Effects of QCD equation of state on Stochastic Gravitational Wave background

Sampurn Anand

Theoretical Physics Division,
Physical Research Laboratory, Ahmedabad, India

In collaboration with: Ujjal Dey & S. Mohanty

6$^{\rm th}$ Feb, 2018

Motivation

- In cosmology, one of the important signal observed so far is CMBR.
- It gives us our earliest electromagnetic view of the state of the universe.
- Information about the universe at the surface of last scattering is contained in it.
- Gravitational waves are NOT electromagnetic radiation like CMBR.
- They carrying information about cosmic objects and events that are not carried by electromagnetic radiation.
- We can investigate some of the early universe phenomenon with GW

Outline

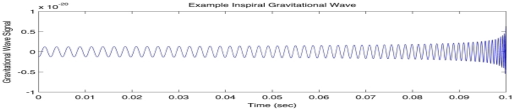
1. Gravitational Waves (GW) and its type
2. Trace anomaly and QCD equation of state
   - GW spectrum with trace anomaly
3. Results
   - GW spectrum with trace anomaly
4. Conclusion

GW and its type

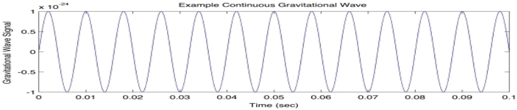
- Distortion in space-time in such a way that the "wave" of distorted space would radiate from the source.
- These ripples in the fabric of space-time are known as Gravitational Wave (GW).

Sources and types of GW

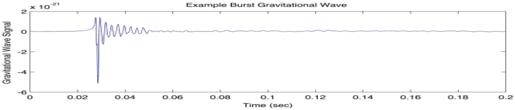
- Compact binary inspiral GW: Binary neutron stars (BNS), binary black hole (BBH), Neutron star Black hole binary (NSBH)



- Continuous GW: spinning massive stars (neutron stars)



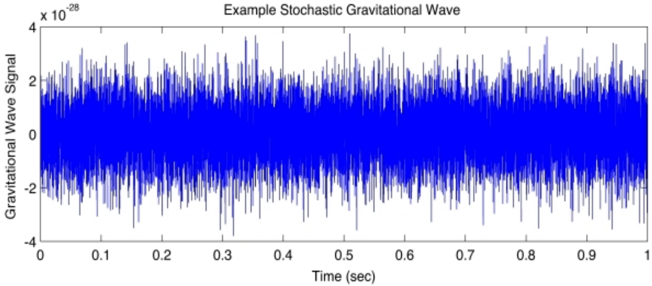
- Burst GW: supernova, GRB



Sources and types of GW contd..

Stochastic GW (SGW):

- Stochastic gravitational waves are the relic gravitational waves from the early evolution of the universe
- These GWs arise from large number of independent and uncorrelated events



https://www.ligo.org/science/GW-Sources

Sources of SGW

- First order Phase transition
  - Bubble collision:
    1OPT occurring explosively, through the nucleation of fast broken phase bubbles, can be a source of GW
  - MHD turbulence:
    Magnetohydrodynamic (MHD) turbulence in the plasma forming after the bubbles have collided.

The GW spectrum

- To estimate the observable GW background today, we propagate the GW from the epoch of phase transition to the current epoch using Boltzmann equation

$$\frac{d}{dt}(\rho_{gw}\, a^4) = 0 \qquad \rightarrow \rho_{gw}\, a^4 = \rho_{*gw}\, a_*^4 \tag{1}$$

- Assuming adiabatic expansion of the universe $\Rightarrow S \propto a^3\, g_s\, T^3$ remains constant, we get

$$\frac{dT}{dt} = -HT\left(1 + \frac{T}{3g_s}\frac{dg_s}{dT}\right)^{-1} \tag{2}$$

  where g_s is the effective number of relativistic degrees of freedom that contributes to the entropy density.

- Integrating Eq. (2) $\Rightarrow \frac{a_*}{a_0} = \exp\left[\int_{T_*}^{T_0} \frac{1}{T}\left(1 + \frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\right]$
- The fractional energy density of the gravitational waves at current epoch is given as

$$\frac{\rho_{\rm gw}}{\rho_{cr}} = \Omega_{\rm gw} = \Omega_{{\rm gw}*}\left(\frac{H_*}{H_0}\right)^2 \exp\left[\int_{T_*}^{T_0} \frac{4}{T}\left(1 + \frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\right] \tag{3}$$

- To evaluate the ratio of the Hubble parameters, we consider the continuity equation, given

$$\dot{\rho}_t = -3H\rho_t\left(1 + p_t/\rho_t\right) \tag{4}$$

with $\rho_t(p_t)$ being the total energy (pressure) density of the universe and dot denotes the derivative with respect to cosmic time.

- In terms of temperature above equation reduces to,

$$\frac{d\rho_t}{\rho_t} = \frac{3}{T}\,(1 + w_{\rm eff})\left(1 + \frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\,, \tag{5}$$

where $w_{\rm eff} = p_t/\rho_t$ is the effective equation of state parameter.

- Integrating above equation leads to

$$\rho_t(T_*) = \rho_t(T_r)\,\exp\left[\int_{T_r}^{T_*} \frac{3}{T}\,(1 + w_{\rm eff})\left(1 + \frac{T}{3g_s}\frac{dg_s}{dT}\right)\, dT\right]. \tag{6}$$

QCD EoS and evolution of the universe

- Using lattice simulation, the equation of state around QCD epoch can be computed using the parametrization of the pressure due to u, d, s quarks and gluons [1]

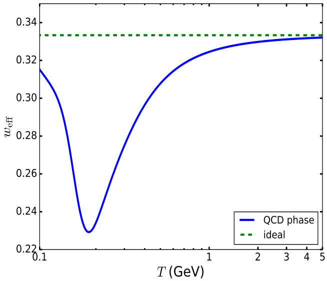
$$\frac{2p}{T^4} = (1 + \tanh[c_\tau(\tau - \tau_0)]) \left(\frac{p_i + a_n/\tau + b_n/\tau^2 + d_n/\tau^4}{1 + a_d/\tau + b_d/\tau^2 + d_d/\tau^4} \right) \quad (7)$$

where $\tau = T/T_c$ with $T_c = 154$ MeV and $p_i = (19\,\pi^2)/36$ is the ideal gas value of p/T^4. $c_\tau = 3.8706$, $a_n = -8.7704$, $b_n = 3.9200$, $d_n = 0.3419$, $a_d = -1.2600$, $b_d = 0.8425$, $d_d = -0.0475$ and $\tau_0 = 0.9761$.

- The energy density can be computed from the trace anomaly [2].

$$\frac{\rho - 3p}{T^4} = T \frac{\partial}{\partial T}(p/T^4) \quad (8)$$

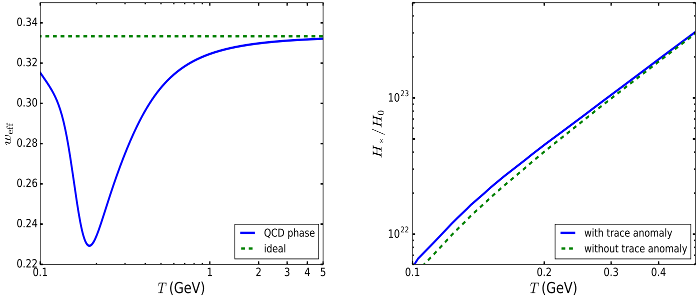
[1] Bazavov *et. al.* PRD **90** (2014) 094503
[2] Cheng. *et. al.* PRD **77**, 014511 (2008)



- Apart from quarks and gluons, contribution to the total energy density and pressure will come from other particles as well.
- energy density and pressure of a non-relativistic particle is exponentially smaller than that of the relativistic particles. Hence, $\rho_{\rm rel} = (\pi^2/30)\big(\sum_{i={\rm bosons}} g_i + \sum_{j={\rm fermions}}(7/8)g_j\big)\, T^4$ and $p_{\rm rel} = \rho_{\rm rel}/3$, respectively.

Using $H_*^2 = \rho_*/(3\, m_p^2)$, we can define [3]

$$\left(\frac{H_*}{H_0}\right)^2 = \Omega_{r0}\left(\frac{a_0}{a_r}\right)^4 \exp\left[\int_{T_r}^{T_*} \frac{3(1+w_{\rm eff})}{T}\left(1+\frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\right], \qquad (9)$$



(a) (b)

[3] Anand *et. al.* JCAP **1703** (2017) no.03, 018

GW spectrum with trace anomaly

$$\Omega_{\rm gw} = \Omega_{r0}\Omega_{{\rm gw}*} \exp\left[\int_{T_*}^{T_r} \frac{4}{T'}\left(1+\frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\right] \times \exp\left[\int_{T_r}^{T_*} \frac{3}{T}(1+w_{\rm eff})\left(1+\frac{T}{3g_s}\frac{dg_s}{dT}\right) dT\right]. \quad (10)$$

We have set : $\Omega_{r0} \simeq 8.5 \times 10^{-5}$ and $T_r = 10^4$ GeV
We also need to know about $\Omega_{{\rm gw}*}$

- It is considered that the QCD transition is just a cross-over. However, this can change in beyond standard model (of particle physics) scenario.
  e.g. A large neutrino chemical potential can make QCD transition first order[4].
- Contribution to $\Omega_{\mathrm{GW}*}$ comes from two important processes at first order phase transition [5]
  - collision of bubble walls:

$$\Omega^{(b)}_{\mathrm{gw}*}(\nu) = \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa_b \alpha}{1+\alpha}\right)^2 \left(\frac{0.11 v^3}{0.42 + v^2}\right) S_b(\nu), \tag{11}$$

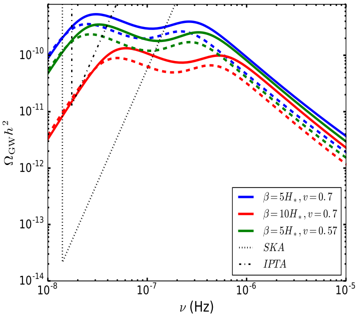
  - Magnetohydrodynamics (MHD) turbulence:

$$\Omega^{(\mathrm{mhd})}_{\mathrm{gw}*}(\nu) = \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\mathrm{mhd}}\, \alpha}{1+\alpha}\right)^{3/2} v\, S_{\mathrm{mhd}}(\nu), \tag{12}$$

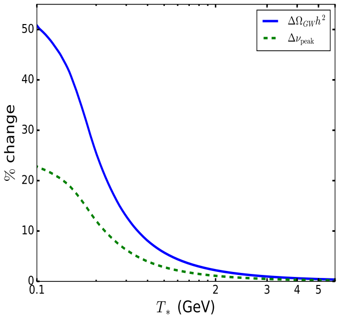
  β^{-1} is the time duration of the phase transition, α is the ratio of the vacuum energy density released in the phase transition to that of the radiation, v is the wall velocity and κ_b denotes the fraction of the latent heat of the phase transition deposited on the bubble wall. The function $S(\nu)$ parametrizes the spectral shape which is given by simulation[6].

[4] Schwarzet. *al.* JCAP **0911** (2009) 025
[5] see Caprini *et. al.* JCAP **1604** (2016) no.04, 001 for detail
[6] Huber *et. al.* JCAP **0805** (2008) 017

Results



(c)



(d)

Anand *et. al.* JCAP **1703** (2017) no.03, 018

GW spectrum with trace anomaly

- The effective equation of state parameter w_{eff}, which depicts the energy content of the universe and hence governs the background evolution, decreases from $1/3$, the ideal value.
- This implies that the density will fall slower than a^{-4}.
- Thus, the Hubble parameter will change slower than T^2.
- which implies that the value of Hubble parameter at the time of transition H_* will be higher than its value obtained without QCD equation of state.
- Therefore, we expect an overall enhancement in the GW signal

Conclusion

- Lattice result have shown a deviation from the ideal gas equation of state during QCD epoch.
- This can alter the evolution of the universe during that phase.
- If QCD transition is a first order phase transition, then the signal of the GW generated during that epoch will be enhanced.
- Enhancement in the amplitude is 50% and 25% in the frequency.

Thank You