Primordial anisotropies from defects during inflation

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Outline

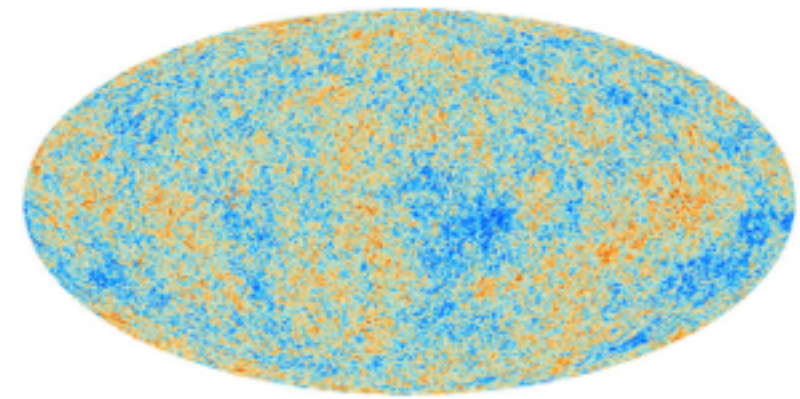
- Motivation for primordial anisotropies/asymmetries
- Motivations for defects in primordial universe
- Domain Wall
- Massive Monopole
- Cosmic String
- Vacuum Bubble Nucleation

Motivation

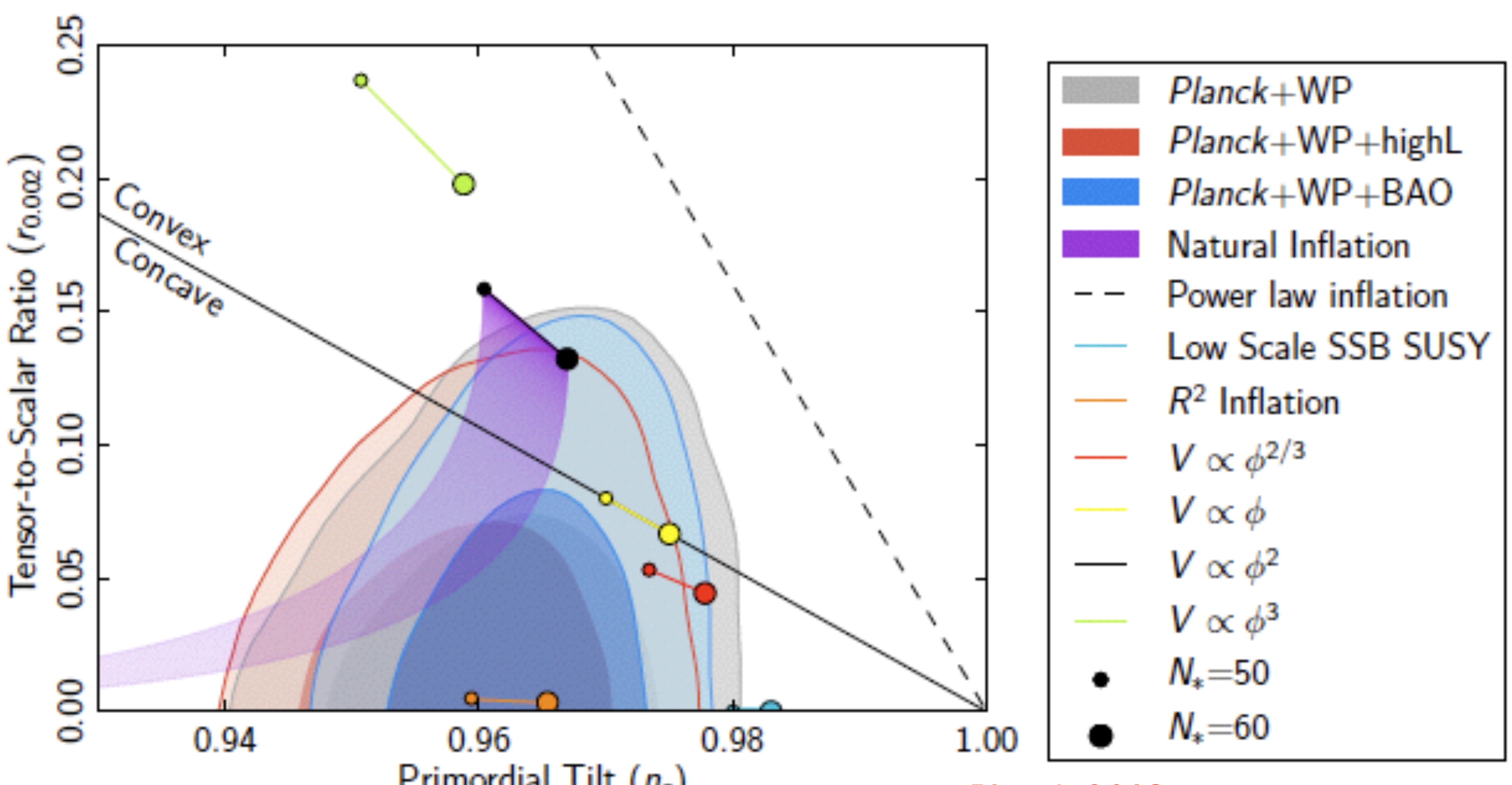
Inflation is the leading paradigm for early Universe and structure formations.

Basics predictions of inflation: The CMB perturbations are

- Nearly scale-invariant
- Nearly Gaussian
- Nearly adiabatic



These predictions are in good agreement with the Planck data.



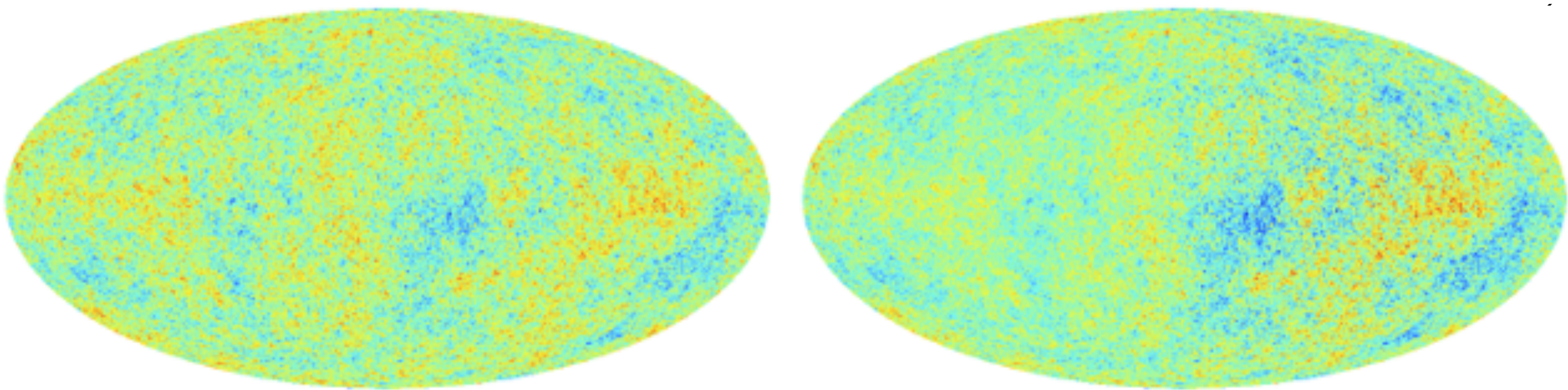
Planck 2013

Asymmetry vs. Anisotropy

Hemispherical Power Asymmetry

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + 2A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$$

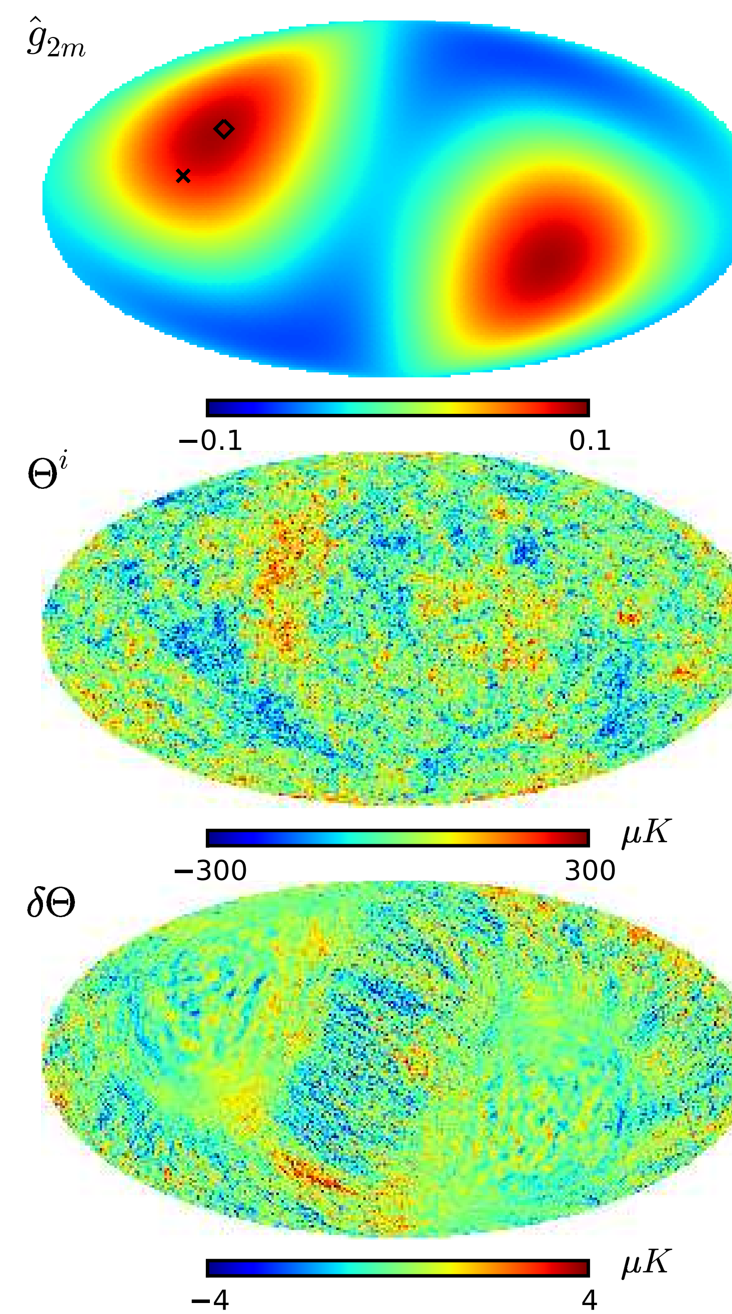
PLANCK : $A = 0.07 \pm 0.02$ for $2 \ll \ell \lesssim 64$



A=0

Seljebotn, 2010

A=0.3



Hanson & Lewis, 2009

Statistical Anisotropy

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}\left(1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}})\right)$$

quadrupole anisotropy: $L = 2, m = 0$

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + g_*\,(\hat{\mathbf{p}}.\hat{\mathbf{k}})^2)$$

Observationally $|g_*| \lesssim 10^{-2}$, Komatsu-Kim, 2013

Anisotropic Inflation from Gauge Field Dynamics

The model contains a $U(1)$ gauge field minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right]$$

Here $1/f(\phi)$ is the time-dependent gauge kinetic coupling.

We turn on the background gauge field $A_\mu = (0, A_x(t), 0, 0)$
The background metric is

$$\begin{aligned} ds^2 &= -dt^2 + e^{2\alpha(t)}\left(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2+dz^2)\right) \\ &= -dt^2 + a(t)^2dx^2 + b(t)^2(dy^2+dz^2) \end{aligned}$$

In this view $H \equiv \dot{\alpha}$ is the average Hubble expansion rate and

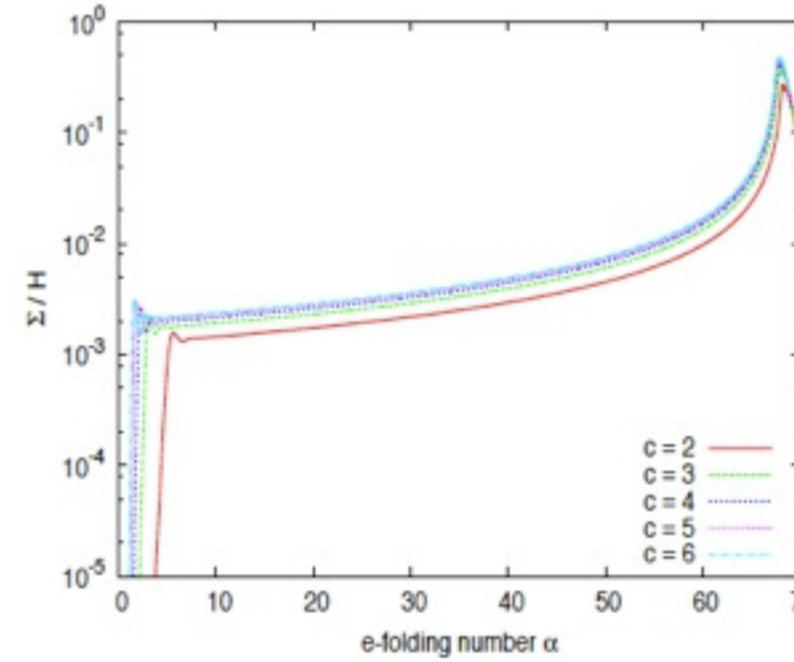
$$H_a \equiv \frac{\dot{a}}{a} \quad , \quad H_b \equiv \frac{\dot{b}}{b}$$

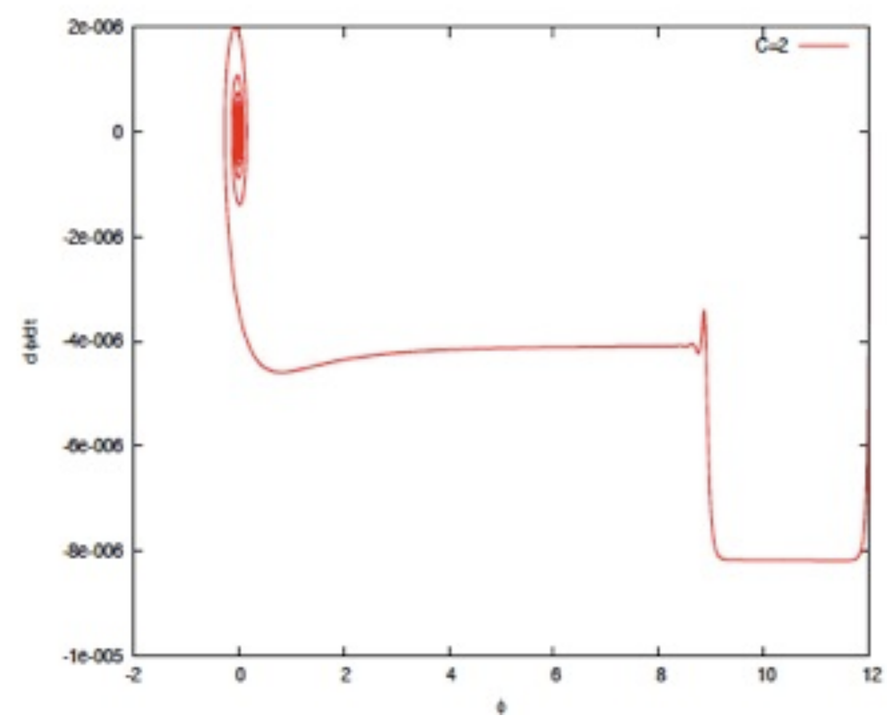
The anisotropy in the system is measured by

$$\frac{\dot{\sigma}}{H} \equiv \frac{H_b - H_a}{H}$$

The background equations are too complicated to be solved !

Watanabe, Kanno, Soda, 09



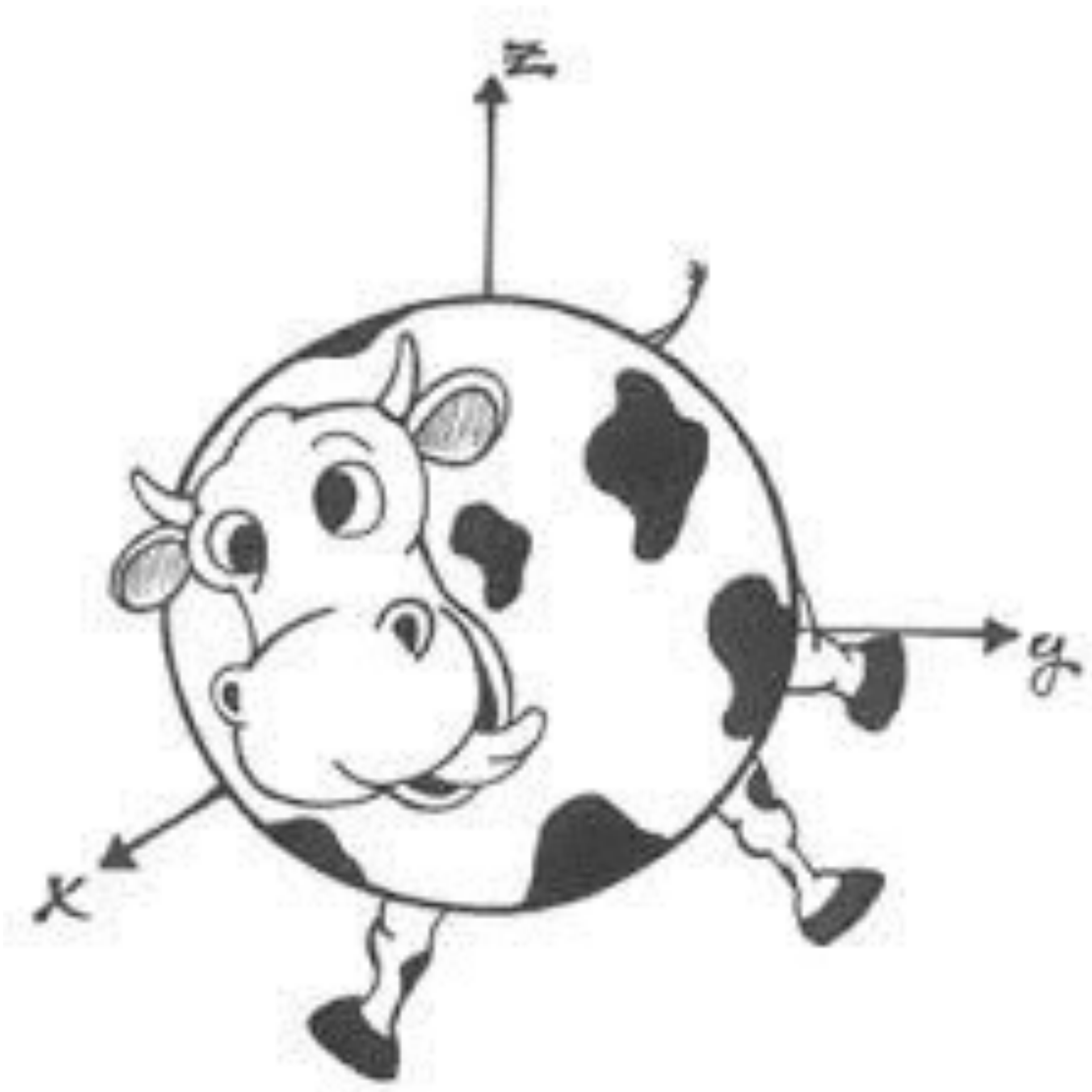


A realization of Power Asymmetry

Barcelona vs. Paris Saint Germain, 2017

Mechanisms to generate primordial anisotropies and asymmetries

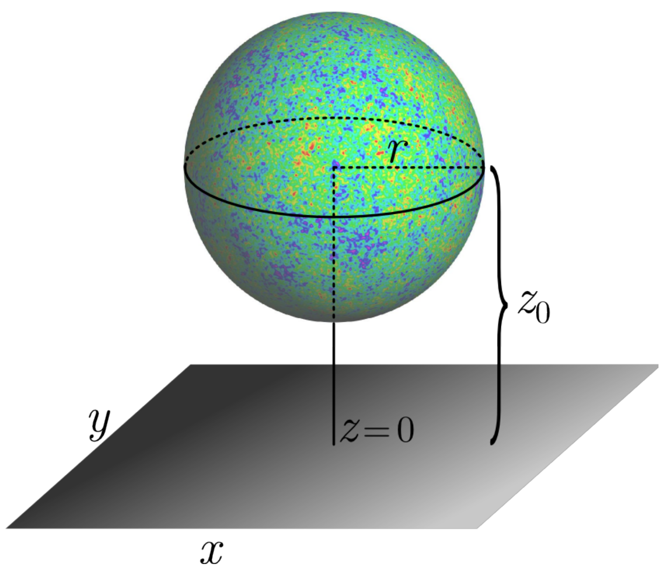
Consider a spherical cow in the vacuum …..



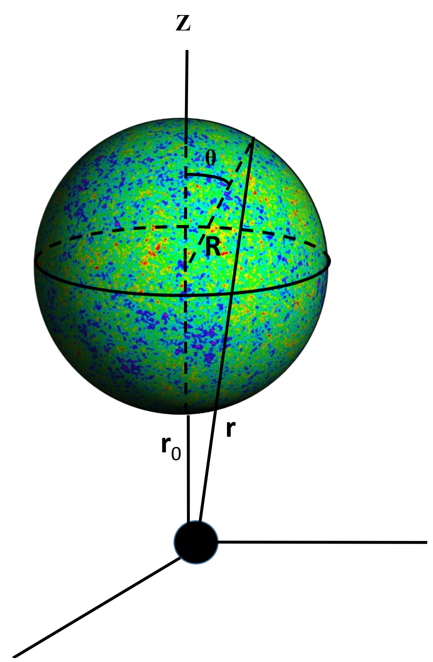
Primordial anisotropies from defects during inflation

We consider various defects during inflation:

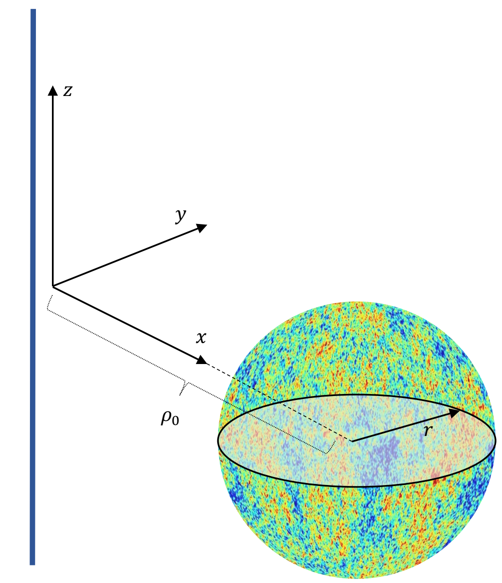
1- Domain walls,



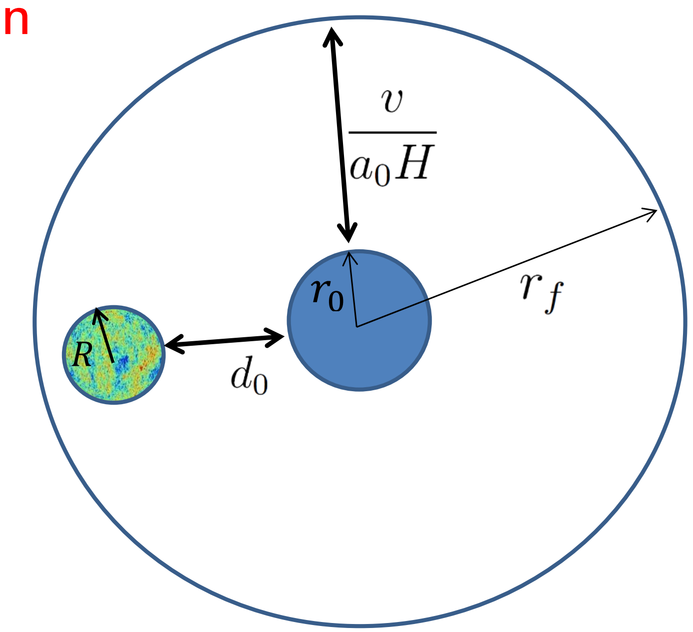
2- Massive defects,

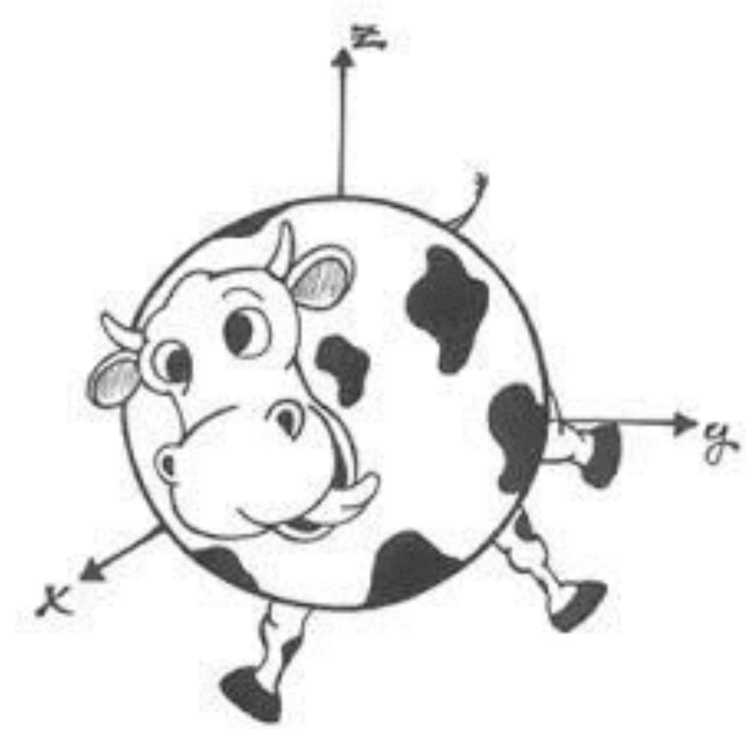


3- Cosmic strings



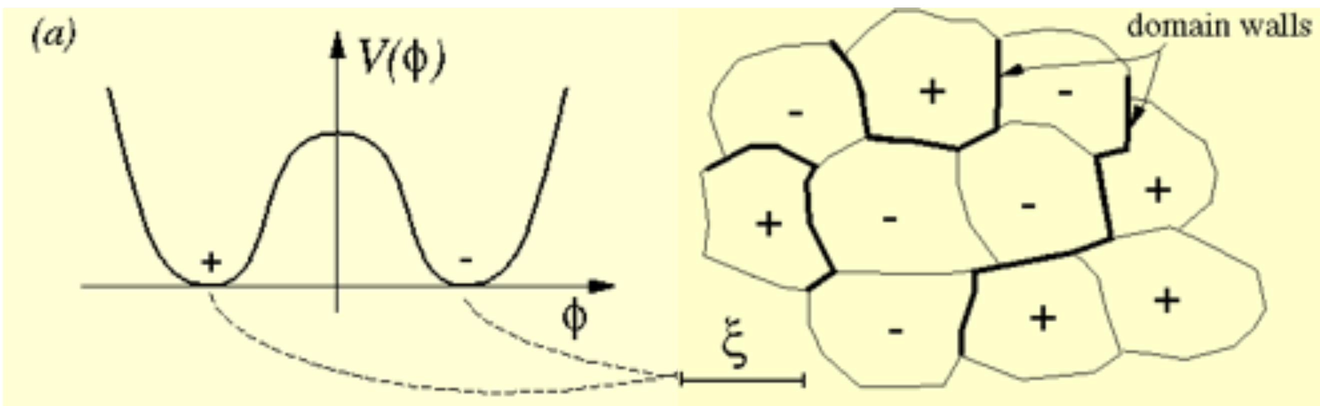
4- Bubble nucleation

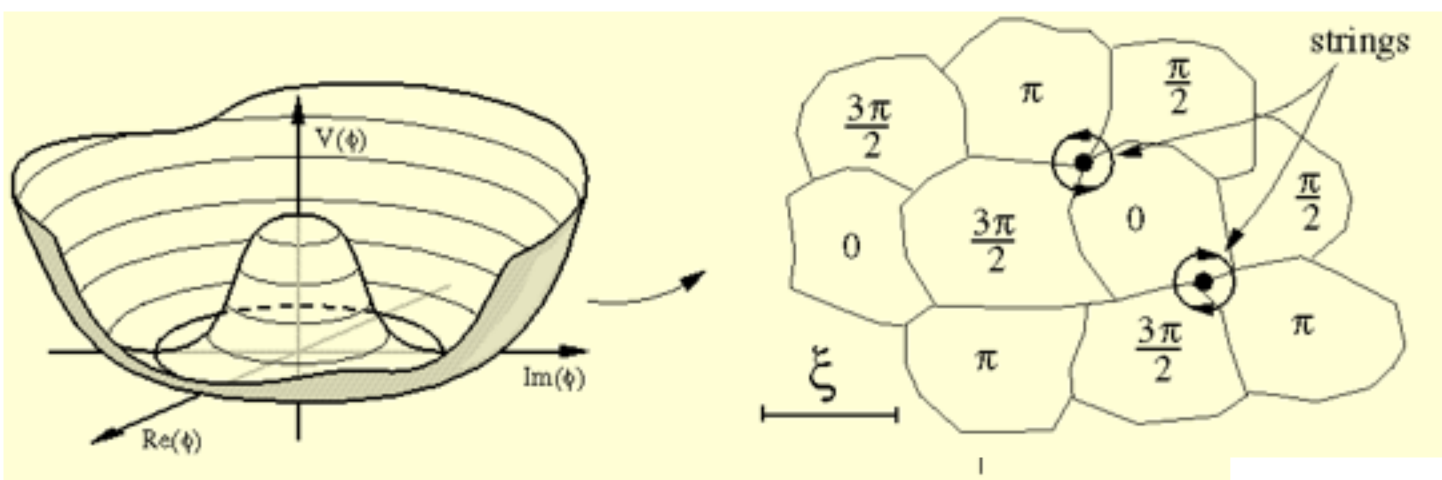




Topological defects in primordial Universe

Kibble Mechanism :
Topological defects are formed from symmetry breaking:



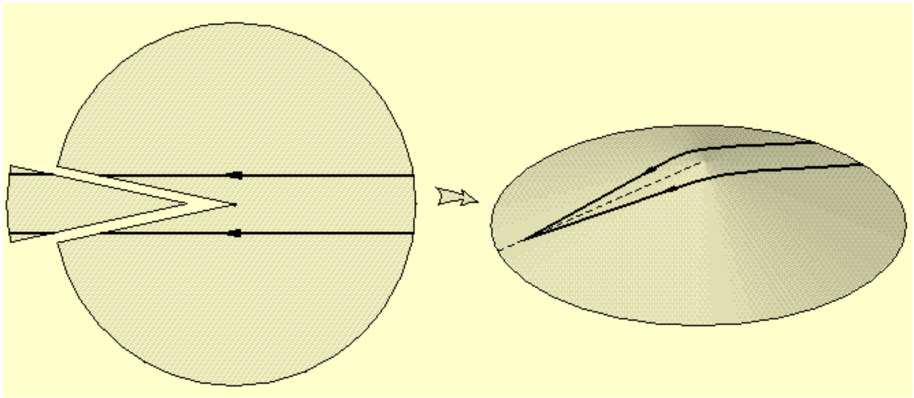


Domain walls and monopolies are cosmologically catastrophic as they rapidly over close the Universe.

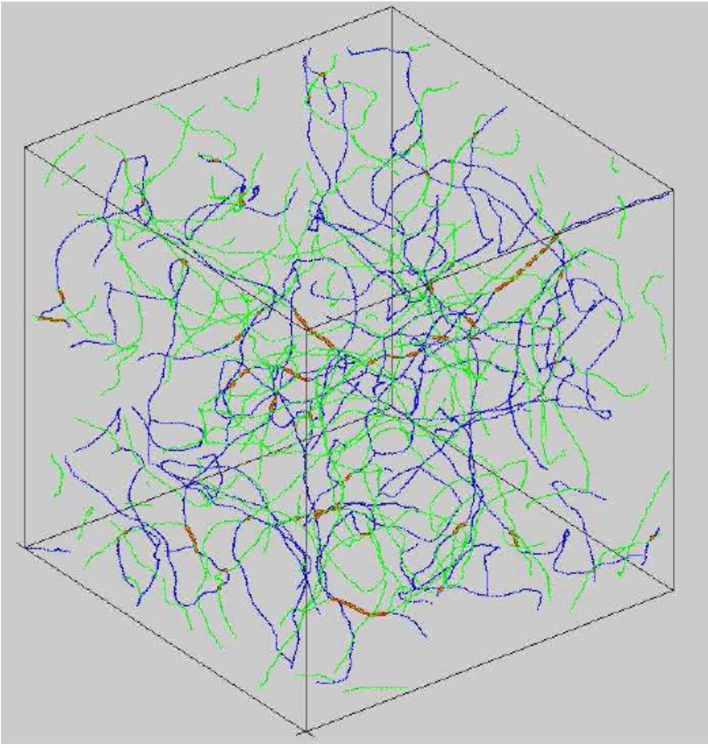
Cosmic strings are viable as they reach the scaling regime. By 1990's cosmic string was a rival candidate to inflation as the origin of perturbations and structure formation in early Universe.

http://www.damtp.cam.ac.uk/research/gr/public/cs_top.html

Motivation for cosmic strings:



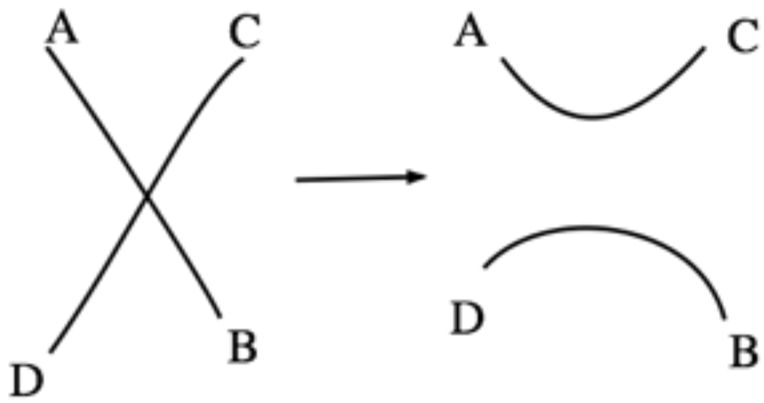
A cosmic string produces a deficit angle around itself. This may be used to observe cosmic string via lensing or via KS effect in CMB maps.



A network of cosmic string reaches the scaling regime in a cosmological background. The constrains from CMB anisotropies suggest the upper bound $G\mu \lesssim 10^{-7}$ for the tension of string μ.

If cosmic strings are from string theory then they are either Fundamental string (F-strings) or D1-brane (D-strings).

The evolution of a network of cosmic string crucially depends on the intercommutation probability P. For ordinary gauge string $P \sim 1$. However, for cosmic superstrings of different types it can be significantly smaller, say $10^{-3} < P < 1$; (Jackson, Jones, Polchinski, hep-th/0405229).



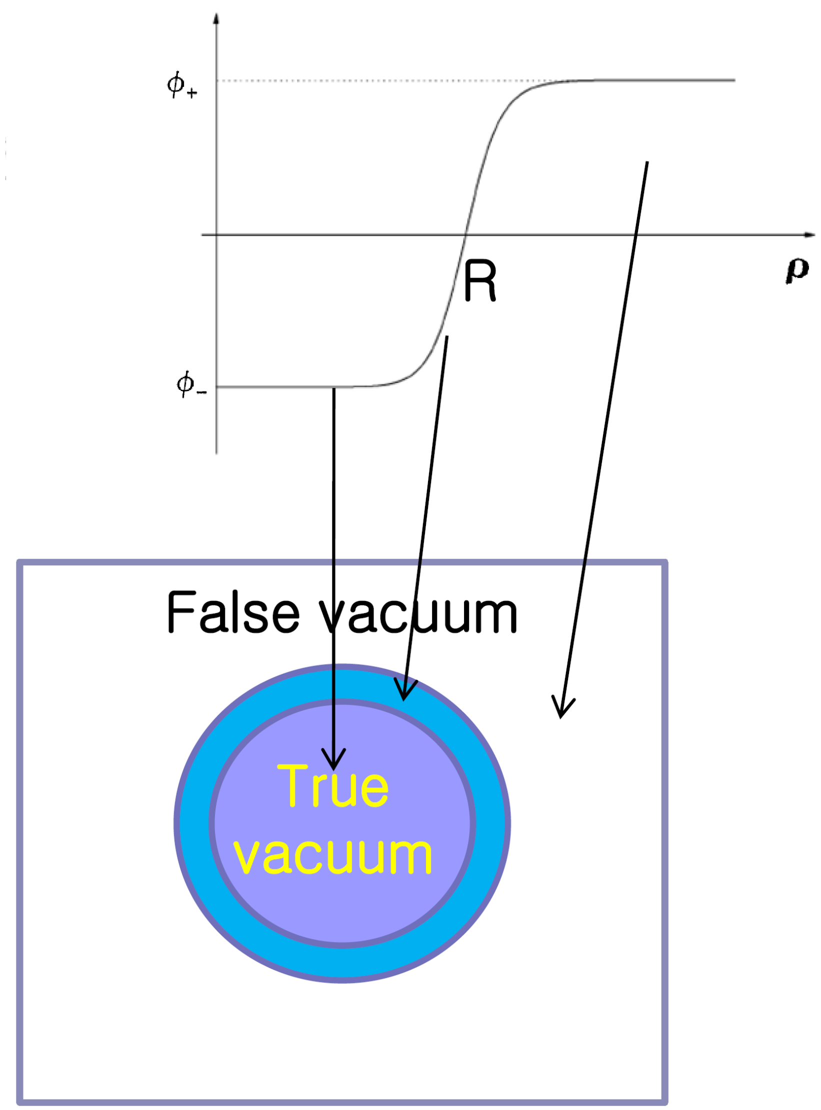
Motivation for bubble nucleation

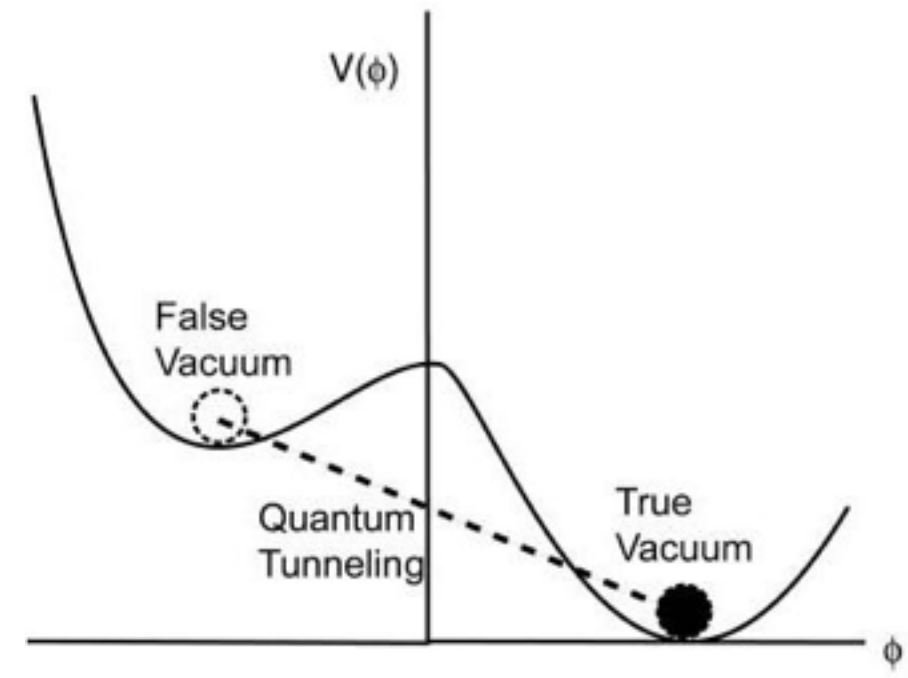
Vacuum bubble nucleation has played important roles in the development of inflationary cosmology and beyond:

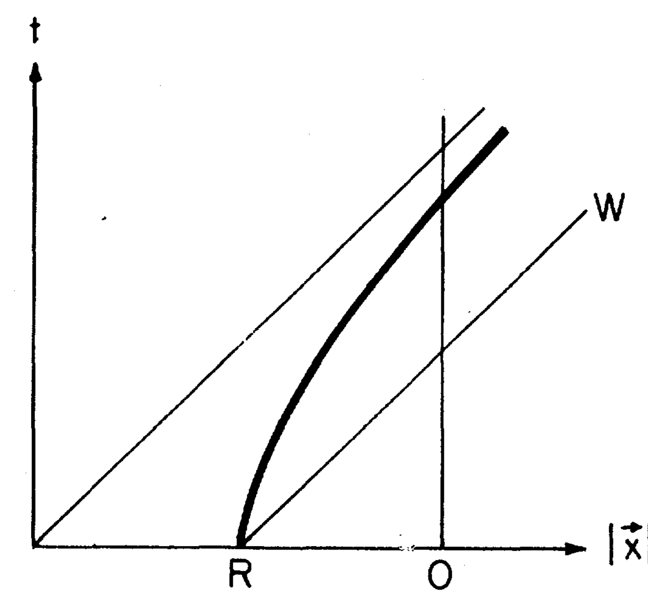
Guth Old Inflation, 1981, Sato et al 1981, 1982.

The basic picture is based on Coleman-de Luccia formalism:

Euclidean classical solutions with the topology of a four-sphere.





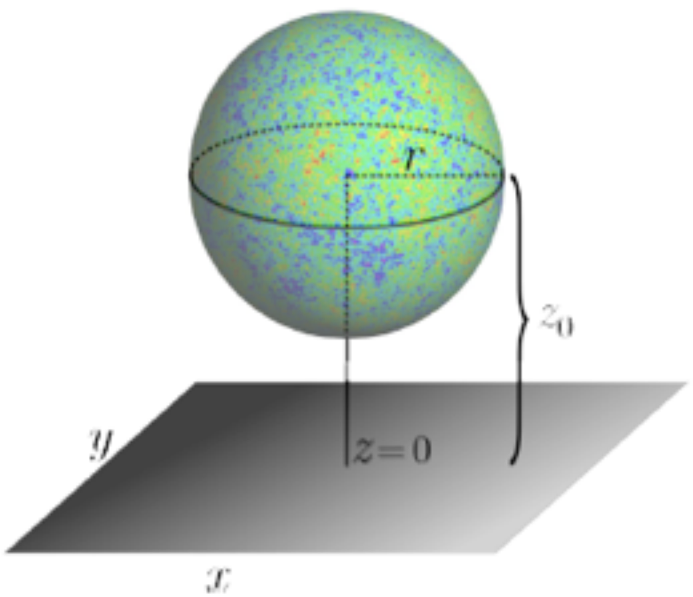


Primordial anisotropies from domain wall

Consider a domain wall in our Hubble patch during inflation. To simplify the analysis assume the DW has zero thickness with tension σ.

Assuming the DW is extended along x-y plane, the metric ansatz is

$$ds^2 = \frac{1}{f(\tau, z)^2}\left(-d\tau^2 + d\mathbf{x}^2\right)$$



The total energy density is

$$T^{\mu}{}_{\nu} = -V\delta^{\mu}{}_{\nu} - \frac{\sigma}{\sqrt{g_{zz}}}\mathrm{diag}\left(1,1,1,0\right)\delta(z).$$

Solving the Einstein fields equations $G_{\mu\nu} = T_{\mu\nu}/M_P^2$ we obtain

$$ds^2 \simeq \frac{1}{\eta^2}\left(-d\eta^2 - 2\beta\,\mathrm{sgn}(z)d\eta dz + dz^2 + dx^2 + dy^2\right)$$

The curvature perturbation as usual is given by $\mathcal{R} = -\frac{H}{\dot{\phi}}\delta\phi$.

The DW modifies the background geometry, affecting the inflaton perturbations

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi\right) = \frac{1}{2H^2\eta^2}\left(\delta\phi'^2 - (\nabla\delta\phi)^2\right) + \frac{\beta}{H^2\eta^2}\mathrm{sgn}(z)\delta\phi'\frac{\partial\delta\phi}{\partial z},$$

S. Jazayeri, Y. Akrami, H. F., A. Solomon, Y. Wang, 1408.3057

Primordial asymmetry from domain wall

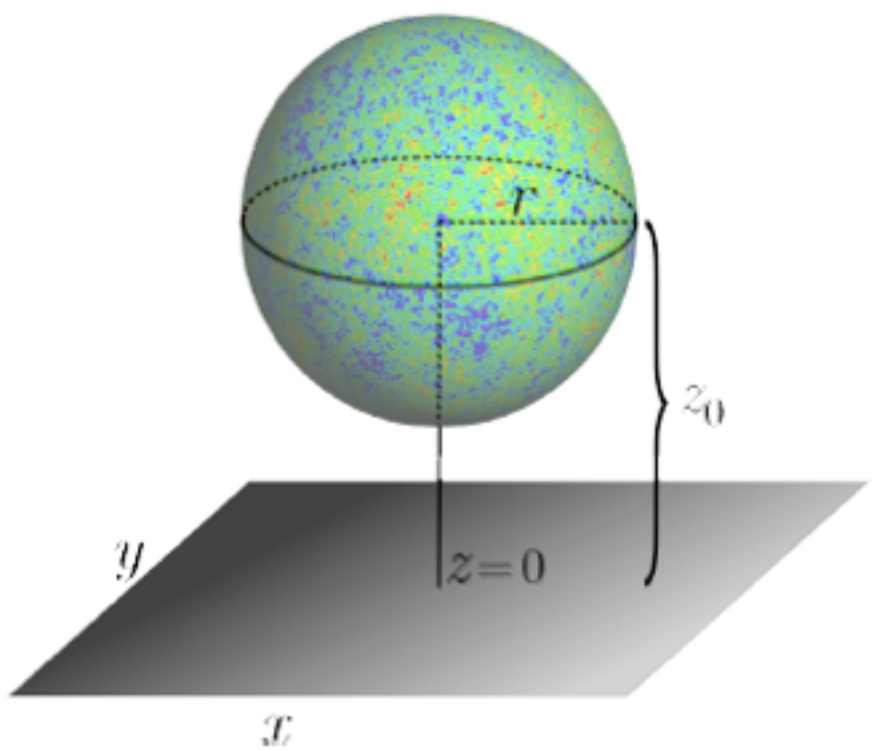
Suppose there exists a domain wall (DW) during inflation.

Treating the effects of DW perturbatively the metric becomes

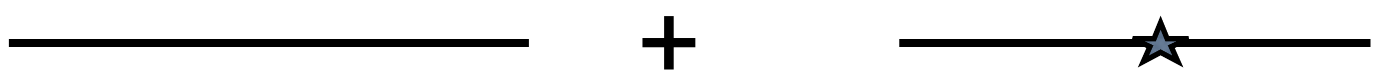
$$ds^2 \simeq \frac{1}{\eta^2}\left(-d\eta^2 - 2\beta\,\mathrm{sgn}(z)\,d\eta dz + dz^2 + dx^2 + dy^2\right) \qquad \beta = \frac{\sigma}{4HM_P^2}$$

We are interested in inflaton power spectrum

$$\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{q}}\rangle = \left(\frac{H}{\dot\phi}\right)^2 \langle \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\rangle$$



The corresponding Feynman diagram is



The corresponding interaction Hamiltonian density is

$$\mathcal{H}_{\mathcal{I}} = -\frac{\beta}{H^2\eta^2}\,\delta\phi'\,\partial_z\delta\phi\,\mathrm{sgn}(z)$$

The correction on inflaton power spectrum is given by

$$\delta\left\langle \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}} \right\rangle \equiv +i\int_{-\infty}^{\tau_e} \left\langle \left[H_I(\eta), \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\right] \right\rangle d\eta$$

the RHS yields

$$-\frac{4\beta}{H^2(2\pi)^4}\int \frac{d\eta}{\eta^2}\int d^2\mathbf{q}'_{||}dq'_z dk'_z \frac{q'_z}{k'_z+q'_z}\mathrm{Im}\left[\left\langle \delta\phi_{\mathbf{q}'}(\eta)\delta\phi'_{\mathbf{k}'}(\eta)\delta\phi_{\mathbf{k}}(\eta_e)\delta\phi_{\mathbf{q}}(\eta_e)\right\rangle\right]$$

The final result for the power spectrum is

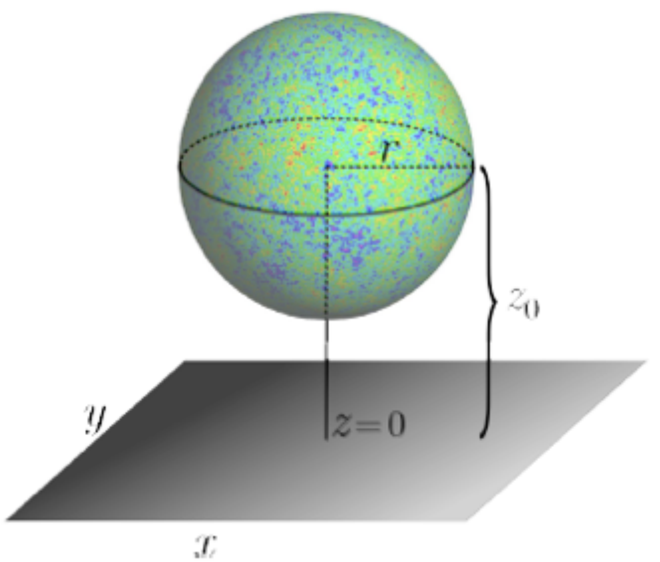
$$\langle \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{q}}\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_0\left[(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q}) - (2\pi)^3\frac{\beta}{2q^3}\frac{k^2q_z+q^2k_z}{k_z+q_z}\delta^2(\mathbf{q}_{||}+\mathbf{k}_{||})\right]$$

As expected the translation invariance along the direction perpendicular to DW is broken.

The Variance in Real Space

$$\begin{aligned}\delta\langle\mathcal{R}^2(\mathbf{x})\rangle &= -\frac{\beta}{4}\mathcal{P}_0\int dk_z dq_z dq_{||}q_{||}\frac{q_{||}^2+k_zq_z}{(q_{||}^2+k_z^2)^{\frac{3}{2}}(q_{||}^2+q_z^2)^{\frac{3}{2}}}e^{i(k_z+q_z)z} \\ &\simeq \beta\mathcal{P}_0 \ln\left|\frac{z}{L}\right| + C\end{aligned}$$

Note that $z = z_0 + r\cos\theta$



S. Jazayeri, Y. Akrami, H. F., A. Solomon, Y. Wang, JCAP, 2014

The Variance Multipoles

Expanding the variance in multipoles $\delta\langle\mathcal{R}^2(\mathbf{x})\rangle = \mathcal{P}_0 \sum_\ell a_\ell P_\ell(\cos\theta)$ we obtain

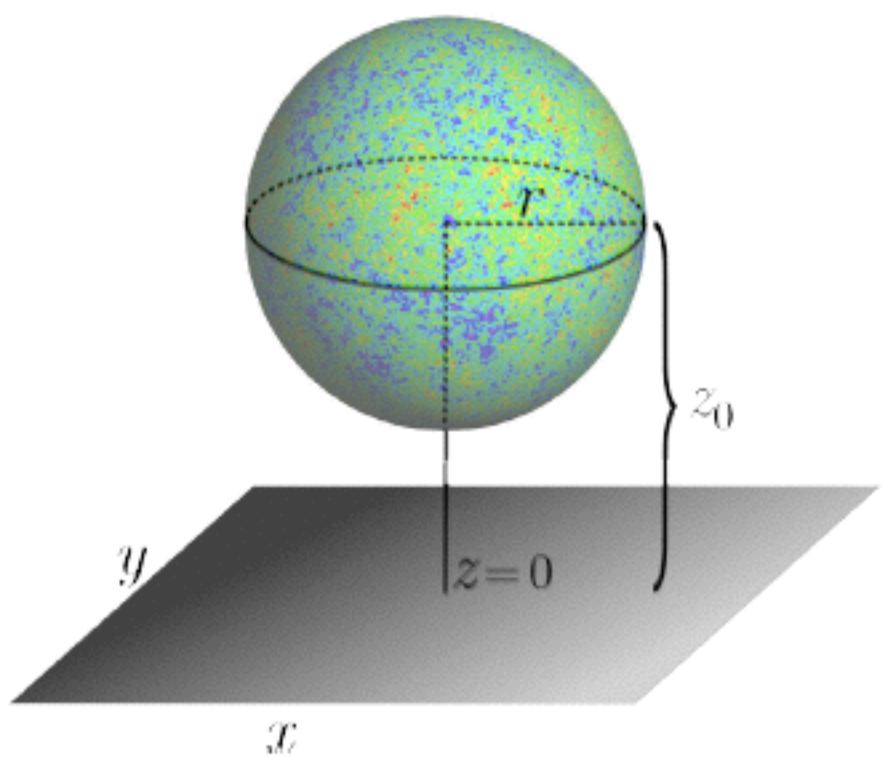
$$a_\ell = \frac{(2\ell+1)\beta}{2}\int_{-1}^{+1} d(\cos\theta) P_\ell(\cos\theta) \ln\left|1+\kappa\cos\theta\right|$$

For the dipole and quadrupole we get

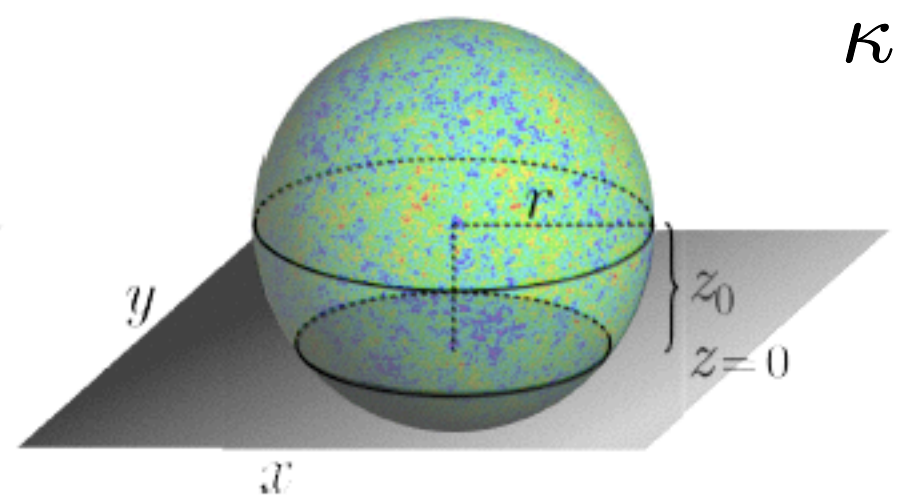
$$\begin{aligned} a_1 &= -\frac{3\beta}{4\kappa^2}\left[(\kappa^2-1)\ln\left|\frac{1-\kappa}{1+\kappa}\right| - 2\kappa\right], \\ a_2 &= \frac{5\beta}{12\kappa^3}\left[3(\kappa^2-1)\ln\left|\frac{1-\kappa}{1+\kappa}\right| + 4\kappa^3 - 6\kappa\right] \end{aligned}$$

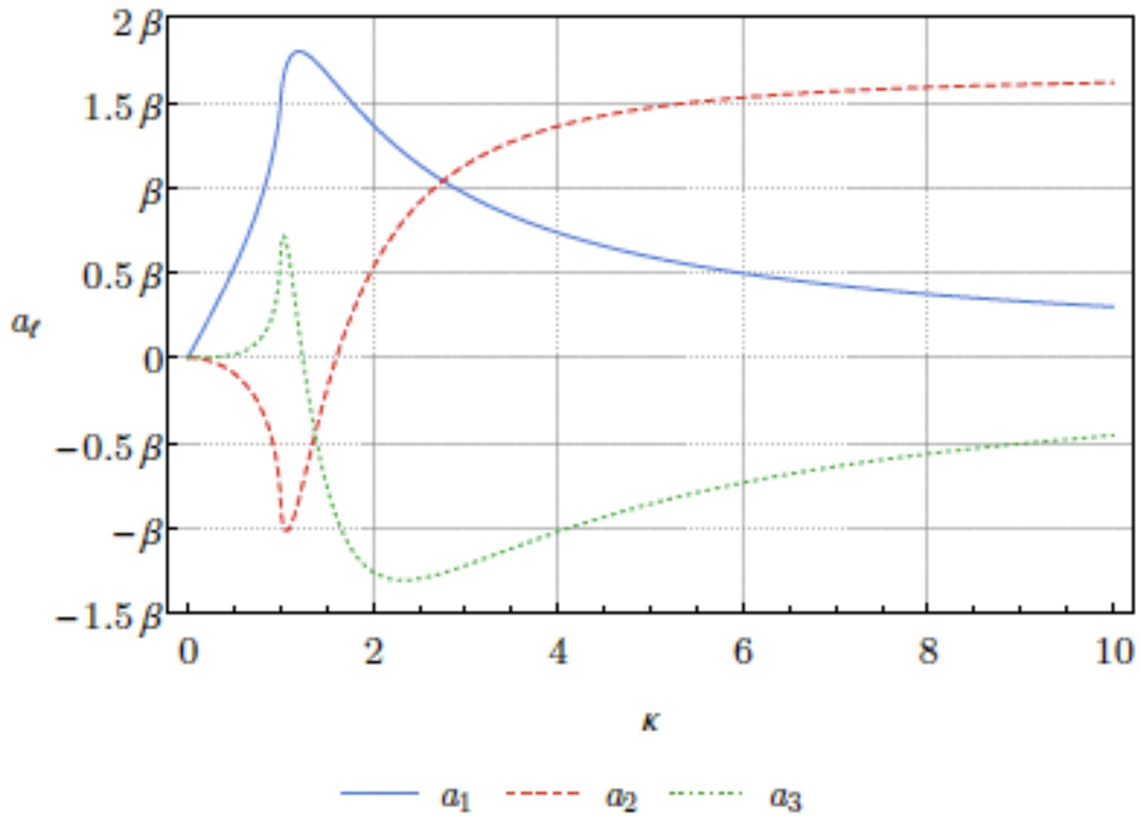
in which $\kappa \equiv \frac{r}{z_0}$ and $z = z_0 + r\cos\theta$.

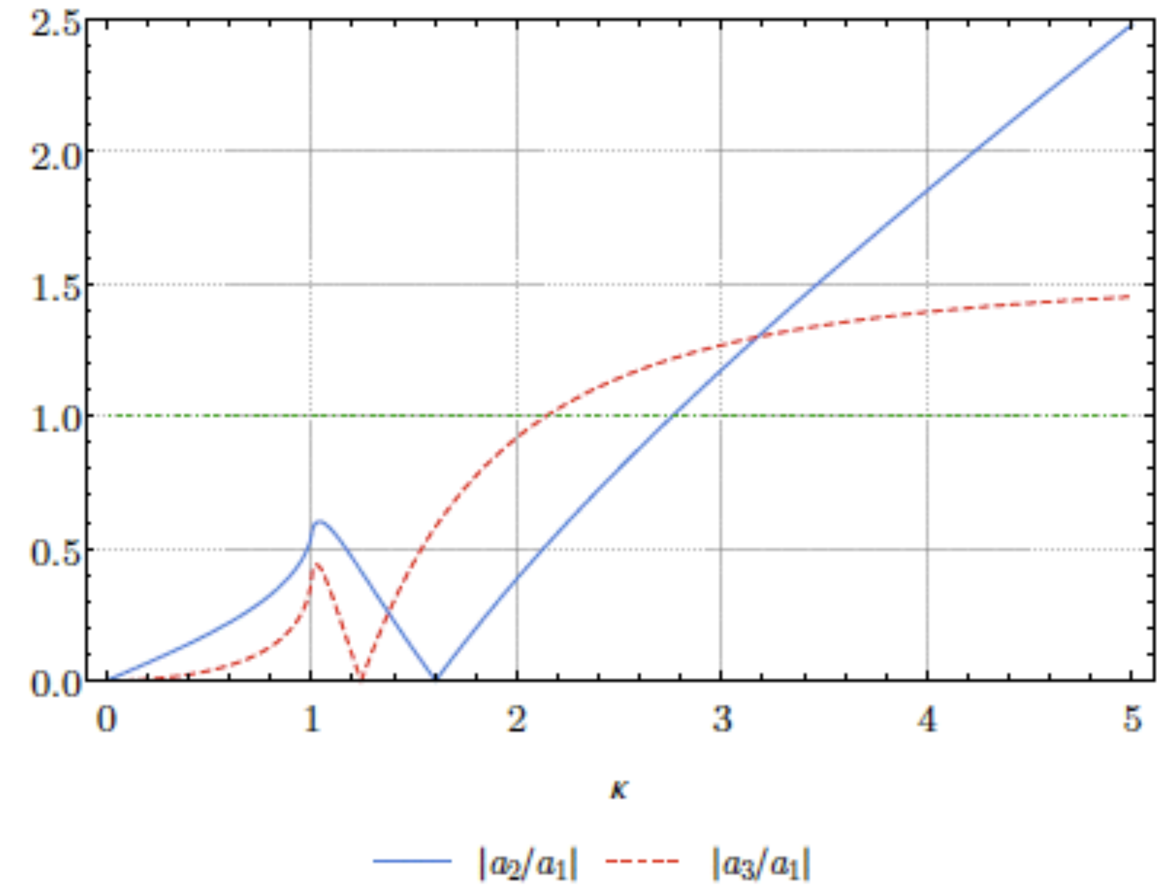
$\kappa < 1$

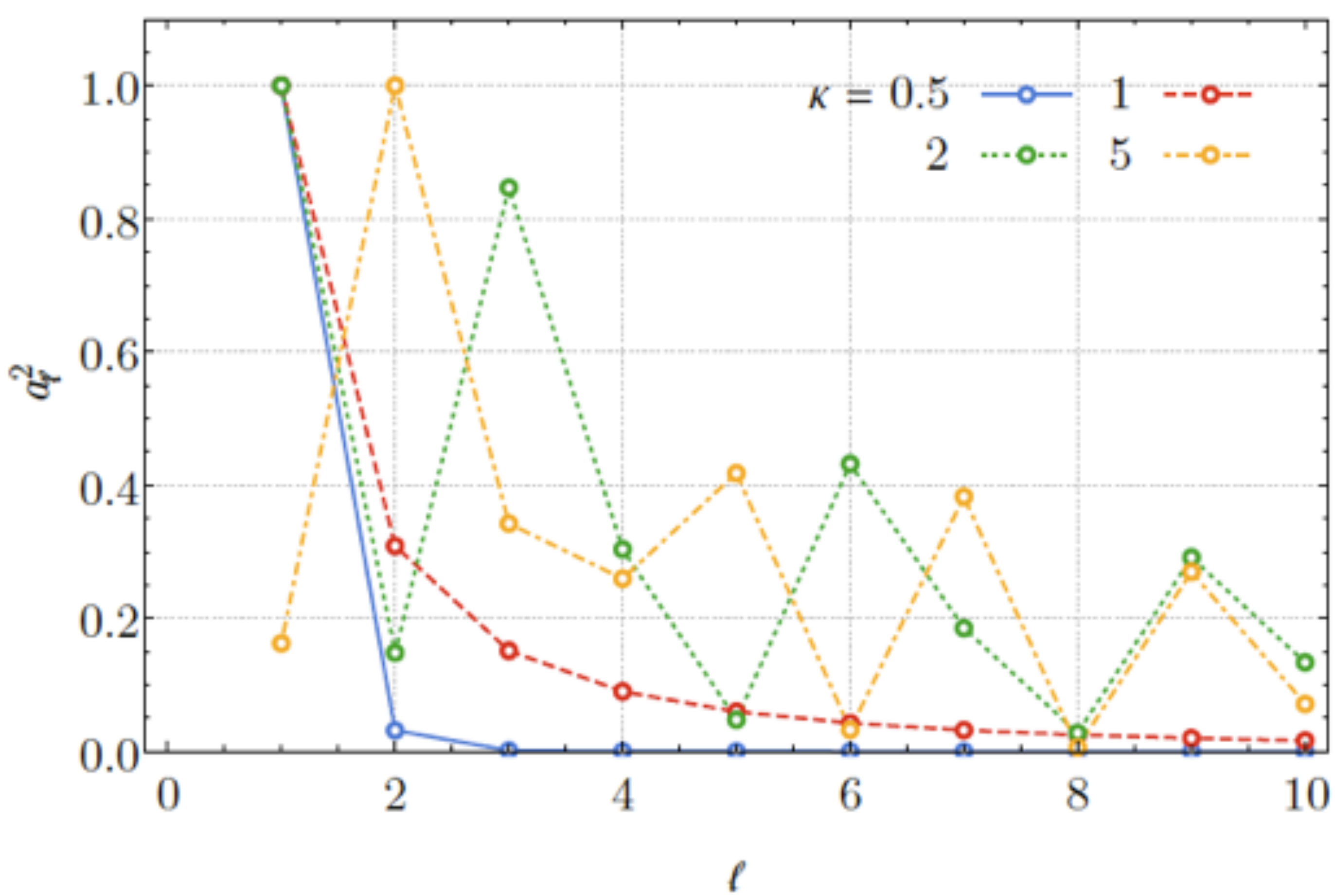


$\kappa > 1$

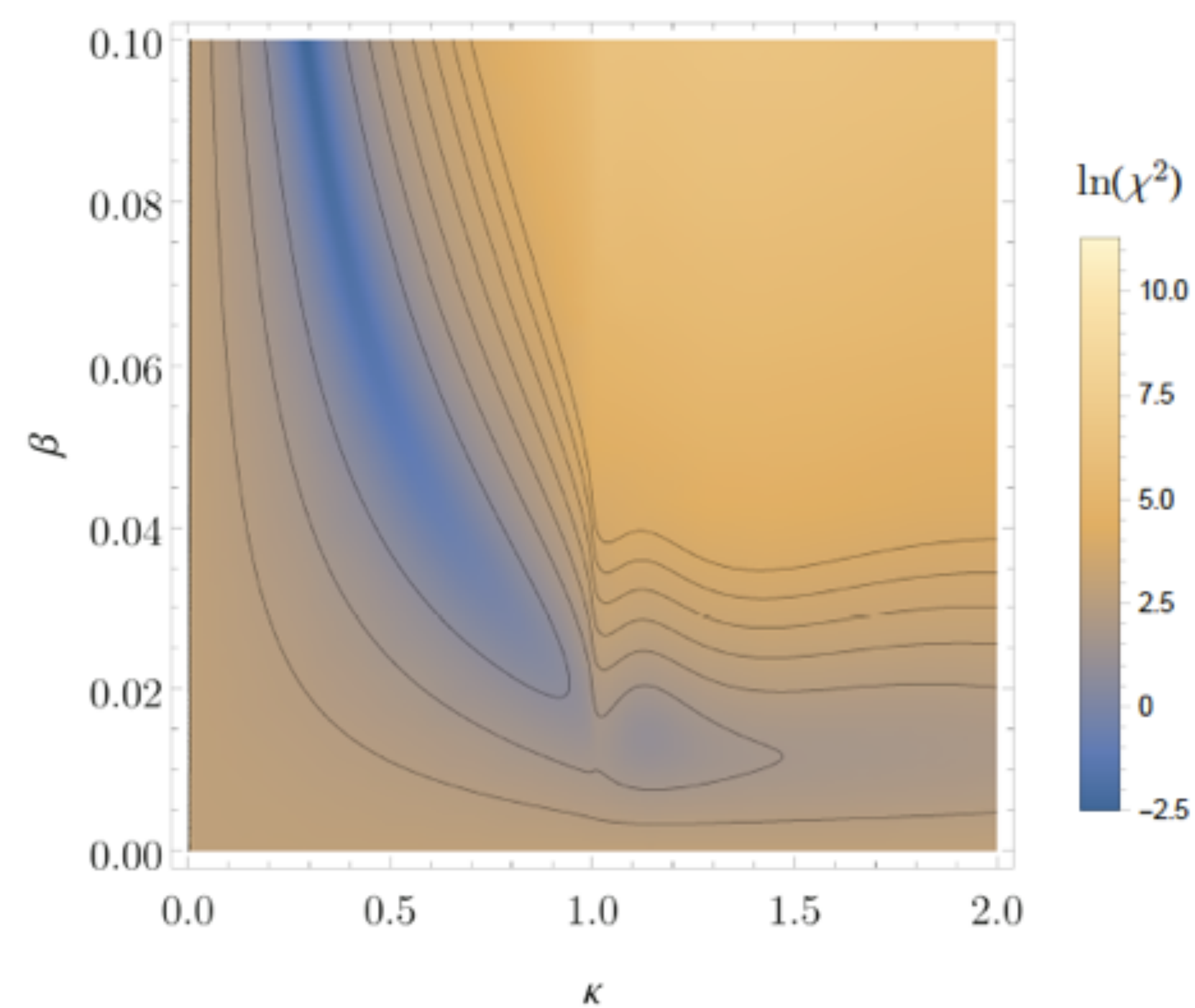








Compare this plot with the results in Akrami et al, arXiv: 1402.0870.



The power spectra for the off-diagonal moments are

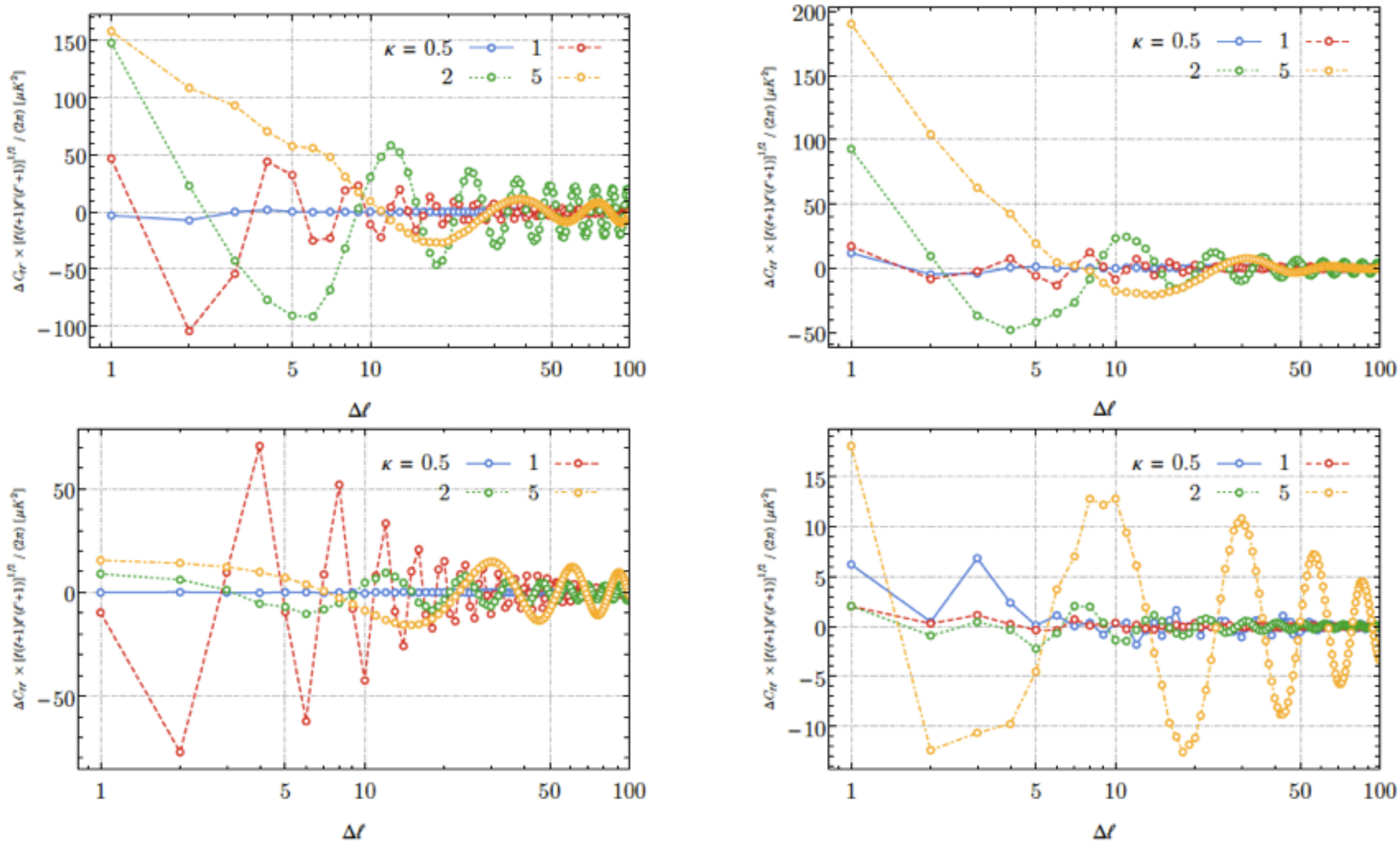


FIG. 7. Contributions of the domain wall to the off-diagonal elements of the CMB correlation matrix $C_{\ell\ell'}$ for two fixed values of ℓ_1, two of m, and four of the parameter κ, plotted against $\Delta\ell = \ell_2 - \ell_1$. **Upper left:** We set $\ell_1 = 3$, $m = 0$. **Upper right:** We set $\ell_1 = m = 3$. **Lower left:** We set $\ell_1 = 50$, $m = 0$. **Lower right:** We set $\ell_1 = m = 50$. In each panel the plotted quantities correspond to $\langle a_{\ell_1,m} a_{\ell_2,m} \rangle$ in Eq. (63), where we set $\ell_2 = \ell_1 + \Delta\ell$.

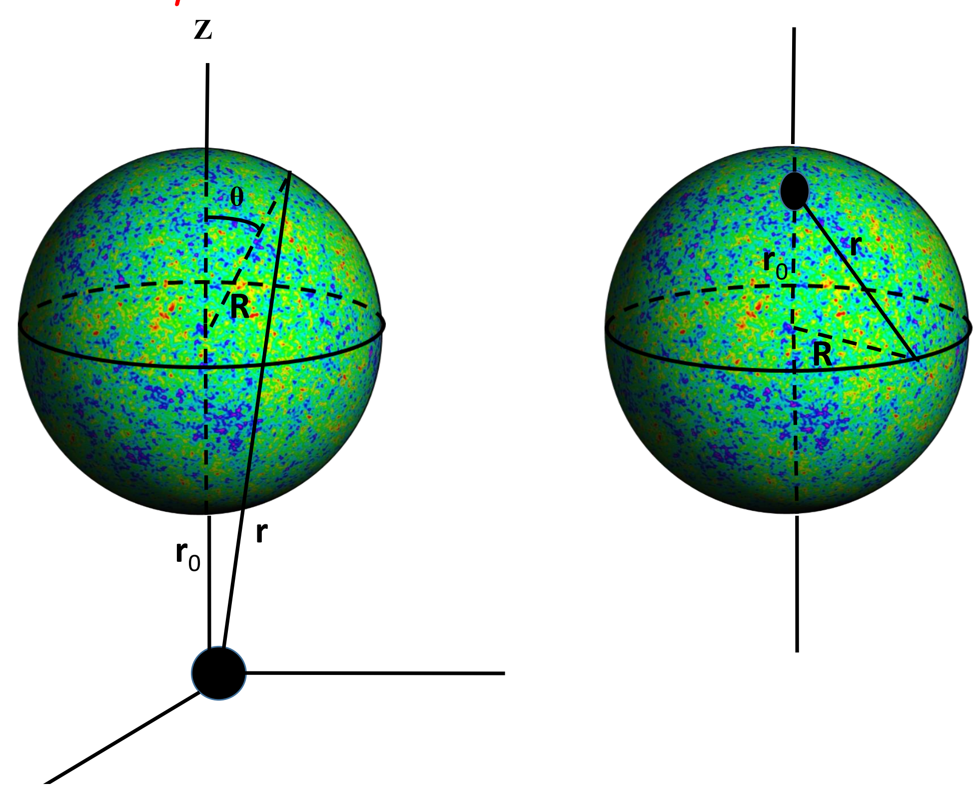
Primordial Inhomogeneities from massive defects

Consider a local massive defects with the total mass M in a inflationary (dS) background

$$ds^2 = -\left(1 - \frac{GM}{2a(t)r}\right)^2 \left(1 + \frac{GM}{2a(t)r}\right)^{-2} dt^2 + a(t)^2 \left(1 + \frac{GM}{2a(t)r}\right)^4 d\vec{\mathbf{x}}^2 .$$

We work in the weak filed approximation : $\mu \equiv GMH \ll 1$.

McVittie, 1933



The interaction Hamiltonian is

$$\mathcal{H}_I = \frac{a^3}{2}\left(-\frac{4MG}{ra} + \frac{35M^2G^2}{4r^2a^2}\right)\delta\dot{\phi}^2 - \frac{a}{2}\left(\frac{M^2G^2}{4r^2a^2}\right)(\nabla\delta\phi)^2 .$$

The inhomogeneous power spectrum is given by

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{q}} \rangle = \left(\frac{H}{\dot{\phi}}\right)^2 \Big(\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{q}} \rangle + \Delta \langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{q}} \rangle \Big).$$

H. F., A. Karami, T. Rostami, JCAP, 2016

The leading interaction Hamiltonian (linear in μ) is

$$H_I = -\frac{2MG}{(2\pi)^3(2\pi^2)}\int \frac{d^3\mathbf{k}d^3\mathbf{q}}{|\mathbf{k}+\mathbf{q}|^2}\delta\phi'(k)\delta\phi'(q).$$

The first order correction in power spectrum is

$$\begin{aligned}\Delta\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\rangle &= -\frac{16\pi\mu}{H^2}\frac{1}{|\mathbf{k}+\mathbf{q}|^2}\int\frac{d\tau}{\tau}\mathrm{Im}\Big[\delta\phi'_q(\tau)\delta\phi'_k(\tau)\delta\phi^*_q(\tau_e)\delta\phi^*_k(\tau_e)\Big] + k\leftrightarrow q \\ &= 0\end{aligned}$$

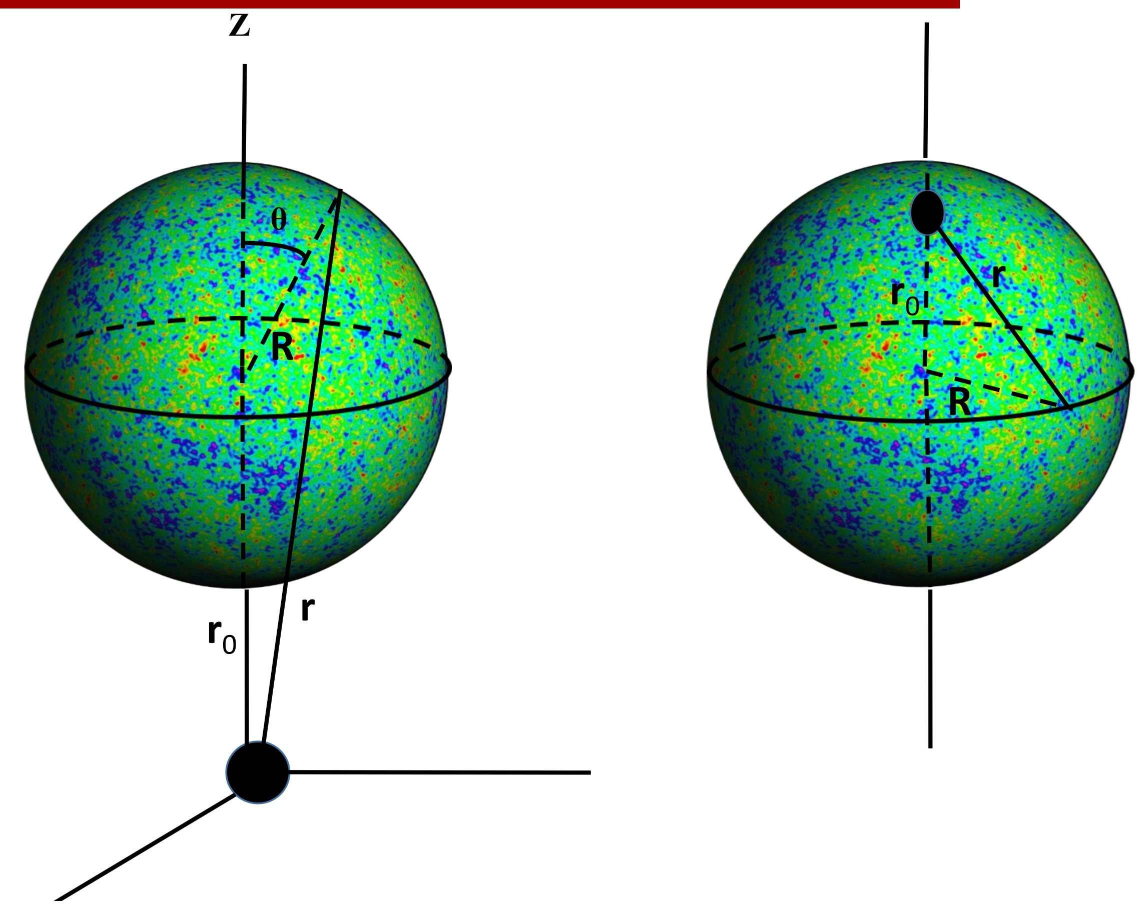
To calculate the corrections in power spectrum we have to work with terms G^2M^2, i.e. second order in μ. Calculating the in-in integrals to second order we obtain

$$\begin{aligned}\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{q}}\rangle = \left(\frac{H^2}{\dot\phi}\right)^2\Big\{&\frac{1}{2k^3}(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q}) - \frac{\mu^2\pi^2}{2|\mathbf{k}+\mathbf{q}|}\Big[\frac{35}{kq(k+q)^3}+\frac{\mathbf{k}.\mathbf{q}\,(k^2+q^2+3kq)}{k^3q^3(k+q)^3}\Big] \\ &+\frac{128\mu^2}{\pi}\int d^3\mathbf{p}\frac{1}{|\mathbf{p}-\mathbf{q}|^2}\frac{1}{|\mathbf{p}+\mathbf{k}|^2}\frac{p^2\left(p^2+2(k+q)p+(k^2+q^2+3kq)\right)}{kq(p+q)^2(p+k)^2(k+q)^3}\Big\}\end{aligned}$$

The induced power spectrum maximally breaks the homogeneity, i.e. there is no $\delta^3(\mathbf{k}+\mathbf{q})$. However, the power spectrum is still isotropic, as expected.

H. F., A. Karami, T. Rostami, JCAP, 2016

Variance



$$\Delta\langle\mathcal{R}^2(\mathbf{r})\rangle \approx \frac{668\mu^2}{8(2\pi)^2}\mathcal{P}_0 \ln\left(\alpha^2 + 1 - 2\alpha\cos\theta\right)$$

$\alpha \equiv r_0/R$ and $r = |\mathbf{x}| = R\sqrt{1+\alpha^2 - 2\alpha\cos\theta}$.

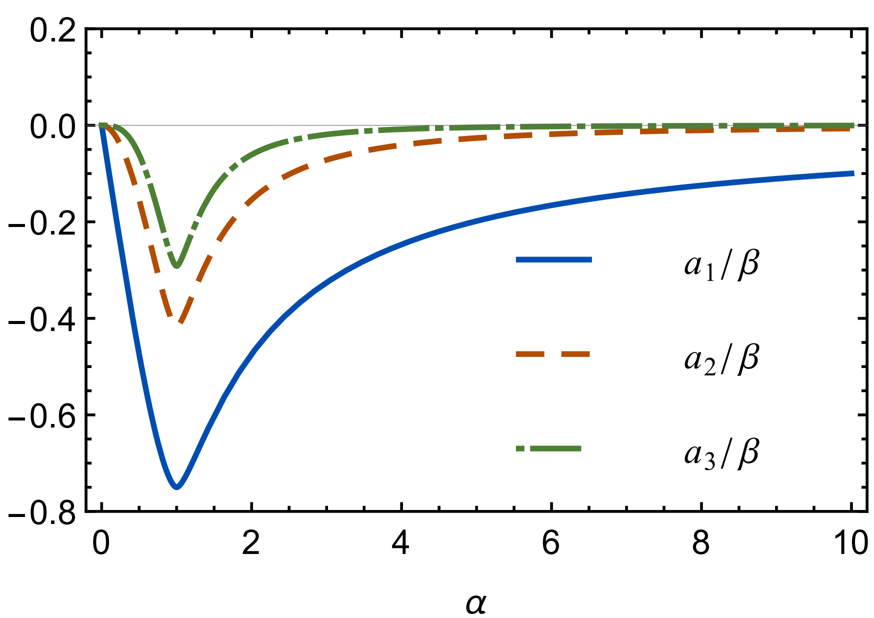
Also

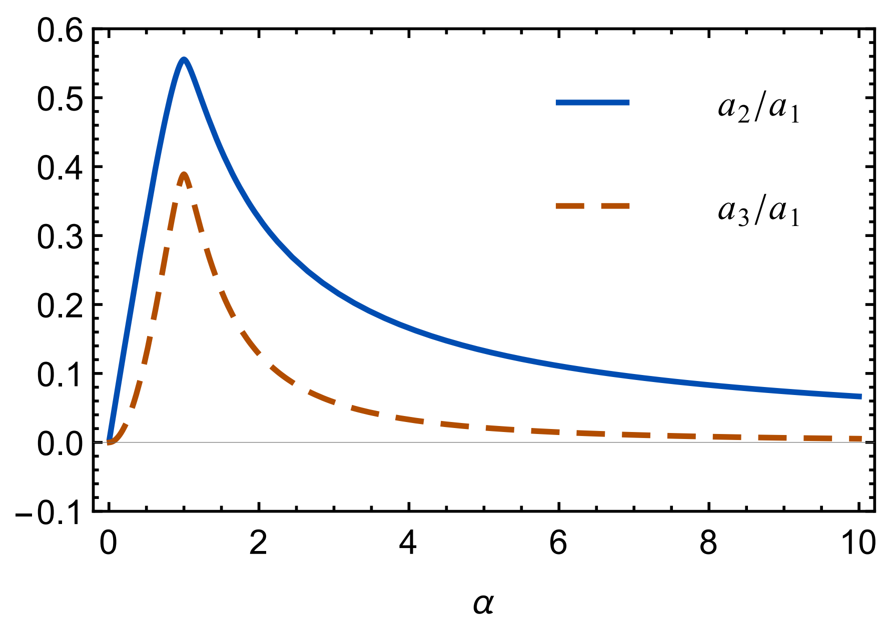
$$a_1 = \frac{3\beta}{16\alpha^2}\left[(\alpha^2-1)^2 \ln\left|\frac{1+\alpha}{1-\alpha}\right| - 2\alpha(1+\alpha^2)\right]$$

$$a_2 = \frac{5\beta}{96\alpha^3}\left[3(\alpha^2-1)^2(1+\alpha^2) \ln\left|\frac{1+\alpha}{1-\alpha}\right| - 2\alpha(3-2\alpha^2+3\alpha^4)\right],$$

$$a_3 = \frac{7\beta}{768\alpha^4}\left[3(\alpha^2-1)^2(5+6\alpha^2+5\alpha^4) \ln\left|\frac{1+\alpha}{1-\alpha}\right| - 2\alpha(15-7\alpha^2-7\alpha^4+15\alpha^6)\right].$$

One can check the curious reflection symmetry in which $\alpha \to 1/\alpha$ and $r_0^{new} = 1/r_0$.





$$\beta \equiv 668\mu^2/4(2\pi)^2.$$

Gravitational waves

The metric perturbations for tensor modes are given by

$$ds^2 = -\left(1 - \frac{GM}{2a(t)r}\right)^2 \left(1 + \frac{GM}{2a(t)r}\right)^{-2} dt^2 + a^2 \left(1 + \frac{MG}{2a(t)\, r}\right)^4 (\delta_{ij} + h_{ij})\, dx^i\, dx^j$$

$$h_{ij}(\mathbf{k}) = \sum_s h^s(\mathbf{k}) e^s_{ij}(\mathbf{k})\,,$$

$$k_i e^s_{ij}(\mathbf{k}) = 0, \qquad e^r_{ij}(\mathbf{k}) e^{s*}_{ij}(\mathbf{k}) = \delta^{rs}, \qquad e^s_{ii}(\mathbf{k}) = 0\,.$$

The interaction Hamiltonian for GW comes from the Einstein-Hilbert term and after integration N and N^i in ADM formalism.

$$H_I^{(1)} = -2\mu\pi \frac{M_P^2}{H} \int d^3\mathbf{x}\, \delta(r)(h_{ij})^2,$$

$$H_I^{(2)} = -2\mu \frac{M_P^2}{H} \int d^3\mathbf{x}\, \partial_j \partial_k \left(\frac{1}{r}\right) h_{ij} h_{ik}\,,$$

$$H_I^{(3)} = \frac{9}{8} \mu M_P^2 H a^2 \int d^3\mathbf{x} \frac{1}{r} h_{ij}^2,$$

$$H_I^{(4)} = \mu \frac{M_P^2 a^2}{8H} \int d^3\mathbf{x} \frac{1}{r} \dot{h}_{ij}^2$$

H. F., A. Karami, T. Rostami, arXiv: 1605. 08338

Because of the isotropy we choose

$$\mathbf{k} = k(0,0,1) \quad \mathbf{q} = q(0, \sin\psi, \cos\psi)$$

The inhomogeneous GW power spectrum is

$$\Delta\Big\langle h^r(\mathbf{k})h^s(\mathbf{q})\Big\rangle^{(i)} = i\int_{-\infty}^{t_e} dt \Big\langle \Big[H_I^{(i)}, h^r(\mathbf{k})h^s(\mathbf{q})\Big] \Big\rangle = -2\,\mathrm{Im}\int_{-\infty}^{t_e} dt \Big\langle H_I^{(i)}, h^r(\mathbf{k})h^s(\mathbf{q})\Big\rangle\,.$$

Calculating the in-in integrals yield

$$\Delta\langle h^\times(\mathbf{k})h^\times(\mathbf{q})\rangle = -\frac{2\pi^2\mu H^2}{M_P^2 k^3 q^3}\cos\psi\left(8 - 9\frac{k^2+q^2}{|\mathbf{k}+\mathbf{q}|^2}\right) - \frac{32\pi^2\mu H^2}{M_P^2}\frac{\sin^2\psi}{k^2q^2|\mathbf{k}+\mathbf{q}|^2}$$

$$\Delta\langle h^+(\mathbf{k})h^+(\mathbf{q})\rangle = \frac{\pi^2\mu H^2}{M_P^2 k^3 q^3}(\cos^2\psi + 1)(8 - 9\frac{k^2+q^2}{|\mathbf{k}+\mathbf{q}|^2}) + \frac{32\pi^2\mu H^2}{M_P^2}\frac{\sin^2\psi\cos\psi}{k^2q^2|\mathbf{k}+\mathbf{q}|^2},$$

The total inhomogeneous tensor power spectrum is

$$\Delta_{\mathrm{total}}\Big\langle h(\mathbf{k})h(\mathbf{q})\Big\rangle = -\frac{\pi^2\mu H^2}{M_P^2 k^3 q^3}(1-\cos\psi)^2\,\frac{k^2+q^2+32kq+16kq\cos\psi}{k^2+q^2+2kq\cos\psi}\,.$$

Unlike the scalar perturbations, the inhomogeneous tensor perturbations is linear in μ.

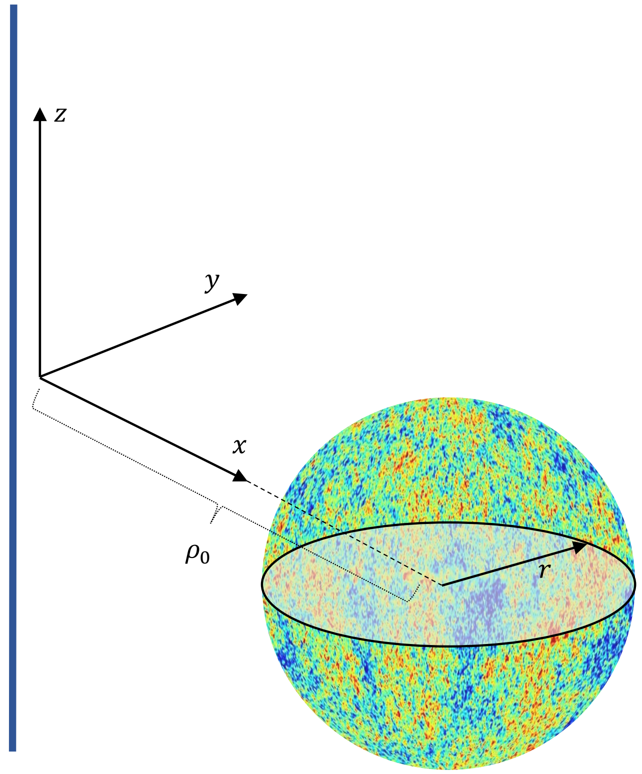
The geometry in the presence of cosmic string is given by

$$ds^2 = -dt^2 + a(t)^2\left(d\rho^2 + (1-4G\mu)^2\rho^2 d\phi^2 + dz^2\right),$$

Or alternatively,

$$ds^2 = -dt^2 + a(t)^2\left(d\mathbf{x}^2 - \frac{\epsilon}{\rho^2}(x^2 dy^2 + y^2 dx^2 - 2x\, y\, dx\, dy)\right)$$

in which $\epsilon = 8G\mu$.



The interaction Hamiltonian is

$$H_I = -\frac{a(t)\epsilon}{2(2\pi)^6}\int d^3\mathbf{x}\, d^3\mathbf{k}\, d^3\mathbf{q}\, \frac{\delta\phi_k\delta\phi_q}{x^2+y^2}\,(yk_x - xk_y)\,(yq_x - xq_y)e^{i(\mathbf{k}+\mathbf{q}).\mathbf{x}}$$

Plugging this inside in-in integrals the corrections from cosmic string is

$$\langle \mathcal{R}_{\mathbf{k}}(t_e)\mathcal{R}_{\mathbf{q}}(t_e)\rangle = \left(\frac{H^2}{\dot\phi}\right)^2\Bigg[\frac{(2\pi)^3}{k^3}\delta^3(\mathbf{k}+\mathbf{q}) - \epsilon\pi\left(\frac{k^2+q^2+kq}{k^3q^3(k+q)}\right)\delta(k_z+q_z)\times$$
$$\times\left[2\pi^2\mathbf{k}_\perp\cdot\mathbf{q}_\perp\delta^2(\mathbf{k}_\perp+\mathbf{q}_\perp) + \frac{2\pi\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{(\mathbf{k}_\perp+\mathbf{q}_\perp)^2} + \frac{4\pi}{(\mathbf{k}_\perp+\mathbf{q}_\perp)^4}(k_xq_y - k_yq_x)^2\right]\Bigg].$$

Sadra Jazayeri, Alireza Vafaei Sadr, H. F., arXiv: 1703.02923

There are two different contributions from cosmic string on CMB:

1- Qudarupole anisotropy

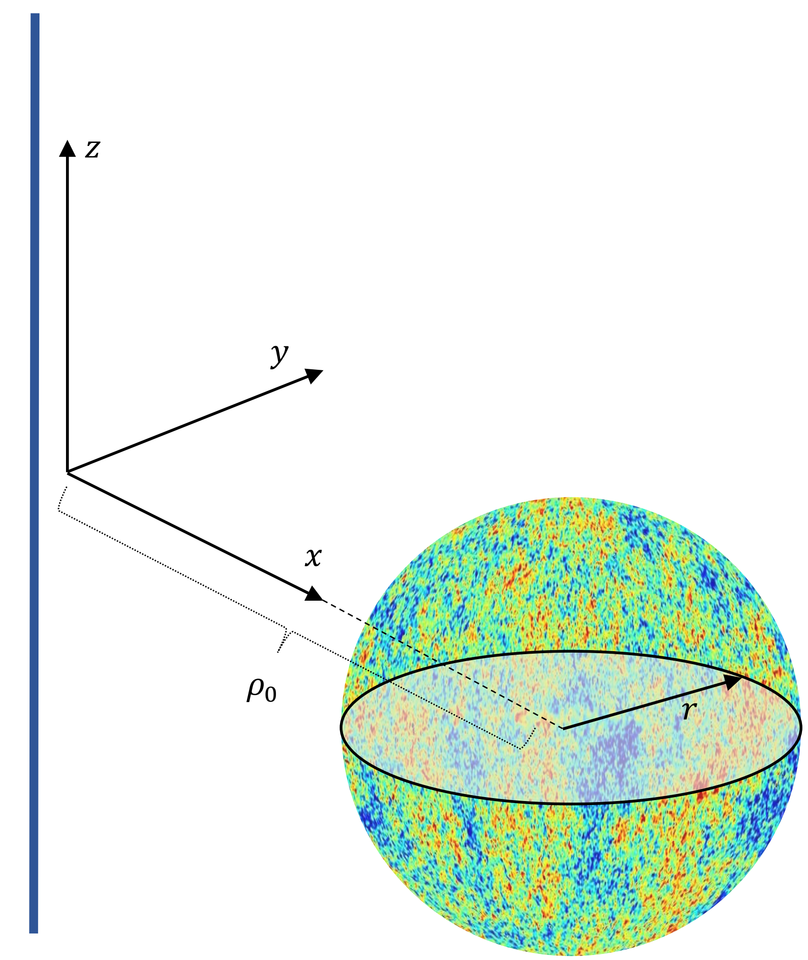
$$g_* = -\frac{3\epsilon}{8} \longrightarrow G\mu \lesssim 10^{-2}$$

Constraints from Planck date implies $g_* \lesssim 10^{-2}$.
As a result, we obtain $\epsilon \lesssim 10^{-2}$.

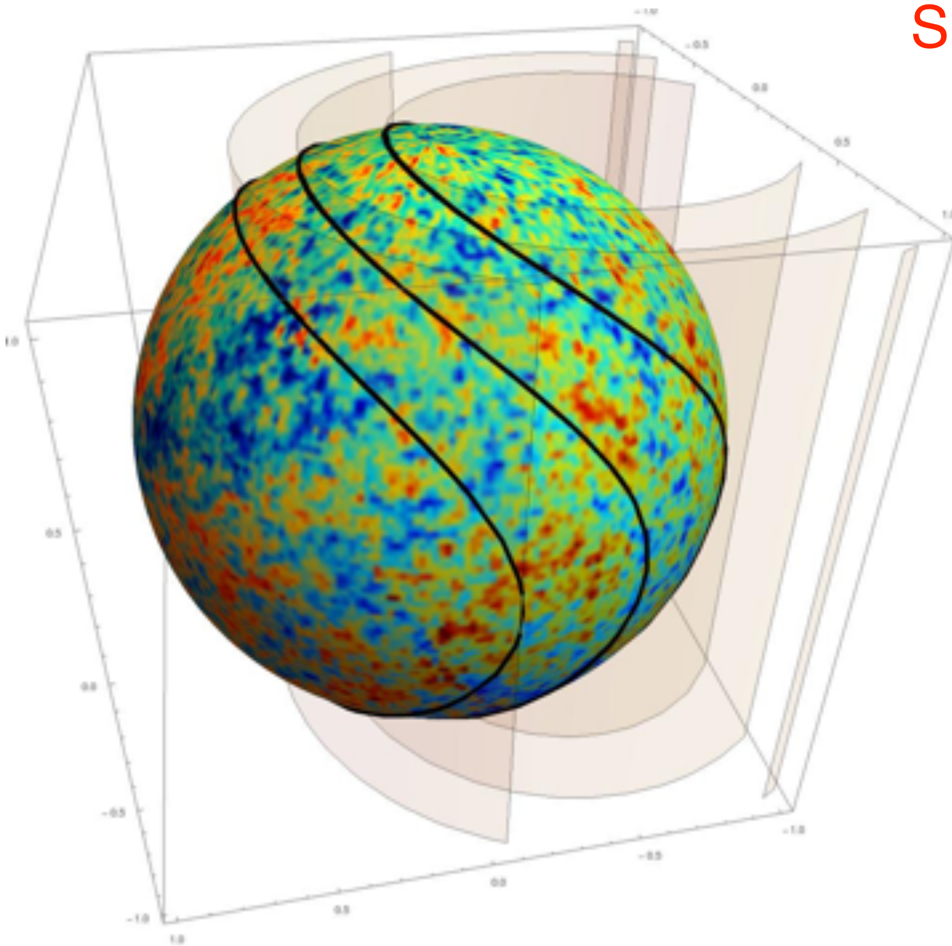
2- Inhomogeneities $\rightarrow$ Power asymmetry

$$\Delta\langle\mathcal{R}(\mathbf{x})^2\rangle_{\text{asym.}} \sim -\frac{\epsilon}{16\pi^3}\left(\frac{H^2}{\dot{\phi}}\right)^2 \ln\left(\frac{\rho}{L}\right)$$

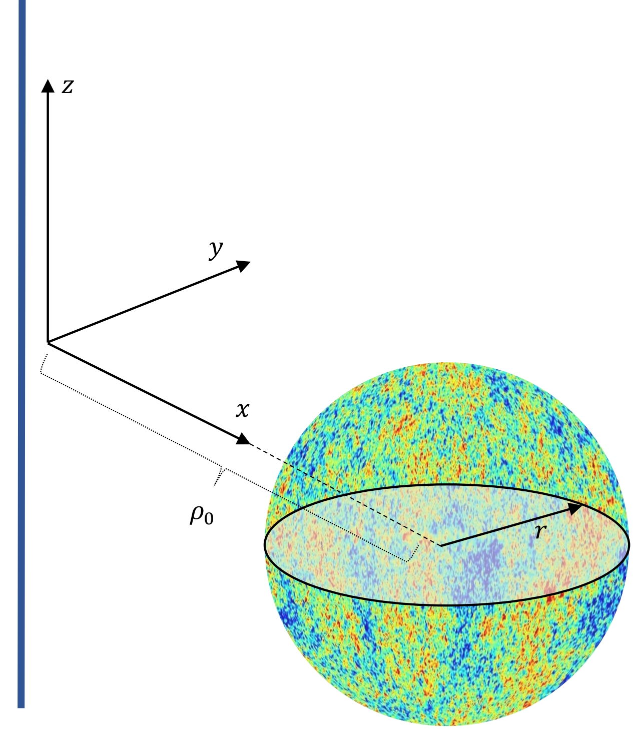
$$\longrightarrow \quad G\mu \sim 10^{-1}$$



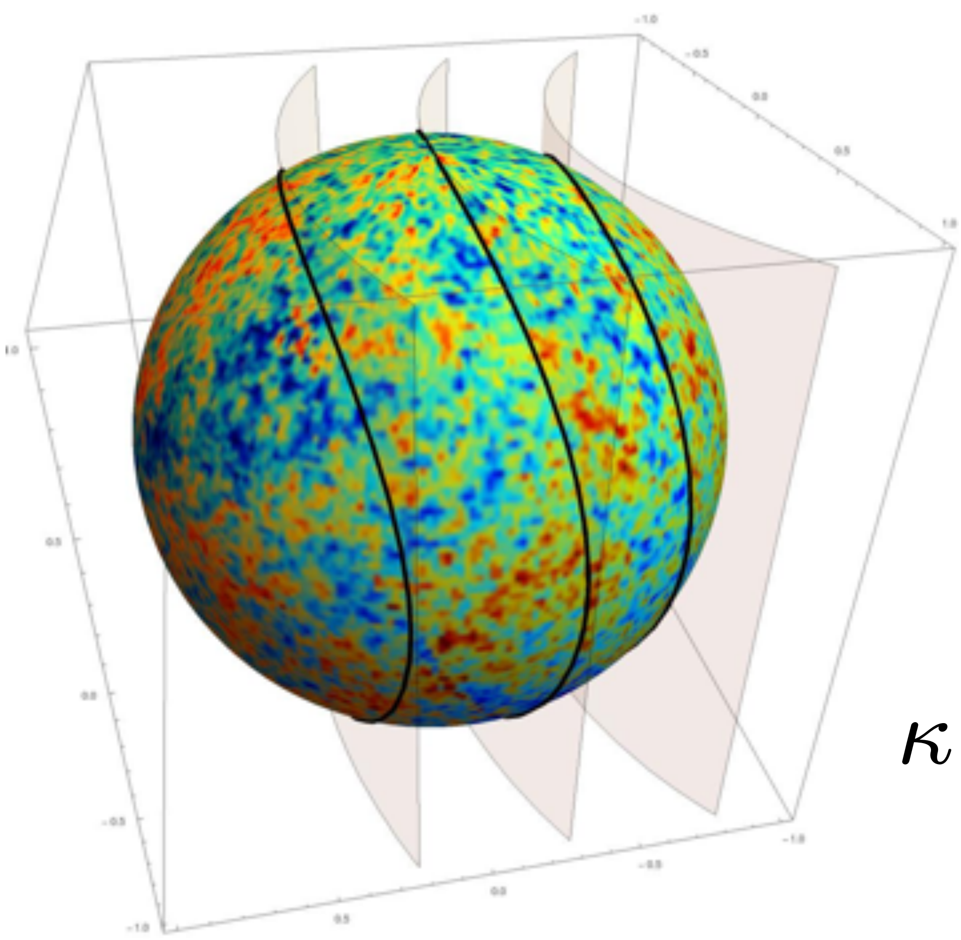
Surface of constant variance



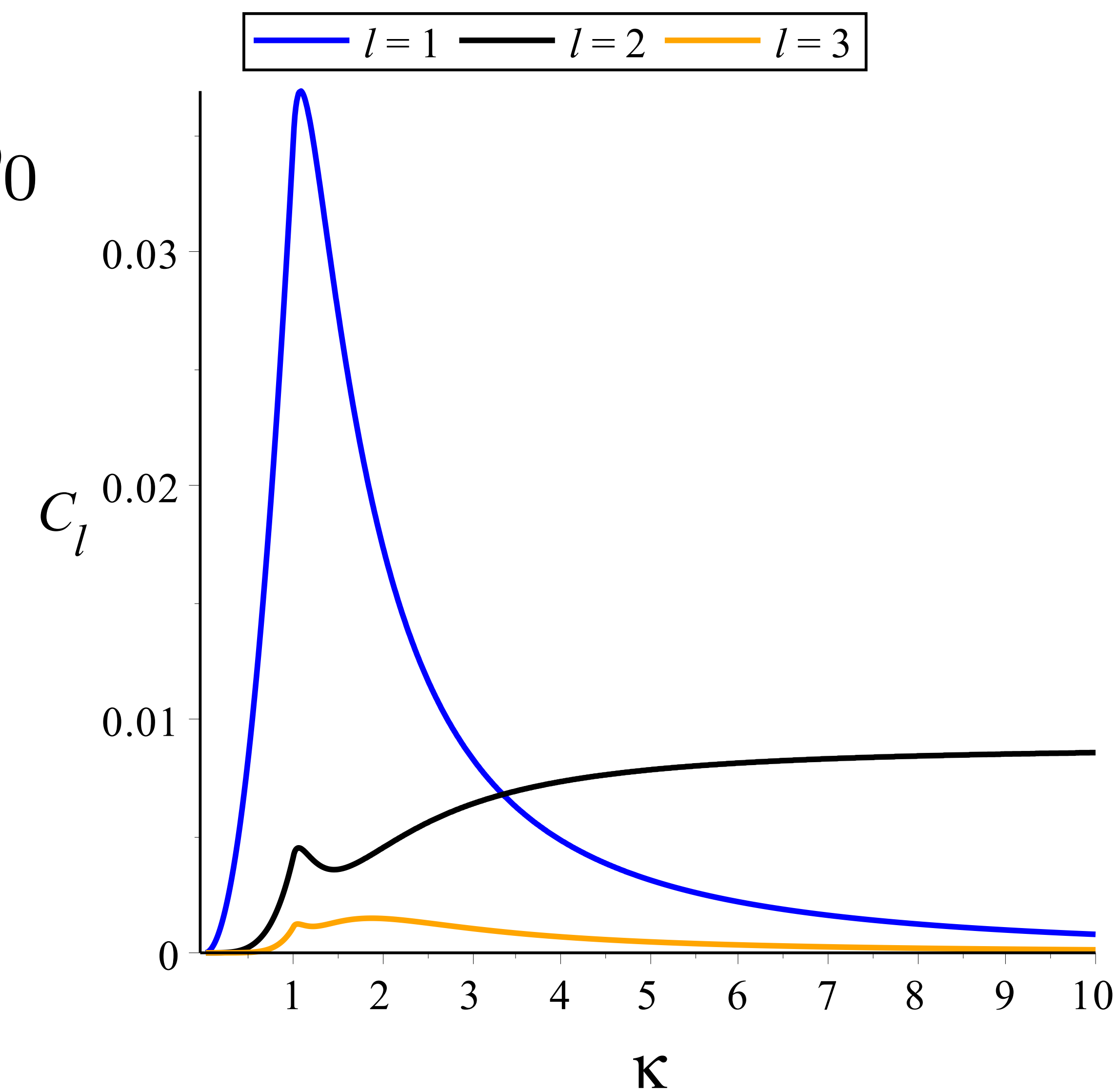
$\kappa = 2$

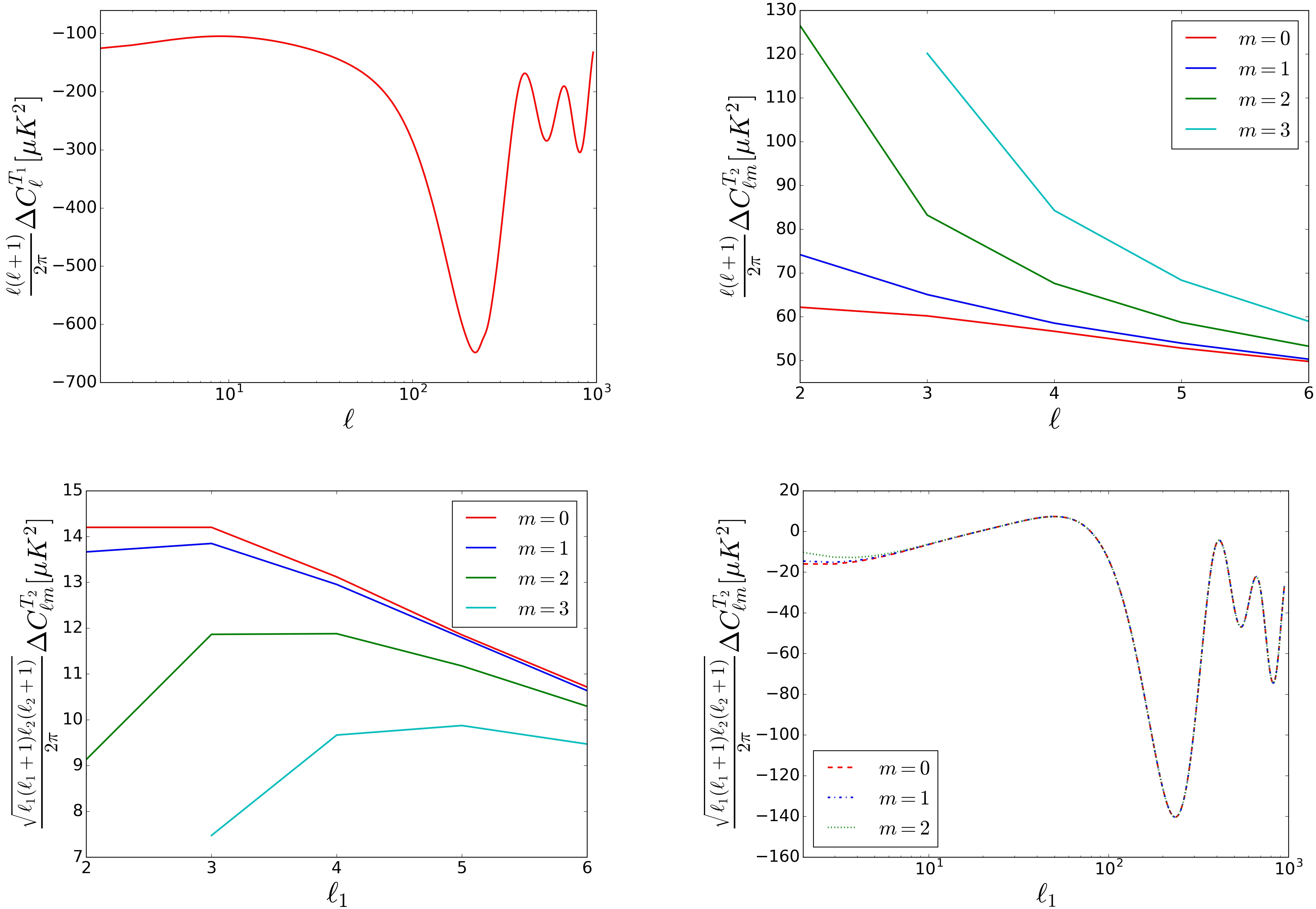


$\kappa \equiv r/\rho_0$



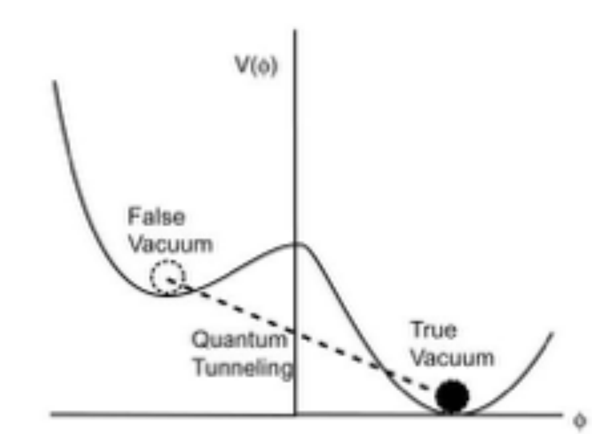
$\kappa = 0.5$



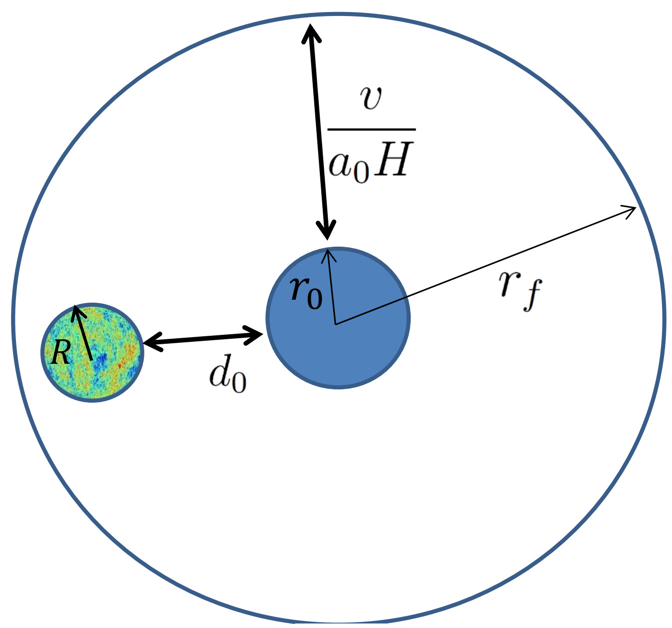


$\frac{\ell(\ell+1)}{2\pi}\Delta C_\ell^{T_1}[\mu K^2]$
ℓ
$\frac{\ell(\ell+1)}{2\pi}\Delta C_{\ell m}^{T_2}[\mu K^2]$
ℓ
$m=0$
$m=1$
$m=2$
$m=3$
$\frac{\sqrt{\ell_1(\ell_1+1)\ell_2(\ell_2+1)}}{2\pi}\Delta C_{\ell m}^{T_2}[\mu K^2]$
ℓ_1
$m=0$
$m=1$
$m=2$
$m=3$
$\frac{\sqrt{\ell_1(\ell_1+1)\ell_2(\ell_2+1)}}{2\pi}\Delta C_{\ell m}^{T_2}[\mu K^2]$
ℓ_1
$m=0$
$m=1$
$m=2$

Bubble nucleation during inflation

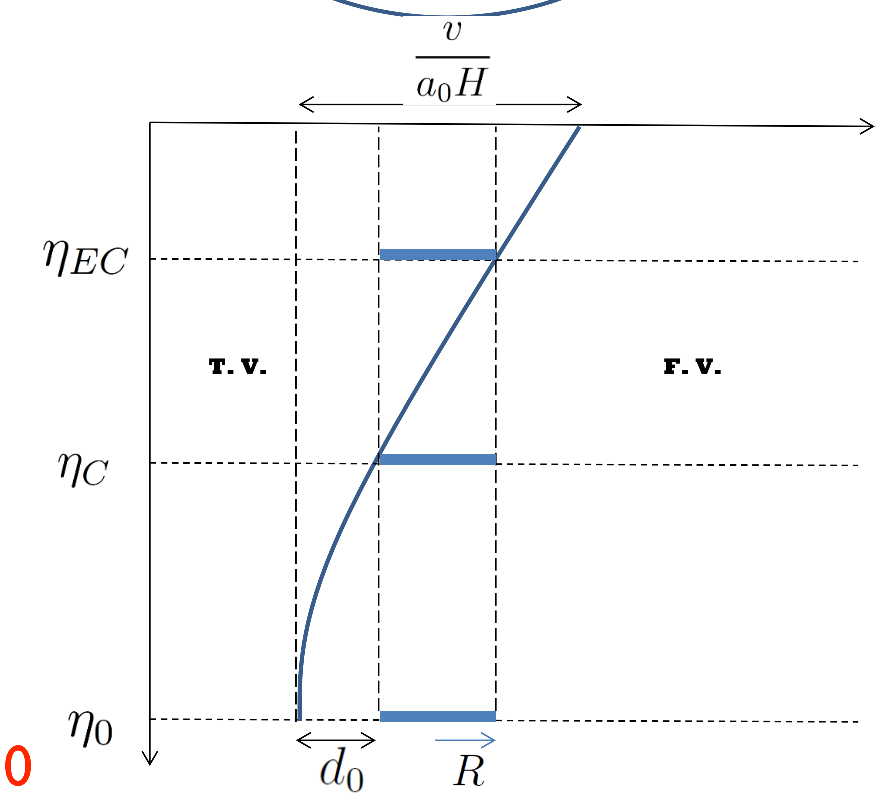


Quantum tunneling from a false vacuum to a true vacuum leads to bubble formation.



The early universe can have a complicated potential with many maxima and minima. This is partly motivated by landsacape hypothesis.

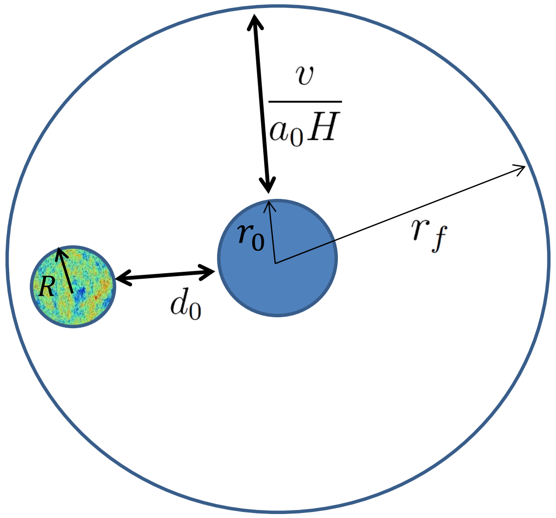
We are interested in a situation that more than one field is involved during inflation: the spectator field ψ and the inflaton field ϕ. The potential along the spectator field has a false vacuum and a true vacuum. Originally ψ is locked in its false vacuum. However, it tunnels to its true vacuum resulting in bubble nucleation during inflation.



The bubble has a small initial radius. It expands relativistically and asymptotically reaches its comoving radius $1/H$.

H. F., Sadra Jazayeri, Asieh Karami, Tahereh Rostami, arXiv: 1707.07550

In the simple approximation in which we work, the bubble divides the spacetime into two nearly dS geometries separated by the bubble wall.



$$ds_+^2 = -dt_+^2 + a_+^2(t_+)\left(dr_+^2 + r_+^2 d\Omega^2\right)$$

$$ds_W^2 = -d\tau^2 + R^2(\tau)d\Omega^2$$

$$ds_-^2 = -dt_-^2 + a_-^2(t_-)\left(dr_-^2 + r_-^2 d\Omega^2\right)$$

After imposing the junction condition the dynamics of the bubble wall is given by

$$R(\tau) = \frac{1}{A}\cosh(A\tau)$$

The geometry of the interior is slightly different than the exterior region given by

$$ds^2 = -dt^2 + \exp(2H_+t)(dr^2 + r^2 d\Omega^2) + \delta g_{\mu\nu}\theta(t-t_0)\theta(R(t)-r)dx^\mu dx^\nu$$

$$\delta g_{00} = -2\epsilon \qquad \delta g_{rr} = 2a^2\epsilon(1+\beta^2) \simeq 2a^2\epsilon \qquad \delta g_{\theta\theta} = \sin^{-2}\theta\,\delta g_{\phi\phi} \simeq 2a^2r^2\epsilon\left(1 - \frac{\beta}{2Hr}\right)$$

The effects of the bubble on inflationary perturbations is:

$$H_I(t) = 2\epsilon\theta(t-t_0)\int_0^{r_W(t)} a^3r^2 dr d\Omega\left[\frac{\delta\dot{\phi}^2}{2}\left(-1 + \frac{\beta}{2Hr}\right) + \frac{(\nabla\delta\phi)^2}{2a^2} - \frac{\beta(\partial_r\delta\phi)^2}{4a^2Hr}\right]$$

The effect of bubble on curvature perturbation is

$$\Delta\Big\langle\mathcal{R}_{\mathbf{k}}(t_e)\mathcal{R}_{\mathbf{q}}(t_e)\Big\rangle = \left(\frac{H^2}{\dot{\phi}}\right)^2 \Delta\Big\langle\delta\phi_{\mathbf{k}}(t_e)\delta\phi_{\mathbf{q}}(t_e)\Big\rangle$$

There are corrections to diagonal parts and the off-diagonal parts

$$\lim_{\mathbf{k}+\mathbf{q}\to 0} \Delta\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\rangle \quad = \quad \frac{-2\pi\epsilon H^2 r_f^3}{3k^3}(2+\frac{7}{k^2 r_f^2})$$

The corrections in diagonal part is

$$\mathcal{P}_{\mathbf{k}} = \mathcal{P}_0\Big(1-\frac{28\pi\epsilon}{3k^2 r_f^2}\Big)$$

The corrections in off-diagonal parts are more complicated

$$|\mathbf{k}+\mathbf{q}|r_f \gg 1$$

$$\cos\alpha = \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$$

$$\begin{aligned}\Delta\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\rangle^{\text{osc}} &= \frac{-2\pi\epsilon H^2 r_f \sin^2\alpha}{kqK^4}\cos(Kr_f)\\ &\times\Big[K\ln(\frac{K+k+q}{K-k-q}) + \frac{2(k+q)(k^2+q^2+kq-K^2)}{K^2-(k+q)^2}\Big]\end{aligned}$$

$$\Delta\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{q}}\rangle^{\text{non-osc}} = \frac{-4\pi\epsilon r_f H^2}{K^2(k+q)kq} + \frac{4\pi\epsilon r_f H^2}{k^2q^2K^4(k+q)}(k^2\cos\alpha + q^2\cos\alpha + 2kq)(k^2+q^2+kq)$$

Conclusion

- Inflation is the leading paradigm for early Universe and for generating
- There are evidences for power asymmetry on CMB maps. However, the statistical significance of this detection is under debate.
- A domain wall during inflation breaks the translation invariance and can generate large scale dependent dipole asymmetry and sub-leading quadrupole and higher multipoles power asymmetry.
- A massive defect maximally breaks translational invariance while leaving the isotropy intact. A scale dependent dipole asymmetry is generated while the higher multipoles can be suppressed.
- Cosmic string induces both statistical anisotropies and power asymmetry. The primary constraint on the tension of strings comes from the quadrupole anisotropy yielding $G\mu \lesssim 10^{-2}$.
- Vacuum bubble from tunneling generates non-trivial power anisotropies which can be tested on CMB.