Generation of FRB by synchrotron maser at the reverse shock of magnetar flare

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Overview

- Synchrotron MASER emission
- Generation of synchrotron MASER emission at pulsar wind termination shock
- Generation of synchrotron MASER emission at magnetar flares
- Verification of the scenario (key parameters and statistics)
- Variability of emission from the far-distance zone

Literature

1. Khangulyan, Barkov & Popov ApJ (2022)
2. Barkov & Popov MNRAS (2022)
3. Khangulyan, Barkov & Popov in preparation (2022)

Synchrotron MASER Emission

Synchrotron self-absorption: $\alpha_\nu = \frac{c^2}{8\pi\nu^2}\int\limits_0^\infty \frac{N(E)}{E^2}\frac{d}{dE}[E^2 P_\nu(E)]\,dE$

is positive (flux attenuation, Wild+1963) for the standard synchrotron emissivity $P_\nu(E) = \frac{\sqrt{3}e^3 B}{mc^2}\frac{\nu}{\nu_c}\int\limits_{\nu/\nu_c}^\infty K_{5/3}(\eta)d\eta$, where $\nu_c = \frac{3eB}{4\pi mc}\left(\frac{E}{mc^2}\right)^2$, which makes impossible synchrotron MASER emission in vacuum. This result, however, doesn't held true in the range of electron energies / photon frequencies where the impact of the medium is important, i.e., the condition $\sqrt{1-\varepsilon_\nu}E \ll mc^2$ fails (e.g. Zheleznyakov 1967). Here ε_ν is dielectric permittivity (strictly speaking, tensor):

$\varepsilon_{\alpha\beta} = \varepsilon_\perp\delta_{\alpha\beta} + (\varepsilon_\parallel - \varepsilon_\perp)b_\alpha b_\beta + \imath g e_{\alpha\beta\gamma}b_\gamma$, where

$$\varepsilon_\perp = 1 - \frac{\omega_{p,e}^2}{\omega^2-\omega_{L,e}^2} - \frac{\omega_{p,i}^2}{\omega^2-\omega_{L,i}^2},\quad \varepsilon_\parallel = 1 - \frac{\omega_{p,e}^2+\omega_{p,i}^2}{\omega^2},\quad g = \frac{\omega_{L,e}\omega_{p,e}^2}{\omega(\omega^2-\omega_{L,e}^2)} - \frac{\omega_{L,i}\omega_{p,i}^2}{\omega(\omega^2-\omega_{L,i}^2)}$$

Plasma frequency: $\omega_{p,e}^2 = \frac{4\pi n_e e^2}{m}$ and $\omega_{p,i}^2 = \frac{4\pi n_i Z_i^2 e^2}{m_i}$

Cyclotron frequency: $\omega_{L,e} = \frac{eB}{mc}$ and $\omega_{L,i} = \frac{eZ_iB}{m_ic}$

Non-relativistic plasma!!!

Dielectric permittivity

Non-relativistic plasma!!!

$\varepsilon_{\alpha\beta} = \varepsilon_\perp \delta_{\alpha\beta} + (\varepsilon_\parallel - \varepsilon_\perp) b_\alpha b_\beta + i g e_{\alpha\beta\gamma} b_\gamma$, where

$$\varepsilon_\perp = 1 - \frac{\omega_{p,e}^2}{\omega^2 - \omega_{L,e}^2} - \frac{\omega_{p,i}^2}{\omega^2 - \omega_{L,i}^2},\quad \varepsilon_\parallel = 1 - \frac{\omega_{p,e}^2 + \omega_{p,i}^2}{\omega^2},\quad g = \frac{\omega_{L,e}\omega_{p,e}^2}{\omega(\omega^2 - \omega_{L,e}^2)} - \frac{\omega_{L,i}\omega_{p,i}^2}{\omega(\omega^2 - \omega_{L,i}^2)}$$

Plasma frequency: $\omega_{p,e}^2 = \dfrac{4\pi n_e e^2}{m} >> \omega_{p,i}^2 = \dfrac{4\pi n_i Z_i^2 e^2}{m_i}$

Cyclotron frequency: $\omega_{L,e} = \dfrac{eB}{mc} >> \omega_{L,i} = \dfrac{eZ_i B}{m_i c}$

If $\omega_{L,e} >> \omega_{p,e}$ the situation is quite complex, one needs to deal with dielectric permittivity tensor is the treatment of synchrotron MASER emission. Limited to numerical experiments?

However, if $\omega_{L,e} << \omega_{p,e}$, the treatment is much simple in the range of frequencies $\omega >> \omega_{L,e}$: $\varepsilon_{\alpha\beta} = \left(1 - \dfrac{\omega_{p,e}^2}{\omega^2}\right)\delta_{\alpha\beta}$ and one can advance with analytic treatment.

Synchrotron MASER Emission

Non-relativistic plasma!!!

In the frequency range $\omega >> \omega_{L,e}$ the synchrotron emissivity is

$$P_\nu(E) = \frac{\sqrt{3}e^3 B}{mc^2\,[1+(\frac{\omega_{p,e}}{2\pi\nu})^2(\frac{E}{mc^2})^2]^{1/2}} \quad \frac{\nu}{\nu_c'} \int_{\nu/\nu_c'}^{\infty} K_{5/3}(\eta)d\eta$$

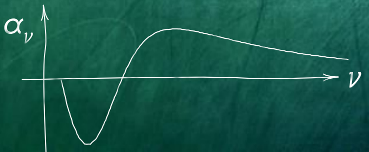
where $\nu_c' = \frac{3eB}{4\pi mc}(\frac{E}{mc^2})^2\,[1+(\frac{\omega_{p,e}}{2\pi\nu})^2(\frac{E}{mc^2})^2]^{-3/2}$. This emissivity should be substituted into equation for the absorption coefficient:

$$\alpha_\nu = \frac{c^2}{8\pi\nu^2}\int_0^\infty \frac{N(E)}{E^2}\frac{d}{dE}[E^2 P_\nu(E)]\,dE$$

which also depends on the electron spectrum $N(E)$. Zheleznyakov (1967) has shown that for mono-energetic electrons and electrons obeying a power-law with sharp low-energy cutoff the absorption coefficient can be negative.

The emission can be amplified if

$$\nu < [\frac{E}{2mc^2}\frac{\omega_{p,e}^3}{\omega_{L,e}}]^{1/2}$$



Synchrotron MASER Emission in Relativistic Plasma

In relativistic plasma, i.e., then not only electrons that produce radiation, but also the background particles have relativistic energies, dielectric permittivity tensor depends on the energy and angular distribution of particles. The simplest assumption is, of course, a mono-energetic distribution of particles. Sazonov (1970) shown that in this case a simple substitution

Larmor frequency $\omega_{L,\cdot} \rightarrow \Omega_{L,\cdot} = \dfrac{mc^2}{E}\omega_{L,\cdot}$

for plasma frequency $\omega_{p,\cdot} \rightarrow \Omega_{p,\cdot} = \sqrt{mc^2/E}\,\omega_{p,\cdot}$

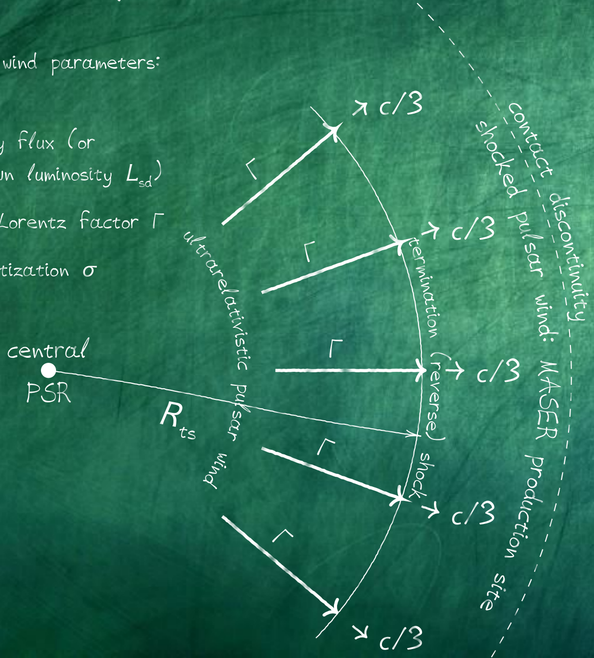
provides the correction for relativistic plasma. Sagiv&Waxman(2002) have also shown that this substitution provides an estimate for the permittivity of relativistic plasma with accuracy of 1% if particles have a power-law distribution of index 2 above E. It means that one can directly use the result by Zheleznyakov (1967), synchrotron MASER emission is generated for

$\nu < [\dfrac{E}{2mc^2}\dfrac{\Omega_{p,e}^3}{\omega_{L,e}}]^{1/2}$ if the following conditions are fulfilled: $\nu >> \dfrac{\Omega_{L,e}}{2\pi}$ and $\Omega_{p,e} >> \Omega_{L,e}$

Synchrotron MASER Emission at Shock Pulsar Wind

Cold pulsar wind parameters:

- Energy flux (or spindown luminosity L_{sd})
- Bulk Lorentz factor Γ
- Magnetization σ



ISM

ISM parameter:

- Density

which defines R_{ts}

Synchrotron MASER Emission at Shock Pulsar Wind

In the shocked pulsar wind the typical energy of electrons is $E \approx \Gamma mc^2$, thus MASER emission is generated at $\nu < [\frac{\Gamma}{2}\frac{\Omega_{p,e}^3}{\omega_{L,e}}]^{1/2}$ if $\nu >> \frac{\Omega_{L,e}}{2\pi}$ and $\Omega_{p,e} >> \Omega_{L,e}$

The latter condition implies: $\frac{\Omega_{L,e}^2}{\Omega_{p,e}^2} = \frac{B^2/(4\pi)}{n_e \Gamma mc^2} << 1 \longrightarrow \sigma << 1$ (small magnetization). For $\nu \approx [\frac{E}{2mc^2}\frac{\Omega_{p,e}^3}{\omega_{L,e}}]^{1/2}$, we also obtain $\nu >> \frac{\Omega_{L,e}}{2\pi}$. It means that synchrotron MASER emission can be generated as the reverse shock formed by weakly magnetized pulsar wind.

Rankie–Hugoniot condition gives

- $\Omega_{p,e} = 2^{3/4}\frac{e}{mc}\sqrt{\frac{(1-\sigma)L_{sd}}{R_{ts}^2 c\Gamma^2}}$
- $\omega_{L,e} = \frac{e}{m_e c}\sqrt{\frac{8\sigma L_{sd}}{R_{ts}^2 c}}$

$$\omega_{max} \approx 2^{-1/8}\frac{e}{m_e c}\sqrt{\frac{L_{sd}}{R_{ts}^2 c}}\frac{(1-\sigma)^{3/4}}{\Gamma\sigma^{1/4}}$$

$$\approx 3\cdot 10^3 \frac{L_{sd,38}^{1/2}}{R_{ts,15}\Gamma_{,3}\sigma_{-2}^{1/4}}\,s^{-1}$$

Only kHz range! It requires unreasonable tuning of the parameters...

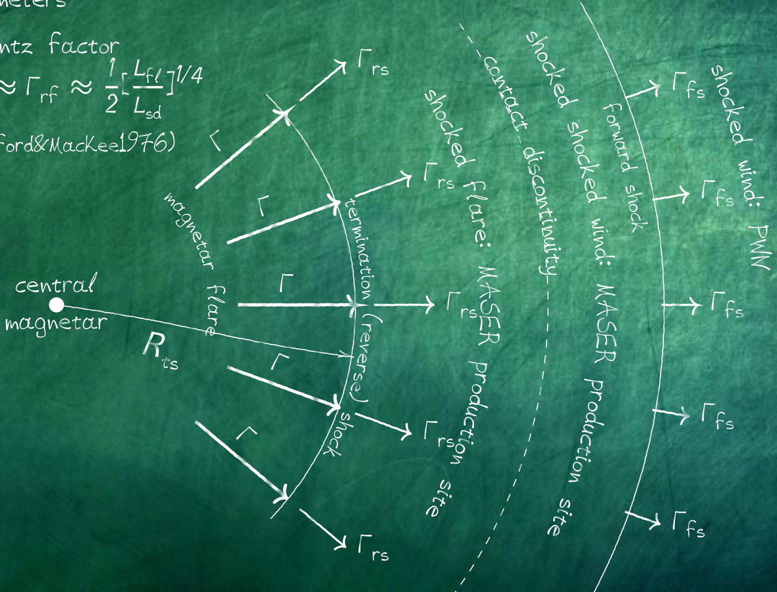
Synchrotron MASER Emission at Magnetar Flare

Key parameters

- Lorentz factor

$$\Gamma_{rf} \approx \Gamma_{rf} \approx \frac{1}{2}\left[\frac{L_{fl}}{L_{sd}}\right]^{1/4}$$

(e.g., Blandford&McKee1976)



Synchrotron MASER Emission at Magnetar Flare

Rankie-Hugoniot condition gives two sets of parameters (for the forward and reverse shock), which allow obtaining the frequencies at which synchrotron MASER emission can be generated

At the reverse shock

$$\omega_{\rm max,rs} \approx 2^{-1/8} \frac{e}{m_e c^{3/2}} \frac{L_{fl}^{3/4}}{L_{\rm sd}^{1/4} R_{\rm ts}} \frac{(1-\sigma_{fl})^{3/4}}{\Gamma_{fl}\sigma_{fl}^{1/4}}$$

$$\approx 3\cdot 10^9 \frac{L_{fl,45}^{3/4}}{L_{\rm sd,35}^{1/4} R_{\rm ts,15}\Gamma_{fl,3}\sigma_{fl,-2}^{1/4}}\,{\rm s}^{-1}$$

At the forward shock

$$\omega_{\rm max,fs} \approx 2^{-1/8} \frac{e}{m_e c} \frac{\sqrt{L_{\rm sd}^{1/2} L_{fl}^{1/2}}}{R_{\rm ts}^2 c} \frac{(1-\sigma)^{3/4}}{\Gamma_{\rm wind}\sigma^{1/4}}$$

$$\approx 3\cdot 10^4 \frac{L_{\rm sd,35}^{1/4} L_{fl,45}^{1/4}}{R_{\rm ts,15}\Gamma_{\rm wind,3}\sigma_{-2}^{1/4}}\,{\rm s}^{-1}$$

- Synchrotron MASER emission at the forward shock can be generated at low frequencies (unlikely?)
- Synchrotron MASER emission at the reverse shock can reach GHz band, but one needs to check if the parameter normalization is reasonable
- Most critically the termination shock radius and magentar flare magnetization

Synchrotron MASER Emission at the Reverse Shock

$$\omega_{\rm max,rs} \approx 3 \cdot 10^9 \frac{L_{fl,45}^{3/4}}{L_{\rm sd,35}^{1/4} R_{\rm ts,15} \Gamma_{fl,3} \sigma_{fl,-2}^{1/4}}\,{\rm s}^{-1}$$

Check of the parameter normalizations:

- Wind luminosity: $10^{35}\,{\rm erg\,s^{-1}}$ seems a reasonable value
- Magnetar flare energy distribution: $dN/dE_{fl} \propto E_{fl}^{-(1.4-2)}$, where the most intense flares are characterized by $10^{47}\,{\rm erg\,s^{-1}}$
- Bulk Lorentz factor: pulsar/magnetar wind are typically characterized by ultrarelativistic bulk speed, $\Gamma_{fl} \sim 10^3$ seems reasonable
- Flare magnetization should be small $\sigma_{fl} << 1$, which is an unusual assumptions (return to this later)
- The termination shock radius is determined by the external pressure, $p_{\rm ext}$ and "averaged" pulsar wind luminosity, $L_{\rm sd}$:

$$R_{\rm ts} \approx \sqrt{\frac{L_{\rm sd}}{4\pi c p_{\rm ext}}} \sim 10^{15} L_{\rm sd,34}^{1/2} p_{\rm ext,-8}^{-1/2}\,{\rm cm}$$

Termination shock radius (SNR evolutionary phase)

$$R_{ts} \approx \sqrt{\frac{L_{sd}}{4\pi c p_{ext}}} \sim 10^{15} L_{sd,34}^{1/2} p_{ext,-8}^{-1/2}\ \mathrm{cm}$$

Is that a reasonable estimate?

- The external pressure can be higher if the magnetar in a binary system (Yoneda+2020). In this case the shock stands closer and this increases the frequency of synchrotron MASER emission (see Barkov&Popov2022)
- For isolated magnetars the PWN surrounding pressure is determined by the state of the SNR
- During early phase, the SNR reverse shock hasn't reached the SNR center, and the magnetar wind may expand almost freely, $R_{ts} >> 10^{15}$ cm, i.e., no GHz MASER emission
- After $\sim 10^3$ yr the reverse shock reaches the centers, and the wind shock is established at

$$R_{ts} \approx 10^{15} L_{sd,35}^{1/2} t_{10.5}^{3/5}\ \mathrm{cm}$$

Termination shock radius (spin-down luminosity)

$$R_{ts} \approx 10^{15} L_{sd,35}^{1/2} t_{10.5}^{3/5} \quad \mathrm{cm}$$

Is that a reasonable normalization?

- To obtain the "averaged" wind luminosity one needs to account for the contribution from the steady wind and for the averaged contribution from the flares if the shock recovery time is long compared to the typical delay between the flares:

$$t_{rec} \sim 3\Delta R/c \sim 3\Delta t_{fl}\sqrt{\frac{L_{fl}}{L_{sd}}}$$

$$L_{sd} = L_{sd} + E_{fl}/T_{fl} \approx 10^{35}(E_{fl,42}T_{fl,7}^{-1} + 0.1L_{sd,34})\,\mathrm{erg\,s^{-1}}$$

Thus, for magnetars reside insider a SNR, the time span during which the radius the wind TS is limited to 10^{15} cm is about 3 kyr. Which is consistent with the age upper limit of $10^{2.5}$ yr derived by Waxman(2017) for persistent radio source associated with FRB 121102.

How many Magnetars and FRB in the Universe?

Number of magnetars is proportional to the star formation rate:

$$\psi(z) = 0.015 \frac{(1+z)^{2.7}}{1+[(1+z)/2.9]^{5.6}} M_\odot \, yr^{-1} Mpc^{-3}$$

(from Madau&Dickinson 2014)

Flare energies obey a power-law distribution: $dN = AE_{fl}^{-5/3} dE_{fl}$, where A is normalized by the total energy release of 10^{48} erg (for a fixed $E_{max} = 10^{48}$). One can detect FRBs from magnetar flares exceeding the limit (FRB takes 1% of energy)

$$E_{lim} \approx 10^{40} \left(\frac{S_{lim}}{0.1\,\mathrm{Jy}}\right)\left(\frac{D_L}{1\,\mathrm{Gpc}}\right)^2 \left(\frac{\Omega}{4\pi\,\mathrm{sr}}\right) \mathrm{erg}$$

This predicts $\sim 5 \cdot 10^8$ detectable FRBs per day. The real rate is $< 10^4$, thus this estimate overshoot the number by a factor of 10^5! If the typical active life-time of a magnetar is 30 kyr, then only 10% of them can produce FRB because of SNR evolutionary phase. The remaining 10^4 factor is consistent with the polar cap relative size: $\sim \frac{R_M}{2R_{lc}} \sim 10^{-4} P_0^{-1}$. Weakly magnetized flares?

Synchrotron MASER Emission at the Reverse Shock

$$\omega_{\mathrm{max,rs}} \approx 3 \cdot 10^9 \frac{L_{fl,45}^{3/4}}{L_{sd,35}^{1/4} R_{ts,15} \Gamma_{fl,3} \sigma_{fl,-2}^{1/4}} \, \mathrm{s}^{-1}$$

Check of the parameter normalizations:

- Wind luminosity: $10^{35}\,\mathrm{erg\,s}^{-1}$ seems a reasonable low-limit value value
- Magnetar flare energy distribution: $dN/dE_{fl} \propto E_{fl}^{-(1.4-2)}$, where the most intense flares are characterized by $10^{47}\,\mathrm{erg\,s}^{-1}$
- The termination shock radius is limited to 10^{15} cm during $3\,\mathrm{kyr}$ for isolated magnetars, i.e., 10% of magnetar active life
- Flare magnetization should be small $\sigma_{fl} << 1$, which is an unusual assumptions. However, flares whose origin at the polar cap of magnetar may escape the magnetosphere without accumulating large magnetic field. I.e., only 10^{-4} flares can generate FRBs
- How to make a FRB at $R \sim 10^{15}$ cm? Duration? Energy spread?

Relativistic blast wave emission

- emission is boosted into a cone of $1/\Gamma$
- ① is detected at τ_1
- ② is detected at τ_2
- geometry gives the signal spread:

$$\tau_2 - \tau_1 \approx \frac{R}{2c\Gamma^2}$$

- Emission lasts T

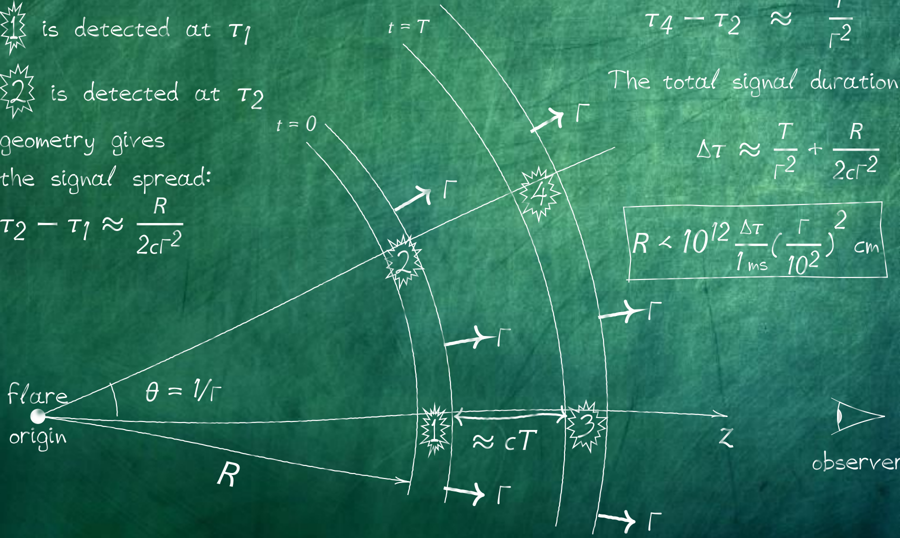
$$\tau_3 - \tau_1 \approx \frac{T}{2\Gamma^2}$$

$$\tau_4 - \tau_2 \approx \frac{T}{\Gamma^2}$$

The total signal duration

$$\Delta\tau \approx \frac{T}{\Gamma^2} + \frac{R}{2c\Gamma^2}$$

$$\boxed{R < 10^{12} \frac{\Delta\tau}{1\,\mathrm{ms}} \left(\frac{\Gamma}{10^2}\right)^2 \mathrm{cm}}$$



What are the underlying assumption?

The consideration in the previous slide is based on

- Geometry ✔
- Special relativity ✔
- Emission is isotropic in the plasma comoving frame ✘

Synchrotron MASER is stimulated emission, i.e., the radiation probability is proportional to the number of photons in the given state. With this one can generate exponentially amplified radiation component. What could be reasons for anisotropy?

- Production region geometry (extension in one direction is significantly larger that in other) ✔
- Strongly anisotropic source of external photons ?

Is Synchrotron MASER anisotropic?

- Production region geometry (extension in one direction is significantly larger that in other) ✔
  It is natural expect that the shock region is significantly shorter in radial direction compared to its lateral extension (especially for large R)
- Strongly anisotropic source of external photons ?
  In the astrophysical context it is natural to use the semi-classical approximation to describe quantum process. Thus the photon states occupation numbers are determined by the photon distribution function in the phase space: $\bar{n}_p = 4\pi^3\hbar^3 f(t,r,p)$. Using the standard relation of the distribution function and emission brightness we obtain: $\bar{n}_p = \frac{c^2 I_\nu}{4\pi\hbar\nu^3}$, i.e., it doesn't depend on the distance to the source! Thus the process might be highly anisotropic $n_{mgs} >> n_{ns} >> n_{pwn}$

Relativistic blast wave emission

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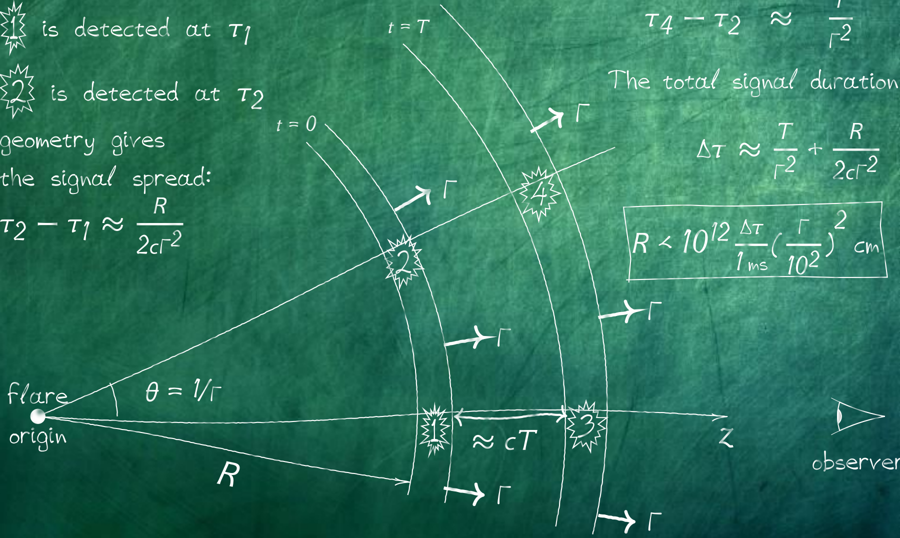

How to get robustly distribution of MASER emission

If the electron cooling is not important, one can simply multiply the distribution of external stimulating photons by the exponential term that depends on the (negative) absorption coefficient and the linear extension of the production region. However, it is clear (exponential factors are too large) that this doesn't provide a reasonable description and one needs to consider processes regulating the electron cooling. Using the standard methods of quantum statistics one can obtain a system of equations for averaged occupation numbers. Under the semi-classical approximations these equations are reduced to

$$\frac{d\bar{n}(|k\rangle)}{dt} = -\bar{n}(|k\rangle)\int d^3p_1 k\frac{\partial\tilde{w}_0}{\partial p_1}\bar{m}(|p_1\rangle)$$

$$\frac{d\bar{m}(|p\rangle)}{dt} = \int d^3k\,\bar{n}(|k\rangle)k\frac{\partial^2\tilde{w}_0}{\partial p\partial k}\frac{\partial\bar{m}(|p\rangle)}{\partial p}$$

Here $\bar{n}(|k\rangle)$ and $\bar{m}(|p\rangle)$ are averaged occupation numbers for corresponding single particle states; and $\tilde{w}_0$ is determined by the probably of corresponding elementary transition.

Summary

- High frequency synchrotron MASER emission can be generated in weakly magnetized relativistic plasma
- While the MASER emission from shock wind appears at relatively low frequencies (kHz), the reverse shock of powerful magnetar flare provides condition for generation of stimulated emission in the GHz band
- Statistical analysis of star formation activity and FRB rate suggests that this mechanism can be responsible for a significant fraction of FRB if one assumes that weakly magnetized flares are generated in the polar cap region
- In framework of this scenario one can obtain the short duration of FRB and narrow frequency interval, if the MASER emission is strongly anisotropic in the plasma comoving frame, which can be caused by bright external sources of stimulating photons or large aspect ratio of the production region