Constructing ghost-free degenerate theories with higher derivatives

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HM, Noui, Suyama, Yamaguchi, Langlois,
JCAP 1607 (2016) 07, 033, [arXiv:1603.09355]
HM, Suyama, Yamaguchi, [arXiv:1711.08125]; in preparation

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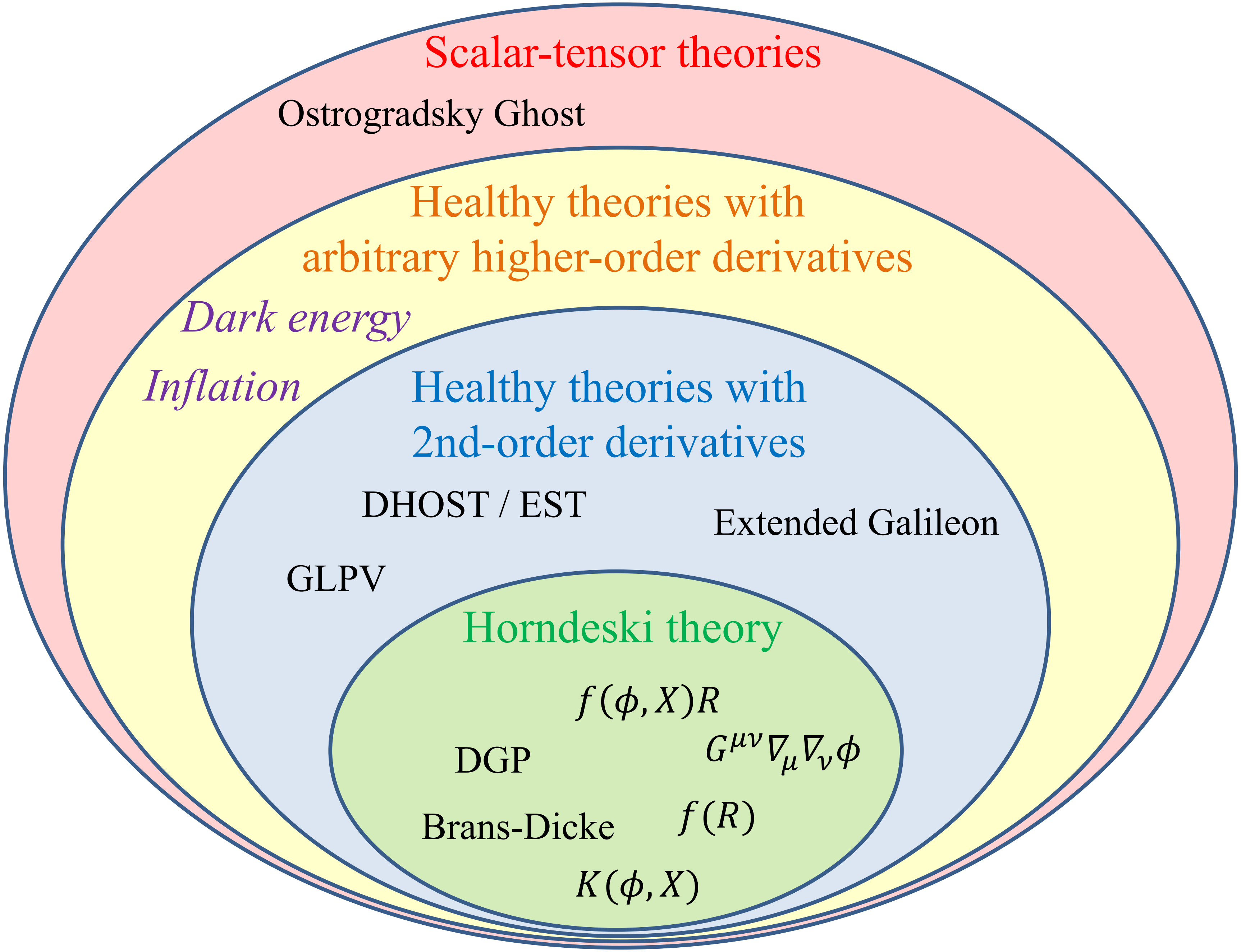
Scalar-tensor theories
Ostrogradsky Ghost
Healthy theories with
arbitrary higher-order derivatives
Dark energy
Inflation
Healthy theories with
2nd-order derivatives
DHOST / EST
Extended Galileon
GLPV
Horndeski theory
$f(\phi, X)R$
DGP
$G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$
Brans-Dicke
$f(R)$
$K(\phi, X)$

Ostrogradsky theorem for $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

Woodard, 1506.02210

- $L = L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$
- $\phi^a = \phi^a(t)$ and $a = 1, \cdots n$
- $K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$ (kinetic matrix)

✓ Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

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$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

- Hamiltonian analysis

$$L_{eq} = L\left(\dot{Q}^a, Q^a, \phi^a\right) + \lambda_a (Q^a - \dot{\phi}^a)$$

$\dot{Q}^a = \ddot{\phi}^n$, $Q^a = \dot{\phi}^n$

$\dot{\phi}^a = Q^a$

$$(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \boldsymbol{\lambda_a}) \leftrightarrow (\boldsymbol{P_a}, \boldsymbol{\pi_a}, \boldsymbol{\rho^a})$$

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

✓ Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

- Hamiltonian analysis

$$L_{eq} = L\left(\dot{Q}^a, Q^a, \phi^a\right) + \lambda_a (Q^a - \dot{\phi}^a)$$

$\dot{\phi}^a$ $=$ $(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \lambda_a)$

$\updownarrow$

$(\boldsymbol{P_a}, \boldsymbol{\pi_a}, \rho^a)$

Canonical momenta

$$\begin{cases} P_a = \dfrac{\partial L}{\partial \dot{Q}^a} \\ \pi_a = -\lambda_a \\ \rho^a = 0 \end{cases}$$

P_a → $\det\left(\frac{\partial P_a}{\partial \dot{Q}^b}\right) \neq 0$

$\Longrightarrow \dot{Q}^a = \dot{Q}^a(P, Q, \phi)$

$\pi_a = -\lambda_a$, $\rho^a = 0$ → Primary constraints (C1)

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

✓Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

- Hamiltonian analysis

$$L_{eq} = L\left(\dot{Q}^a, Q^a, \phi^a\right) + \lambda_a\left(Q^a - \dot{\phi}^a\right)$$

$\dot{\phi}^a$

$\parallel$

$(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \lambda_a)$

$\updownarrow$

$(\boldsymbol{P_a}, \boldsymbol{\pi_a}, \rho^a)$

Canonical momenta

$$P_a = \frac{\partial L}{\partial \dot{Q}^a} \rightarrow \det\left(\frac{\partial P_a}{\partial \dot{Q}^b}\right) \neq 0$$

$$\Longrightarrow \dot{Q}^a = \dot{Q}^a(P, Q, \phi)$$

$$\pi_a = -\lambda_a$$

$$\rho^a = 0$$

$\rightarrow$ Primary constraints (C1)

$$\{\pi_a + \lambda_a, \rho^b\} = \delta_a^b$$

$\Longrightarrow$ Second class. No secondary constraints (C2)

$\Longrightarrow$ n healthy + n ghost DOFs

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

✓Ostrogradsky theorem

$\det K \neq 0 \Longrightarrow H$ is unbounded

- Hamiltonian analysis

$$L_{eq} = L\left(\dot{Q}^a, Q^a, \phi^a\right) + \lambda_a (Q^a - \dot{\phi}^a)$$

$\dot{\phi}^a$ $\|$ $(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \lambda_a)$ $\updownarrow$ $(\boldsymbol{P_a}, \boldsymbol{\pi_a}, \rho^a)$

Canonical momenta

$$\begin{cases} P_a = \dfrac{\partial L}{\partial \dot{Q}^a} & \rightarrow \det\left(\dfrac{\partial P_a}{\partial \dot{Q}^b}\right) \neq 0 \\ & \Longrightarrow \dot{Q}^a = \dot{Q}^a(P, Q, \phi) \\ \pi_a = -\lambda_a & \\ \rho^a = 0 & \rightarrow \text{Primary constraints (C1)} \end{cases}$$

Hamiltonian

$$H = H_0(P, Q, \phi) + \pi_a Q^a$$

π_a shows up only linearly. H is unbounded.

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

✓Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

? No-ghost condition

$$K_{ab} = 0 \overset{?}{\Longrightarrow} H \text{ is bounded}$$

✓Ostrogradsky theorem

$\det K \neq 0 \Longrightarrow H$ is unbounded

$\Updownarrow$

✓$K_{ab} = 0 \Longleftarrow H$ is bounded

↕ Different

? No-ghost condition

$K_{ab} = 0 \overset{?}{\Longrightarrow} H$ is bounded

... though it is a part of no-ghost conditions

"1st degeneracy condition" (DC1)

$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

HM, Suyama, 1411.3721

$\dot{\phi}^a$

$||$

$(\boldsymbol{Q}^{\boldsymbol{a}}, \boldsymbol{\phi}^{\boldsymbol{a}}, \lambda_a)$

$\updownarrow$

$(P_a, \boldsymbol{\pi}_{\boldsymbol{a}}, \rho^a)$

$H = H_0 + \pi_a Q^a$

- DC1: $K_{ab} = 0$

$\Longrightarrow$ Additional C1: $\Psi_a \equiv P_a - F_a(Q, \phi) = 0$

✓ Fixed

Still π_a is not fixed.

$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

HM, Suyama, 1411.3721

$\dot{\phi}^a$

$||$

$(\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \lambda_a)$

$\updownarrow$

(P_a, π_a, ρ^a)

$$H = H_0 + \pi_a Q^a$$

- DC1: $K_{ab} = 0$

$\Longrightarrow$ Additional C1: $\Psi_a \equiv P_a - F_a(Q,\phi) = 0$

✓ Fixed

- DC2: $M_{ab} \equiv \{\Psi_a, \Psi_b\} = 0$

$\Longrightarrow$ C2: $\Upsilon_n \equiv \pi_a - G_a(Q,\phi) = 0$

✓ Fixed

✓ We eliminated all the ghosts. H is bounded.

✓ The most general Lagrangian: $L \sim G(\dot{\phi}^a, \phi^a)$

✓Ostrogradsky theorem

$$\det K \neq 0 \Longrightarrow H \text{ is unbounded}$$

✓No-ghost condition (DC1 & DC2)

$$K_{ab} = 0 \ \& \ M_{ab} = 0 \Longrightarrow H \text{ is bounded}$$

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$$

✓Ostrogradsky theorem updated

$\det K \neq 0$ or $\det M \neq 0 \Longrightarrow H$ is unbounded

✓No-ghost condition (DC1 & DC2)

$K_{ab} = 0$ & $M_{ab} = 0 \Longrightarrow H$ is bounded

$$K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$$

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$$

✓ Ostrogradsky theorem updated

$\det K \neq 0$ or $\det M \neq 0 \Longrightarrow H$ is unbounded

✓ No-ghost condition (DC1 & DC2)

$K_{ab} = 0$ & $M_{ab} = 0 \Longrightarrow H$ is bounded

Highest

Next-highest

✓ EL eq

$$\cancel{K_{ab}}\ddddot{\phi}^b + (\cancel{\dot{K}_{ab}} + \cancel{M_{ab}})\dddot{\phi}^b = (\text{terms up to } \ddot{\phi}^a)$$

$\Longrightarrow$ 2nd-order system

Arbitrary higher-order derivatives

HM, Suyama, 1411.3721

- $L = L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^{a})$
- $\phi^{a} = \phi^{a}(t)$ and $a = 1, \cdots n$
- $K_{ab} \equiv \frac{\partial^2 L}{\partial \phi^{a(d)} \partial \phi^{b(d)}}$, $M_{ab} \equiv \frac{\partial^2 L}{\partial \phi^{a(d)} \partial \phi^{b(d-1)}} - \frac{\partial^2 L}{\partial \phi^{b(d)} \partial \phi^{a(d-1)}}$

✓Ostrogradsky theorem updated

$\det K \neq 0$ or $\det M \neq 0 \Longrightarrow H$ is unbounded

$K_{ab} = 0$ ⟶ ✓ ~~$\phi^{a(2d)}$~~ from EL eq Highest

$M_{ab} = 0$ ⟶ ✓ ~~$\phi^{a(2d-1)}$~~ from EL eq Next-highest

- Still remain ghosts from lower (> 2) derivatives.

Eliminating Ostrogradsky ghost

✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721

- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

Eliminating Ostrogradsky ghost

✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721

- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

$$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$$

$$\dot{\phi}^a$$
$$\parallel$$
$$\left(\boldsymbol{Q}^a, \boldsymbol{\phi}^a, \boldsymbol{q}^i, \lambda_a\right)$$
$$\updownarrow$$
$$(\boldsymbol{P}_a, \boldsymbol{\pi}_a, \boldsymbol{p}_i, \rho^a)$$

- Hamiltonian analysis

$$L_{eq} = L\left(\dot{Q}^a, Q^a, \phi^a; \dot{q}^i, q^i\right) + \lambda_a(Q^a - \dot{\phi}^a)$$

Canonical momenta

$$\begin{cases} P_a = L_{\dot{Q}^a} \\ p_i = L_{\dot{q}^i} \\ \pi_a = -\lambda_a \\ \rho^a = 0 \end{cases}$$

$\pi_a = -\lambda_a$, $\rho^a = 0$ → Primary constraints (C1)

Hamiltonian

$$H = H_0(P, Q, \phi, p, q) + \pi_a Q^a$$

π_a shows up only linearly. H is unbounded.

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

$$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$$

$$\dot{\phi}^a = (\boldsymbol{Q^a}, \boldsymbol{\phi^a}, \boldsymbol{q^i}, \lambda_a) \leftrightarrow (P_a, \pi_a, \boldsymbol{p_i}, \rho^a)$$

$$H = H_0 + \pi_a Q^a$$

$$\det L_{\dot{q}^i \dot{q}^j} \neq 0$$

- DC1: $L_{\dot{Q}^a \dot{Q}^b} - L_{\dot{q}^i \dot{Q}^a} L^{-1}_{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}^b} = 0$

$\Longrightarrow$ Additional C1: $\Psi_a \equiv P_a - F_a(Q, \phi, p, q) = 0$

✓ Fixed

- DC2: $M_{ab} \equiv \{\Psi_a, \Psi_b\} = 0$

$\Longrightarrow$ C2: $\Upsilon_n \equiv \pi_a - G_a(Q, \phi, p, q) = 0$

✓ Fixed

✓ We eliminated all the ghosts. H is bounded.

✓ EL eqs $\Longrightarrow$ 2nd-order system

✓ Applies for a wide class of theories

Applications

Crisostomi, Klein, Roest, 1703.01623

- ✓ Field theory in flat spacetime
- ✓ SU(2) — Allys, Peter, Rodriguez, 1609.05870
- ✓ Boson-Fermion — Kimura, Sakakihara, Yamaguchi, 1704.02717
- ✓ Scalar-tensor theories — Langlois, Noui, 1510.06930, 1512.06820

  Crisostomi, Koyama, Tasinato, 1602.03119

  Achour, Langlois, Noui, 1602.08398

  Achour, Crisostomi, Koyama, Langlois, Noui, Tasinato, 1608.08135
- ✓ Vector-tensor theories

  Kimura, Naruko, Yoshida, 1608.07066
- ✓ Tensor theories

  Crisostomi, Noui, Charmousis, Langlois, 1710.04531

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi},\dot{\phi},\phi) \Longrightarrow L(\dot{\phi},\phi)$
- ✓ $L(\ddot{\phi}^a,\dot{\phi}^a,\phi^a) \Longrightarrow L(\dot{\phi}^a,\phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)},\phi^{a(d-1)},\dots,\phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi},\dot{\phi},\phi;\dot{q},q)$
- ✓ $L(\ddot{\phi},\dot{\phi},\phi;\dot{q}^i,q^i) \;\sim\; \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a,\dot{\phi}^a,\phi^a;\dot{q}^i,q^i) \;\sim\; \phi^a + g_{\mu\nu}$

HM, Suyama, Yamaguchi, 1711.08125; in prep.

- $L(\dddot{\psi},\ddot{\psi},\dot{\psi},\psi;\dot{q}^i,q^i)$
- $L(\dddot{\psi}^n,\ddot{\psi}^n,\dot{\psi}^n,\psi^n;\ddot{\phi}^a,\dot{\phi}^a,\phi^a;\dot{q}^i,q^i)$
- $L(\phi^{i_d(d+1)},\dots;\phi^{i_{d-1}(d)},\dots;\dots;\dot{\phi}^{i_0},\phi^{i_0})$

HM, Suyama, Yamaguchi, 1711.08125

Quadratic model with $\dddot{\psi}^n, \dot{q}^i$

$\det A_{ij} \neq 0$
$\det c_{nm} \neq 0$

$$L = \frac{1}{2} a_{nm} \dddot{\psi}^n \dddot{\psi}^m + \frac{1}{2} b_{nm} \ddot{\psi}^n \ddot{\psi}^m + \frac{1}{2} c_{nm} \dot{\psi}^n \dot{\psi}^m$$
$$+ \frac{1}{2} d_{nm} \psi^n \psi^m + e_{nm} \dddot{\psi}^n \ddot{\psi}^m + f_{nm} \ddot{\psi}^n \dot{\psi}^m$$
$$+ \frac{1}{2} A_{ij} \dot{q}^i \dot{q}^j + \frac{1}{2} B_{ij} q^i q^j + C_{ij} \dot{q}^i q^j + \alpha_{ni} \dddot{\psi}^n \dot{q}^i$$

$\psi^n(t)$
$q^i(t)$

Equivalent form

$$L_{eq} = L(\dot{Q}, Q, R, \psi, \dot{q}, q) + \xi_n(\dot{\psi}^n - R^n) + \lambda_n(\dot{R}^n - Q^n)$$

$\dot{Q} = \dddot{\psi}^n$, $Q = \ddot{\psi}^n$, $R = \dot{\psi}^n$

Canonical momenta

$$\begin{cases} P_{Q^n} = a_{nm} \dot{Q}^m + \alpha_{ni} \dot{q}^i + e_{nm} Q^m \\ p_i = \alpha_{ni} \dot{Q}^n + A_{ij} \dot{q}^j + C_{ij} q^j \\ P_{R^n} = \lambda_n, \quad \pi_{\psi^n} = \xi_n \\ \rho_{\lambda_n} = 0, \quad \rho_{\xi_n} = 0 \end{cases}$$

→ Primary constraints (C1)

HM, Suyama, Yamaguchi, 1711.08125

Quadratic model with $\dddot{\psi}^n, \dot{q}^i$

$$\ddot{\psi}^n = Q^n, \quad \dot{\psi}^n = R^n$$

$$(\boldsymbol{Q^n}, \boldsymbol{R^n}, \boldsymbol{\psi^n}, \boldsymbol{q^i}, \lambda_n, \xi_n) \updownarrow (P_{Q^n}, P_{R^n}, \pi_{\psi^n}, \boldsymbol{p_i}, \rho_{\lambda_n}, \rho_{\xi_n})$$

$$H = H_0 + P_{R^n} Q^n + \pi_{\psi^n} R^n$$

- DC1: $a_{nm} - \alpha_{ni} A^{ij} \alpha_{jm} = 0$

$\Longrightarrow$ Additional C1: $\Psi_n \equiv P_{Q^n} - \cdots = 0$

✓ Fixed

- DC2: $\{\Psi_n, \Psi_m\} = -2[e_{nm} - \cdots] = 0$

$\Longrightarrow$ C2: $\Upsilon_n \equiv P_{R^n} - \cdots = 0$

✓ Fixed

- DC3: $\{\Upsilon_n, \Psi_m\} = -b_{nm} - \cdots = 0$

$\Longrightarrow$ C3: $\Lambda_n \equiv \pi_{\psi^n} - \cdots = 0$

✓ Fixed

We eliminated all the ghosts? ***No!***

HM, Suyama, Yamaguchi, 1711.08125

Quadratic model with $\dddot{\psi}^n, \dot{q}^i$

$\ddot{\psi}^n \quad \dot{\psi}^n$

$\parallel \quad \parallel$

$(Q^n, \boldsymbol{R^n}, \boldsymbol{\psi^n}, \boldsymbol{q^i}, \lambda_n, \xi_n)$

$\updownarrow$

$(P_{Q^n}, P_{R^n}, \pi_{\psi^n}, \boldsymbol{p_i}, \rho_{\lambda_n}, \rho_{\xi_n})$

$$H = H_0 + P_{R^n} Q^n + \pi_{\psi^n} R^n$$

$\downarrow$ All DCs and Cs

H: linear in $Q^n \Longrightarrow$ *Hidden ghosts appeared*

HM, Suyama, Yamaguchi, 1711.08125

Quadratic model with $\dddot{\psi}^n, \dot{q}^i$

$\ddot{\psi}^n \quad \dot{\psi}^n$

$\quad \| \quad\quad \|$

$(Q^n, \boldsymbol{R^n}, \boldsymbol{\psi^n}, \boldsymbol{q^i}, \lambda_n, \xi_n)$

$\updownarrow$

$(P_{Q^n}, P_{R^n}, \pi_{\psi^n}, \boldsymbol{p_i}, \rho_{\lambda_n}, \rho_{\xi_n})$

$$H = H_0 + P_{R^n} Q^n + \pi_{\psi^n} R^n$$

↓ All DCs and Cs

H: linear in Q^n $\Longrightarrow$ *Hidden ghosts appeared*

- DC4: $\{\Lambda_n, \Psi_m\} = 2(f_{nm} - \cdots) = 0$

$\Longrightarrow$ C4: $\Omega_n \equiv c_{nm} Q^m - \cdots = 0$

✓ Fixed

- Condition to complete Dirac procedure:

$$\det Z_{nm} \equiv \det\{\Omega_n, \Psi_m\} \neq 0$$

✓ We eliminated all the ghosts. H is bounded.

HM, Suyama, Yamaguchi, 1711.08125

Quadratic model with $\dddot{\psi}^n, \dot{q}^i$

- Dirac matrix

$$D = \begin{array}{c|cccccc} & \overbrace{\Phi_\beta \quad \bar{\Phi}_\beta \quad \Psi_m}^{C1} & & & \Omega_m^{\,C4} & \Upsilon_m^{\,C2} & \Lambda_m^{\,C3} \\ \hline \Phi_\alpha & 0 & -\mathbf{1} & * & * & * & * \\ \bar{\Phi}_\alpha & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ \Psi_n & * & 0 & 0 & -Z_{mn} & 0 & 0 \\ \Omega_n & * & 0 & Z_{nm} & * & * & * \\ \Upsilon_n & * & 0 & 0 & * & 0 & Z_{mn} \\ \Lambda_n & * & 0 & 0 & * & -Z_{nm} & * \end{array}$$

$\det Z_{nm} \neq 0$

$\Longrightarrow \det D \neq 0$; All constraints are second class

$\Longrightarrow$ Healthy $(N + I)$ DOFs

✓ L can be transformed to a lower-derivative L.

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- ✓ $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \;\sim\; \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \;\sim\; \phi^a + g_{\mu\nu}$

HM, Suyama, Yamaguchi, 1711.08125; in prep.

- ✓ $L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$
- ✓ $L(\dddot{\psi}^n, \ddot{\psi}^n, \dot{\psi}^n, \psi^n; \ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$
- ✓ $L(\phi^{i_d(d+1)}, \dots; \phi^{i_{d-1}(d)}, \dots; \dots; \dot{\phi}^{i_0}, \phi^{i_0})$

Summary

Ostrogradsky ghosts appear as

- $L \ni$ 2nd-order time derivatives $\Longrightarrow H$: linear in P

which can be removed by degeneracy conditions.

The analysis of $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$ applies for a wide class of model buildings.

We found that for quadratic model with $\dddot{\psi}^n, \dot{q}^i$

- $L \ni$ 3rd-order time derivatives $\Longrightarrow H$: linear in P, Q

We constructed the first ghost-free model with 3rd-order time derivatives in L.

The analyses of general L and field theory are work in progress.