CONFORMAL SYMMETRY IN STANDARD MODEL AND GRAVITY

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Stefano Lucat and T. Prokopec, arXiv:1705.00889 [gr-qc]; 1709.00330 [gr-qc];1606.02677 [hep-th]
T. Prokopec, Leonardo da Rocha, Michael Schmidt, Bogumila Swiezewskae-Print: arXiv:1801.05258 [hep-ph] +

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CONTENTS

PHYSICAL MOTIVATION

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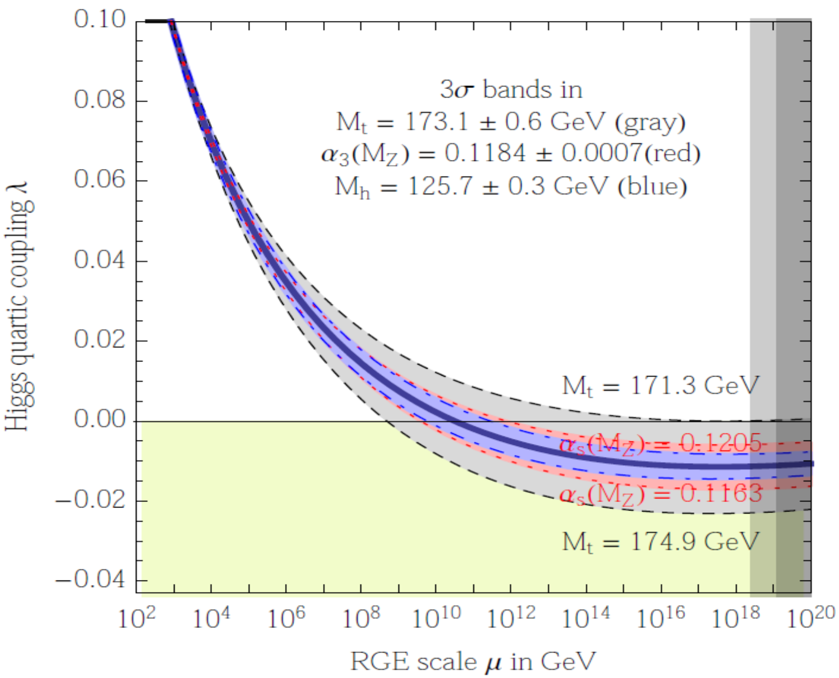
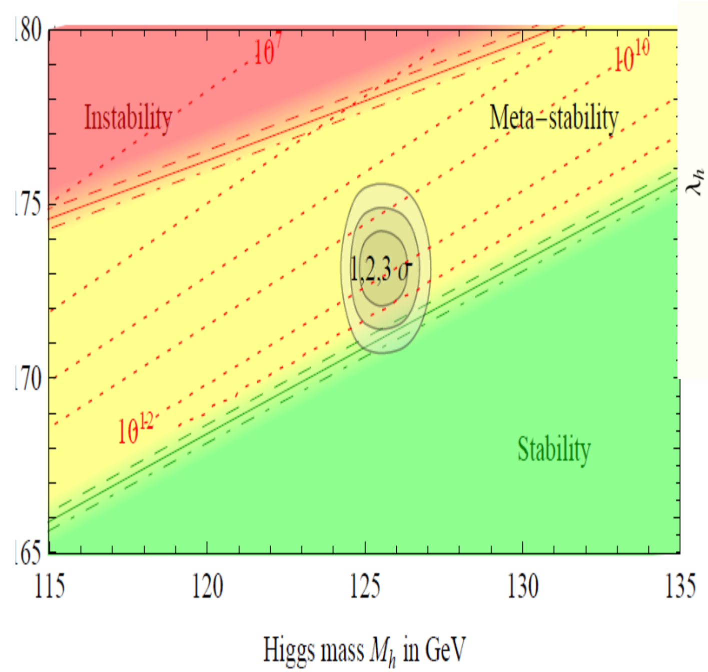
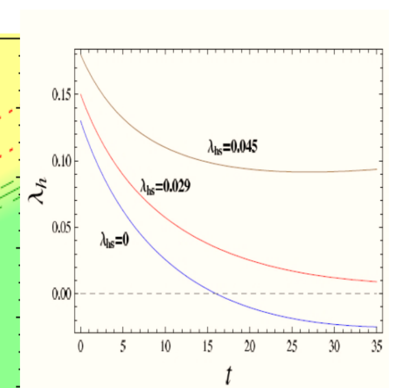
- AT LARGE ENERGIES THE STANDARD MODEL IS ALMOST CONFORMALLY INVARIANT.
- HIGGS MASS AND KINETIC TERMS BREAK THE SYMMETRY
- OBSERVED HIGGS MASS: $m_H = 125.3\text{GeV}$ is close to the stability bound
- STABILITY BOUND: $m_H \approx 130\text{GeV}$: CAN BE ATTAINED BY ADDING SCALAR

Oleg Lebedev, e-Print: arXiv:1203.0156 [hep-ph]







Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 1205.6497 [hep-ph]

THEORETICAL MOTIVATION

THEORETICAL MOTIVATION IN SM

- HIGGS MASS TERM RESPONSIBLE FOR GAUGE HIERARCHY PROBLEM

- IF WE COULD FORBID IT BY SYMMETRY, THE GHP WOULD BE SOLVED

- THIS SYMMETRY COULD BE WEYL SYMMETRY IMPOSED CLASSICALLY

- HIGGS MASS COULD BE GENERATED DYNAMICALLY BY THE COLEMAN-WEINBERG (CW) MECHANISM

THEORETICAL MOTIVATION IN GRAVITY

- THE SYMMETRY IS BROKEN BY THE NEWTON CONSTANT AND COSMOLOGICAL TERM, G & Λ.

- G & Λ ARE RESPONSIBLE FOR GRAVITATIONAL HIERARCHY PROBLEM.

- SCALAR DILATON & CARTAN TORSION CAN RESTORE WEYL SYMMETRY IN CLASSICAL GRAVITY.

- G & Λ CAN BE GENERATED BY DILATON CONDENSATION INDUCED BY QUANTUM EFFECTS akin to THE COLEMAN-WEINBERG MECHANISM.

- IF GRAVITY IS CONFORMAL IN UV, IT MAY BE FREE OF SINGULARITIES (BOTH COSMOLOGICAL AND BLACK HOLE).

WEYL SYMMETRY IN CLASSICAL GRAVITY

CARTAN EINSTEIN THEORY

- POSITS THAT FERMIONS (& SCALARS) SOURCE SPACETIME TORSION.
- TORSION IS CLASSICALLY A CONSTRAINT FIELD (NOT DYNAMICAL, DOES NOT PROPAGATE)

$\Rightarrow$ CARTAN EQUATION CAN BE INTEGRATED OUT, RESULTING IN THE KIBBLE-SCIAMA THEORY

Lucat, Prokopec, e-Print: arXiv:1512.06074 [gr-qc]

$\Rightarrow$ THIS THEORY PROVIDES ADDITIONAL SOURCE TO STRESS-ENERGY, WHICH CAN CHANGE BIG-BANG SINGULARITY TO A BOUNCE.

- CARTAN-EINSTEIN THEORY CAN BE MADE CLASSICALLY CONFORMAL!

Lucat & Prokopec, arxiv:1606.02677 [hep-th]

CLASSICAL WEYL SYMMETRY

- WEYL TRANSFORMATION ON THE METRIC TENSOR

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\theta(x)} g_{\mu\nu} \qquad \mathrm{d}\tau \to \mathrm{d}\tilde{\tau} = e^{\theta(x)} \mathrm{d}\tau$$

- GENERAL CONNECTION Γ, TORSION TENSOR T, CHRISTOFFEL CON $\mathring{\Gamma}$

$$\Gamma^{\lambda}{}_{\mu\nu} = T^{\lambda}{}_{\mu\nu} + T_{\mu\nu}{}^{\lambda} + T_{\nu\mu}{}^{\lambda} + \mathring{\Gamma}^{\lambda}{}_{\mu\nu}$$

$\delta\mathring{\Gamma}^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{(\alpha}\partial_{\beta)}\theta,$ ASSUME: $\delta\Gamma^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{\alpha}\partial_{\beta}\theta \;\Rightarrow\; \delta T^{\mu}{}_{\alpha\beta} = \delta^{\mu}{}_{[\alpha}\partial_{\beta]}\theta$

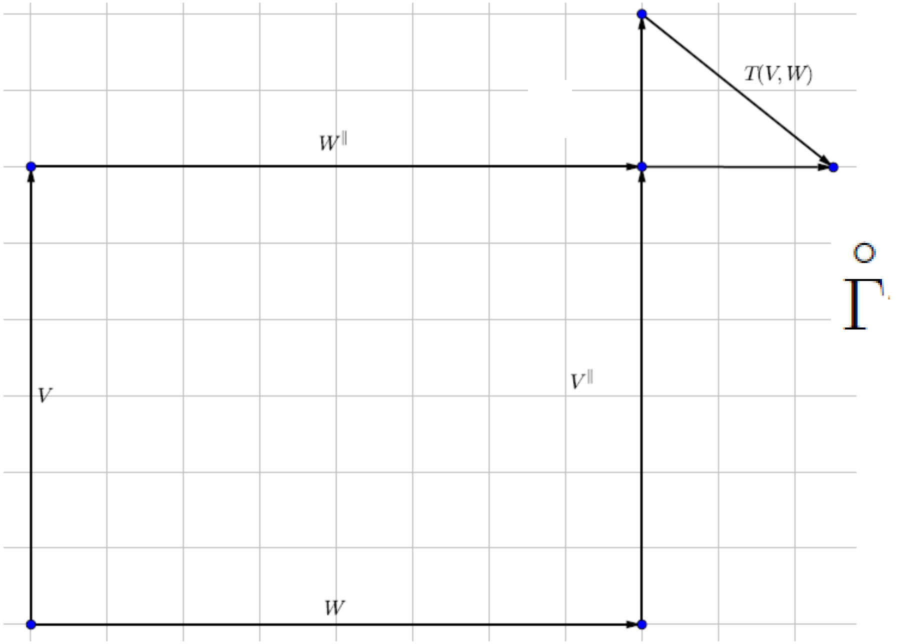
- RIEMANN TENSOR IS INVARIANT: $\delta R^{\alpha}{}_{\beta\gamma\delta} = 0$
- THIS IMPLIES THAT THE VACUUM EINSTEIN EQUATION IS WEYL INV:

$$G_{\mu\nu} = 0, \qquad \delta G_{\mu\nu} = 0$$

GEOMETRIC VIEW OF TORSION

- (VECTORIAL) TORSION TRACE 1-FORM:

$$\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$$



- TRANSFORMS AS A VECTOR FIELD:

$$\mathcal{T} \to \mathcal{T} + \mathrm{d}\theta$$

- WHEN A VECTOR IS PARALLEL-TRANSPORTED, TORSION TRACE INDUCES A LENGTH CHANGE: CRUCIAL IN WHAT FOLLOWS

PARALLEL TRANSPORT AND JACOBI EQUATION

- GEODESIC EQUATION:

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau}\equiv\frac{dx^\lambda}{d\tau}\nabla_\lambda\frac{dx^\mu}{d\tau}=0$$

$$\Gamma^\lambda{}_{\mu\nu}=T^\lambda{}_{\mu\nu}+T_{\mu\nu}{}^\lambda+T_{\nu\mu}{}^\lambda+\mathring{\Gamma}^\lambda{}_{\mu\nu}$$

$\mathring{\Gamma}$ = LEVI-CIVITA

$$T[X,Y]=-\frac{1}{2}(\nabla_X Y-\nabla_Y X-[X,Y])$$

$$T^\lambda{}_{\mu\nu}=\Gamma^\lambda{}_{[\mu\nu]}=\frac{1}{2}\left(\Gamma^\lambda{}_{\mu\nu}-\Gamma^\lambda{}_{\nu\mu}\right)$$

→ TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$)

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau}=0\Rightarrow e^{-2\theta(x)}\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau}=0$$

NB: TRANSFORMATION OF $d\tau$ COMPENSATED BY TRANSFORMATION OF Γ !

- JACOBI EQUATION (JACOBI FIELDS $J\perp\dot\gamma$) AND RAYCHAUDHURI EQ:

$$\nabla_{\dot\gamma}\nabla_{\dot\gamma}J+2\nabla_{\dot\gamma}T[\dot\gamma,J]=R[\dot\gamma,J]\dot\gamma$$

→ ALSO TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$) UNDER WEYL TR

- SUGGESTS TO DEFINE A GAUGE INVARIANT PROPER TIME:

$(d\tau)_{g.i.}=\exp\left(-\int_{x_0}^{x}T_\mu dx^\mu\right)d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS!

T. Prokopec, het Lam, e-Print: arXiv:1606.01147

CONFORMAL SYMMETRY AND OBSERVATIONS

CONFRONTING OBSERVATIONS

$$\Gamma_{EFF} \supset -\int d^4x\sqrt{-g}\,\{[\alpha(\mu)\bar{R}^2 + \beta(\mu)\phi^2 R] + \gamma(\mu)T_{\alpha\beta}\,T^{\alpha\beta}\}$$

► $T_{\alpha\beta}$=TORSION (TRACE) FIELD STRENGTH

EARLY COSMOLOGY

- INFLATIONARY MODELS GENERATED BY CONDENSATION OF SCALARON, DILATON OR TORSION TRACE MAY HAVE SPECIFIC FEATURES.
- PRELIMINARY RESULTS: CAN GET (quasi)de SITTER UNIVERSE AND NEARLY SCALE INVARIANT SCALAR SPECTRUM.
- STRONG 1st ORDER EW PT $\Rightarrow$ GW PRODUCTION & BARYOGENESIS

J. REZACEK, B. SWIEZEWSKA and T. PROKOPEC, in progress

LATE COSMOLOGY

- CAN BE TESTED BY STUDYING e.g. DARK ENERGY AND STRUCTURE FORMATION, POSSIBLY DARK MATTER CANDIDATE.
- TORSION TRACE (AND MIXED TORSION) CAN BE DETECTED BY CONVENTIONAL GRAVITATIONAL WAVE DETECTORS

Stefano Lucat and T. Prokopec, arXiv:1705.00889 [gr-qc]

GRAVITATIONAL DETECTORS

GRAVITATIONAL WAVES

- GRAVITATIONAL WAVES

$$\frac{d^2 J^i}{dt^2} = \frac{1}{2}\ddot{h}_{ij}(t, \vec{x}) J^j$$

$$\text{Plus polarization: } h_{xx} = -h_{yy} = h_+ \cos(\omega t - kz)$$

$$J^x(t, z) = J^x_{(0)}\left[1 + (h_+/2)\cos(\omega t - kz)\right]$$

$$\text{Cross polarization: } h_{xy} = h_{yx} = h_\times \cos(\omega t - kz)$$

$$J^x(t, z) = J^x_{(0)} + (h_\times/2) J^y_{(0)} \cos(\omega t - kz)]$$

DETECTORS FOR TORSION WAVES

► GW INTEFEROMETERS such as aLIGO/VIRGO

- TORSION TRACE

$$\ddot{J}^i = J^0 \dot{\mathcal{T}}^i + J^j \partial_j \mathcal{T}^i \qquad \mathcal{T}^i = \mathcal{T}^i_{(0)} \cos(\omega t - kz)$$

► LONGITUDINAL $\mathcal{T}^i_{(0),L} = \delta^i_z \frac{\omega}{m}\,, \ \mathcal{T}^0_{(0),L} = -\frac{\|\vec{k}\|}{m}$

○ DETECTOR RESPONSE

$$\Delta J^z_{(0)} = 0\,, \qquad \Delta J^{x,y}_{(0)} = -\frac{c^2 k}{\omega^2} \mathcal{T}^{x,y}_{(0),T} J^z_{(0)} \approx -\frac{c}{m} \mathcal{T}^{x,y}_{(0),T} J^z_{(0)}$$

► TRANSVERSE $\mathcal{T}^i_{(0),T} = \frac{1}{\sqrt{2}} \left(\delta^i_x \pm \delta^i_y\right)\,, \ \mathcal{T}^0_{(0),T} = 0$

○ DETECTOR RESPONSE

$$\Delta J^z_{(0)} = -\frac{c^2 k}{\omega^2} \mathcal{T}^z_{(0),L} J^z_{(0)} \approx -\frac{c}{\omega} \mathcal{T}^z_{(0),L} J^z_{(0)}\,, \qquad \Delta J^{x,y}_{(0)} = 0\,.$$

- GRAVITATIONAL WAVES vs TORSION WAVES: a comparsion

► PHASE SHIFT ¼ PERIOD

► FREQUENCY DEPENDENCE

► TORSION TRACE (L) COUPLES TO

TORSION SOURCES

- E.G.: TORSION TRACE: LONGITUDINAL MODE $\mathcal{T}_\mu = \partial_\mu \theta$
  - ► ITS MASS IS PROTECTED BY THE CONFORMAL WARD=TAKAHASHI, (see talk of Stefano Lucat)

$$\Box\theta = \frac{8\pi G_N}{c^4}\frac{T^\mu_\mu}{6}\,,\ \Box h_{ij} = \frac{8\pi G_N}{c^4} T_{ij}$$

  - ► THIS IMPLIES ABOUT 1 order of magnitude suppression when compared with the amplitude of gravitational waves, i.e.

$$\frac{\theta}{h_{ij}} \sim \frac{e^2}{2}$$

    - ○ e=sources excentricity (can be 0.5)

  - ► DETECTABLE BY THE NEXT GENERATION OF OBSERVATORIES such as EINSTEIN TELESCOPE.

CONCLUSIONS AND OUTLOOK

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- CHALLENGE 1: USE FRG METHODS TO STUDY HOW THIS THEORY DIFFERS FROM THE USUAL GRAVITY, i.e. WHETHER IT IS ASYMPTOTICALLY SAFE/ADMITS UV COMPLETION.

- CHALLENGE 2: CONFRONT THIS THEORY AS MUCH AS POSSIBLE WITH OBSERVATIONS

- CHALLENGE 3: CAN WE GET RID OF (COSMOLOGICAL AND BLACK HOLE) SINGULARITIES?

HINT: RECALL: $(d\tau)_{g.i.} = \exp\left(-\int_{x_0}^{x} T_\mu dx^\mu\right) d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS