Gaining information about inflation via the reheating era

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Outline

CORE collaboration: arXiv:1612.08270
J. Martin, CR and V. Vennin: arXiv:1609.04739, arXiv:1603.02606, arXiv:1410.7958

Reheating-consistent observable predictions

Single field example

- Dynamics given by $(\kappa^2 = 1/M_{\mathrm{P}}^2)$

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \mathcal{L}(\phi)\right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

- Can be used to describe:
  - Minimally coupled scalar field to General Relativity
  - Scalar-tensor theory of gravitation in the Einstein frame

    the graviton' scalar partner is also the inflaton (HI, RPI1,. . .)

- Everything can be consistently solved in the slow-roll approximation
  - Background evolution $\phi(N)$ where $N \equiv \ln a$
  - Linear perturbations for the field-metric system $\zeta(t, \boldsymbol{x})$, $\delta\phi(t, \boldsymbol{x})$

- Slow-roll = expansion in terms of the Hubble flow functions [Schwarz 01]

$$\epsilon_0 \equiv \frac{H_{\mathrm{ini}}}{H}\,, \qquad \epsilon_{i+1} \equiv \frac{\mathrm{d}\ln|\epsilon_i|}{\mathrm{d}N} \qquad \text{measure deviations from de-Sitter}$$

Decoupling field and space-time evolution

- Friedmann-Lemaître equations in e-fold time (with $M_{\mathrm{P}}^2 = 1$)

$$\begin{cases} H^2 = \dfrac{1}{3}\left(\dfrac{1}{2}\dot{\phi}^2 + V\right) \\ \dfrac{\ddot{a}}{a} = -\dfrac{1}{3}\left(\dot{\phi}^2 - V\right) \end{cases} \Rightarrow \begin{cases} H^2 = \dfrac{V}{3 - \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2} \\ -\dfrac{\mathrm{d}\ln H}{\mathrm{d}N} = \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases} \Leftrightarrow \begin{cases} H^2 = \dfrac{V}{3-\epsilon_1} \\ \epsilon_1 = \dfrac{1}{2}\left(\dfrac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases}$$

- Klein-Gordon equation in e-folds: relativistic kinematics with friction

$$\frac{1}{3-\epsilon_1}\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi} \quad \Leftrightarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{3-\epsilon_1}{3-\epsilon_1+\dfrac{\epsilon_2}{2}}\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

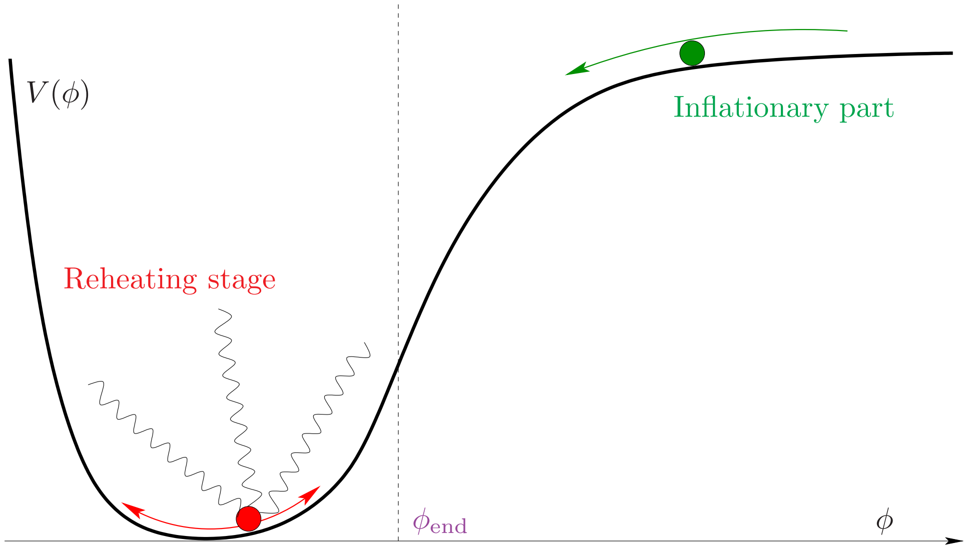
- Slow-roll approximation: all $\epsilon_i = \mathcal{O}(\epsilon)$ and $\epsilon_1 < 1$ is the definition of inflation ($\ddot{a} > 0$)
  - ✦ The trajectory can be solved for N

$$N - N_{\mathrm{end}} \simeq \int_{\phi}^{\phi_{\mathrm{end}}} \frac{V(\psi)}{V'(\psi)}\,\mathrm{d}\psi$$

Reheating-consistent observable predictions

The end of inflation and after

- Accelerated expansion stops for $\epsilon_1 > 1$ $(\ddot{a} < 0)$ at $N = N_{\rm end}$
  - Naturally happens during field evolution (graceful exit) at $\phi = \phi_{\rm end}$
    $$\epsilon_1(\phi_{\rm end}) = 1$$
  - Or, there is another mechanism ending inflation (tachyonic or field-curvature instability) and $\phi_{\rm end}$ is a model parameter that has to be specified
- The reheating stage: everything after $N_{\rm end}$ till radiation domination
  - Basic picture $\longrightarrow$
  - But in reality a very complicated process, microphysics dependent
  - Reheating duration is usually unknown:
    $$\Delta N_{\rm reh} \equiv N_{\rm reh} - N_{\rm end}$$



Redshift at which reheating ends

- Denoting $N = N_{\rm reh}$ the end of reheating = beginning of radiation era
  - If thermalized, and no extra entropy production: $a_{\rm reh}^3 s_{\rm reh} = a_0^3 s_0$

$$\begin{cases} s_{\rm reh} = q_{\rm reh}\dfrac{2\pi^2}{45}T_{\rm reh}^3 \\ \rho_{\rm reh} = g_{\rm reh}\dfrac{\pi^2}{30}T_{\rm reh}^4 \end{cases} \Rightarrow \quad \frac{a_0}{a_{\rm reh}} = \left(\frac{q_{\rm reh}^{1/3}\, g_0^{1/4}}{q_0^{1/3}\, g_{\rm reh}^{1/4}}\right)\frac{\rho_{\rm reh}^{1/4}}{\rho_\gamma^{1/4}}$$

$$\text{or} \quad 1 + z_{\rm reh} = \left(\frac{\rho_{\rm reh}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

- Depends on $\rho_{\rm reh}$ and $\tilde{\rho}_\gamma \equiv \mathcal{Q}_{\rm reh}\rho_\gamma$
  - Energy density of radiation today: $\rho_\gamma = 3\dfrac{H_0^2}{M_{\rm P}^2}\Omega_{\rm rad}$
  - Change in the number of entropy and energy relativistic degrees of freedom (small effect compared to $\rho_{\rm reh}/\rho_\gamma$)

$$\mathcal{Q}_{\rm reh} \equiv \frac{g_{\rm reh}}{g_0}\left(\frac{q_0}{q_{\rm reh}}\right)^{1/4}$$

Redshift at which inflation ends

- Depends on the redshift of reheating

$$1 + z_{\mathrm{end}} = \frac{a_0}{a_{\mathrm{end}}} = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}(1 + z_{\mathrm{reh}}) = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}\left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma}\right)^{1/4} = \frac{1}{R_{\mathrm{rad}}}\left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - ✦ The reheating parameter $R_{\mathrm{rad}} \equiv \frac{a_{\mathrm{end}}}{a_{\mathrm{reh}}}\left(\frac{\rho_{\mathrm{end}}}{\rho_{\mathrm{reh}}}\right)^{1/4}$
  - ✦ Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{\mathrm{rad}} = 1$

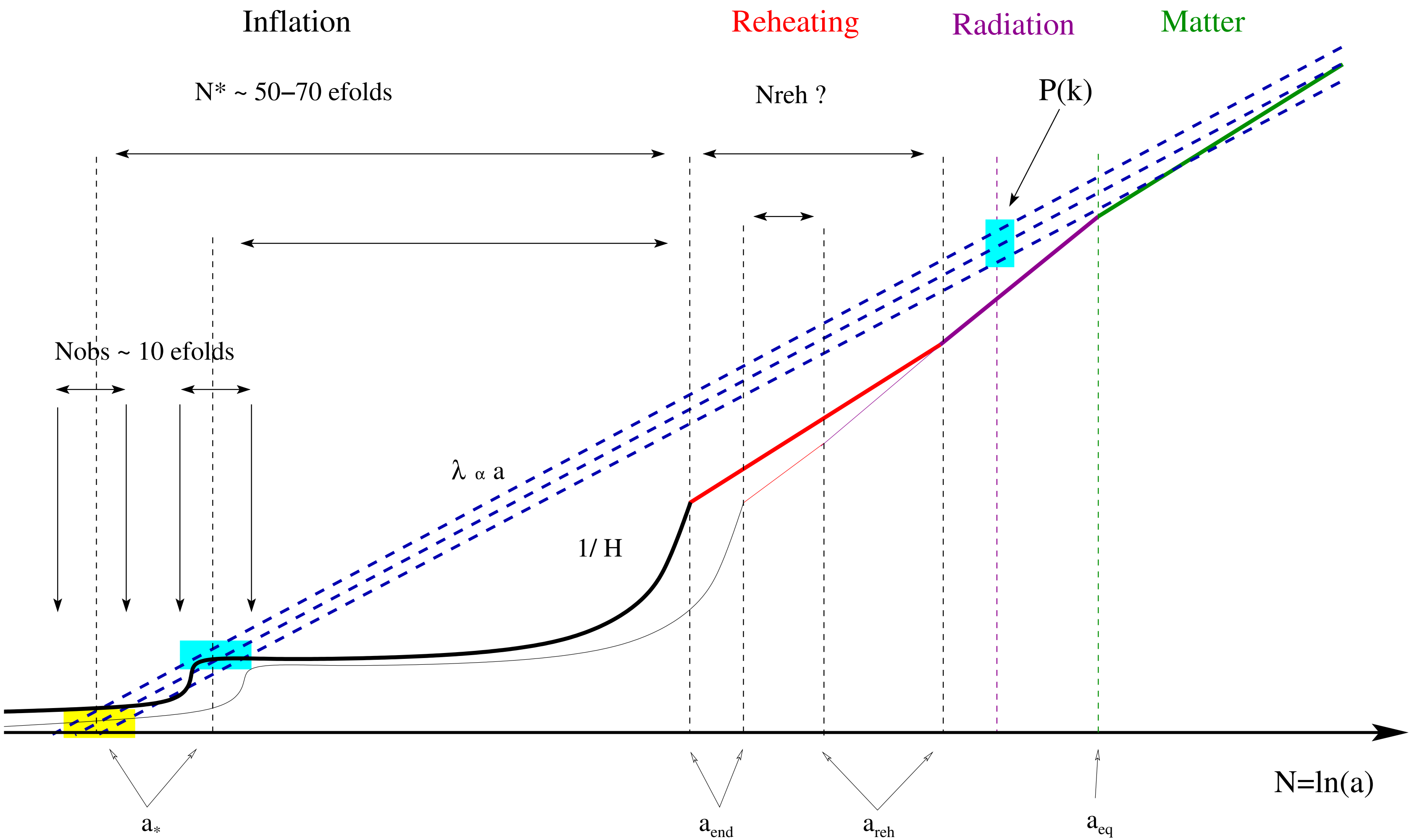
- R_{rad} can be expressed in terms of $(\rho_{\mathrm{reh}}, \overline{w}_{\mathrm{reh}})$ or $(\Delta N_{\mathrm{reh}}, \overline{w}_{\mathrm{reh}})$

$$\ln R_{\mathrm{rad}} = \frac{\Delta N_{\mathrm{reh}}}{4}(3\overline{w}_{\mathrm{reh}} - 1) = \frac{1 - 3\overline{w}_{\mathrm{reh}}}{12(1 + \overline{w}_{\mathrm{reh}})} \ln\left(\frac{\rho_{\mathrm{reh}}}{\rho_{\mathrm{end}}}\right)$$

where $\overline{w}_{\mathrm{reh}} \equiv \frac{1}{\Delta N_{\mathrm{reh}}} \int_{N_{\mathrm{end}}}^{N_{\mathrm{reh}}} \frac{P(N)}{\rho(N)} \mathrm{d}N$

- A fixed inflationary parameters, z_{end} can still be affected by R_{rad}

Reheating effects on inflationary observables



- Model testing: reheating effects must be included!

Inflationary perturbations in slow-roll

- Equations of motion for the linear perturbations

$$\left.\begin{aligned}\mu_{\mathrm{T}} &\equiv a h \\ \mu_{\mathrm{S}} &\equiv a\sqrt{2}\phi_{,N}\zeta\end{aligned}\right\} \Rightarrow \mu''_{\mathrm{TS}} + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathrm{TS}} = 0$$

- Can be consistently solved using slow-roll and pivot expansion [Stewart:1993, Gong:2001, Schwarz:2001, Leach:2002, Martin:2002, Habib:2002, Casadio:2005, Lorenz:2008, Martin:2013, Beltran:2013]

$$\begin{aligned}\mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\mathrm{P}}^2 \epsilon_{1*}} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 + \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*} \\ & + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*} \\ & + \left[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*}\right]\ln\left(\frac{k}{k_*}\right) \\ & + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*}\right]\ln^2\left(\frac{k}{k_*}\right)\Bigg\},\end{aligned}$$

$$\begin{aligned}\mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\mathrm{P}}^2}\Bigg\{ & 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2\right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2\right]\epsilon_{1*}\epsilon_{2*} \\ & + \left[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*}\right]\ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right)\ln^2\left(\frac{k}{k_*}\right)\Bigg\}\end{aligned}$$

- Notice that: $H_* \equiv H(\Delta N_*)$ and $\epsilon_{i*} \equiv \epsilon_i(\Delta N_*)$ with $k_*\eta(\Delta N_*) = -1$

The power law parameters

- From the observable point of view, one defines spectral index, running, tensor-to-scalar ratio, . . .

$$n_{\mathrm{S}} - 1 \equiv \left.\frac{\mathrm{d}\ln \mathcal{P}_\zeta}{\mathrm{d}\ln k}\right|_{k_*}, \qquad \alpha_{\mathrm{S}} \equiv \left.\frac{\mathrm{d}^2 \ln \mathcal{P}_\zeta}{\mathrm{d}(\ln k)^2}\right|_{k_*}, \qquad r \equiv \left.\frac{\mathcal{P}_h}{\mathcal{P}_h}\right|_{k_*}$$

- They are read-off from the previous slow-roll expression

$$n_{\mathrm{S}} = 1 - 2\epsilon_{1*} - \epsilon_{2*} - (3+2C)\epsilon_{1*}\epsilon_{2*} - 2\epsilon_{1*}^2 - C\epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right)$$
$$\alpha_{\mathrm{S}} = -2\epsilon_{1*}\epsilon_{2*} - \epsilon_{2*}\epsilon_{3*} + \mathcal{O}\left(\epsilon^3\right)$$
$$r = 16\epsilon_{1*}\left(1 + C\epsilon_{2*}\right) + \mathcal{O}\left(\epsilon^3\right)$$

- One has to know the functions $\epsilon_i(\Delta N_*)$ and the value of ΔN_* to make predictions

Hubble-flow functions from the potential

- One would prefer a "slow-roll" hierarchy based on $V(\phi)$ only

$$\epsilon_{v_0}(\phi) \equiv \sqrt{\frac{3}{V(\phi)}}\,, \qquad \epsilon_{v_{i+1}}(\phi) \equiv \frac{\mathrm{d}\ln \epsilon_{v_i}(\phi)}{\mathrm{d}\tilde{N}} \quad \text{with} \quad \frac{\mathrm{d}}{\mathrm{d}\tilde{N}} \equiv -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}$$

- Can be mapped with the Hubble flow hierarchy

$$\epsilon_{v_0} = \frac{\epsilon_0}{\sqrt{1-\epsilon_1/3}}\,, \qquad \epsilon_{v_1} = \epsilon_1\left(1+\frac{\epsilon_2/6}{1-\epsilon_1/3}\right)^2$$

$$\epsilon_{v_2} = \epsilon_2\left[1+\frac{\epsilon_2/6+\epsilon_3/3}{1-\epsilon_1/3}+\frac{\epsilon_1\epsilon_2^2}{(3-\epsilon_1)^2}\right]\,, \qquad \epsilon_{v_3} = \cdots$$

- Inversion can only be made perturbatively

$$\epsilon_1 = \epsilon_{v_1} - \frac{1}{3}\epsilon_{v_1}\epsilon_{v_2} - \frac{1}{9}\epsilon_{v_1}^2\epsilon_{v_2} + \frac{5}{36}\epsilon_{v_1}\epsilon_{v_2}^2 + \frac{1}{9}\epsilon_{v_1}\epsilon_{v_2}\epsilon_{v_3} + \mathcal{O}\left(\epsilon^4\right)$$

$$\epsilon_2 = \epsilon_{v_2} - \frac{1}{6}\epsilon_{v_2}^2 - \frac{1}{3}\epsilon_{v_2}\epsilon_{v_3} - \frac{1}{6}\epsilon_{v_1}\epsilon_{v_2}^2 + \frac{1}{18}\epsilon_{v_2}^3 - \frac{1}{9}\epsilon_{v_1}\epsilon_{v_2}\epsilon_{v_3} + \frac{5}{18}\epsilon_{v_2}^2\epsilon_{v_3} + \frac{1}{9}\epsilon_{v_2}\epsilon_{v_3}^2 + \frac{1}{9}\epsilon_{v_2}\epsilon_{v_3}\epsilon_{v_4} + \mathcal{O}\left(\epsilon^4\right)$$

Solving for the time of pivot crossing

- To make inflationary predictions, one has to solve $k_*\eta_* = -1$

$$\frac{k_*}{a_0} = \frac{a(N_*)}{a_0}H_* = e^{N_*-N_\mathrm{end}}\frac{a_\mathrm{end}}{a_0}H_* = \frac{e^{\Delta N_*}H_*}{1+z_\mathrm{end}} = e^{\Delta N_*}R_\mathrm{rad}\left(\frac{\rho_\mathrm{end}}{\tilde{\rho}_\gamma}\right)^{-\frac{1}{4}}H_*$$

- Defining $N_0 \equiv \ln\left(\frac{k_*}{a_0}\frac{1}{\tilde{\rho}_\gamma^{1/4}}\right)$ (number of e-folds of deceleration)
  - ✦ This is a non-trivial integral equation that depends on: model + how inflation ends + reheating + data

$$-\left[\int_{\phi_\mathrm{end}}^{\phi_*}\frac{V(\psi)}{V'(\psi)}\mathrm{d}\psi\right] = \ln R_\mathrm{rad} - N_0 + \frac{1}{4}\ln(8\pi^2 P_*) - \frac{1}{4}\ln\left\{\frac{9}{\epsilon_1(\phi_*)[3-\epsilon_1(\phi_\mathrm{end})]}\frac{V(\phi_\mathrm{end})}{V(\phi_*)}\right\}$$

  - ✦ Result: one gets ϕ_*, or equivalently ΔN_*, as a function of inflationary model parameters and R_rad

Solving exactly for the perturbations

- Inflationary dynamics given by ($\kappa^2 = 1/M_{\mathrm{P}}^2$)

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \mathcal{L}(\phi)\right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

- Knowing $V(\phi)$ + FLRW gives $\phi(N)$ (background); in turns $\phi(N)$ gives the evolution of $\mu_{\mathrm{S}}(\eta, \boldsymbol{k}) \equiv a\sqrt{2}\dot{\phi}\zeta(\eta, \boldsymbol{k})$

$$\dot{\phi} \equiv \frac{\mathrm{d}\phi}{\mathrm{d}N} \quad \Rightarrow \ddot{\mu}_{\mathrm{S}} + \left(1 - \frac{1}{2}\dot{\phi}^2\right)\dot{\mu}_{\mathrm{S}} + \frac{1}{H^2}\left[\left(\frac{k}{a}\right)^2 - \frac{(a\dot{\phi})''}{a^3\dot{\phi}}\right]\mu_{\mathrm{S}} = 0$$

  - ✦ ζ is conserved after Hubble exit $\Rightarrow \mathcal{P}_\zeta(k)$

- What is the actual value of k/a to plug into this equation?

$$k/a = (k/a_0)(1 + z_{\mathrm{end}})e^{N_{\mathrm{end}} - N}$$

  - ✦ The input are k/a_0 (in Mpc^{-1}) and R_{rad}

- Exact integration requires R_{rad} (multifields included) [astro-ph/0605367, astro-ph/0703486, arXiv:1004.5525]

The optimal reheating parameter

- Defining the rescaled reheating parameter [astro-ph/0605367]

$$\ln R_{\mathrm{reh}} \equiv \ln R_{\mathrm{rad}} + \frac{1}{4} \ln \rho_{\mathrm{end}}$$

- "Magic" cancellation: R_{reh} absorbs the dependency in P_* (valid out of slow-roll and for multifields)

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*} \frac{V(\psi)}{V'(\psi)} \mathrm{d}\psi\right] = \ln R_{\mathrm{reh}} - N_0 - \frac{1}{2} \ln\left[\frac{9}{3-\epsilon_1(\phi_{\mathrm{end}})} \frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right]$$

- What are the possible values of R_{reh}?
  - ✦ Within a given microphysics model, R_{reh} would be a function of coupling constants and inflationary parameters
  - ✦ Without any information, assuming $-1/3 < \overline{w}_{\mathrm{reh}} < 1$ and $\rho_{\mathrm{nuc}} \equiv (10\,\mathrm{MeV})^4 < \rho_{\mathrm{reh}} < \rho_{\mathrm{end}}$

$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3} \ln \rho_{\mathrm{end}}$$

Example with Higgs and Starobinski inflation

- Same potential: $V(\phi) \propto \left(1 - e^{-\sqrt{2/3}\,\phi/M_\mathrm{P}}\right)^2$
  - Starobinski Inflation: $\rho_\mathrm{reh}^{1/4} \simeq 10^9\,\mathrm{GeV}$ [Terada et al., arXiv:1411.6746]
  - Higgs Inflation: $\rho_\mathrm{reh}^{1/4} \lesssim 10^{13}\,\mathrm{GeV}??$ [Garcia-Bellido et al., arXiv:0812.4624]

Example with Higgs and Starobinski inflation

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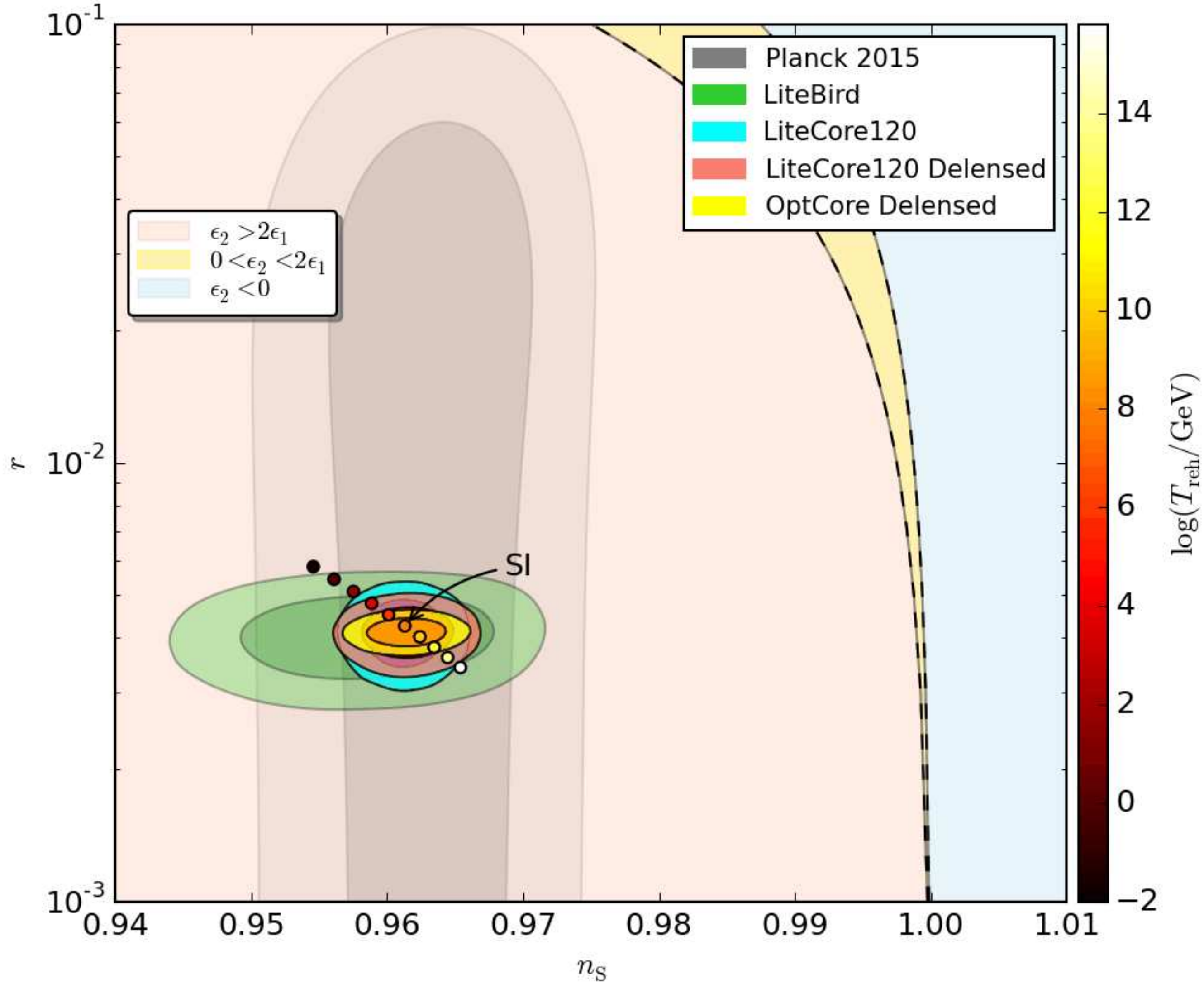


Alternative parametrizations?

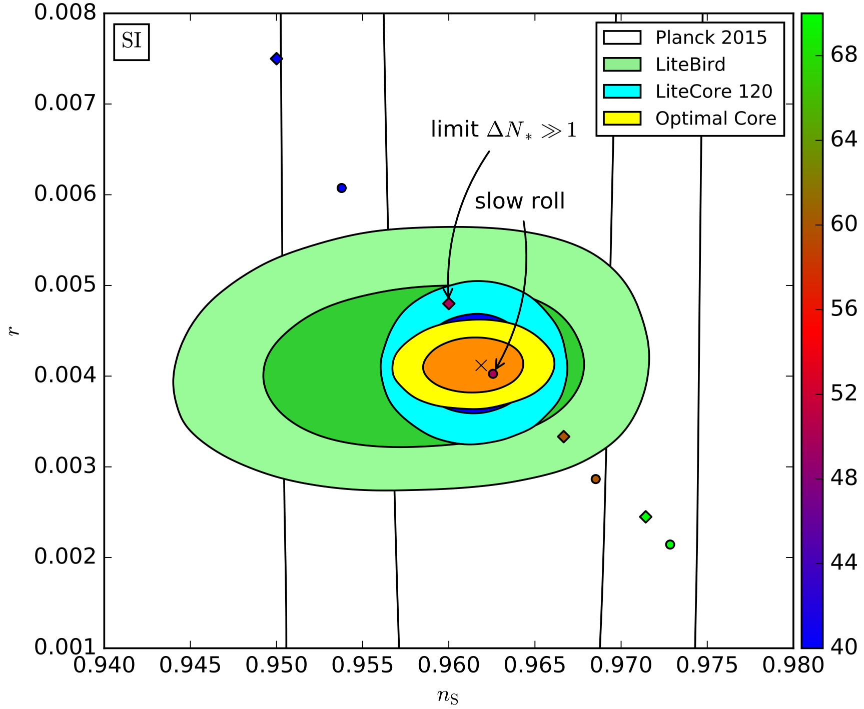
- The large ΔN_* limit (when it exists) leads to inaccurate predictions

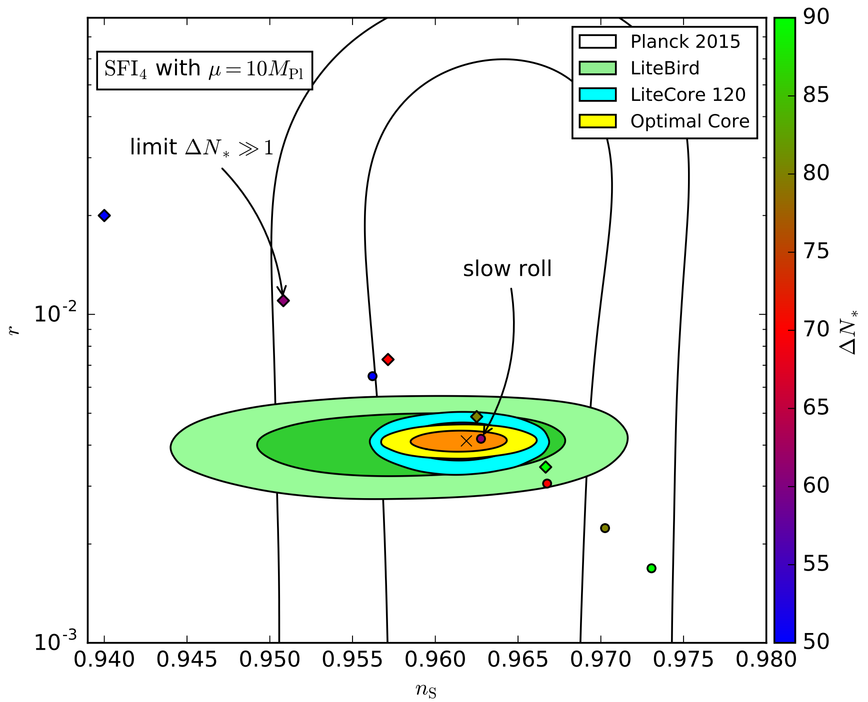
Starobinski Inflation

$$V(\phi) \propto \left(1 - e^{-\sqrt{2/3}\,\phi/M_{\rm P}}\right)^2$$

Quartic Small Field Inflation

$$V(\phi) \propto 1 - (\phi/\mu)^4$$





- ΔN_* without a potential is unpredictable...

CMB constraints on reheating

Data analysis in model space

- Data should be analyzed within the parameter space of each model, including the reheating parameter: $(\boldsymbol{\theta}_{\rm inf}, R_{\rm reh})$
- Using the public code ASPIC of Encyclopaedia Inflationaris [arxiv:1303.3787]

$$(\boldsymbol{\theta}_{\rm inf}, R_{\rm reh}) \longrightarrow \texttt{ASPIC} \longrightarrow \epsilon_{i*} \longrightarrow \begin{cases} \mathcal{P}_\zeta(k) \\ \mathcal{P}_h(k) \end{cases} \longrightarrow \begin{cases} \texttt{CAMB} \\ \texttt{CLASS} \end{cases} \longleftrightarrow \text{CMB data}$$

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\rm Pl}}+\frac{A_{\rm I}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\rm Pl}^2}\left[1+\alpha\frac{\phi^2}{M_{\rm Pl}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\rm Pl}^2}\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\rm Pl}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\rm Pl}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\rm Pl}}e^{-\phi/M_{\rm Pl}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\rm Pl}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\rm Pl}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\rm Pl}}\left\vert e^{\sqrt{2/3}\phi/M_{\rm Pl}}-1\right\vert^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\rm Pl})^2}{\alpha+(\phi/M_{\rm Pl})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-\left(3+\alpha^2\right)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)\right]$
CNBI	1	1	$M^4\left[\left(3-\alpha^2\right)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$
II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\rm Pl}}\exp\left(-\beta\frac{\phi}{M_{\rm Pl}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^\alpha\exp\left[-\beta(\phi/M_{\rm Pl})^\gamma\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\rm Pl}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\rm Pl}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\rm Pl}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\rm Pl}}\right)+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\rm Pl}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[\left(3+\alpha^2\right)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\rm Pl}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\rm Pl}}\right)\right]\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^2+\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$
RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\rm Pl}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\rm Pl}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\rm Pl}}\right)\right]\right\}^2}$

Speeding up posterior and evidence calculations

- Effective likelihood for slow-roll inflation
  - Requires only one complete data analysis (COSMOMC) to get

$$\mathcal{L}_{\mathrm{eff}}(D|P_*, \epsilon_{i*}) = \int p(D|\boldsymbol{\theta}_{\mathrm{cosmo}}, P_*, \epsilon_{i*})\pi(\boldsymbol{\theta}_{\mathrm{cosmo}})\mathrm{d}\boldsymbol{\theta}_{\mathrm{cosmo}}$$

  - Use machine-learning algorithm to fit its multidimensional shape
  - For each model $\mathcal{M}$ and their parameters $\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}$

$$p(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}|D, \mathcal{M}) = \frac{\mathcal{L}_{\mathrm{eff}}[D|P_*(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}), \epsilon_{i*}(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}})]\pi(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}|\mathcal{M})}{p(D|\mathcal{M})}$$

- All posteriors on $(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}})$ can be obtained from $\mathcal{L}_{\mathrm{eff}}$
- Marginalizing $\mathcal{L}_{\mathrm{eff}}$ over $(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}})$ gives the Bayesian evidence
- In practice
  - BAYASPIC $\equiv$ ASPIC + MULTINEST + $\mathcal{L}_{\mathrm{eff}}$ [arXiv:1312.2347]
  - 1 cpu-hour per model $\mathcal{M}$

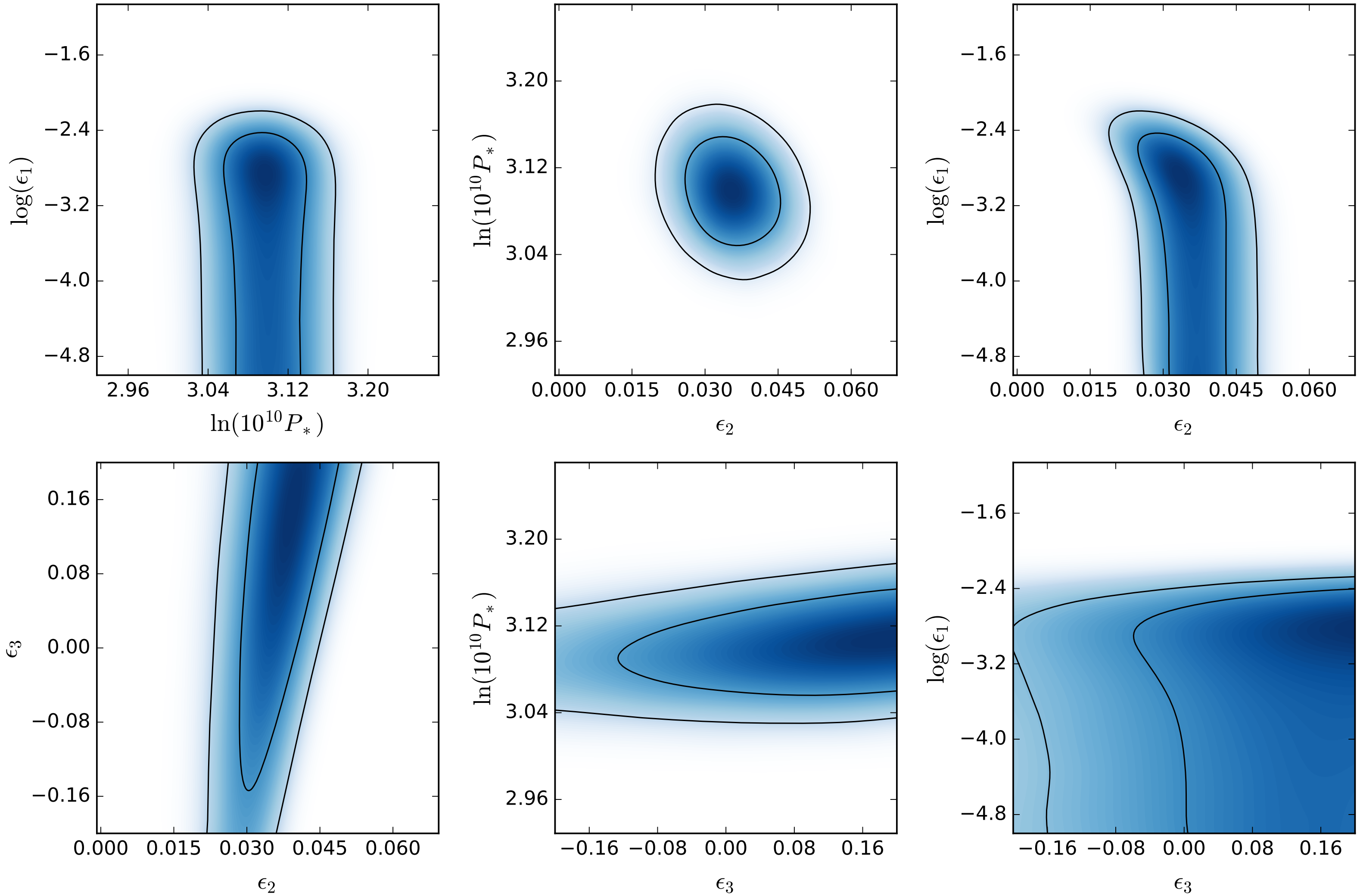
Planck 2015 + BICEP2/KECK data

- Marginalizing over instrumental, astro and cosmo parameters
  - ✦ With polarization TT and $TE + B = 32$ dimensions

$$\boldsymbol{\theta}_{\mathrm{cosmo}} = \{\Omega_{\mathrm{b}} h^2, \Omega_{\mathrm{dm}} h^2, 100\theta_{\mathrm{MC}}, \tau,$$
$$y_{\mathrm{cal}}, A_{B,\mathrm{dust}}, \beta_{B,\mathrm{dust}}, A^{\mathrm{CIB}}_{217}, \xi^{\mathrm{tSZ,CIB}}, A^{\mathrm{tSZ}}_{143},$$
$$A^{\mathrm{PS}}_{100}, A^{\mathrm{PS}}_{143}, A^{\mathrm{PS}}_{143\times 217}, A^{\mathrm{PS}}_{217}, A^{\mathrm{kSZ}}, A^{\mathrm{dust}TT}_{100}, A^{\mathrm{dust}TT}_{143},$$
$$A^{\mathrm{dust}TT}_{143\times 217}, A^{\mathrm{dust}TT}_{217}, A^{\mathrm{dust}EE}_{100}, A^{\mathrm{dust}EE}_{100\times 143}, A^{\mathrm{dust}EE}_{100\times 217},$$
$$A^{\mathrm{dust}EE}_{143}, A^{\mathrm{dust}EE}_{143\times 217}, A^{\mathrm{dust}EE}_{217}, A^{\mathrm{dust}TE}_{100}, A^{\mathrm{dust}TE}_{100\times 143},$$
$$A^{\mathrm{dust}TE}_{100\times 217}, A^{\mathrm{dust}TE}_{143}, A^{\mathrm{dust}TE}_{143\times 217}, A^{\mathrm{dust}TE}_{217}, c_{100}, c_{217}\}.$$

Planck 2015 + BICEP2/KECK data

- Marginalizing over instrumental, astro and cosmo parameters
  - ✦ With polarization TT and $TE + B = 32$ dimensions



Posteriors on the reheating parameter

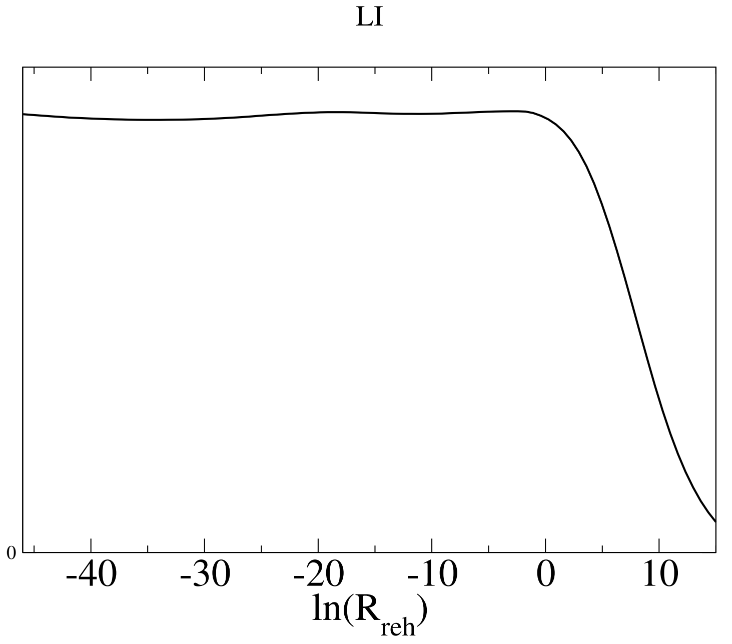
- For each model, we use the most generic parameterization: R_{reh}
  - ✦ Prior choice: Jeffreys' on $R_{\mathrm{reh}} \Leftrightarrow$ flat on $\ln R_{\mathrm{reh}}$ with:

$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3} \ln \rho_{\mathrm{end}}$$

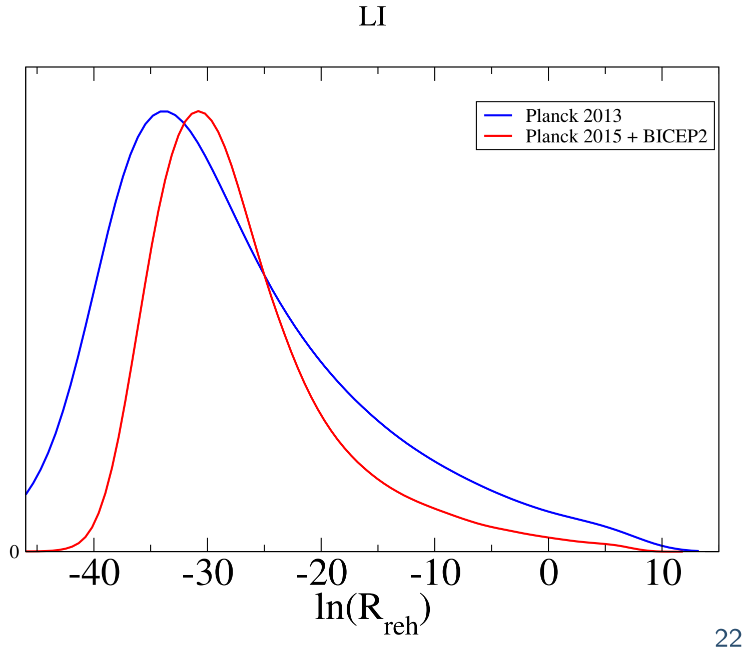
  - ✦ Planck data put non-trivial constraints on many models

- Examples: LI with $V(\phi) = M^4 (1 + \alpha \ln \phi)$

prior



posterior



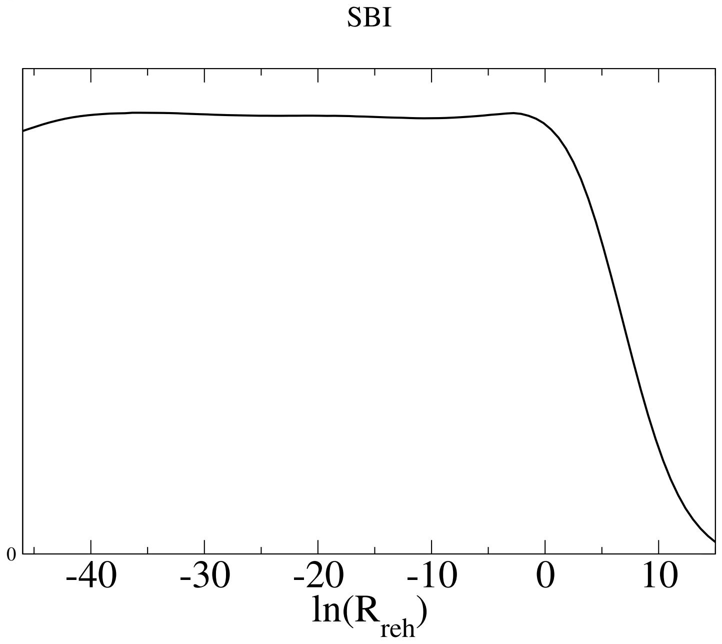
Posteriors on the reheating parameter

- For each model, we use the most generic parameterization: R_{reh}
  - Prior choice: Jeffreys' on $R_{\mathrm{reh}} \Leftrightarrow$ flat on $\ln R_{\mathrm{reh}}$ with:

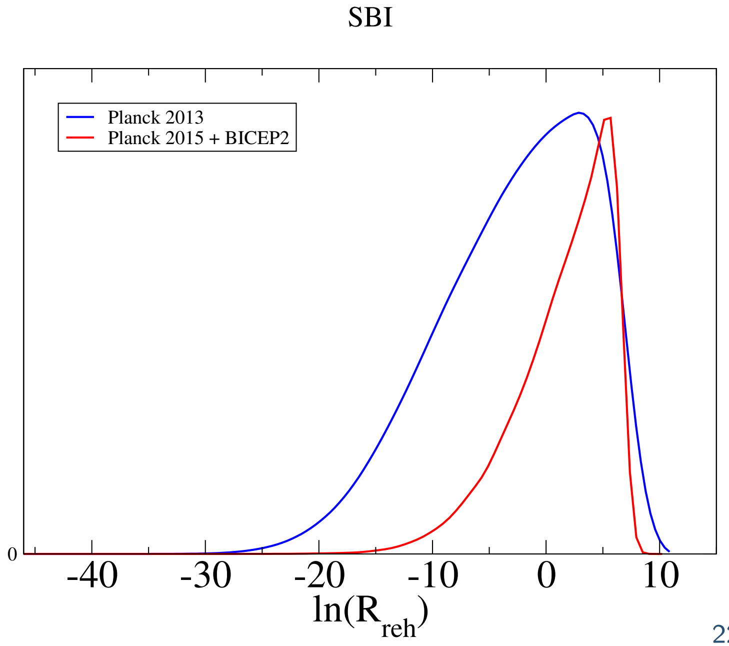
$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3} \ln \rho_{\mathrm{end}}$$

  - Planck data put non-trivial constraints on many models
- Examples: SBI with $V(\phi) = M^4 \left[1 + \phi^4 \left(-\alpha + \beta \ln \phi\right)\right]$

prior



posterior



Kullback-Leibler divergence

- Measure of information gain for the reheating parameter

$$D_{\mathrm{KL}} = \int P(\ln R_{\mathrm{reh}}|D) \ln \left[\frac{P(\ln R_{\mathrm{reh}}|D)}{\pi(\ln R_{\mathrm{reh}})}\right] \mathrm{d} \ln R_{\mathrm{reh}}$$

- Compute D_{KL} for about 200 models of inflation $\mathcal{M}_i$?
  - But some models provide a very poor fit to the data
  - Can be quantified by the Bayesian Evidence

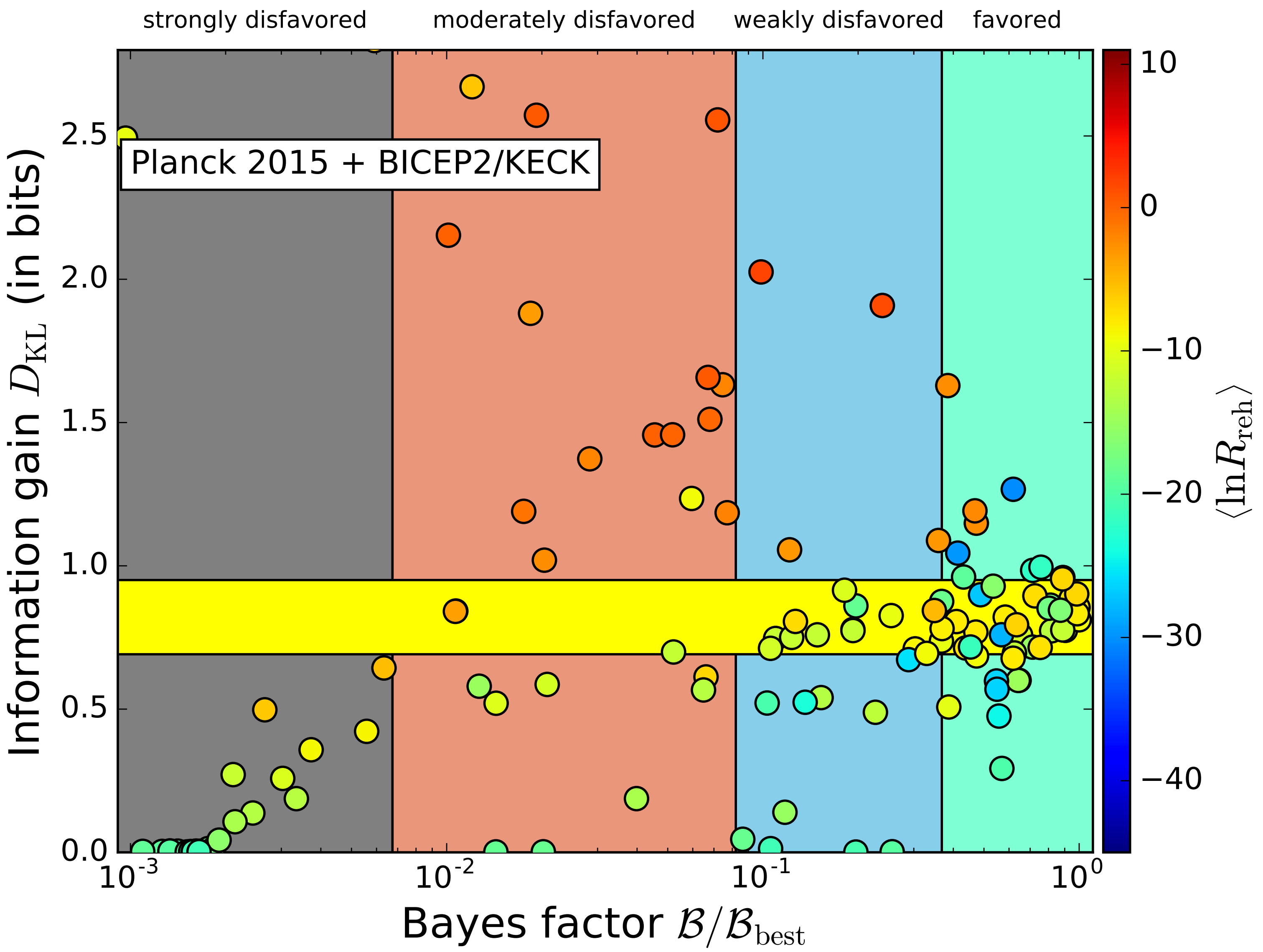
$$p(\mathcal{M}_i|D) = \pi(\mathcal{M}_i) \int \mathcal{L}_{\mathrm{eff}}\, \pi(\boldsymbol{\theta}_{\mathrm{inf}})\pi(\ln R_{\mathrm{reh}})\mathrm{d}\boldsymbol{\theta}_{\mathrm{inf}} \mathrm{d} \ln R_{\mathrm{reh}}$$

  - We use Bayes Factors (relative scale of Evidences) with non-commital priors

$$\mathcal{B}_i \equiv \frac{p(\mathcal{M}_i|D)}{\sup_j \left[p(\mathcal{M}_j|D)\right]} = \frac{p(D|\mathcal{M}_i)}{\sup_j \left[p(D|\mathcal{M}_j)\right]}$$

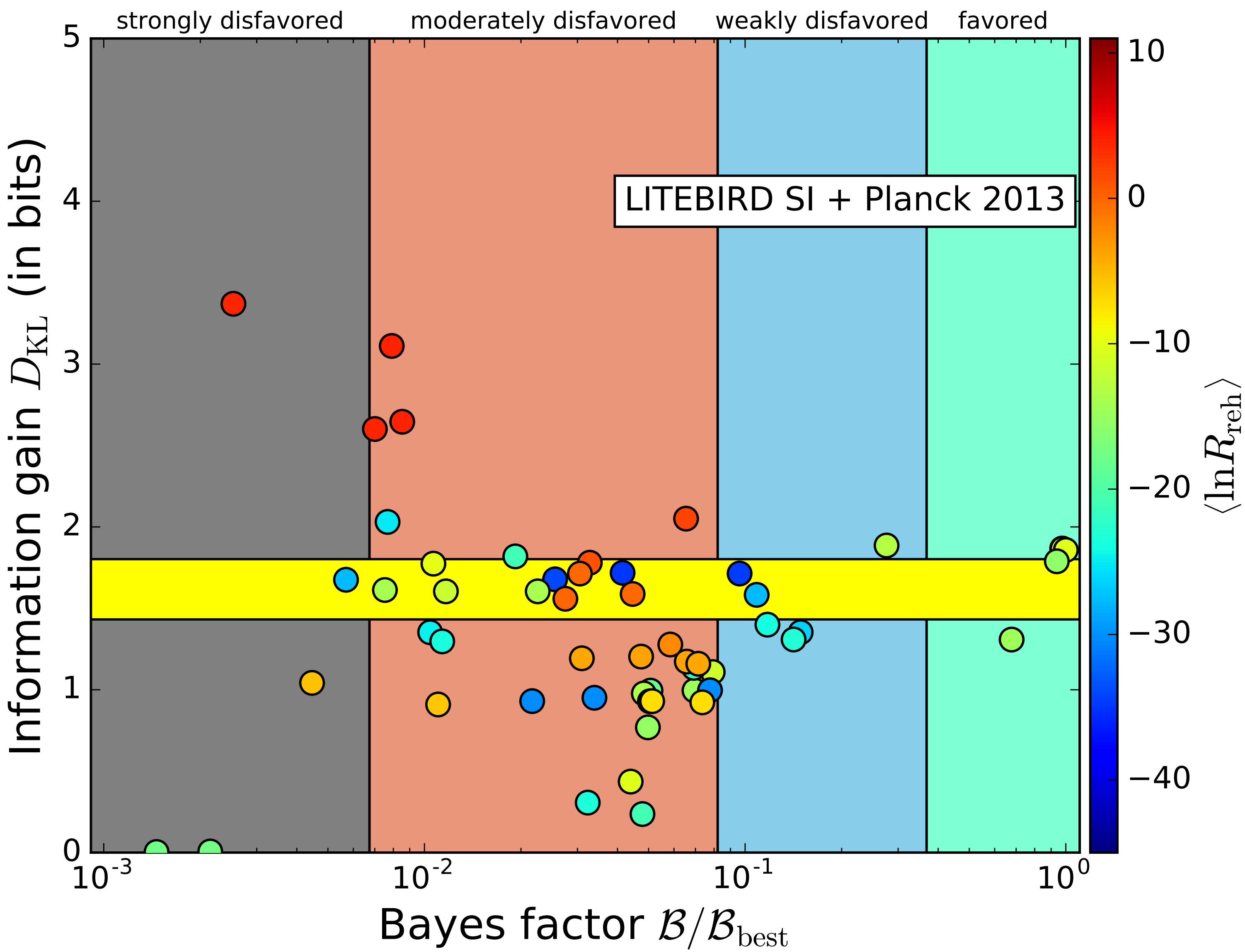
Information gain from current CMB data

- Evidence weighted $\langle D_{\mathrm{KL}} \rangle \equiv \sum_i P(\mathcal{M}_i|D) D_{\mathrm{KL}}(\mathcal{M}_i) \simeq 0.82$



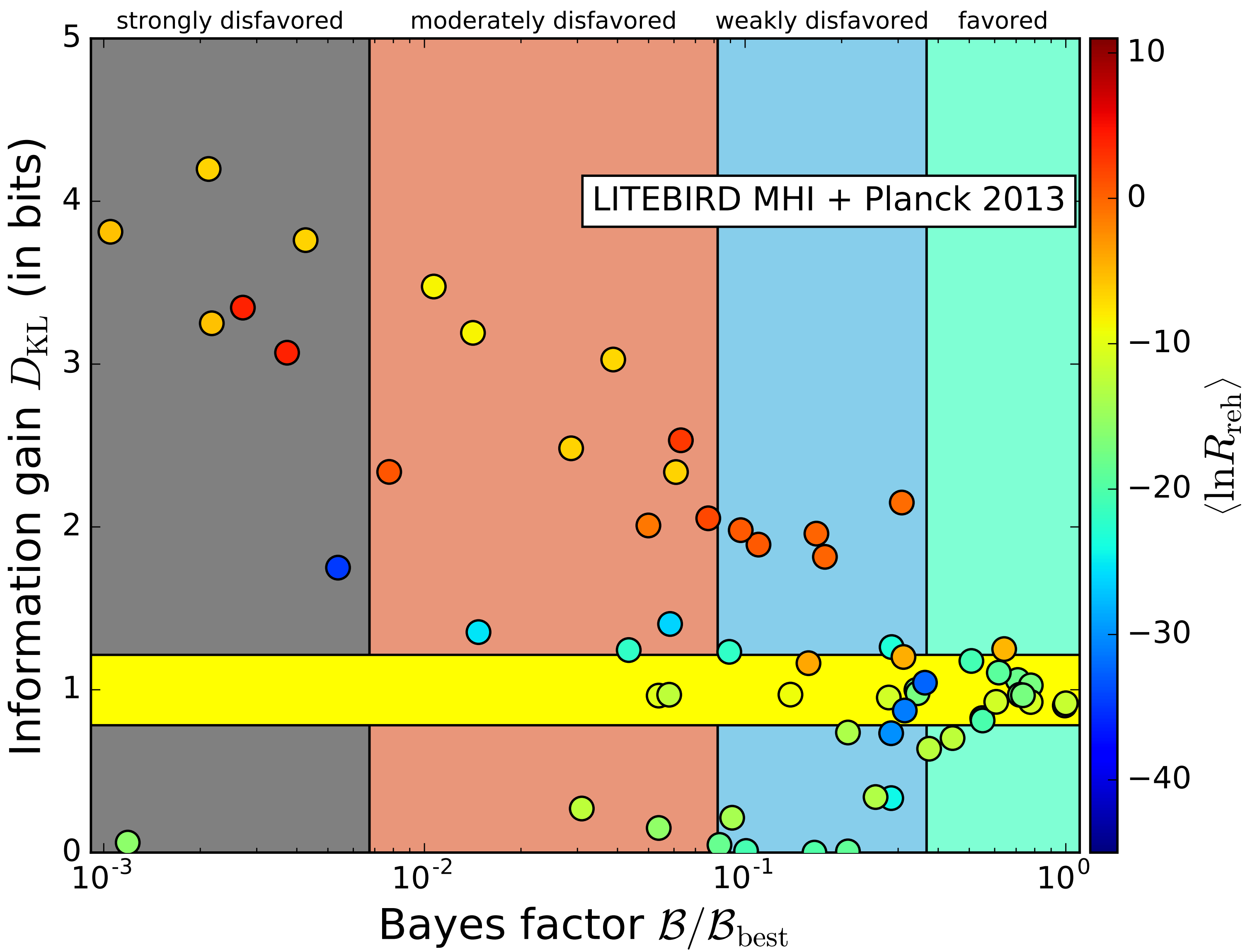
Information gain from future CMB data

- LITEBIRD with B-mode detection



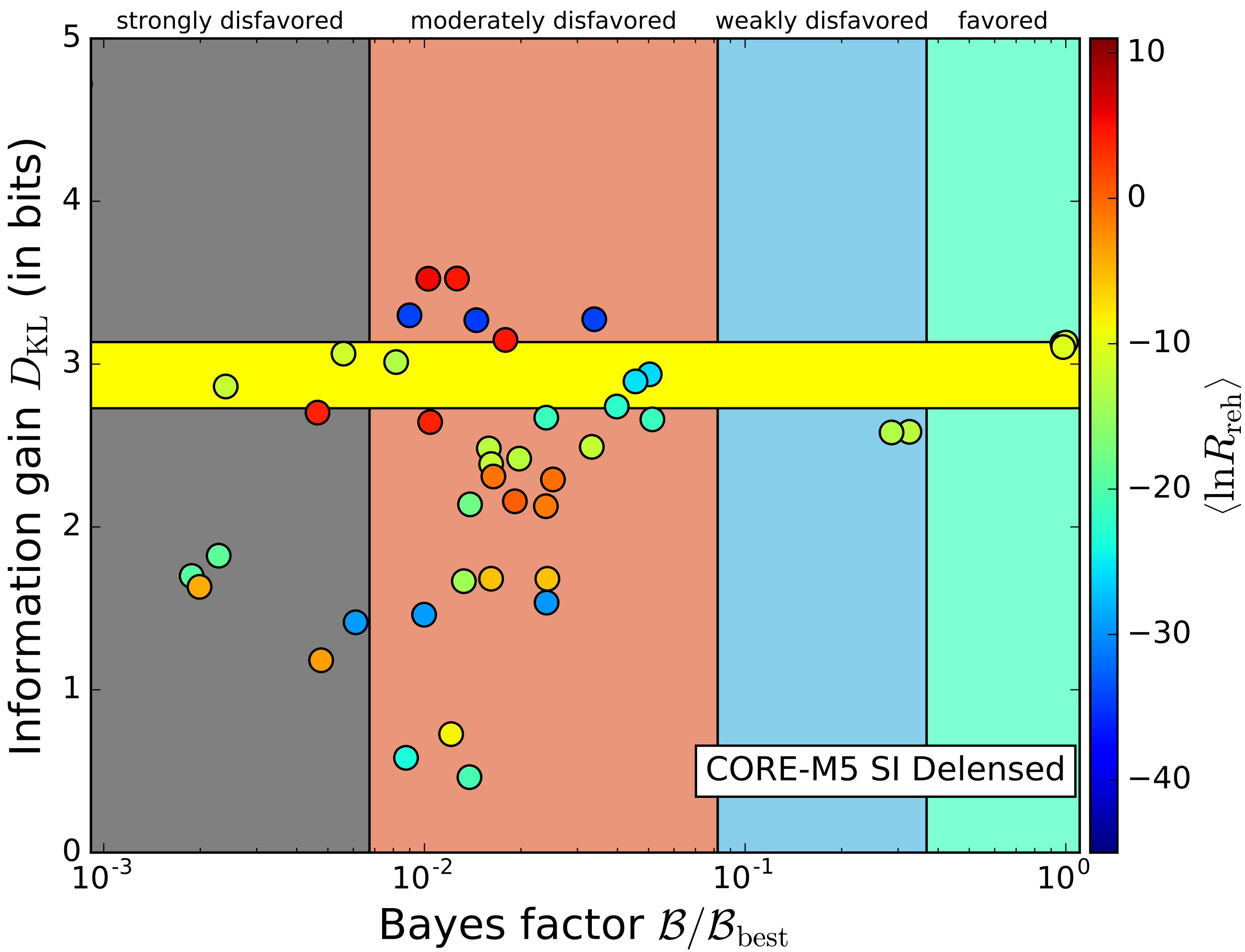
Information gain from future CMB data

- LITEBIRD without B-mode detection



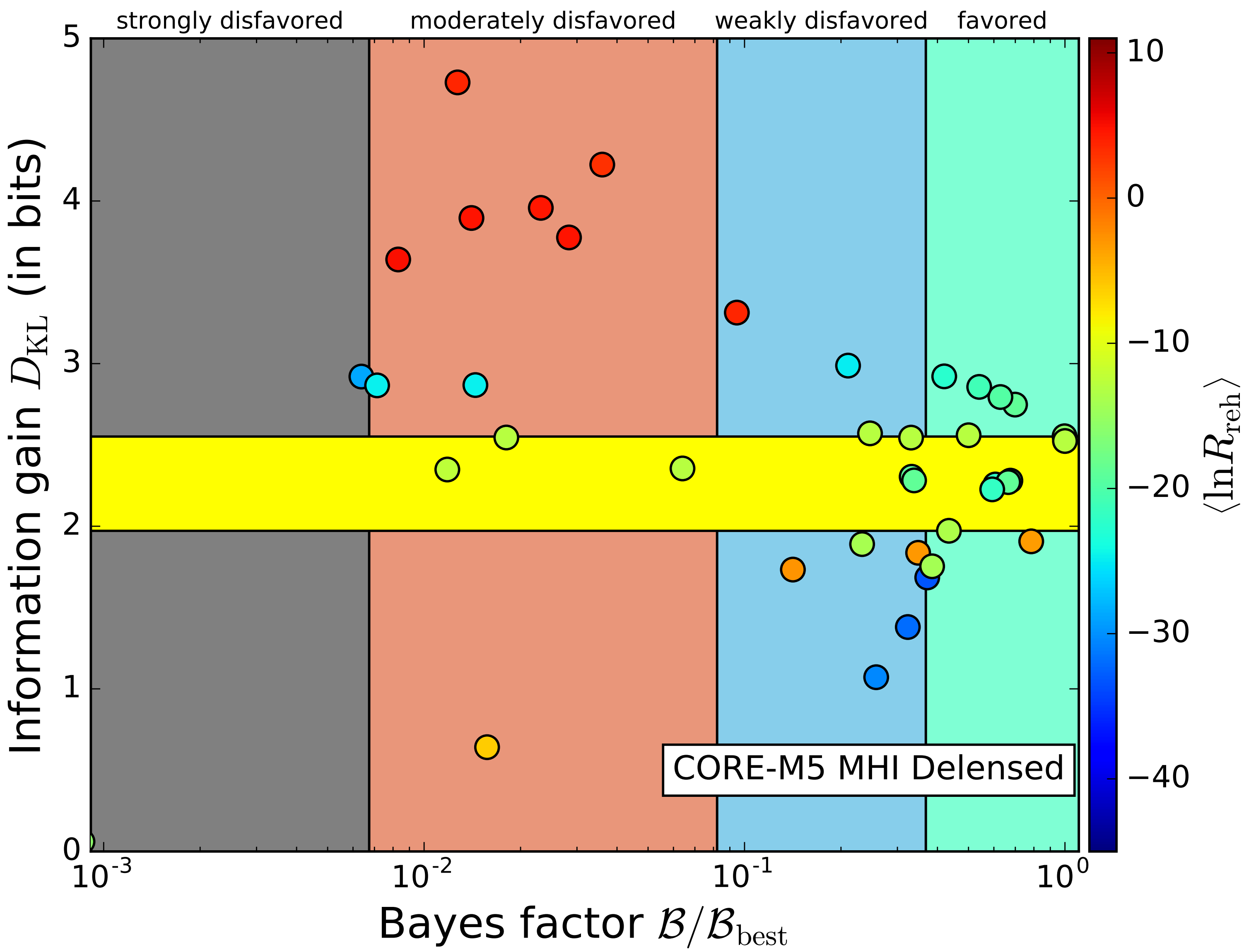
Information gain from future CMB data

- CORE with B-mode detection



Information gain from future CMB data

- CORE without B-mode detection



Conclusion

- Current CMB data constrain reheating by 1 bit
  - 1 bit = answers if $R_{\rm reh}$ is small or large
  - 1 bit = amount of information contained in one letter [Shannon:1951]
- Many models would be more severely constrained (or ruled-out) if reheating predictions could be (or would have been) done

$$\ln R_{\rm rad} = \frac{\Delta N_{\rm reh}}{4}(3\overline{w}_{\rm reh} - 1)$$

- Additional X-era,late-time entropy production,. . . are undistinguishable from the CMB and structure formation point of view
  - Effective parameter: $R_{\rm reh} \longrightarrow R_{\rm reh} R_{\rm X} R_{\rm Y}$
  - But can be disambiguated with GW direct detection: arXiv:1301.1778
- Euclid and large scale galaxy surveys will provide even more information on the reheating (work in progress. . .)