Stable cosmology in chameleon bigravity

Yota Watanabe (Kavli IPMU, YITP)

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with A. DeFelice,
S. Mukohyama,
M. Oliosi

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Introduction: GW era has come!

- Direct detection of Gravitational Wave (GW)
  : New test of gravitational theories
  - in the strong-field regime
  - propagated on cosmological scale

- Important to study theoretical consistency of a model which predicts different phenomena from GR

- A theory of massive spin-2 field is interesting
  - Theory construction is nontrivial
  - Different GW waveform could be detected

Fierz, Pauli (1939)
van Dam, Veltman (1970), Zakharov (1970)
Vainshtein (1972), Boulware, Deser (1972)
de Rham, Gabadadze, Tolley (2010)

Bigravity

Hassan, Rosen 1109.3515

Ghost-free theory of massive spin-2 field with FLRW sol.

$$S[g_{\mu\nu}, f_{\mu\nu}] = S_{\mathrm{EH},g} + S_{\mathrm{EH},f} + S_{\mathrm{int}} + S_{\mathrm{mat}}$$

$$S_{\mathrm{EH},g} = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g]$$

$$S_{\mathrm{EH},f} = \frac{\kappa M_g^2}{2}\int d^4x\sqrt{-f}R[f]$$

$$S_{\mathrm{int}} = M_g^2 m^2 \int d^4x\sqrt{-g}\Sigma_{i=0}^4 \beta_i U_i$$

β_i: constant

$$U_0 = 1, \qquad U_1 = T_1, \qquad U_2 = \frac{1}{2}(T_1^2 - T_2),$$

$$U_3 = \frac{1}{6}(T_1^3 - 3T_1^2 + 2T_3),$$

$$U_4 = \frac{1}{24}(T_1^4 - 6T_1^2 T_2 + 3T_2^2 + 8T_1T_3 - 6T_4),$$

$$T_n = \mathrm{Tr}[s^n], \qquad s = \sqrt{g^{\mu\nu} f_{\nu\sigma}}$$

- One massless tensor, one massive tensor
  → "graviton oscillation" analogous to ν oscillation

DeFelice, Nakamura, Tanaka 1304.3920

Narikawa, Ueno, Tagoshi, Tanaka, Kanda, Nakamura 1412.8074

- Cosmological constant is included in S_{int}

Chameleon bigravity

DeFelice, Mukohyama, Uzan 1702.04490

- Original bigravity is valid
  - 1) for (Energy scale) $\lesssim \left(M_g m^2\right)^{1/3}$
    : m must be large, but then phenomena are almost the same as GR, not interesting
  - 2) for $H^2 \lesssim m_{\mathrm{Tensor}}^2$: cannot be applied to early universe
  - 3) with fine-tuning to pass solar-system tests (Vainshtein screening) keeping m small

DeFelice, Nakamura, Tanaka 1304.3920

- Chameleon extension
  - i) Introduce scalar ϕ
  - ii) $\beta_i \to \beta_i(\phi)$
  - iii) Couple matter to $\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$

Khoury, Weltman 0309300, 0309411

→ Potential minimum of ϕ depends on matter density ρ

↓

$m_{\mathrm{Tensor}}^2(\beta_i) \propto \rho$

Outline of our work

- Stability condition of chameleon bigravity was studied only around de Sitter DeFelice, Mukohyama, Uzan 1702.04490

- Our work: DeFelice, Mukohyama, Oliosi, YW 1711.04655
  - derive stability conditions
    1) of rad/ mat era under homogeneous perturbations
    2) of inhomogeneous perturbations around FLRW
  - 3) numerical realization of stable cosmology (not compared with obs. data)

(1/3) Stability of rad/mat era

- Rad/mat era must last long enough.
  Scaling solution of each era
  : every term in background EoM has the same time dependence for $a(t) \sim t^{2/n}$

$$n = \begin{cases} 4 \text{ (rad)} \\ 3 \text{ (mat)} \end{cases}$$

$$\begin{cases} \ln h = \ln h_0 - \frac{n}{2} N_e + \epsilon\, h^{(1)} \\ \varphi = \frac{n}{\lambda} N_e (1 + \epsilon\, \varphi^{(1)}) \\ \xi = \bar{\xi}(1 + \epsilon\, \xi^{(1)}) \\ c = c^{(0)}(1 + \epsilon\, c^{(1)}) \end{cases}$$

$\mathcal{O}(\epsilon)$: perturbations

$$ds_g^2 = -dt^2 + a^2 d\boldsymbol{x}^2$$

$$ds_f^2 = \xi^2(-c^2 dt^2 + a^2 d\boldsymbol{x}^2)$$

$h = H/m$, $\varphi = \phi/M_g$,

$N_e = \ln a$, $' = d/dN_e$

$\bar{\xi}, c^{(0)}$: constants

EoM

$$\begin{cases} \varphi^{(1)\prime\prime} + \left(1 + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_r \varphi^{(1)} = 0 \text{ (rad)} \\ \varphi^{(1)\prime\prime} + \left(\frac{3}{2} + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_m \varphi^{(1)} = 0 \text{ (mat)} \end{cases}$$

Stability conditions

$$\mathcal{A}_r > 0, \mathcal{A}_m > 0$$

(2/3) Stability of inhomo. pert.

- Flat FLRW + inhomogeneous perturbation

$$ds_g^2 = -\mathcal{N}^2 dt^2 + \gamma_{ij}(\mathcal{N}^i dt + dx^i)(\mathcal{N}^j dt + dx^j)$$

$$ds_f^2 = -\tilde{\mathcal{N}}^2 dt^2 + \tilde{\gamma}_{ij}(\tilde{\mathcal{N}}^i dt + dx^i)(\tilde{\mathcal{N}}^j dt + dx^j)$$

$$\phi = \bar{\phi} + \delta\phi \qquad \psi_{\mathrm{rad/mat}} = \bar{\psi}_{\mathrm{rad/mat}} + \delta\psi_{\mathrm{rad/mat}}$$

$$\mathcal{N} = N(1+\Phi)$$

$$\tilde{\mathcal{N}} = N\xi c(1+\tilde{\Phi})$$

$$\gamma_{ij} = a^2\delta_{ij} + \delta\gamma_{ij}$$

$$\tilde{\gamma}_{ij} = \xi^2 a^2 \delta_{ij} + \delta\tilde{\gamma}_{ij}$$

- Tensor sector

$\delta\gamma_{ij} = h_{ij}, \quad \delta\tilde{\gamma}_{ij} = \tilde{h}_{ij}$: Transverse traceless

$$\mathcal{L}_T^{(2)} = \frac{M_g^2 N a^3}{8}\left[\frac{\dot{h}^{ij}\dot{h}_{ij}}{N^2} - \frac{k^2}{a^2}h^{ij}h_{ij} + \frac{\kappa\xi^2}{c}\left(\frac{\dot{\tilde{h}}^{ij}\dot{\tilde{h}}_{ij}}{N^2} - c^2\frac{k^2}{a^2}\tilde{h}^{ij}\tilde{h}_{ij}\right) - m^2\Gamma\left(h-\tilde{h}\right)^{ij}\left(h-\tilde{h}\right)_{ij}\right]$$

$$\Gamma = -[\beta_1\xi + (1+c)\beta_2\xi^2 + c\beta_3\xi^3]$$

No-ghost condition for $\tilde{h}_{ij}$ → $\boxed{c > 0}$

$$m_T^2 = \frac{c+\kappa\xi^2}{\kappa\xi^2}m^2\Gamma$$

(2/3) Stability of inhomo. pert.

➢Vector sector

$$\mathcal{N}^i = B^i, \qquad \delta\gamma_{ij} = \frac{1}{2}(\partial_i E_j + \partial_j E_i),$$

$$\tilde{\mathcal{N}}^i = \tilde{B}^i, \qquad \delta\tilde{\gamma}_{ij} = \frac{1}{2}(\partial_i \tilde{E}_j + \partial_j \tilde{E}_i),$$

: Transverse modes

⇓ Integrate out non-dynamical modes $B_i, \tilde{B}_i$

$$\mathcal{L}_V^{(2)} = \frac{M_g^2 N a^3}{8} \mathrm{A} \left[\frac{\dot{\mathcal{E}}^i \dot{\mathcal{E}}_i}{N^2} - \left(c_V^2 \frac{k^2}{a^2} + m_V^2 \right) \mathcal{E}^i \mathcal{E}_i \right]$$

$$\mathrm{A} = \frac{m^2 \kappa \xi^2 J k^2}{(c+1)\kappa\xi k^2/a^2 + 2m^2(c+\kappa\xi^2)J}$$

$$\mathcal{E}_i = E_i - \tilde{E}_i$$

$$m_V^2 = m_T^2$$

$$c_V^2 = \frac{(c+1)\Gamma}{2\xi J}$$

$$J = -[\beta_1 + 2\beta_2\xi + \beta_3\xi^2]$$

No-ghost condition → $J > 0$

No-gradient-instability condition→ $\Gamma > 0$

(2/3) Stability of inhomo. pert.

- Scalar sector

$$\Phi, \quad \mathcal{N}^i = \partial^i B, \quad \delta\gamma_{ij} = 2\Psi\delta_{ij} + \left(\partial_i\partial_j - \frac{\Delta}{3}\delta_{ij}\right)E,$$

$$\tilde{\Phi}, \quad \tilde{\mathcal{N}}^i = \partial^i \tilde{B}, \quad \delta\tilde{\gamma}_{ij} = 2\tilde{\Psi}\delta_{ij} + \left(\partial_i\partial_j - \frac{\Delta}{3}\delta_{ij}\right)\tilde{E},$$

⇓ Integrate out non-dynamical modes $\Phi, \tilde{\Phi}, B, \tilde{B}, \tilde{\Psi}$
Gauge $\Psi = E = 0$

$$\mathcal{L}_S^{(2)} = \frac{Na^3}{2}\left[\frac{\dot{\mathcal{Y}}^T}{N}\mathcal{K}\frac{\dot{\mathcal{Y}}}{N} + \frac{\dot{\mathcal{Y}}^T}{N}\mathcal{F}\mathcal{Y} - \mathcal{Y}^T\mathcal{F}\frac{\dot{\mathcal{Y}}}{N} - \mathcal{Y}^T\mathcal{M}\mathcal{Y}\right], \quad \mathcal{Y} = (\tilde{E}\ \delta\phi\ \psi_{\rm rad}\ \psi_{\rm mat})^T$$

- ○ No-ghost condition: Eigenvalues of $\mathcal{K}$ >0 in high k limit
  → Null-Energy Condition for fluids & one nontrivial condition
- ○ No-grad-instability condition:

$$\det\left[c_s^2\frac{k^2}{a^2}\mathcal{K} + \mathcal{M}\right]_{\text{high } k} = 0 \quad \Rightarrow \quad \boxed{c_s^2 > 0}$$

(3/3) Numerical realization

- Simple couplings

$$\beta_i(\phi) = -c_i \mathrm{e}^{-\lambda\varphi}, \quad A(\phi) = \mathrm{e}^{\beta\varphi}$$

c_i, λ, β: constants

Approximate scaling solution for mat. dom.→ $\beta \approx 0$

- Example parameters (The other parameters are determined by EoM)

$$c_{\mathrm{ini}} = 1.01, \quad c_V^2 = 1, \quad c_1 c_3 - c_2^2 = 1, \quad c_1 + 2c_2 + c_3 = 1,$$

$$\Omega_{\Lambda,\mathrm{ini}} = 10^{-30}, \quad \Omega_{\mathrm{m,ini}} = 10^{-5}, \quad \Omega_{\varphi\mathrm{kin,ini}} = \frac{3}{200}, \quad \Omega_{\mathrm{GravPot,ini}} = \frac{1}{200},$$

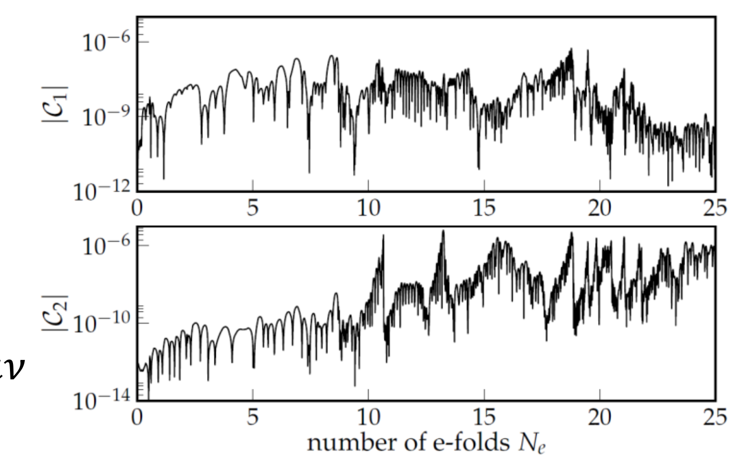
$$\beta = 10^{-2}, \quad \lambda = 40/3,$$

- Constraints $\mathcal{C}_1, \mathcal{C}_2$

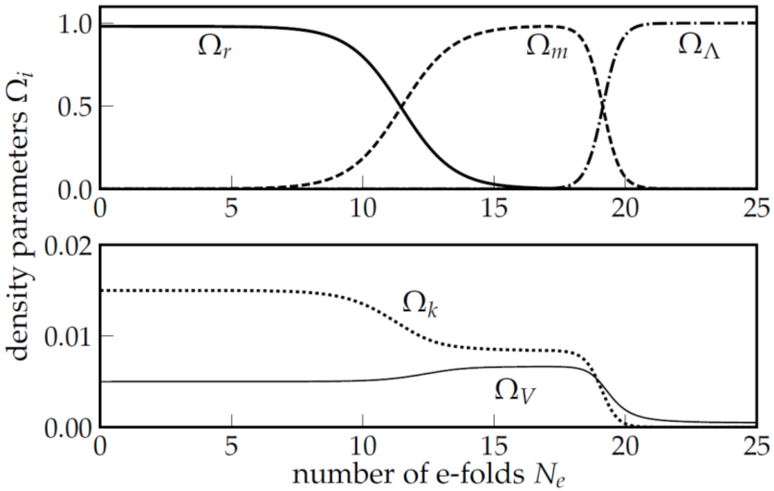
$$\mathcal{C}_1 = \frac{1 - \Sigma_i \Omega_i}{1 + \Sigma_i |\Omega_i|}$$

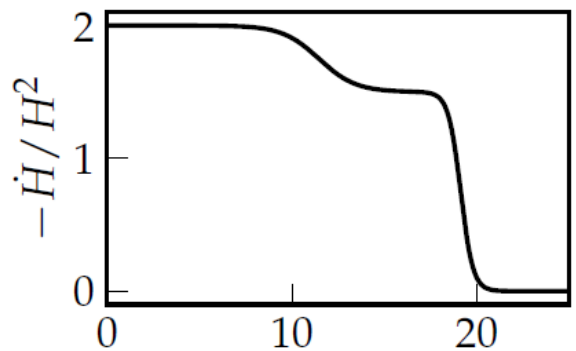
:Normalized Friedmann eq for $g_{\mu\nu}$

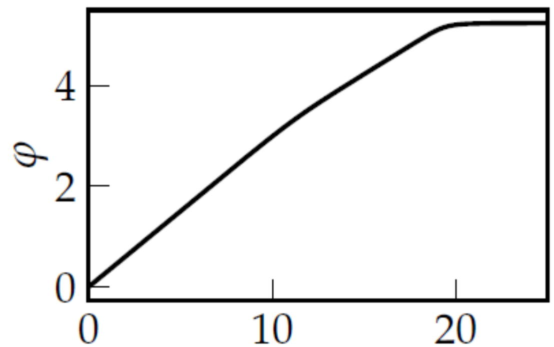
Similarly define $\mathcal{C}_2$ for $f_{\mu\nu}$

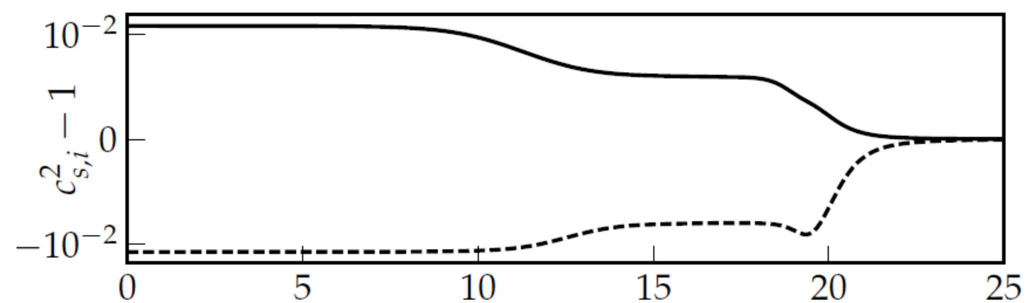


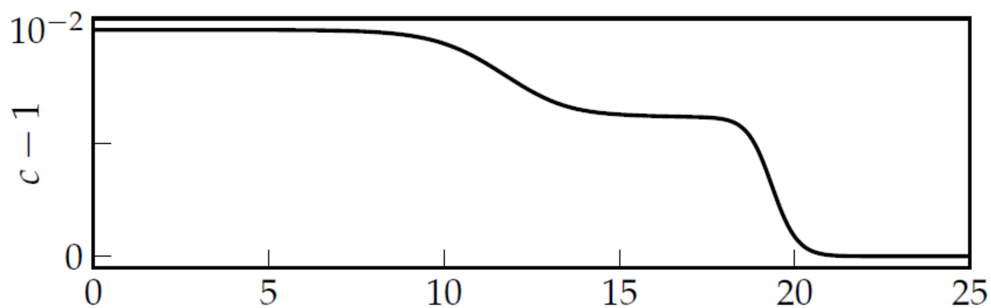
(3/3) Numerical realization

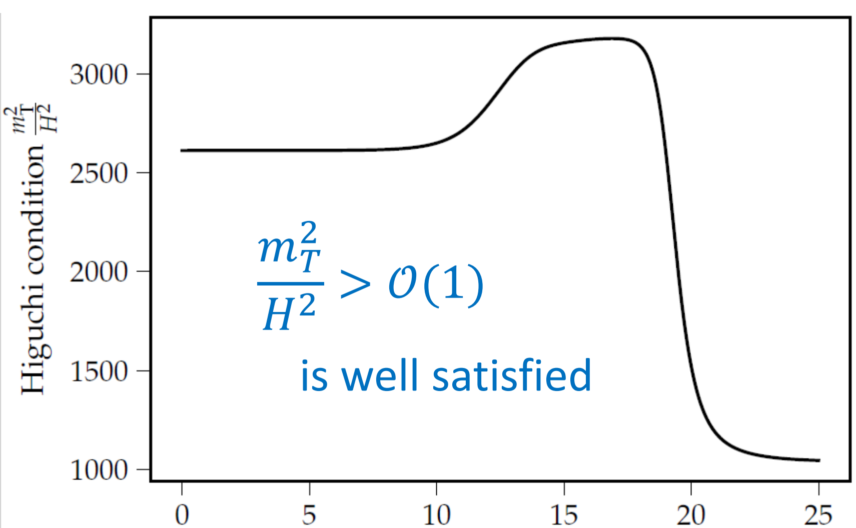












$$\frac{m_T^2}{H^2} > \mathcal{O}(1)$$

is well satisfied

Summary

- Bigravity: nontrivial theory of a massive spin-2 field
  But not valid at early universe/solar system
  keeping mass small w/o fine-tuning
- Chameleon bigravity: introduce ϕ,
  its potential minimum depends on environment

  $\rightarrow$ Graviton mass depends on environment
- Derived stability conditions
  1) of rad/mat era under homogeneous perturbation
  2) of inhomogeneous perturbation around FLRW
- Numerically realize stable cosmology