

「冷却原子系ポーラロンと周辺の話題」 仲野英司@高知大

内容

1. 電子ポーラロン … LLP理論
2. 冷却原子系ポーラロン : Bose polaron
 - 弱・中間結合領域
 - 強結合領域と少数粒子相関
3. 冷却原子系ポーラロン : Fermi polaron
 - Chevy ansatz
 - 核物質中の少数アルファ粒子
4. まとめ、関連研究

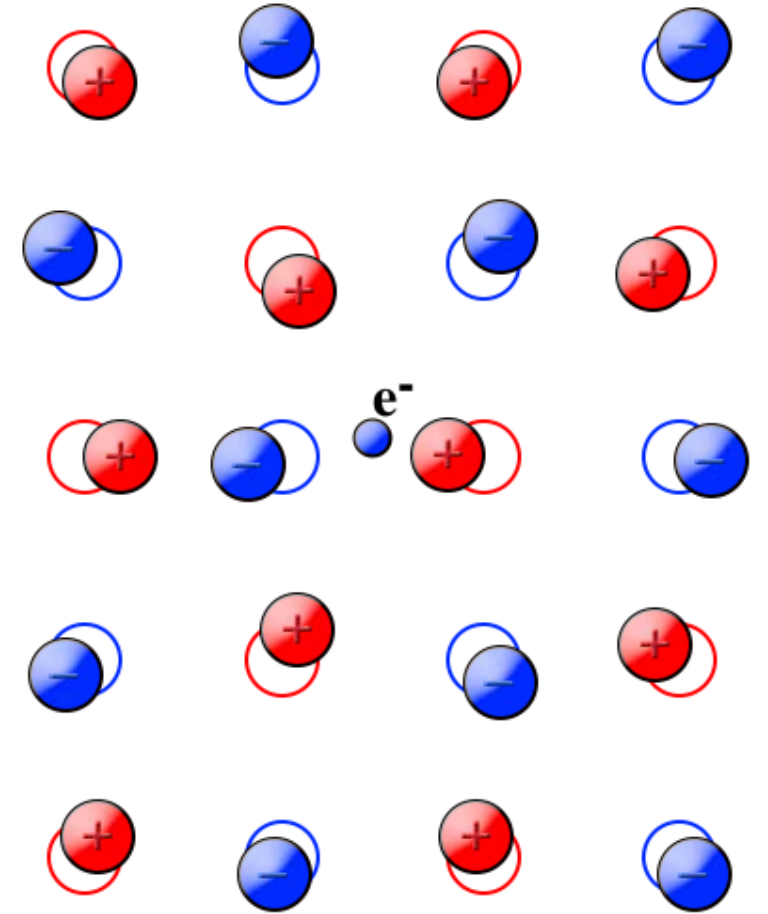
1. 電子ポーラロン ('48 Landau, Pekar)

Hamiltonian for 'single' electron in ionic crystal

$$H = H_0 + H_{int}$$

$$H_0 = -\frac{\nabla^2}{2m} + \sum_q \omega_q C_q^\dagger C_q$$

$$H_{int} = e\Phi(x) = \sum_q g_q C_q e^{iqx} + h.c.$$

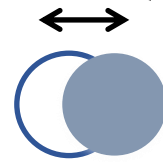


Electron-phonon Interaction from dielectric polarization $P(x)$

from wiki

$$-\nabla\Phi(\mathbf{x}) = E(x) = -4\pi P(x) \propto \delta R(x) \quad (\text{continuum approx.})$$

Maxwell eq.



Lee-Low-Pines theory (LLP '53)

Ground state property for 'single' Polaron

1. Eigenvalue problem $H|\psi\rangle = E|\psi\rangle$
2. Variation method, using unitary transformations, S and T

S : a gauge transformation : comoving frame of impurity

T : to generate a coherent state of phonons

$$S = \exp[-i \sum_q q \cdot x C_q^\dagger C_q]$$

A gauge transformation:

$$S^{-1} C_q S = e^{-iqx} C_q$$

$$S^{-1} C_q^\dagger S = e^{iqx} C_q^\dagger$$

Total momentum \rightarrow impurity momentum

$$S^{-1} (-i\nabla + \sum_q q C_q^\dagger C_q) S = -i\nabla$$

$$H = H_0 + H_{int}$$

$$H_0 = -\frac{\nabla^2}{2m} + \sum_q \omega_q C_q^\dagger C_q$$

$$H_{int} = \sum_q g_q C_q e^{iqx} + h.c.$$

$$S^{-1} H S = H_0' + H_{int}'$$

$$H_0' = \frac{(-i\nabla - \sum_q q C_q^\dagger C_q)^2}{2m} + \sum_q \omega_q C_q^\dagger C_q$$

$$H_{int}' = \sum_q g_q C_q + h.c.$$

Transformed Hamiltonian commutes with fermion momentum:

$$[S^{-1} H S, -i\nabla] = 0 \quad -i\nabla \rightarrow P \text{ conserved quantity}$$

$$T = \exp[\sum_q f_{q;P} C_q^\dagger - h.c.]$$

We assume a trial state: $|\phi_P\rangle = T|0\rangle$ where $|0\rangle$: Phonon's Fock vacuum

$f_{q;P}$: variation parameter = phonon probability amplitude

$$T^{-1}C_qT = C_q + f_{q;P} \Rightarrow \langle 0|T^{-1}C_qT|0\rangle = f_{q;P}$$

$$E_P = \langle \psi_P | H' | \psi_P \rangle, \quad |\psi_P\rangle = e^{ixP} |\phi_P\rangle$$

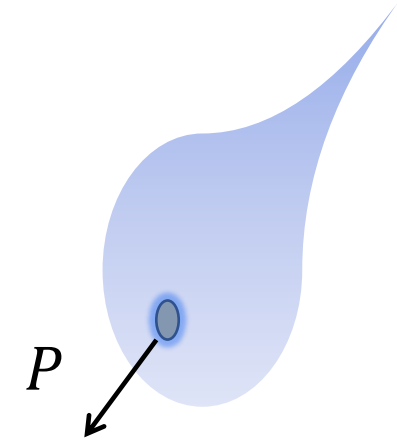
- variation condition $\frac{\delta E_P}{\delta f_{q;P}} = 0$

$$f_{q;P} = -g_q \left[\omega_q + \frac{q^2 - 2(1 - \eta)q \cdot P}{2m_f} \right]^{-1}$$

Properties of a mobile polaron:

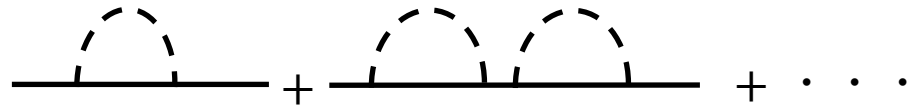
- η = momentum drag parameter

$$\eta P = \sum_{q \neq 0} q |f_{q;P}|^2 \quad (\text{self-consistent eq.})$$



$0 < \eta < 1$: *share ratio of P by phonons*

$$\eta = \frac{s}{1+s} \quad s = O(g_q^2)$$



- Effective mass

$$E_P = E_0 + \frac{P^2}{2m_{eff}} + O(P^4) \quad m_{eff} = \frac{m_f}{1-\eta}$$

- Quasi-particle residue

$$z = |\langle 0 | \psi_P \rangle|^2 = e^{-\sum_q |f_{q;P}|^2}$$

ポーラロンにおける励起ボゴリューボフ・フォノンの運動量分布

$$N_k = \langle x; P | C_k^\dagger C_k | x; P \rangle = |f_k|^2,$$

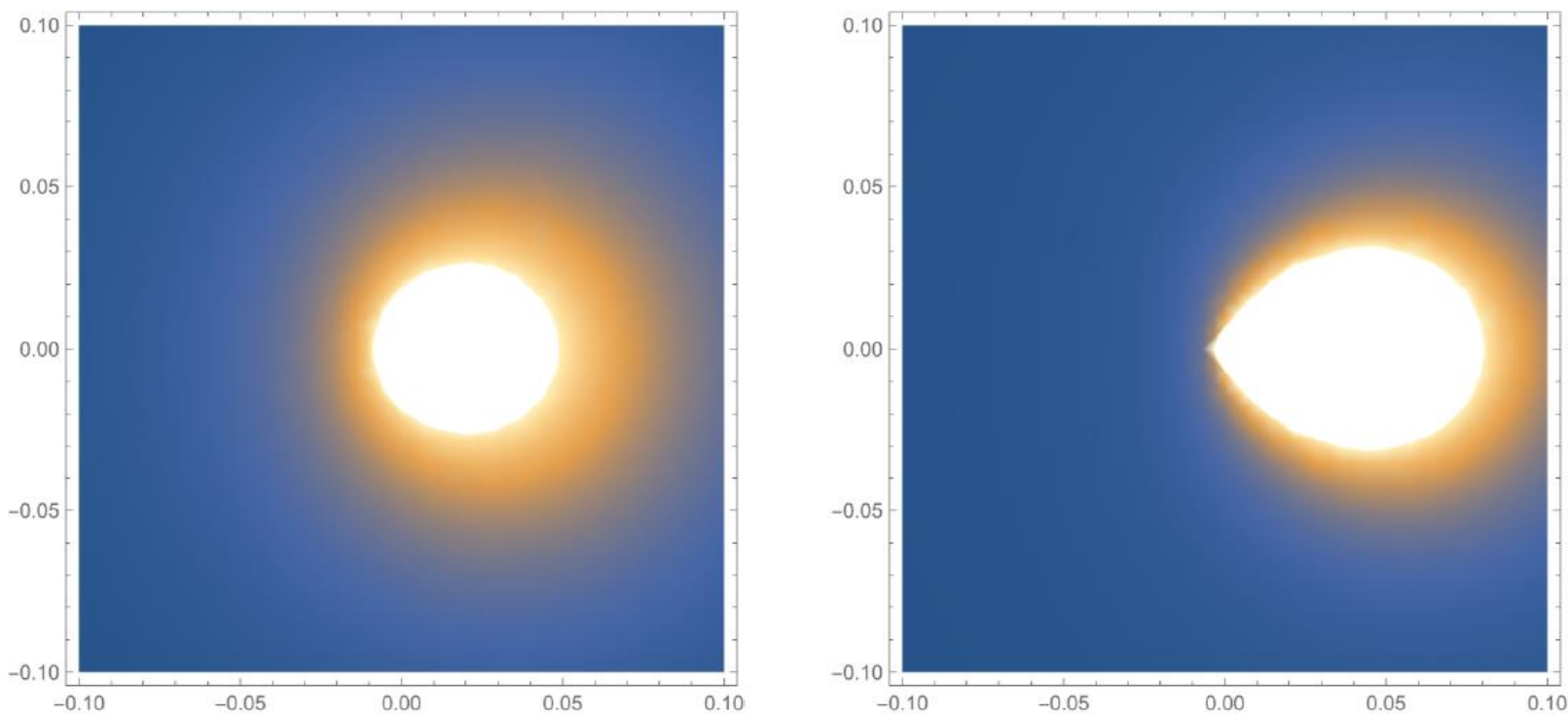


Figure: $|f_q|^2$ の q_x - q_y 平面分布 $P_x = 0.2$ (左)、 $P_x = 0.4$ (右) $a = -0.05$ 、 $a_b = 0.01$ 、 $\xi = 10$ 、以上すべて $n_0^{1/3}$ で無次元化、 $R = 5$ 。

大体実験と同じ大きさのパラメタ。

In what situation this method works well?

- In **Heavy** impurity limit → **the exact solution**

$$H_0' = \frac{(P - \sum_q q C_q^\dagger C_q)^2}{2m_f} + \sum_q \omega_q C_q^\dagger C_q$$

$$H_{int}' = \sum_q g_q C_q + h.c.$$

- For **not so fast** impurity (or shock wave → **phonon radiation**)

$$f_{q;P} = -g_q \left[\omega_q + \frac{q^2 - 2(1 - \eta)q \cdot P}{2m_f} \right]^{-1}$$

$$\omega_q + \frac{q^2 - 2(1 - \eta)q \cdot P}{2m_f} \simeq |q| \left(v_{ph} - \frac{1 - \eta}{m_f} |P| \cos \theta \right) \implies \frac{|P|}{m_{eff}} < v_{ph}$$

$$\omega_q \sim v_{ph}|q|, \quad v_{ph}: \text{Phonon velocity}$$

- Variation method \Rightarrow weak / intermediate coupling regime
(by Tomonaga '46 for Nucleon – meson system)

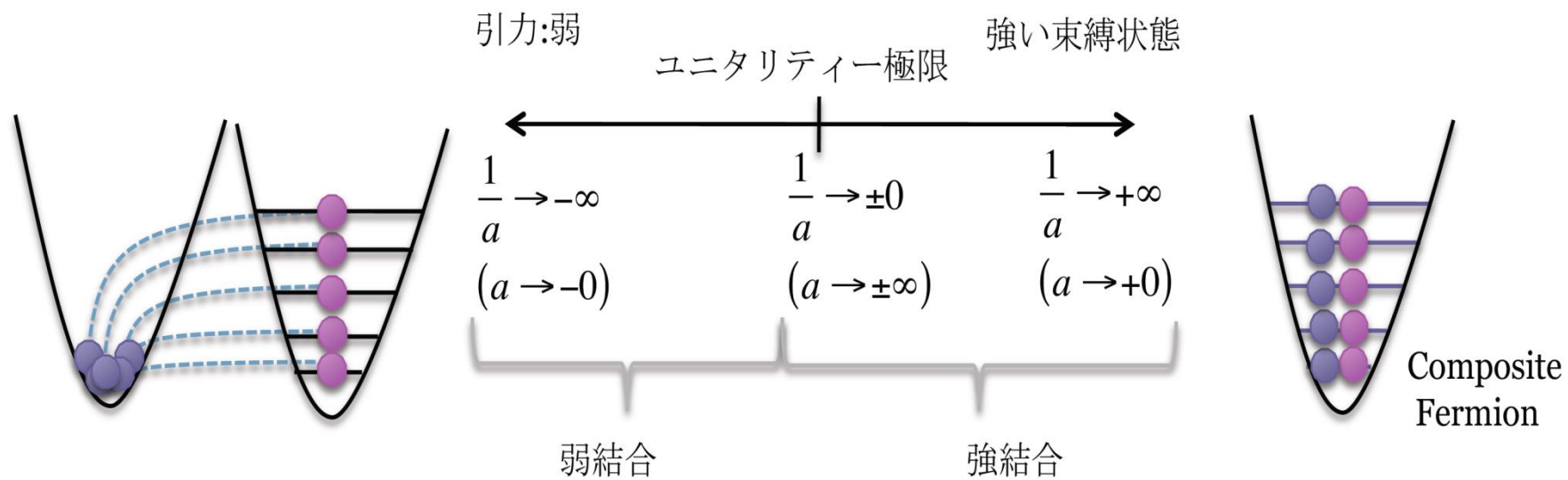
detour: A **many-body** extension of LLP theory (EN-Yabu, PRB '16)

もともとの動機: dilute and heavy fermions in Bose gas  $\ll n_f^{-\frac{1}{3}}$

n_f : Fermion density

ユニタリ変換でどこまで媒質の素励起の平均場を取り入れられるか、フェルミオンの希薄極限でLLPを再現できるか？

$$U = e^S = \exp \left[\sum_{q,P} (f_{q;P} a_{P-q}^\dagger a_P C_q^\dagger - f_{q;P}^* a_P^\dagger a_{P-q} C_q) \right]$$



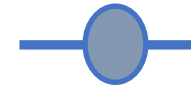
U Transformation of Hamiltonian :

$$U^{-1} H U \simeq H^{(mf)} + H'^{(2)} + H'^{(4)} + H'^{(no)}$$

Higher order terms

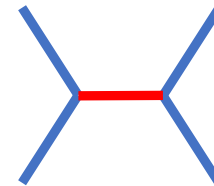
$$H'^{(2)} = \sum_P \left[\frac{(P - u_P)^2}{2m_f} + \sum_{q \neq 0} \left(\omega_q + \frac{q^2}{2m_f} \right) f_{q;P}^2 - 2 \sum_{q \neq 0} g_q f_{q;P} \right] a_P^\dagger a_P,$$

same as single polaron energy in LLP



$$H'^{(4)} = - \sum_{q, P, Q} \{ \omega_q f_{q;P} f_{q;Q} + g_q (f_{q;P} + f_{q;Q}) \}$$

$$\times a_P^\dagger a_{Q-q}^\dagger a_{P-q} a_Q,$$

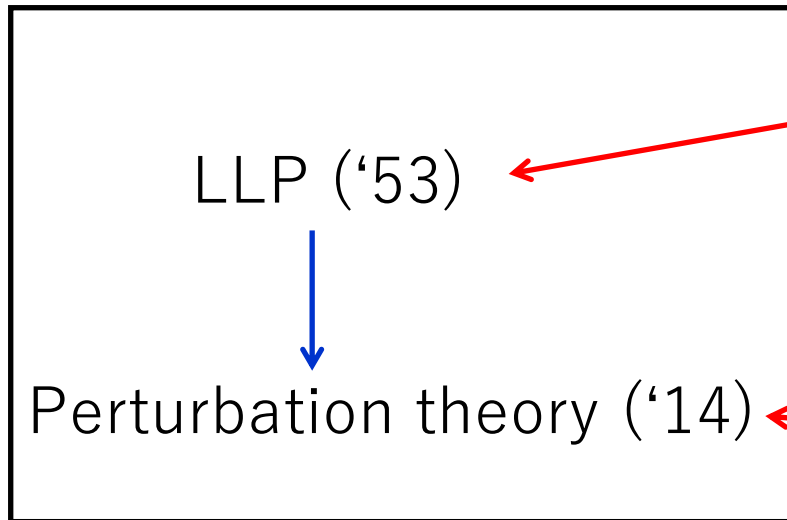


Interaction between polarons
(Yukawa potential in probe approx.)

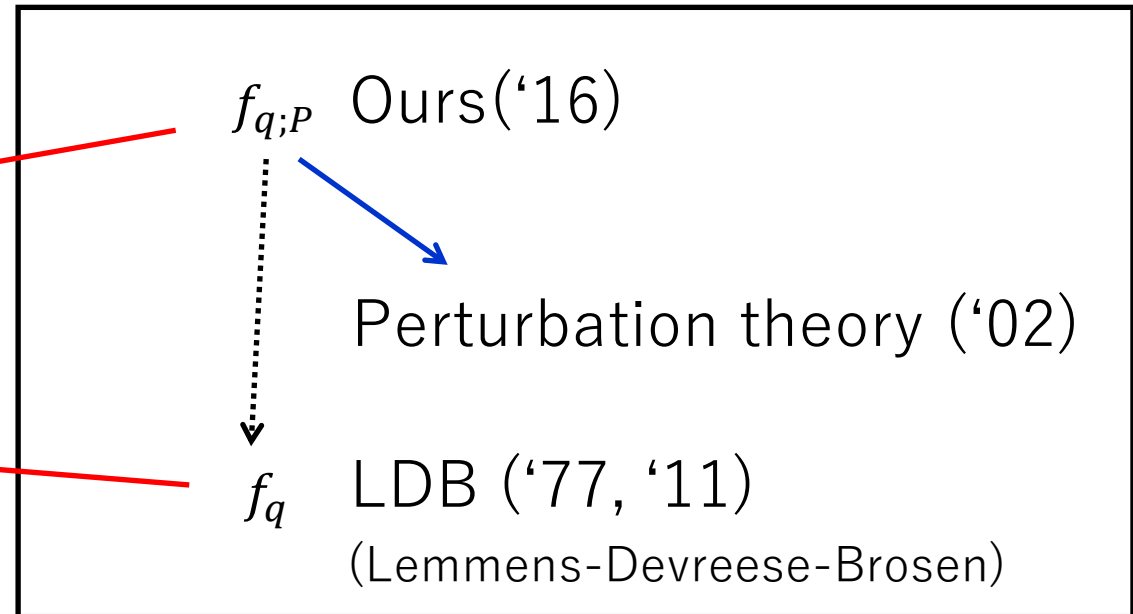
- S. Tomonaga, Prog. Theor. Phys. {¥bf 1}, 83, 109 (1946); {¥bf 2}, 6 (1947),
- T. Miyazima and S. Tomonaga, Sci. Papers of the Int. Whys.-Chem. Res. {¥bf 40}, 274 (1943)

Relation among theories :

- Single polaron



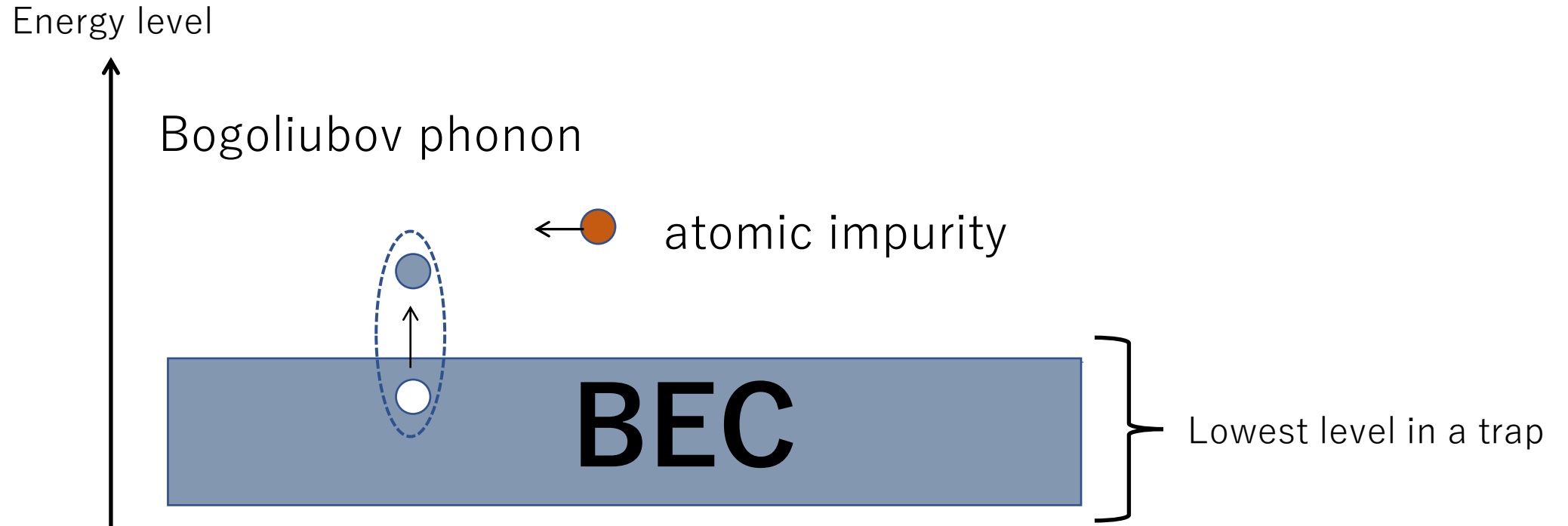
- Many-body polaron



Perturbation theory = 2nd order of Yukawa coupling



冷却原子系ポーラロン : Bose polaron

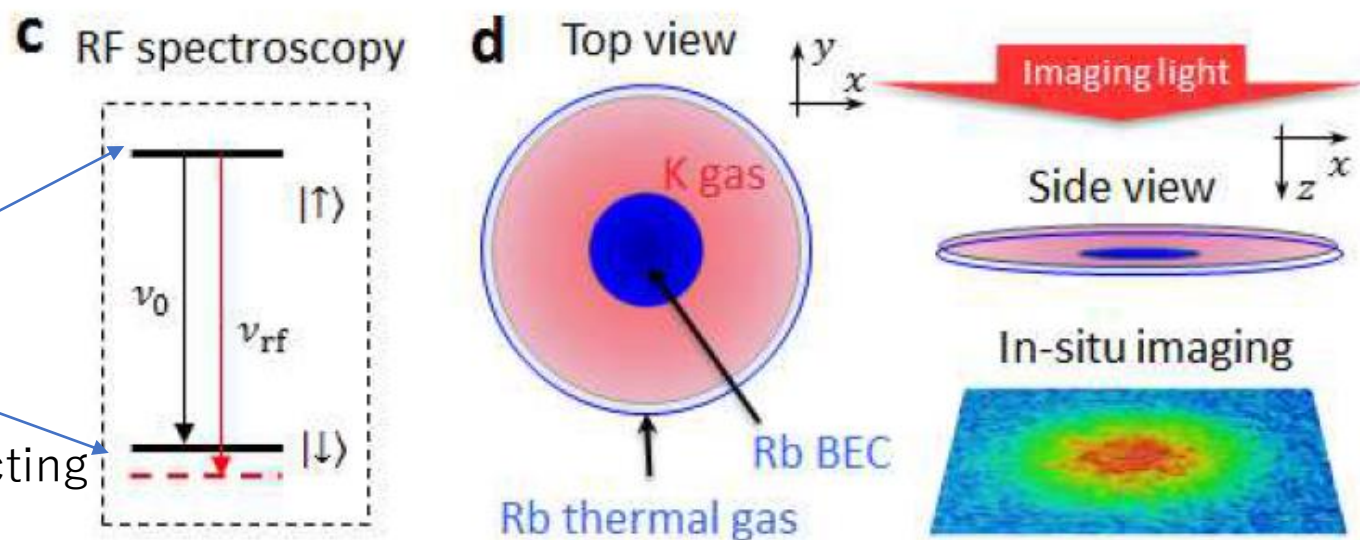
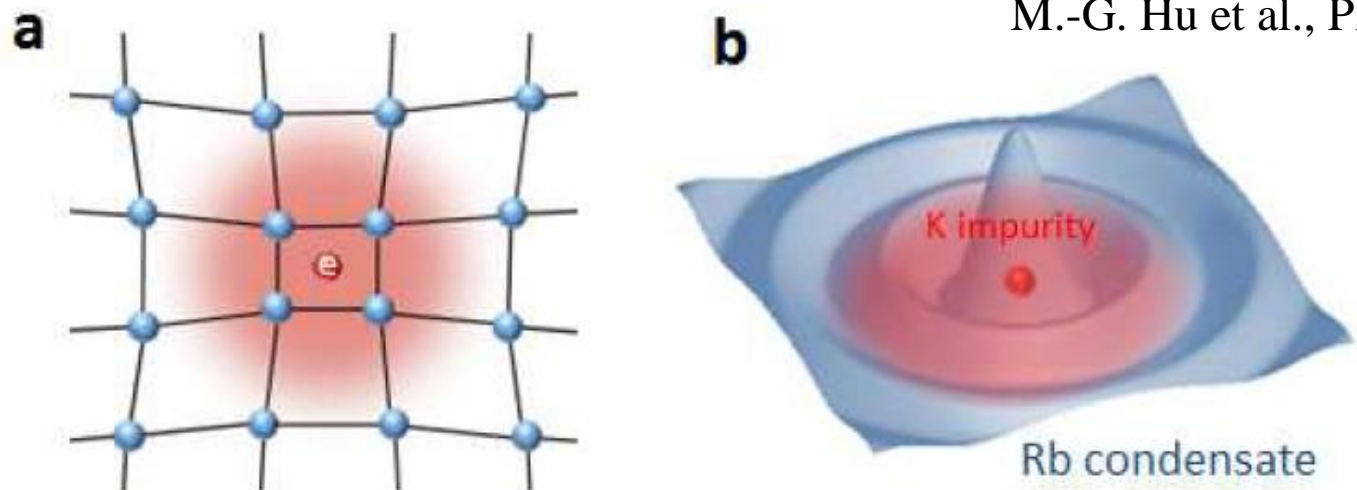


Atomic impurity in BEC = Bose polaron

冷却原子系ポラロンの実験

Bose polaron experiment (K impurity in Rb BEC gas)


M.-G. Hu et al., PRL 117 (2016) 055301



Potassium hyper fine states: Interacting/non-interacting with bosons

Low energy effective Hamiltonian for impurities in Bose gas

b_q : **boson** , a_q : **Impurity**

$$\mathcal{H} = \sum_p \left(\frac{p^2}{2m_f} a_p^\dagger a_p + \frac{p^2}{2m_b} b_p^\dagger b_p \right) + \frac{1}{V} \sum_{k,p,q} \left\{ g_{bf} a_{p+q}^\dagger b_{k-q}^\dagger b_k a_p + \frac{1}{2} g_{bb} b_{p+q}^\dagger b_{k-q}^\dagger b_k b_p \right\}$$


Coupling constants g_{bf}, g_{bb} in terms of **s-wave scattering length** via LS eq.:

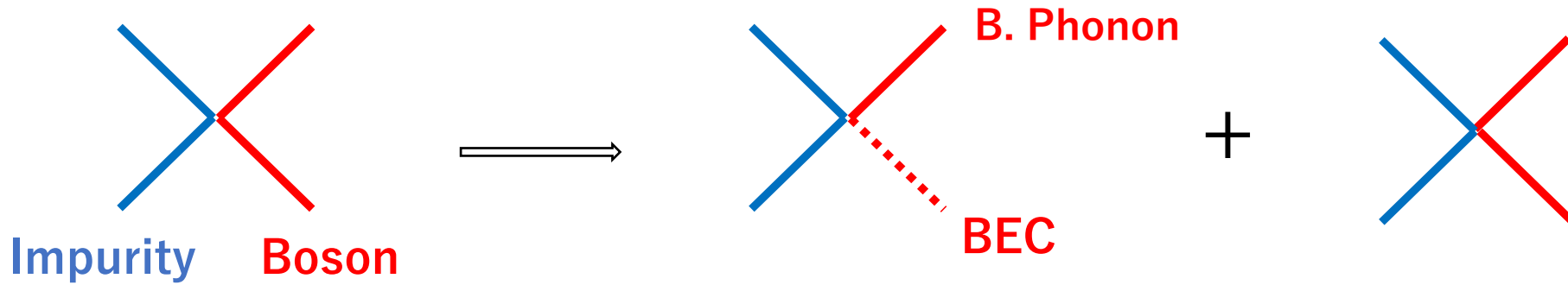
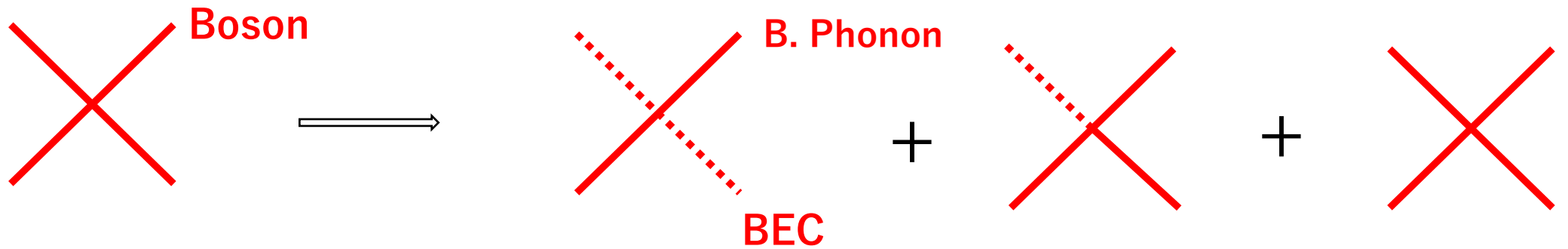
$$\frac{m_{ij}}{2\pi a_{ij}} = \frac{1}{g_{ij}} + \int_p \frac{1}{p^2/(2m_i) + p^2/(2m_j)}, \quad (i, j = b, f),$$

$$m_{ij} = \frac{m_i m_j}{m_i + m_j}$$

c.f. Fano-Feshbach resonance scattering

媒質は低温でBECだからBogoliubov Phonon (素励起)で書き直すと、

$$\begin{pmatrix} b_{-k} \\ b_k^\dagger \end{pmatrix} = \begin{pmatrix} u_k^* & -v_k \\ -v_k^* & u_k \end{pmatrix} \begin{pmatrix} C_{-k} \\ C_k^\dagger \end{pmatrix}$$



電子ポーラロンと同じYukawa型

強結合で多体相関に効く

Fock空間の波動関数による解析：

一般的な BEC + 不純物粒子系の状態 (ポーラロン?)

$$|\psi(t)\rangle = \sum_p f_p^{(0)}(t) a_p^\dagger |0\rangle + \sum_{p,q_1} f_{p,q_1}^{(1)}(t) a_p^\dagger C_{q_1}^\dagger |0\rangle + \sum_{p,q_1,q_2} f_{p,q_1,q_2}^{(2)}(t) a_p^\dagger C_{q_1}^\dagger C_{q_2}^\dagger |0\rangle + \dots \quad (121)$$

となる。ゲージ変換した後では、不純物粒子の自由度を消せるので、

$$|\psi(t)\rangle = f^{(0)}(t) |0\rangle + \sum_{q_1} f_{q_1}^{(1)}(t) C_{q_1}^\dagger |0\rangle + \sum_{q_1,q_2} f_{q_1,q_2}^{(2)}(t) C_{q_1}^\dagger C_{q_2}^\dagger |0\rangle + \dots \quad (122)$$

これまで用いてきたコヒーレント状態変分法では

$$\begin{aligned} |\Psi(t)\rangle &= e^{\sum_q f_q(t) C_q^\dagger - h.c.} |0\rangle = e^{-\sum_q |f_q|^2/2} e^{\sum_q f_q(t) C_q^\dagger} |0\rangle \\ &= e^{-\sum_q |f_q|^2/2} \left[|0\rangle + \sum_{q_1} f_{q_1}(t) C_{q_1}^\dagger |0\rangle + \sum_{q_1,q_2} f_{q_1}(t) f_{q_2}(t) C_{q_1}^\dagger C_{q_2}^\dagger |0\rangle + \dots \right] \quad (123) \end{aligned}$$

LLP平均場理論の結果 1 :

B. Phonon間残留相互作用は無視

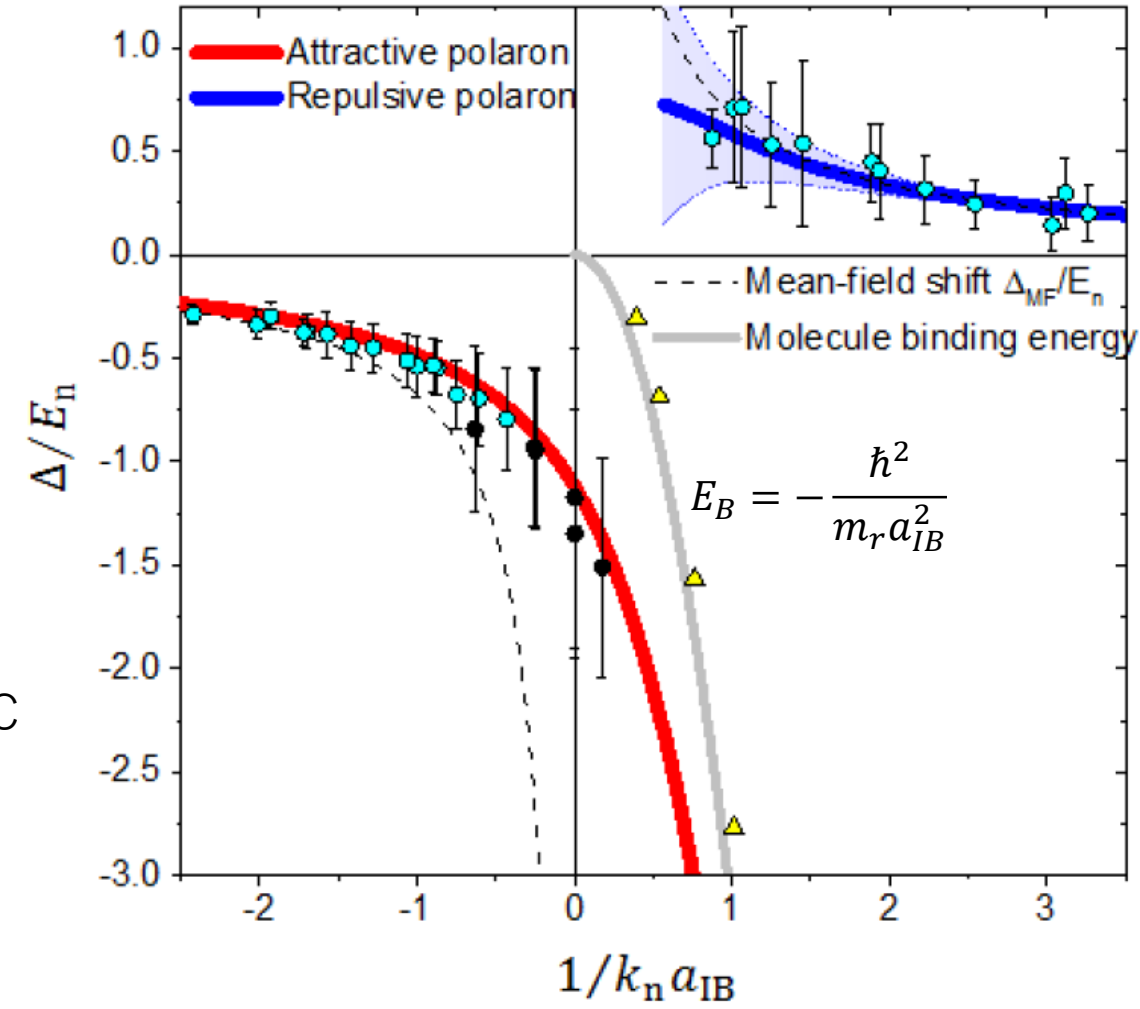
$$E_{pol}(\mathbf{P}) = \frac{(1 - \eta^2)P^2}{2M} + \frac{2\pi}{m_r} \frac{n}{a^{-1} - \tilde{a}^{-1}}$$

\uparrow
 in-medium shift by BEC

ここで $\tilde{a}^{-1} = \frac{2\pi}{m_r} V^{-1} \sum_q \left(\frac{2m_r}{q^2} - \frac{W_q^2}{\Omega_q} \right)$ は \mathbf{P} の関数である。

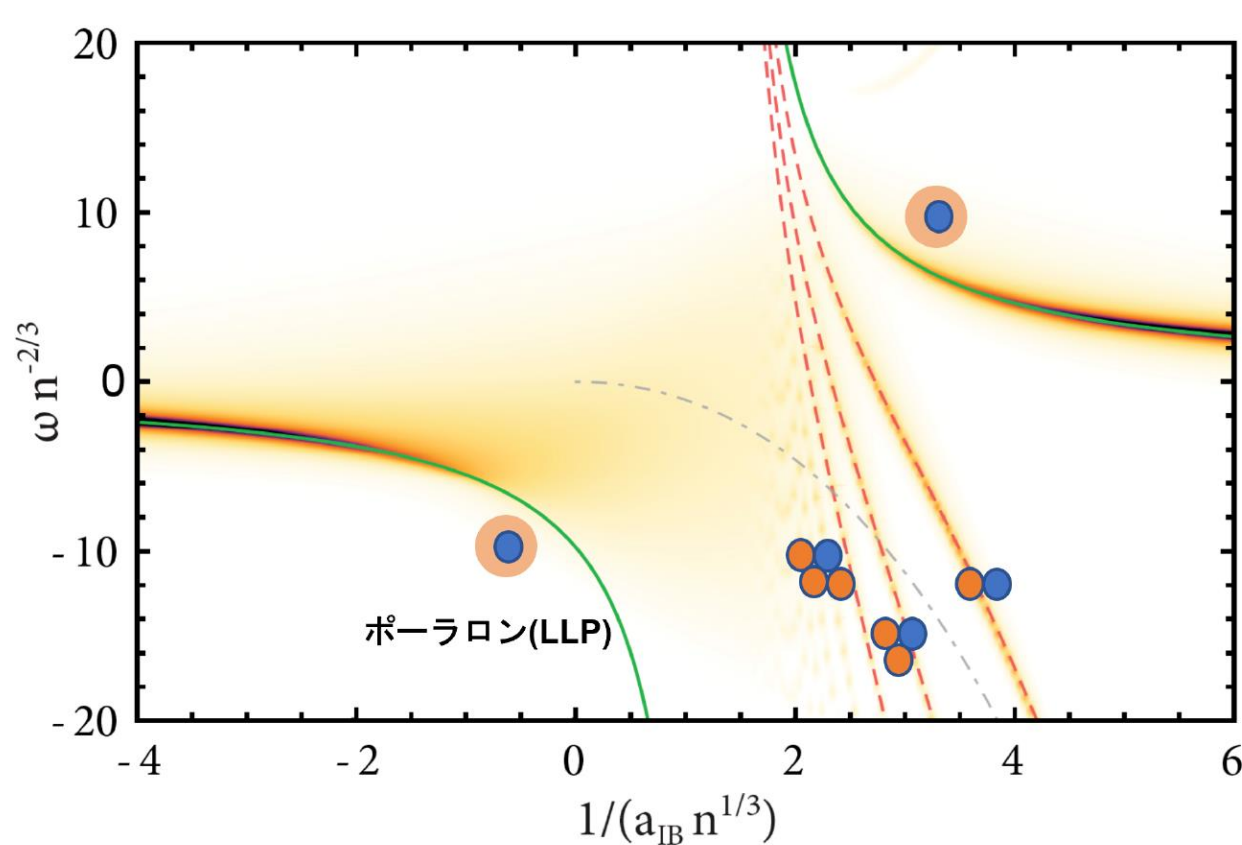
湯川型のみで求めた結果は、上記結果において $\mathcal{O}(a^2)$ と一致する:

$$\frac{2\pi}{m_r} \frac{n_0}{a^{-1} - \tilde{a}^{-1}} = \frac{2\pi n_0}{m_r} \frac{a}{1 - a\tilde{a}^{-1}} = \frac{2\pi n_0}{m_r} a \left(1 + a\tilde{a}^{-1} + \dots \right)$$



LLP、トランケーション方法の結果、例 1

e.g. Quantum Dynamics of Ultracold Bose Polarons
Yulia E. Shchadilova, Richard Schmidt, Fabian Grusdt, and Eugene Demler
Phys. Rev. Lett. 117, 113002 – Published 7 September 2016



- 有限質量の不純物粒子
- B.Phonon間残留相互作用は無視

In-medium 2-body problem
for B.phonon and Polaron:

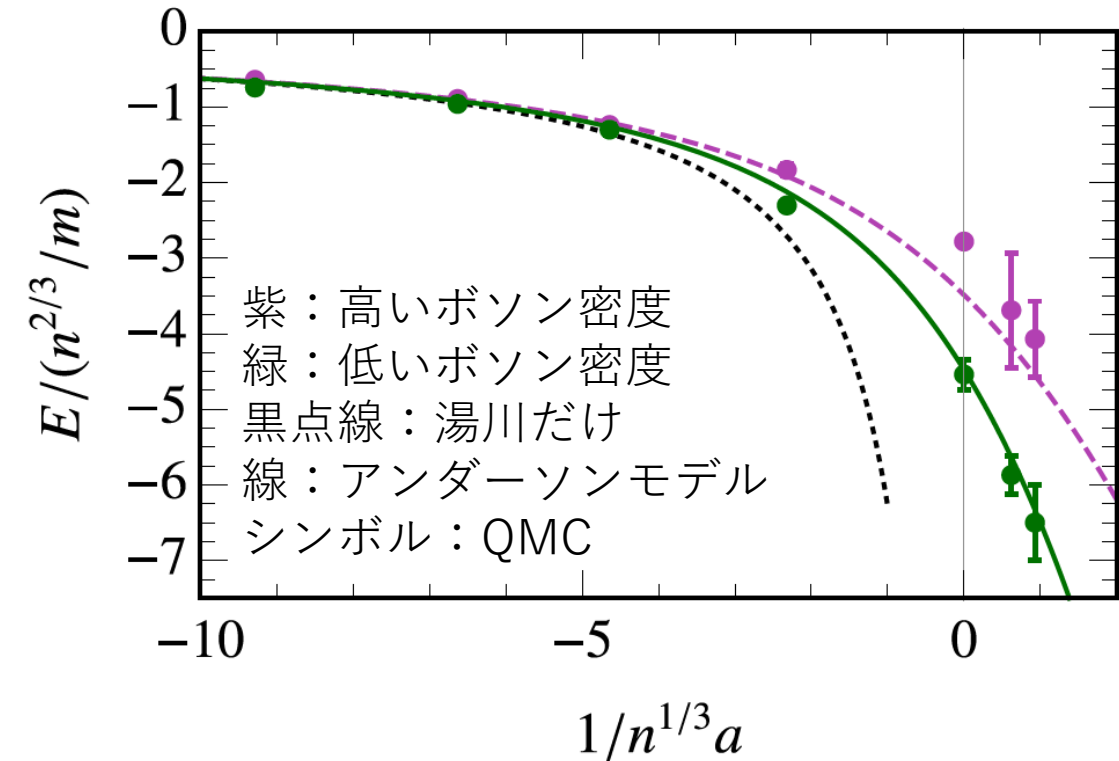
$$|\Psi'(t)\rangle = \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t) \hat{b}_{\mathbf{k}}^{\dagger} |\Psi_{\text{pol}}\rangle$$

:

- from ポーラロン to 不純物粒子と素励起の強い粒子相関
- ボソンの素励起はいくつでもくっつくのか？

QMC, トランケーション方法の結果、例 2

J. Levinsen, L. A. Peña Ardila, S. M. Yoshida, and M. M. Parish, PRL **127**, 033401(2021)



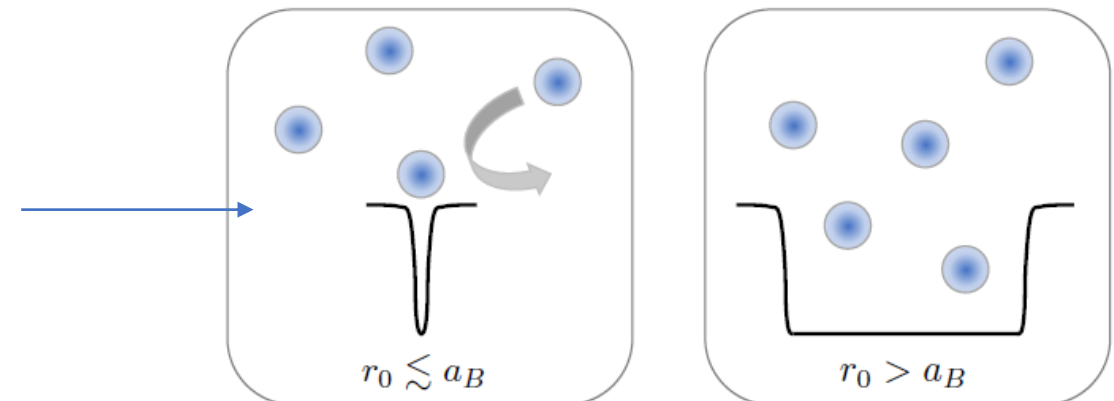
- ポテンシャル・レンジが十分小さければ、高々ボソン 1 個までで十分、2 個以上はブロックされる。

重い不純物粒子、Boson間相互作用あり：

$$|\Psi\rangle = \left(\alpha_0 + \sum_{\mathbf{k} \neq 0} \alpha_{\mathbf{k}} b_{\mathbf{k}}^\dagger + \frac{1}{2} \sum_{\mathbf{k}_1, \mathbf{k}_2 \neq 0} \alpha_{\mathbf{k}_1 \mathbf{k}_2} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger \dots \right) |\Phi\rangle,$$

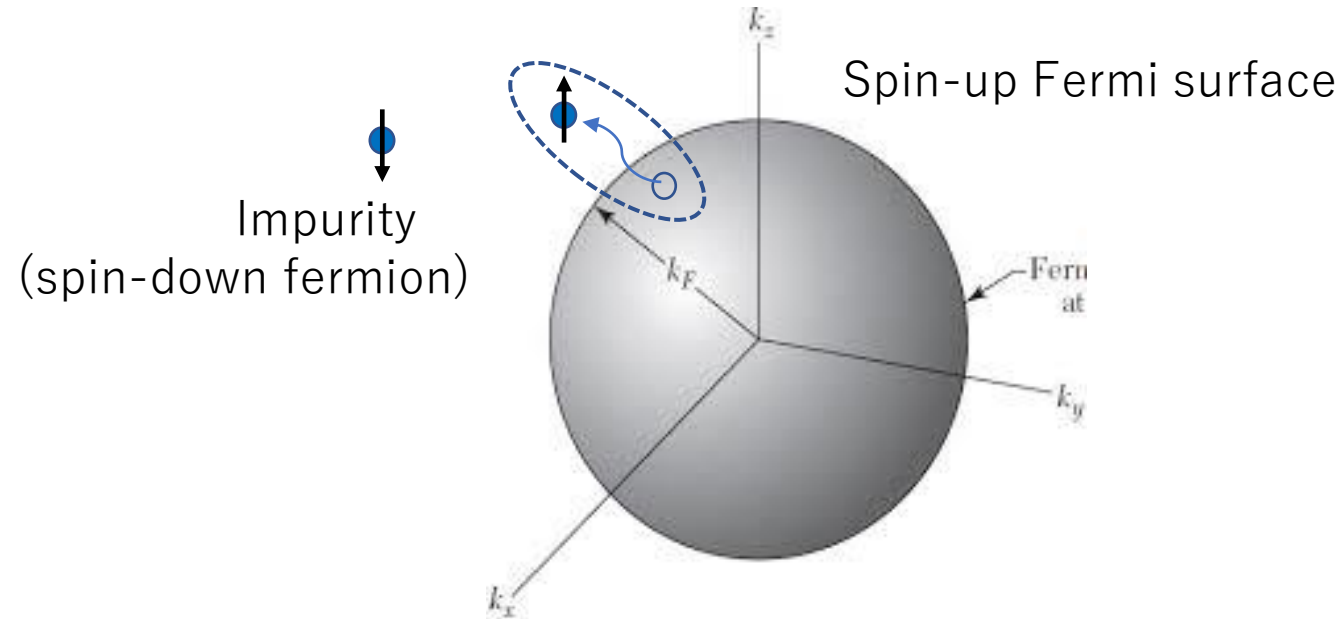
$$E = n \left[\frac{m}{2\pi a} + \sum_{\mathbf{k}} \left(\frac{1}{\epsilon_{\mathbf{k}} + G_{\mathbf{k}}} - \frac{1}{\epsilon_{\mathbf{k}}} \right) \right]^{-1}$$

Boson-boson correlation included



冷却原子系ポーラロン : Fermi polaron

Perfectly (quasi)spin-imbalanced Fermi gas (Chevy '06, others)



$$\hat{H} = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} \hat{a}_{\mathbf{k}\uparrow}^\dagger \hat{a}_{\mathbf{k}\uparrow} + E_{\mathbf{k}} \hat{a}_{\mathbf{k}\downarrow}^\dagger \hat{a}_{\mathbf{k}\downarrow} \right) + \frac{g_0}{V} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \hat{a}_{\mathbf{k}+\mathbf{q}\uparrow}^\dagger \hat{a}_{\mathbf{k}'-\mathbf{q}\downarrow}^\dagger \hat{a}_{\mathbf{k}'\downarrow} \hat{a}_{\mathbf{k}\uparrow}$$

$$|\psi\rangle = \phi_0 |\mathbf{p}\rangle_{\downarrow} |0\rangle_{\uparrow} + \sum_{q < k_F}^{k > k_F} \phi_{\mathbf{q}\mathbf{k}} |\mathbf{p} + \mathbf{q} - \mathbf{k}\rangle_{\downarrow} c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{q}\uparrow} |0\rangle_{\uparrow}$$

Fermi polaron experiment (^6Li hyperfine states)

From F. Scazza et al., PRL 118 (2017) 083602

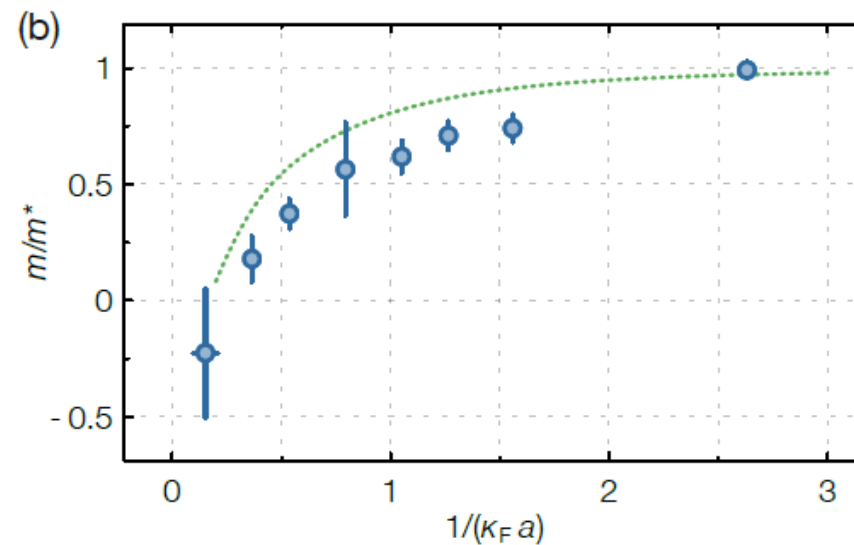
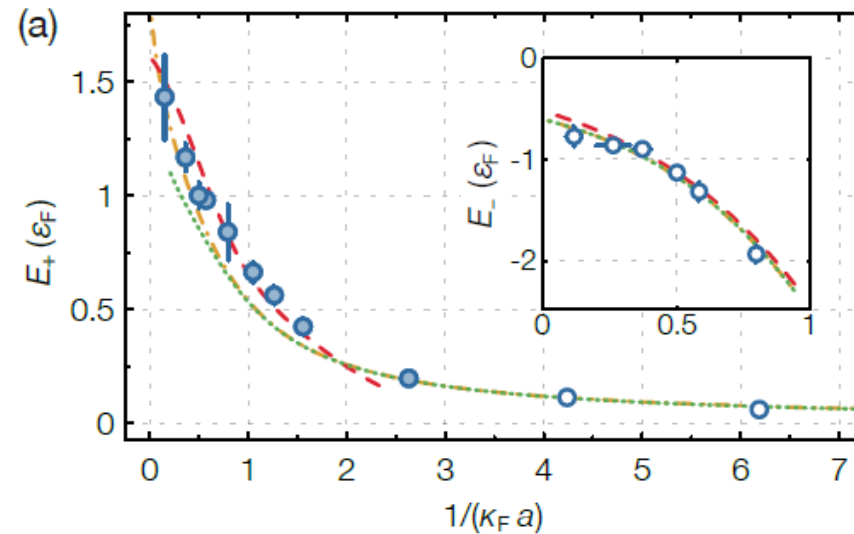
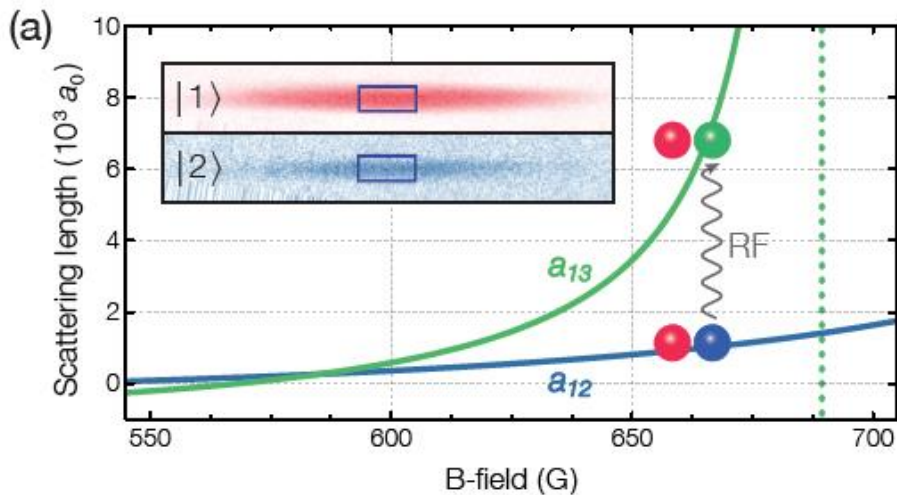
Zeeman states (quasi-spin):

|1> Majority (medium Fermi gas)

|2,3> Minority (impurity atoms)

$$N_2/N_1 \sim 0.05$$

不純物粒子-媒質粒子間の散乱長



変分法(Chevy+Dirac型) : single particle-hole excitation on majority Fermi surface

$$|\Psi\rangle = F_0|\psi_0\rangle + \sum_{k>,p<,s} F_{k,p}^s a_{s,k}^\dagger a_{s,p} |\psi_0\rangle,$$



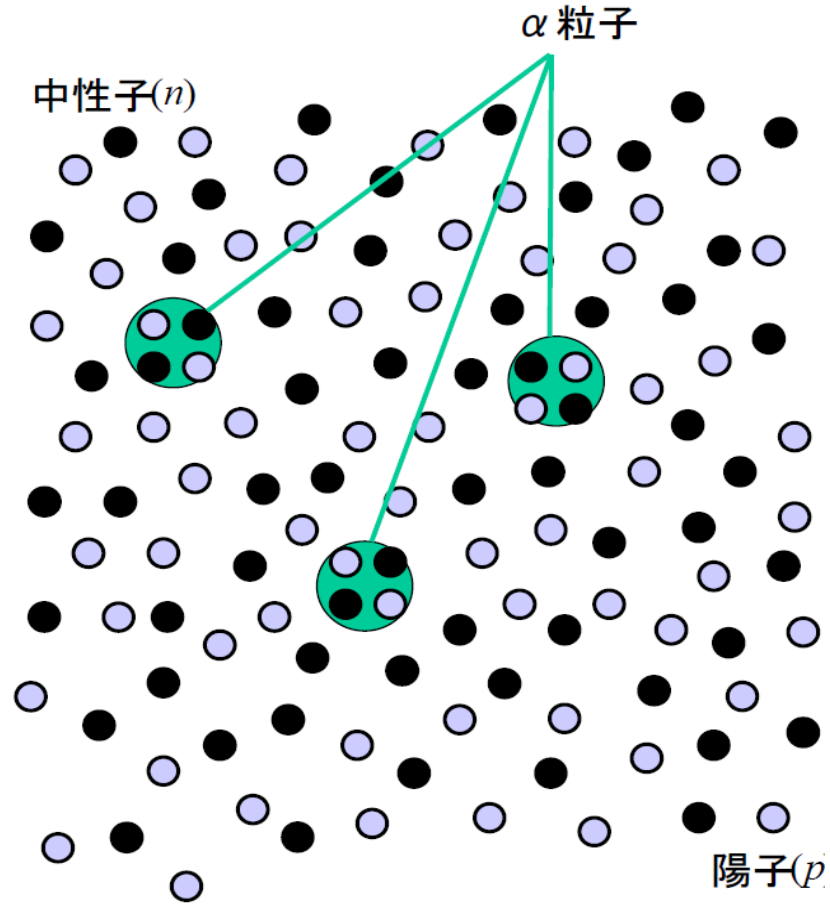
Fermi polaronのエネルギー分散式 (方程式)

$$\omega = \frac{P^2}{2M} + \Sigma(\omega, P),$$
$$\text{with } \Sigma(\omega, P) = \sum_{p<,s} \frac{1}{\frac{m_r}{2\pi a_s} - \sum_{k>} \left(\frac{1}{\omega - \Omega_{k,p;P}^s} + \frac{2m_r}{k^2} \right) - \sum_{q<} \frac{2m_r}{q^2}},$$

Attractive branch の例 : $E \cong -0.6 \varepsilon_E$ at unitarity limit (Chevy '06)

Repulsive branch の例：冷たい中性子物質中のアルファ粒子

(w/Horiuchi, Iida PRC '20)



(飯田さんより)

左図はアイソスピンのバランスしてる



アイソスピン完全非対称系を考える：

中性子密度 $\gg \gg$ 陽子密度



極限

中性子物質中にアルファ粒子1つ入れて、その準粒子描像（媒質効果）を調べる

真空中、 α -中性子の低エネルギー散乱のPhase shift:

s -wave 斥力的

p -wave ($P_{3/2}$) resonance

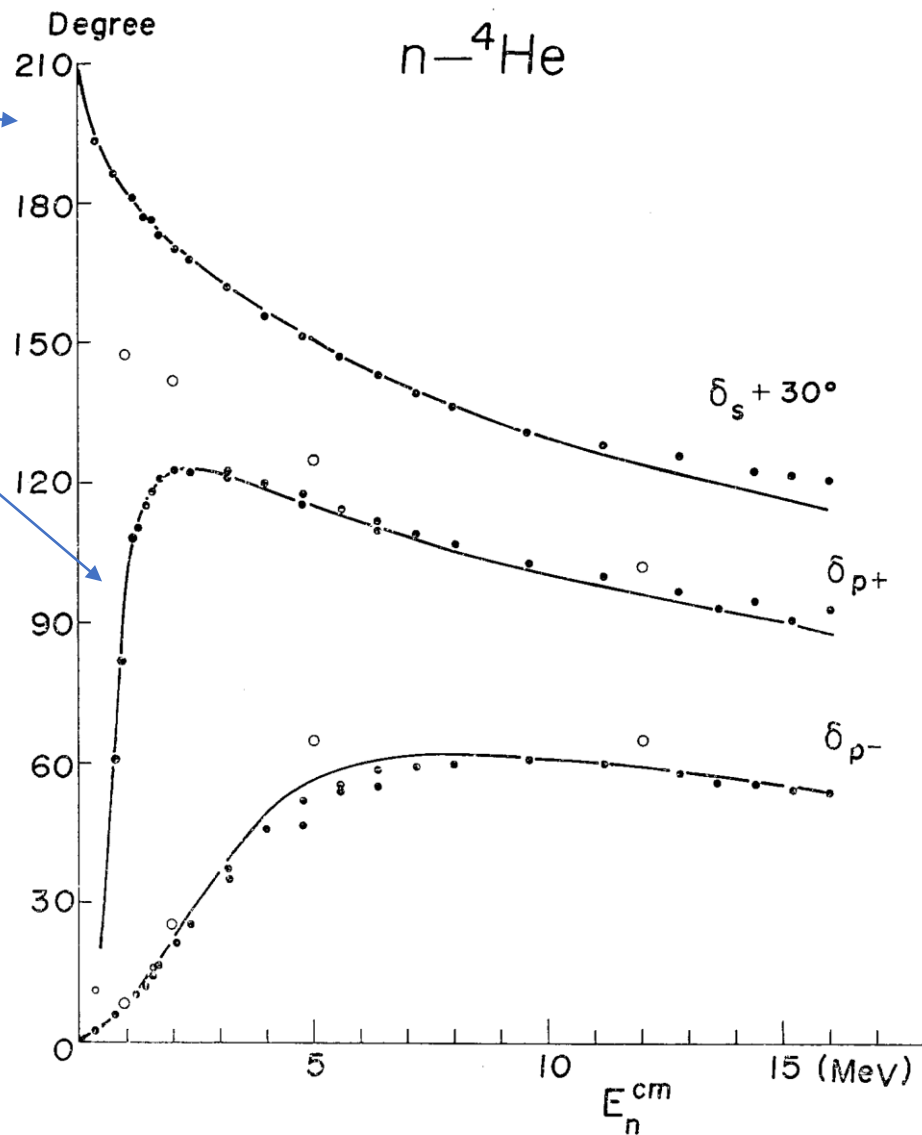
共鳴散乱エネルギーを
中性子フェルミエネルギーに換算すると、

$$k_F < 0.25 \text{ fm}^{-1} \rightarrow \rho < 0.003\rho_0,$$

Progress of Theoretical Physics, Vol. 61, No. 5, May 1979

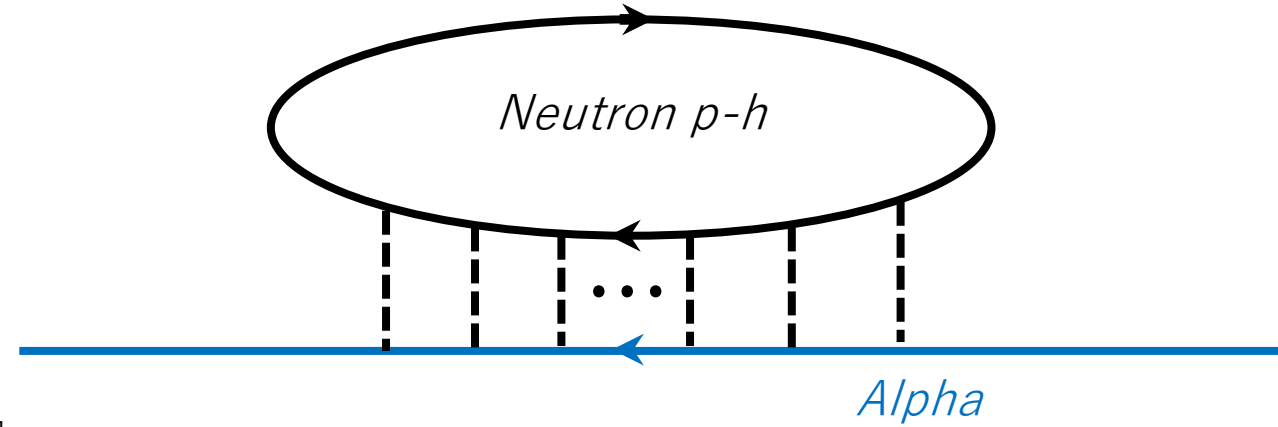
Microscopic Study of Nucleon- ^4He Scattering and Effective Nuclear Potentials

Hiroyuki KANADA, Tsuneo KANEKO, Shinobu NAGATA*
and Morikazu NOMOTO



Chevy 変分法(+ analytic cont. upper half plane) =

Self-energy by a ladder type 2-body correlation



中性子物質中におけるアルファ粒子の準粒子描像

$$G^R(\omega, P) = \frac{1}{\omega + i0 - \frac{P^2}{2M} - \Sigma(\omega + i0, P)} \sim \frac{Z_P}{\omega - E_P + i\Gamma_P},$$

エネルギー分散式

$$E_P = \frac{P^2}{2M} + \text{Re}\Sigma(E_P + i0, P),$$

準粒子留数

$$Z_P = \left[1 - \text{Re} \frac{\partial \Sigma(\omega + i0, P)}{\partial \omega} \Big|_{\omega=E_P} \right]^{-1}$$

崩壊幅

$$\Gamma_P = -Z_P \text{Im}\Sigma(E_P + i0, P),$$

有効質量

$$M^* = \frac{M}{Z} \left[1 + M \frac{\partial^2 \text{Re}\Sigma(\omega + i0, P)}{\partial P^2} \Big|_{\omega=E, P=0} \right]^{-1}$$

数値計算：入力パラメータ

- Scattering length from alpha-neutron Low energy scattering

H. Kanada, T. Kaneko, S. Nagata, and M. Nomoto, Prog. Theor. Phys. **61**, 1327 (1979).

Reproduce the experimental phase shift by 堀内さん

散乱長： $a_{\text{ref}} = 2.64 \text{ fm}$ 有効レンジ： $r_0 = 1.43 \text{ fm}$
(repulsive branch)

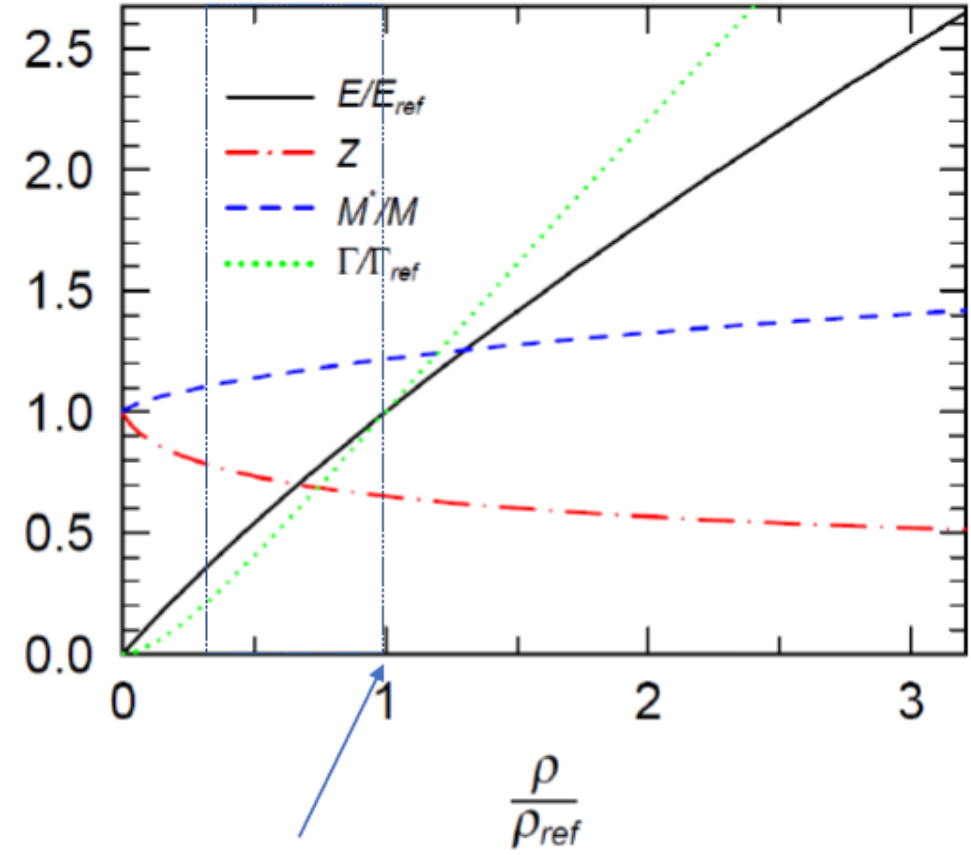
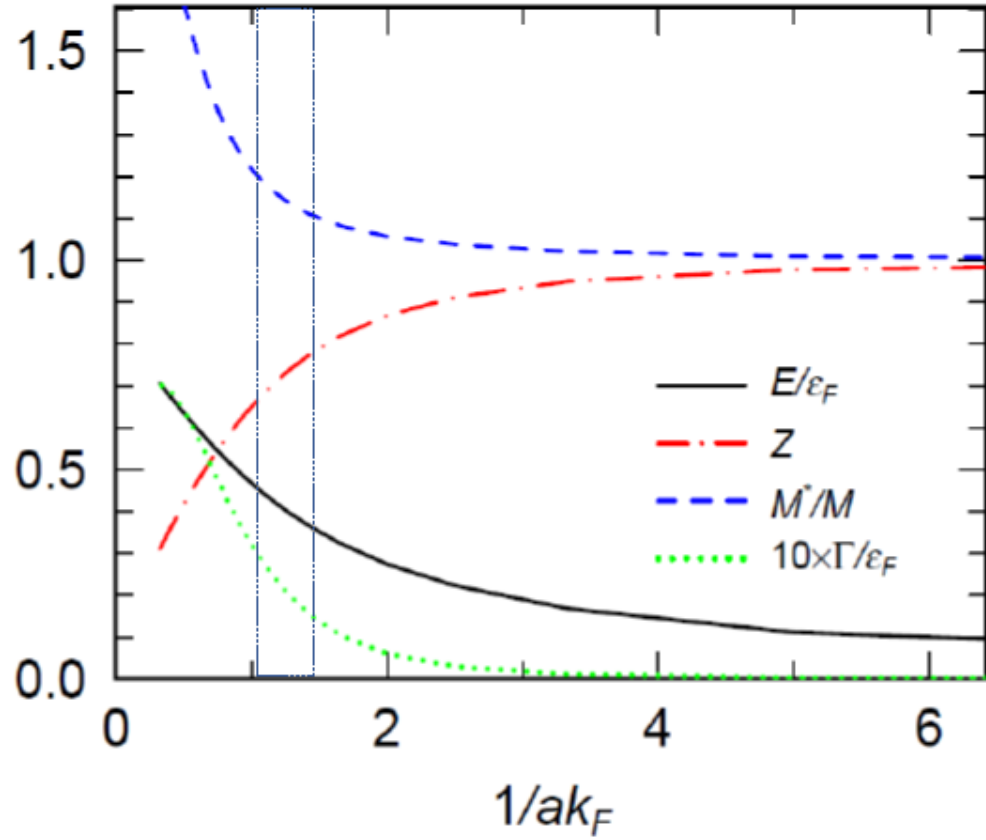
- 中性子物質の密度(\sim supernova core, $<$ neutron skin of heavy nuclei)

$$\rho_{\text{ref}} = 0.01 \rho_0 \quad \rho_0 = 0.16 \text{ fm}^{-3} \text{ is the normal nuclear density.}$$

$$a_{\text{ref}} k_{F\text{ref}} = 0.95 \simeq 1, \quad r_0 k_{F\text{ref}} = 0.51$$

だいたい強結合領域

数値計算結果：s波近似のために低密度領域に制限される



中性子密度： $\rho = 0.003 \sim 0.01 \rho_0$ (縦破線範囲)

$\sim 0.01 \rho_0$

通常核密度： $\rho_0 = 0.16 \text{ fm}^{-3}$

- E : 静止エネルギー < 1.26 MeV
- Z : 留数 > 0.65
- M^*/M : 有効質量比 1.1 ~ 1.2
- Γ : 崩壊幅 < 0.0086 MeV

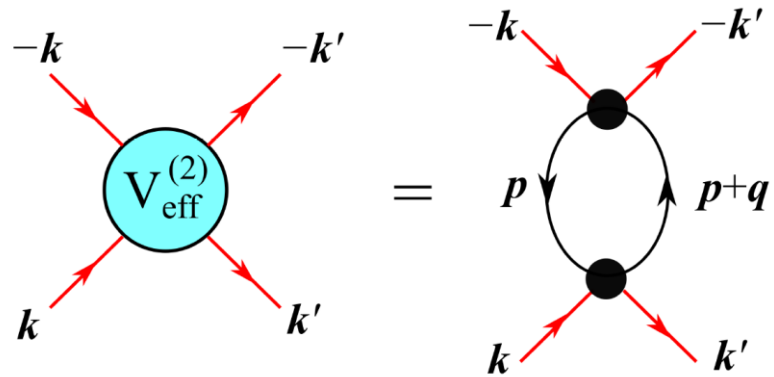
⇒ α 粒子の準粒子描像、悪くない。

有効質量10%~20%増。

準粒子 α の応用例：2 α , 3 α states in cold neutron matter,

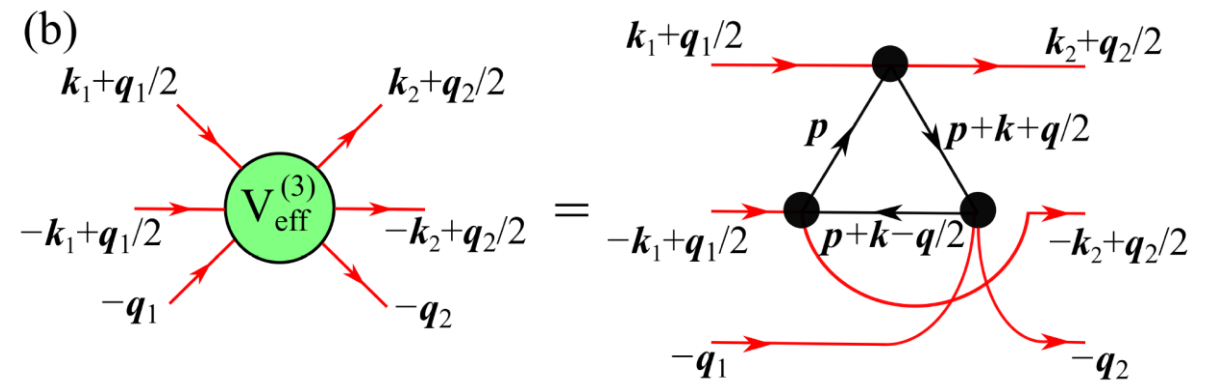
[Hajime Moriya](#), [Hiroyuki Tajima](#), [Wataru Horiuchi](#), [Kei Iida](#), [Eiji Nakano](#)
[arXiv:2005.13196](#), [arXiv:2106.14469](#),

媒質中性子 1 ループ媒介の 2,3 アルファ粒子間有効相互作用 (田島さん@東大)



$$V_{\text{eff}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{m}{8\pi^3 \hbar^2} \left(\frac{2\pi \hbar^2 a}{m_r} \right)^2 \times \frac{(2k_F r) \cos(2k_F r) - \sin(2k_F r)}{r^4},$$

有限レンジ、ほぼ引力：RKKY相互作用



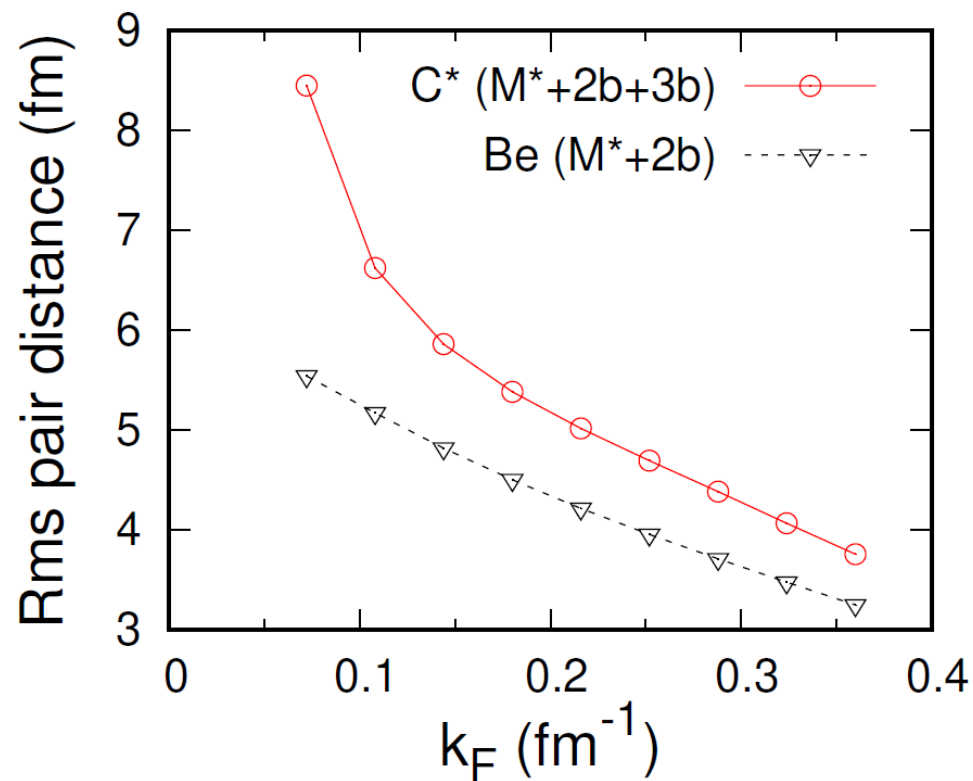
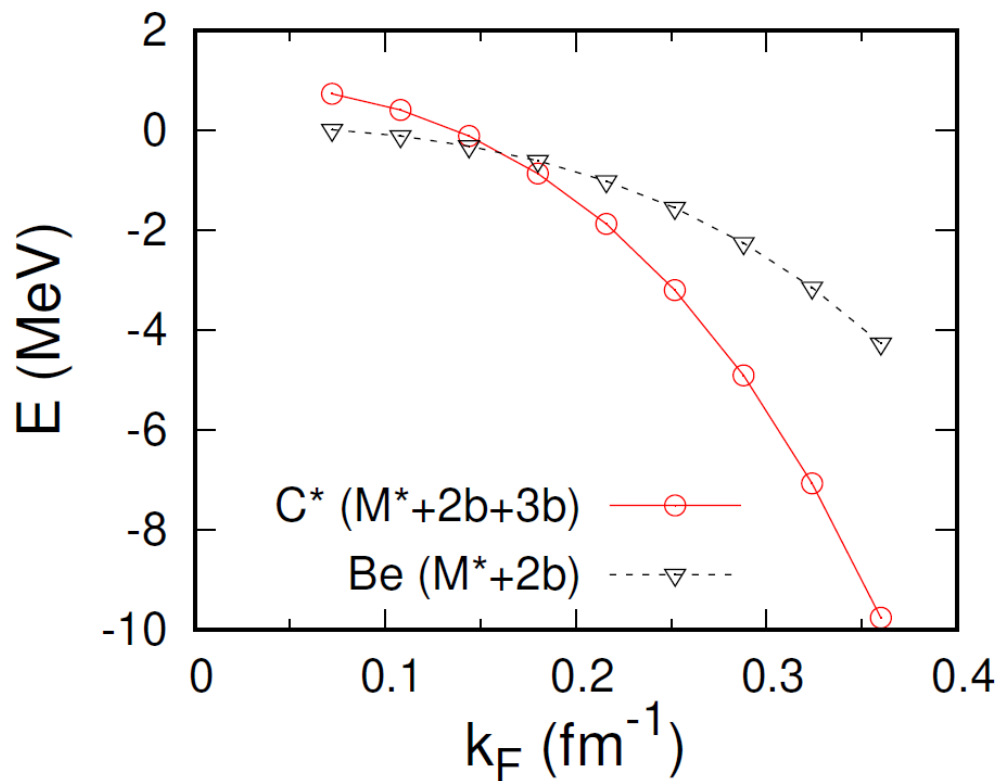
$$V_{\text{eff}}^{(3)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \frac{m^2}{\pi^2 \hbar^4 k_F} \left(\frac{2\pi \hbar^2 a}{m_r} \right)^3 \delta(\mathbf{x}_1) \delta(\mathbf{x}_2)$$

ほぼ斥力：短距離の 3 体ポテンシャルエネルギーに換算

媒質中のアルファ少数系の有効ハミルトニアン：

$$H = \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2M^*} - T_{\text{cm}} + \sum_{i<j=1}^3 \left[U_{ij}^{(2)} + V_{\text{eff};ij}^{(2)} \right] + U^{(3)} + V_{\text{eff}}^{(3)}$$

結果：2アルファ状態（Be）、3アルファ状態（Hoyle状態）エネルギー vs 中性子フェルミ運動量（森谷さん@北大）



考察： M^* は10%~20%増加 \Rightarrow ゆらぎ抑制局在化で束縛しやすくなる傾向。 $k_F \sim 0.1 \text{ fm}^{-1}$ くらいで束縛。

まとめ

1. 電子ポーラロン … LLP理論(素励起の平均場理論)
2. 冷却原子系ポーラロン (s wave)

Bose polaron … 弱・中間結合領域 \Rightarrow LLP 理論

強結合領域 \Rightarrow 少数粒子相関(\neq ポーラロン)

Fermi polaron … 引力枝の例: Chevy ansatz (1p-1h) = 梯子近似の自己エネルギー

斥力枝の例: 核物質中の α 粒子、および、 2α 、 3α 状態

3. 関連研究… ● Helium 5 (alpha-neutron P_{3/2} resonance-molecule transition) in cold neutron matter ([arXiv:2207.13907](https://arxiv.org/abs/2207.13907), [Hiroyuki Tajima](#), et.al) 、
● Bose polaron in trap ([arXiv:1610.00421](https://arxiv.org/abs/1610.00421), [arXiv:1904.08799](https://arxiv.org/abs/1904.08799), [arXiv:1901.02847](https://arxiv.org/abs/1901.02847)) 、
● Fermi polaron in dipolar (anisotropic) medium([arXiv:2010.15558v2](https://arxiv.org/abs/2010.15558v2))、 . . .

‘Bosonic’ Anderson model :

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \nu_0 d^{\dagger} d + \lambda \sum_{\mathbf{k}} (d^{\dagger} b_{\mathbf{k}} + b_{\mathbf{k}}^{\dagger} d) + \frac{U}{2} d^{\dagger} d^{\dagger} d d, \quad (\text{S30})$$

where we assume that the operator d is bosonic and we take the limit $U \rightarrow +\infty$ at the end of the calculation.

全部ボソン、 b -ボソンは状態を変えて d -ボソン（不純物にとらわれた b : dimerを想定）になり得る、
が、 d 状態にあるボソンは、たかだた1粒くらい（ $U \rightarrow \infty$; blockade効果に類似）。

Resonance-to-bound transition of ${}^5\text{He}$ in neutron matter and its analogy with heteronuclear Feshbach molecule

[Hiroyuki Tajima](#), [Hajime Moriya](#), [Wataru Horiuchi](#), [Kei Iida](#), [Eiji Nakano](#), [arXiv:2207.13907](#)

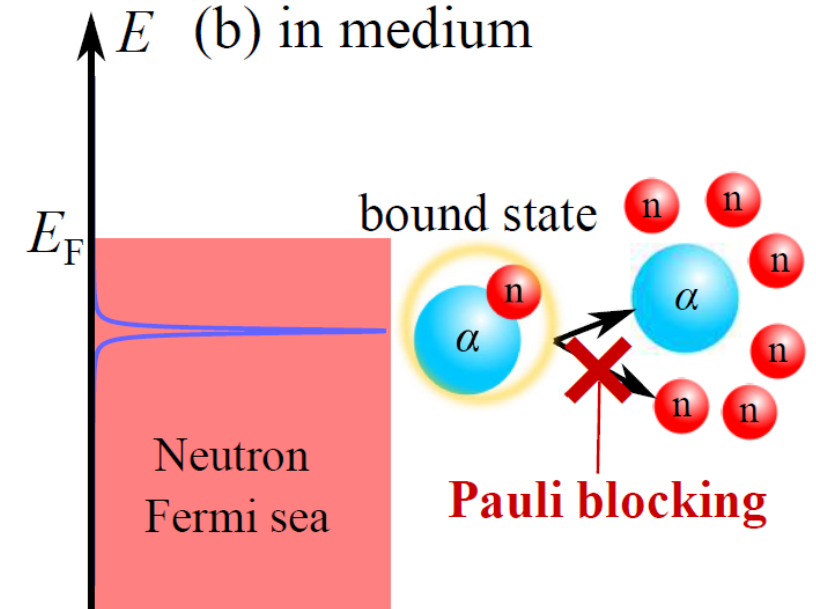
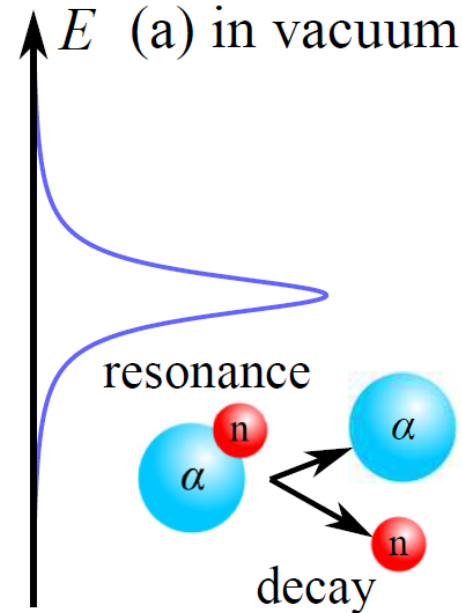
Two channel model: Neutron + Alpha \Leftrightarrow Helium 5 ($J=3/2$)

$$H = H_{0,\nu} + H_{0,\alpha} + H_{0,\Phi} + V,$$

$$H_{0,\nu} = \sum_{\mathbf{k}} \sum_{s_z = \pm 1/2} \xi_{\mathbf{k},\nu} \nu_{\mathbf{k},s_z}^\dagger \nu_{\mathbf{k},s_z},$$

$$H_{0,\alpha} = \sum_{\mathbf{q}} \xi_{\mathbf{q},\alpha} \alpha_{\mathbf{q}}^\dagger \alpha_{\mathbf{q}},$$

$$H_{0,\Phi} = \sum_{\mathbf{P}, J, J_z} (\xi_{\mathbf{P},\Phi, J} + E_{\Phi, J}) \Phi_{\mathbf{P}, J, J_z}^\dagger \Phi_{\mathbf{P}, J, J_z}$$



$$V = \sum_{J=1/2, 3/2} \sum_{J_z, s_z, m} \sum_{\mathbf{P}, \mathbf{k}} \left(k g_{\mathbf{k}, J} \sqrt{\frac{4\pi}{3}} Y_m^{\ell=1}(\hat{\mathbf{k}}) \langle 1, m; 1/2, s_z | J, J_z \rangle \Phi_{\mathbf{P}, J, J_z}^\dagger \nu_{\mathbf{k}+\mathbf{P}/2, s_z} \alpha_{-\mathbf{k}+\mathbf{P}/2} + \text{h.c.} \right)$$

T matrix in medium *for* Helium 5 in $J=3/2$ channel = sum of bubbles

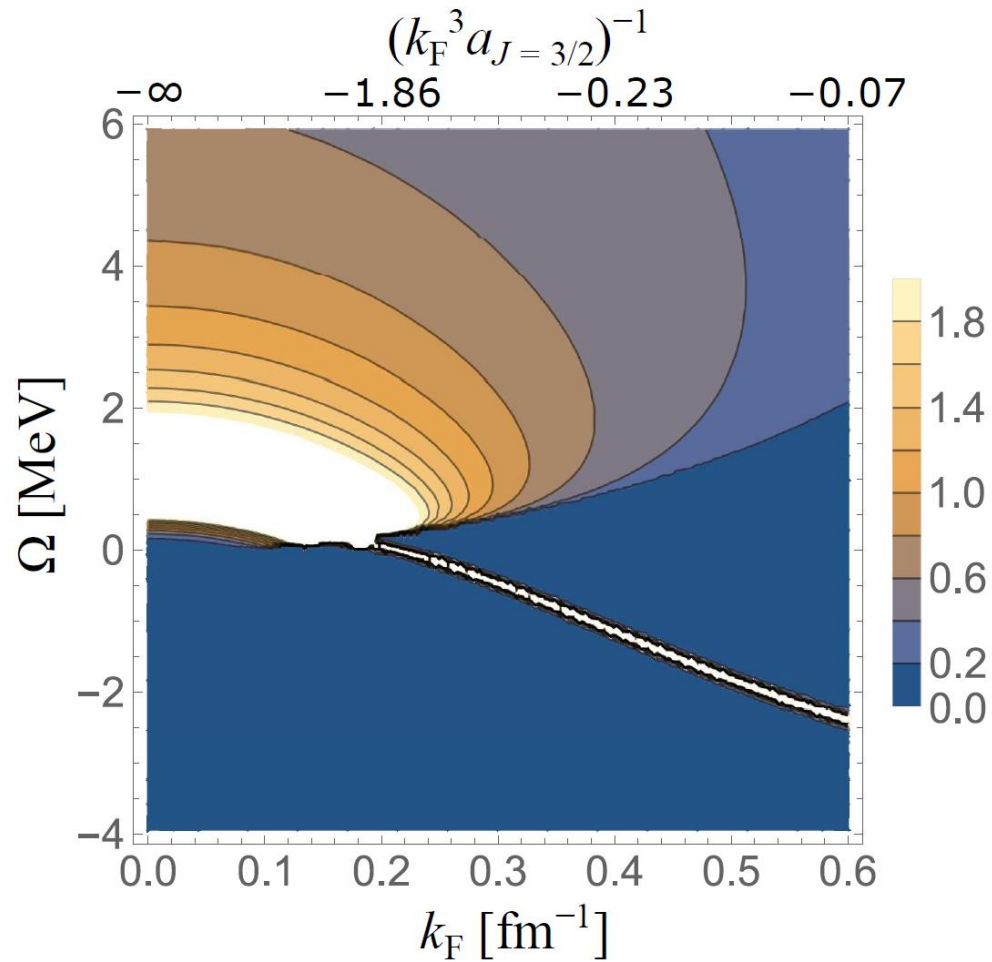
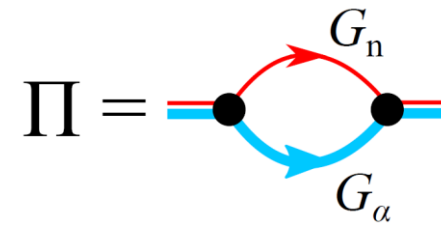


FIG. 4. Contour plot of the $J^\pi = 3/2^-$ ${}^5\text{He}$ spectral function $A_{J=3/2}(\Omega)$ in the unit of fm in the plane of the energy Ω and the neutron Fermi momentum k_F . On the top of the figure, we show the value of the dimensionless coupling parameter $(k_F^3 a_{J=3/2})^{-1}$ in terms of the p -wave scattering volume, which is frequently used in cold atomic physics.

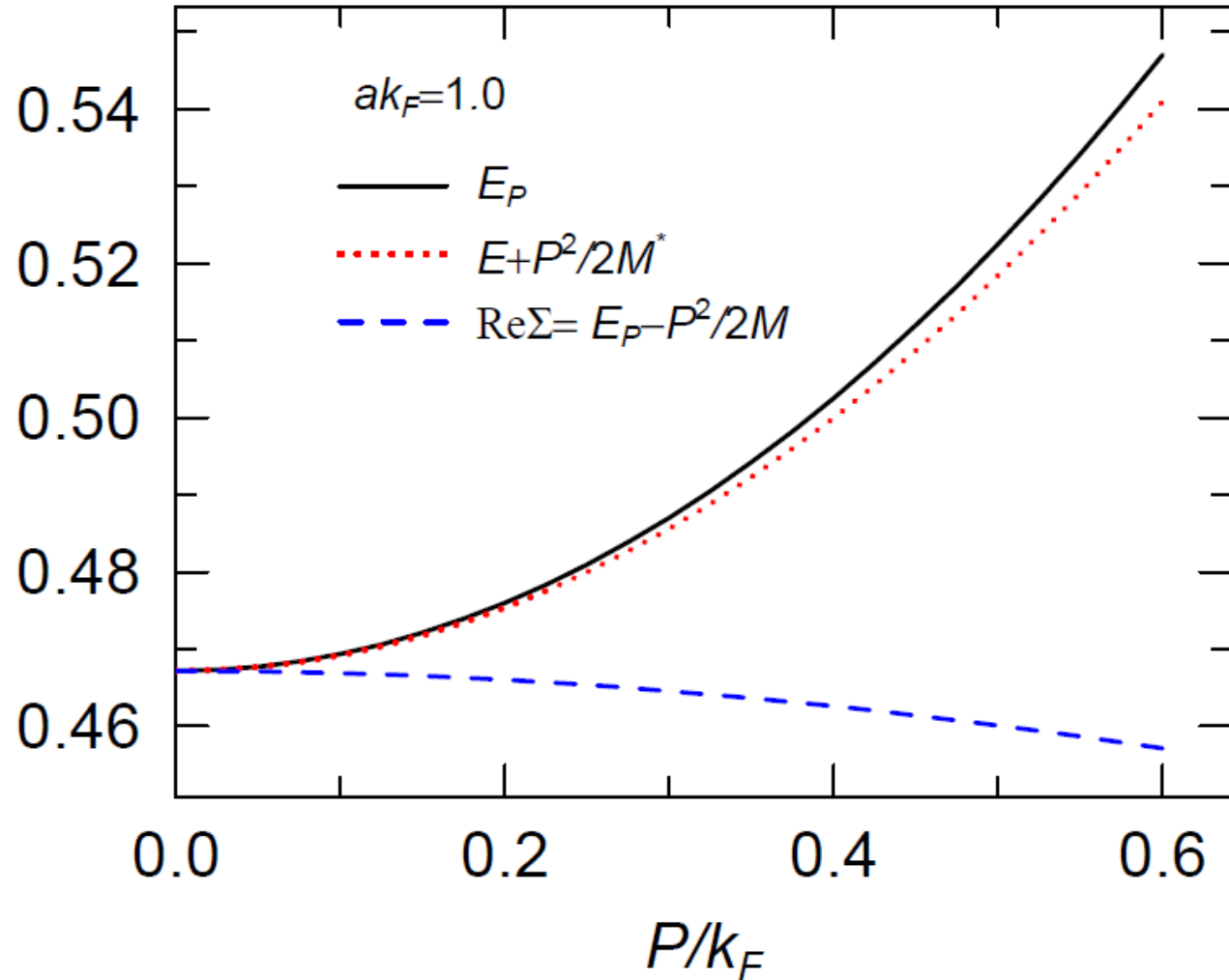
Point like alpha particle valid up to $k_F = 0.67$ fm
(Pauli blocking effect inside alpha/binding energy)

T. Sogo, G. Roepke, and P. Schuck,
PRC 82, 034322 (2010).

計算結果 1 エネルギー分散関係

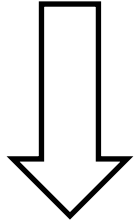
$$\frac{M}{m} = 4, T = 0, \rho_{ref} = 0.01\rho_0$$

エネルギー分散関係



Single polaron :

$$U = STS^{-1} = e^{Q(r)} = \exp \left[\sum_{k \neq 0} (e^{-ik \cdot r} f_k C_k^\dagger - f_k^* e^{ik \cdot r} C_k) \right]$$



P dependence
comes from the
state

2nd quantization (in *not unique* way) :

$$Q(r) \rightarrow \sum_i Q(r_i) = \int_r \hat{n}_f(r) Q(r)$$

$$\hat{n}_f(x) = \psi^\dagger(x) \psi(x) \quad \psi(x) = V^{-1/2} \sum_p e^{ipx} a_p$$

$$U = e^S = \exp \left[\sum_{q,P} (f_{q;P} a_{P-q}^\dagger a_P C_q^\dagger - f_{q;P}^* a_P^\dagger a_{P-q} C_q) \right]$$

P dependence
assigned by hand

U Transformation of fermion :

$$U^{-1}a_P U \simeq \frac{1}{V} \sum_Q \int_{x,y} e^{-iP \cdot x + iQ \cdot y} \langle x | e^{\hat{A}} | y \rangle a_Q$$

$$= a_p + \dots \quad \text{not a shift operation}$$

interactions among fermions ←

$$\langle x | e^{\hat{A}} | y \rangle = \delta(x - y) + A(x, y) + \frac{1}{2!} \int_z A(x, z) A(z, y) + \dots$$

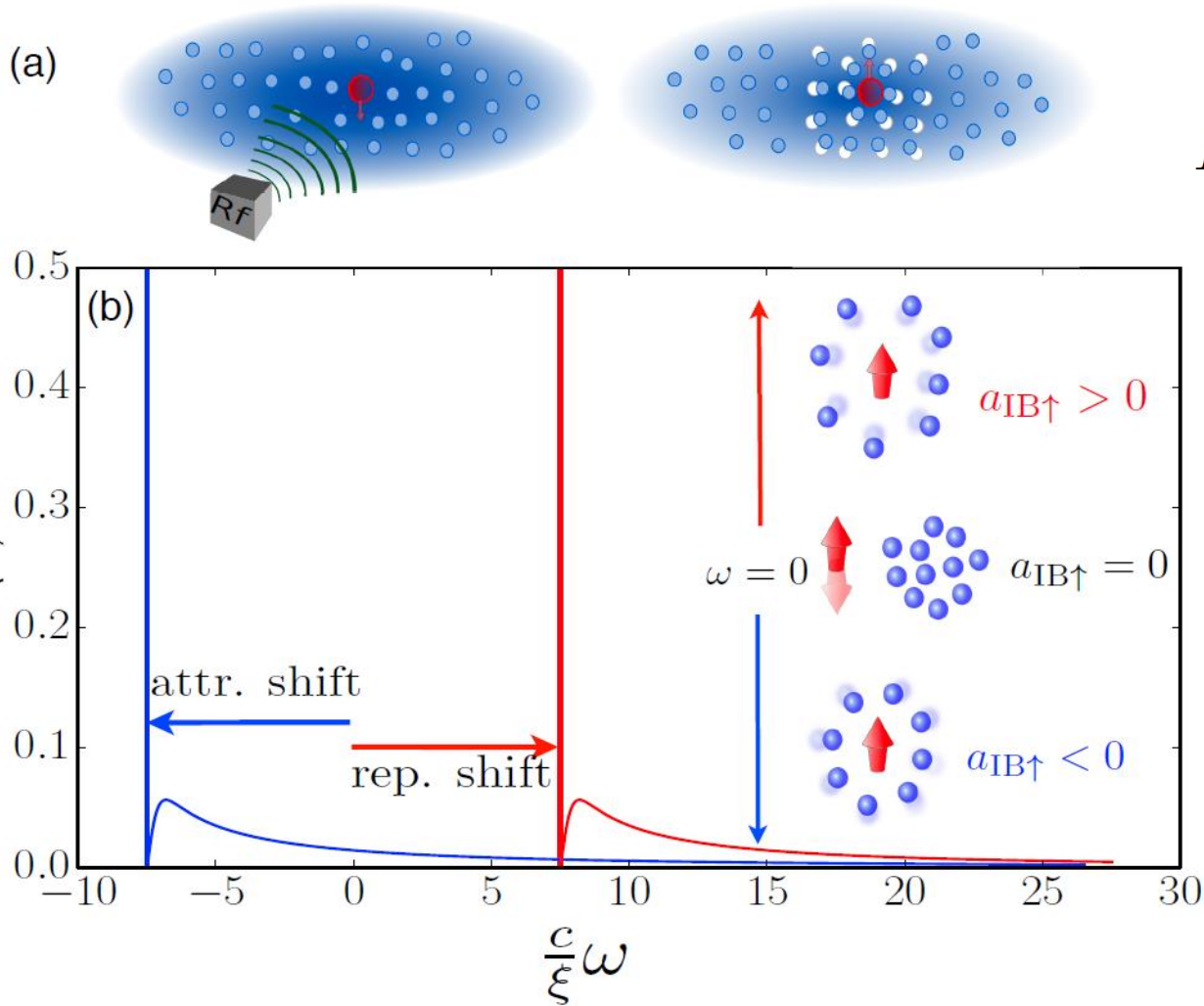
$$A(x, y) = \frac{1}{V} \sum_{k,0} e^{i(Q-k)x} e^{-iQy} \{ f_{k;Q} C_k^\dagger - f_{-k;Q-k} C_{-k} \}$$

$$+ \frac{1}{V} \sum_{k,P,Q} e^{i(Q-k)x} e^{-iQy} \{ f_{k;Q} f_{k;P} - f_{-k;Q-k} f_{-k;P-k} \} a_P^\dagger a_{P-k}$$

U Transformation of phonon :

$$U^{-1}C_q U = C_q + \sum_P f_{q;P} a_{P-q}^\dagger a_P$$

RF spectroscopy : response of the pseudo-spin flip to ex. pert.



$$I(p, \omega) = \sum_n |\langle n_{\uparrow p} | \hat{V}_{\text{RF}} | i_{\downarrow p} \rangle|^2 \delta(\omega - (E_{n\uparrow} - E_{i\downarrow}))$$

$$I(p, \omega) = \text{Re} \frac{1}{\pi} \int_0^\infty dt e^{i\omega t} A_p(t)$$

$$A_p(t) = e^{iE_{i\downarrow}t} \langle i_{\uparrow p} | e^{-i(\mathcal{H}_b + \mathcal{H}_I + H_{\text{int}\uparrow})t} | i_{\uparrow p} \rangle$$

$$\begin{aligned}
I(p, \omega) &= \text{Re} \frac{1}{\pi} \int_0^\infty dt e^{i(\omega + E_{i\downarrow})t} \langle i_{\uparrow p} | e^{-i\tilde{\mathcal{H}}_{\uparrow} t} | i_{\uparrow p} \rangle \\
&= \sum_m \text{Re} \frac{1}{\pi} \int_0^\infty dt e^{i(\omega + E_{i\downarrow} - E_{m\uparrow})t} |\langle m_{\uparrow p} | i_{\uparrow p} \rangle|^2 \\
&= \text{Re} \frac{1}{\pi} \int_0^\infty dt e^{i(\omega - \Delta_0)t} Z_{\uparrow\downarrow} \times \\
&\quad \left(1 + \sum_{m \neq 0} e^{i\Delta_m t} \frac{|\langle m_{\uparrow p} | i_{\uparrow p} \rangle|^2}{Z_{\uparrow\downarrow}} \right) \quad (30)
\end{aligned}$$

with

$$Z_{\uparrow\downarrow} = |\langle 0_{\uparrow p} | i_{\uparrow p} \rangle|^2, \Delta_m = E_{m\uparrow} - E_{i\downarrow}, \quad (31)$$

$S(t)$ の計算はかなり難しいので、ここではこれまでのコヒーレント状態変分法を時間依存に拡張して近似する：系の時間発展を

$$|\Psi_{\text{pol}}(t)\rangle = e^{\sum_k f_k(t) C_k^\dagger - h.c.} |0\rangle$$

と近似すると、

$$\begin{aligned} S(t) &= \langle BEC | e^{-iHt} | BEC \rangle \\ &\simeq \langle 0 | \Psi_{\text{pol}}(t) \rangle \\ &= e^{-\sum_k |f_k(t)|^2 / 2} \end{aligned}$$

と書ける。つまり、時間発展のヒルベルト空間をまたコヒーレント状態で張られる空間に射影している。 $f_k(t)$ の振る舞いがわかれば、いろいろとわかる。 $f_k(t)$ の運動方程式は、Dirac 型時間依存変分法をもちいる：

$$L = \langle \Psi_{\text{pol}}(t) | i\partial_t - H | \Psi_{\text{pol}}(t) \rangle, \quad \rightarrow \frac{\delta L}{\delta f_k} = 0$$

$$if_k = g\sqrt{n}W_k + \left(\omega_k + \frac{k^2}{2m}\right) f_k + \frac{g}{2} \left[W_k \sum_{k'} W_{k'} (f_{k'} + f_{k'}^*) + W_k^{-1} \sum_{k'} W_{k'}^{-1} (f_{k'} - f_{k'}^*) \right]$$

運動方程式の解 $f_k(t)$ は、エネルギー変分解と違って、**複素数**（エネルギー保存からの要請）になることに注意されたい。

不純物 + 少数ボソン系の結果：

普通はこんな関数でフィットできそうだが、

$$E = -N_B \varepsilon_b + U N_B (N_B - 1) / 2,$$

$$-\varepsilon_b = -1/2 m a^2 \quad (\text{不純物 + ボソン 1 粒の束縛エネルギー})$$

$$U \sim a_B / m r_0^3$$

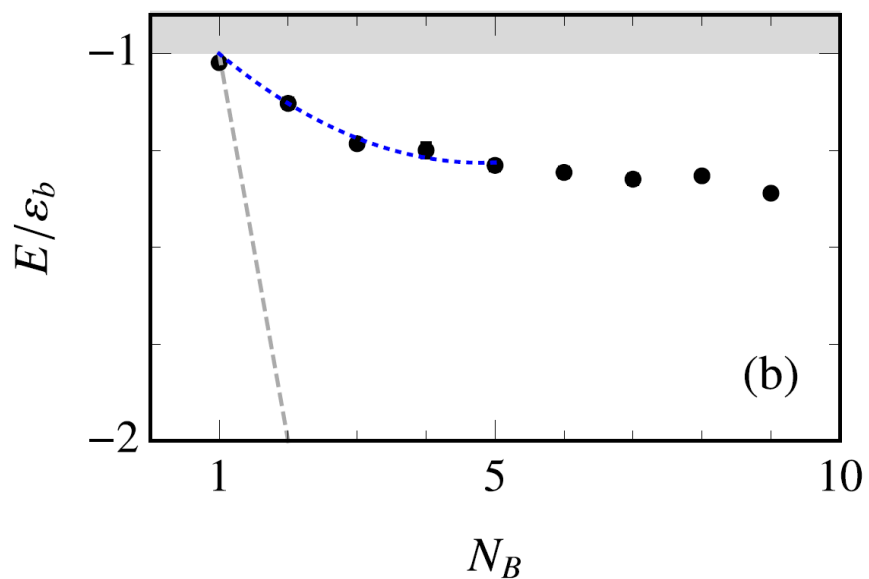
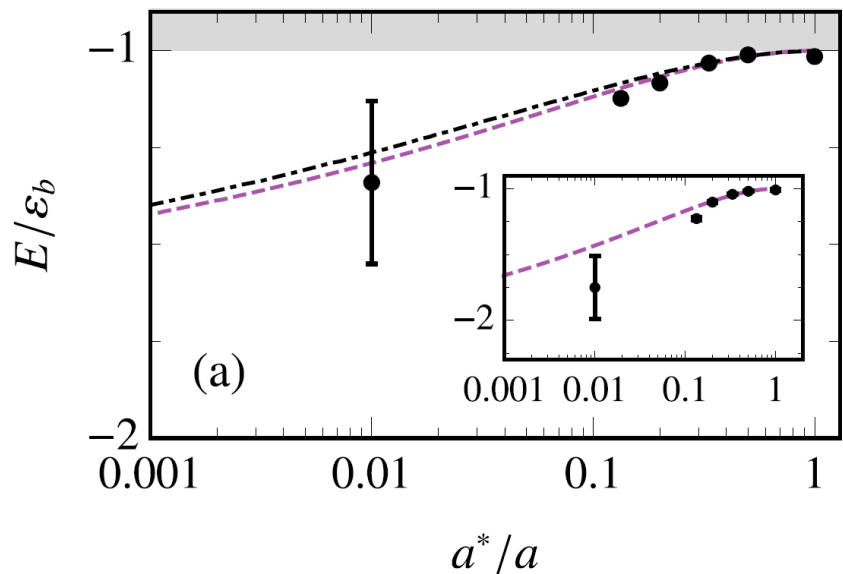
以下の方がうまくいく、

$$E = -\varepsilon_b - (N_B - 1) \varepsilon_T + \frac{U}{2} (N_B - 1) (N_B - 2), \quad (2)$$

不純物 + ボソン 2 粒の束縛エネルギー $\ll \varepsilon_b$
(Dimer + ボソン 1 粒)

$$U \sim a_B / m a^3$$

先行研究から、Dimer-ボソン・ポテンシャルの有効レンジ $\sim a$



$a > a^*$: 不純物 + ボソン 1 つ以上の束縛状態ができる閾値

FIG. 2. (a) Trimer energy as a function of inverse scattering length obtained from QMC (black circles), bosons with attractive contact interaction (black dot dashed line), and the Anderson model (purple dashed line). The inset compares

ボソンボソン相互作用は有効的に斥力 (準安定状態だけ)

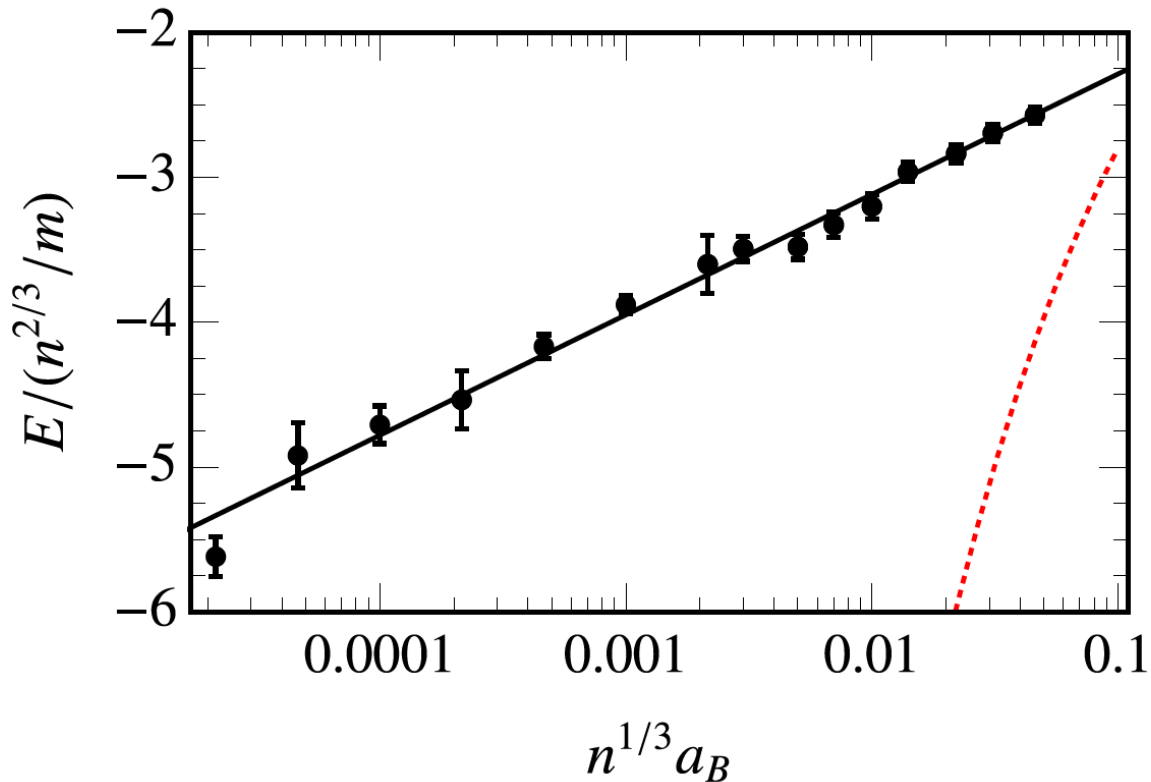


FIG. 4. Bose polaron ground-state energy in the unitarity regime of impurity-boson interactions, $1/a = 0$. The QMC results (symbols) are consistent with a logarithmic dependence of the form $E^{\text{QMC}} \simeq 0.36 \ln(0.019n^{1/3}a_B)n^{2/3}/m$ (solid line). The dashed red line is the prediction of the coherent state ansatz within the Bogoliubov approximation [13].

At unitarity $1/a = 0$, the polaron energy takes the universal form

$$E = -f(n^{1/3}a_B)n^{2/3}/m, \quad (6)$$

where $f(x)$ is a dimensionless function. When $a_B \rightarrow 0$ at fixed density, we know that $E \rightarrow -\infty$, while in the zero-density limit $n \rightarrow 0$, we must have $E \rightarrow 0$ since there are no bound states. Thus, in the limit $n^{1/3}a_B \rightarrow 0$, we require $f(x) \rightarrow \infty$ slower than $\sim 1/x^2$. Indeed, our QMC results reveal a *logarithmically* slow dependence $f(x) \sim -\ln(x)$, as shown in Fig. 4. This behavior is difficult to fully capture within the truncated basis approach [32] since it requires an increasingly larger number of boson excitations as $n^{1/3}a_B \rightarrow 0$. On the other

Bogoliubov近似とはB.フォノン間の残留相互作用を落としたもの。