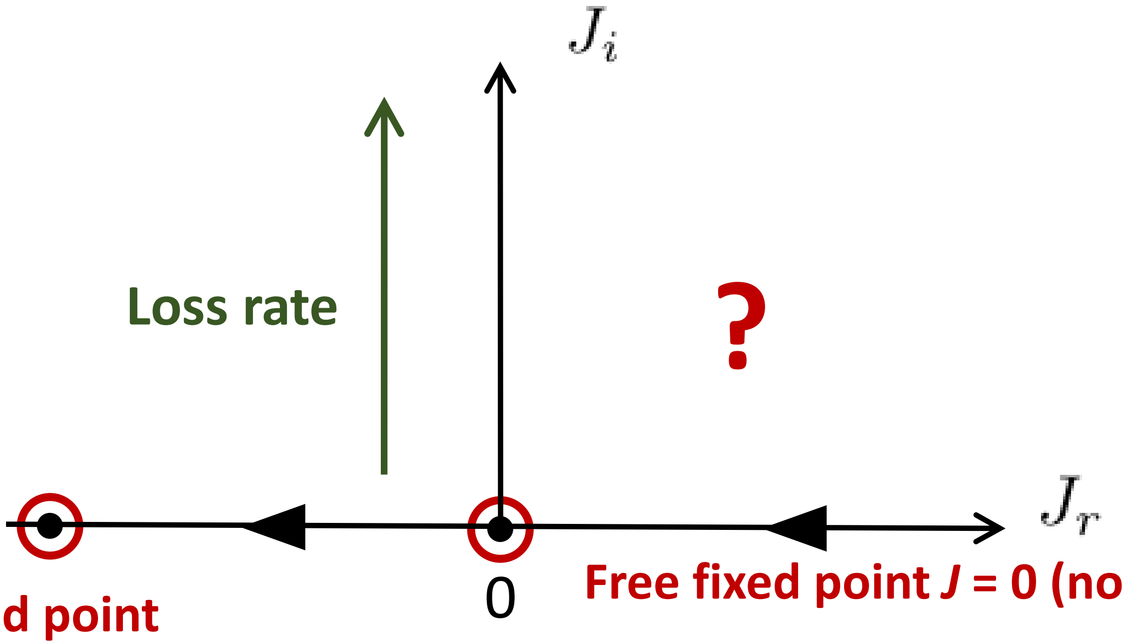
- **Renormalization group** [MN, N. Kawakami, and M. Ueda, PRL 121, 203001 (2018)]

$$\frac{dJ}{d\ln D} = \rho_0 J^2 + \frac{\rho_0^2}{2} J^3,$$

(ρ_0: DOS at the Fermi energy)

(two-loop RG)

$$\boxed{(J = J_r + iJ_i)}$$



Renormalization group

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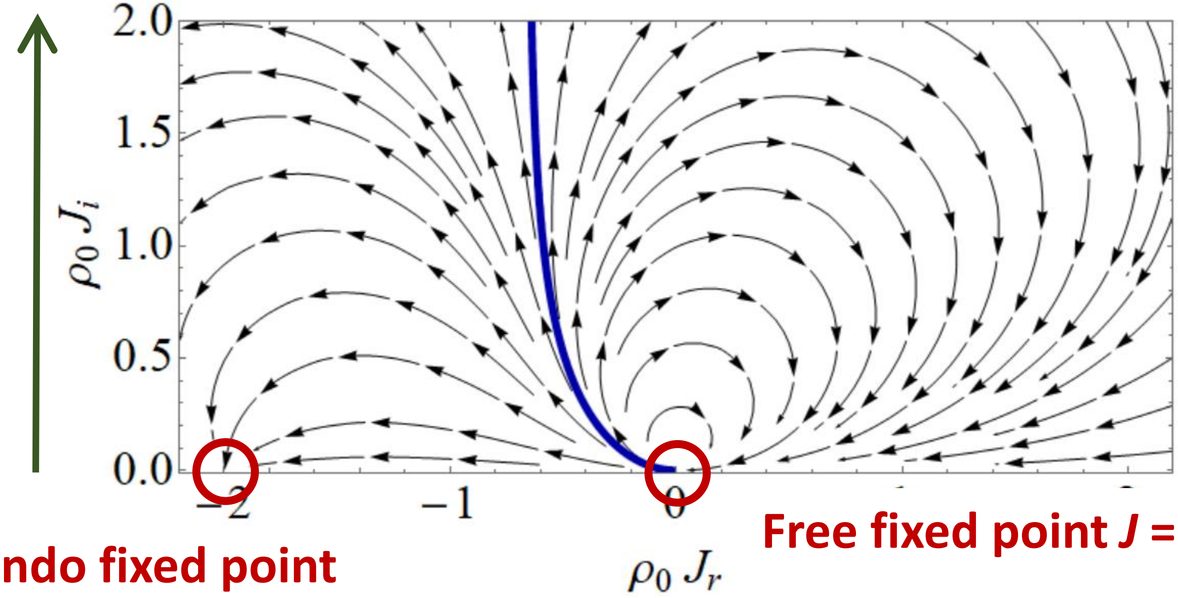
Critical line



Loss rate

Kondo fixed point

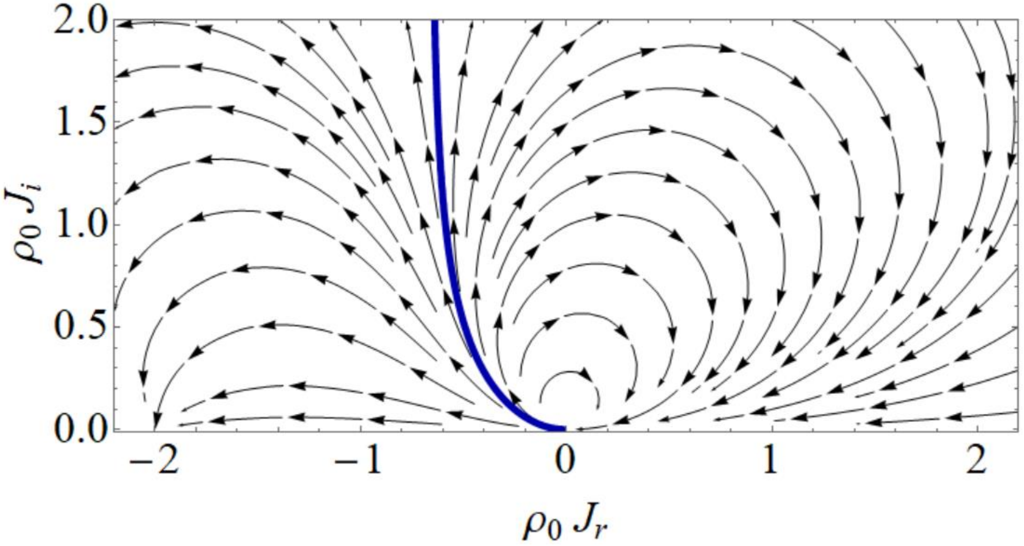
Free fixed point *J* = 0 (no Kondo effect)

A new type of RG flow: *reversion* (RG backflow)

Non-Hermitian quantum phase transition induced by inelastic collisions

Violation of *g*-theorem

■ Anomalous "reversion" of RG flow



■ RG flow in **Hermitian** quantum impurity systems

→ Ground state degeneracy must decrease along the RG flow

(*g*-theorem) [Affleck and Ludwig, PRL 67, 161 (1991)]

"Reversion" of RG flow is unique to non-Hermitian systems

(violation of *g*-theorem)

Bethe ansatz solution

- To confirm the RG prediction:

  Exact solution of the non-Hermitian Kondo model

  [MN, N. Kawakami, and M. Ueda, PRL 121, 203001 (2018)]

- Non-Hermitian generalization of **Bethe ansatz**

  [Hermitian case: Andrei (1980), Wiegmann (1981)]

$$e^{ik_j L} = e^{-i\pi\rho_0 J/2} \prod_{\alpha=1}^{M} \frac{\lambda_\alpha + i/2}{\lambda_\alpha - i/2} \quad (j = 1, \cdots, N),$$

$$\left(\frac{\lambda_\alpha + i/2}{\lambda_\alpha - i/2}\right)^N \left(\frac{\lambda_\alpha + 1/g + i/2}{\lambda_\alpha + 1/g - i/2}\right) = -\prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \quad (\alpha = 1, \cdots, M),$$

where $g = -\tan(\pi\rho_0 J)$

$k_j \ (j = 1, \cdots, N)$: quasi-momenta of conduction fermions
(*N*: # of particles)

$\lambda_\alpha \ (\alpha = 1, \cdots, M)$: spin rapidity of ↓ spin electrons
(*M*: # of ↓ spins)

Bethe ansatz solution

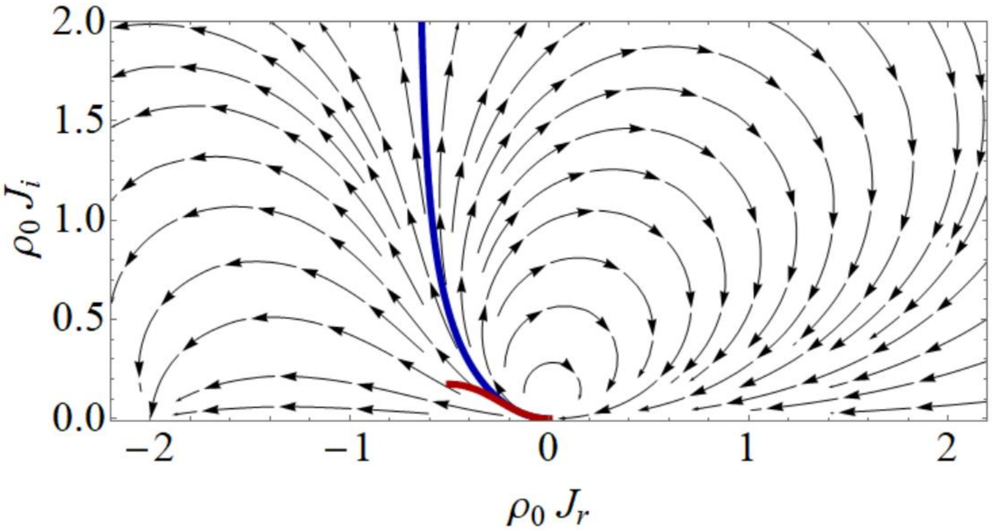
- **Impurity magnetization *M_i*** of the ground state

◇ $|\mathrm{Im}(1/\tan(\pi\rho_0 J))| < 1/2$

$\Longrightarrow M_i = 0$ **Kondo singlet solution**

◇ $|\mathrm{Im}(1/\tan(\pi\rho_0 J))| > 1/2$

$\Longrightarrow M_i = 1/2$ **Non-Kondo solution!!**



Blue: critical line from RG

Red: critical line from Bethe ansatz

Good agreement at weak coupling

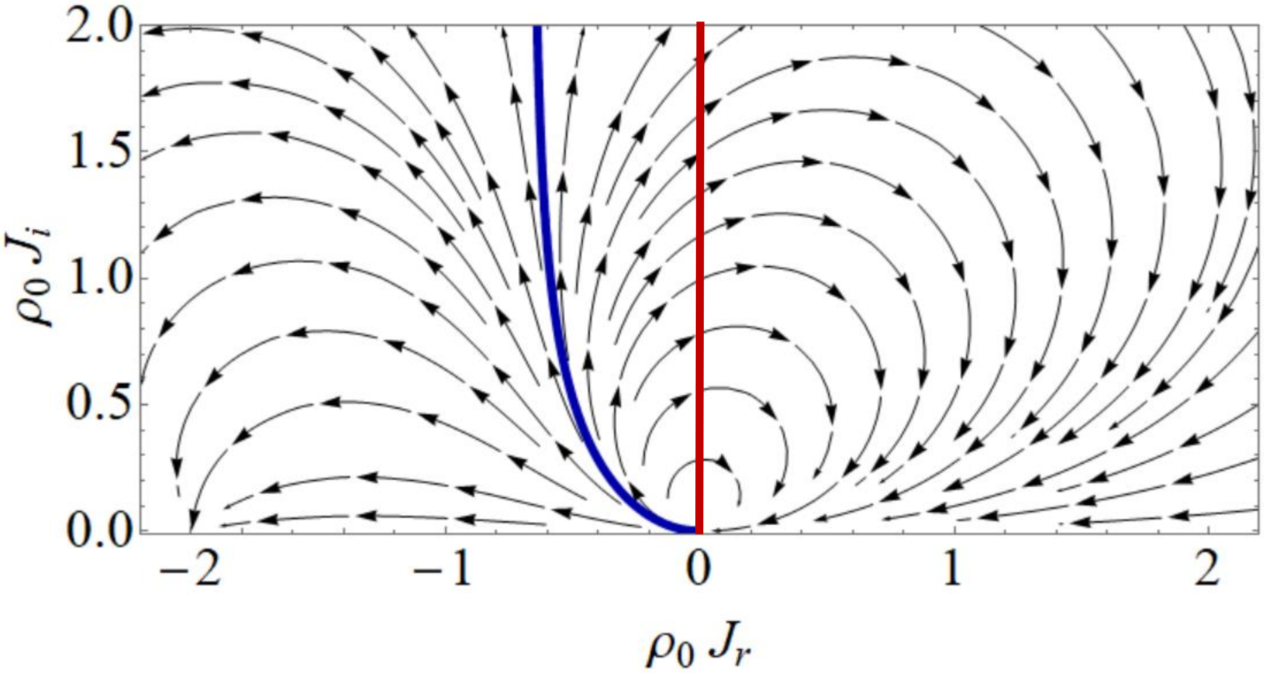
Confirming the exotic RG flow from the exact solution!

[MN, N. Kawakami, and M. Ueda, PRL 121, 203001 (2018)]

Energy scale of the non-Hermitian Kondo effect

- Non-Hermitian renormalization group

Define a characteristic scale : $J_r(T_{\mathrm{Kdiss}}) = 0$



$$T_{\mathrm{Kdiss}} \simeq \frac{D}{\sqrt{2}}\sqrt{|\rho_0 J|}\exp\left[\frac{J_r}{\rho_0|J|^2}\right]$$

Non-Hermitian generalization of Kondo temperature

Kondo temperature reflects dissipation

- Signature of the emergent energy scale of the non-Hermitian RG flow

[M. Hasegawa, MN, and K. Saito, arXiv:2111.07771]

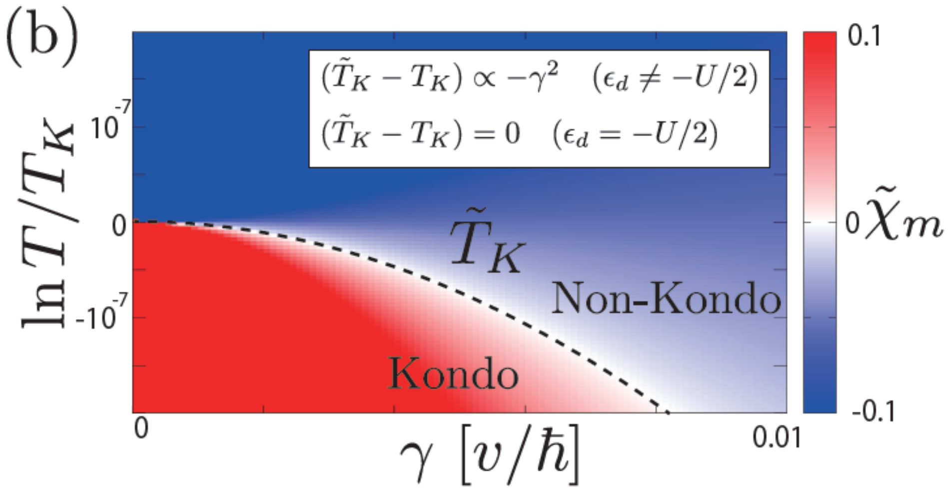
Quantum dot + measurement backaction

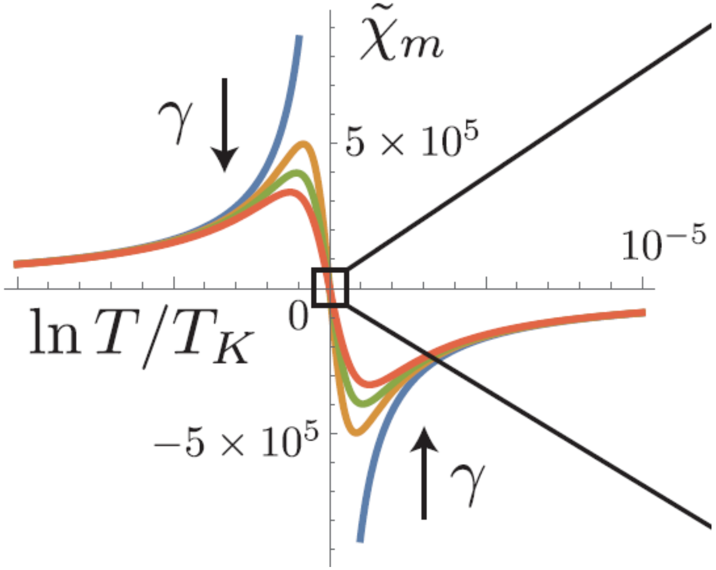
→ effective non-Hermitian Kondo model

Spin susceptibility

(perturbation theory, bath is equilibrium)

$$\tilde{\chi}_m \simeq -\frac{\chi_{m,0}}{\sqrt{3}a}\mathrm{Re}\left[\frac{aJ\nu}{1+aJ\nu\ln T/D}\right]$$

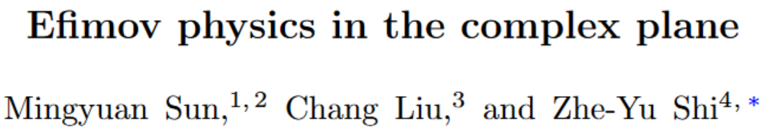




Susceptibility reflects T_K of non-Hermitian RG flow!

Non-Hermitian universality

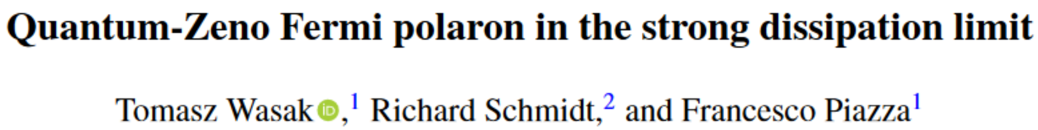
- Loss is ubiquitous in cold atoms
- Non-Hermitian physics → extension of parameters to **complex plane**
- Recent theoretical works
  - Efimov effect + loss [Zhou & Cui, PRR 3, 043225 (2021); Sun *et al.*, arXiv:2109.11206]



Efimov physics in the complex plane

Mingyuan Sun,[1,2] Chang Liu,[3] and Zhe-Yu Shi[4,*]

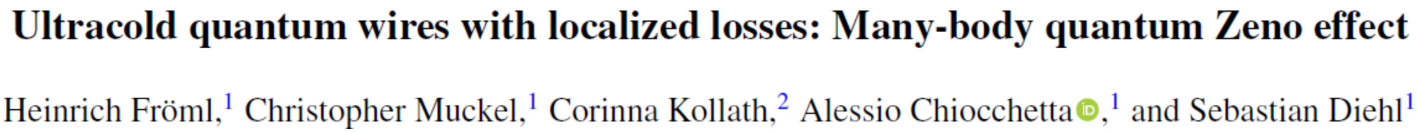
  - Polaron + loss [Wasak *et al.*, PRR 3, 013086 (2021)]



Quantum-Zeno Fermi polaron in the strong dissipation limit

Tomasz Wasak,[1] Richard Schmidt,[2] and Francesco Piazza[1]

  - Dissipative impurity (localized loss) [Fröml *et al.*, PRB 101, 144301 (2020)]



Ultracold quantum wires with localized losses: Many-body quantum Zeno effect

Heinrich Fröml,[1] Christopher Muckel,[1] Corinna Kollath,[2] Alessio Chiocchetta,[1] and Sebastian Diehl[1]

Non-Hermitian universality in dissipative systems?

Outline

1. Introduction
    - Universality of the Kondo effect

2. Kondo effect in ultracold atoms

3. New universality of the Kondo effect
    - Non-Hermitian Kondo effect

4. Summary

Summary

- Universality of the Kondo effect

  Renormalization group flow & boundary critical phenomena

- Kondo effect in ultracold atoms

  Alkaline-earth-like atoms: current status & prospects

- Non-Hermitian Kondo effect

  Inelastic collisions = complex-valued non-Hermitian interactions

  Exotic RG flow: non-Hermitian universality?

MN and N. Kawakami, Phys. Rev. Lett. 115, 165303 (2015)

MN, N. Kawakami, and M. Ueda, Phys. Rev. Lett. 121, 203001 (2018)

M. Hasegawa, MN, and K. Saito, arXiv:2111.07771