# Statistical Modelling of the Cosmological Dispersion Measure

Ryuichi Takahashi (Hirosaki U)

with Kunihito Ioka (Kyoto U)

A. Mori & K. Funahashi (Hirosaki U)

# Abstract


- We study the basic statistics of the cosmological dispersion measure (DM) such as its mean, variance, probability distribution, and angular correlation function using the state-of-the-art hydrodynamic simulations, IllustrisTNG300.

- We then provide an analytical model of the DM statistics using fitting functions of the cosmological free-electron distribution calibrated with TNG300.

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# Statistical modelling of the cosmological dispersion measure

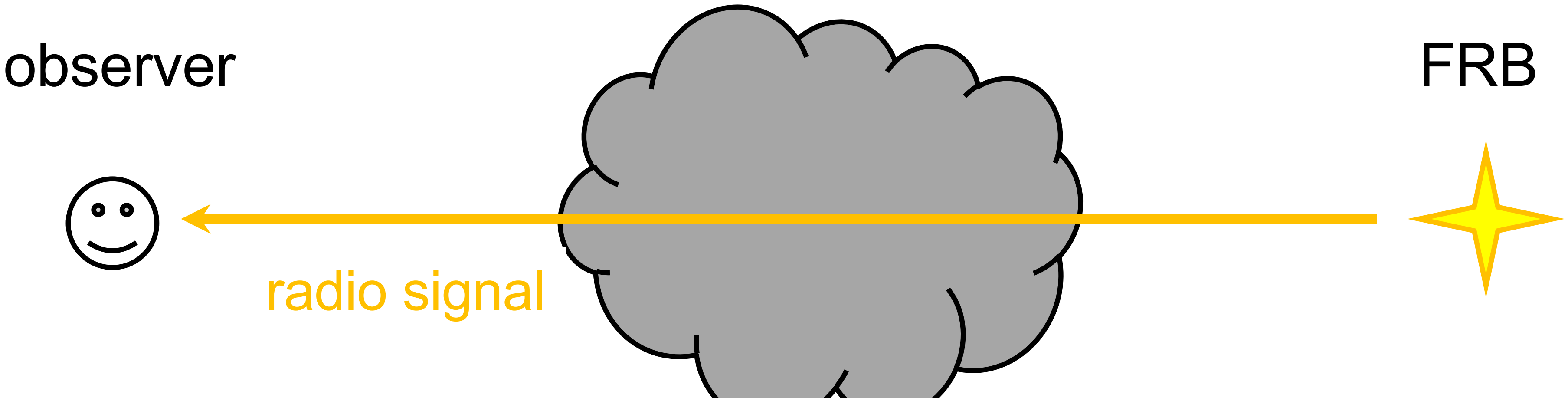
Ryuichi Takahashi,[1★] Kunihito Ioka,[2] Asuka Mori[1] and Koki Funahashi[1]

[1] *Faculty of Science and Technology, Hirosaki University, 3 Bunkyo-cho, Hirosaki, Aomori 036-8561, Japan*
[2] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

# Cosmological dispersion measure (DM)

(Ioka 2003; Inoue 2004)

ionized gas in IGM & intervening galaxies




gas is almost (~90%) fully ionized after H & He reionization

$$\mathrm{DM} = \int d\ell \, n_{\mathrm{e}}$$ (= column density of free electrons)

$n_{\mathrm{e}}$: free-electron number density

$\ell$ : distance along the line of sight

excluding the Milky Way & host-galaxy contributions

# Previous theoretical woks on the DM statistics

- analytical approach

  using the halo model (e.g., Cooray & Sheth 2002)

  (e.g., McQuinn 2014; Masui & Sigurdson 2015; Shirasaki+ 2017; Madhavacheril+ 2019; Dai & Xia 2020)

  the free-electron distribution in halos is assumed

  need to be validated with numerical simulations

- numerical-simulation approach

  using cosmological hydrodynamic simulations

  (e..g, Dolag+ 2015; Zhu+ 2018; Pol+ 2019; Jaroszynski 2019; Batten+ 2020; Zhang+ 2021)

  the most reliable tool to investigate the non-linear free-electron distribution in the universe

## Main purpose of our study

**1) model the two-point statistics of DM using the hydrodynamic simulations, IllustrisTNG300**

first measure the free-electron abundance & the power spectrum of its spatial fluctuations

because the DM statistics are fully determined by the free-electron statistics

**2) validate our analytical model against mock DM measurements**

# IllustrisTNG300 (e.g., Nelson+ 2018)

- the state-of-the-art hydrodynamical simulations
- follow gravitational evolution & astrophysical processes

  (star formation, gas cooling, stellar & AGN feedback)

- simulation data is publicly available at z=0-12
- **reproduce fee-electron distribution in the universe**

| | $N_{\rm baryon}$ | $N_{\rm dark}$ | $m_{\rm baryon}\,(h^{-1}{\rm M}_\odot)$ | $m_{\rm dark}\,(h^{-1}{\rm M}_\odot)$ |
|---|---|---|---|---|
| TNG300-1 | $2500^3$ | $2500^3$ | hydro. runs | |
| TNG300-2 | $1250^3$ | $1250^3$ | | |
| TNG300-3 | $625^3$ | $625^3$ | | |
| TNG300-1-Dark | – | $2500^3$ | ← dark-matter-only run | |

box size L=205Mpc/h (~300Mpc)

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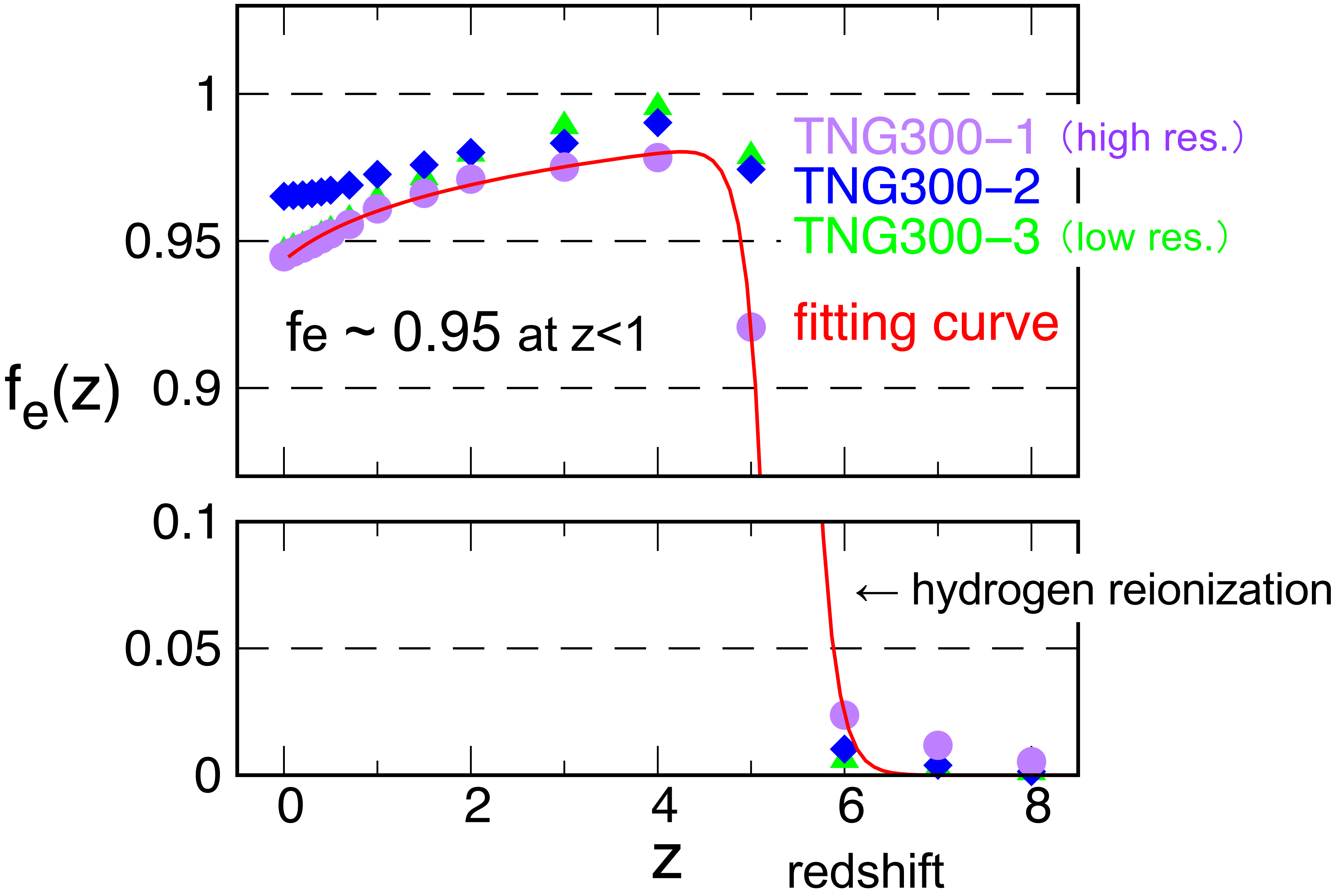
| | $N_{\rm baryon}$ | $N_{\rm dark}$ | $m_{\rm baryon}\,(h^{-1}{\rm M}_\odot)$ | $m_{\rm dark}\,(h^{-1}{\rm M}_\odot)$ |
|---|---|---|---|---|
| TNG300-1 | $2500^3$ | $2500^3$ | $7.4\times10^6$ | $4.0\times10^7$ |
| TNG300-2 | $1250^3$ | $1250^3$ | $6.0\times10^7$ | $3.2\times10^8$ |
| TNG300-3 | $625^3$ | $625^3$ | $4.8\times10^8$ | $2.5\times10^9$ |
| TNG300-1-Dark | – | $2500^3$ | – | $4.7\times10^7$ |

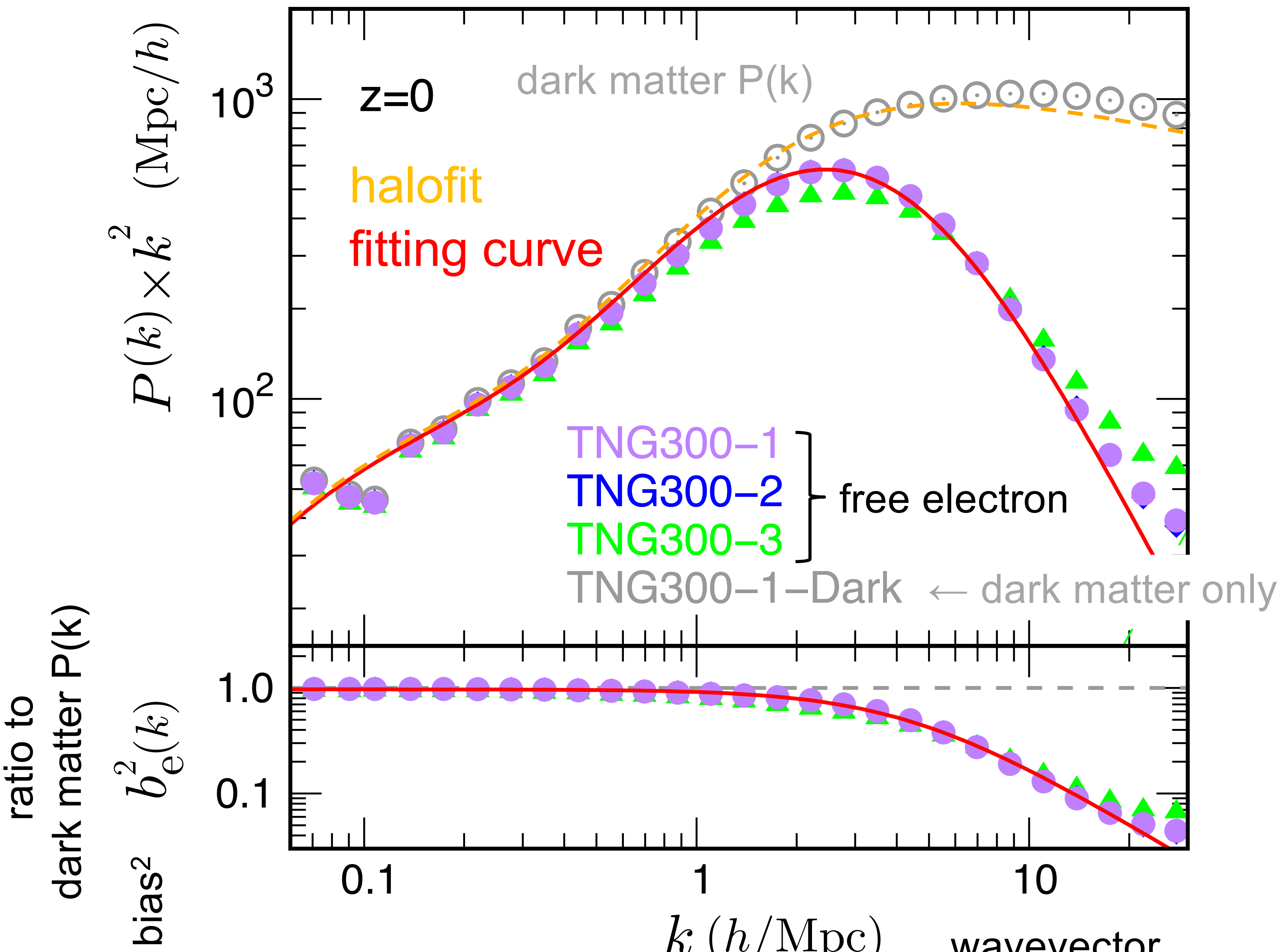
box size L=205Mpc/h (~300Mpc)

# free-electron fraction

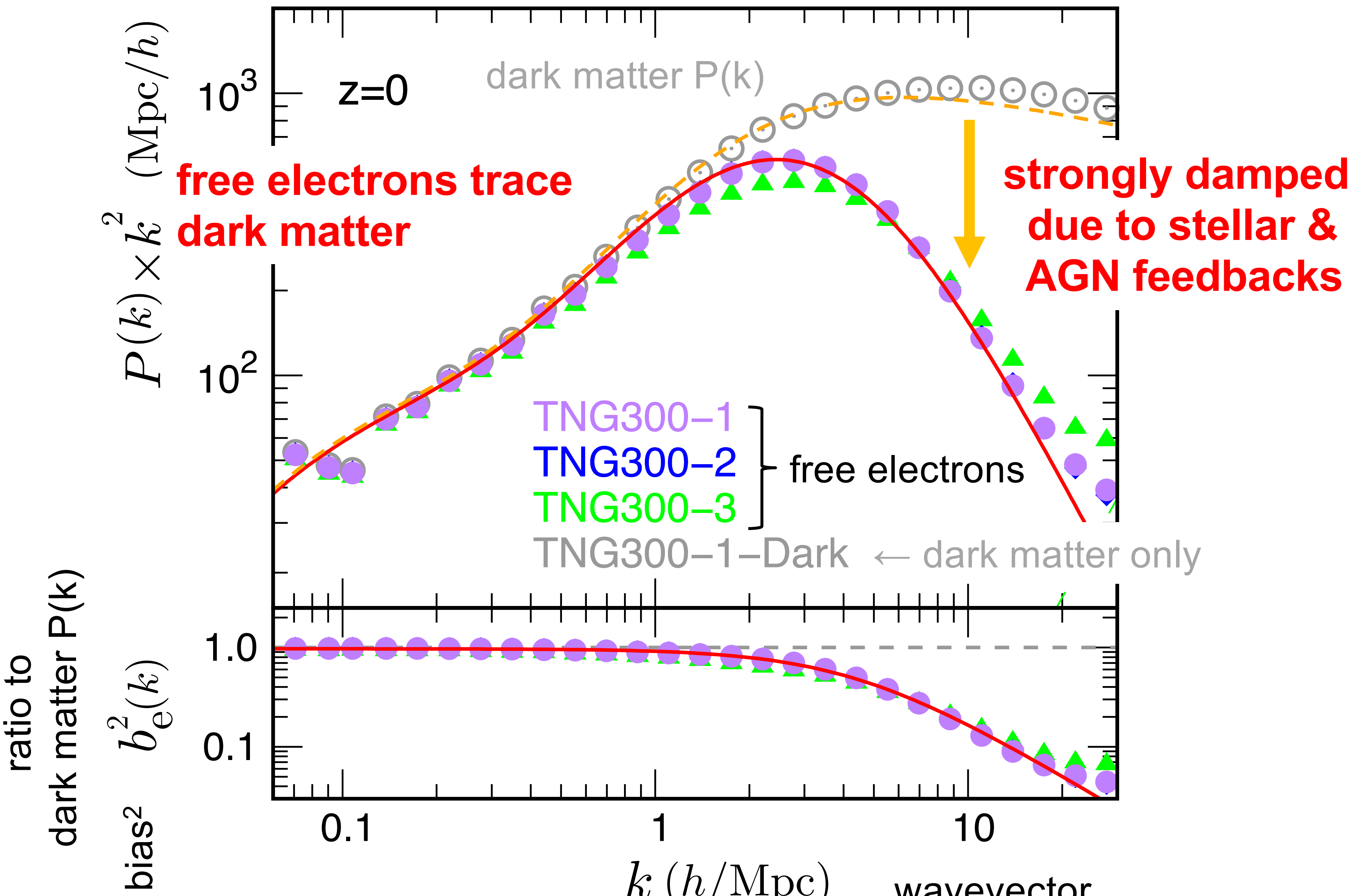
=(# of free electrons)/(total # of electrons)



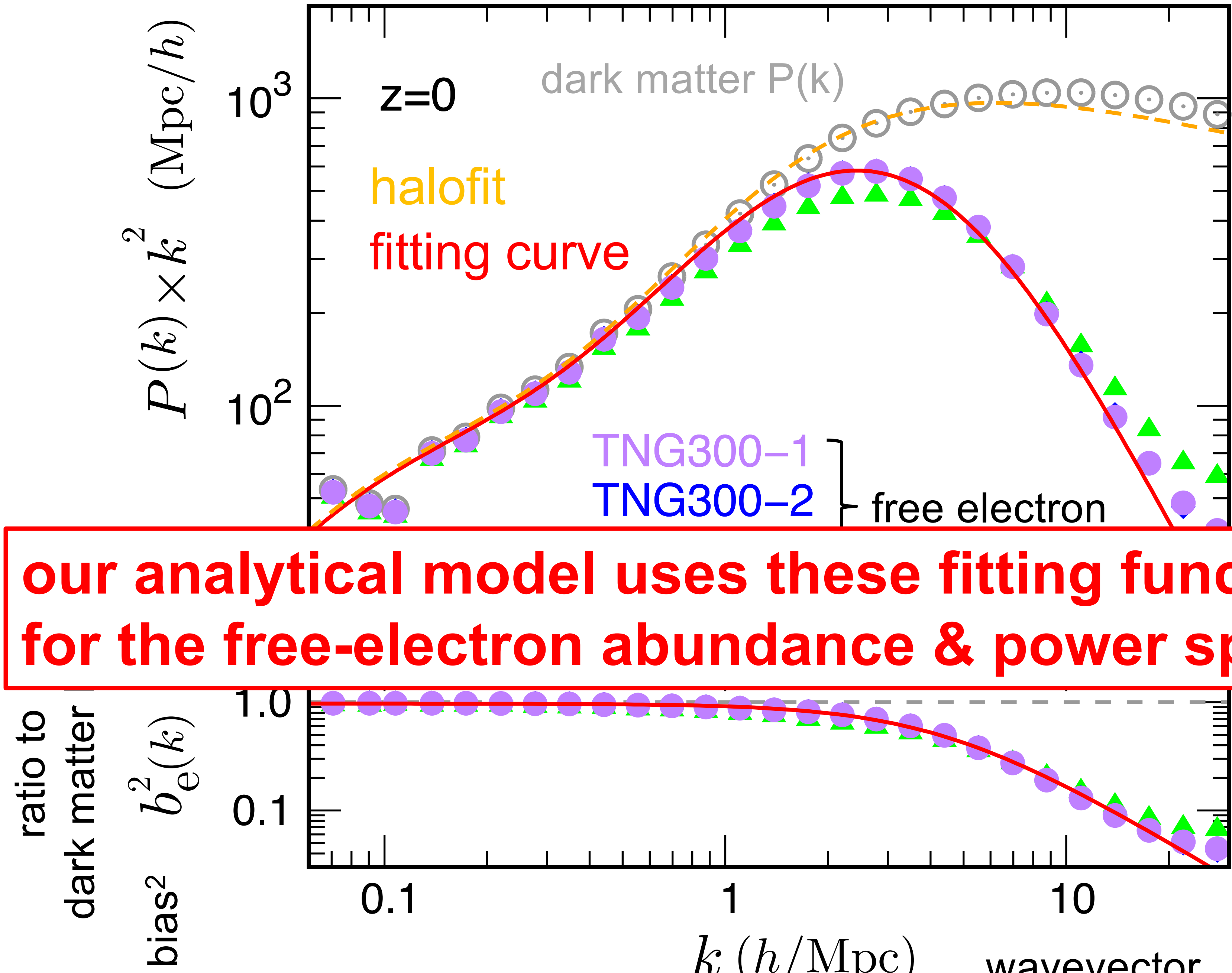
# fee-electron power spectrum



# fee-electron power spectrum



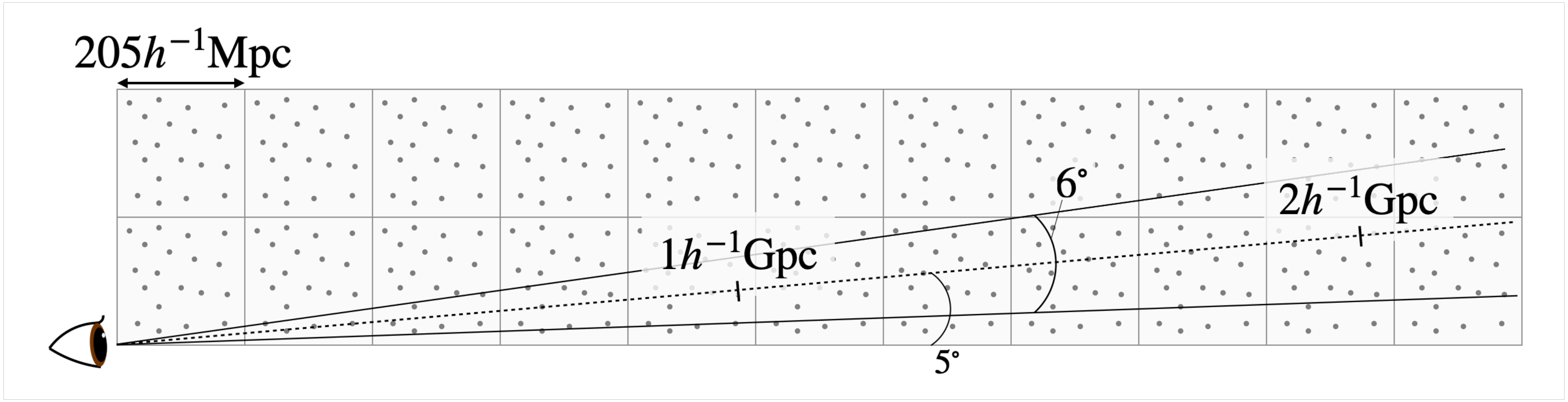
# fee-electron power spectrum




**our analytical model uses these fitting functions for the free-electron abundance & power spectrum**

# Making mock sky maps of DM

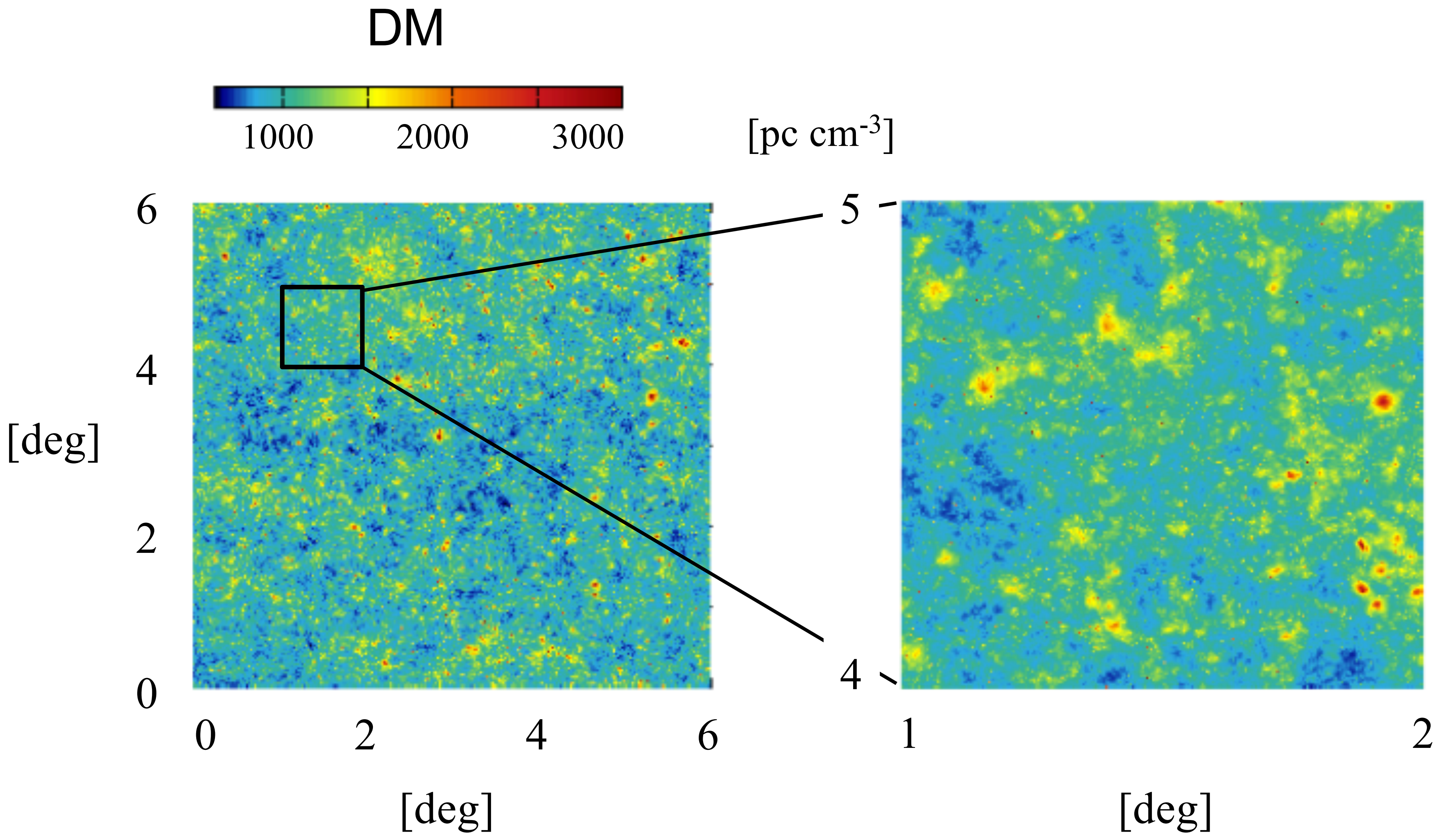
based on a standard ray-tracing technique



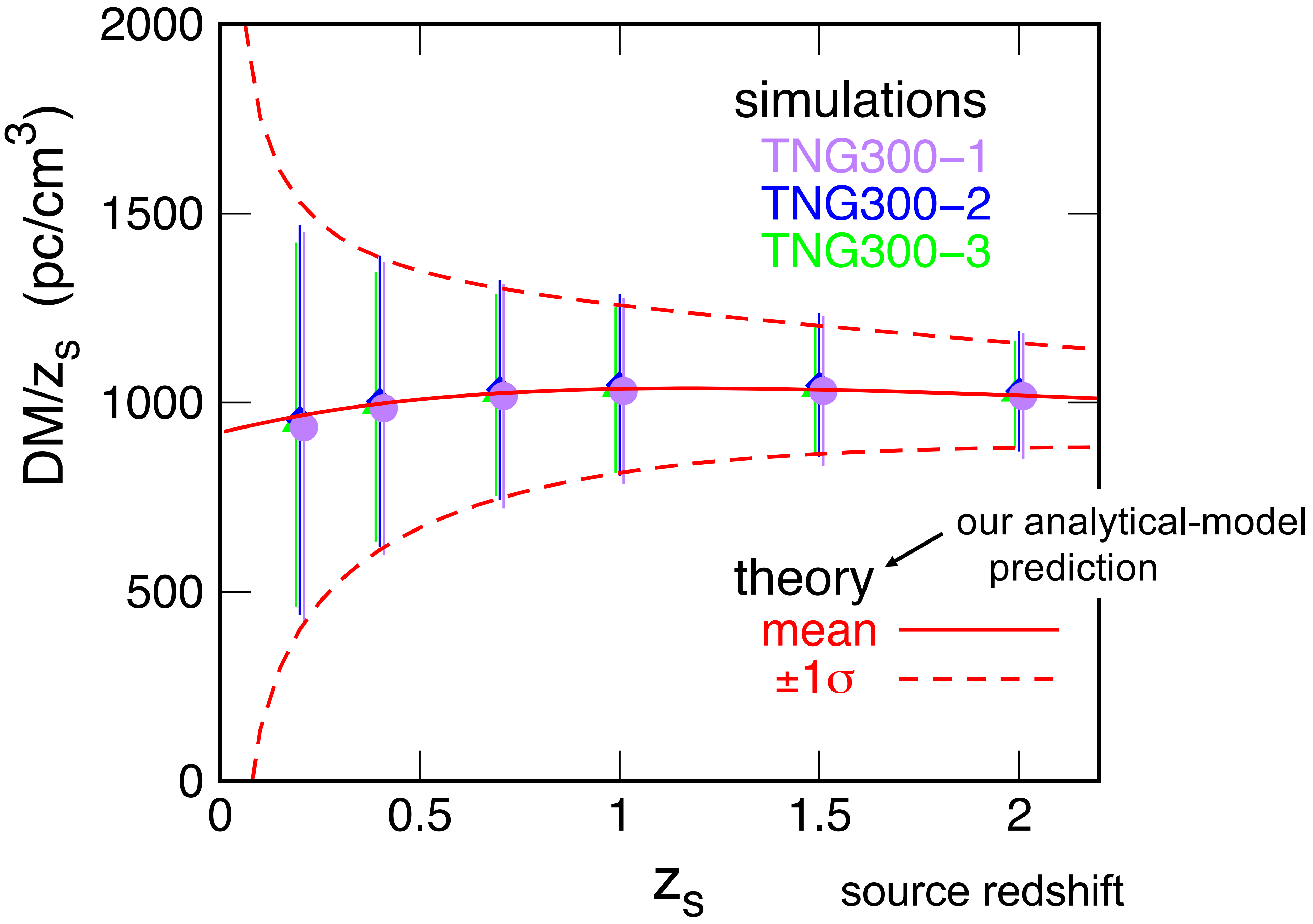

1. place simulation boxes along the line of sight
2. emit $5400^2$ light rays in a field of view 6x6 $deg^2$

   (angular resolution = 4arcsec = 6deg/5400)
3. calculate free-electron column density along each ray path

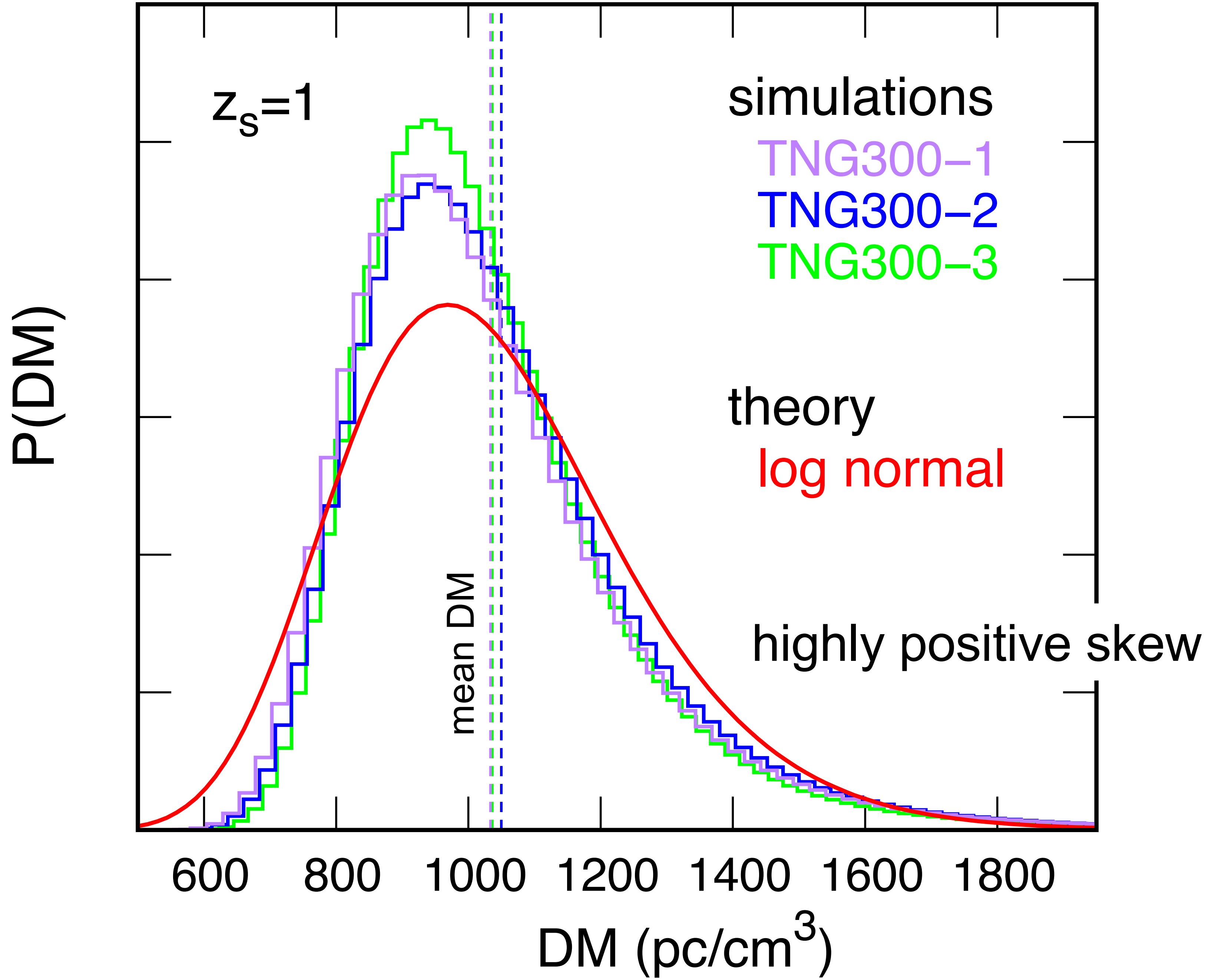
# Contour map of DM at $z_s$=1



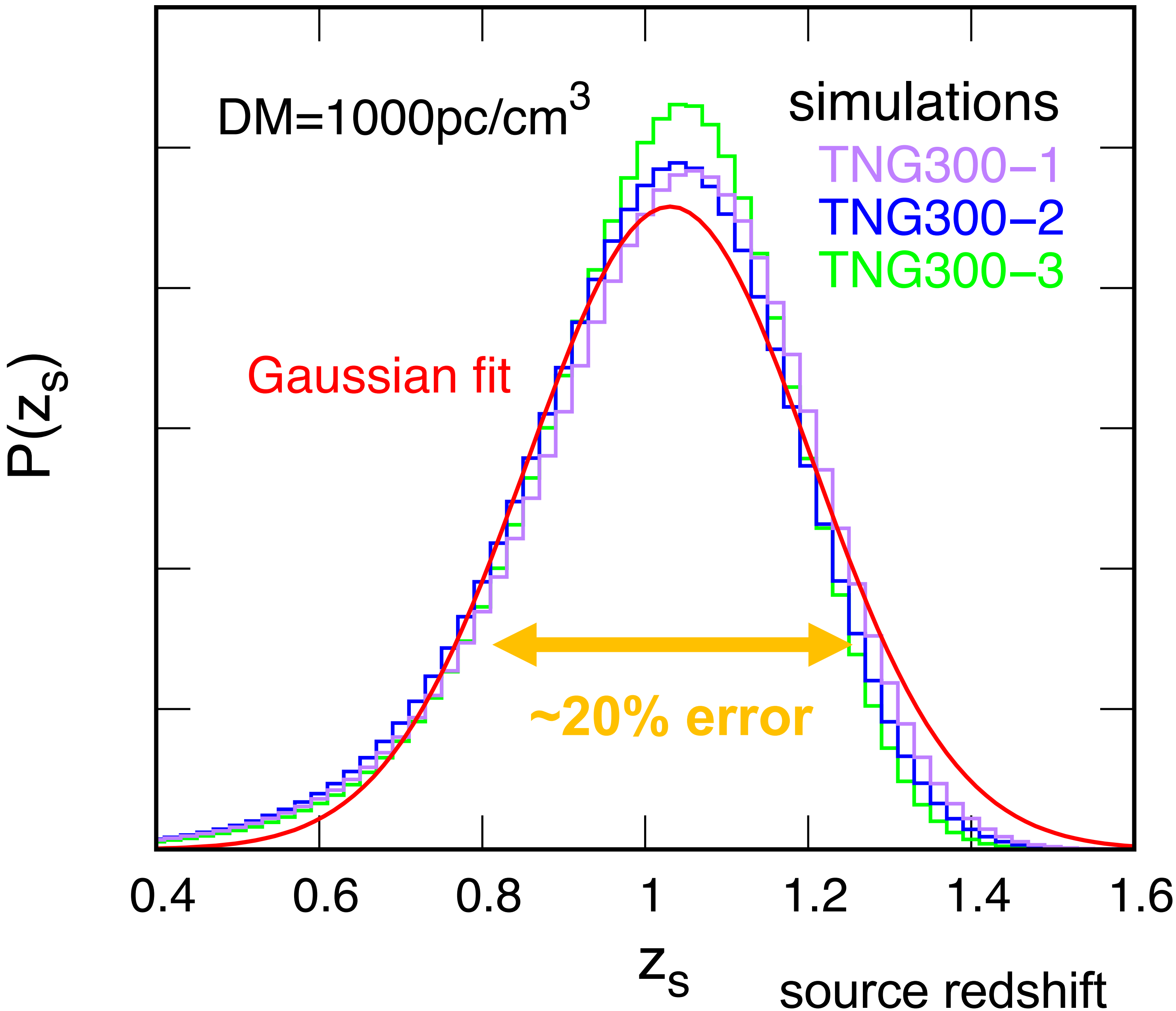
# mean & variance of DM



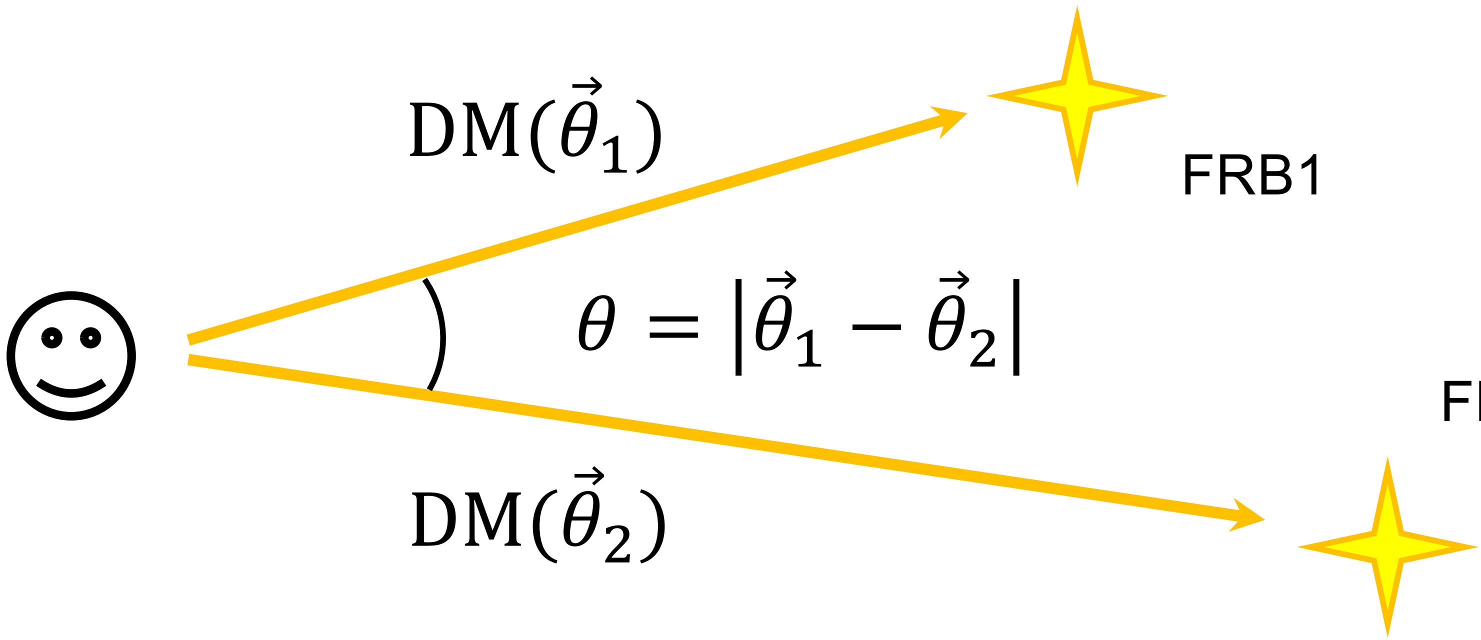
# probability distribution of DM



# probability distribution of source redshift for a given DM



# angular two-point correlation function of DM

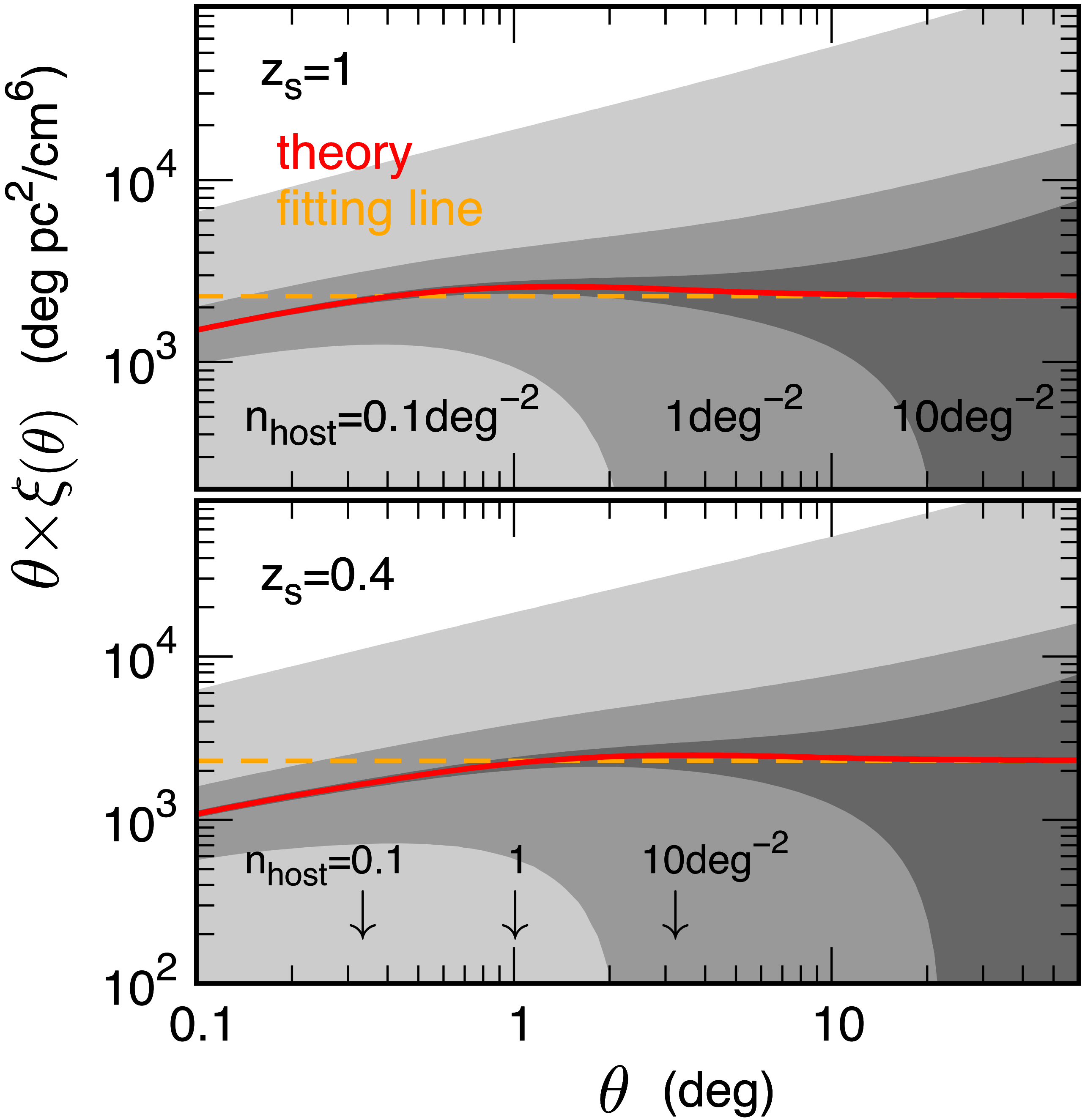



$$\xi(\theta) \equiv \left\langle \mathrm{DM}\left(\vec{\theta}_1\right)\mathrm{DM}\left(\vec{\theta}_2\right)\right\rangle - \langle \mathrm{DM}\rangle^2$$

$$\approx 2400\left(\frac{\theta}{\mathrm{deg}}\right)^{-1} \ \mathrm{pc}^2/\mathrm{cm}^6 \quad \text{for } \theta \gtrsim 1 \text{ deg} \ \& \ z_s \gtrsim 0.3$$

$\xi(\theta)$ will be detected using thousands of FRBs

# angular correlation function for full-sky measurement




shaded region:
1σ error

determined by
survey area &
shot noise

single power-law fit

$$\xi(\theta) \approx 2400 \left(\frac{\theta}{\text{deg}}\right)^{-1}$$

(pc$^2$/cm$^6$)

# Numerical code written in C

freely available

computes

the free-electron statistics（its abundance & power spectrum）

the DM statistics（its variance, angular power spectrum & angular correlation function）

Cornell University

arXiv.org > astro-ph > arXiv:2010.01560

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*[Submitted on 4 Oct 2020 (v1), last revised 2 Feb 2021 (this version, v2)]*

## Statistical modelling of the cosmological dispersion measure

Ryuichi Takahashi, Kunihito Ioka, Asuka Mori, Koki Funahashi

We have investigated the basic statistics of the cosmological dispersion measure (DM) -- such as its mean, variance, probability distribution, angular power spectrum and correlation function -- using the state-of-the-art hydrodynamic simulations, IllustrisTNG300, for the fast radio burst (FRB) cosmology. To model the DM statistics, we first measured the free-electron abundance and the power spectrum of its spatial fluctuations. The free-electron power spectrum turns out to be consistent with the dark matter power spectrum at large scales, but it is strongly damped at small scales ($\lesssim 1$Mpc) owing to the stellar and active galactic nucleus feedback. The free-electron power spectrum is well modelled using a scale-dependent bias factor (the ratio of its fluctuation amplitude to that of the dark matter). We provide analytical fitting functions for the free-electron abundance and its bias factor. We next constructed mock sky maps of the DM by performing standard ray-tracing simulations with the TNG300 data. The DM statistics are calculated analytically from the fitting functions of the free-electron distribution, which agree well with the simulation results measured from the mock maps. We have also obtained the probability distribution of source redshift for a given DM, which helps in identifying the host galaxies of FRBs from the measured DMs. The angular two-point correlation function of the DM is described by a simple power law, $\xi(\theta) \approx 2400(\theta/\mathrm{deg})^{-1}\,\mathrm{pc}^2\,\mathrm{cm}^{-6}$, which we anticipate will be confirmed by future observations when thousands of FRBs are available.

Comments: 15 pages, 16 figures, accepted for publication in MNRAS; minor corrections; a numerical code for the DM statistics is available at this http URL
Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; General Relativity and Quantum Cosmology (gr-qc)
DOI: 10.1093/mnras/stab170
Cite as: arXiv:2010.01560 **[astro-ph.CO]**
(or arXiv:2010.01560v2 **[astro-ph.CO]** for this version)

# Conclusions

- **We provided an analytical two-point statistical model of DM calibrated with IllustrisTNG300**

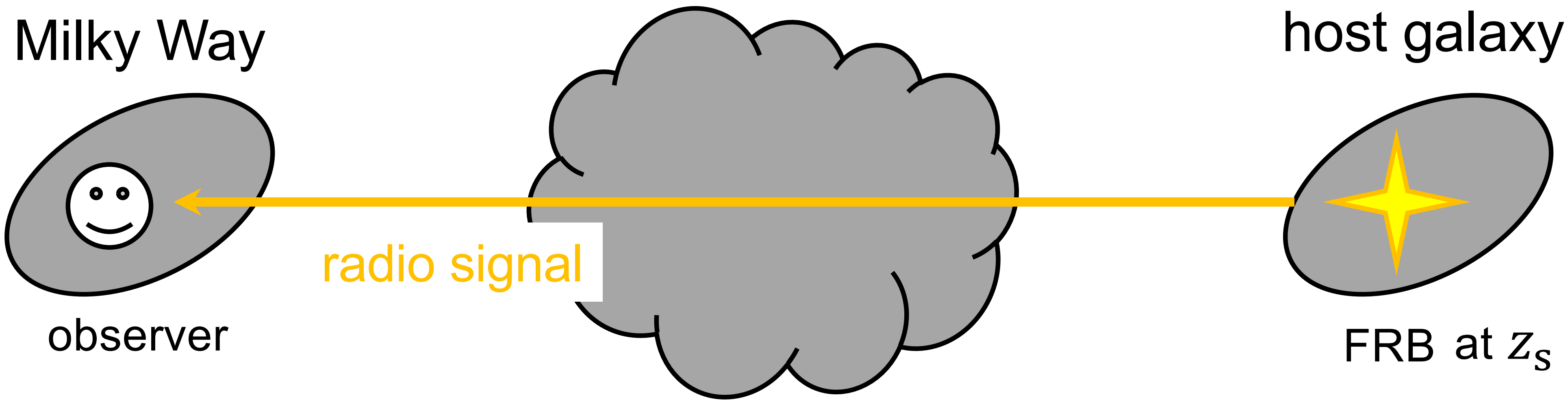
  based on free-electron abundance & its power spectrum

- **We validated it against mock DM measurements**

- **We provided a source redshift-DM relation, which will be useful for inferring source redshift from measured DM**

- **Angular correlation function of DM will be measured when >$10^3$ FRBs are available**

ionized gas in IGM & intervening galaxies




$\mathrm{DM_{MW}} \approx$ $30 - 200\ \mathrm{pc/cm^3}$

estimated from
pulsar observations

$\mathrm{DM} \approx$ $1000 z_s\ \mathrm{pc/cm^3}$

increase $\propto z_s$

$$\mathrm{DM_{host}} = \frac{\mathrm{DM_{host,rest\ frame}}}{1 + z_s}$$

decrease $\propto (1 + z_s)^{-1}$

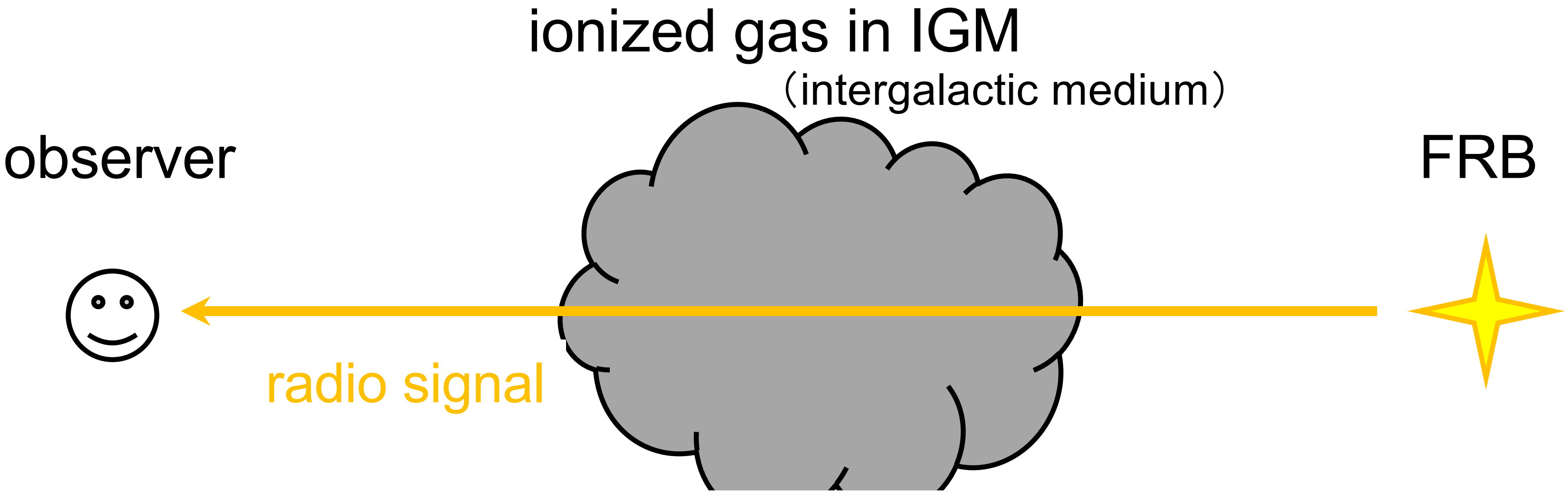
cosmological DM dominates at $z_s \gtrsim 0.3$

$\mathrm{DM} \propto \Delta t\, \nu^2$

time lag $\Delta t \propto 1 + z_s$

frequency $\nu \propto (1 + z_s)^{-1}$

# Cosmological dispersion measure (DM)




gas is almost (~90%) fully ionized after H & He reionization

**DM is an unique probe of cosmological baryon distribution (including "missing baryons")**

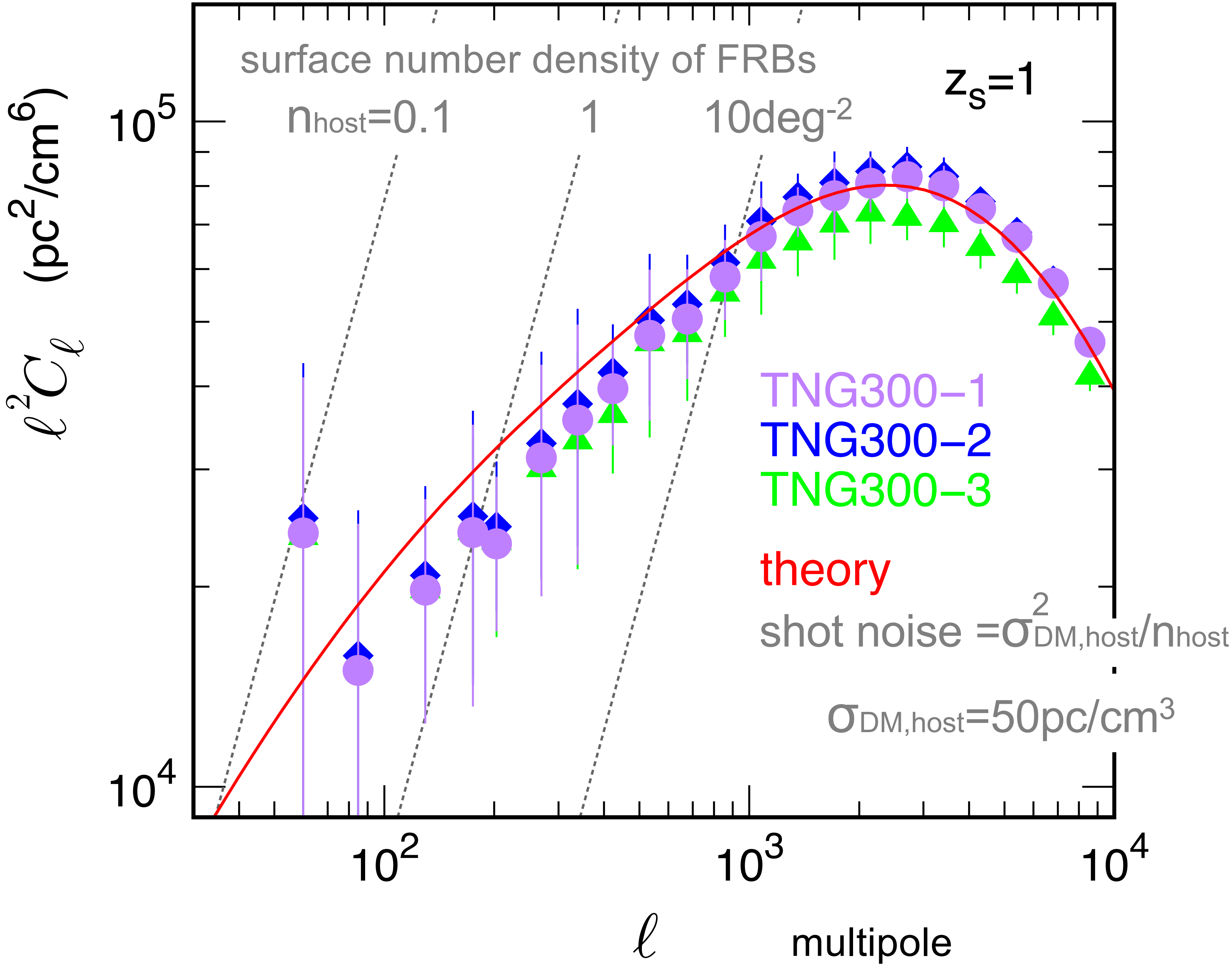
(Ioka 2003; Inoue 2004)

$$\mathrm{DM} = \int d\ell \, n_{\mathrm{e}}$$ (= column density of free electrons)

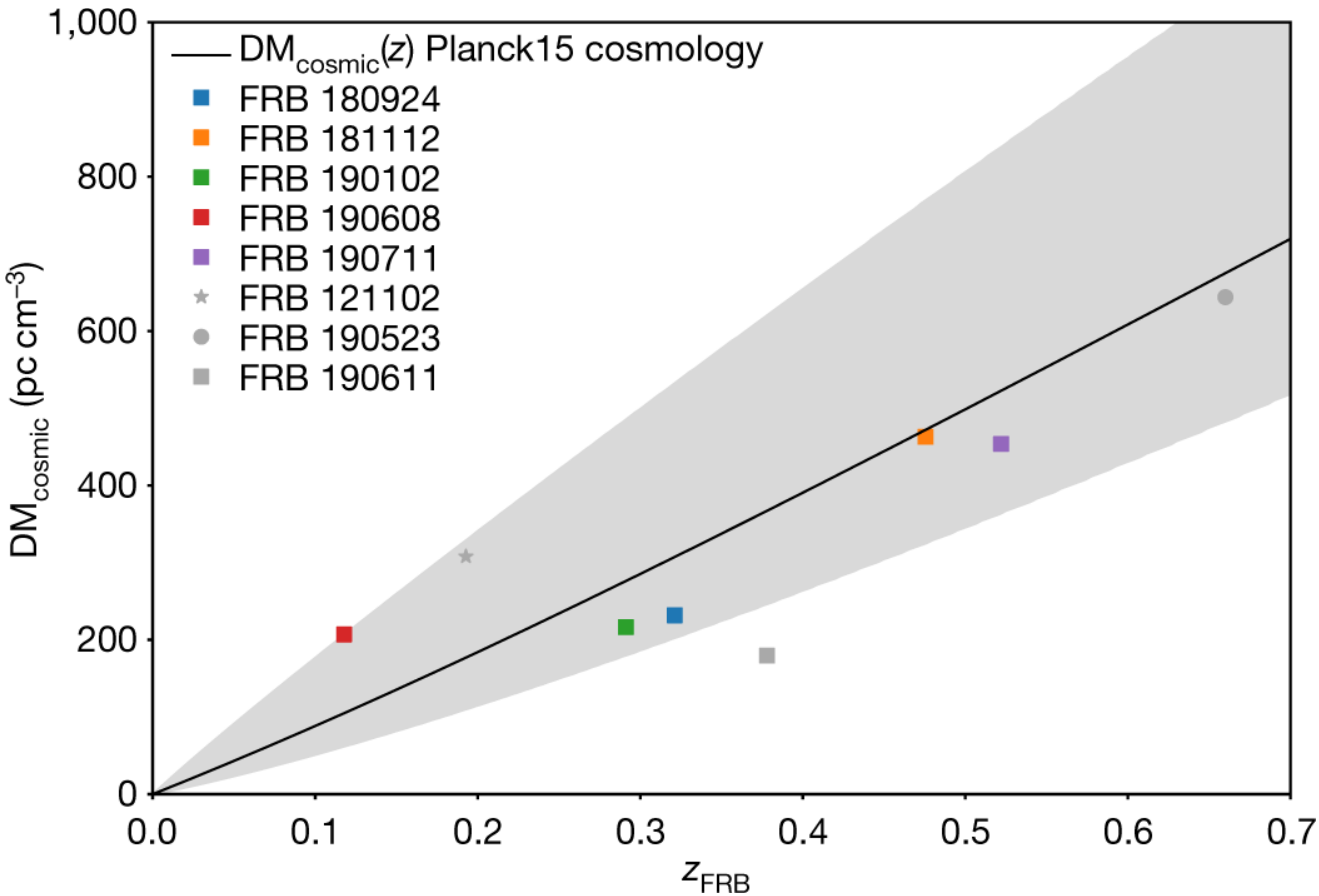
$n_{\mathrm{e}}$: free-electron number density

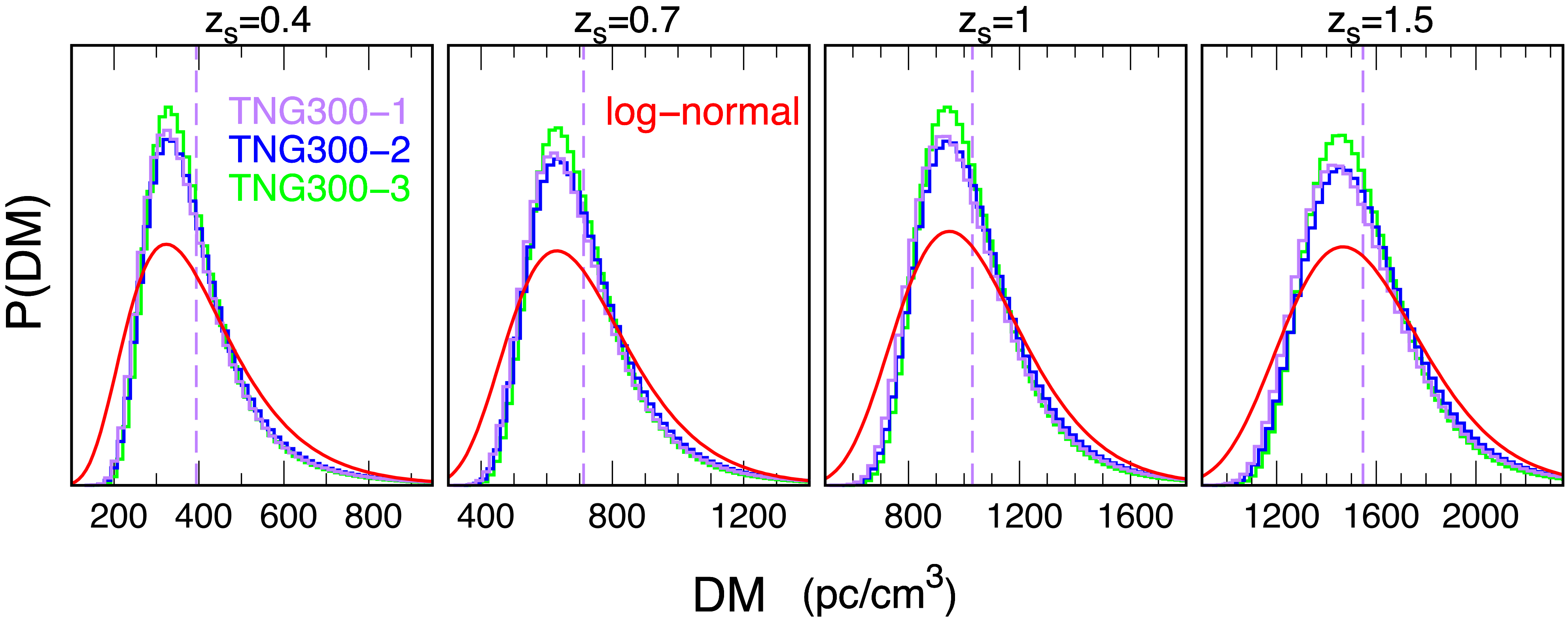
$\ell$ : distance along the line of sight

# angular power spectrum of DM

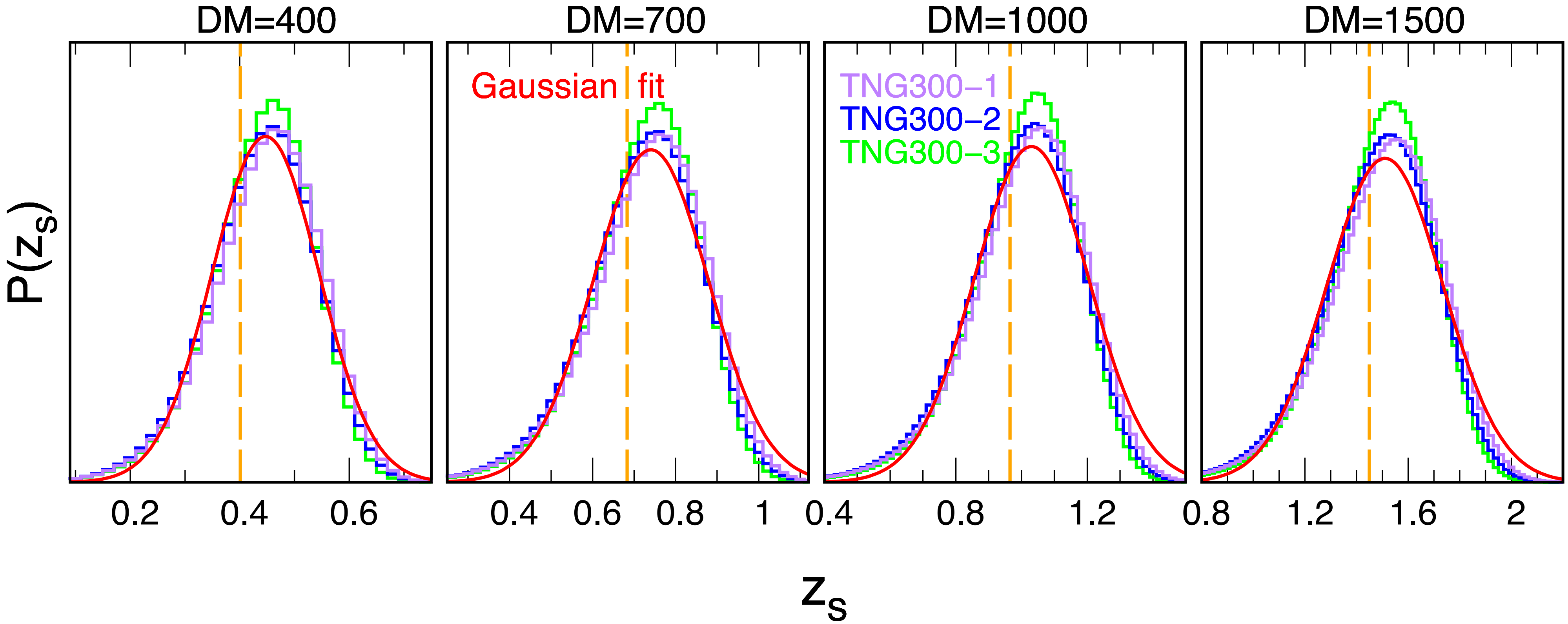


Macquart+ 2020





$z_s$=0.4
$z_s$=0.7
$z_s$=1
$z_s$=1.5
TNG300−1
TNG300−2
TNG300−3
log−normal
P(DM)
200
400
600
800
400
800
1200
800
1200
1600
1200
1600
2000
DM (pc/cm$^3$)



DM=400
DM=700
DM=1000
DM=1500
Gaussian fit
TNG300-1
TNG300-2
TNG300-3
P(z_s)
z_s
0.2
0.4
0.6
0.4
0.6
0.8
1
0.4
0.8
1.2
0.8
1.2
1.6
2

# probability distribution of DM

## comparison with previous fitting formulas