Primordial Black Holes: the morphology of cosmological perturbations

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PBHs: A bit of history

- In the early universe **large amplitude perturbations of the metric** can collapse into **Primordial Black Holes (PBHs)** [**Zeldovich** & **Novikov** (1967); **Hawking** (1971)] characterized by a wide range of masses (from the Planck mass to 10^6 $M_\odot$ for PBHs formed at the Nucleosynthesis).

- **Hawking evaporation effect** (1974) has been inspired by the idea of PBH formation which could be small as particles and quantum effects need to taken into account. PBHs smaller than 10^{15} grams would evaporate by now via Hawking evaporation, becoming **possible sources of Gamma Ray Burst**, **Cosmic Rays, evaporation remnants as cold dark matter**.

- The threshold amplitude of PBH formation ($\delta_c \sim c_s^2$) measured at horizon crossing time **Carr** (1975) tell us if a perturbation collapse into a **PBH** or bounce and disperse into the surrounding medium. This has been confirmed by **full relativistic numerical simulations** **Nadezin**, **Novikov** & **Polnarev** (1978), **Niemeyer** & **Jedamzik** (1998, 1999); **Musco et al.** (2005, 2007, 2009, 2013)] suggesting that **critical collapse** (scaling law) might apply in the early universe, in particular during the radiation dominated era ($c_s^2 = 1/3$). **Harada, Nakama et al.** (2013, 2014, 2015) have calculated an analytical threshold for PBH formation, proposing also a phenomenological parameterisation of PBH threshold in terms of initial density shapes.

$$ds^2 = -a^2\, dt^2 + b^2\, dr^2 + R^2 d\Omega^2 \qquad ds^2 = -f^2\, du^2 - 2fb\, dr\, du + R^2 d\Omega$$

COSMIC TIME $\qquad f\, du = a\, dt - b\, dr \qquad$ **NULL TIME**

Cosmic time:

$$D_t \equiv \frac{1}{a}\left(\frac{\partial}{\partial t}\right) \qquad D_r \equiv \frac{1}{b}\left(\frac{\partial}{\partial r}\right)$$

$$U \equiv D_t R \qquad \Gamma \equiv D_r R$$

$$D_t U = -\left[\frac{\Gamma}{(e+p)} D_r p + \frac{M}{R^2} + 4\pi R p\right]$$

$$D_t \rho = -\frac{\rho}{\Gamma R^2} D_r (R^2 U)$$

$$D_t e = \frac{e+p}{\rho} D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_r a = -\frac{a}{e+p} D_r p$$

$$D_r M = 4\pi R^2 \Gamma e$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

Null time:

$$D_t \equiv \frac{1}{f}\left(\frac{\partial}{\partial u}\right) \qquad D_k \equiv D_r + D_t$$

$$D_t U = -\frac{1}{1-c_s^2}\left[\frac{\Gamma}{(e+p)} D_k p + \frac{M}{R^2} + 4\pi R p + \right.$$
$$\left. + c_s^2 \left(D_k U + \frac{2U\Gamma}{R}\right)\right]$$

$$D_t \rho = \frac{\rho}{\Gamma}\left[D_t U - D_k U - \frac{2U\Gamma}{R}\right]$$

$$D_t e = \left(\frac{e+p}{\rho}\right) D_t \rho$$

$$D_t M = -4\pi R^2 p U$$

$$D_k \left[\frac{(\Gamma + U)}{f}\right] = -4\pi R (e+p) f$$

$$D_k M = 4\pi R^2 [e\Gamma - pU],$$

$$\Gamma = D_k R - U = 1 + U^2 - \frac{2M}{R}$$

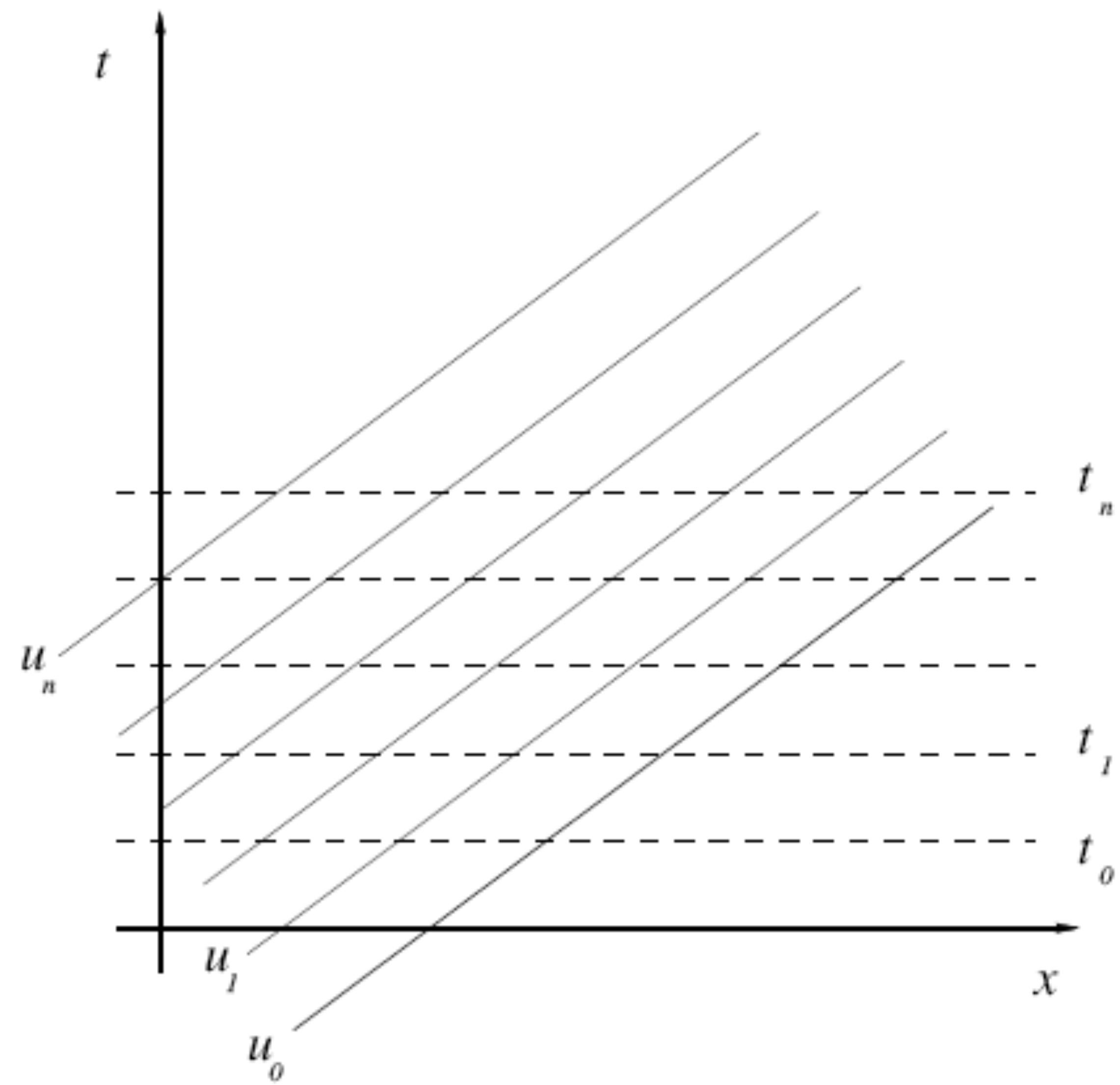
Numerical Results: the method

- Simulations are performed using a **Lagrangian spherically symmetric GR hydro code with an adaptive grid** (AMR).

- We set initial conditions using a **cosmic time coordinate** t.

- We transfer those onto a **null foliation** of the space time, then evolved using an **observer time coordinate** u.

- The **formation of a PBH is seen by a distant external observer** (the singularity is hidden by the asymptotic formation of the apparent horizon).



Equation of State

energy density: $e = \rho(1 + \epsilon)$ pressure: $p = (\gamma - 1)\rho\epsilon$

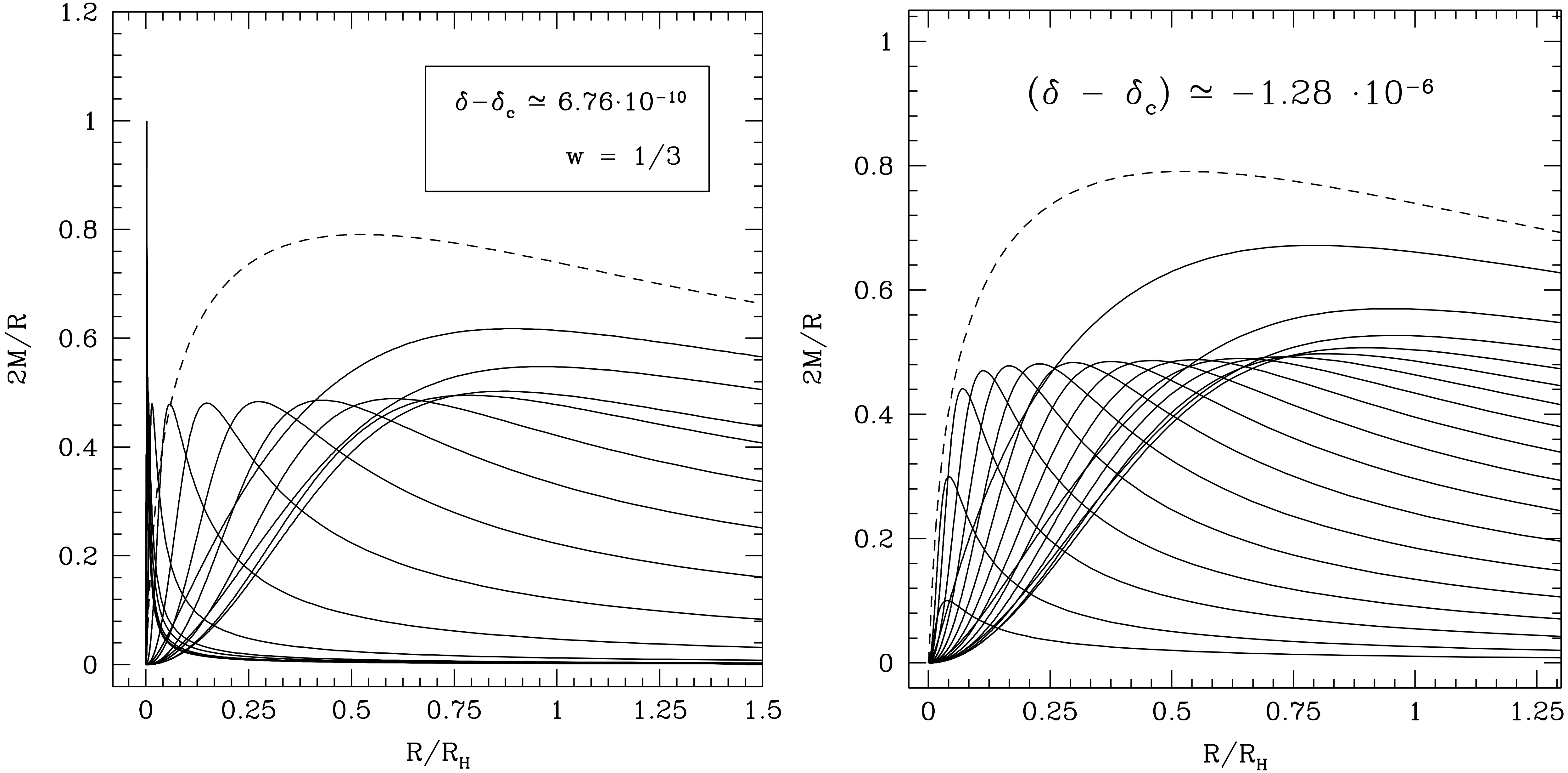
rest mass density

adiabatic index - particle degree of freedom

specific internal energy (velocity dispersion)

- Barotropic fluid (no rest mass density): $p = we$ with $w \in [0, 1]$
  - radiation dominated era: $w = 1/3$ RADIATION $(\gamma = 4/3)$
  - matter dominated era: $w = 0$ DUST $(\gamma = 1)$
- Polytropic fluid: $p = K(s)\rho^{\gamma}$ $(\gamma = 5/3,\ 4/3,\ 2)$
  - If the fluid is adiabatic (no entropy change): $K(s) = K$ (constant)

Numerical Results: PBH formation/bounce



IM, J. Miller - CQG (2005, 2009)

Background model & Curvature profile

- The **unperturbed solution,** describing an expanding homogeneous universe**,** is given by the FRW metric: $K = \pm 1, 0$ is the **curvature parameter**, $s(t)$ is the **scale factor**, and $R = s(t)r$ **circumferential radial coordinate / areal radius**.

$$ds^2 = -dt^2 + s^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$$

- In the linear regime of cosmological perturbations, pure growing modes on the super horizon scale can be described by a time independent curvature profile (**quasi-homogeneous / gradient expansion solution**).

$K(r)$ or $\zeta(\tilde{r})$ $\qquad\qquad r = \tilde{r}e^{\zeta(\tilde{r})}$

$$\boxed{K(r)r^2 = -\tilde{r}\zeta'(\tilde{r})\left[2 + \tilde{r}\zeta'(\tilde{r})\right]}$$

PBH formation: setting the problem

- Defining the scale of the cosmological perturbations as R_0 and the **cosmological horizon** scale as $R_H := 1/H_b$. In the linear regime of supra horizon growing modes we can construct a **small parameter** $\epsilon(t) << 1$ as:

$$\epsilon(t) := \frac{R_H}{R_0} \propto \left(\frac{t}{t_0}\right)^{\frac{(1+3w)}{3(1+w)}} \qquad H^2 = \frac{8\pi}{3} e_b \Rightarrow R_H = \frac{1}{H}$$

- Pure growing modes are given by $f(r,t) = f_0 \left[1 + \epsilon^2(t)\tilde{f}(r)\right] \quad \epsilon \ll 1$

$$a = 1 + \epsilon^2 \tilde{a} \qquad e = e_b \left(1 + \epsilon^2 \tilde{e}\right)$$

$$b = \frac{\partial_r R}{\sqrt{1 - K(r) r^2}} \left(1 + \epsilon^2 \tilde{b}\right) \qquad U = HR\left(1 + \epsilon^2 \tilde{U}\right)$$

$$R = s(t) r \left(1 + \epsilon^2 \tilde{R}\right) \qquad M = \frac{4}{3}\pi e_b R^3 \left(1 + \epsilon^2 \tilde{M}\right)$$

- In the **linear regime**, when $\epsilon << 1$, the **curvature profile is time independent** because pressure gradients are negligible, and can be used as the only independent source of perturbations.

Density profile & perturbation amplitude

- The density profile is expressed in terms of the curvature profile :

$$\frac{\delta e}{e_b} = \left(\frac{1}{sH}\right)^2 \frac{3(1+w)}{5+3w} \left[K(r) + \frac{r}{3}K'(r)\right]$$

$$\frac{\delta e}{e_b} = -\left(\frac{1}{sH}\right)^2 \frac{2(1+w)}{5+3w} e^{-2\zeta(\tilde{r})} \left[\zeta''(\tilde{r}) + \zeta'(\tilde{r})\left(\frac{2}{\tilde{r}} + \frac{1}{2}\zeta'(\tilde{r})\right)\right]$$

- The perturbation amplitude can measured as the mass excess inside a certain radius (normalized with respect $\varepsilon = 1$) :

$$\delta(r) := \frac{1}{V}\int_0^r 4\pi \frac{\delta e}{e_b} r'^2 \, dr' \qquad V = \frac{4}{3}\pi r^3$$

- The radius of the perturbation is identified as the location where the perturbation reaches its maximum compactness:

$$\mathcal{C} := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = f(w)K(r)r^2 \qquad f(w) = \frac{3(1+w)}{5+3w}$$

$$r_m \ : \ C'(r_m) = 0 \quad \Rightarrow \quad \delta(r_m) + \frac{r_m}{2}\delta'(r_m) = 0$$

$$K(r_m) + \frac{r_m}{2}K'(r_m) = 0 \qquad \zeta'(\tilde{r}_m) + \tilde{r}_m\zeta''(\tilde{r}_m) = 0$$

$$\delta_m = f(w)K(r_m)r_m^2 = -f(w)\tilde{r}_m\zeta'(\tilde{r}_m)[2 + \tilde{r}_m\zeta'(\tilde{r}_m)]$$

$$\boxed{\frac{\delta e}{e_b}(r_m) = \frac{\delta_m}{3}}$$

Specifying the curvature profile

$$K(r) = \mathcal{A}\left(\frac{r}{\Delta}\right)^n \exp\left[-\frac{1}{2}\left(\frac{r}{\Delta}\right)^{2\alpha}\right]$$

$$n \geq 0$$

$$\alpha > 0$$

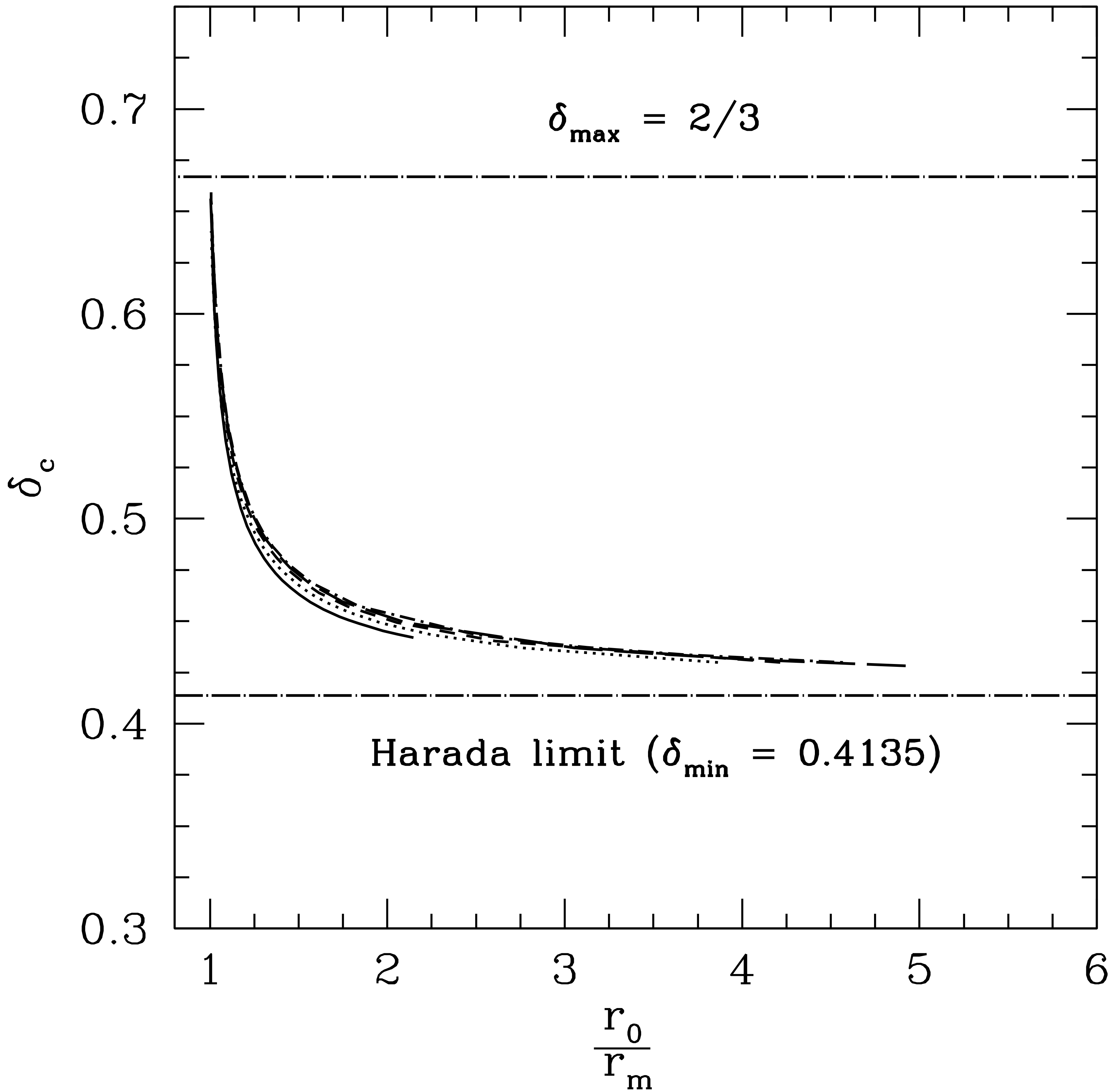
$$\frac{\delta e}{e_b} = f(w)\left[1 + \frac{n}{3} - \frac{\alpha}{3}\left(\frac{r}{\Delta}\right)^{2\alpha}\right] K(r)$$

$$\delta_m = f(w)\left(\frac{n+2}{\alpha}\right)^{(n+2)/2\alpha} \exp\left(-\frac{n+2}{2\alpha}\right) \mathcal{A}\Delta^2$$

$$\frac{r_0}{r_m} = \left(\frac{n+3}{n+2}\right)^{1/2\alpha}$$

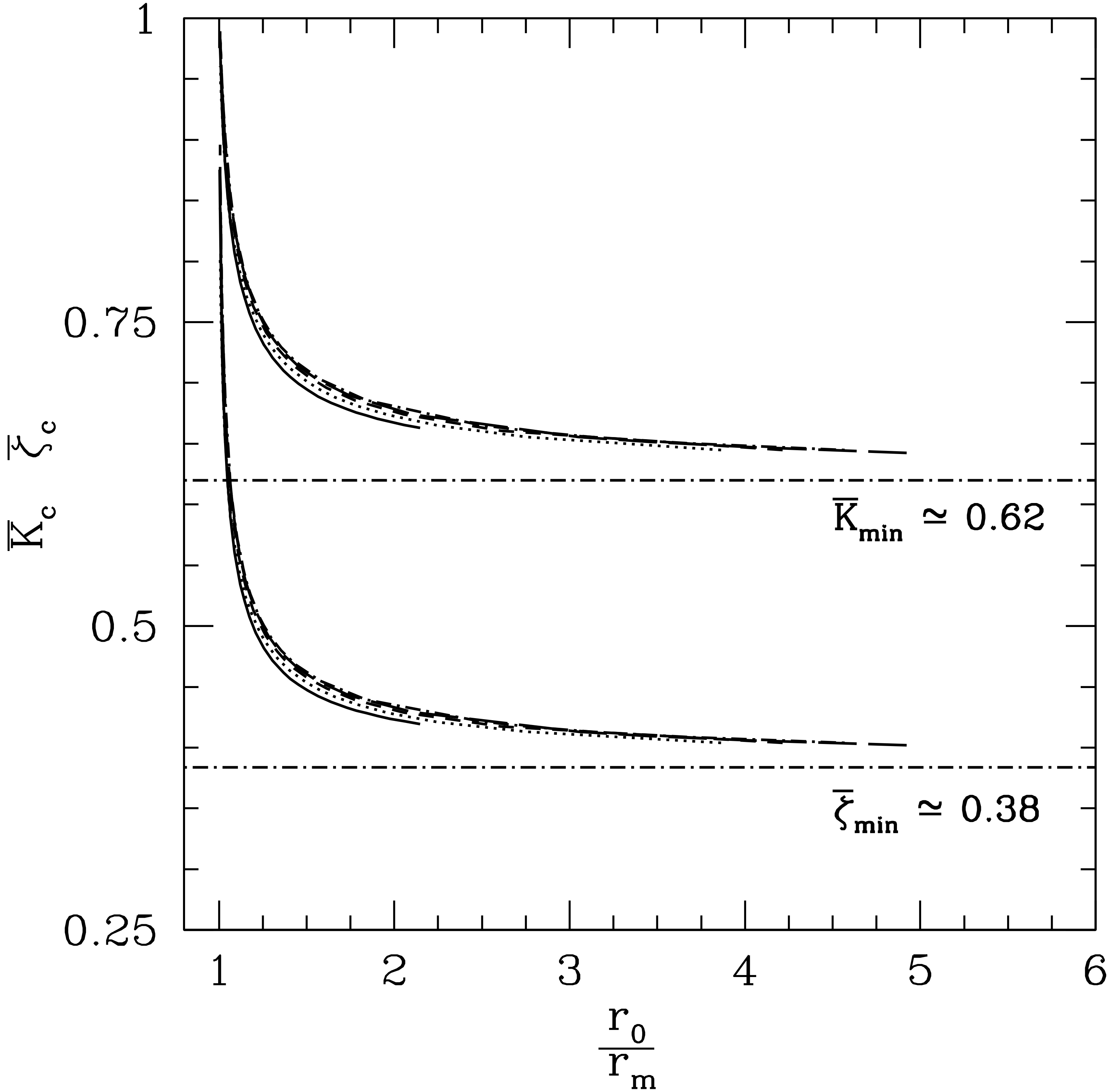
<u>**measuring the steepness of the profile**</u>

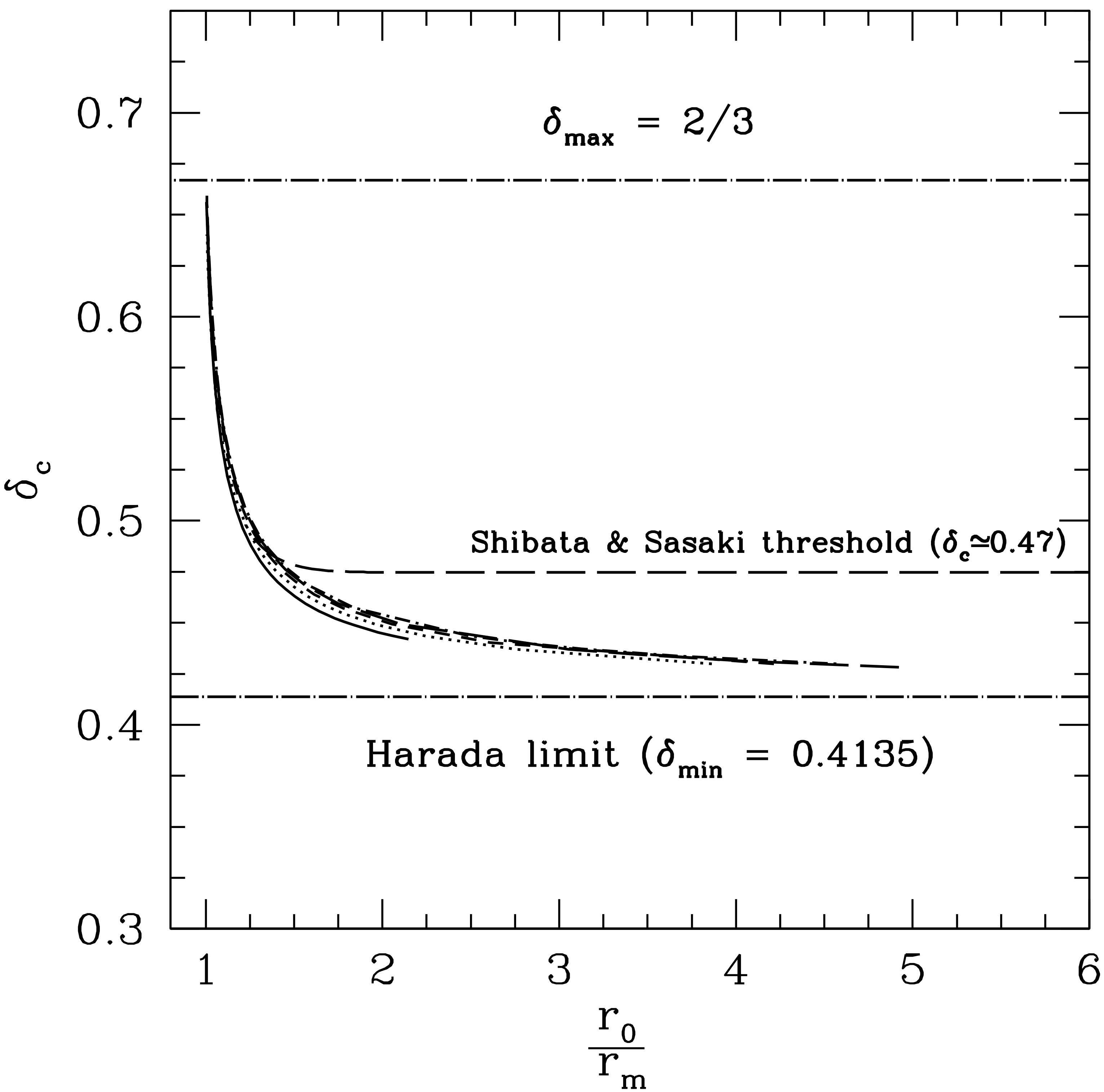
PBH threshold against profile steepness



$$\bar{K} = K(r_m)r_m^2 \quad \bar{\zeta} = -\tilde{r_m}\zeta'(\tilde{r}_m)$$

$$\boxed{\delta_c = f(w)\bar{K}_c = f(w)\bar{\zeta}_c[2-\bar{\zeta}_c]}$$





To characterize the pressure gradients of a density profile we need more than one parameter!!!

$$\delta_c\left[\frac{r_0}{r_m}, \frac{\delta e}{e_b}\left(\frac{r_p}{r_m}\right)\right]$$

IM - in preparation

Conclusions & Future perpectives

- With the Misner-Sharp equations (cosmic time slicing) we have studied initial condition for PBH formation corresponding to pure growing modes in the early Universe (radiation dominated era).

- The choice of the equation of state determines the final virialized structure of the collapse. **Pressure and curvature profiles plays a key role determining the particular value of the threshold for PBH formation.**

- **PBH formation is characterised by non liner curvature profile,** the linear approximation used by ***Green, Liddle, Malik, Sasaki*** (2004) does not gives accurate results. **In terms of *ζ(r)* the threshold is given by its first derivative at the length scale of the perturbation.**

- **The shape effects need to be described by more than one parameter.**

$$\bar{\zeta} = -\tilde{r}_m \zeta'(\tilde{r}_m)$$

$$\delta_c \left[\frac{r_0}{r_m}, \frac{\delta e}{e_b}\left(\frac{r_p}{r_m} \right) \right]$$

- Cosmological consequences of numerical results: ***S.Young, IM, C.Byrnes - in preparation.***