Probing the Universe through the Stochastic GW Background

Towards optimal detection

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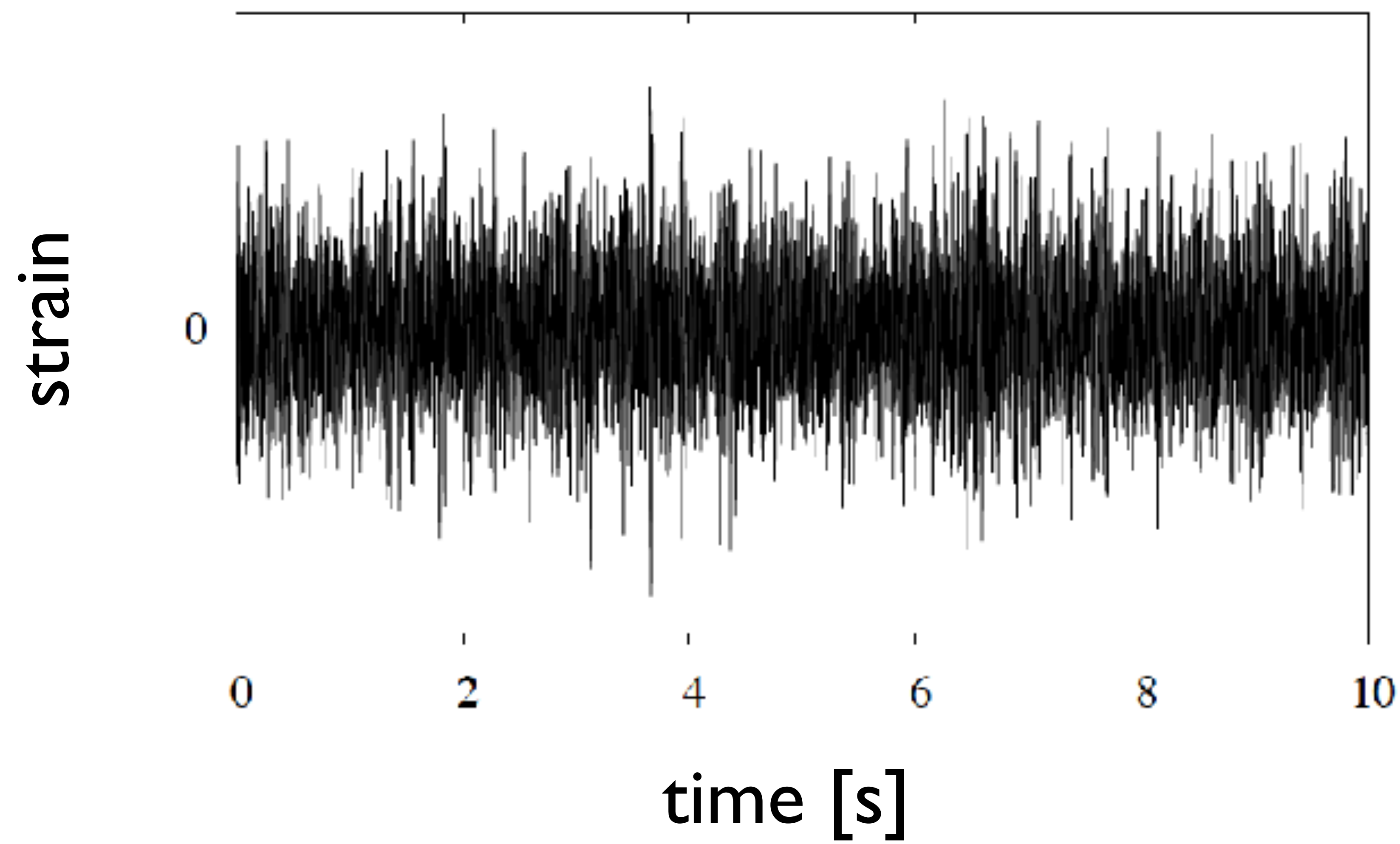
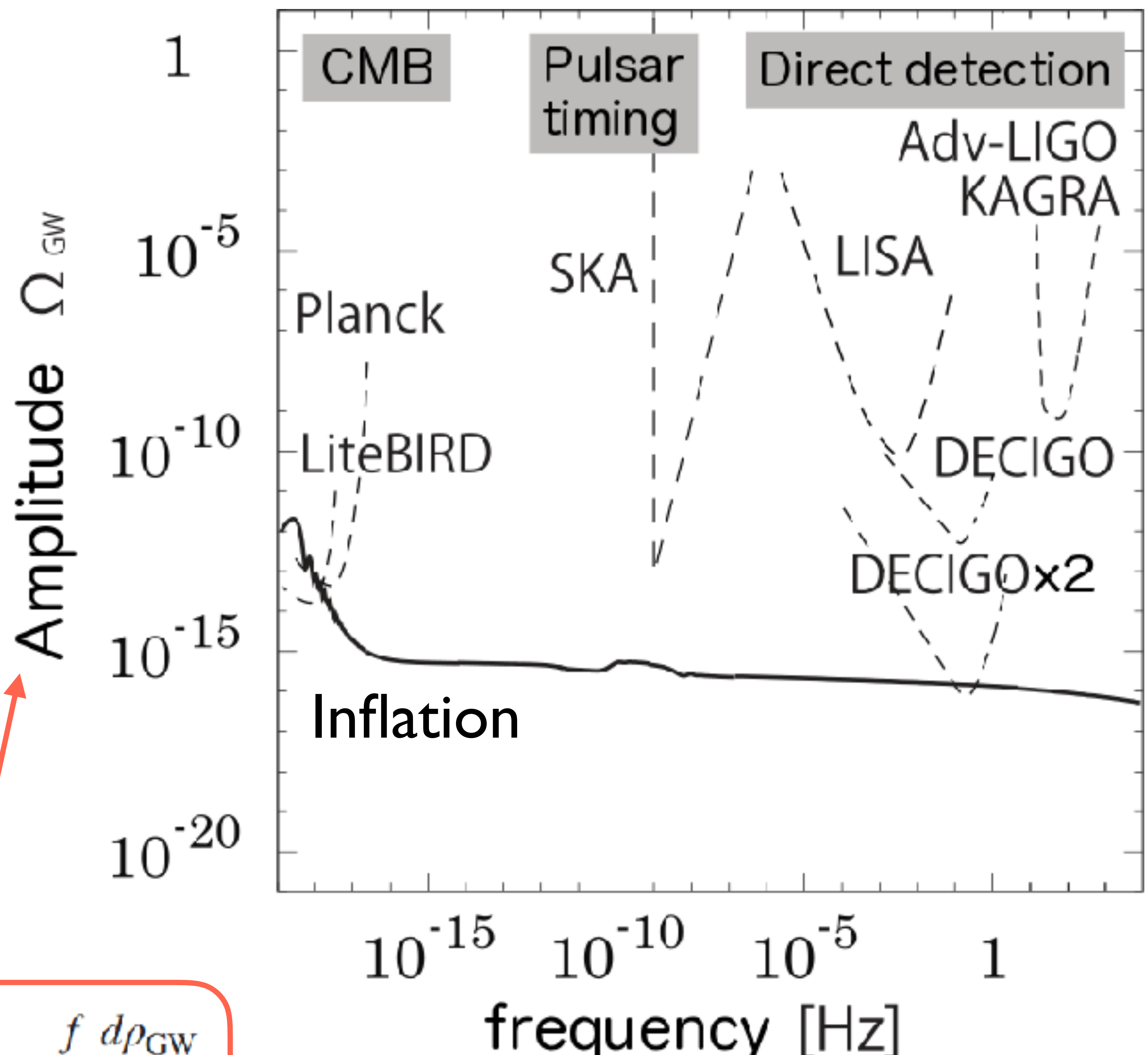
Stochastic GW background



random phase & no directional dependence

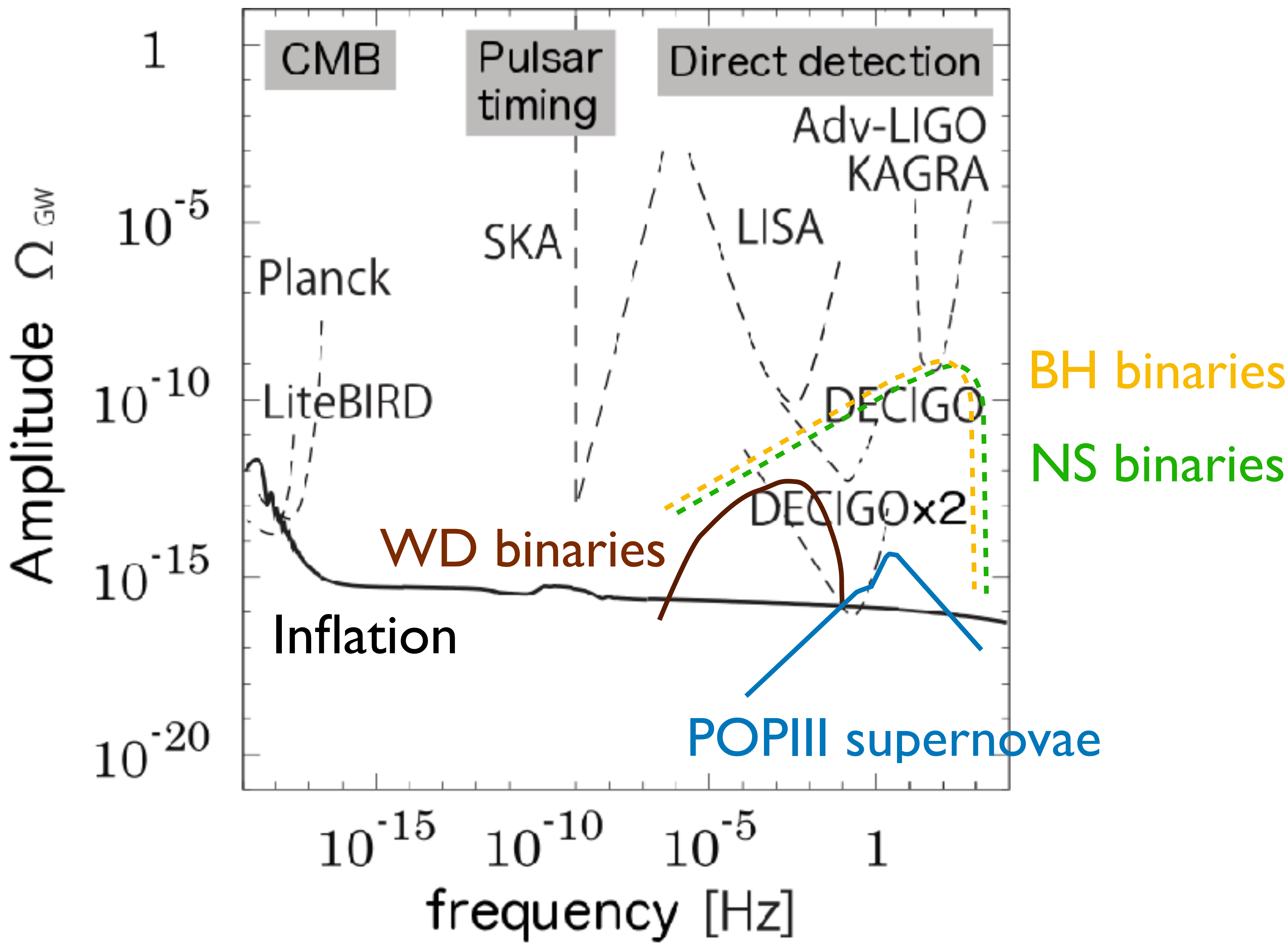
- Overlapped astrophysical GWs $\Delta T < f^{-1}$
- GWs from the early Universe

Sensitivity curves for GW background

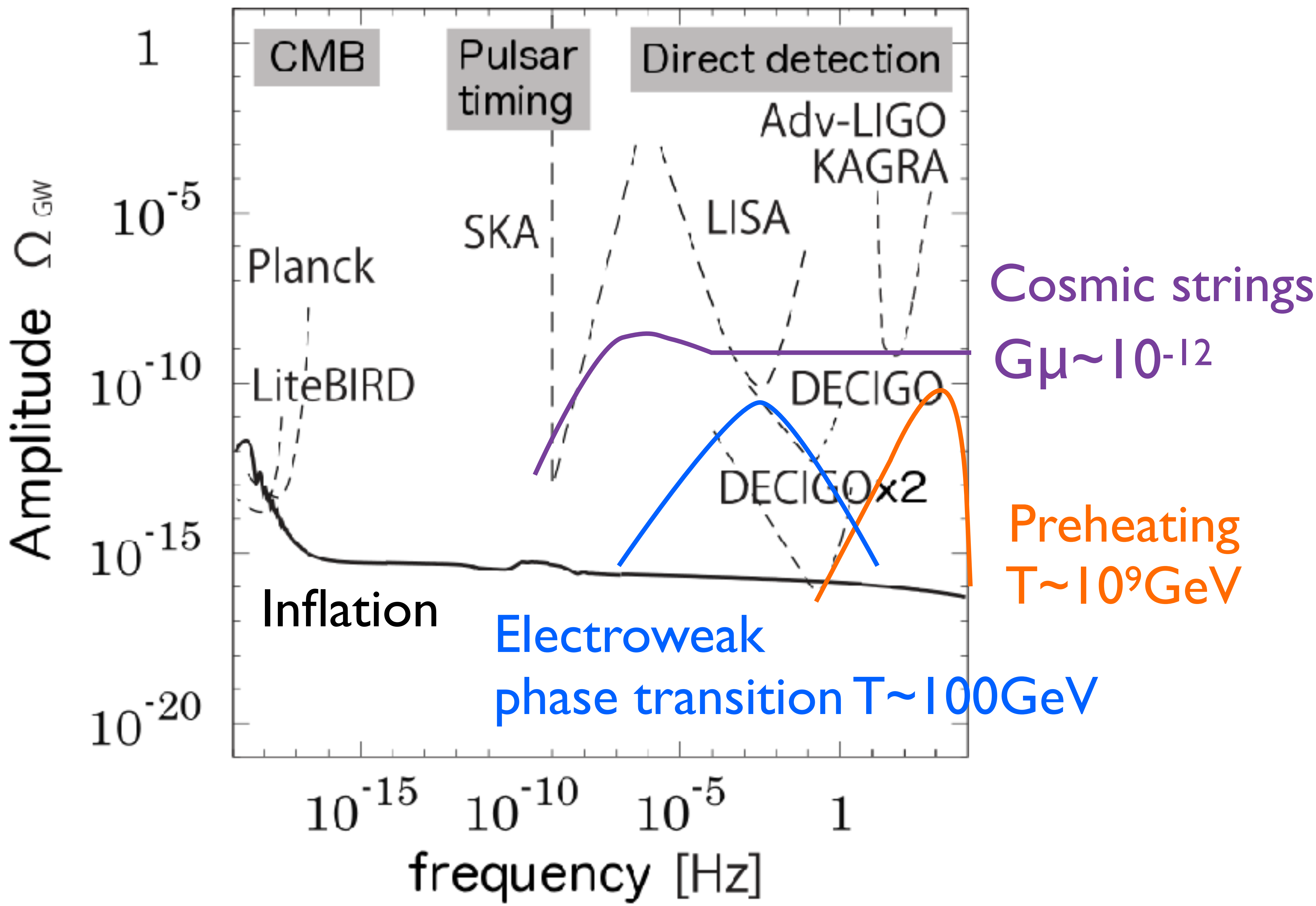


$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df}$$

Astrophysical GW background



Cosmological GW background



How to detect a stochastic background

Cross Correlation

detector1	↔	detector2
$s_1(t) = h(t) + n_1(t)$		$s_2(t) = h(t) + n_2(t)$

s: observed signal
h: gravitational waves
n: noise

$$\langle S \rangle = \int_{-T/2}^{T/2} dt \, \langle s_1(t) s_2(t) \rangle$$

$$= \int_{-T/2}^{T/2} dt \, \langle h^2(t) + \underline{h(t) n_2(t) + n_1(t) h(t) + n_1(t) n_2(t)} \rangle$$

no correlations → 0

$$= \int_{-T/2}^{T/2} dt \, \underline{\langle h^2(t) \rangle}$$

GW signal

(for detector at the same location)

Optimal filtering

Ref. Allen & Romano, PRD 59, 102001 (1999)

$$S := \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \, s_1(t) s_2(t') \underline{Q(t,t')}$$

filter function

<u>Signal in Fourier space</u>

$$\mu := \langle S \rangle = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \, \delta_T(f-f') \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle \tilde{Q}(f')$$

<u>Noise in Fourier space</u>

$$\sigma^2 := \langle S^2 \rangle - \langle S \rangle^2 \approx \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \, \delta_T(f-f') \delta_T(k-k') \langle \tilde{n}_1^*(f) \tilde{n}_1(-k) \rangle \langle \tilde{n}_2^*(-f') \tilde{n}_2(k') \rangle \tilde{Q}(f') \tilde{Q}(k')$$

<u>Signal-to-noise ratio</u>

$$\mathrm{SNR}^2 = \frac{\mu^2}{\sigma^2} \approx \left(\frac{3H_0^2}{10\pi^2} \right)^2 T \frac{\left(\underline{\tilde{Q}}, \frac{\gamma(|f|)\Omega_{\mathrm{gw}}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)} \right)^2}{\underline{(\tilde{Q}, \tilde{Q})}}$$

$$(A,B) := \int_{-\infty}^{\infty} df \, A^*(f) B(f) P_1(|f|) P_2(|f|)$$

Maximized when

$$\tilde{Q}(f) = \lambda \frac{\gamma(|f|)\Omega_{\mathrm{gw}}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}$$

$P_i(|f|)$:noise spectrum

$$\langle \tilde{n}_i^*(f) \tilde{n}_i(f') \rangle = \frac{1}{2} \delta(f-f') P_i(|f|)$$

$\gamma(|f|)$:overlap reduction function (determined by detector positions)

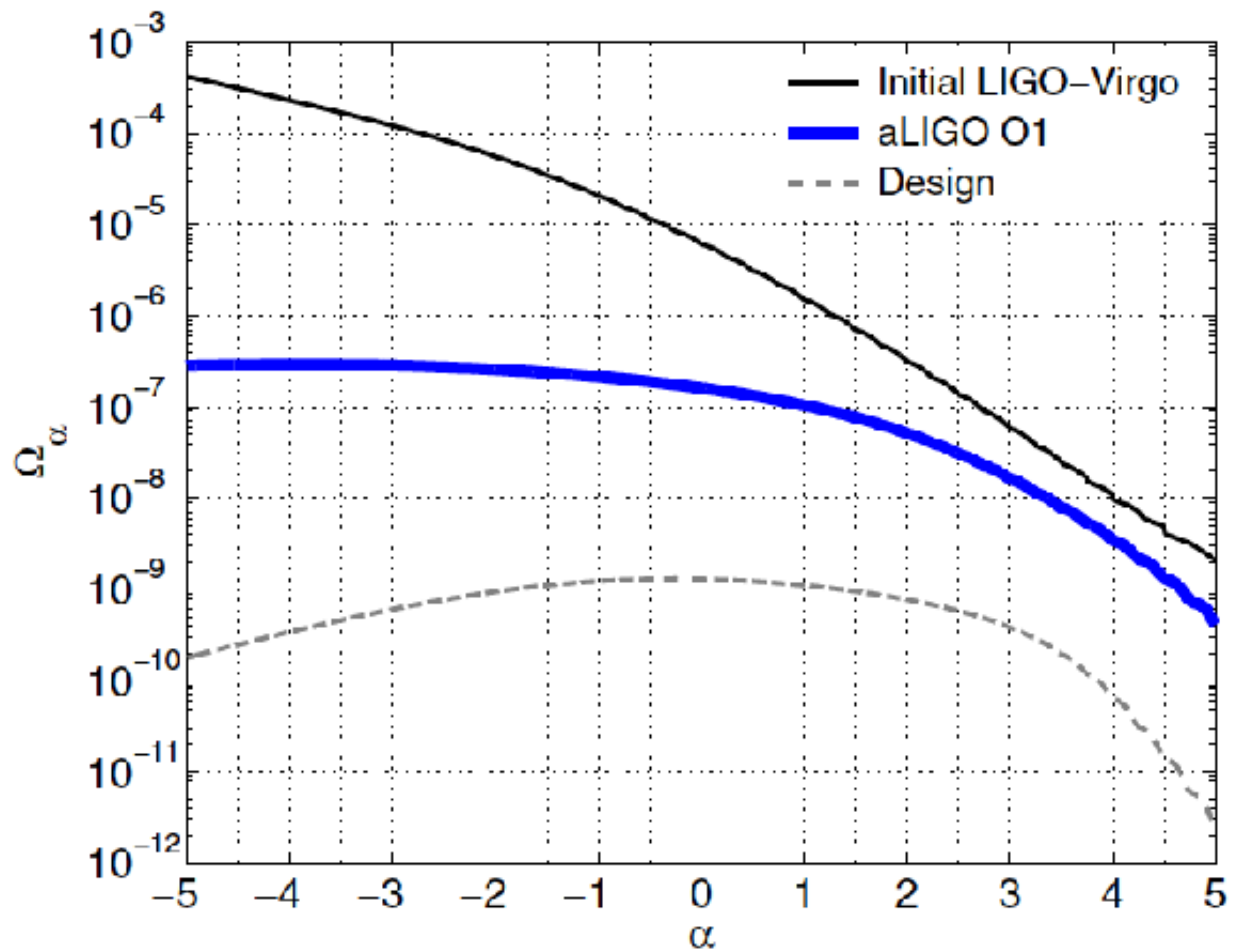
We need template
= spectral shape

"Upper Limits on the Stochastic Gravitational-Wave Background from Advanced LIGO's First Observing Run", LIGO & Virgo Collaboration, PRL. 118, 121101 (2017)

parametrized by
a single power law

$$\Omega_{\rm GW}(f) = \Omega_\alpha \left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

$$f_{\rm ref} = 25{\rm Hz}$$

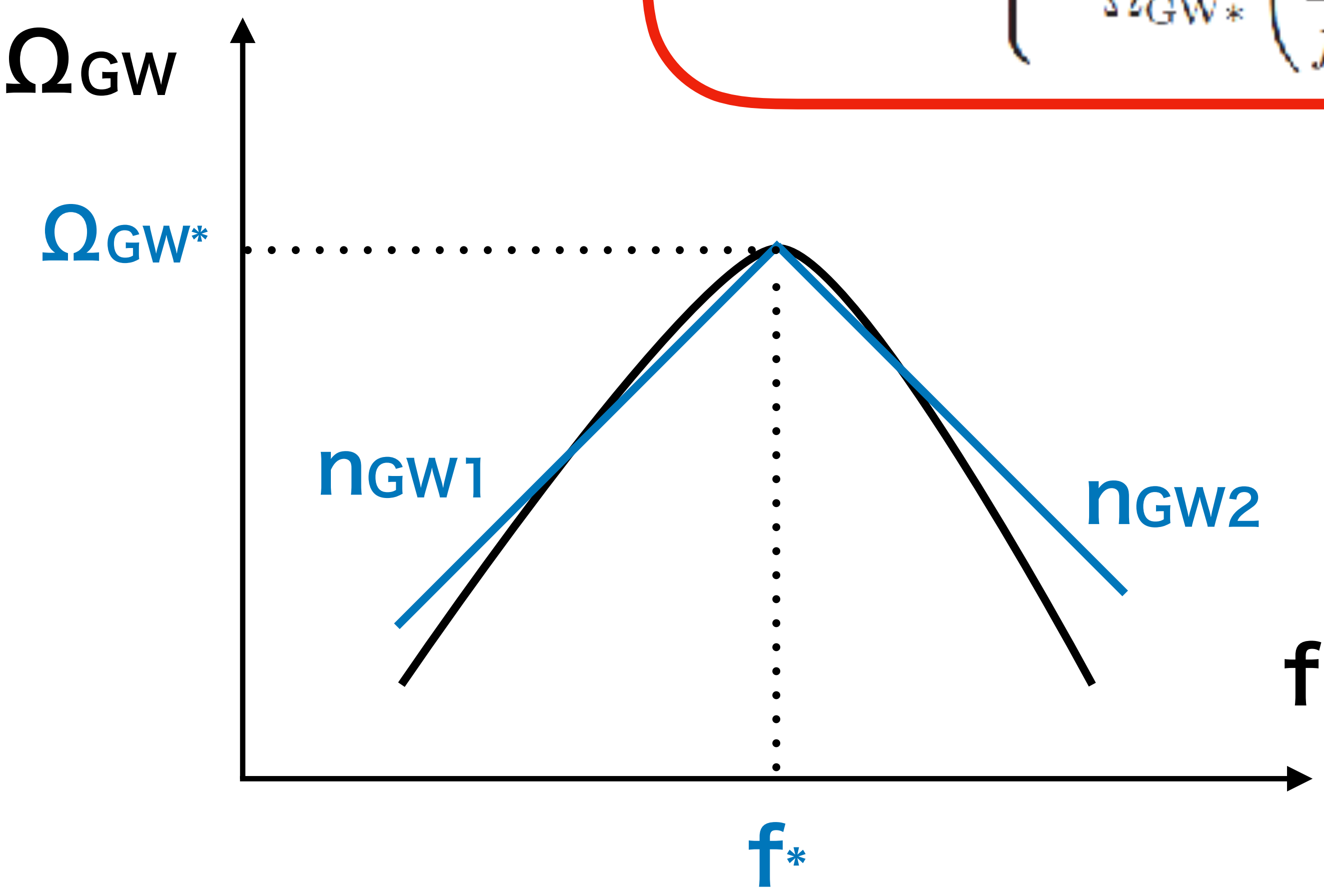


Idea

Many models of stochastic background predict a peaked shape

Is broken-power law better for fitting?

$$\Omega_{GW}(f) = \begin{cases} \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW1}} & \text{for} \quad f < f_*, \\ \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW2}} & \text{for} \quad f > f_*, \end{cases}$$



<u>example</u>

- Phase transition

  $n_{GW1}=3$, $n_{GW2}=-2$
- Preheating

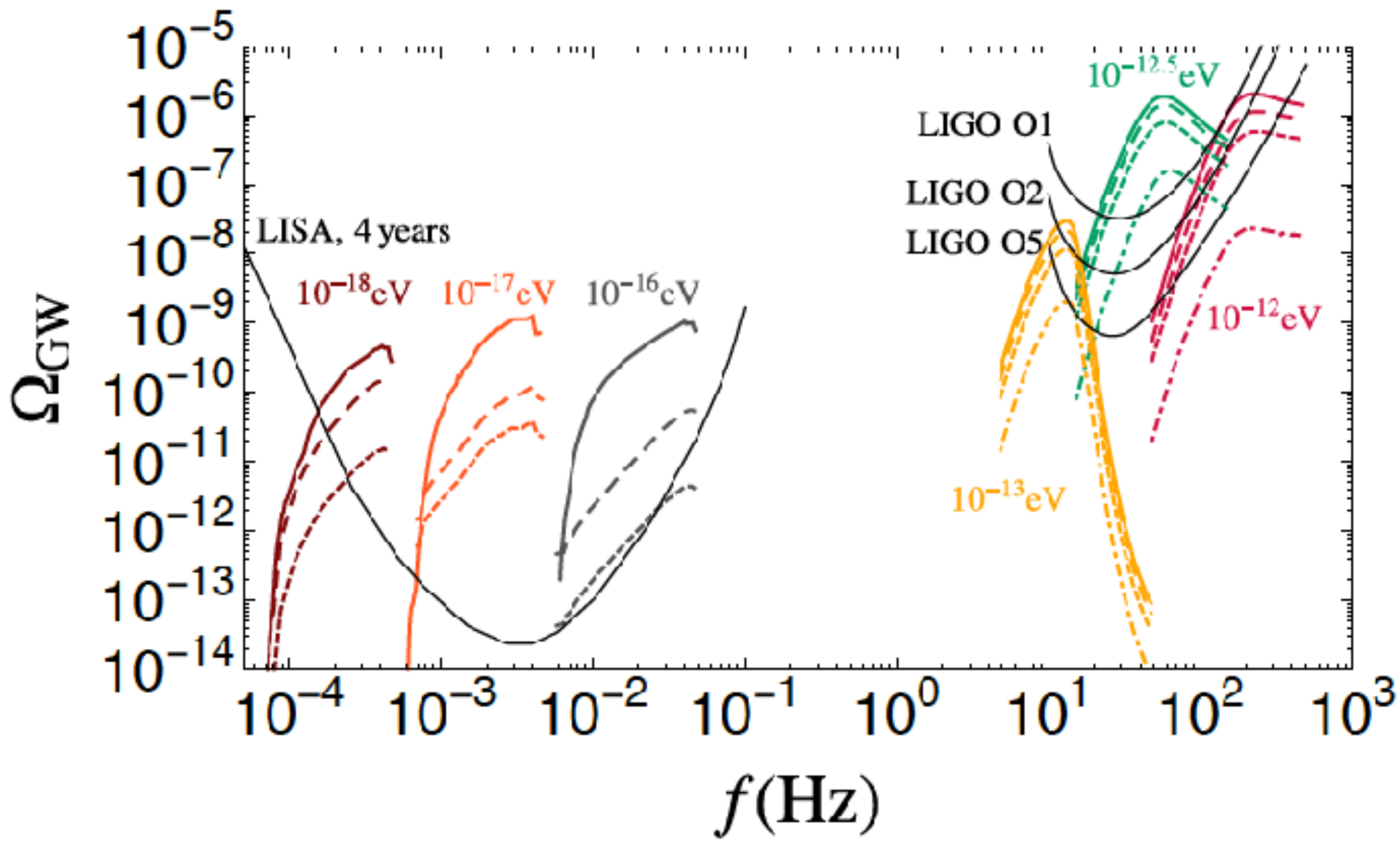
  $n_{GW1}=3$, n_{GW2}: exponential cutoff

$f_* \sim$ energy scale of the event

Spectral shape is important information to identify generation mechanism

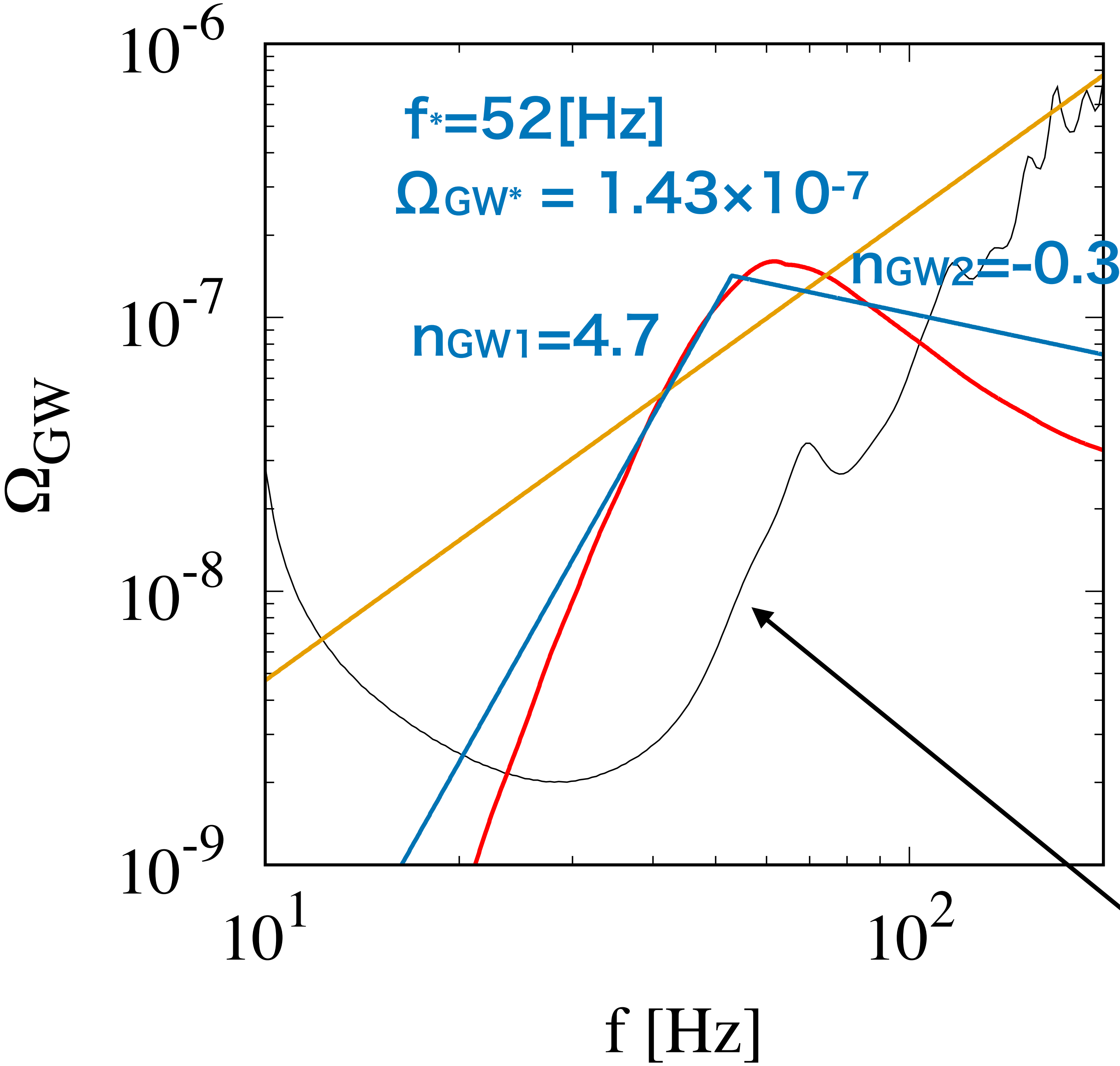
Example

GWB from superradiant instabilities
(Ultralight scalar fields around spinning black holes)



"Stochastic and resolvable gravitational waves from ultralight bosons"
Brito et al. PRL 119, 131101 (2017)

Template fitting



← single power-law fitting

$n = 2.3$

Ω_{GW^*} (at 25Hz) = 1.25×10^{-8}

← broken power-law fitting

← true signal

LIGO+VIRGO+KAGRA design

single: SNR=70.7, $\delta\chi^2$=1440

broken: SNR=80.0, $\delta\chi^2$=47.4

(perfect template: SNR=80.3)

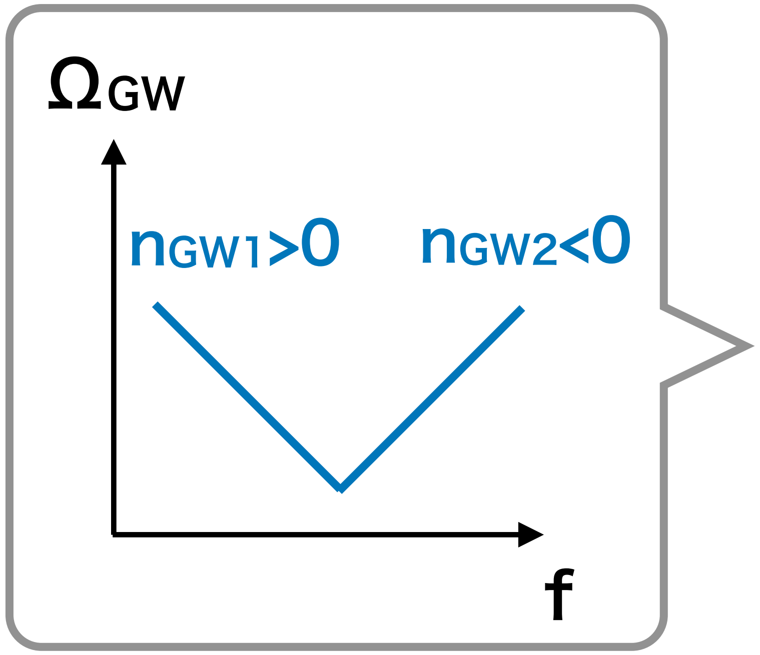
~10% loss of signal-to-noise ratio →

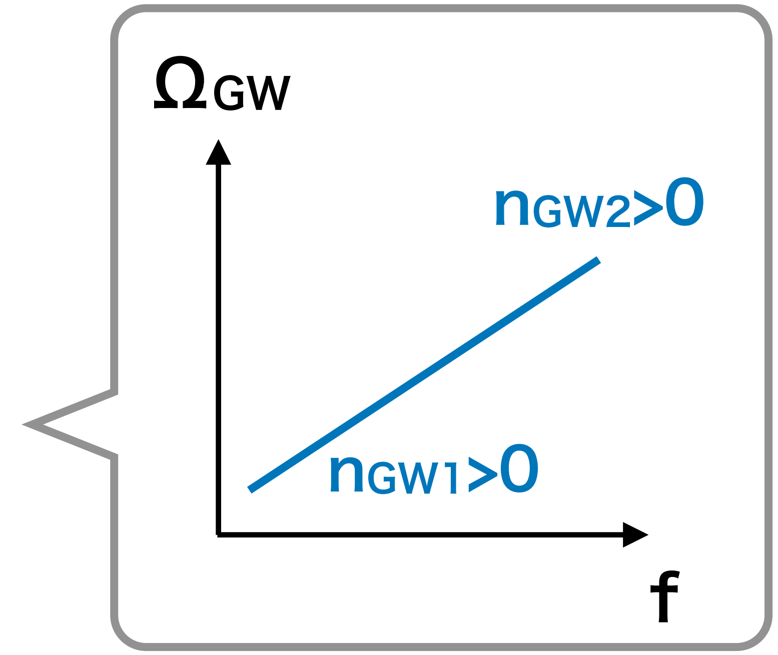
$\delta\chi^2$ shows single is bad fit

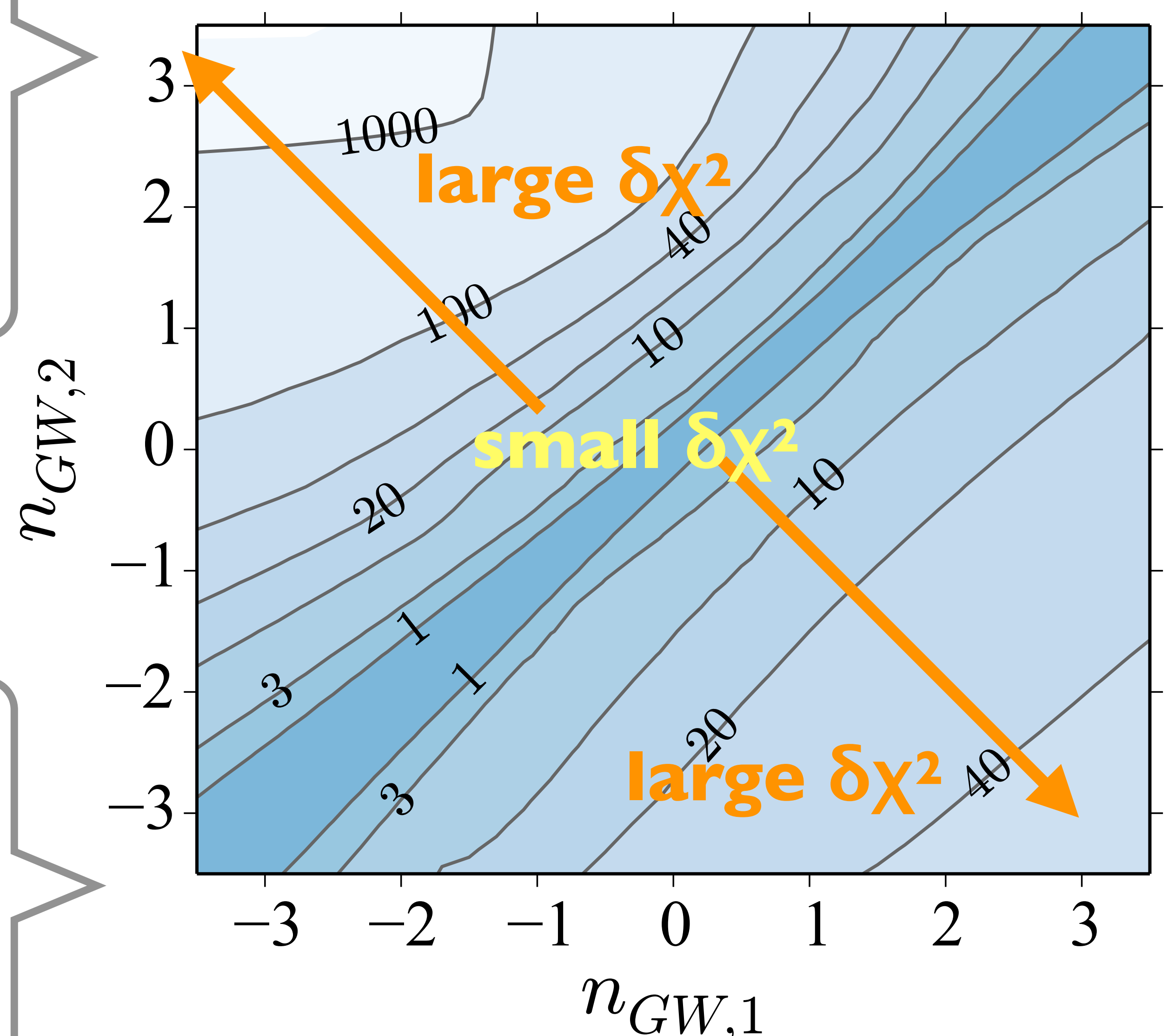
$\delta\chi^2_{single} - \delta\chi^2_{broken}$

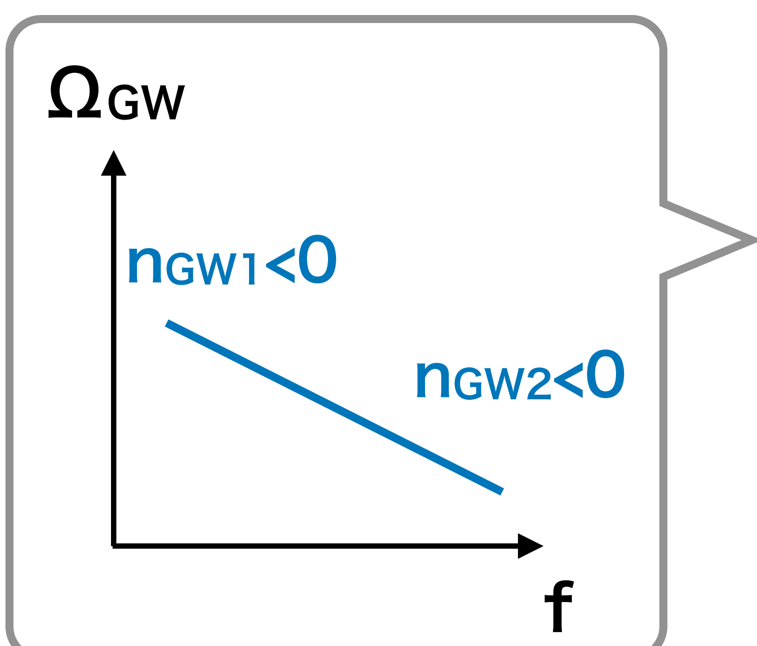
$\Omega_{GW*} = 10^{-8}$

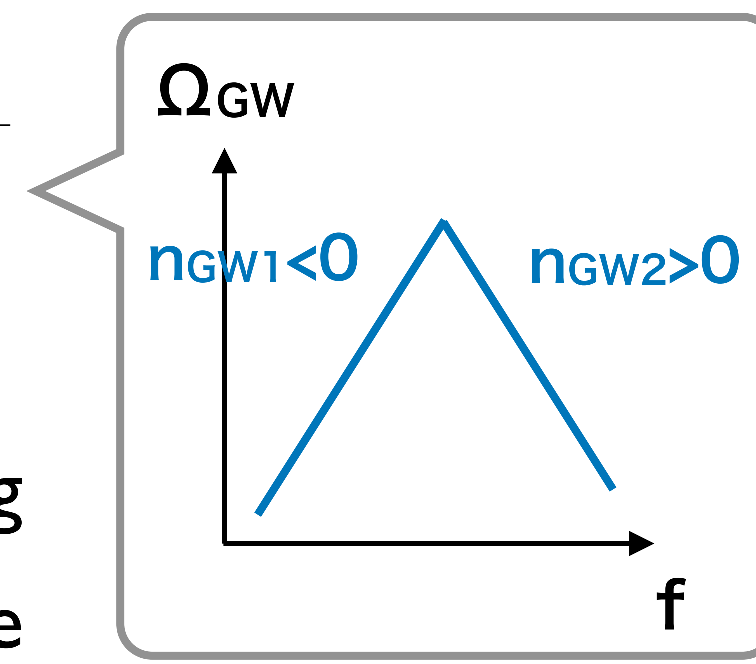
$f_* = 30\text{Hz}$









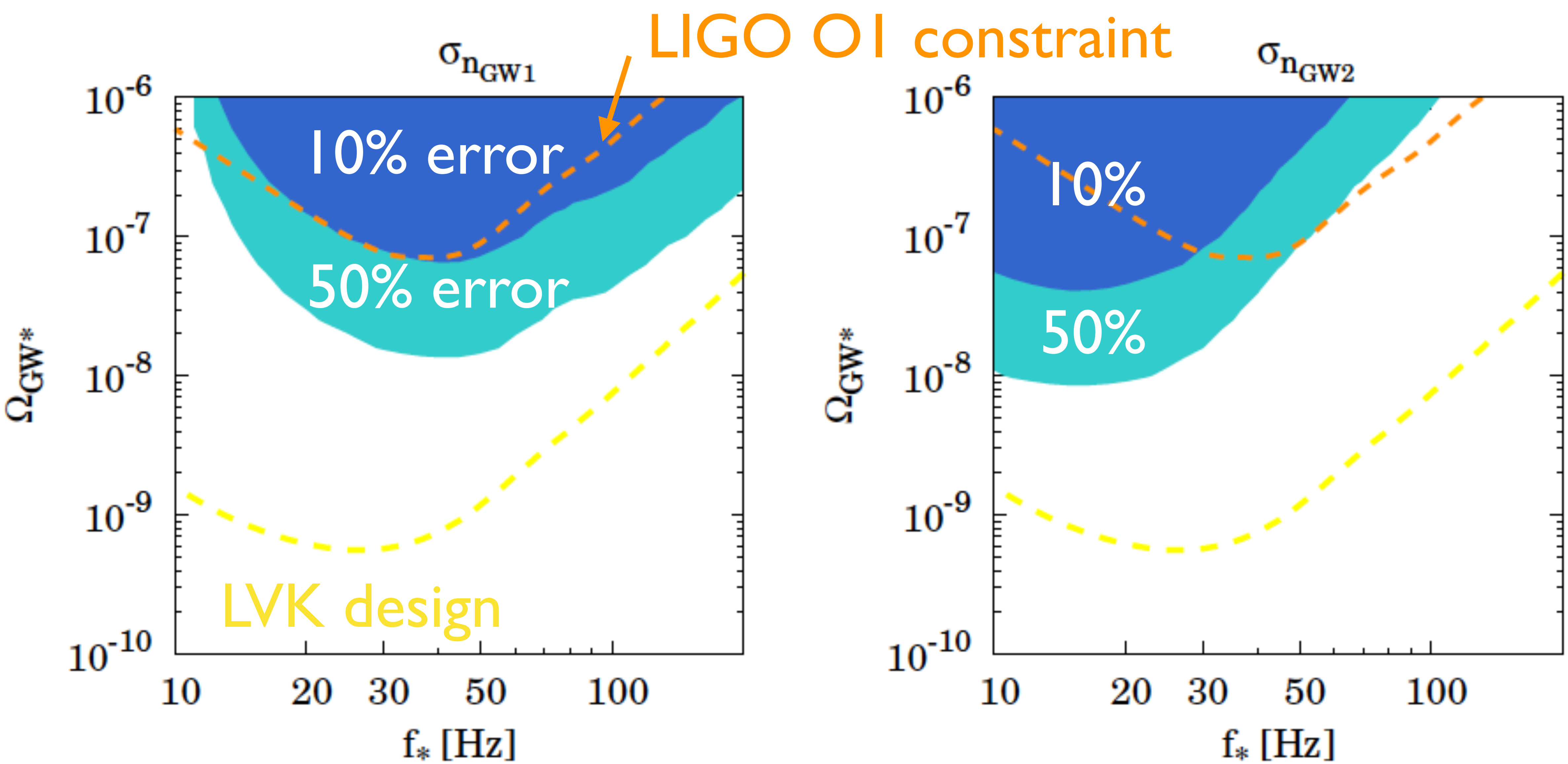


Broken power-law improves fitting

→ better measurement of shape

How accurately can we measure the tilt?

Prediction by Fisher analysis for $n_{GW1} = 3$, $n_{GW2} = -2$



$n_{GW1} = 3.0 \pm\ ?$ $n_{GW2} = -2.0 \pm\ ?$

1. Large amplitude is necessary to measure the tilt
2. The error also depends on the peak position

How accurately can we measure the tilt?

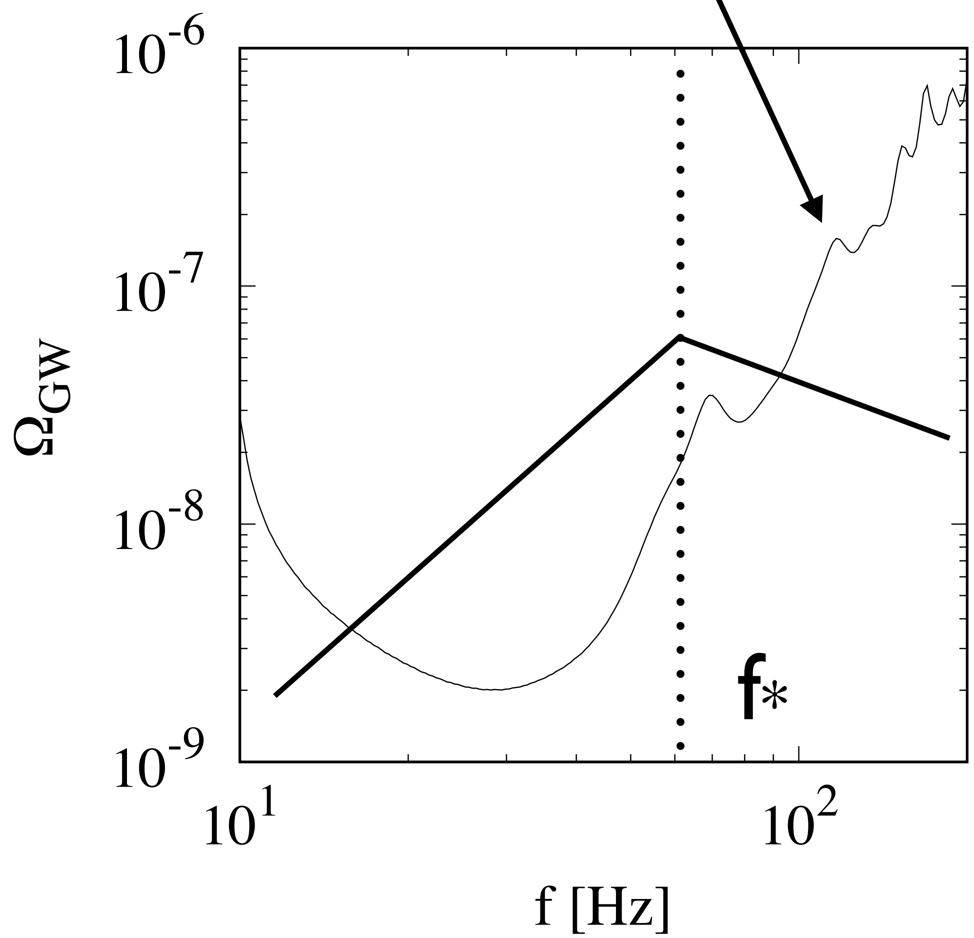
$\sigma_{n_1, n_2} \propto \mathrm{SNR}^{-1}$

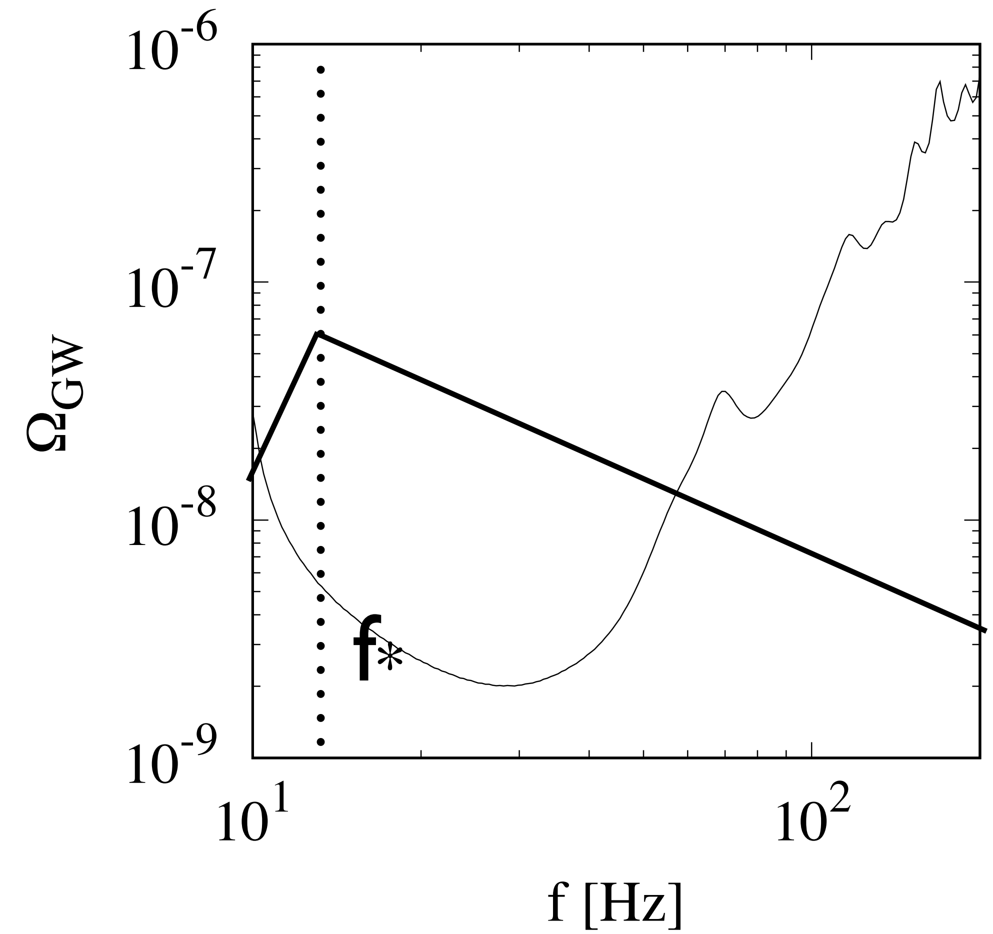
$$\mathrm{SNR} \approx \frac{3H_0^2}{10\pi^2}\sqrt{T}\left[\int_{-\infty}^{\infty} df \frac{\gamma^2(|f|)\Omega_{gw}^2(|f|)}{f^6 P_1(|f|)P_2(|f|)}\right]^{1/2}$$

Sensitivity curve $\propto \frac{10\pi^2}{3H_0^2}\left[\frac{f^5 P_1(|f|)P_2(|f|)}{T\Delta\log f\gamma^2(|f|)}\right]^{1/2}$

integration in frequency domain

SNR>2 for in each frequency bin Δlogf = 0.1

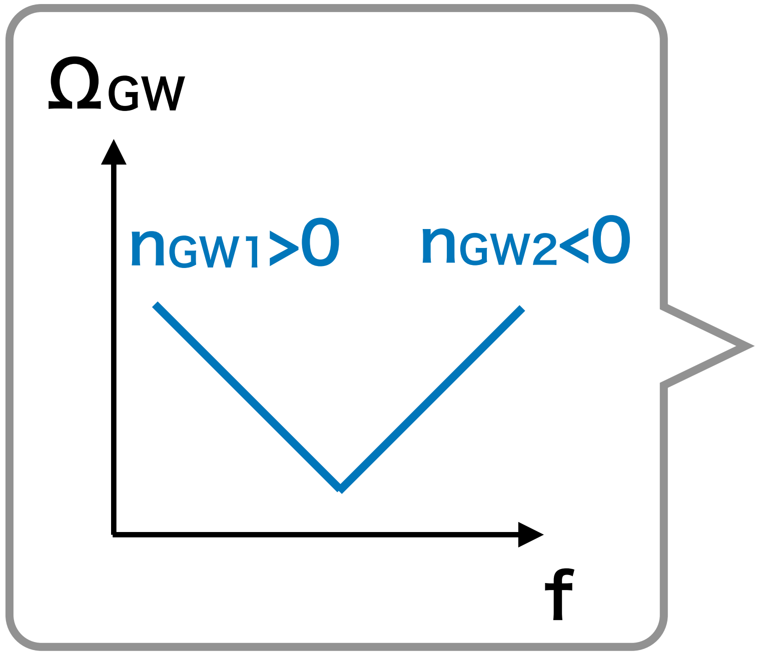




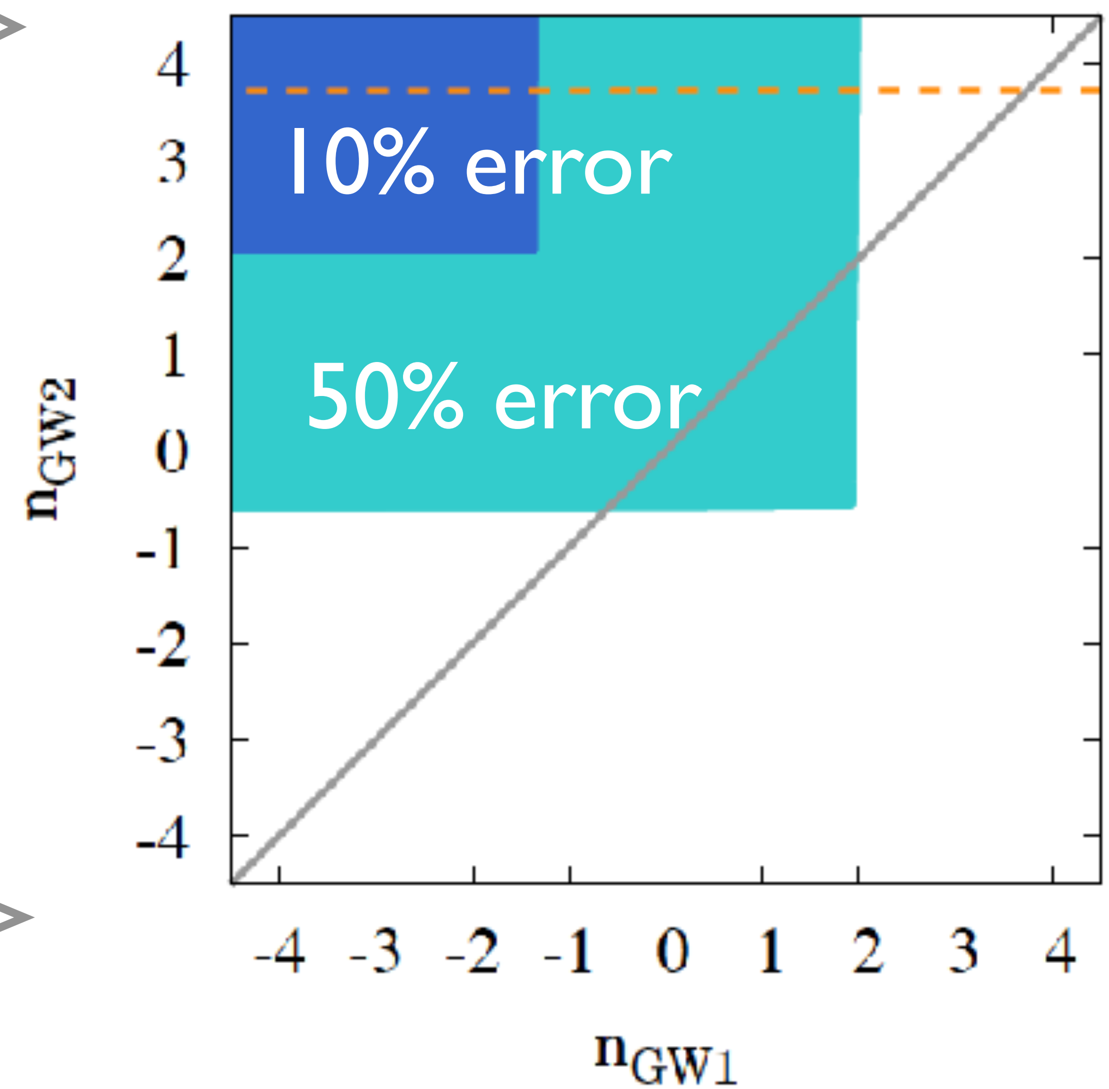
n_{GW1} is determined accurately

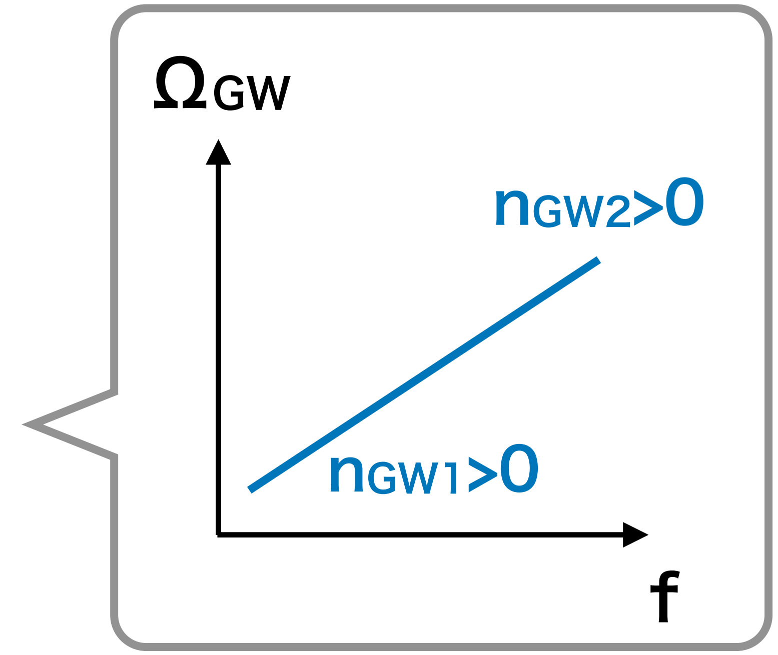
n_{GW2} is determined accurately

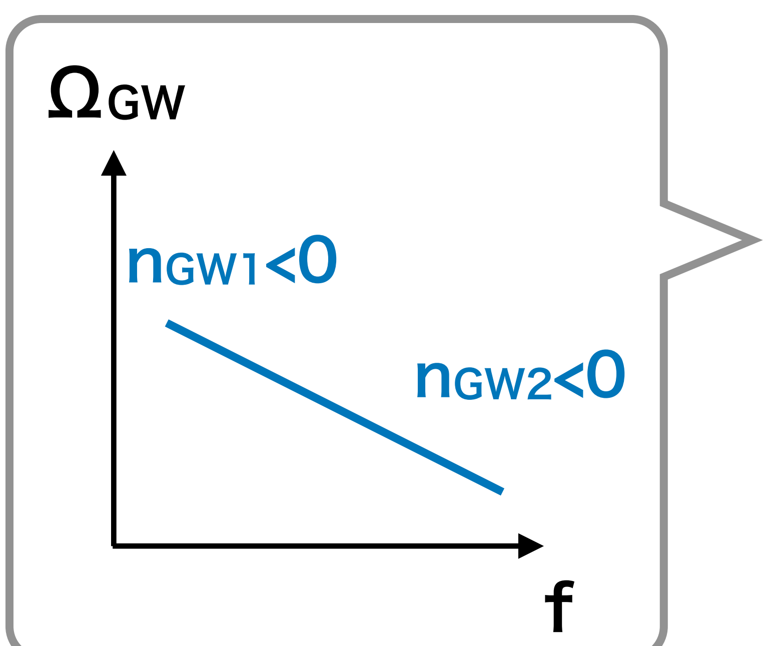
General expectation

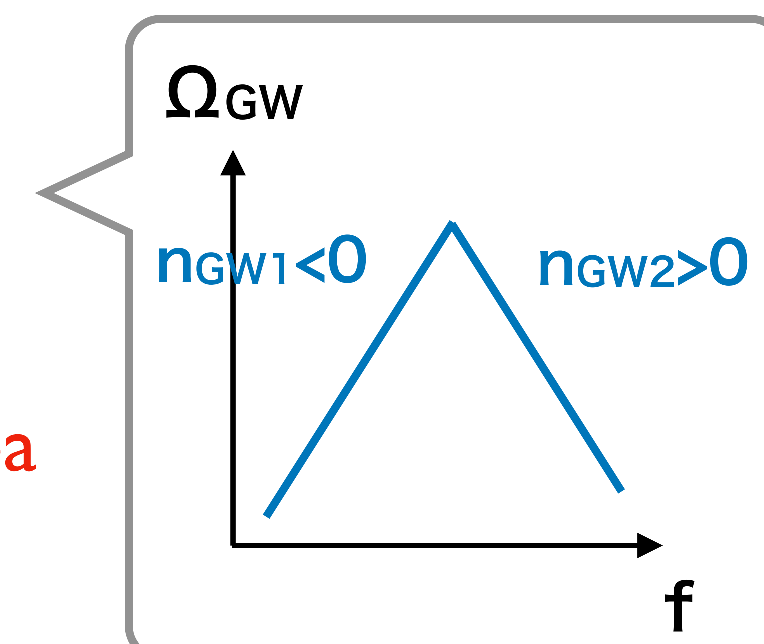


$$\Omega_{GW*} = 10^{-8}$$
$$f_* = 30\text{Hz}$$







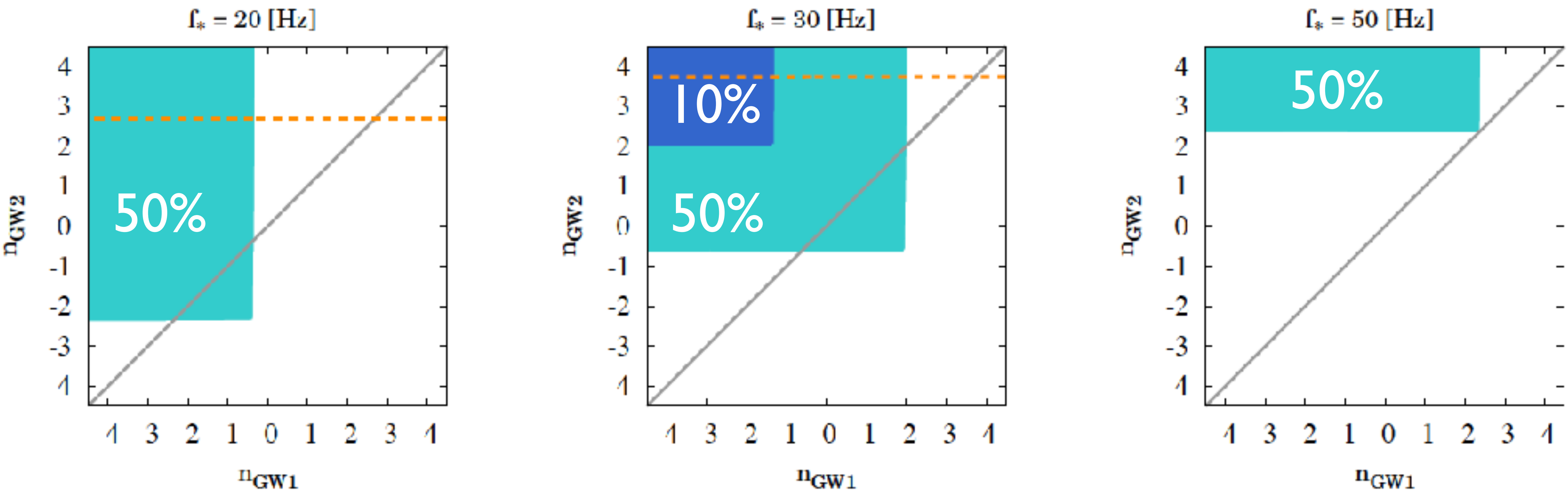


Larger amplitude increases the area

$$\sigma_{n_1,n_2} \propto SNR^{-1} \propto \Omega_{GW*}^{-1}$$

Peak frequency dependence

$$\Omega_{GW*} = 10^{-8}$$



n_{GW2} is determined accurately

n_{GW1} is determined accurately

Discussion

- Fitting by broken power-law is more time consuming

  single: 1free parameter (n_{GW})
  broken: 3 free parameter (n_{GW1}, n_{GW2}, f_*)

- Strategy?
  1. GW search by single power-law
  2. Fitting by broken power-law

  High SNR detection is necessary for the 2nd step

- Same discussion holds for DECIGO
  → More chance to detect GW background

Summary

- Detection of a stochastic GW background is the next challenging step for GW science

- It's searched by matched filtering so we need to prepare templates (= spectral shape)

- We made quantitive estimations on broken-power law fitting and found that it dramatically improves $\delta\chi^2$

- We also made estimation on how accurately the spectral told can be determined. Precise fitting of spectral shape would help to identify the generation mechanism