Primordial GWs sourced by gauge field

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with A.Ito (Kobe U.) (in progress)
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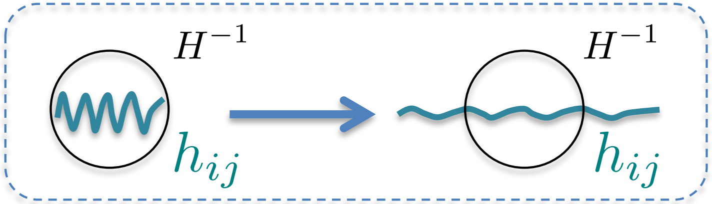
Primordial GWs from Inflation

- Metric tensor modes provided by the inflationary expansion of spacetime.

$$g_{ij} = a(\tau)^2[\delta_{ij} + \boxed{h_{ij}}]$$



Power spectrum of vacuum fluctuations: $\langle \hat{h}^A_{\boldsymbol{k}} \hat{h}^A_{\boldsymbol{k}'} \rangle = (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} \mathcal{P}^A_h(k),$

$$\mathcal{P}^+_h = \mathcal{P}^\times_h = \left. \frac{H^2}{\pi^2 M_p^2} \right|_{k=aH}$$

(scale-invariant & isotropic)

- This relation is valid only if the dominant contributions of GWs are vacuum modes.

- From the theoretical point of view, however, there is a room to suppose another source of GWs caused by **matter sectors in early universe**:

Inflation with gauge field (today's talk)

- We explore the generation of PGWs sourced by gauge field **kinetically coupled with scalar field**, which are potentially testable with future **CMB mission or GW detectors.**

$$f(\sigma)^2 F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Gauge field can grow up at super horizon scale due to the time variation of gauge kinetic function and enhance coupled metric tensor modes.

$$\Box h_{ij} = S_{ij}(F, \chi) \quad \rightarrow \quad h_{ij}^{+\times}(\boldsymbol{k}) = h_{ij}^{v}(k) + h_{ij}^{s+\times}(\boldsymbol{k})$$

$$\rightarrow \mathcal{P}_h^{+\times} = \mathcal{P}_h^{v}(k) + \boxed{\mathcal{P}_h^{s+\times}(k,\theta)}$$

(scale-variant & isotropic!)

Particle production of gauge field

EOM for the mode function of gauge field:

$$(fA_k)'' + \left(k^2 - \frac{n(n-1)}{\tau^2}\right) fA_k = 0$$

$f \propto \tau^n, \ f''/f = n(n-1)/\tau^2$

$\tau = -(Ha)^{-1}$: conformal time

f''/f

- **Tachyonic instability can occur at super-horizon scales.**

$(|k\tau| \to 0)$

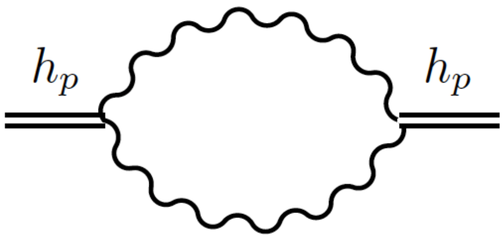
- We can define the electric field as $E_i \equiv -\dfrac{f}{a^2} A_i'$
- At superhorizon scales, it reads

$$E_k = i\frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}} H^2 \left(\frac{2}{k}\right)^{3/2} \left(\frac{2}{-k\tau}\right)^{n-2}$$

$$\leftrightarrow |E_k| \propto a(\tau)^{n-2}$$

- **It grows up exponentially when n>2 is satisfied.**

GWs sourced by gauge boson



$$\left[\partial_\tau^2 - \nabla^2 - \frac{2}{\tau^2}\right]\psi_{ij} = -\frac{a^3}{M_p}(E_iE_j + B_iB_j)^{TT}$$

$$\psi_{ij} \equiv \frac{M_p}{2} a h_{ij}$$

Power Spectrum of sourced GWs:

$$\mathcal{P}_{h \ \mathrm{source}} \simeq \frac{H^4}{M_p^4} A(n) \underline{\exp(2(n-2)\Delta N(k))}$$

$(\Delta N$: time interval when particle production of gauge field occurs$)$

- Particle production of gauge fields sources GWs.
- **The amplitude is exponentially red-tilted.**

Model building

Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - U(\varphi) - \frac{1}{2}(\partial_\mu\sigma)^2 - V(\sigma) - \frac{1}{4}f(\sigma)^2 F_{\mu\nu}F^{\mu\nu}\right]$$

φ : inflaton, σ : spectator

Assumption

✓ Only spectator field couples to gauge field directly.
(→suppresses an overproduction of curvature perturbation)

Configuration of gauge kinetic function

$$f(\sigma) = \exp\left[\frac{\sigma}{\Lambda}\right]$$
$$\propto a(t)^{-n} \qquad \left(n \equiv -\frac{\dot{f}}{Hf} = -\frac{\dot{\sigma}}{H\Lambda}\right)$$

Background dynamics (i)

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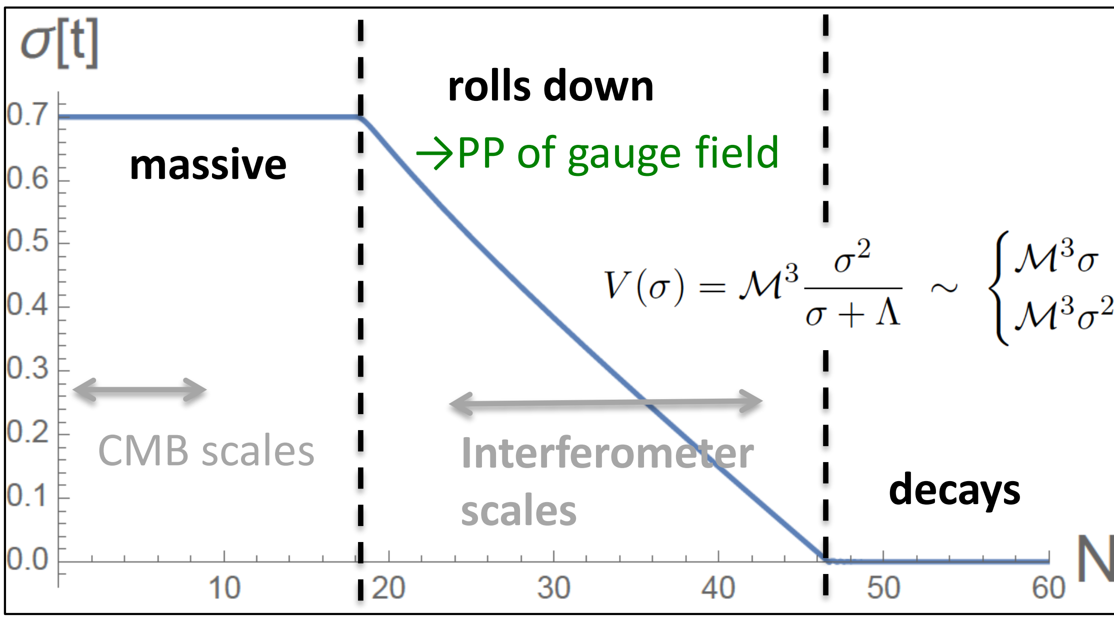
(i) gauge field is amplified on small scales

Time evolution of spectator field



$$V(\sigma) = \mathcal{M}^3 \frac{\sigma^2}{\sigma + \Lambda} \sim \begin{cases} \mathcal{M}^3 \sigma & (\sigma \gg \Lambda) \\ \mathcal{M}^3 \sigma^2 / \Lambda & (\sigma \ll \Lambda) \end{cases}$$

- Particle production of gauge field occurs around interferometer scales

PGWs sourced by gauge field

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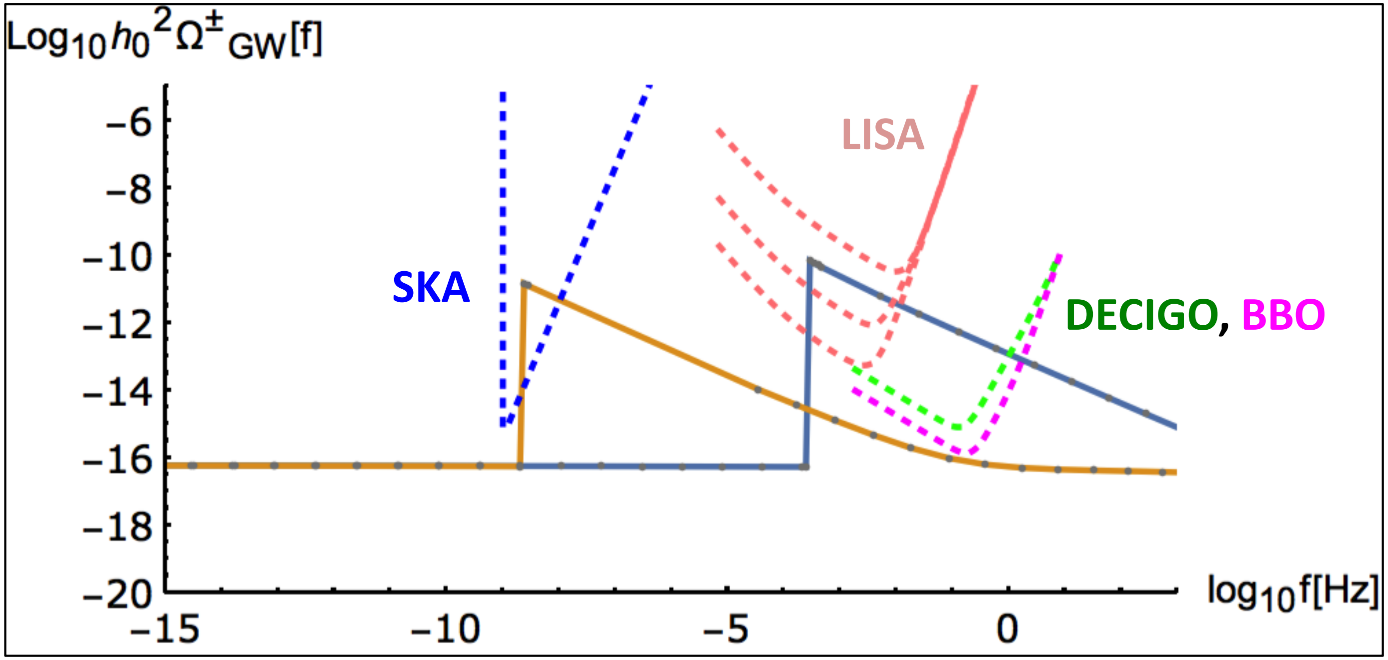
in progress

Energy density of Present GWs as a function of frequency



$(\Lambda = 10^{-2} M_p)$

$(n \gtrsim 2, \Delta N \sim 30)$

- Sourced PGWs are potentially testable with future pulsar timing array mission or space-based interferometers.

Background dynamics (ii)

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(ii) gauge field is amplified on large scales

→Anisotropic attractor solution with spectator field can be realizable.

c.f. anisotropic inflation
Watanabe *et al*. 2009

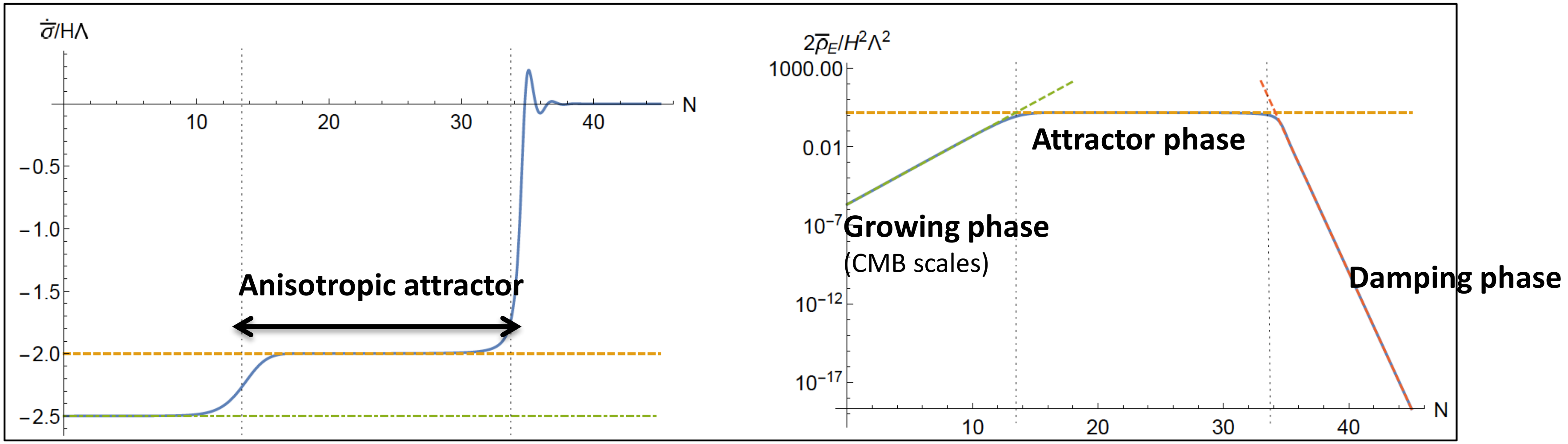
$$A_i(t,\boldsymbol{x}) = \bar{A}_i(t) + \delta A_i(t,\boldsymbol{x})$$

$$\ddot{\bar{\sigma}} + 3H\dot{\bar{\sigma}} + \bar{V}' = \frac{2}{\Lambda}\bar{\rho}_E$$

$$\bar{\rho}_E \equiv \frac{\bar{I}^2}{2a^2}\dot{\bar{A}}_i^2$$

$$V(\sigma) = \mathcal{M}^3\frac{\sigma^2}{\sigma+\Lambda} \sim \begin{cases}\mathcal{M}^3\sigma & (\sigma \gg \Lambda)\\ \mathcal{M}^3\sigma^2/\Lambda & (\sigma \ll \Lambda)\end{cases}$$

Time evolution of spectator and energy density of background gauge field



Statistically anisotropic GWs

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$$\delta A_i(t,\boldsymbol{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left[e_i^X(\hat{\boldsymbol{k}})\delta A_{\boldsymbol{k}}^X(t) + i e_i^Y(\hat{\boldsymbol{k}})\delta A_{\boldsymbol{k}}^Y(t) \right] \qquad \dot{\boldsymbol{A}} \propto \hat{\boldsymbol{z}}$$

EOMs for GWs

$$\left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right] h_k^+ = \frac{4\sqrt{\bar\rho_E}}{aM_{\rm Pl}^2}\sin\theta \left[\bar I \delta \dot A_k^X - a\sqrt{2\bar\rho_E}\sin\theta \frac{\delta\sigma_k}{\Lambda}\right]$$

$$\left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right] h_k^\times = \frac{4\sqrt{\bar\rho_E}}{aM_{\rm Pl}^2}\sin\theta\, \bar I \delta \dot A_k^Y,$$

$$\cos\theta \equiv \boldsymbol{k}\cdot\dot{\boldsymbol{A}}/(|\boldsymbol{k}||\dot{\boldsymbol{A}}|)$$

(note: GWs and gauge fields are mixed via the background vector field)

Power spectrum of sourced GWs $\quad k_{\rm CMB} \ll k_A$

$$\mathcal{P}_h^{(s)} = \frac{1}{2}\mathcal{P}_h^{(\rm vac)}\left(\left|\frac{\psi_{(s)}^+}{\psi_{(\rm vac)}}\right|^2 + \left|\frac{\psi_{(s)}^\times}{\psi_{(\rm vac)}}\right|^2\right),$$

$$= \frac{2H^2}{\pi^2 M_{\rm Pl}^2}\left(1-\cos^2\theta+\cos^4\theta-\cos^6\theta\right)\left[\Delta n\,\tilde\gamma(n)\frac{\Lambda}{M_{\rm Pl}}\left(\frac{k_A}{k}\right)^{\Delta n}\left(N_A - \frac{1}{3}\right)\right]^2$$

$$\mathcal{P}_h^+ \propto \cos^4\theta(1-\cos^2\theta), \qquad \mathcal{P}_h^\times \propto 1-\cos^2\theta$$

Detectability of GWs

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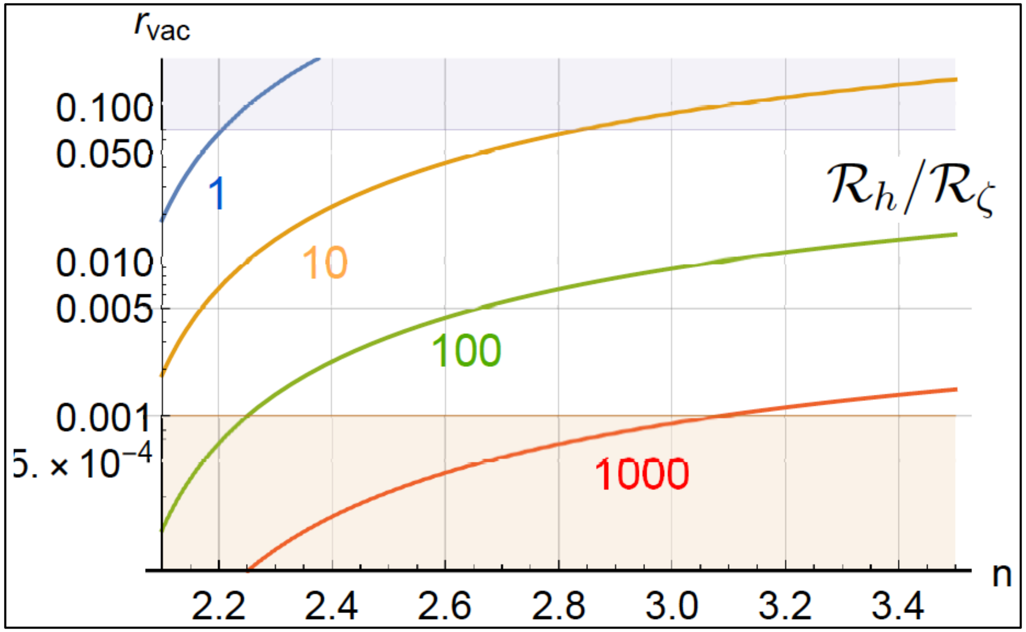
- We define the ratio between vacuum contribution and sourced one to the power spectra of curvature perturbation and GWs:

$$\mathcal{R}_\zeta \equiv \mathcal{P}_\zeta^{(s)}/\mathcal{P}_\zeta^{(\mathrm{vac})}, \qquad \mathcal{R}_h \equiv \mathcal{P}_h^{(s)}/\mathcal{P}_h^{(\mathrm{vac})}$$

- In order to have detectable GWs without producing too much ζ , one needs

$$\mathcal{R}_\zeta \ll 1, \qquad \mathcal{R}_h \gtrsim 1$$



$$\frac{\mathcal{R}_h}{\mathcal{R}_\zeta} = \frac{8\Delta n^2}{n^2 r_{\mathrm{vac}}} \frac{1+\cos^4\theta}{\left(1-\frac{\Delta n}{n}\cos^2\theta\right)^2}$$

Summary & Outlook

- ✓ We develop the possibility of generating scale-dependent or statistically anisotropic PGWs sourced by U(1) gauge field which is kinetically coupled to spectator field.
- ✓ We find that it could be potentially testable with upcoming CMB observations, GW detectors and pulsar timing arrays.
- ■ Analysis of scalar perturbation (possibility of PBH formation) has to be studied. (case (i))
- ■ Further studies on other potential forms based on a dedicated model building are also fascinating. (case (ii))
- ■ …