Holographic Screens in Flat Spacetime

— Towards Holographic Dark Matters in Late Universe

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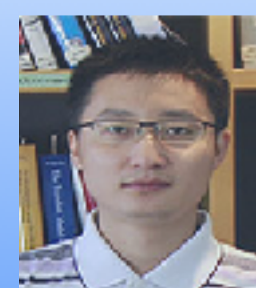
Rindler Fluid
1705.05078
[JHEP, 1801]



S. Khimphun, **B.-H. Lee** (Sogang), **C.-Y. Park** (APCTP)

de-Sitter Fluid
1712.09326

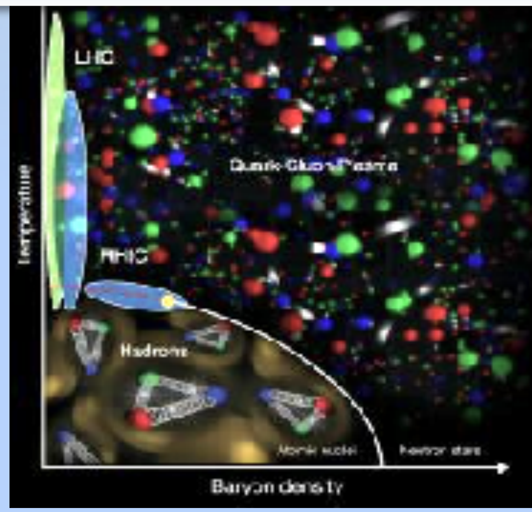




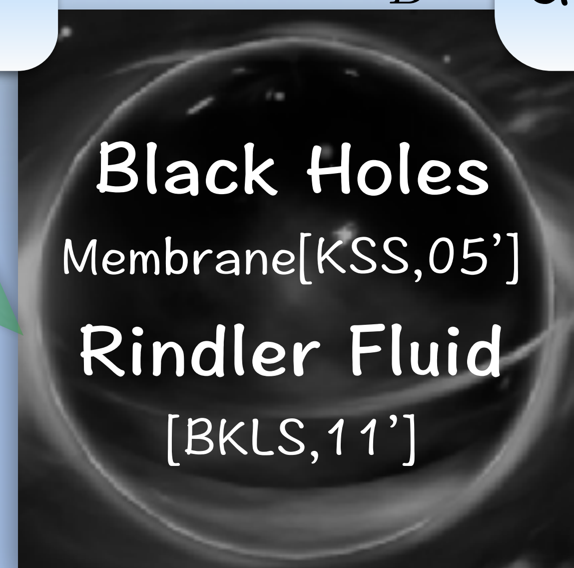
R.-G. Cai (ITP-CAS), **S.-C. Sun** (NTU), **Y.-L. Zhang** (APCTP)

Motivations: What is the Most Perfect Fluid in the World?

Quark Gluon Plasma
in RHIC ['08] & LHC ['16]

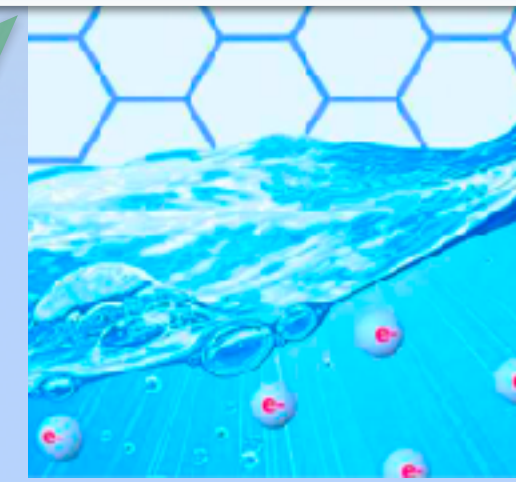


$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$



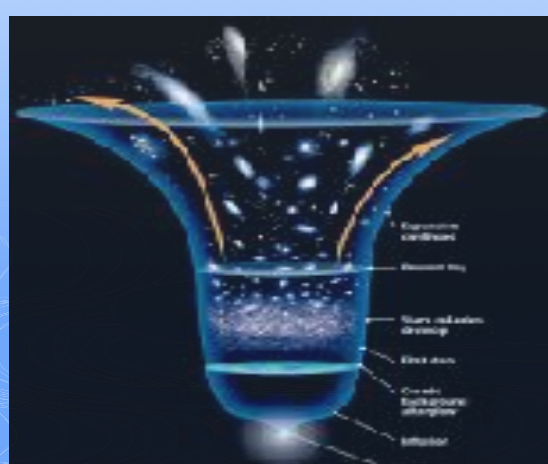
Black Holes
Membrane[KSS,05']
Rindler Fluid
[BKLS,11']

Quantum Critical Liquid
Graphene ['09] & Semi-Metal['16]

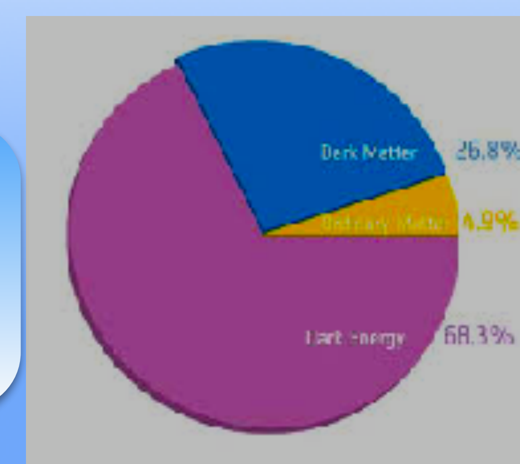


$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$



Cosmological Fluid [CSZ,'17]
Dark Matter & Dark Energy



Figures Credit: RHIC & Google

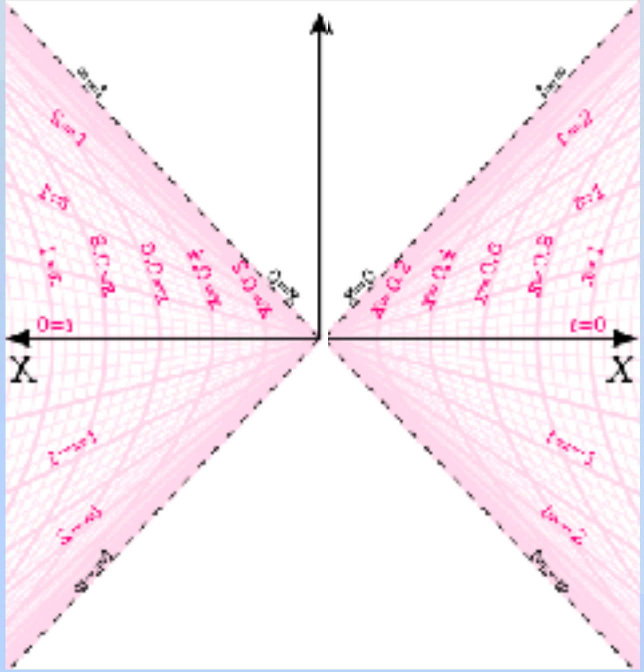
Holographic Screens in Flat Spacetime

— Rindler Screen & de-Sitter Screen

I. Black Holes & Rindler Fluid

— Accelerating Screen

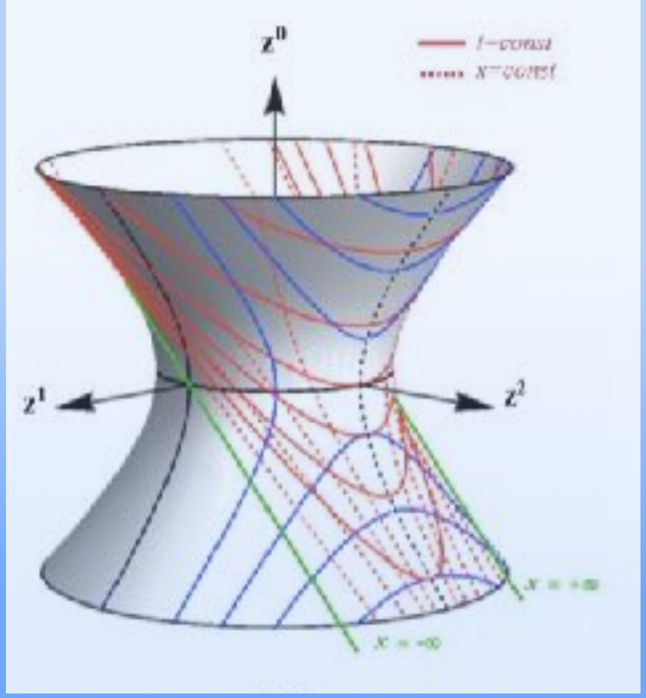
— Relation to AdS/CFT



II. Dark Matter Fluid & dS Membrane

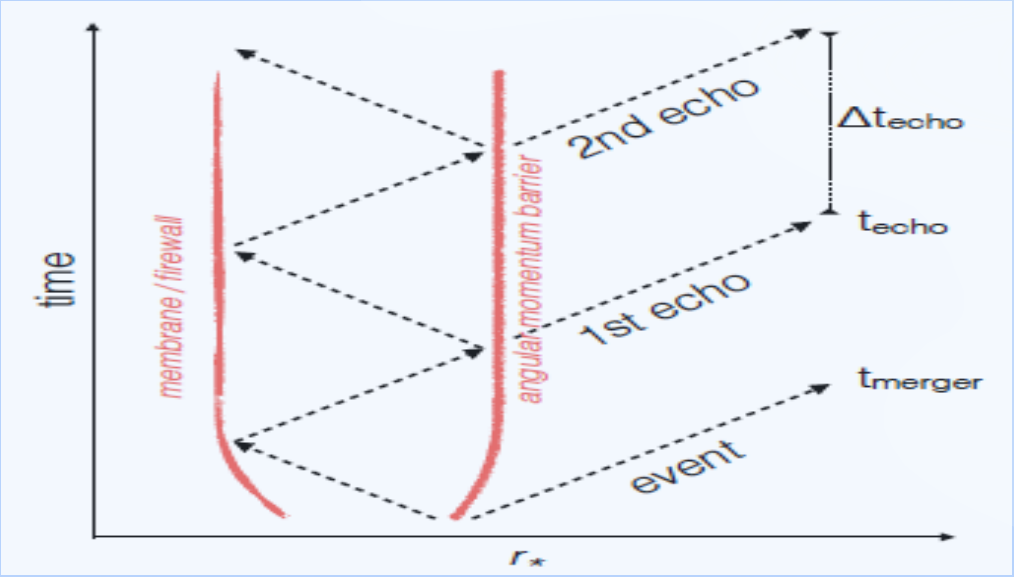
— de-Sitter & FRW Screens

— Relation to DGP Brane-world



Membrane paradigm(1980s): Effective Fluid

T. Doumer & K. Thorne, ⋯

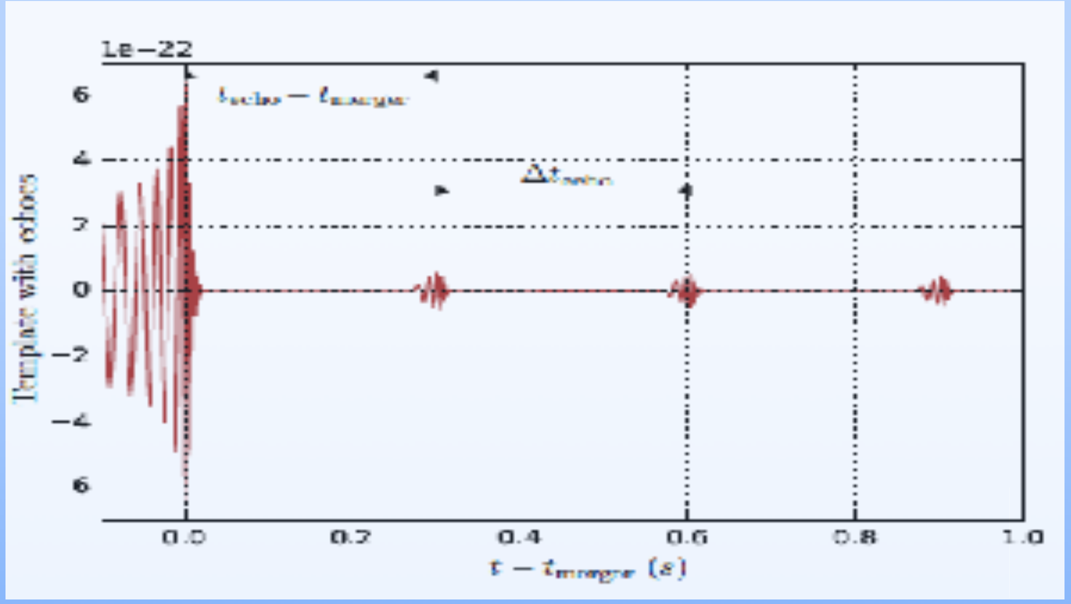


Effective Description

$$\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab})$$

Membrane on Stretched horizon

Viscosity & Conductivity



Echoes from the Abyss [1612.00266 PRD'17]

Holographic Properties of Gravity

(2010s) Gravity/Entanglement: Effective Metric

(2000s) AdS/CFT Duality: Black Hole in a Box

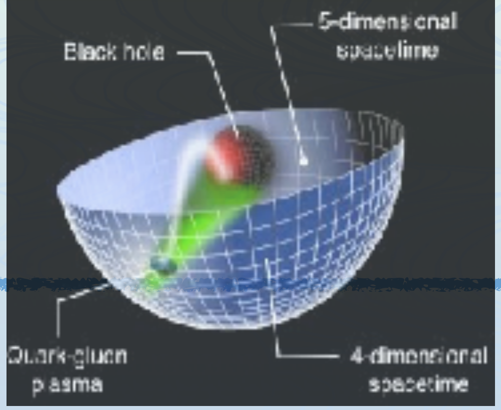
Astrophysical Black Holes

(1990s) Holographic Principle: Horizon Encoding

(1980s) Membrane Paradigm: Effective Fluid

(1970s) Hawking Radiation: Thermodynamics

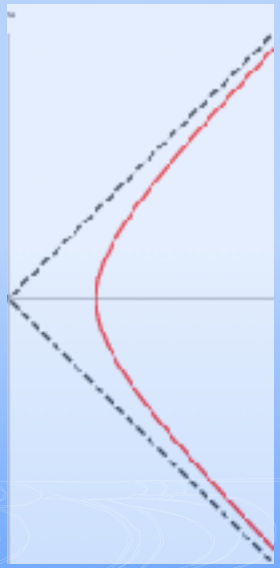
Motivations for the Accelerating Screen

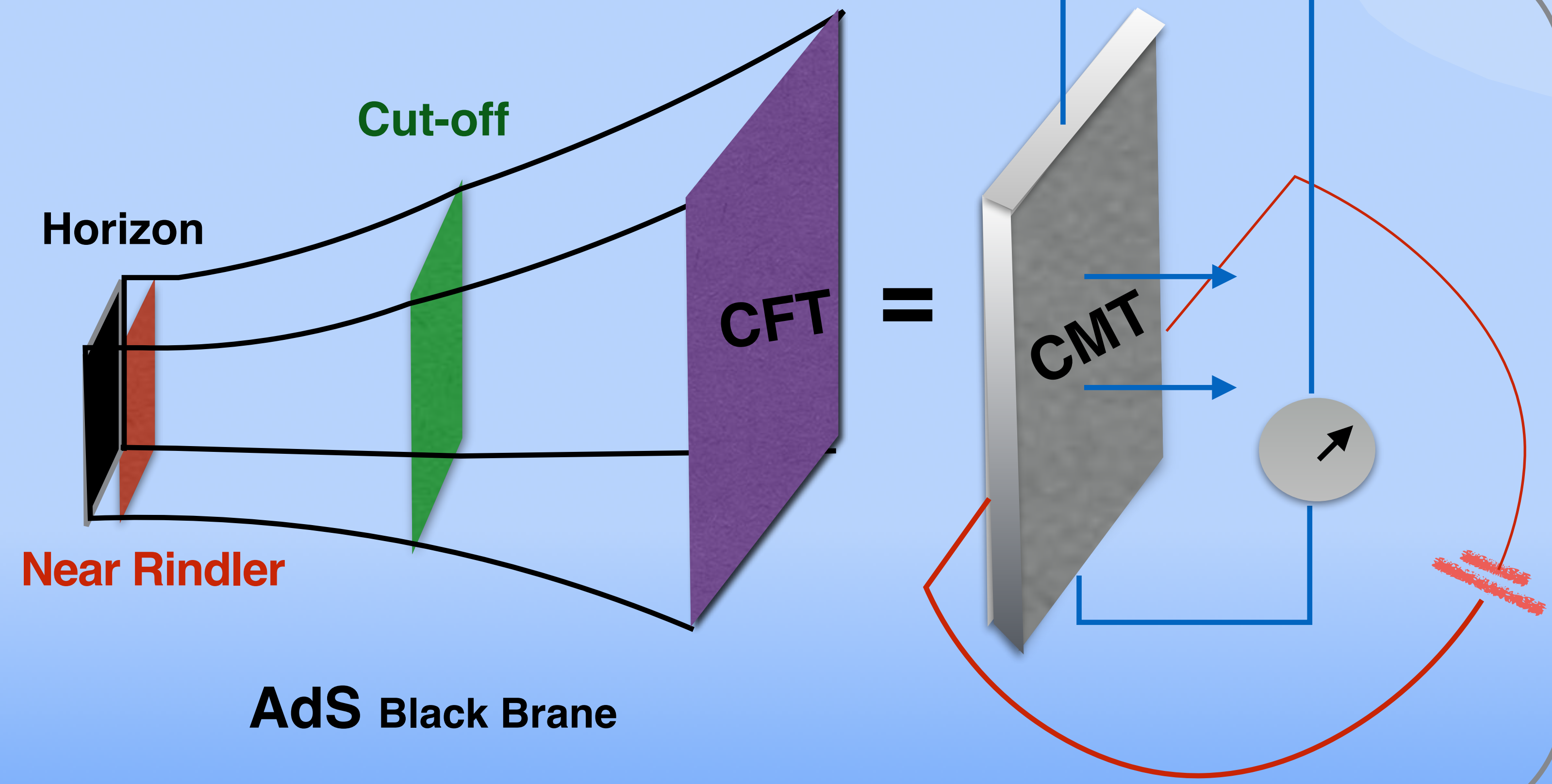


Extremal Charged BH

$AdS_2/CFT_1 \times R_p$ & Non-Fermi Liquid ← Near Horizon ← **Cutoff AdS / Effective CMT?** → Near Boundary → **AdS /CFT&CMT**

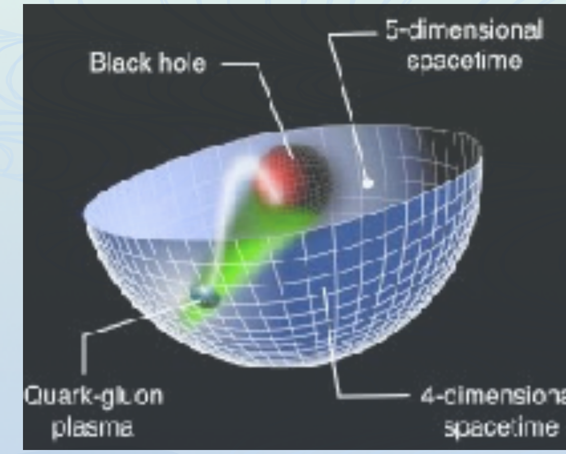
Cutoff AdS / Effective CMT? → Finite Temperature → **Rindler Space/ Special CMT** → **Membrane Black Holes**

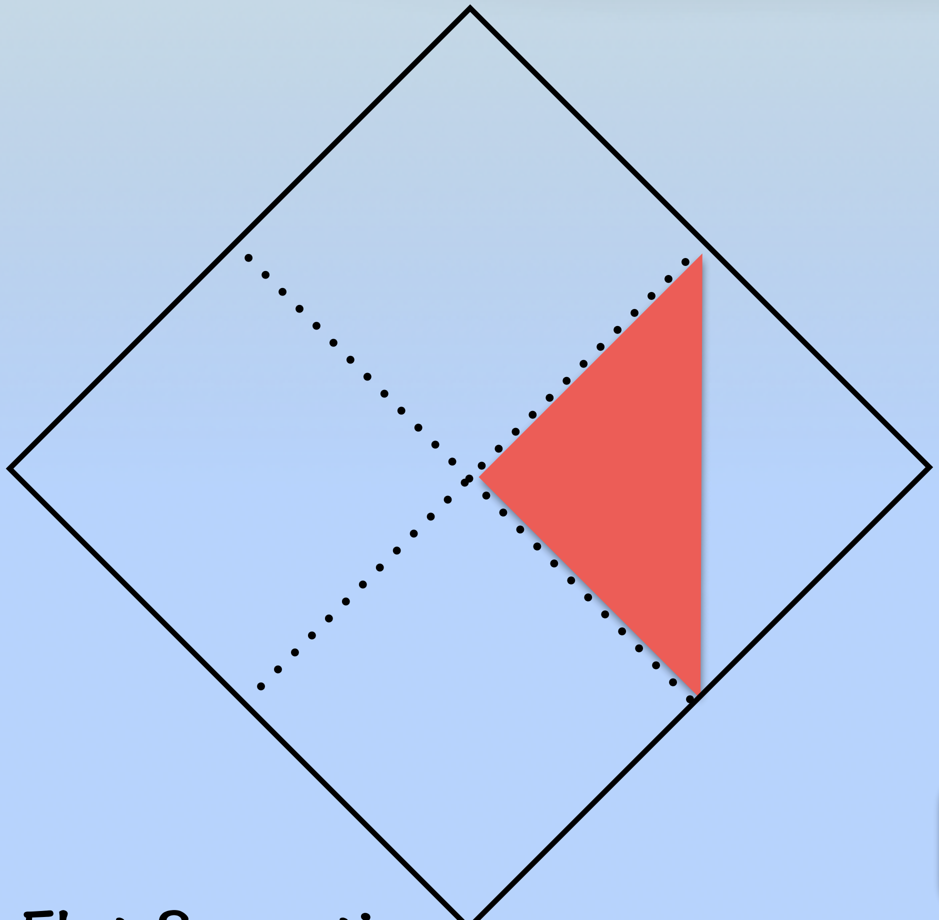




Wilsonian Approach to Fluid/Gravity Duality [Bredberg, Keeler, Lysov, Strominger, '11]

From AdS/CFT to Holographic Rindler Fluid
— with an Accelerating Screen





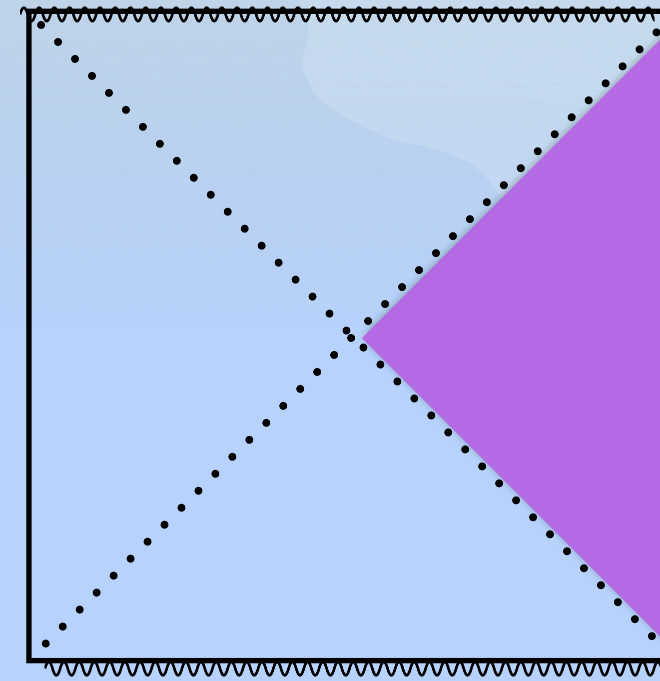
Flat Spacetime

Holographic Screen

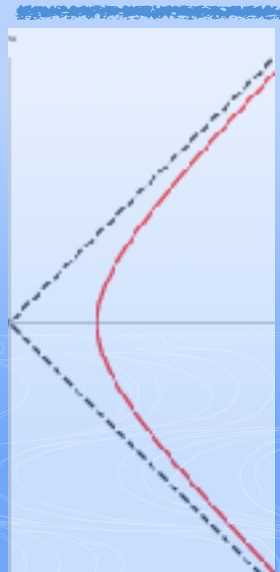
The Time-like boundary

What is Rindler Fluid?

Fluid dual to Rindler spacetime



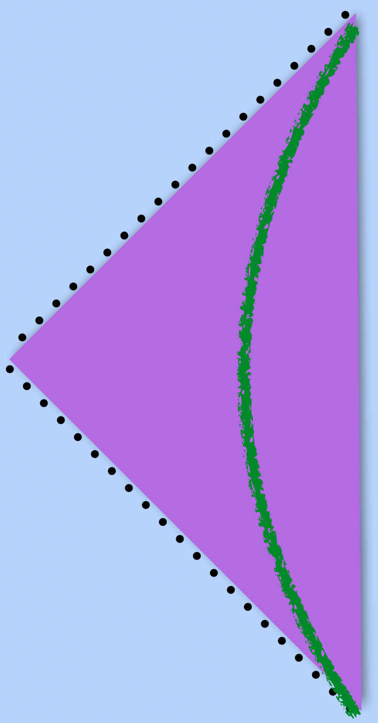
AdS Spacetime



Navier-Stokes Equations:	Bredberg, Keeler, Lysov, Strominger ['10,'11]
Fluid/Gravity Expansion:	Compere, McFadden, Skenderis, Taylor ['11,'12]
Entropy Current and Constraint:	Chirco, Eling, Liberati, Meyer, Oz ['12,'13]
Comparison with AdS/Fluid:	Matsuo, Natsuume, Ohta, Okamura ['12,13]
Rindler Fluid and Recurrence Relation	Cai, Li, Yang, Zhang ['13,'14]
Rindler Fluid with Momentum Relaxtion	Khimphun, Li, Park, Zhang ['17]

Cutoff AdS Fluid with Momentum Relaxation

Rindler Fluid ← Near Horizon — **Cutoff AdS/Fluid** — Near Boundary → **AdS/CFT Fluid**



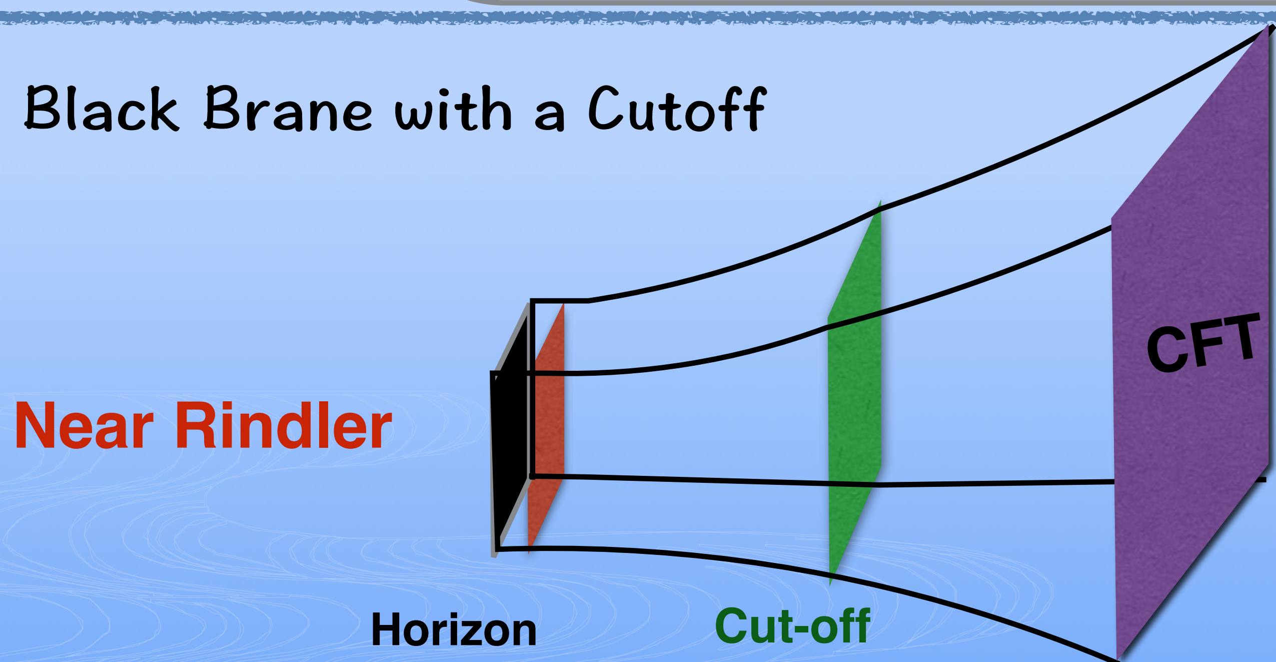
Ward Identity

$$\partial_t \langle \mathcal{T}^t{}_i \rangle + \partial_i \tilde{\mathcal{P}} = -\bar{\tau}_c^{-1} \langle \mathcal{T}^t{}_i \rangle - \ell_c k^2 \partial_t v_i + \cdots \,, \qquad \bar{\tau}_c^{-1} = \frac{k^2 \tilde{s}_c}{4\pi(\tilde{\mathcal{E}} + \tilde{\mathcal{P}})} .$$

Thermal Conductivity & Relaxation Rate

$$\tilde{\kappa}_\omega = \frac{1}{1 - \mathbb{i}\omega\tau_c} \frac{4\pi \tilde{s}_c \tilde{T}_c}{k^2}, \qquad \tau_c^{-1} = \frac{k^2}{4\pi \tilde{T}_c} \left[1 - \frac{\ell_c T_c}{s_c} \frac{k^2}{T_c^2} \right] + O(k^6).$$

AdS Black Brane with a Cutoff



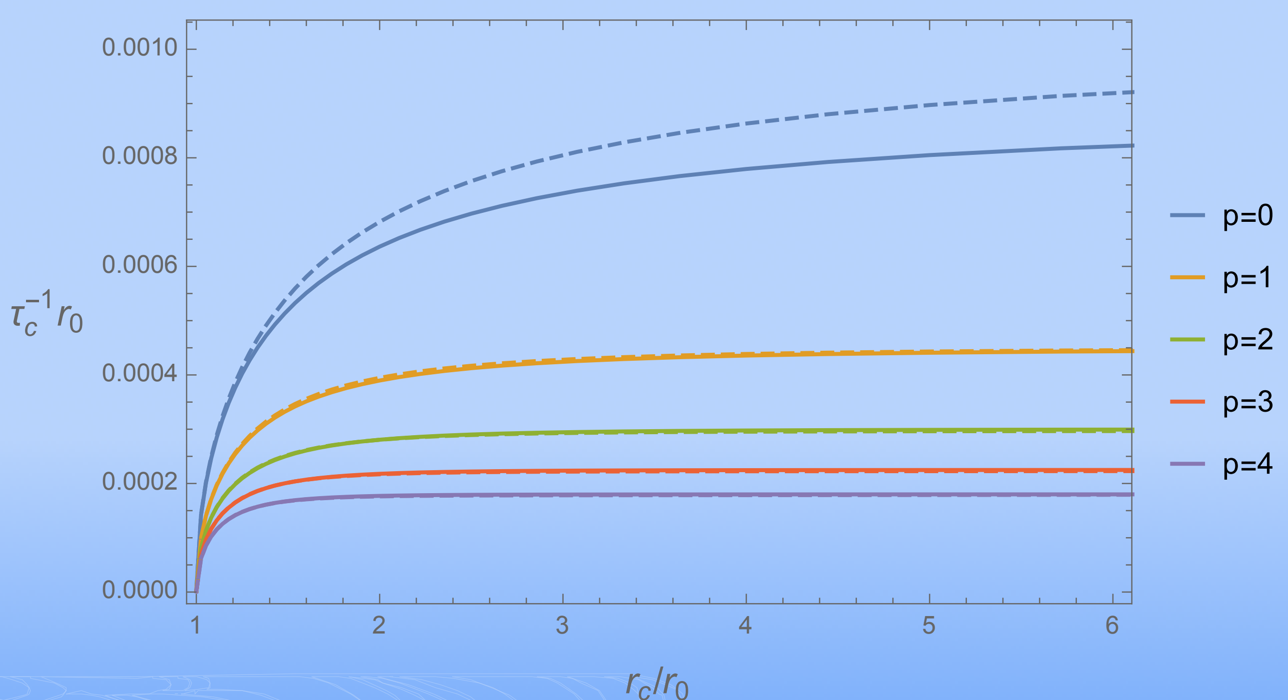
Sub-Leading Correction

$$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1) \left[\tilde{\xi}_p(r_c) - \frac{r_c \tilde{\xi}_p'(r_c)}{(p-1)} \right] ,$$

Running From Conformal Fluid to Rindler Fluid

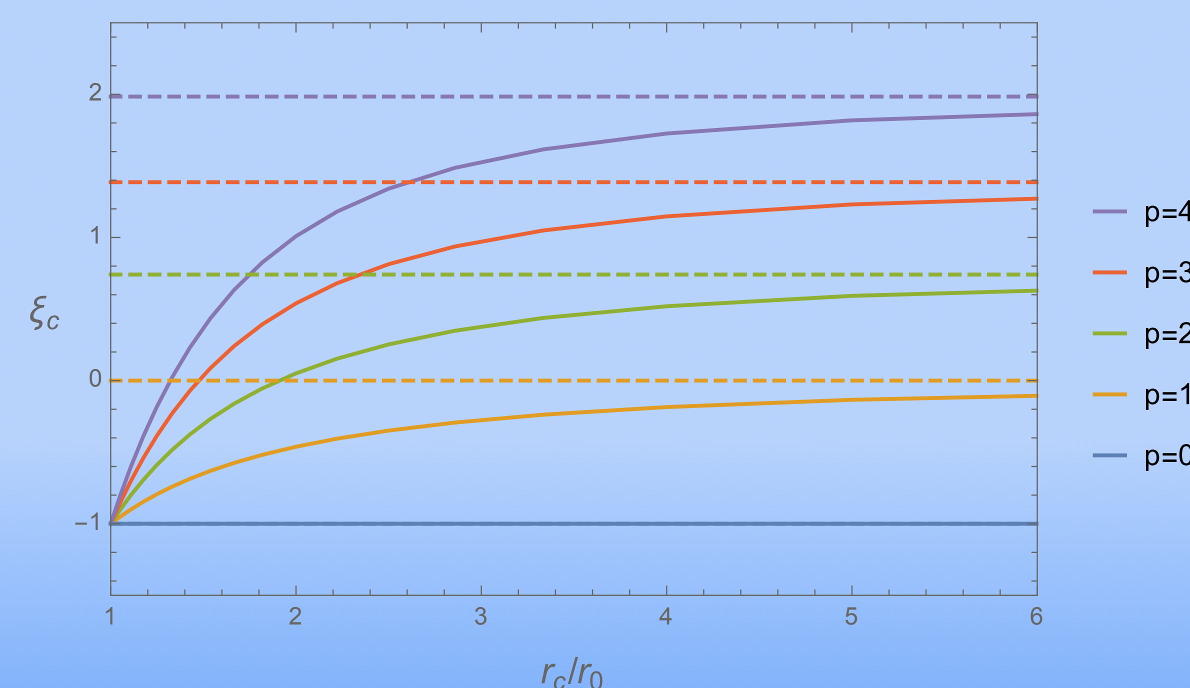
$$S_{\rm Rindler} = S_{\rm CFT} - S_{\rm AdS}|^{\infty}_{r_0+\epsilon} \qquad S_{\rm Cutoff} = S_{\rm CFT} - S_{\rm AdS}|^{\infty}_{r_c} \qquad S_{\rm AdS} = S_{\rm CFT}$$

Momentum Relaxation Rate



$$\tau_c^{-1} = \frac{k^2}{4\pi \tilde{T}_c}\left(1 - \xi_c \frac{k^2}{\tilde{T}_c^2}\right), \qquad \xi_c = \frac{\ell_c T_c}{s_c}.$$

Sub-leading Corrections

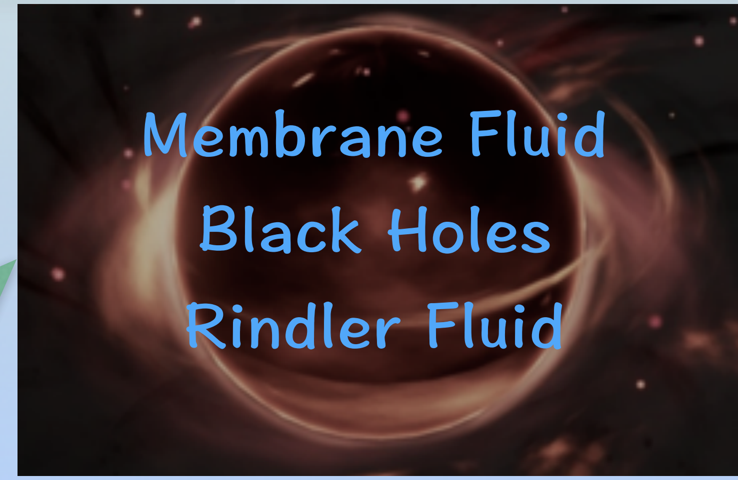


$$\tilde{\xi}_p(r) \equiv \int_{r_0}^{r} \frac{d\tilde{r}\, r_0^2}{\tilde{r}^3 f(\tilde{r})}\left(1 - \frac{r_0^{p-1}}{\tilde{r}^{p-1}}\right).$$

Universal Holographic Properties of Horizon

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$

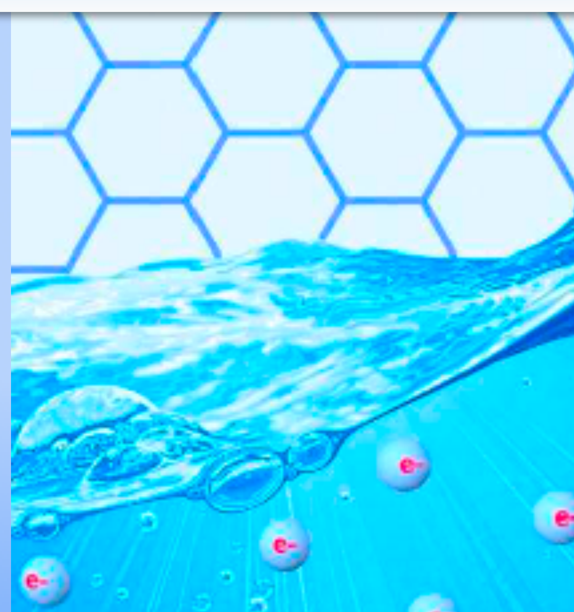


$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$

$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

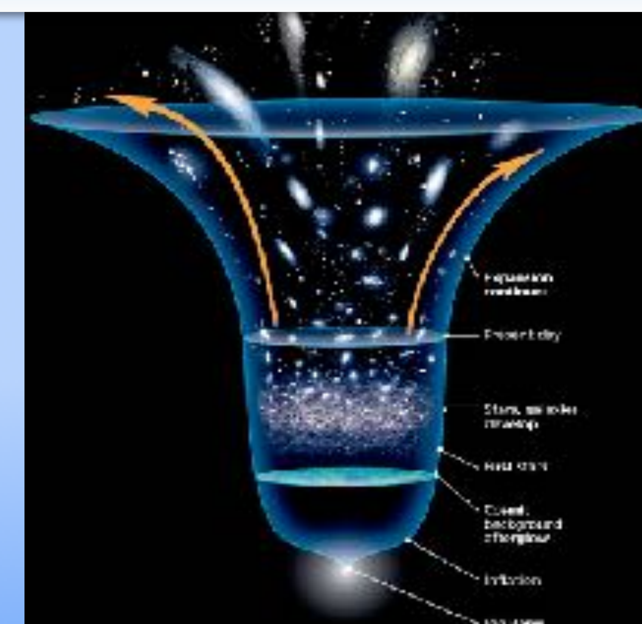
Quantum Critical Liquid

Graphene & Semi-Metal & QGP



Cosmological Fluid

Dark Matter & Energy



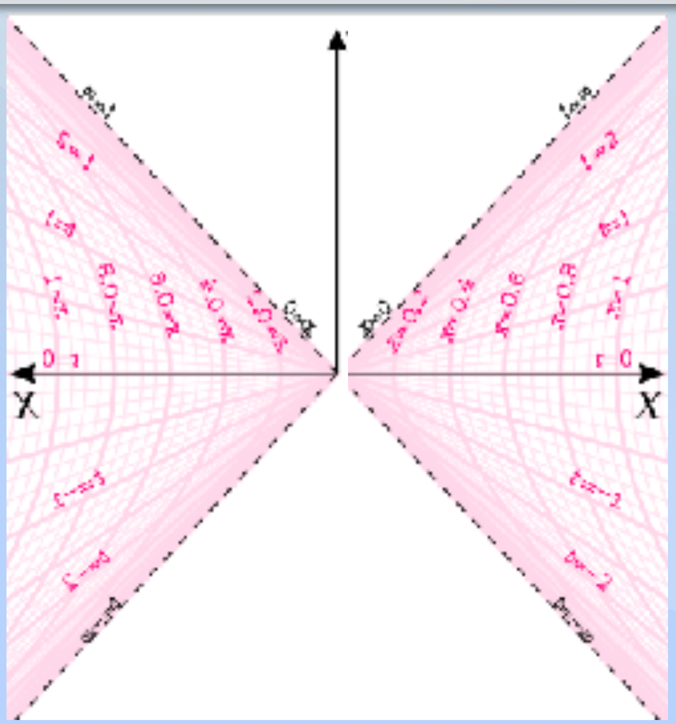
Figures Credit: Nature & Wiki

Holographic Screens in Flat Spacetime

Holographic Rindler Fluid

Accelerating Screen in Flat Spacetime

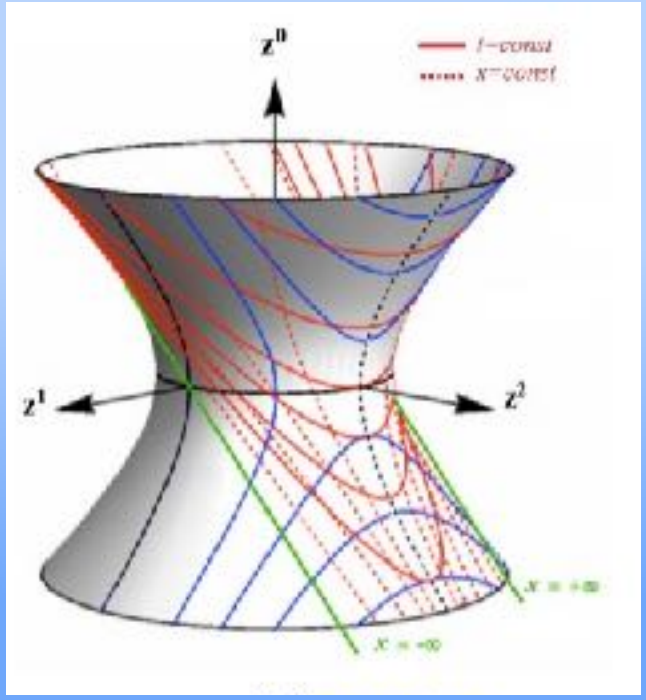
From Conformal Fluid to Rindler Fluid



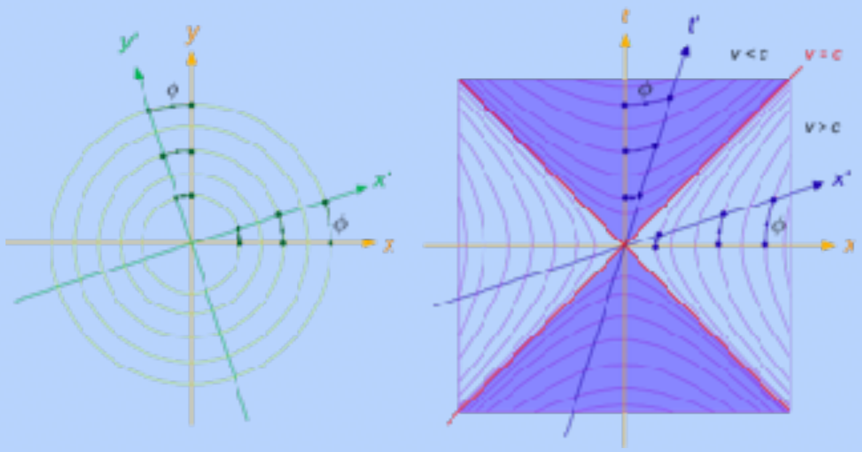
Holographic de-Sitter Fluid

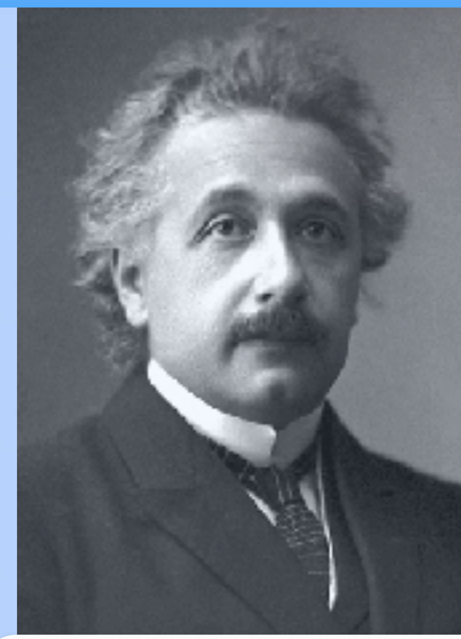
de-Sitter & FRW Screen

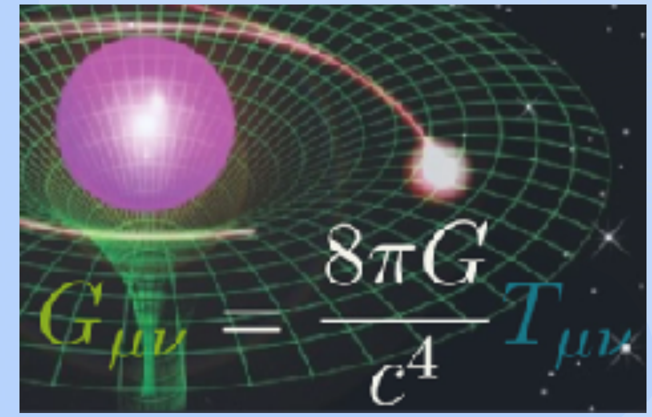
Relation to DGP brane world Models



From Einstein's Gravity to Dark Universe







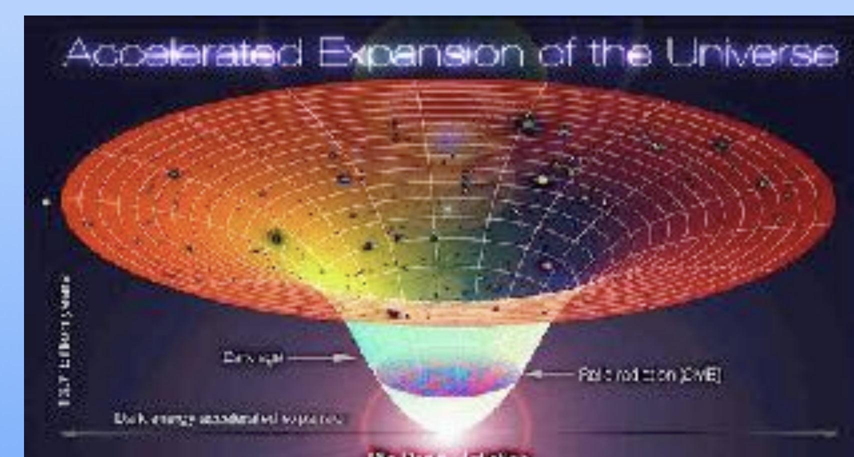
Newton's Gravity → Special Relativity(1905) → **Einstein's Gravity (1915)**

Einstein's Gravity (1915) → **Gravitational Waves (2016)**

Einstein's Gravity (1915) → **Black Holes**

Einstein's Gravity (1915) ⇢ **Dark Matters**

Einstein's Gravity (1915) ⇢ **Dark Energy**

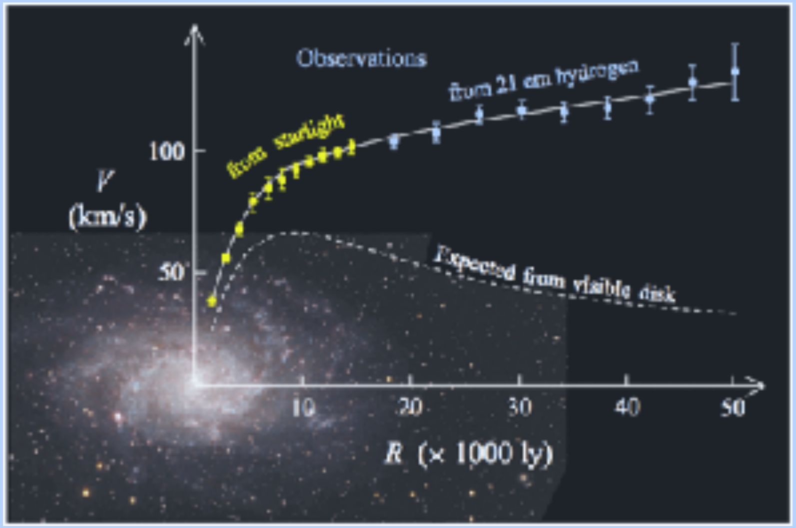


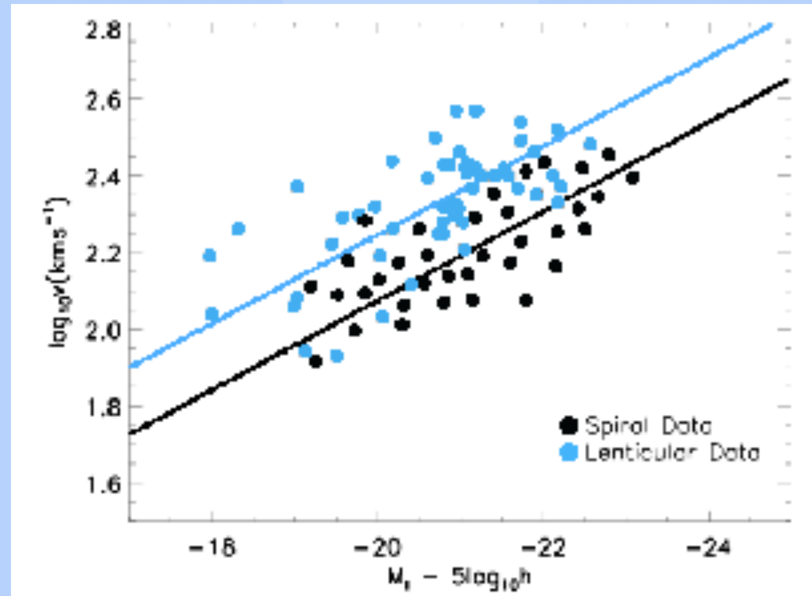
NATURE and Nature's Laws lay hid in Night: God said, "Let Newton be!" and all was light. — Alexander Pope

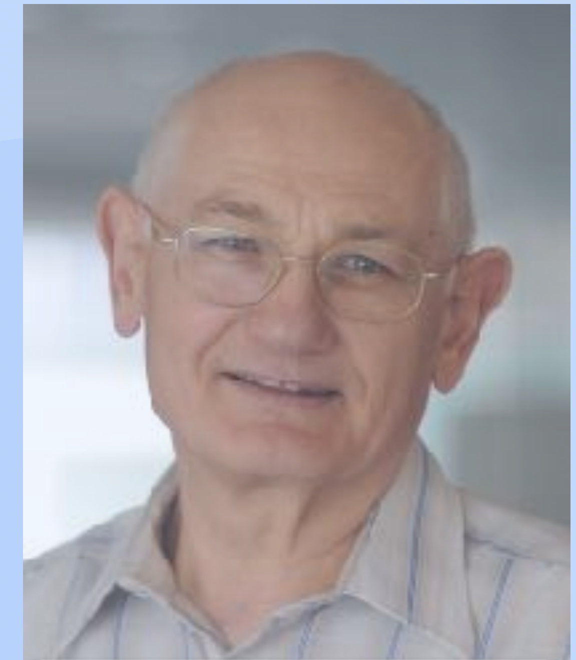
It did not last: the Devil howling: "Ho! Let Einstein be!" restored the status quo. — J. C. Squire

Figures credit: Wiki

From Observation to Milgrom's MOND (Modified Newton Dynamics)







Galaxy Rotation Curve (1970s) → **Tully–Fisher Relation (1977)** → **Milgrom's MOND (1983)**

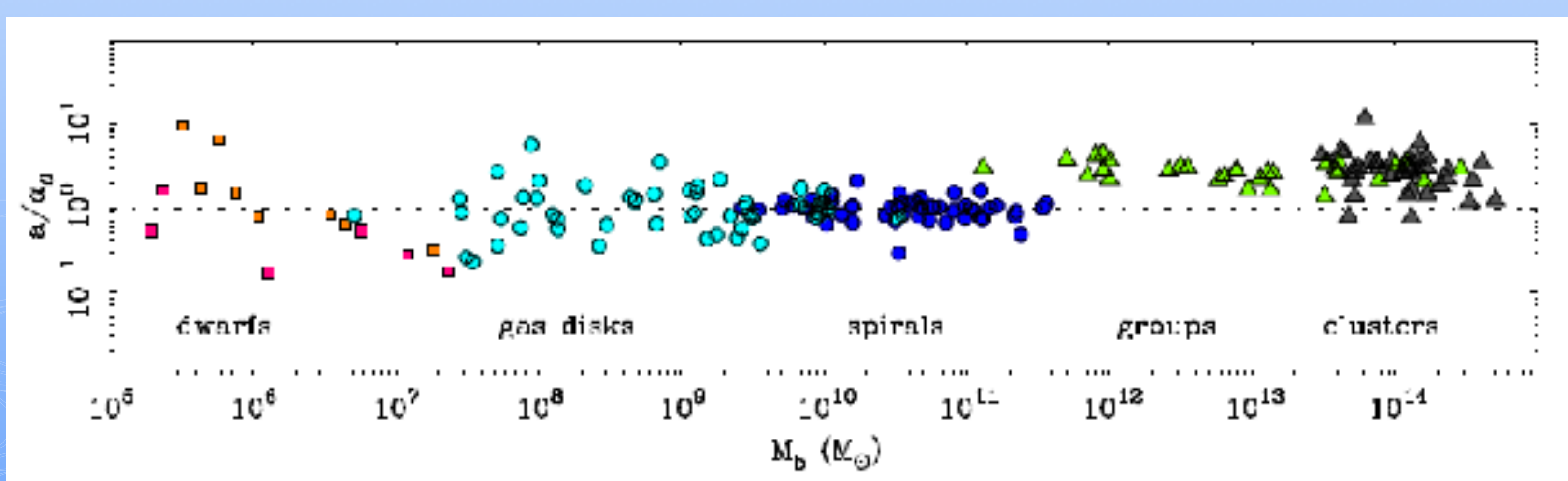
Tully–Fisher Relation (1977) ⇢ **Dark Matter**

$$v_f^4 \simeq a_0 G_N M_B$$

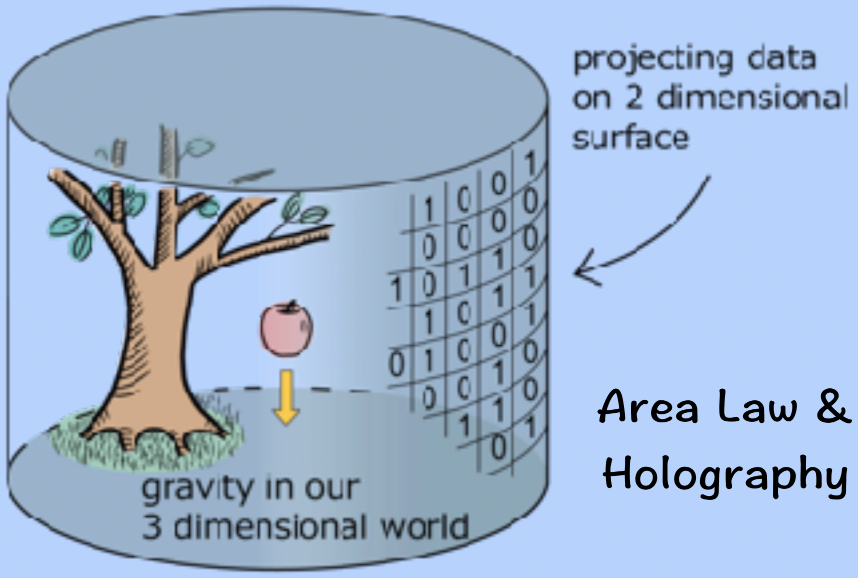
$$F_N = ma\,\mu\left(\frac{a}{a_0}\right)$$

Dark Matter

$$a_0 \simeq \sqrt{\Lambda}$$

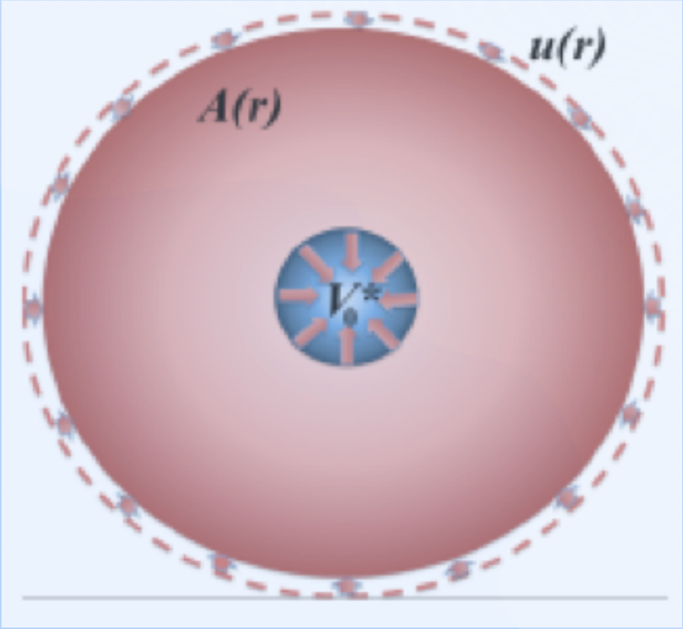


From Verlinde's Gravity to Dark Universe



Area Law &
Holography

Volume Law &
Entanglement



Entropy Gravity (2010)

Velinde's Gravity (2016)

$$\int_0^r \frac{G M_D^2(r')}{r'^2} dr' = \frac{M_B(r) a_0 r}{6}.$$

Tully–Fisher relation

$$g_D(r) = \sqrt{a_M g_B(r)}$$

$$a_M = \frac{a_0}{6}$$

Cluster of galaxies

$$\overline{\rho}_D^2(r) = \left(4 - \overline{\beta}_B(r)\right) \frac{a_0}{8\pi G} \frac{\overline{\rho}_B(r)}{r}$$

$$a_0 = c H_0$$

Parameters in **LCDM**

$$\Omega_D^2 = \frac{4}{3} \Omega_B$$

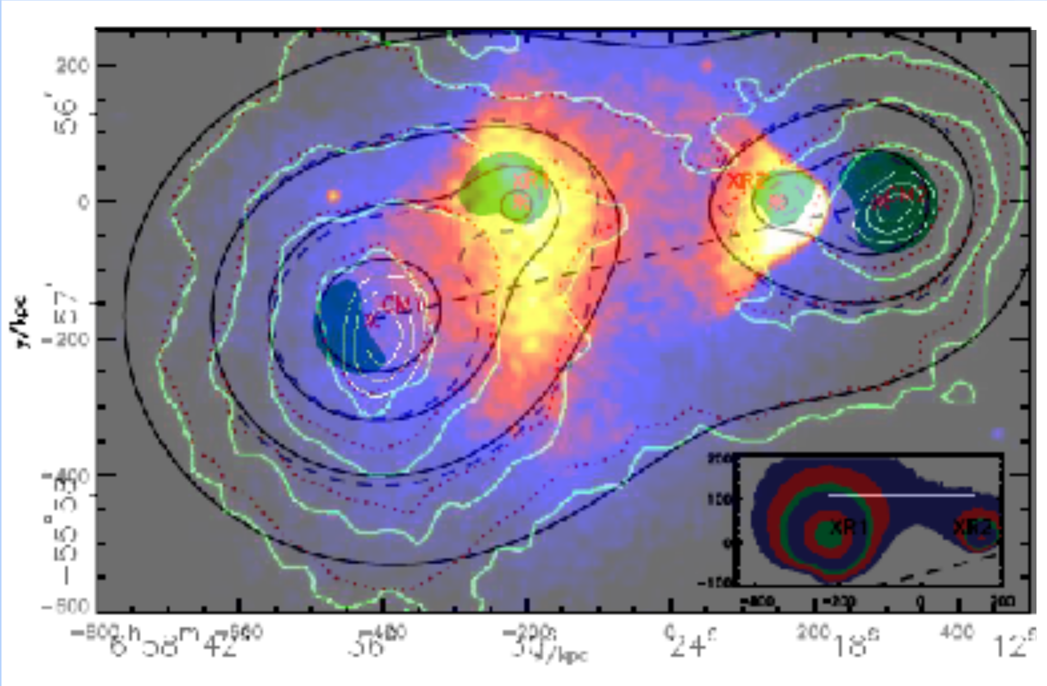
No Covariant Equations of Motion!

Figures credit: Wiki

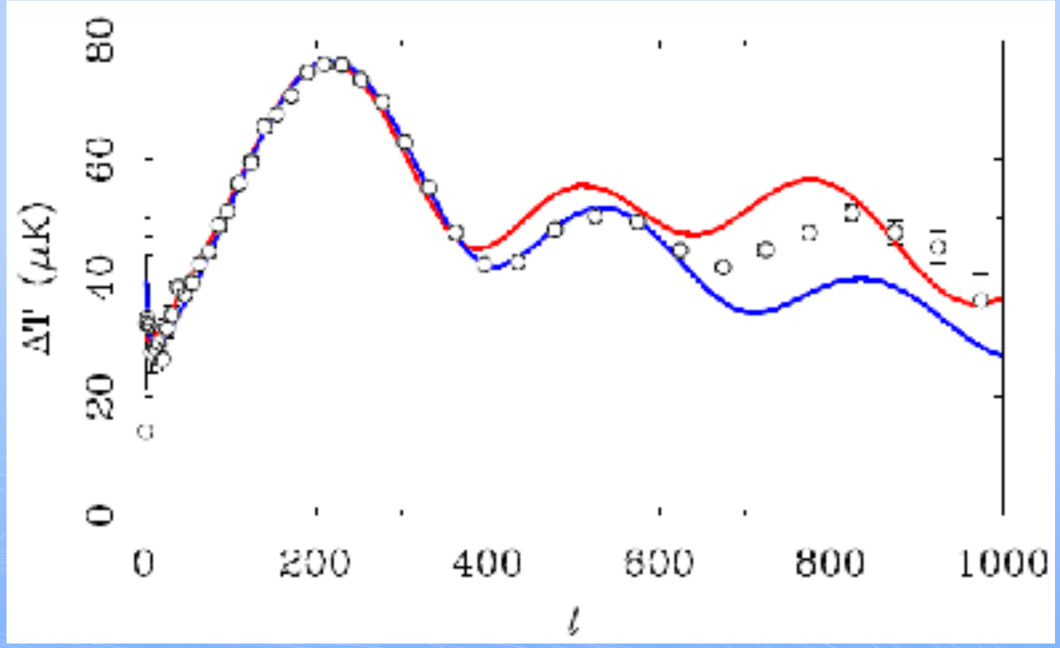
20 years after MOND

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho.$$

Famaey & McGaugh,
Living Rev.Rel. 15 (2012) 10



Bullet Clusters



Acoustic Power Spectrum of CMB

Table 2: Observational tests of MOND.

Observational Test	Successful	Promising	Unclear	Problematic
Rotating Systems				
solar system			X	
galaxy rotation curve shapes	X			
surface brightness $\propto \Sigma \propto a^2$	X			
galaxy rotation curve fits	X			
fitted M_*/L	X			
Tully–Fisher Relation				
baryon based	X			
slope	X			
normalization	X			
no size nor Σ dependence	X			
no intrinsic scatter	X			
Galaxy Disk Stability				
maximum surface density	X			
spiral structure in LSBGs	X			
thin & bulgeless disks		X		
Interacting Galaxies				
tidal tail morphology		X		
dynamical friction			X	
tidal dwarfs	X			
Spheroidal Systems				
star clusters			X	
ultrafaint dwarfs			X	
dwarf Spheroidals	X			
ellipticals	X			
Faber Jackson relation	X			
Clusters of Galaxies				
dynamical mass				X
mass–temperature slope	X			
velocity (bulk & collisional)		X		
Gravitational Lensing				
strong lensing	X			
weak lensing (clusters & LSS)			X	
Cosmology				
expansion history			X	
geometry			X	
big bang nucleosynthesis	X			
Structure Formation				
galaxy power spectrum			X	
empty voids		X		
early structure		X		
Background Radiation				
first:second acoustic peak	X			
second:third acoustic peak				X
detailed fit				X
early re-ionization	X			

Constrains on MOND from Gravitational waves

Chesler & Loeb, arXiv:1704.05116 [PRL, '17]

1) The Speed of gravitational waves

Constraint of energy loss rate from ultra-high energy cosmic rays

2) Linear equations of motion in the weak-field limit

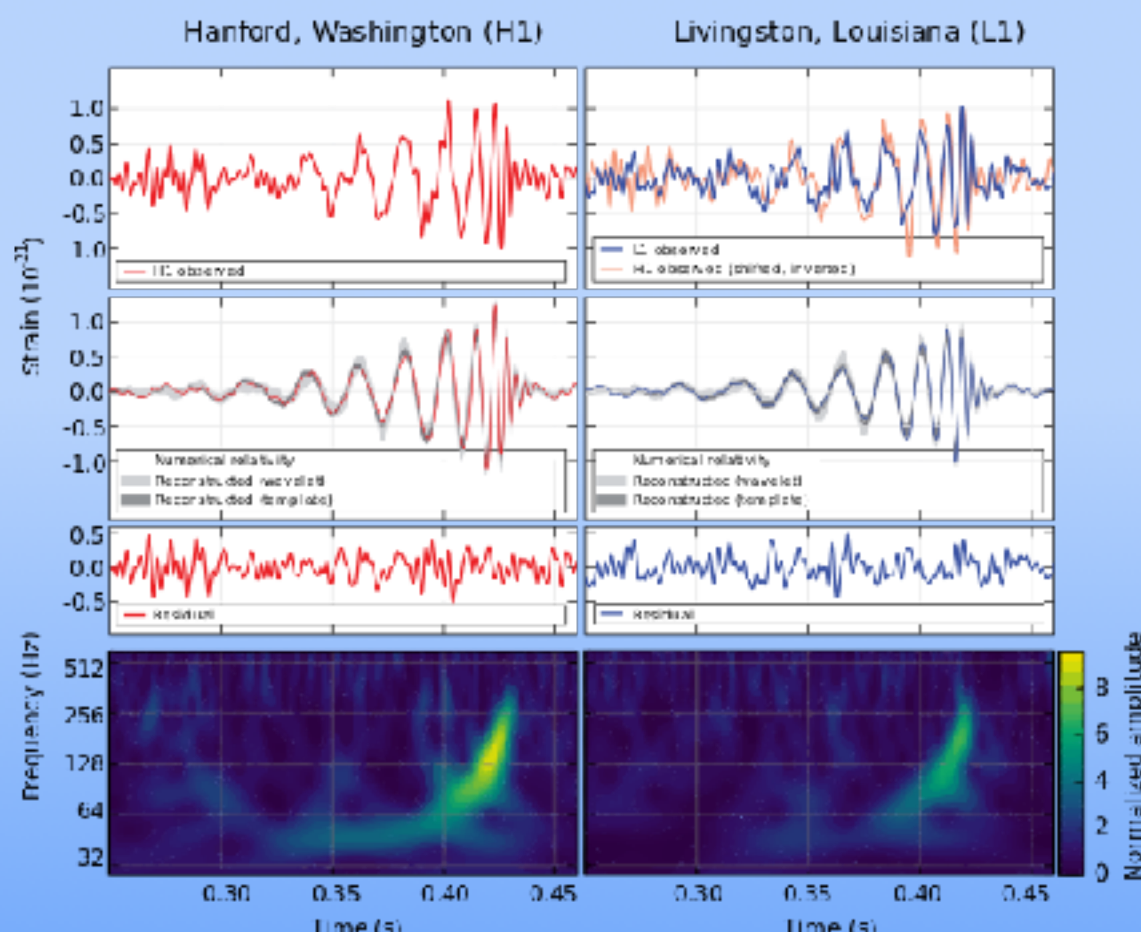
The observed gravitational waveforms from LIGO, which are consistent with Einstein's gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,\left[R + \mathcal{M}^2\mathcal{F}(\tfrac{\mathcal{K}}{\mathcal{M}^2}) + \lambda(A^2{+}1)\right] + S_{\rm mat}$$

Einstein-Aether theory (2004, Bekenstein)

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} &= \mathcal{T}_{\mu\nu} + 8\pi G T^{\rm mat}_{\mu\nu}, \\ \nabla_\alpha[\mathcal{F}' J^\alpha_{\ \beta}] - \mathcal{F}' y_\beta &= 2\lambda A_\beta, \end{aligned}$$

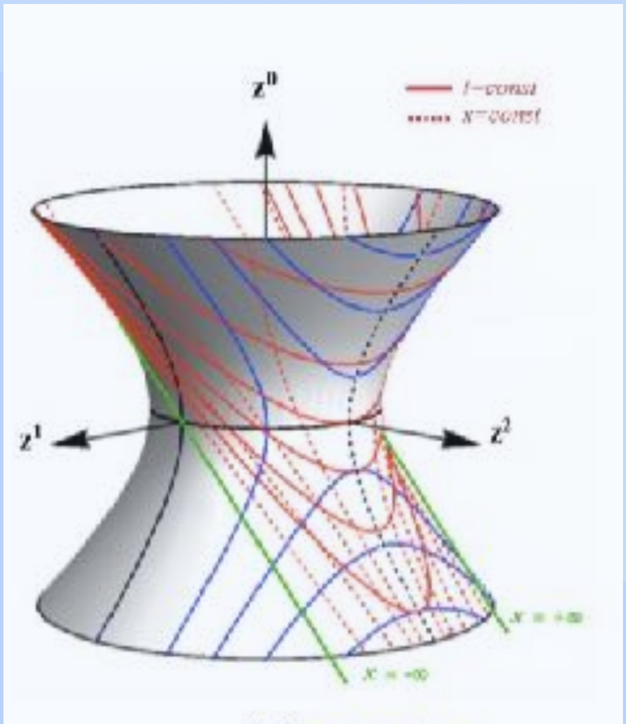
$$\begin{aligned} \mathcal{T}_{\alpha\beta} &= \tfrac{1}{2}\nabla_\sigma\{\mathcal{F}'[J_{(\alpha}{}^\sigma A_{\beta)} - J^\sigma{}_{(\alpha}A_{\beta)} - J_{(\alpha\beta)}A^\sigma]\} \\ &- \mathcal{F}' Y_{\alpha\beta} + \tfrac{1}{2}g_{\alpha\beta}\mathcal{M}^2\mathcal{F} + \lambda A_\alpha A_\beta, \end{aligned}$$



Holographic dS Universe? — de-Sitter Screen

1) Holographic Stress Tensor — Dark Sectors

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa_4 T_{\mu\nu} + \kappa_4 \langle \mathcal{T} \rangle_{\mu\nu}, \qquad \langle \mathcal{T} \rangle_{\mu\nu} \equiv \frac{1}{\kappa_4 L} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \right)$$



Modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \frac{1}{L} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \right) = \kappa_4 T_{\mu\nu}$$

Hamiltonian constraints

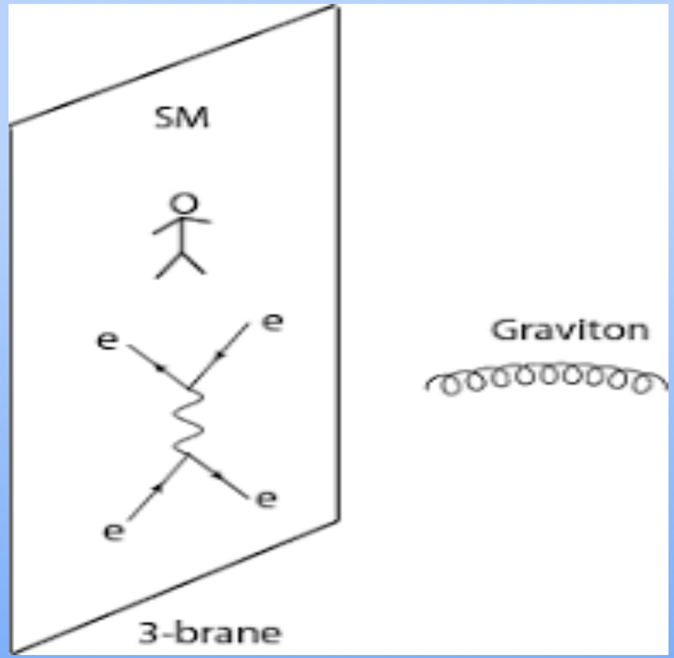
$$\mathcal{K}^2 - \mathcal{K}_{\mu\nu} \mathcal{K}^{\mu\nu} = R + 2\, G^{(d+1)}_{MN} \mathcal{N}^M \mathcal{N}^N,$$

2) Embedding in higher dimensions — Brane Worlds

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \mathcal{T}^{\mathcal{M}}_{\mu\nu} + T^B_{\mu\nu},$$

$$\mathcal{T}^{\mathcal{M}}_{\mu\nu} \equiv (\mathcal{K} g_{\mu\sigma} - \mathcal{K}_{\mu\sigma}) \mathcal{K}^{\sigma}{}_{\nu} + \mathcal{M}_{\mu\nu} - \frac{1}{2} \left(\mathcal{K}^2 - \mathcal{K}_{\rho\sigma} \mathcal{K}^{\rho\sigma} \right) g_{\mu\nu},$$

$$\mathcal{M}_{\mu\nu} \equiv g_{\mu}{}^{M} g_{\nu}{}^{N} R^{(d+1)}_{MN} - g_{\mu}{}^{M} \mathcal{N}^P g_{\nu}{}^{N} \mathcal{N}^Q R^{(d+1)}_{MPNQ}.$$



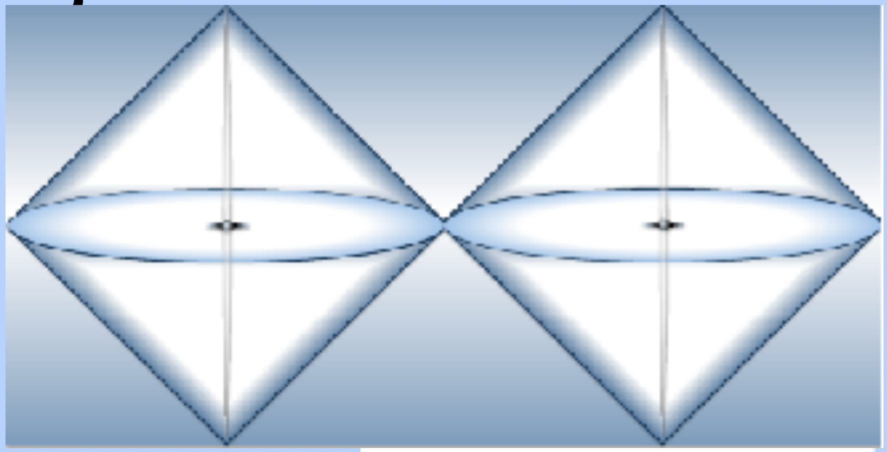
Ref: 1106.2476 [Living Rev. '10]

Compare with Verlinde's Emergent Universe

Gravitational quantity	Elastic quantity	Correspondence
Newtonian potential Φ	displacement field u_i	$u_i = \Phi n_i / a_0$
gravitational acceleration g_i	strain tensor ε_{ij}	$\varepsilon_{ij} n_j = -g_i / a_0$
surface mass density Σ_i	stress tensor σ_{ij}	$\sigma_{ij} n_j = \Sigma_i a_0$
mass density ρ	body force b_i	$b_i = -\rho\, a_0 n_i$
point mass m	point force f_i	$f_i = -m\, a_0 n_i$

Holographic Universe vs. Emergent Universe?

$$\frac{\mathcal{T}^2}{d-1} - \mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu} = -\frac{\rho_\Lambda c^2}{d-1}(T + \mathcal{T}).$$

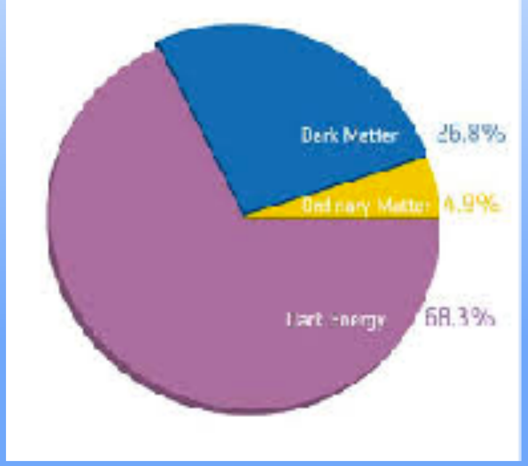


Constrain Equations

$$\Delta_V \equiv \Omega_D^2 - \frac{4}{3}\Omega_B \simeq 0.36\%\,,$$

$$\Delta_{CSZ} \equiv \Omega_D^2 - \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B) \simeq -0.34\%\,.$$

R.G. Cai, S. Sun, Y.L. Zhang, 1712.09326



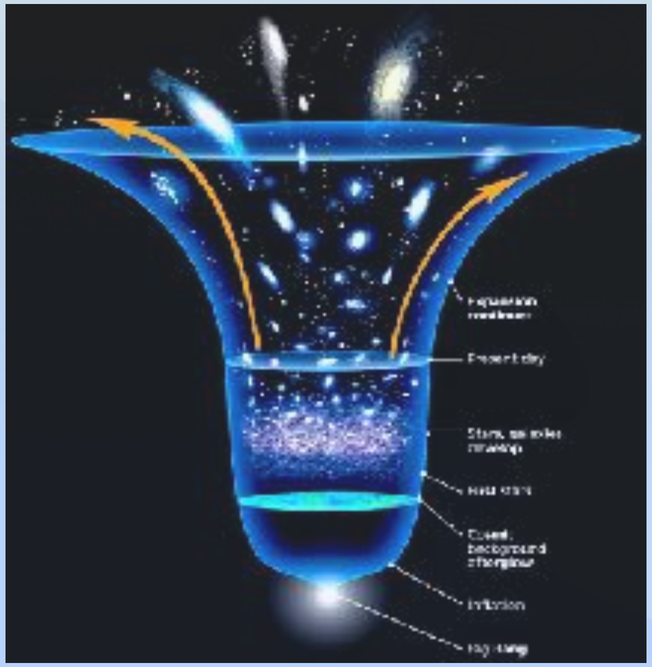
LCDM Universe?

$$H(a)^2 = H_0^2\left[\Omega_\Lambda + (\Omega_D + \Omega_B)a^{-3} + \Omega_R a^{-4}\right]$$

FRW Screen in a Flat Bulk

$$\mathcal{S}_5 = \frac{1}{2\kappa_5}\int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-\tilde{g}}\,\mathcal{R} + \frac{1}{\kappa_5}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,\mathcal{K},$$

$$\mathcal{S}_4 = \frac{1}{2\kappa_4}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,R + \int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\mathcal{L}_M.$$



FRW Screen

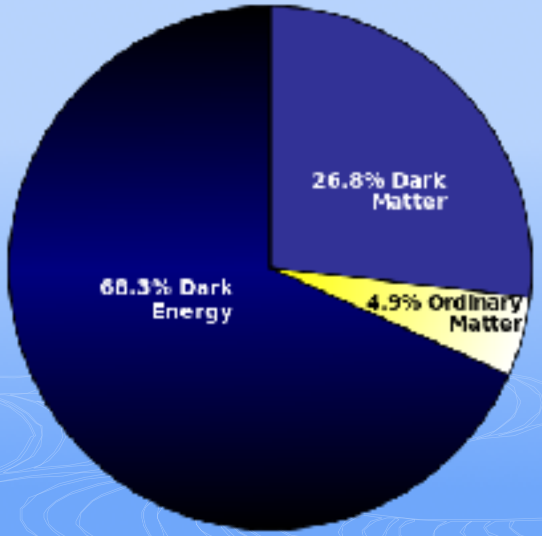
$$\mathrm{d}s_4^2 = -\,c^2\mathrm{d}t^2 + a(t)^2\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2\right]$$

Friedmann eq.

$$\frac{H(t)^2}{H_0^2} \simeq \frac{\Omega_B}{a(t)^3} + \Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}\right]^{1/2}$$

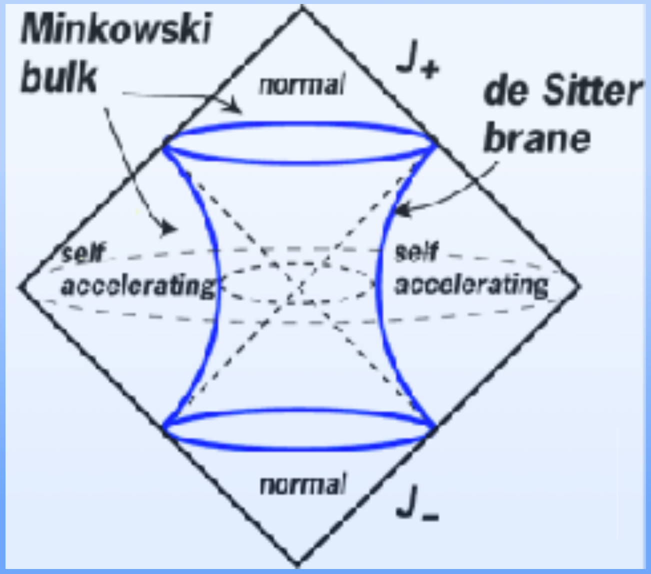
Ref: 1712.09326 [Cai, Sun, Zhang]

DGP BraneWorld



$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_M}{a(t)^3} + \Omega_\ell^{1/2}\frac{H(t)}{H_0},$$

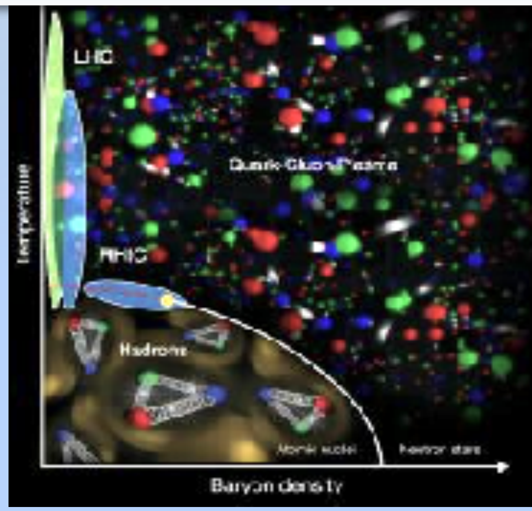
$$\dot{\rho}_\imath(t) = -3H(t)\left[\rho_\imath(t) + p_\imath(t)/c^2\right],$$



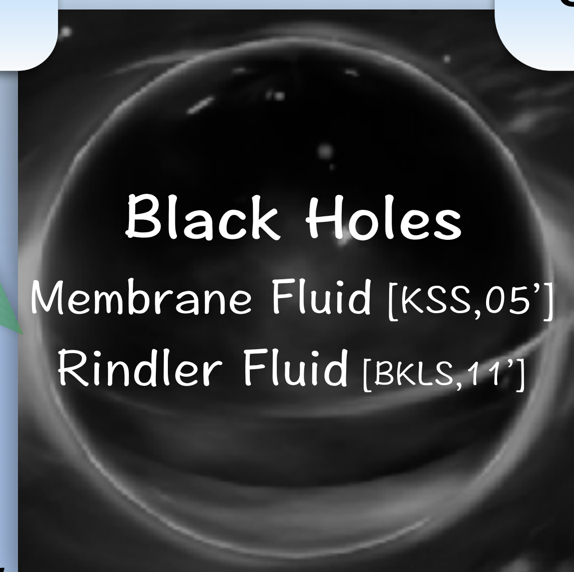
Ref: 1106.2476 [Living Rev. '10]

Summary & Conclusion

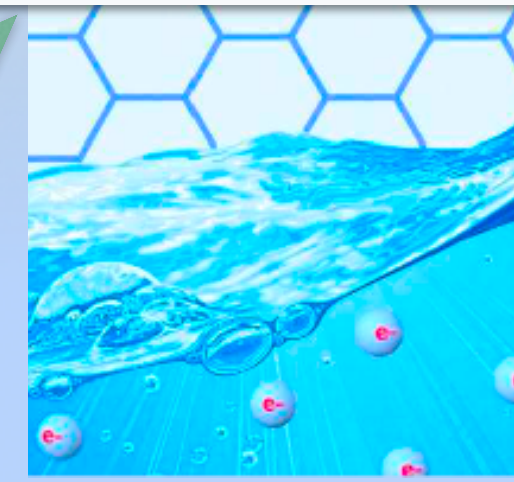
Quark Gluon Plasma
RHIC ['08] & LHC ['16]



$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$



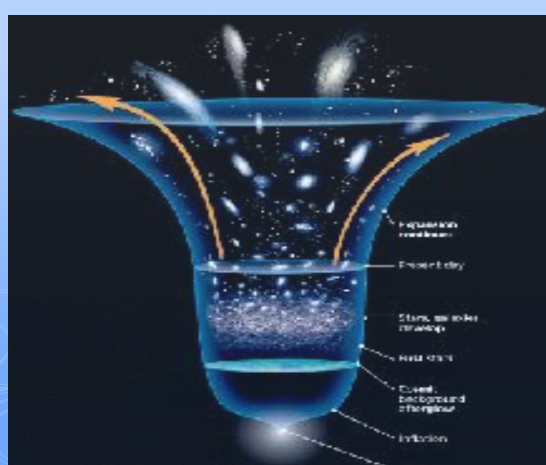
Quantum Critical Liquid
Graphene ['09] & Semi-Metal['16]



$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$

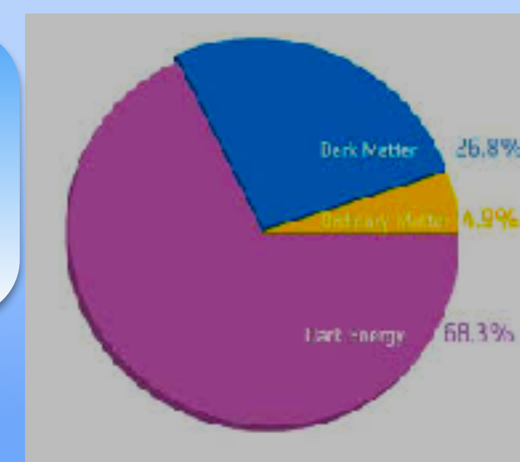
$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$



Cosmological Fluid [CSZ,'17]
Dark Matter & Dark Energy

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$



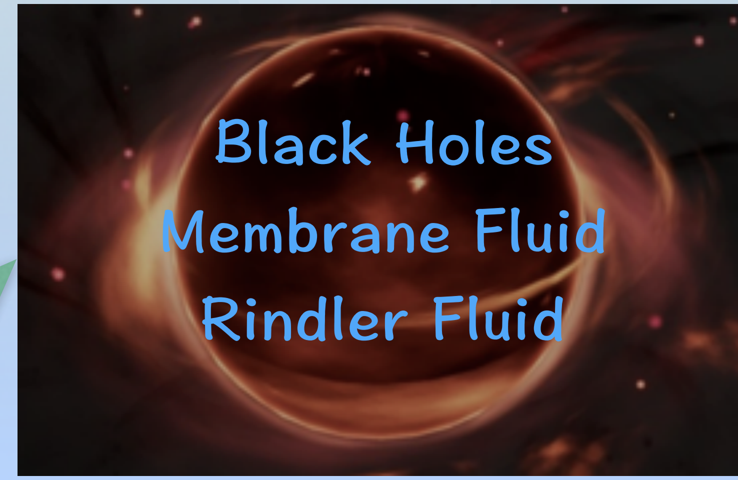
Ref: 1712.09326 [Cai, Sun, Zhang]

Figures Credit: RHIC & Google

Summary & Outlook

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$



$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$

$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

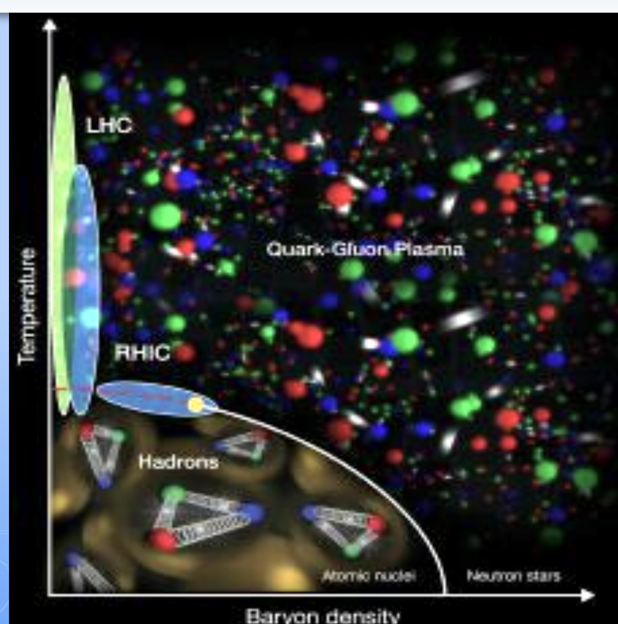
Quark Critical Liquid

QGP in RHIC ['08] & LHC ['16]

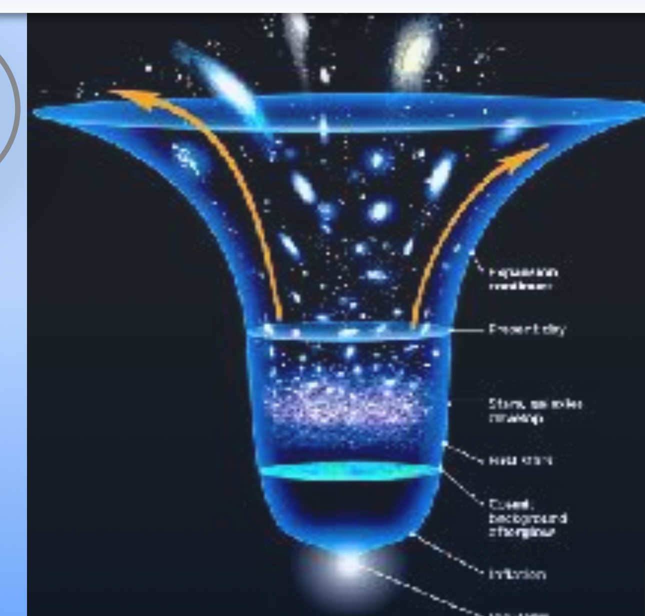
?

Cosmological Fluid

Dark Matter['70s] & Energy['90s]



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$



Thanks for All Your Attention!

Ref: 1712.09326 [Cai, Sun, Zhang]

Figures Credit: Nature & Wiki

Navier-Stokes Equations and Hydrodynamics

Incompressible Navier-Stokes Equations

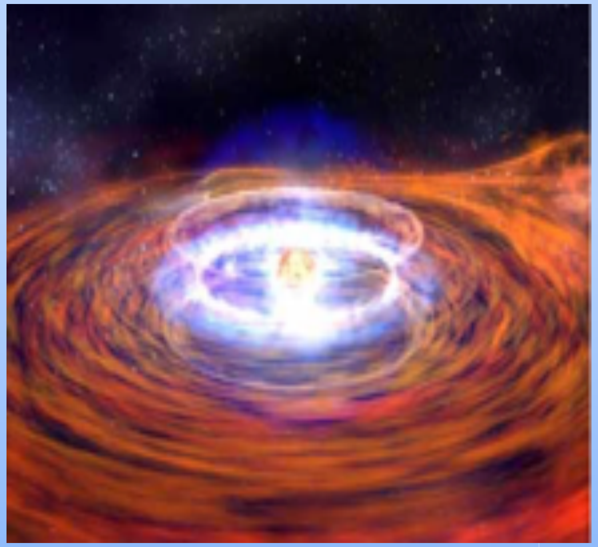
$$\partial_i P + \partial_\tau v_i + v^j \partial_j v_i - \nu \partial^2 v_i = 0, \qquad \partial_i v^i = 0,$$

Relativistic Hydrodynamics

$$T^{ab}(x) = \mathcal{E}(x) u^a(x) u^b(x) + \mathcal{P}(x) P^{ab}(x) + \Pi^{ab}_{\langle \partial \rangle}(x)$$

$$\Pi^{ab}_{(1)} = -2\,\eta\,\sigma^{ab} - \zeta\,\theta\,P^{ab} \qquad \theta \equiv \nabla_c\, u^c,$$

$$\sigma^{ab} = \nabla^{\langle a}\, u^{b\rangle} \equiv P^{ac}\, P^{bd}\left(\nabla_{(c}\, u_{d)} - \frac{1}{p}\,\theta\, P_{cd}\right),$$



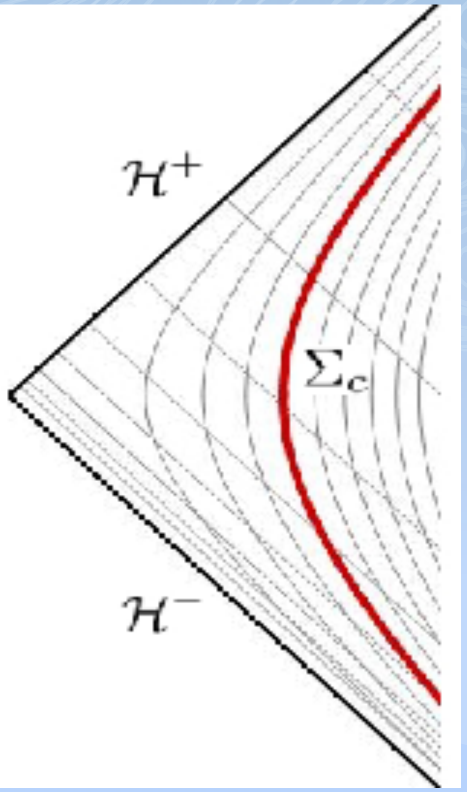
Rindler Hydrodynamics

- Induced metric

$$ds^2_{p+1} = \gamma_{ab} dx_a dx^b = -r_c d\tau^2 + dx_i dx^i.$$

- Dual Fluid:

$$T_{ab} = 2(K\gamma_{ab} - K_{ab}).$$



- Constraint equations

$$2G_{\mu b} n^\mu |_{\Sigma_c} = 2(\partial^a K_{ab} - \partial_b K) = 0 \Longrightarrow \partial^a T_{ab} = 0,$$

$$2G_{\mu\nu} n^\mu n^\nu |_{\Sigma_c} = (K^2 - K_{ab}K^{ab}) = 0 \Longrightarrow T^2 - p\, T_{ab}T^{ab} = 0,$$

Bredberg, Keeler, Lysov, Strominger (JHEP 07 (2012) 146)

What is the Most Perfect Fluid in the World?

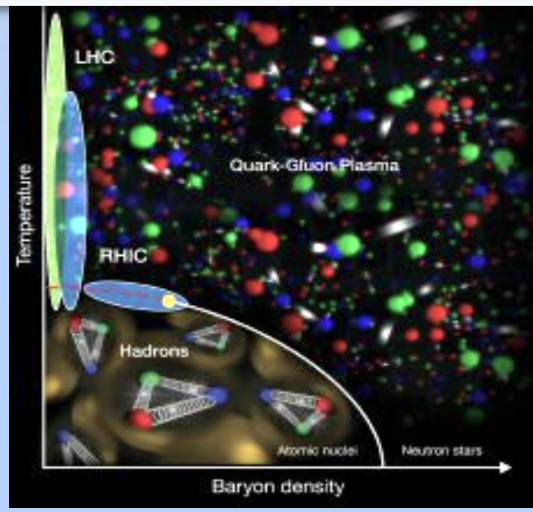
Quark Gluon Plasma

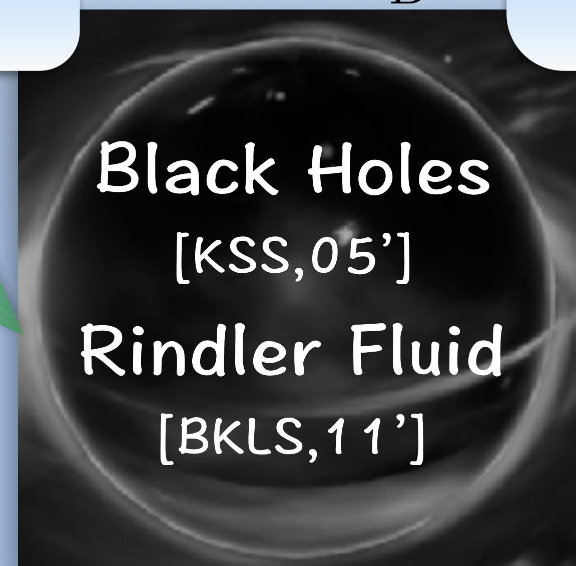
in RHIC [08’] & LHC [16’]

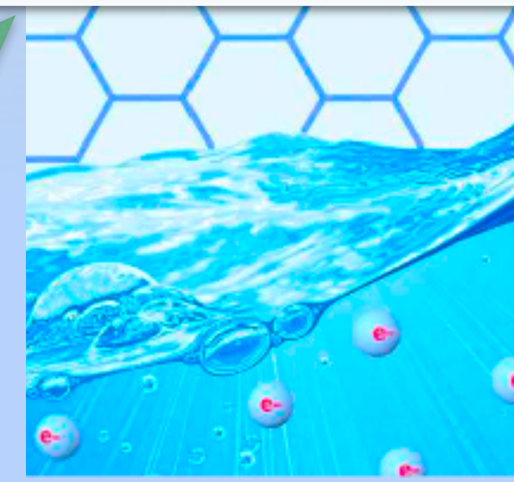
$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Quantum Critical Liquid

Graphene [09’] & Semi-Metal[16’]

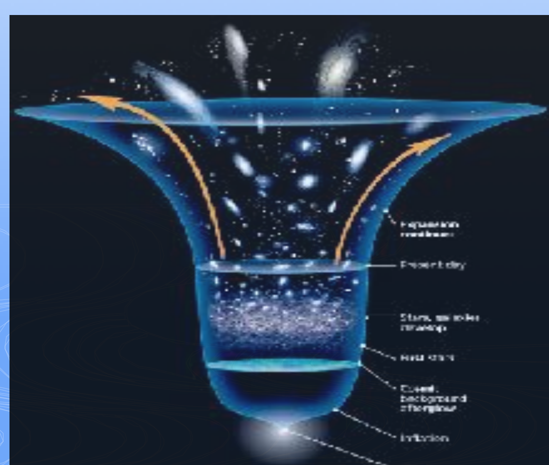






$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\Big(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\Big)}$$

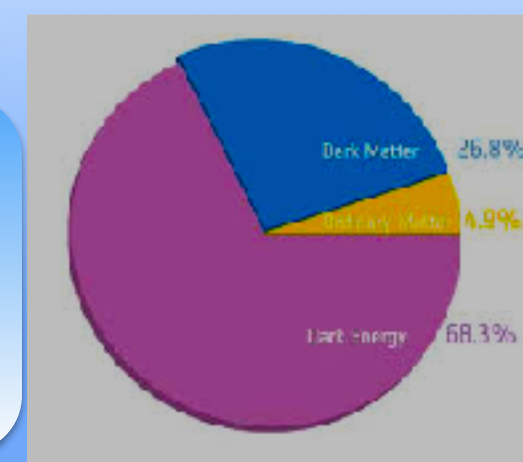
$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$



Dark Fluid in the Universe?

Cosmological Fluid [CSZ,17’]

[1712.09326, Cai, Sun, Zhang]



Figures Credit: RHIC & Google