Dark energy & Modified gravity in scalar-tensor theories

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Astroparticules
et Cosmologie

Introduction

- So far, **GR** seems compatible with all observations.

- Several motivations for exploring **modified gravity**
  - Quantum gravity effects
  - Explain cosmological acceleration (or possibly dark matter)
  - Explore alternative gravitational theories
  - Testing gravity

- Many models of **dark energy** & **modified gravity:**
  quintessence, K-essence, f(R) gravity, massive gravity…

- **Generalized framework** for scalar-tensor theories, allowing for **2nd order derivatives** in their Lagrangian

Traditional scalar-tensor theories

- Simplest extensions of GR: add a **scalar field**

$$S = \int d^4x\sqrt{-g}\left[F(\phi)^{(4)}R - Z(\phi)\partial_\mu\phi\partial^\mu\phi - U(\phi)\right] + S_m[\psi_m; g_{\mu\nu}]$$

- **K-essence/ k-inflation**: non standard kinetic term

$$S = \int d^4x\,\sqrt{-g}\left[\frac{M_P^2}{2}\,{}^{(4)}R + P(X,\phi)\right]$$

$$X \equiv \nabla_\mu\phi\,\nabla^\mu\phi$$

Higher order scalar-tensor theories

- Traditional scalar-tensor theories : $\mathcal{L}(\nabla_\lambda \phi, \phi)$

- Generalized theories with second order derivatives

$$\mathcal{L}(\nabla_\mu \nabla_\nu \phi,\ \nabla_\lambda \phi,\ \phi)$$

- In general, they contain an **extra degree of freedom**, expected to lead to **Ostrogradsky instabilities**

$$L(\ddot{q}, \dot{q}, q)$$

- But there are exceptions...

Horndeski theories

Horndeski 74

(a.k.a. Generalized Galileons)

- Combination of the Lagrangians

$$L_2^H = G_2(\phi, X)$$
$$L_3^H = G_3(\phi, X)\,\Box\phi$$
$$L_4^H = G_4(\phi, X)\,{}^{(4)}R - 2G_{4X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$
$$L_5^H = G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})$$

with
$$X \equiv \nabla_\mu\phi\nabla^\mu\phi$$
$$\phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$$

- **Second order equations of motion** for the scalar field and the metric

- They contain 1 scalar DOF and 2 tensor DOF.

  No dangerous extra DOF !

Beyond Horndeski & DHOST theories

- Extensions "beyond Horndeski" Gleyzes, DL, Piazza &Vernizzi '14

$$L_4^{\rm bH} \equiv F_4(\phi, X)\,\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$

$$L_5^{\rm bH} \equiv F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

leading to **third order** equations of motion.

- Earlier hint: disformal transformation of Einstein-Hilbert
Zumalacarregui & Garcia-Bellido '13

- Even if EOM are higher order, **no extra DOF** if the Lagrangian is "**degenerate**".

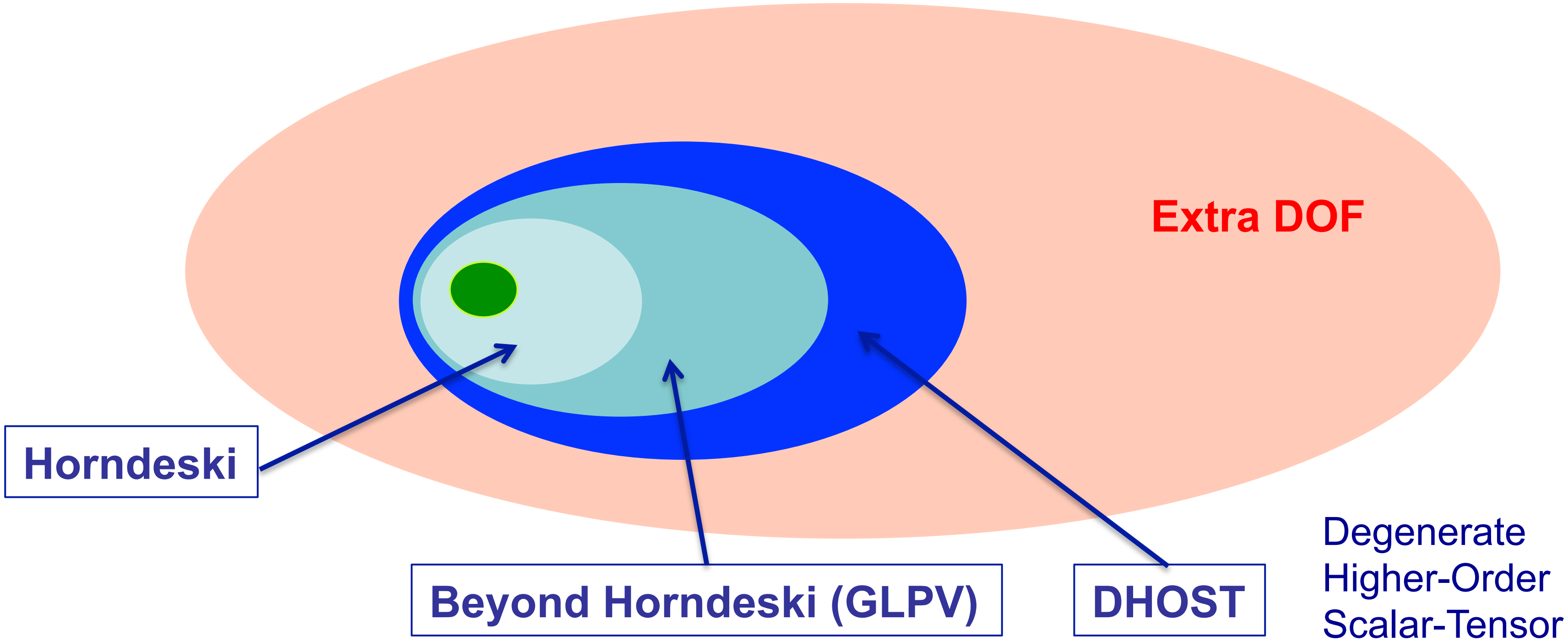
➡ **DHOST** theories DL & K. Noui '15

(Degenerate Higher-Order Scalar-Tensor)

Higher order scalar-tensor theories

- Traditional theories: $\mathcal{L}(\nabla_\lambda \phi, \phi)$

- Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi,\ \nabla_\lambda \phi,\ \phi)$



Degenerate Lagrangians

DL & K. Noui ‘1510

- **Scalar-tensor theories**: scalar field + metric
- Simple toy model: $\phi(x^\lambda) \to \phi(t)\,, \quad g_{\mu\nu}(x^\lambda) \to q(t)$
- Lagrangian

$$L = \frac{1}{2} a\, \ddot{\phi}^2 + b\, \ddot{\phi}\, \dot{q} + \frac{1}{2} c\, \dot{q}^2 + \frac{1}{2} \dot{\phi}^2 - V(\phi, q)$$

- Equations of motion are higher order

  (4th order if a nonzero, 3rd order if a=0)

Degrees of freedom

- Introduce the auxiliary variable $Q \equiv \dot{\phi}$

$$L = \frac{1}{2} a \, \dot{Q}^2 + b \, \dot{Q} \dot{q} + \frac{1}{2} c \, \dot{q}^2 + \frac{1}{2} Q^2 - V(\phi, q) - \lambda (Q - \dot{\phi})$$

- Equations of motion

$$a \, \ddot{Q} + b \, \ddot{q} = Q - \lambda \qquad \dot{\phi} = Q \, , \quad \dot{\lambda} = -V_\phi$$

$$b \, \ddot{Q} + c \, \ddot{q} = -V_q$$

- If the Hessian matrix is **invertible**, one finds **3 DOF.**

$$M \equiv \left(\frac{\partial^2 L}{\partial v^a \partial v^b} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

[6 initial conditions]

Degrees of freedom

- If the Hessian matrix is degenerate, i.e. $ac - b^2 = 0$ then only **2 DOF** (at most**).**

$$M \equiv \left(\frac{\partial^2 L}{\partial v^a \partial v^b} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

[$\ddot{\phi}$ can be absorbed in $\dot{x} \equiv \dot{q} + \frac{b}{c}\ddot{\phi}$]

- **Hamiltonian analysis**: primary constraint and secondary constraint

[$p_a = \frac{\partial L}{\partial v^a}(v)$ cannot be inverted]

Generalization (classical mechanics)

Motohashi, Noui, Suyama, Yamaguchi & DL 1603

[See also Klein & Roest 1604]

- Consider a general Lagrangian

$$L(\ddot{\phi}^\alpha, \dot{\phi}^\alpha, \phi^\alpha; \dot{q}^i, q^i) \qquad \alpha = 1, \cdots, n; \; i = 1, \cdots, m$$

In general, **2n+m DOF**. But the n extra DOF can be eliminated by requiring:

1. **Primary conditions** (n **primary** constraints)

$$L_{\dot{Q}^\alpha \dot{Q}^\alpha} - L_{\dot{Q}^\alpha \dot{q}^i} (L^{-1})^{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}^\beta} = 0$$

2. **Secondary conditions** (n **secondary** constraints)

$$L_{\dot{Q}^\alpha \dot{\phi}^\beta} - L_{\dot{Q}^\beta \dot{\phi}^\alpha} = 0 \qquad \text{if} \quad m = 0$$

- Third-order time derivatives... Motohashi, Suyama, Yamaguchi 1711

Quadratic DHOST theories

DL & Noui '1510

- Consider all theories of the form

$$S[g,\phi] = \int d^4x \sqrt{-g} \left[f_2 {}^{(4)}R + C_{(2)}^{\mu\nu\rho\sigma} \, \nabla_\mu \nabla_\nu \phi \, \nabla_\rho \nabla_\sigma \phi \right]$$

where $f_2 = f_2(X,\phi)$ and $C_{(2)}^{\mu\nu\rho\sigma}$ depends only on ϕ and $\nabla_\mu \phi$.

- All possible contractions of $\phi_{\mu\nu} \, \phi_{\rho\sigma}$?

e.g. $g^{\mu\nu} g^{\rho\sigma} \phi_{\mu\nu} \, \phi_{\rho\sigma} = (\Box\phi)^2$ or $\phi^\mu \phi^\nu \phi^\rho \phi^\sigma \phi_{\mu\nu} \, \phi_{\rho\sigma} = (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$

In summary: $C_{(2)}^{\mu\nu\rho\sigma} \, \phi_{\mu\nu} \, \phi_{\rho\sigma} = \sum a_A(X,\phi) \, L_A^{(2)}$

$$L_1^{(2)} = \phi_{\mu\nu} \, \phi^{\mu\nu} \, , \quad L_2^{(2)} = (\Box\phi)^2 \, , \quad L_3^{(2)} = (\Box\phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu \, , \quad L_5^{(2)} = (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

Quadratic DHOST theories

- Lagrangians of the form

DL & Noui '1510

$$L = f_2(X,\phi)\,{}^{(4)}R + \sum_{A=I}^{5} a_A(X,\phi)\, L_A^{(2)}$$

which depend on 6 arbitrary functions.

- **Degeneracy** yields **three conditions** on the 6 functions.

- Classification: 7 subclasses (4 with $f_2 \neq 0$, 3 with $f_2 = 0$)

[See also Crisostomi et al '1602; Ben Achour, DL & Noui '1602; de Rham & Matas '1604]

- This includes, in particular, $L_4^{\rm H}$ and $L_4^{\rm bH}$

$$f_2 = G_4\,, \quad a_1 = -a_2 = 2G_{4X} + XF_4\,, \quad a_3 = -a_4 = 2F_4$$

Cubic DHOST theories

[Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '1608]

- Action of the form

$$S[g,\phi] = \int d^4x\,\sqrt{-g}\left[f_3\,G^{\mu\nu}\phi_{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta}\right]$$

depends on eleven functions: $C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta} = \sum_{i=1}^{10} b_i(X,\phi)\,L_i^{(3)}$

- This includes the Lagrangians L_5^{H} and L_5^{bH} .

- 9 degenerate subclasses: 2 with $f_3 \neq 0$, 7 with $f_3 = 0$

- 25 **combinations of quadratic and cubic theories** (out of 7x9) are degenerate.

Disformal transformations

- Transformations of the metric [Bekenstein '93]

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X,\phi)\, g_{\mu\nu} + D(X,\phi)\, \partial_\mu\phi\, \partial_\nu\phi$$

- Starting from an action $\tilde{S}\left[\phi, \tilde{g}_{\mu\nu}\right]$, one can define the new action

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}\left[\phi, \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \phi_\mu \phi_\nu\right]$$

- Disformal transformation of quadratic DHOST theories ?

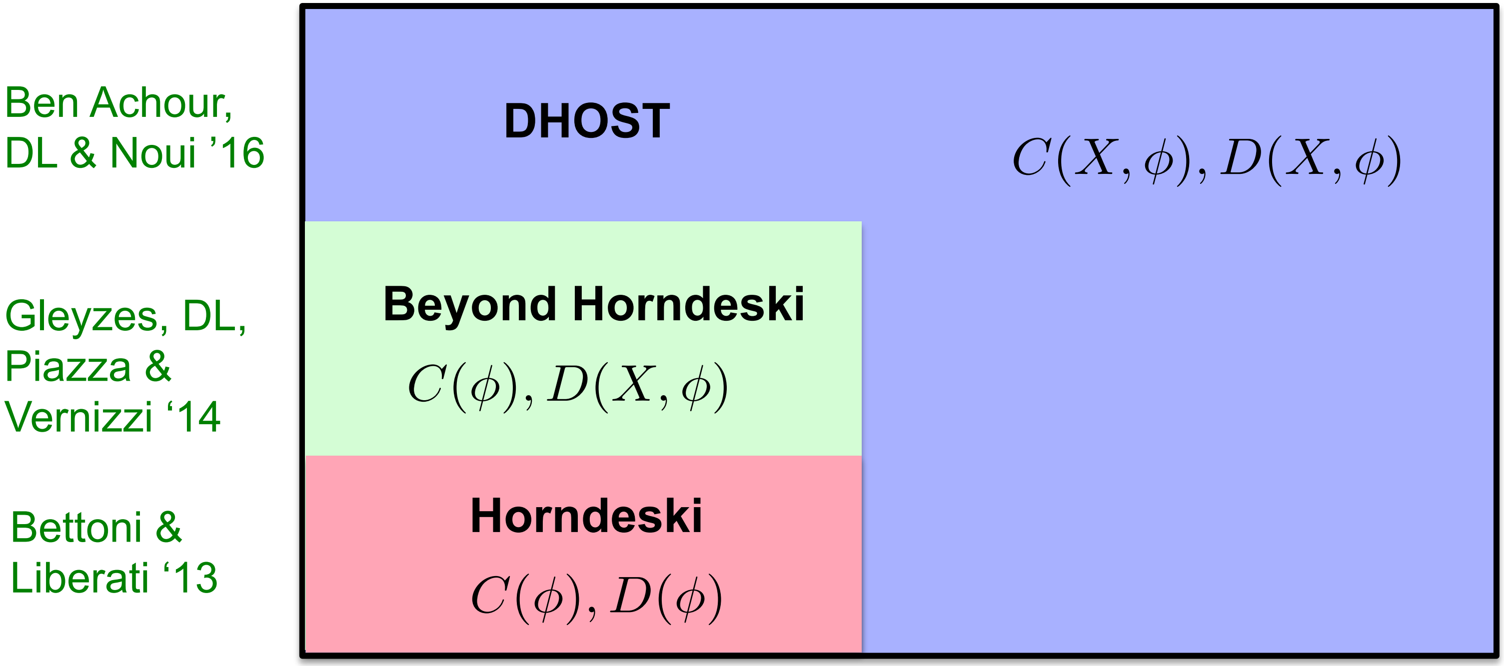
$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{f}_2\, {}^{(4)}\tilde{R} + \sum_I \tilde{a}_I\, \tilde{L}_I^{(2)}\right]$$

The structure of DHOST theories is preserved and all seven subclasses are stable.

[Ben Achour, DL & Noui '1602]

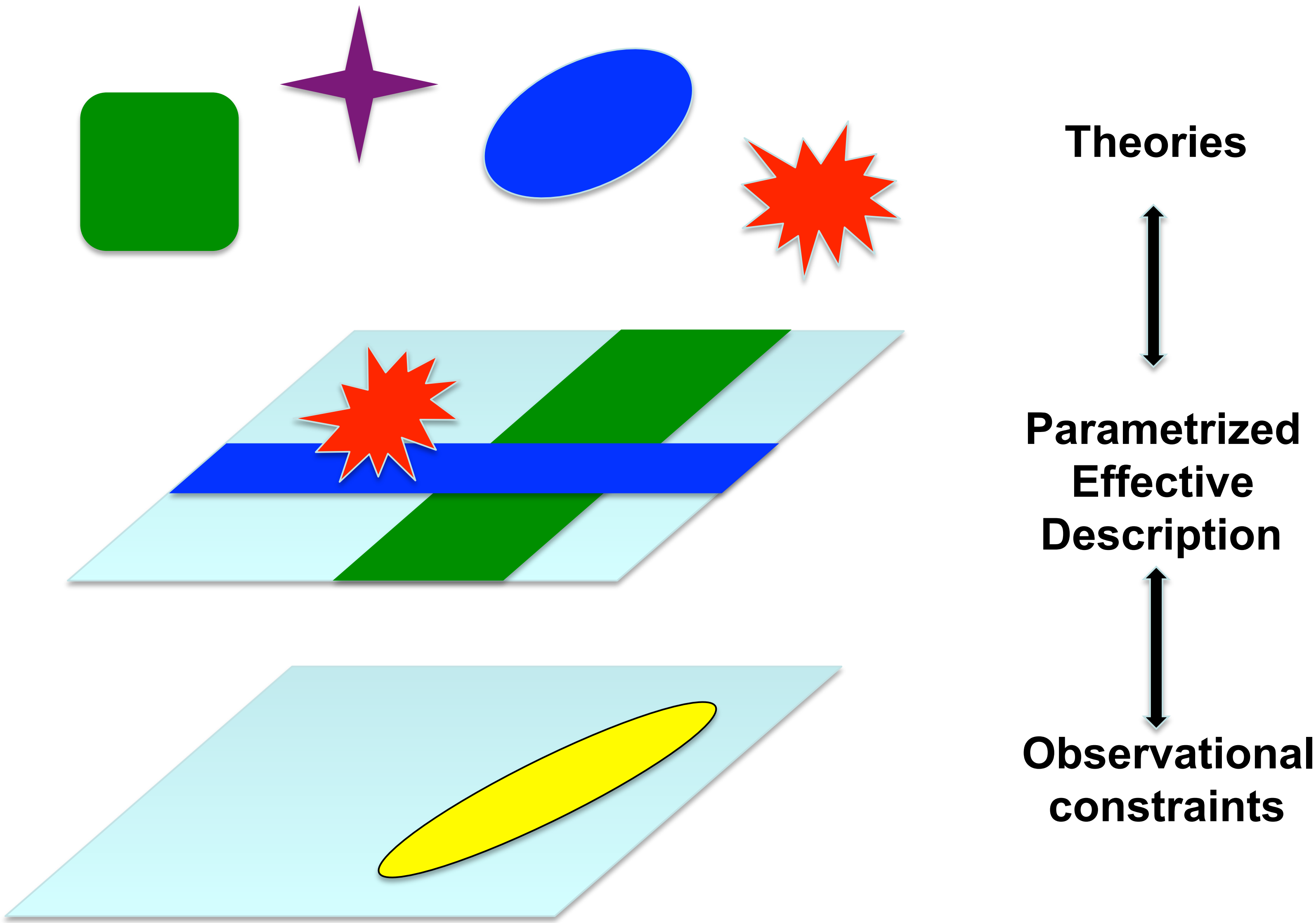
Disformal transformations

- Stability under $g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \partial_\mu \phi\, \partial_\nu \phi$



- When **matter** is included (with minimal coupling), two disformally related theories are **physically inequivalent** !

Cosmology:
Effective description of Dark Energy & Modified Gravity


Theories
Parametrized
Effective
Description
Observational
constraints

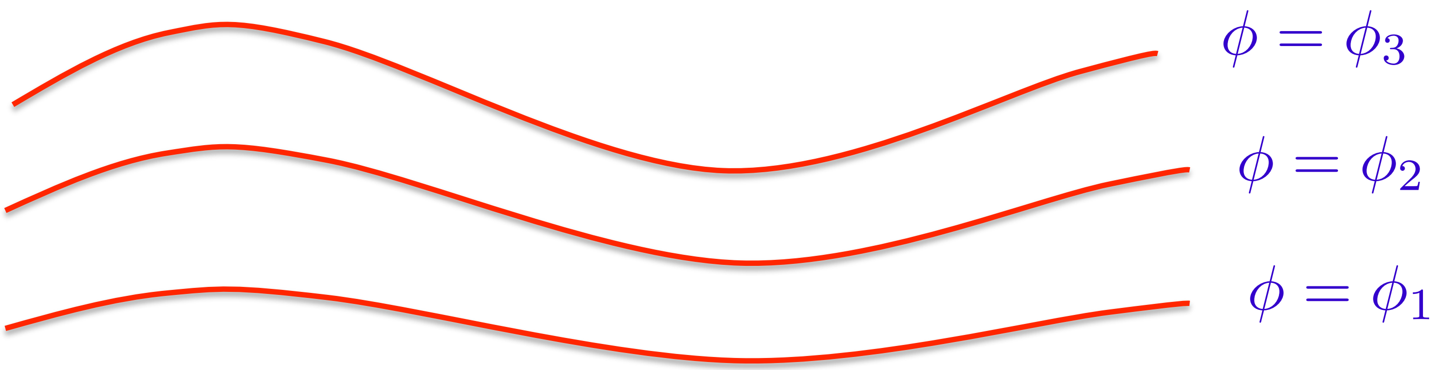
Effective description of Dark Energy

[See e.g review: Gleyzes, DL & Vernizzi 1411.3712]

- Restriction: **single scalar field** models

- The scalar field defines a **preferred slicing**

  Constant time hypersurfaces = uniform field hypersurfaces



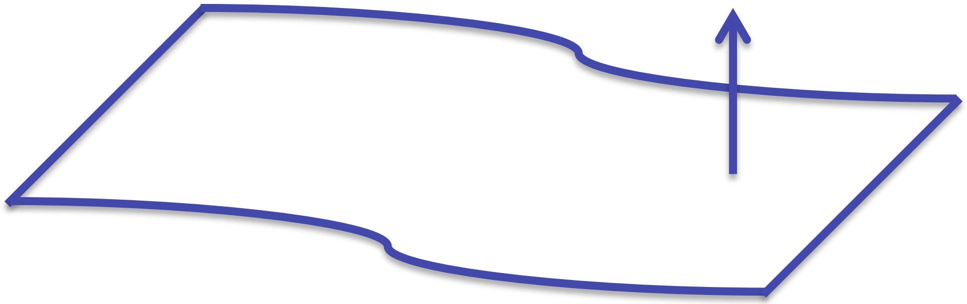
- All perturbations embodied by the metric only

Uniform scalar field slicing

- **3+1 decomposition** based on this preferred slicing

- Basic ingredients
  - **Unit vector normal** to the hypersurfaces

$$n^\mu = -\frac{\nabla^\mu \phi}{\sqrt{-(\nabla\phi)^2}}$$

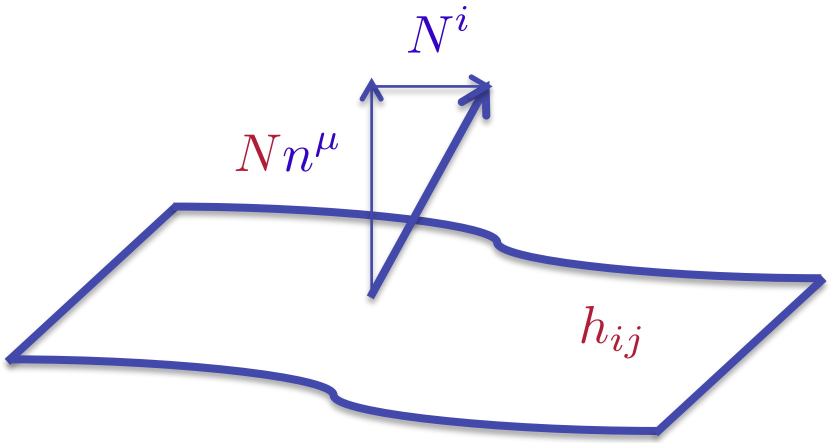


  - **Projection** on the hypersurfaces: $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

ADM formulation

- **ADM decomposition of spacetime**

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$



Extrinsic curvature:

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

Intrinsic curvature: R_{ij}

$$X \equiv g^{\mu\nu}\nabla_\mu \phi \nabla_\nu \phi = -\frac{\dot{\phi}^2(t)}{N^2}$$

- **Generic Lagrangians of the form**

$$S_g = \int d^4x\, N\sqrt{h}\, L(N, K_{ij}, R_{ij}; t)$$

Homogeneous background & linear perturbations

- **Background** $$ds^2 = -\bar{N}^2(t)\,dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j$$

$$\bar{L}(a,\dot{a},\bar{N}) \equiv L\left[K^i_j = \frac{\dot{a}}{\bar{N}a}\,\delta^i_j,\, R^i_j = 0,\, N = \bar{N}(t)\right]$$

- **Perturbations:** $\delta N \equiv N - \bar{N}\,,\ \delta K^i_j \equiv K^i_j - H\delta^i_j\,,\ \delta R^i_j \equiv R^j_i$

- Expanding the Lagrangian $L(q_A)$ with $q_A \equiv \{N, K^i_j, R^i_j\}$

yields $$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A}\delta q^A + \frac{1}{2}\frac{\partial^2 L}{\partial q_A \partial q_B}\delta q_A \delta q_B + \dots$$

- The **quadratic** action describes the **dynamics of linear perturbations**

Horndeski & beyond Horndeski

- Quadratic action

Gleyzes, DL, Piazza & Vernizzi '13,
[notation: Bellini & Sawicki '14]

$$S^{(2)} = \int dx^3 dt\, a^3 \frac{M^2}{2} \Big[\delta K^i_j \delta K^j_i - \delta K^2 + \alpha_K H^2 \delta N^2 + 4\,\alpha_B H\, \delta K\, \delta N + (1+\alpha_T)\delta_2\left(\frac{\sqrt{h}}{a^3}R\right) + (1+\alpha_H) R\, \delta N \Big]$$

$$\alpha_M \equiv \frac{d \ln M^2}{H\, dt}$$

	α_K	α_B	α_M	α_T	α_H
Quintessence, K-essence	✓				
Kinetic braiding, DGP	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

Scalar degree of freedom

- Scalar perturbations: $\delta N\,, \quad N_i \equiv \partial_i \psi\,, \quad h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$

- Quadratic action for the **physical degree of freedom**:

$$S^{(2)} = \frac{1}{2} \int dx^3 dt\, a^3 \left[\mathcal{K}_t\, \dot{\zeta}^2 + \mathcal{K}_s\, \frac{(\partial_i \zeta)^2}{a^2} \right]$$

$$\mathcal{K}_t \equiv \frac{\alpha_K + 6\alpha_B^2}{(1+\alpha_B)^2}\,, \quad \mathcal{K}_s \equiv 2M^2 \left\{ 1 + \alpha_T - \frac{1+\alpha_H}{1+\alpha_B} \left(1 + \alpha_M - \frac{\dot{H}}{H^2} \right) - \frac{1}{H} \frac{d}{dt} \left(\frac{1+\alpha_H}{1+\alpha_B} \right) \right\}$$

- Stability (neither ghost nor gradient instability)

$$\mathcal{K}_t > 0 \qquad\qquad c_s^2 \equiv -\frac{\mathcal{K}_s}{\mathcal{K}_t} > 0$$

Tensor degrees of freedom

- Quadratic action for the **tensor modes**:

$$S_\gamma^{(2)} = \frac{1}{2}\int dt\, d^3x\, a^3 \left[\frac{M^2}{4}\dot{\gamma}_{ij}^2 - \frac{M^2}{4}(1+\alpha_T)\frac{(\partial_k \gamma_{ij})^2}{a^2}\right]$$

- Stability:

$M^2 > 0$ and $c_T^2 \equiv 1 + \alpha_T > 0$

Extension to DHOST theories

DL, Mancarella, Noui & Vernizzi '1703

- Quadratic action in terms of **9 functions of time**

$$S_{\rm quad} = \int d^3x\, dt\, a^3\, \frac{M^2}{2} \left\{ \delta K_{ij}\delta K^{ij} - \left(1+\frac{2}{3}\alpha_{\rm L}\right)\delta K^2 + (1+\alpha_T)\left(R\frac{\delta\sqrt{h}}{a^3} + \delta_2 R\right) \right.$$

$$\left. + H^2\alpha_K \delta N^2 + 4H\alpha_B \delta K \delta N + (1+\alpha_H) R\delta N + 4\beta_1 \delta K \delta\dot{N} + \beta_2 \delta\dot{N}^2 + \frac{\beta_3}{a^2}(\partial_i \delta N)^2 \right\}$$

9-3=6 independent coefficients

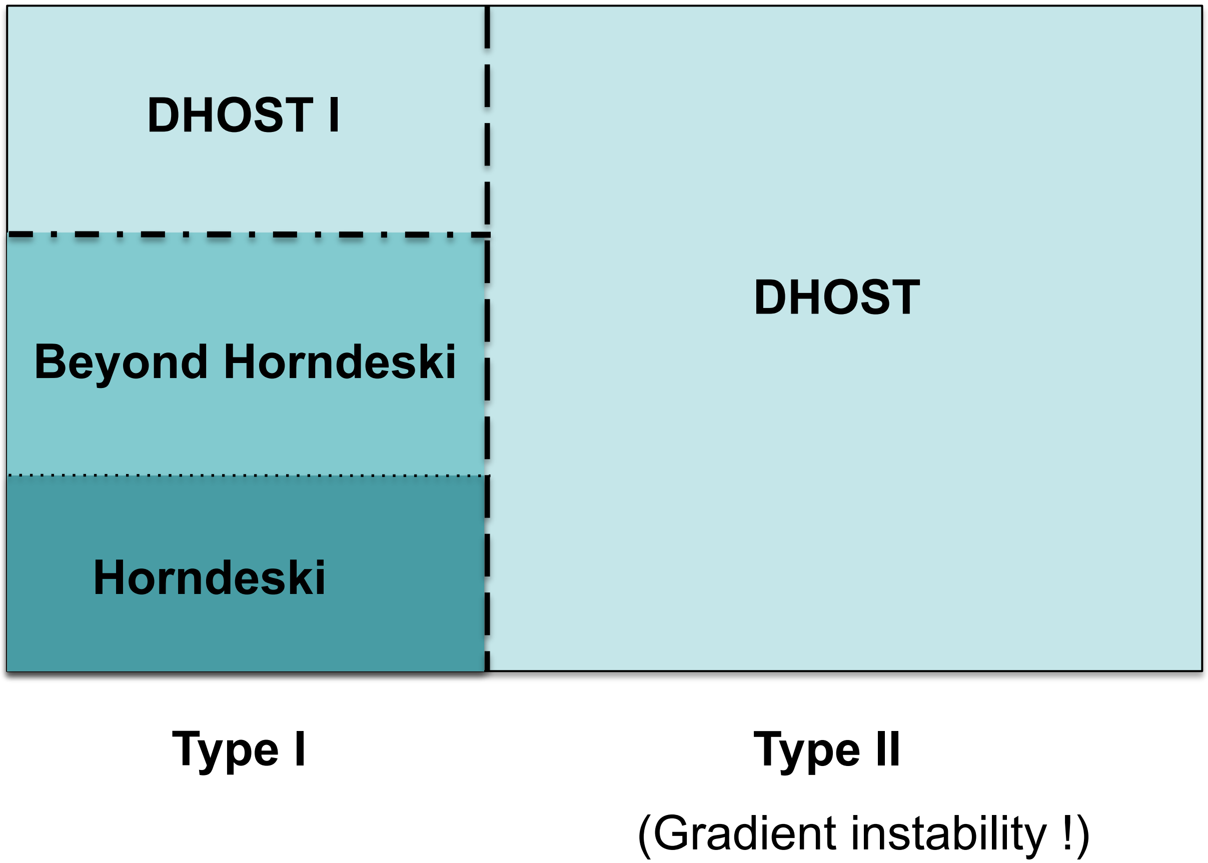
- **Degeneracy conditions:** 2 categories

$$\mathcal{C}_{\rm I}:\ \alpha_{\rm L} = 0\,,\ \beta_2 = -6\beta_1^2\,,\ \beta_3 = -2\beta_1\left[2(1+\alpha_{\rm H}) + \beta_1(1+\alpha_{\rm T})\right]$$

$$\mathcal{C}_{\rm II}:\ \beta_1 = -(1+\alpha_{\rm L})\frac{1+\alpha_{\rm H}}{1+\alpha_{\rm T}}\,,\ \beta_2 = -6(1+\alpha_{\rm L})\frac{(1+\alpha_{\rm H})^2}{(1+\alpha_{\rm T})^2}\,,\ \beta_3 = 2\frac{(1+\alpha_{\rm H})^2}{1+\alpha_{\rm T}}$$

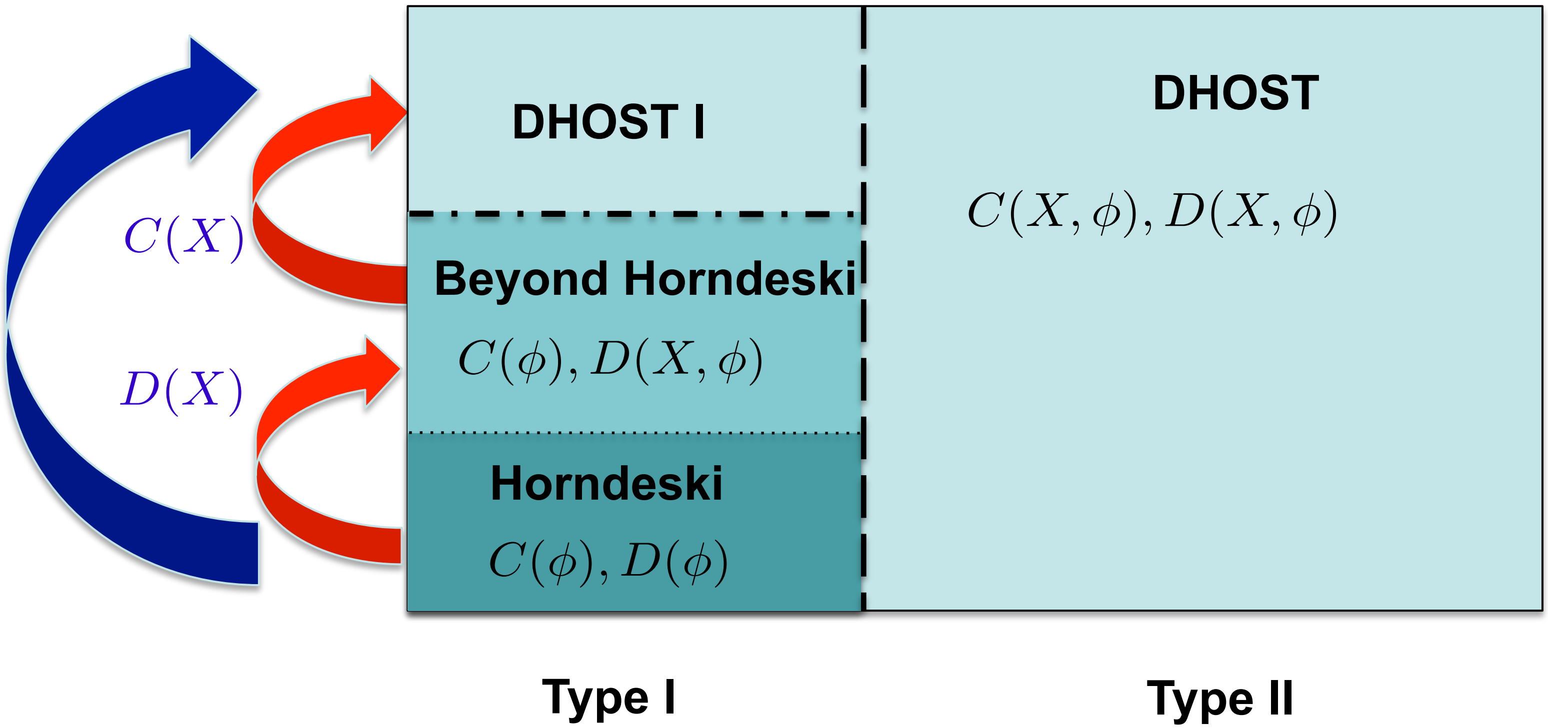
$\mathcal{C}_{\rm II}$: gradient instability either in the scalar or the tensor sector

Scalar-tensor theories



Disformal transformations

- Disformal transformations: $\tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \partial_\mu\phi\, \partial_\nu\phi$



Disformal transformations

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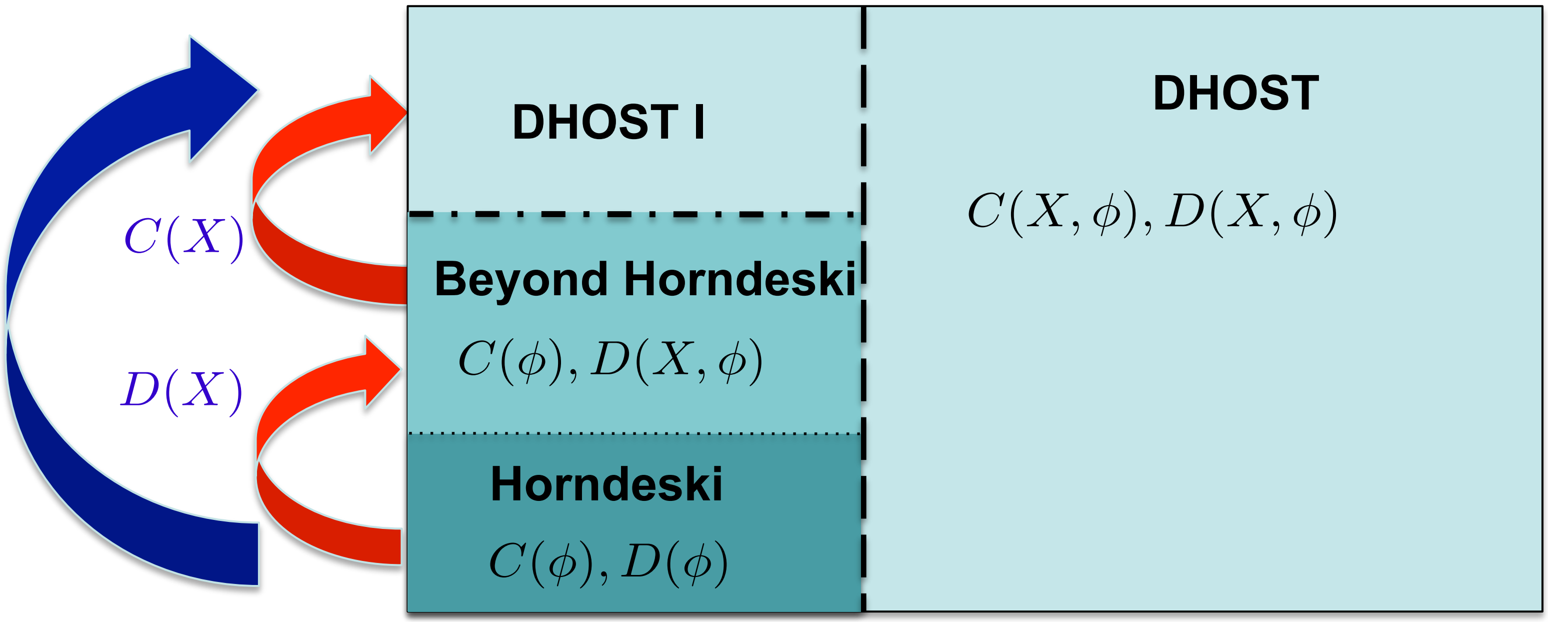

- Mimetic gravity & extensions (non-invertible transf) are DHOST theories of type II (some of type I too) and all **unstable.**

DL, Mancarella, Noui & Vernizzi ’1802 [see also Takahashi & Kobayashi ‘1708]

DHOST theories after GW170817

DHOST theories after GW170817

- Constraint on the speed of gravitational waves:

$$\alpha_T < 10^{-15}$$

- Assuming $\alpha_T = 0$ holds exactly, this implies

1. Quadratic terms: $a_1 = 0$

$$L_{\rm ADM} = (f - X a_1) K_{ij} K^{ij} - f\,^{(3)}R$$

2. No cubic term (for type I theories)

- Remain quadratic DHOST theories of type I with $a_1 = 0$

DHOST theories with c_g= c

- Taking into account the degeneracy conditions,

$$a_1 = a_2 = 0\,,$$
$$a_4 = \frac{1}{8f_2}\left[48 f_{2X}^2 - 8(f_2 - X f_{2X})a_3 - X^2 a_3^2\right]\,,$$
$$a_5 = \frac{1}{2f_2}\left(4f_{2X} + X a_3\right) a_3$$

2 free functions

- **Total Lagrangian**

$$L_{c_g=1}^{\rm DHOST} = f_2(X,\phi)\,{}^{(4)}R + P(X,\phi) + Q(X,\phi)\,\Box\phi$$
$$+ a_3(X,\phi)\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi + a_4(X,\phi)\phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}$$
$$+ a_5(X,\phi)(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2$$

4 free functions of X and ϕ (as in Horndeski without $c_g = 1$!)

Horndeski and Beyond Horndeski with $c_g = c$

- Remaining Beyond Horndeski theories

$$a_1 = 2G_{4X} + XF_4 = 0 \qquad \Longrightarrow \qquad F_4 = -\frac{2}{X} G_{4X}$$

$$S[g,\phi] = \int d^4x \sqrt{-g} \left\{ f(\phi, X)\, R - \frac{4}{X} f_X \left[(\Box\phi)\phi^\mu \phi_{\mu\nu}\phi^\nu - \phi^\mu \phi_{\mu\nu}\phi^{\nu\rho}\,\phi_\rho \right] \right\}$$

- Remaining Horndeski theories

$$G_2(X,\phi)\,, \quad G_3(X,\phi)\,, \quad G_4(\phi)$$

Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui '1711 [see also Crisostomi & Koyama '1711 and Dima & Vernizzi '1712]

- Quasi-static approximation on scales $r \ll H^{-1}$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)\delta_{ij}dx^i dx^j$$

$$\phi = \phi_c(t) + \chi(r)$$

- Equations of motion for χ, Φ and Ψ
  - Scalar equation
  - Metric equations

- Matter source: spherical body with density $\rho(r)$

Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui '1711 [see also Crisostomi & Koyama '1711 and Dima & Vernizzi '1712]

- Gravitational laws

$$\mathcal{M}(r) \equiv 4\pi \int_0^r \bar{r}^2 \rho(\bar{r}) \mathrm{d}\bar{r}$$

$$\frac{d\Phi}{dr} = \frac{G_\mathrm{N}\,\mathcal{M}(r)}{r^2} + \Xi_1\, G_\mathrm{N}\,\mathcal{M}''(r)\,,$$

$$\frac{d\Psi}{dr} = \frac{G_\mathrm{N}\,\mathcal{M}(r)}{r^2} + \Xi_2 \frac{G_\mathrm{N}\,\mathcal{M}'(r)}{r} + \Xi_3\, G_\mathrm{N}\,\mathcal{M}''(r)$$

with $(8\pi G_\mathrm{N})^{-1} \equiv 2f\,(1+\Xi_0)$

where the coefficients Ξ_I are given in terms of f, f_X, a_3 and $\dot{\phi}_c$

- Breaking of the Vainshtein screening **inside matter** !

already noticed for Beyond Horndeski (GLPV) Kobayashi, Watanabe & Yamauchi '14

Gravitation in DHOST with $c_g = c$

- The four coefficients Ξ_I depend on only 2 parameters

$$\Xi_0 = -\alpha_{\rm H} - 3\beta_1 \, , \qquad \Xi_1 = -\frac{(\alpha_{\rm H} + \beta_1)^2}{2(\alpha_{\rm H} + 2\beta_1)} \, ,$$

$$\Xi_2 = \alpha_{\rm H} \, , \qquad \Xi_3 = -\frac{\beta_1(\alpha_{\rm H} + \beta_1)}{2(\alpha_{\rm H} + 2\beta_1)} \, .$$

- Constraints on the coefficients

$$\Xi_0 = \frac{G_{\rm gw}}{G_{\rm N}} - 1$$

Hulse-Taylor binary pulsar: $|\Xi_0| < 10^{-2}$

Beltran Jimenez, Piazza & Velten 1507

Stars: $-\frac{1}{12} < \Xi_1 \lesssim 0.2$

[Saito, Yamauchi, Mizuno, Gleyzes & DL '15]

[Sakstein 15]

Gravitational lensing for the other coefficients…

Conclusions

- **DHOST theories** provide a very general framework to describe scalar-tensor theories with higher derivatives.

  Systematic classification of "**degenerate**" theories that contain a single scalar DOF. They include and extend Horndeski and "beyond Horndeski" theories as particular cases.

- **Drastic reduction of viable models after GW170817.**

- These theories of modified gravity can be tested & constrained via cosmology (future LSS observations) and astrophysics.